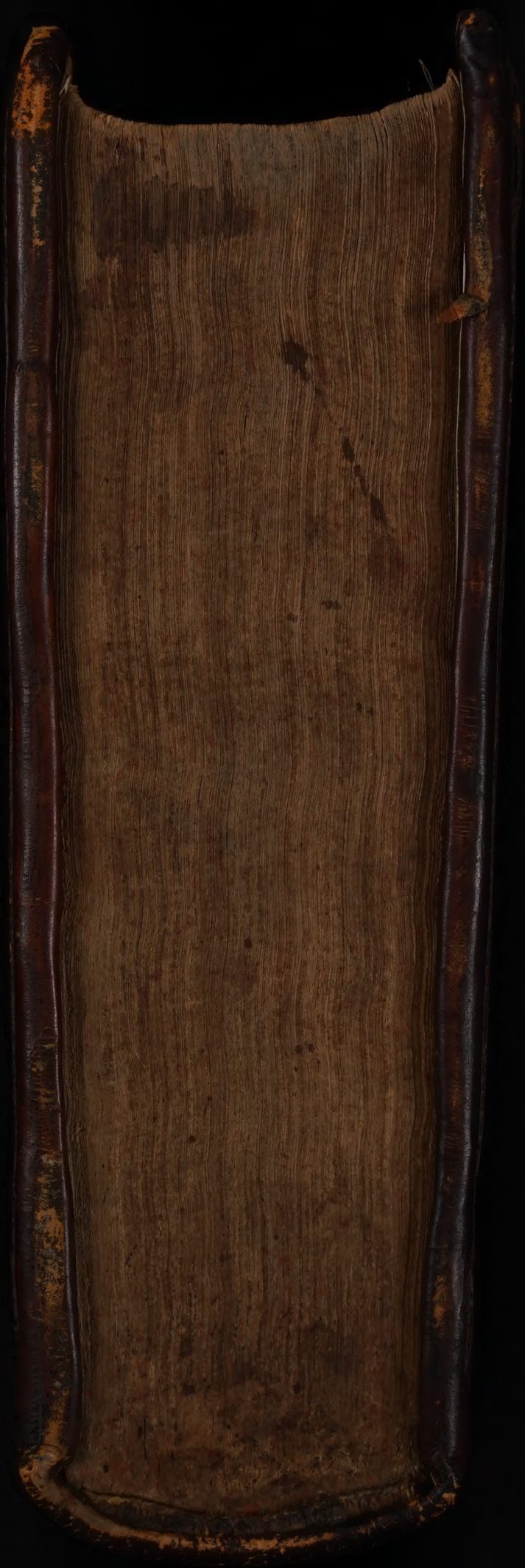
CURSUS
MATHEMATICUS.

W. LEYBOURNE.

1690



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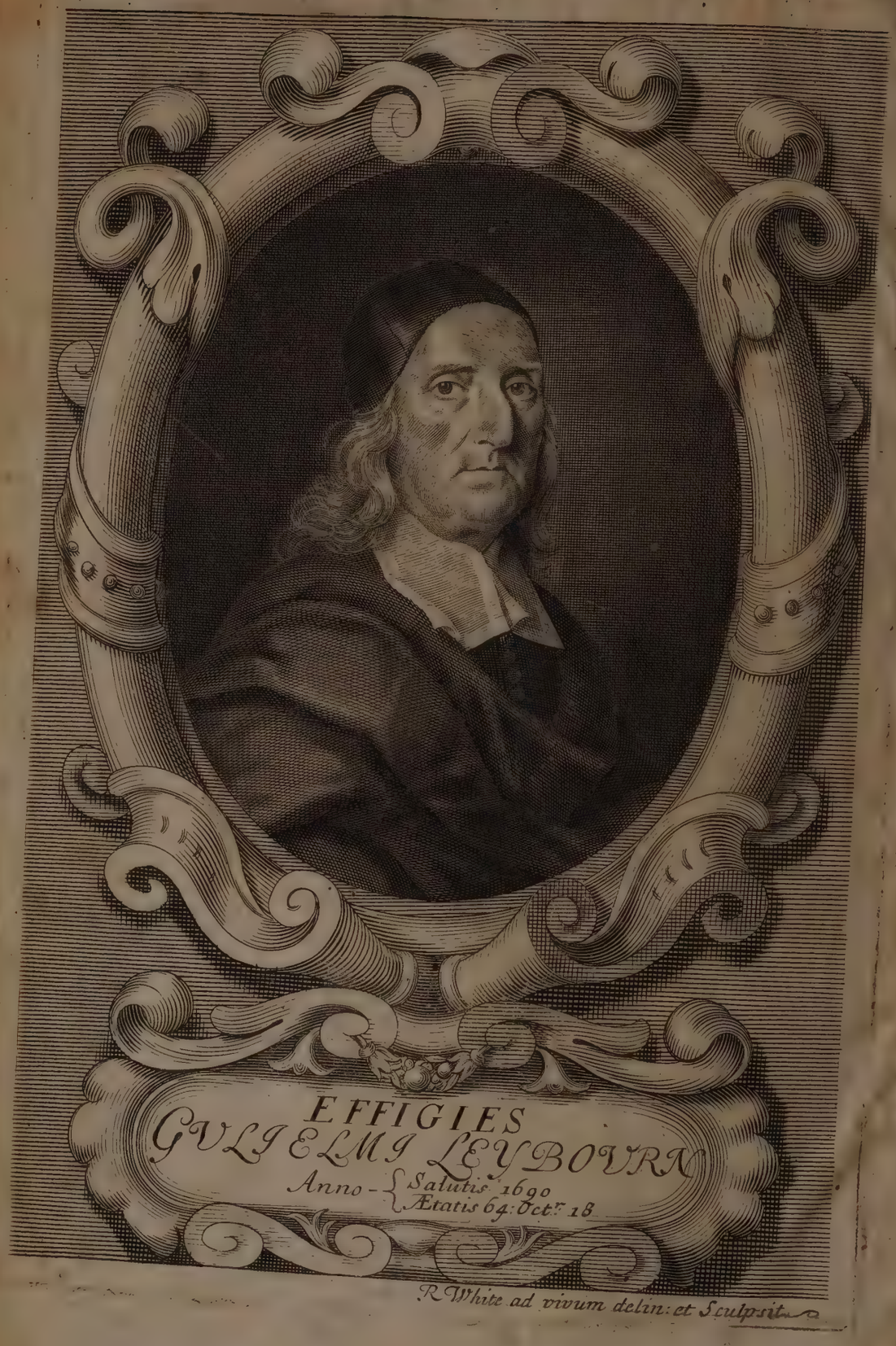

EFFIGIES
GVLIELMI LEYBOVRN
Anno - { Salutis 1690
Ætatis 64: Octr. 18

R. White ad vivum delin: et Sculpsit

Cursus Mathematicus.

MATHEMATICAL SCIENCES,

In Nine BOOKS.

James Comprehending Rubins

ARITHMETICK,
Vulgar, Decimal, { *Instrumental, Algebraical.*

GEOMETRY,
Plain, { *Solid.*

COSMOGRAPHY,
Cœlestial, { *Terrestrial.*

ASTRONOMY,
Theorical, { *Practical.*

NAVIGATION,
Plain, { *Spherical.*

TRIGONOMETRY,
Rectilinear, { *Circular.*

WITH THE

Description, Construction, and Use of *Geometrical* and *Nautical*

INSTRUMENTS:

AND

The Doctrine of *TRIANGLES* applied to Practice in MENSURATIONS of all kinds:

AS IN

Longimetria, Planometria, Military Architecture, } Or { Taking of Heights, Distances, &c. / Surveying of Land, Woods, Roads, &c. / Fortification.

And in *Navigation, Dialling,* &c.

All which are disposed in such Method and Order as they ought to be Read and Practised, by such as would attain to a competent Proficiency in them by their own Industry.

By *WILLIAM LEYBOURN,* Philomath.

Nil tam difficile est quod non Solertia vincat.

LONDON,
Printed for *Thomas Basset, Benjamin Tooke, Thomas Sawbridge, Awnsham* and *John Churchill.* MDCXC.

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THE

PREFACE

TO THE

READER:

And principally to the Young Students in these Sciences.

I *Shall not trouble my* Reader *(in this place) to give him an account of the* Antiquity, Progress, Excellency, *and* Utility *of the* Mathematical Sciences, *nor of such* Parts *or* Members *of them, as are contained in this Volume; for that the* General Survey *of the* Work *following this* Preface, *and the* Procems *before the several* Books *and* Parts *hereof, will give him ample satisfaction in those particulars.*

This which I have now published, was by me many Years since designed; well knowing (by my own experience) that such a Work *in the* English Tongue, *(for there are several in other* Languages,*) could not but be very acceptable to such as are any ways* Mathematically *affected or inclined. And for such as are delighted with* Recreations *of this kind, and have not much time to spend in the Study of them, such are here fitted: For the several* Parts *hereof are so methodically disposed, in such an Order as they ought to be* Read *and* Practised *by such as would (in short time) attain to a* competent Proficiency *in them by their own* Industry.

And although there are divers Books *extant of several* Branches *and* Parts *of the* Mathematicks, *and many such by me published, yet the* Money *laid out in the* Purchase, *and the* Time *that must necessarily be expended in* reading *of them, are both here much abbreviated, this Book not having reference to any other than it self.*

The most of those Tractates *of mine already published, were first written for my own private Use, and the Use of such of my Friends and others that had a desire to be instructed by me in some of these* Sciences, *not intending to expose them to publick View; yet some of them (at several times) were (by a civil Violence) extorted from me, and so made publick; all which have found good acceptation in the World, and upon that account I of late accquainted several of my* Mathematical *Friends with my former design, who earnestly importuning me to pursue the same, I have*

adven-

adventured to range them together, as they are in the following CURSUS; *But how well I have performed the Work, I muſt leave that to general Cenſure, knowing that*

Pro captu Lectoris habent ſua fata Libelli.

And now a Word or two to the Young Student *in theſe* Sciences. *I ſhall uſe no Arguments to commend the* Mathematicks, *or go about to prefer them before the* Dogmaticks; *for that were but to write in the praiſe of* Hercules; *yet I will ſay thus much of them, that although ſome* Bodily Exerciſes, *as* Riding, Leaping, Ringing, *&c. may to ſome* Bodies *conduce more to* Health; *and ſome* Mental Labours *to others, as* Merchandiſing, Adventuring, Uſury, *&c. more to* Wealth, *yet none affords the* Mind *more* Pleaſure *with leſs* Repentance *than theſe do; and therefore in a dull Solitude or vacancy of Buſineſs, (both which may happen to* Scholars, Gentlemen, *and others,) theſe are amiable* Company, *which yield a* delightfull *and* innocent expence *of vacant* Hours.

But it is a frequent Queſtion, What Profit is there in theſe ſublime and hard Studies? *This needs no Anſwer, becauſe it imports the* Ignorance *or* Idleneſs *of the Asker, or both; as firſt, for ſuch as make it their buſineſs to write or ſpeak in a* Quærying *way, what do they doe but declare their* Ignorance, *otherwiſe they need not ask what they already know: And Secondly, If ſome kind of* Meat *or* Phyſick *ſhould be recommended to a Man with which he is yet altogether unacquainted; if he ſhould firſt ask whether they would pleaſe his* Palate, *or agree with his* Stomach? *the* Queſtion *is abſurd; for he cannot know that till he have* taſted *and* digeſted; *ſo the* Idleneſs *of the* Asker *may from hence be diſcovered; for there is no* Profit *to be gained by any* Science, *except the* Science *be firſt gained by* Induſtry. *As for the* difficulty *of theſe* Sciences, *it muſt be granted, that the firſt Aſpect of them may ſeem uncouth and difficult,* (Radices doctrinæ amaræ ſunt, Fructus tamen dulces,) *yet there is no reaſon why any ſhould be deterr'd hereby, but rather animated with deſire to go on as far as others, or (at leaſt) to arrive to a competent Proficiency in them; in order to which I hope this* CURSUS *will, as* Ariadne's *Thread, lead them through the intricate* Labyrinth *of theſe* Studies; *which that it may do, is the earneſt deſire of him who is a Lover of the* Mathematicks,

W. Leybourn.

A

GENERAL VIEW
OF THE
WHOLE WORK,
Analytically disposed by way of
CONTENTS.

The Whole containeth IX. BOOKS.

I. *Arithmetick*,
II. *Geometry*,
III. *Astronomy Pract.*
IV. *Trigonometry*,
V. *Cosmography*,
VI. *Geometrical Instrum.*
VII. *Navigation*,
VIII. *Dialling*,
IX. *Astronomy Theor.*

BOOK I.
Of
Arithmetick.

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	Fractions,		36		55
	Extraction of Roots, Square and Cube,		55		70
	Tables of both, and their Use,		71		84
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Decimal, in	Reduction of Fractions by Pen and Tables,	From	85	To	91
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	Interest: Compound,		118		143
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		The Cosmical or Mundane *Systems* of the World, ancient and modern,		429		435
		The *Fixed Stars*, their Number, Constellations, Figure, Magnitudes, Distances, Motions, *&c.* and Tables to them relating,	From	435	To	445
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		Of New Stars, Comets, *&c.*		451		453

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			Page		Page
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		Its Division by Parts, Kingdoms, Provinces, Empires, *&c.* and by *Zones*, *Climes*, and *Parallels*,	456		464
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			Page	Page
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A Miſcellaneous *APPENDIX.* — From 891 To 905

A Triangular Canon, Logarithmical and Logiſtical, with the Artificial Sines, Tangents, and Complements Arithmetical, of Sines, to every Degree and Minute of the Quadrant; — From 906 To the End.

The Errours committed are thus to be amended.

Page	Line	Read
		BOOK I.
34	40	*Anſwer, or the*
78	15	*of the firſt*
81	23	*of the Rope*
82	25	*and the Solidity*
		BOOK II.
207	*ult.*	BEC
208	21	EFGD
209	43	AK *and* KD
210	26	*and* NP
ibid.	*ibid.*	LM *and* O
211	12	*Multiplex*
220	29	EG
222	29	AE *and* CF
ibid.	*ibid.*	EB *and* FD
225	11,12	EF *and* FH
229	46	BG *and* CH
230	23	*Right Angle*
ibid.	27	*Square made of*
231	17	*Angle* ACB
ibid.	20	ACB, *or*
233	34	, *as*
ibid.	35	DC
235	20	*Permutation,* AC
ibid.	21	*is to* DE,
241	3	*Fig.* XXVII.
247	13	*But the Angle* KBC
251	35	AB *and*
262	5	*and* EG
ibid.	*ibid.*	*given Line* B
ibid.	36	*any three Points*
264	51	N *to* O, *alſo*
265	49	*the Line* CF
267	31	*of joining* V,
ibid.	47	MLKG,
ibid.	*ult.*	*ſhall* VW
269	20	*of three of them*
273	38	*Point* E,
275	9	*towards* E,
279	6	*inclining*
308	42	ſerè.
310	43	XS *and* XV
311	1	AOB, DOE
ibid.	28	.318310 ſerè.
313	13	1.17557
317	24	*The Logar. of*
ibid.	45	324
322	20	*Baſe* EF
		BOOK III.
360	1	*with* 69 *&* 69
ibid.	44	*cut by the*
ibid.	45	*being in* 69
362	24	P ☉ S
363	13	FDG
ibid.	14	CFM
ibid.	23	*from* O 51 *deg.*
367	10	*Sun is* 20 *deg. and in*
368	14, 15	for ♒ ♍ read ♍ ♒
378	23	*bring* 21 *deg.*
399	7	*the Half Tangents*
402	13	*the Limb, it*
404	*ult.*	*Hour Line of Six*
409	9	*or* 12 *of*
410	2	W *to* f *gives*
411	53	dele *each of them*
421	28	*Aſcentional Difference,*
424	5, 7	30 *deg.* 30 *min.*

Page	Line	Read
		BOOK IV.
434	15	Venus *and* Mercury
435	32	*(or Manger)*
457	9	*Of a* Peninſula
ibid.	13	*Of an* Iſthmus
		BOOK V.
482	29	*Equicrural*
483	38	8 *and* 6.
484	6	*Angles Proportional*
488	7	643.26
ibid.	39	*Angles*
488[2]	20	*Angle* CBA
489	41	*Angle* ACB
ibid.	42	*Angle* GAB
492[2]	27	BDC
497[2]	12	*where* CB
502	47	*viz.* D *or* C
505[2]	18	*at the Baſe*
506	8	*Angle at* Z
ibid.	16	*Angle* P
ibid.	45	*Side* PZ
Ib.[2]	22	dele *that whole Line.*
Ib.[2]	25	*of* SB
507	20	*Angle* BZS
508	42	*Hypotenuſe* CB
ib.[2]	38	*forme the*
509	24	*Sines of their Angles are*
		BOOK VI.
549	38	BAC
550	44	32 *deg.* 30 *min.*
553	3, 8	B
ibid.	6	A
ibid.	19,21	ADC
ibid.	22	DAC
ibid.	25	AD
557	45	*are conſti-*
558	11,16	28 *deg.* 30 *min.*
ibid.	42	77 *deg.* 52 *min.*
ibid.	46	dele 57 *deg.* 51 *min.*
559	51	*S.* Helens *at* M
562	32	163 *deg.* 15 *min.*
567	*ult.*	*all the Angles*
580	4	HIKL *&*
587	21	dele *below*
ibid.	33	*Defence* QG
		BOOK VII.
625	8, 9	*of* ♒ Aquarius
ibid.	*pen.*	Aquarius
635	6	AC 54.3
636	6	56 *deg.*
638	10	*from* B *to* C
646	6, 11, 31	for 29.41 read 22.22
ibid.	24	*Then for the*
ibid.	*ult.*	32.26 *Leag.*
648	*pen.*	A *to* C
653	11	*to* AB
655	35	*draw a Line*
ibid.	*ibid.*	50 *deg.*
661	6	*foregoing Table*
664[2]	11	OP *and* AQ
ibid.	50 51	*through* X
666	6	*in the Departure.*
668	18	*of* 50 *deg.*
ibid.	31	*equal to* CN,
ib.[2]	26	*Places* L *and* P, *is* LPT, *and*

Page	Line	Read
669[2]	43	*The Angle* SPL 70 *deg.*
679	30,40	*Firſt after Leap-Tear*
688[2]	20	48—10
		BOOK VIII.
698	3	*Procem*
710[2]	10	*may thereby, by*
ibid.	20	dele *becauſe*
701[2]	7	*Plains*
704	18	dele *done in the*
706	23	*reſolving*
ib.[2]	29	*Plains is the*
ibid.	30	12, *and muſt*
708	49	*To the Tang. of* 13 *deg.*
709[2]	15	*Circle near* F,
716[2]	18	*from* B *to* 11—
719	33, 36, 38	for H read K
ibid.	40	*in the Point* H, *and draw the Lines* GH *and* KH. *And now, &c.*
		BOOK IX.
730	13	*are explained*
735	21	*and in its*
737	6	*reckoned ſo many*
738	47	*Semidiametres*
741	53	*South to North*
767	27	*from the Equinox*
791	36	*to be* 3′
800	23	*Fig.* XXXVI.
801	22	*have one to*
ibid.	29	*find it ſuch*
809	10	XVI *Tables, and*

In ſeveral Places read *Sine* for *Sign.* Alſo *Tolomey's* for *Timocheris.*

Figures omitted to be quoted in the Margin.

484	10	*Fig.* VII.
ibid.	23	*Fig.* VIII.
486	10	*Fig.* X.
492[2]	30	*Fig.* II.
508	2	*Fig.* II.
517	14	*Fig.* V.
550	21,40	*Fig.* III. *and* IV.
601	10	*Fig.* VI.
659	13	*Fig.* VII.
670	28	*Fig.* XII.
671	41	*Fig.* XII.

Page 656, Column 9 and 10, Read thus:

22	0	1354
	6	1360
	12	1367
	18	2373
	24	1380
	30	1386
	36	1393
	42	1399
	48	1406
	54	1412

Page 96, Read as followeth:

In this Example there ſhould have been thirteen Places of Parts cut off, but three were ſufficient for the occaſion.

```
2598790625
 896371111
----------
2598790891
 2598798 9
  2598799
  1819159
    77964
    15592
     2339
      208
----------
290381.941
```

In Page 367, Line 11, 12, 13, 14, 15, Read thus:

	deg.	min.			deg.	min.	
When the Sun is in	16	00	♌	the Sun's Declination will be found to be	16	00	North
	25	00	♍		19	05	South
	29	00	♒		11	52	South
	13	00	♈		05	09	North

Page 372 and 373, Probl. XVIII. *is to be Read thus:*

The Latitude (51 deg. 30 min.) *the Sun's Place in the* Ecliptick, (29 deg. 00 min. *of* Taurus,) *and his Altitude* (12 deg.) *being given, to find,*

I. *The Sun's* Azimuth *at any time.*

Practice. THE Globe being rectified, *&c.* the *Quadrant of Altitude* fixed, and brought to the *Horizon*, turn 29 *deg.* of *Taurus* towards the Eaſt, if in the Morning; or towards the Weſt, if in the Evening, till it comes to lie juſt under 12 *deg.* of the *Quadrant* of *Altitude*; and then note at what degree in the *Horizon* the *Quadrant* of *Altitude* reſteth, which will be at 17 *deg.* 8 *min.* from the Eaſt, (if in the Morning,) or at 17 *deg.* 8 *min.* from the Weſt (if in the Afternoon,) towards the North: And this *Azimuth*, if reckoned by the Points of the Compaſs upon the *Horizon*, will be *E.* by *N.* 5 *deg.* 53 *min.* Northerly, (if in the Morning;) or *W.* by *N.* 5 *deg.* 53 *min.* Northerly, (if in the Evening.) Now if you count the Degrees of the *Horizon* between the Quadrant of Altitude and the North part of the Meridian, you ſhall find them to be 72 *deg.* 52 *min.* which is the *Azimuth* from the North: And if you count them from the South part of the Meridian, you ſhall find them to be 107 *deg.* 08 *min.* which is the *Azimuth* from the South.

CURSUS MATHEMATICUS.

BOOK I.

OF ARITHMETICK.

In Four PARTS.

CONTAINING

I. *Natural*, or *Vulgar*, *Arithmetick*, in *Whole Numbers* and *Fractions:* Explaining the Principal *Species* and *Rules* thereof, and the *Reaſon* of their ſeveral *Operations*; and the *Extraction* of *Roots*, **Geometrically demonſtrated.**

II. *Decimal Arithmetick*; by *Tens*, *Hundreds*, *&c.* Shewing the Excellency thereof (above *Vulgar Arithmetick*) in *Mathematical Operations*; as in all kind of *Menſurations*, both of *Plains* and *Solids*; and alſo in *Anatociſm*, or *Intereſt*, both *Simple* and *Compound:* With the Making and Uſe of *Tables* of both kinds, for any *Rate* or *Time* propoſed.

III. *Artificial Arithmetick*, both *Logarithmetical* and *Logiſtical:* Wherein the *Nature* and *Conſtruction* of thoſe *Artificial Numbers* called *LOGARITHMS*, is fully declared; and the *Uſes* of them illuſtrated in the ſeveral *Rules* of *Arithmetick*, in *Anatociſm*, and in *Extracting* of *Roots* to the *higheſt Power.*

IV. *Inſtrumental Arithmetick:* Shewing how the ſeveral *Rules* of *Arithmetick* may be performed: Three ſeveral ways, *viz.*

I. By *Inſpection* only, by *Decimal Scales* to reduce Vulgar *Fractions* to *Decimal Parts*, and thoſe again to *Vulgar Fractions.*

II. By *Rhabdologia*, or *Speaking Rods*, vulgarly called *NEPEIR's* BONES.

III. By *Logarithmetical Scales*, and by the application of Compaſſes thereunto, to work all *Proportions Arithmetical* and *Geometrical.*

By *WILLIAM LEYBOURN*, Philomath.

LONDON,
Printed *Anno Dom.* MDCXC.

Cursus Mathematicus.

BOOK I.

NATURAL, or VULGAR, ARITHMETICK.

PART I.

BRIEFLY

Explaining the Principal *Species* and *Rules* thereof, in *Whole Numbers* and *Fractions*; and the Reason of their Operations Geometrically demonstrated.

Proœme.

Of the Antiquity, Progress, Excellency, and Use of Arithmetick.

FOR *the* Antiquity *of* Arithmetick, *it is generally conceived to be co-aged with the World, or the Infancy thereof; for, in the beginning of Time, the Omnipotent Creatour and Artificer of the World laid the Structure of it in* Number, Weight, *and* Measure, *as the Wise Man hath it.*

The first Practitioners in this Art, are recorded to be Seth, *and his succeeding Generation, in imitation of their Parent, whom they esteemed as a God: For* Josephus, *in the third Chapter of his first Book of Antiquities, affirms, That his Race were of ingenious Dispositions, and the Inventors of* Astronomy: *And as* Dubartus *hath it,*

Old *Seth*, (saith *Heber*,) *Adam*'s Scholar yerst,
(Who was the Scholar of his Maker first,)
Having attain'd to know the *Course*, and *Sites*,
Th' *Aspects*, and *Greatness* of Heav'ns glistring *Lights*,
He taught his Children, whose industrious Wit,
Through diligence, grew excellent in it:
For while their Flocks on flow'ry Shoars they kept
Of th' Eastern Flouds, while others soundly slept,
They, living lusty, thrice the Age of Ravens,
Observ'd the twinkling Wonders of the Heavens;
And on their Grandsire's firm and goodly Ground,
A sumptuous Building they in time do found.

Which were Two Pillars, *one of* Brick, *and the other of* Stone: *For,*

Phalec and *Heber*, as they wandred, fand
A huge high Pillar, which upright did ſtand;
And afterward another, nigh as great,
But not ſo ſtrong, ſo ſtately, nor ſo neat;
For on the flow'ry Field it lay all flat,
Built but of Brick, of ruſty Tiles and Slat;
Whereas the firſt was builded fair and ſtrong,
Of Jaſper ſmooth, and Marble laſting long.

From them this Knowledge deſcended unto Noah, *and after the Floud, (with the reparation of Mankind,) reviv'd again by the induſtry of the* Syrians, *who were expert in all* Mathematical Sciences, *of which* Arithmetick *is* Primum Mobile. *The* Phænicians, *by* Strabo, Lib. 6. *are recorded to be famous in* Arithmetick, Navigation, *and all* Warlike Arts. *The* Chaldæans *inſtructed* Abraham, *and he the* Egyptians, *in* Arithmetick, *and the* Motions *of the* Heavens. *From hence theſe Sciences did arrive in* Greece, *and from learned* Athens *tranſported to* Rome; *and from thence diſperſed over* Europe, *unto the* Britains, *where all Learning flouriſhed, untill ſuppreſſed by Hereſies, and Heatheniſh Impieties. In the time of the* Engliſh Saxons *Religion returned again, with the* Liberal Sciences, *and all kinds of Learning, attending upon her Train.* England *twice acknowledged to be the learned Tutour unto* France; *and ſince that, all Arts have flouriſhed here, though leſs in the* Theory *than in the* Practick Part, *every one not being born to be an* Artiſt; *according to the Adagie,*

Non cuivis Homini contingat adire Corinthum.

The Uſe *and* Excellency *of* Arithmetick *is manifeſted by many ancient Writers, and grave* Philoſophers; *and this* Science *of Numbers by them placed as the* Primum Mobile *to all other* Mathematical Sciences: *For it clears all Difficulties in* Quantities; *it proves all* Angles, Lines, *and* Superficies; *it meaſures the Magnitudes of all* Bodies, *the* Gravity *and* Proportion *of* Weights; *it diſcovers the* Harmony *of* Sounds *in* Muſick; *it ſolves all* enigmatical Problems *in* Geometry, Aſtronomy, Navigation, Architecture, *either* Military *or* Civil; *and wonderfull ſecrets in the* Optick Sciences. *I will inſtance in a few: And firſt,* Archytas Tarentinus, *one of* Plato's *Diſciples in* Geometry, *ſo renowned and recorded to Poſterity for his Knowledge in* Arithmetick, *that the ingenious Poet* Horace, Lib. 1. Ode 28. *thus writes of him,*

Te Maris, & Terræ, Numeroque carentis Arenæ
Menſorem, cohibent, Archyta,
Pulveris exigui prope littus parva Matinum
Munera. ———

Alſo, how aſſiſtant was Arithmetick *to* Archimedes *in examining the* Crown *made for the King of* Sicilia, *into the Gold whereof the Maker had mixed other adulterate Metal; which, how much pure Gold was therein, and how much adulterate Metal, this famous Artiſt diſcovered without defacing of the Crown.*

But, beſides the general Uſe of Arithmetick in Mundane *Affairs, as in* Merchandiſing, Buying, Selling, *&c. it is of excellent Uſe in the compoſition of* Simples, *for the making of Medicines, in reſpect of the* Temperature *in any of the Four Degrees,* viz. Hot, Cold, Dry, Moiſt. *Infinite are the Uſes to which this Science is applicable; but leaving them to be applied to ſuch Uſes as every Man ſhall have occaſion, I proceed to the Practice thereof.*

CHAP.

CHAP. I.
Of ARITHMETICK in General.

Definition. ARITHMETICK is the *Art* of *Numbering well*, and to the attaining thereof these things following are required to be known:

First, The *Number* of the *Figures*, or *Characters*.

Secondly, Their *Kinds*; as single, or joined with others.

Thirdly, Their *Values*.

First, For the *Number* of the Figures, or Characters, they are *Nine*, thus *Named* and *Characterised*,

Their *Names*, One, Two, Three, Four, Five, Six, Seven, Eight, Nine.

Their *Characters*, 1 2 3 4 5 6 7 8 9

To which must be added an [0] which is called a *Cypher*.

Secondly, For their *Kinds*, and those are *Three*; namely, *Digit* Numbers, *Article* Numbers, *Mixt* or *Compound* Numbers.

I. All Numbers consisting of one single *Figure*, or *Character*, are called *Digit Numbers*; as, 1, 2, 7, 3, 9, *&c.*

II. If a *Cypher*, or *Cyphers*, be added to any *Digit Number*, it maketh it an *Article Number*; as, 70, 30, 60, 50, or 200, 600, 800, *&c.*

III. If many *Digits* alone, or many *Digits* and *Cyphers* stand together, (in one rank or row,) promiscuously placed one among the other, such *Numbers* are called *Mixt*, or *Compound Numbers*; and such are these, 26, 34, 672, 403, 8706, *&c.*

The *Number*, *Names*, *Characters*, and *Kinds* of *Numbers* being thus far explained, the next thing to be known is their *Value*, being thus promiscuously disposed; and that shall be shewed in the following Chapter.

CHAP. II.
Of NUMERATION.

Definition. NUMERATION is that part of *Arithmetick* which teacheth how to Read any number of Figures that shall be set or written down; or how to Write, or set down in Figures, any Summ, or Number, that shall be required.

For the effecting of this you are to consider, that the *Progression* of these Figures, (many of them being set together,) is *Decimal*, or by *Tens*: For every *Figure* standing to the *Left hand*, is encreased *Ten* times the *Value* of that Figure which standeth to the *Right* of it.

And so if one Figure stand singly by itself, it signifieth simply itself; as 7 standing alone, signifieth only *Seven*; and 4 alone, only *Four*.

But if a *Cypher* be set to any one of the Nine *Digits*, (to the Right hand of it,) it increaseth the Digit to *Ten* times his Value; as, a Cypher set on the right hand of 7 thus, [70] it maketh the 7 *Seventy*; being set to 4 thus, [40] it maketh the 4 *Forty*; to 6 thus, [60] *Sixty*, *&c.*

And if any two *Digits* be placed together, that which standeth on the *Right* hand, signifieth only itself, but that Figure which standeth to the *Left* hand, signifieth its own Value *Ten times*, and must be so read or expressed. Thus these two Figures 7 and 4 being placed together thus, [74] the 4 to the Right hand signifieth only *Four*; but the 7 to the Left hand signifieth ten times 7, that is *Seventy*, and must be so called; and these two Figures together must be read *Seventy Four*. In like manner,

8 3	must be read	*Eighty three.*
6 2		*Sixty two.*
9 1		*Ninety one.*
4 5		*Forty five*, &c.

Again, If three *Digits* stand together, that towards the Right hand signifieth only *itself*; the second, or middle Figure, *Ten times* itself; and the third, next the Left hand, *One hundred times itself*. Thus these three Figures 7, 6, and 3, standing to-

gether thus, [763] the 3 ſignifieth only *Three*, the 6 *Sixty*, and the 7 *Seven hundred*, and muſt be ſo read or expreſſed; namely, *Seven hundred Sixty Three*. In like manner theſe following Numbers,

727	muſt be thus read,	*Seven hundred Twenty Seven.*
469		*Four hundred Sixty Nine.*
302		*Three hundred and Two.*
888		*Eight hundred Eighty Eight.*

And let this ſuffice for the *Reading* or *Expreſſing* of *Numbers* conſiſting of *Three Figures*, or *Places*, which may properly be called a *Period*: For he that can number or read *three Figures* properly, may (by the ſame reaſon) read or number as many as he pleaſes, and give unto them what appellation he will.

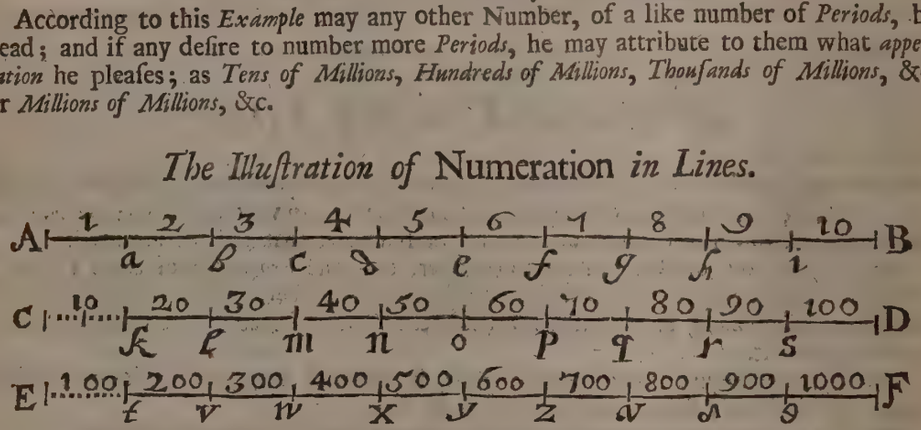
But, Three *Periods* being ſufficient for any occaſion in *Mundane Affairs*, I ſhall therefore add a ſhort *Table* conſiſting of *Three Periods*; the Firſt of *Unites*, the Second of *Thouſands*, and the Third of *Millions*.

III. Period of Millions.			II. Period of Thouſands.			I. Period of Unites.		
One Hundred	Ten	One	One Hundred	Ten	One	One Hundred	Ten	One
7	9	3	2	6	1	4	7	2

This *Table*, conſiſting of *Three Periods*, hath under the Firſt *Period* this Number, [472] or *Four hundred Seventy Two* Unites; under the Second *Period* this Number, [261] or *Two hundred Sixty One* Thouſands; under the Third *Period* this Number, [793] or *Seven hundred Ninety Three* Millions: And ſo the Three Numbers ſtanding together as one entire Number, being diſtinguiſhed by a *Point* between every *Period*, ſtand thus, [793.261.472] and are thus to be read, Seven hundred Ninety Three *Millions*, Two hundred Sixty One *Thouſands*, Four hundred Seventy Two *Unites*.

According to this *Example* may any other Number, of a like number of *Periods*, be read; and if any deſire to number more *Periods*, he may attribute to them what *appellation* he pleaſes; as *Tens of Millions*, *Hundreds of Millions*, *Thouſands of Millions*, &c. or *Millions of Millions*, &c.

The Illuſtration of Numeration *in Lines.*



As Numbers are increaſed *Ten times* their Value by the Addition of one *Cypher* to the Number, towards the *Right hand*, ſo *Lines* are alſo encreaſed decimally, by *ſub-diviſions*: For if One *Unite* of a *Line* divided into *Ten equal Parts*, be ſub-divided into *Ten* ſmaller equal Parts, that increaſeth the Line in *value Ten times*; and if one of thoſe *ſub-diviſions* ſhall again be divided into *Ten* ſmaller parts, that ſhall encreaſe the *Line* yet *Ten* times more, and ſo on.

For *Example*: Let the Line A B be ſuppoſed to be One Statute Foot, divided into Ten equal Parts, as A *a b c d e f g h i* B, then is every one of thoſe Diviſions, as A *a*, one Tenth part of a Foot; A *b*, two Tenth parts of a Foot; and ſo the whole Line, A B, ten Tenth parts, or one whole Foot. But if one Unite of the Line C D were divided into Ten parts, as C *k* is, then the whole Line C D would be Ten Foot. And again, if One of the Tenths thereof, as E *t*, were divided again into Ten parts then E *t* would be 100 parts, and the whole Line E F 1000 Feet.

Having thus laid the Foundation, by giving an account of the *Characters*, *Names*, *Kinds*, *Degrees* of *Places*, and how to *Read* or *Expreſs* any *Summ* or *Number* of *Figures*, orderly placed together, our next work ſhall be to ſhew how to work the principal *Species*, or *Rules*, of *Arithmetick*; which are (generally taken to be) Four; *viz.*

1. *Addi-*

1. *Addition*, } {3. *Multiplication*,
2. *Subtraction*, } {4. *Diviſion*.

Which indeed are but *Two*; for *Multiplication* is but a compendious way of *Addition*, and *Subtraction* a Compendium of *Diviſion*; For it is poſſible for a Man to *Multiply* or *Divide* (and that by the Pen) the *largeſt* and moſt *difficult Summ* that can be propoſed, although he cannot tell by *Memory* that 8 times 3, or 3 times 8, is 24; or 6 times 6 is 36; or 8 times 9 is 72, *&c.* But (becauſe I would not be thought to be ſingular) I ſhall retain the uſual way, as others have done before me, beginning with the Firſt.

CHAP. III.
Of ADDITION.

Definition. ADDITION, being the Firſt *Species* of *Arithmetick*, is of Two kinds, namely, *Simple* and *Compound*; and it teacheth how to bring divers ſmaller *Summs* into one entire or *Groſs Summ*, which is called the *Aggregate*, *Total*, or *Groſs Summ*.

Simple Addition teacheth how to add divers Summs of one *Kind*, *Name*, or *Denomination*, (as *Men*, *Pounds*, *Ships*, &c.) together, and bring them into one entire or *groſs Summ*.

Compound Addition teacheth how to add divers *Summs* of different *Kinds*, *Names*, or *Denominations*, (as *Pounds*, *Shillings*, and *Pence*; *Hundred Weights*, *Quarters*, *Pounds*, and *Ounces*; *Quarters*, *Buſhels*, *Pecks*; *Years*, *Months*, *Days*, *Hours*, and *Minutes*,) together, and to bring them into one entire, *Total*, or *Groſs Summ*.

For the Working of *Simple Addition*, you muſt ſet the *Numbers* to be added together one under the other; that is to ſay, *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, *&c.* and for the performance thereof this is the *RULE*:

Having placed your numbers to be added in due order, one under another, draw a Line under them, and begin at the lowermoſt Figure toward your right hand, and add that to the next Figure above, and the ſumm of them to the next Figure above that; proceeding in this order, till you have added the whole Line, or File, together; which when you have done, conſider how many Tens are contained in that Line; and for every Ten keep one Unite *in your mind, to be added to the next File, or Row; but if there be any odd* Digits, *you muſt ſet them beneath the Line, juſt under the Line you added together: Having thus finiſhed the Addition of one Line, proceed to the next, and from thence to the third, and ſo forward, be there never ſo many.*

§ I. *ADDITION of Numbers of* One Denomination.

Example 1. Let the Numbers given to be added together be 7832, 5609, 376, 8547, having thus placed them in order one under another, as in the Margin is done, draw a Line under them; then begin your Addition at the lowermoſt Figure towards your right hand, ſaying, 7 and 6 is 13, and 9 is 22, and 2 is 24; now (becauſe in 24 there are two *Tens*, and 4 remaining) place the 4 under the Line, and carry the two *Tens* to the next row of *Tens*, ſaying, 2 which I carried and 4 make 6, and 7 makes 13, and 3 makes 16; in which row there is but one *Ten* contained, and 6 remaining, which 6 ſet under the Line, and carry the *Ten* to the next row of *Hundreds*, ſaying, 1 that I carried and 5 make 6, and 3 makes 9, and 6 makes 15, and 8 makes 23; in which 23, *Ten* is contained two times, and 3 remaining; the 3 place under the Line, and carry the two *Tens* to the next row of *Thouſands*, ſaying, 2 which I carried and 8 make 10, and 5 makes 15, and 7 makes 22, in which *Ten* is contained two times, and 2 remaining, which 2 ſet under the Line; and becauſe there is never another row to be added (to which you ſhould carry the two *Tens*) therefore ſet 2 down alſo under the Line towards the left hand, as you ſee done in the margin: So the *total* or *groſs Summ* of theſe numbers, being added together, is 22364.

Thouſands.	*Hundreds.*	*Tens.*	*Unites.*
7	8	3	2
5	6	0	9
	3	7	6
8	5	4	7
2	2	3	6	4

Example

Example 2. *A Man hath in his Ortyard* 136 *Apple Trees*, 76 *Pear Trees*, 107 *Cherry Trees, and* 36 *Plum Trees, and he desires readily to know how many Trees he hath in all.*

Apple Trees	136
Pear Trees	76
Cherry Trees	107
Plum Trees	36
Trees in all	355

Place your numbers one under another, as in the margin, and then begin to add them together at your right hand, saying, 6 and 7 make 13, and 6 makes 19, and 6 makes 25; place 5 under the line, and carry 2 to the next row, saying, 2 and 3 is 5, and 7 is 12, and 3 is 15, place 5 under the line, and carry 1 to the next row, saying, 1 and 1 is 2, and 1 is 3; which 3 set under the line, and (because there were no *Tens* contained in that line, therefore) the total is 355, and so many *Trees* are in the *Ortyard*.

Other Examples *for Practice.*

95432	321	9161	50423	3020
76100	1986	235	16572	161
2570	23	72	83109	312
832	1107	9	4973	69
Total 174934	Total 3437	Total 9477	Total 155077	Total 3562

§II. *ADDITION of Numbers of* Divers Denominations.

I. ADDITION *of Englisb Money.*

The most usual Coins used in *England* are *Pounds*, *Shillings*, *Pence*, *Farthings*, of which

4 Farthings, 12 Pence, 20 Shillings } make { 1 Penny, 1 Shilling, 1 Pound } thus Charactered, { *d.*, *s.*, *li.*

For a Farthing we use *q*.

For the performance of Addition of divers Denominations this is the *RULE*:

Place all Numbers of the same Denomination one directly under another, as Pounds *under* Pounds, Shillings *under* Shillings, Pence *under* Pence, *and* Farthings *under* Farthings: *Then draw a Line under them, and begin your Addition with the least Denomination first, observing how many times the next greater Denomination is contained in that least, and for every time carry one unite to the next Denomination, as before you did the Tens, setting down the remainder, if any be; then adding the next Denomination together, take notice how many times the next greater Denomination is contained in the lesser, carrying for every time one to the next greater Denomination; thus proceeding till you have gone over all the Denominations, be they never so many.*

li.	*s.*	*d.*	*q.*
37	16	9.	3.
21	09.	8.	1
13	12	9	2
72	19	3	2

Example 1. Let the numbers to be added together be 37 *li.* 16 *s.* 9 *d.* 3 *q.* 21 *li.* 9 *s.* 8 *d.* 1 *q.* 13 *li.* 12 *s.* 9 *d.* 2 *q.* Place the numbers as in the margin, draw a Line under them, and begin with the least Denomination (which in this Example is *farthings*) first, saying, 2 *q.* and 1 *q.* is 3 *q.* and 3 *q.* is 6 *q.* which is one penny, and 2 *q.* remaining; which 2 *q.* place under the Line, and carry the one penny to the next row, which is the place of pence; saying, 1 *d.* and 9 *d.* is 10 *d.* and 8 *d.* is 18 *d.* which is 1 *s.* and 6 *d.* (*Now against the* 8, *make a prick with your Pen, for your better remembrance, to signifie that there is one shilling to be carried to the place of shillings,*) and then go on, and say, 6 *d.* and 9 *d.* is 15 *d.* which is 1 *s.* and 3 *d.* therefore against 9 make a prick with your Pen, and (because that is the last number) set down the odd 3 *d.* under the place of pence, and (seeing you find two pricks in the line of pence, therefore) carry 2 *s.* to the place of shillings; saying, 2 *s.* which I carried and 12 *s.* is 14 *s.* and 9 *s.* is 23 *s.* which is 1 *li.* and 3 *s.* remaining; make a prick against 9, and going on, say, 3 *s.* and 16 *s.* is 19 *s.* which (seeing there are no more numbers to be added, and being also less than 20 *s.*) set under the line, and finding one prick in the line of shillings, therefore carry one to the place of pounds; saying, 1 which I carried and 3 is 4, and 1 is 5, and 7 is 12; set down the 2 under the line, (as in Addition of Numbers of one Denomination,) and carry one to the next row; saying, 1 that I carried and 1 is 2, and 2 is 4, and 3 is is 7, which, being the last, set down, and so the total, or gross Summ is 72 *li.* 19 *s.* 3 *d.* 2 *q.*

Example

Example 2. Let the numbers to be added be 29 *li.* 16 *s.* 8 *d.* 32 *li.* 17 *s.* 9 *d.* 81 *li.* 13 *s.* 11 *d.* and let it be required to find the total, or groſs ſumm. Here in this Example the leaſt Denomination is *pence*, therefore begin with them, and ſay, 11 *d.* and 9 *d.* is 20 *d.* which is 1 *s.* and 8 *d.* make a prick againſt the 9, and ſay, 8 *d.* and 8 *d.* is 16 *d.* that is 1 *s.* and 4 *d.* make a prick againſt the 8, and ſet down the odd 4 *d.* then (becauſe there are two pricks in the line of pence) you muſt carry 2 *s.* to the place of ſhillings, ſaying, 2 *s.* which I carry, and 13 *s.* is 15 *s.* and 17 *s.* is 32 *s.* which is 1 *li.* 12 *s.* make a prick againſt 17, and ſay, 12 *s.* and 16 *s.* is 28 *s.* make a prick againſt 16, and (becauſe there are no more numbers to be added) ſet down the odd 8 *s.* under ſhillings, and (ſeing there are two pricks in the line of ſhillings) carry 2 to the place of pounds; ſaying, 2 and 1 is 3, and 2 is 5, and 9 is 14, ſet down 4 and carry 1 to the next line; and ſay, 1 and 8 is 9, and 3 is 12, and 2 is 14, which (becauſe it is the laſt) you ſet down, ſo is the total, or groſs ſumm 144 *li.* 8 *s.* 4 *d.*

li.	*s.*	*d.*
29	16.	8.
32	17.	9.
81	13	11
144	08	04

Other Examples *for Practice.*

li.	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*
29	18.	7	3	36	2	8.	45	12.	6	2	321	17	6.
63	11.	2.	1.	29	0	2	209	10	11.	1.	83	2.	5
229	4	0	2	31	16.	9.	67	9.	10.	3	56	11	10.
3	7	10	1	6	2	5	618	17	5	0	412	6	3
326	11	8	3	103	2	0	941	10	9	2	873	18	0

II. ADDITION *of Troy-weight.*

Troy-weight is a weight uſed in *England*, by the which is weighed *Bread*, *Gold*, *Silver*, *Pearl*, &c. the moſt uſual denominations of which weight are *Pounds*, *Ounces*, *Penny-weights*, and *Grains*; of which

24 Grains, 20 Penny-weight, 12 Ounces } make { 1 Penny-weight, 1 Ounce, 1 Pound, } thus charactered, { *pw.* *ou.* *lb.*

For a Grain we write *gr.*

The Addition of *Troy-weight* (and conſequently of any other weight or meaſure whatſoever, either domeſtick or foreign) differeth nothing at all from the Addition of Engliſh Coin laſt taught, if the affinity of one denomination to another be firſt known.

For, whereas in Money, becauſe 12 *d.* make 1 *s.* you therefore obſerve how many twelves there are in the addition of your pence, and for every 12 you add 1 ſhilling to the place of ſhillings; ſo in the Addition of *Troy-weight*, knowing that 24 *gr.* make one *penny-weight*, you muſt therefore in the addition of *Grains* of *Troy-weight* obſerve how many times 24 you find in your line of *Grains*, and for every 24, carry one to the place of *penny-weights*; likewiſe, in the addition of *penny-weights*, you muſt conſider how many times 20 is contained in your line, and for every 20 carry one to the place of *ounces*, (becauſe 20 penny-weights make an ounce;) alſo, in the addition of *Ounces* Troy, you muſt obſerve how many times 12 you find in your line of *ounces*, and for every 12 carry one to the place of *pounds*; then, laſtly, add your *pounds* together, as numbers of one denomination.

Example: Let the numbers to be added together be 7 *lb.* 11 *ou.* 13 *pw.* 19 *gr.* 6 *lb.* 7 *ou.* 16 *pw.* 19 *gr.* 3 *lb.* 7 *ou.* 9 *pw.* 6 *gr.* Place your numbers as in Addition of Money, each under other, according to their reſpective denominations, as in the margin; then draw a line under them, and begin your Addition with the leaſt denomination firſt, *viz.* Grains, ſaying, 6 *gr.* and 19 *gr.* is 25 *gr.* which is one penny-weight and one grain; make a prick againſt 19, and carry the odd grain to the number above, ſaying, 1 *gr.* and 19 *gr.* is 20 *gr.* which (becauſe it is leſs than one penny-weight) ſet under the line; then finding one prick in the line of grains, therefore carry one to the place of penny-weights, ſaying, 1 and 9 is 10, and 16 is 26, which is 1 ounce and 6 *pw.* make a prick againſt 16, and ſay 6 and 13 is 19, which (being leſs than an ounce) ſet under the line; then for the one prick carry 1 to the place of ounces, ſaying,

lb.	*ou.*	*pw.*	*gr.*
7	11.	13	19
6	07.	16.	19.
3	07	09	06
18	02	19	20

saying, 1 and 7 is 8, and 7 is 15, which is 1 pound and 3 ounces; make a prick at 7, and say, 3 and 11 is 14, which is 1 pound and 2 ounces; make a prick against 11, and set down the 2 ounces, and for the two pricks carry 2 pounds to the place of pounds; saying, 2 and 3 is 5, and 6 is 11, and 7 is 18, which set under the place of pounds; so is your Addition ended, and the summ is 18 *lb.* 2 *ou.* 19 *pw.* 20 *gr.*

Other Examples *for Practice.*

lb.	*ou.*	*pw.*	*gr.*
32	9.	12	16
17	11.	6	9.
34	8.	15.	10
8	10	4	7
94	3	18	18

lb.	*ou.*	*pw.*	*gr.*
0	10	17	11
0	6.	0	5
0	0	19.	8.
0	5	2	19
1	10	19	19

lb.	*ou.*	*pw.*	*gr.*
617	09.	18.	23.
64	11.	03	16
12	06	19.	04.
96	10.	12	20
792	02	14	15

III. ADDITION *of* Avoirdupois *little weight.*

There is another kind of weight most commonly used in *England*, called *Avoirdupois little weight*, by which is weighed all sorts of Wares or Merchandise, garblable, as *Sugar*, *Pepper*, *Cloves*, &c. This weight is commonly divided into these denominations, *Pounds*, *Ounces*, and *Drams*, of which

16 Drams, 16 Ounces } make { 1 Ounce, 1 Pound } thus charactered { *ou.* *lb.*

For a Dram we write *dr.*

In the Addition of *Avoirdupois-weight*, you must observe the very same method and order as in *Money* and *Troy-weight*, having due respect to the quantity of the denominations; as in the Addition of *drams*, to make a prick at every 16, setting down the remainder, and for ever prick carrying an unite to the next place. The preceding Rules being so copious in this particular, I shall forbear to make any verbal illustration, but only give you some *Examples* ready wrought, together with the most usual parts into which the *Weights* and *Measures* now used in *England* are divided; which to the ingenious will be sufficient.

Examples *of* Addition *of* Avoirdupois *little weight.*

lb.	*ou.*	*dr.*
12	11.	09
76	05	12.
32	10.	00
91	07.	13.
32	13	07
246	00	09

lb.	*ou.*	*dr.*
06	13.	07
05	09.	12
06	03	09.
10	00	00
05	07	09
34	02	05

lb.	*ou.*	*dr.*
576	15.	12.
734	11.	10
29	13	14.
68	04.	11.
12	11	12
1422	09	11

lb.	*ou.*	*dr.*
825	12.	07
631	07.	11.
312	13	12
44	11.	03.
89	04	13
1904	01	14

IV. ADDITION *of* Avoirdupois *great weight.*

There is a weight commonly used in *England*, by which is weighed all commodities that are sold by the hundred, as *Corins*, *Wool*, *Flesh*, *Butter*, *Cheese*, and the like, the which hundred weight containeth 112 pounds, and the hundred weight is divided into *Quarters*, *Pounds*, and *Ounces*; so that

16 Ounces, 28 Pounds, 4 Quarters } make { 1 Pound, 1 Quarter of a Hundred, 1 Hundred weight, } thus charactered, { *lb.* *qr.* *C.*

For an Ounce we write *ou.*

Examples *of* Addition *of* Avoirdupois *great weight.*

C.	*qr.*	*lb.*	*ou.*
37	3.	21	12.
9	1	06	03
33	2	20.	00
10	0	00	00
12	3.	07	03
103	2	27	02

C.	*qr.*	*lb.*	*ou.*
5	1	07	07
3	2.	18.	06.
0	1	06	08
11	3.	04	00
6	1	10	05
27	1	18	10

C.	*qr.*	*lb.*	*ou.*
763	1.	26.	11.
149	2	19.	03
68	3.	01	14.
18	0	24	09
95	3.	27.	02
1096	0	15	07

I might

I might farther proceed to give you Examples of Addition of common *Engliſh Meaſures*, *viz.* of *Long Meaſures*, *Liquid Meaſures*, and *Dry Meaſures*; as alſo of *Time*, *Motion*, &c. but the preceding Examples being of ſufficient extent, I ſhall forbear to trouble either my ſelf or the Reader with that which I conceive ſuperfluous: Only, before I leave *Addition*, I will give you a brief view of the moſt uſual *Meaſures* uſed in *England*, which take as followeth:

V. *Of* Liquid Meaſures.

Liquid Meaſures are thoſe by which all ſorts of Liquid Subſtances are meaſured, of which, (according to the Statute of 12 *Hen.* 7. *chap.* 5.) a *Pint* is the leaſt, from which the greater *Liquid Meaſures* are deduced, according as is expreſſed in the Table following:

2 Pints	make	1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
8 Gallons		1 Firkin of Ale, Sope, or Herrins.
9 Gallons		1 Firkin of Beer.
10 ½ Gallons		1 Firkin of Salmon, or Eels.
2 Firkins		1 Kilderkin.
2 Kilderkins		1 Barrel.
42 Gallons		1 Tierce of Wine.
63 Gallons.		1 Hogſhead.
2 Hogſheads		1 Pipe or Butt.
2 Pipes or Butts		1 Tun of Wine.

VI. *Of* Dry Meaſures.

Dry Meaſures are thoſe by which all kind of Dry Subſtances are meaſured, as *Corn*, *Salt*, *Coles*, *Sand*, &c. of which a Pint is the leaſt.

2 Pints	make	1 Quart.
2 Quarts		1 Pottle.
2 Pottles		1 Gallon.
2 Gallons		1 Peck.
4 Pecks		1 Buſhel Land Meaſure.
5 Pecks		1 Buſhel Water Meaſure.
8 Buſhels		1 Quarter.
4 Quarters		1 Chaldron.
5 Quarters		1 Wey.

VII. *Of* Long Meaſures.

Long Meaſure is that by which is meaſured *Cloth*, *Land*, *Board*, *Glaſs*, *Pavement*, *Tapeſtry*, &c. of which Meaſures (according to the Statute of 33 *Ed.* 1. and 25 *El.*) a *Barley Corn* is the leaſt: So that

3 Barly Corns	make	1 Inch.
12 Inches		1 Foot.
3 Feet		1 Yard.
3 Feet 9 Inches		1 Ell.
6 Feet		1 Fathom.
5 ½ Yards, or 16 ½ Feet		1 Pole or Perch.
40 Perches		1 Furlong.
8 Furlongs		1 Engliſh Mile.

VIII. *Of* Time.

Time conſiſteth of *Years*, *Months*, *Weeks*, *Days*, *Hours*, and *Minutes*: So that

60 Minutes	make	1 Hour.
24 Hours		1 Day Natural.
7 Days		1 Week.
4 Weeks		1 Month of 28 Days.
13 Months, 1 day, and 6 hours		1 Year.

Examples *of* Time.

Y.	*M.*	*W.*	*D.*	*H.*	*Min.*	*Y.*	*M.*	*W.*	*D.*	*H.*	*Min.*
21	4	3.	5.	20.	34	68	10.	1.	3	18	56.
16	9.	2	4.	13	21.	15	8	0	1.	22.	12
3	11.	1.	6.	9.	16	82	4	3	5	8.	2
2	4	0	4	16	27	9	11.	2.	6.	22	18
44	4	1	0	10	38	176	9	0	3	23	28

IX. *Of* Motion.

Motion is reckoned by *Signs*, *Degrees*, *Minutes*, *Seconds*, *Thirds*, &c. Of which

60 Thirds ‴
60 Seconds
60 Minutes
30 Degrees
} make { 1 Second,
1 Minute,
1 Degree,
1 Sign,
} thus charactered { ″
′
Deg.
Sig.

Examples *of* Motion.

Sig.	*Deg.*	′	″	‴	*Sig.*	*Deg.*	′	″	‴
9	21	17.	20.	05	15	28.	18	12.	2
4	00	12	11	42.	8	12.	4.	7	20.
0	08.	17	26	11	7	29.	1	2	41
8	21	36	18	21	9	24	56	48	2
21	21	23	16	19	42	4	20	10	5

X. *Of* Apothecaries Weight.

The Weights used by *Apothecaries* are *Grains*, *Scruples*, *Drams*, and *Ounces*, of which

20 Grains
3 Scruples
8 Drams
12 Ounces
} make { 1 Scruple,
1 Dram,
1 Ounce,
1 Pound,
} thus charactered, { ℈.
ʒ.
℥.
℔.

By help of these Tables, and the Rules and Cautions before expressed, any Man may make Addition of any of the above-said Measures one with another, or of any other, either *domestick* or *foreign*; and therefore I shall forbear to illustrate them by *Examples*, but leave them to every Man's own Practice, and thus I conclude *Addition*.

The Proof *of* ADDITION.

The best *Proof* of *Addition* is by *Subtraction*; but that (supposed) yet not known how to perform, rest satisfied till it be; which, (as the second *Species* of *Arithmetick*) next follows, at the Close of which, the *Proofs* of both shall be exemplarily demonstrated.

CHAP. IV.
Of SUBTRACTION.

Definition. SUBTRACTION is the taking of one or more small Summs out of one greater, as 7 *s.* out of 12 *l.* or 37 *li.* out of 100 *li.* or 137 Feet out of 983 Feet, and the like.

As in *Addition*, the Summs to be added may be either of one, or of divers denominations, so likewise they may be in *Subtraction*, and the manner of placing them is the same; for you must set *Unites* under *Unites*, *Tens* under *Tens*, *Hundreds* under *Hundreds*, &c.

§ I. *Of SUBTRACTION of Numbers of* One Denomination.

Example 1. *Let it be required to ſubtract* 234 *out of* 986.

Place the numbers one under another, as you ſee done in the Margin; draw a line under them, and begin with the firſt figure towards your right hand, which is 4; ſaying, take 4 from 6, and there remains 2; place 2 under the line, and go to the next figure, which is 3; ſaying, take 3 from 8, and there remains 5; place 5 under the line, and go to the next figure, which is 2; ſaying, 2 out of 9, and there remains 7; place 7 under the line, and your Subtraction is ended: And it is evident by the work, that if you take 234 out of 986, there will remain 752; which you may thus prove: For if you add the 234 to 752, you ſhall find the ſumm of that addition to be 986, which is equal to the whole ſumm from which 234 was ſubtracted.

From	986
Subtract	234
Remains	752

Example 2. *Let it be required to ſubtract* 2976 *out of* 96527.

Place the numbers one under another, as in the Margin you ſee done; then draw a line under them, and begin with the firſt figure towards your right hand; ſaying, take 6 out of 7, and there remains 1; place 1 under the line, and proceed to the next figure; ſaying 7 out of 2 I cannot, (wherefore you muſt always add [10] to the number above, which in this Example is 2, and it makes it 12,) therefore take 7 out of 12, and there remains 5; place 5 under the line, and (becauſe you added 10 to the 2 to make it 12, you muſt) carry an Unite to the next figure; ſaying, 1 which I carried and 9 is 10, take 10 out of 5, which you cannot; therefore you muſt add 10 to 5, and it makes 15; and ſay, 10 out of 15, and there remains 5; place 5 under the line, and (becauſe you added 10 to 5, to make it 15, you muſt therefore) carry an unite to the next figure; ſaying, 1 which I carried and 2 is 3; take 3 out of 6, and there remains 3; place 3 under the line; and becauſe there are no more figures to be ſubtracted from the number above, you muſt ſay, nothing from 9 and there remains 9; ſet the 9 under the line, and your Subtraction is ended; and is proved by adding 2976 to 93551, which together make 96527.

From	96527
Subtract	2976
Remains	93551
Proof	96527

Other Examples *for Practice proved.*

	li.	*Reams of Paper.*		*Sheep.*	
Lent	5762	*Bought*	9765	*From*	1000
Paid	378	*Sold*	6529	*Take*	394
Reſts to pay	5384	*Unſold*	3236	*Remains*	606
Proof	5762	*Proof*	9765	*Proof*	1000

§ II. *SUBTRACTION of Numbers of* Different Denominations.

This is but the converſe of *Addition* of Numbers of *Different Denominations*; and to perform it, this is the *RULE:*

In Subtraction *of Numbers of divers Denominations, you muſt obſerve the ſame order as in* Addition; *namely, to place every number in due order, with reſpect to the denomination, as* Pounds *under* Pounds, Shillings *under* Shillings, *&c. the greater number always uppermoſt, and drawing a line under them, begin with the leaſt denomination firſt, ſubtracting it from the line above, and ſetting the* remainder *under the line, as in whole numbers; but if the* Pence *or* Shillings *in the upper row be ſmaller than thoſe in the lower row, you muſt add* 12 d. *or* 20 s. *to the ſmaller number, that ſo Subtraction may be made, as by the* Examples *following will appear.*

Example 1. *Let it be required to ſubtract* 38 li. 12 s. 8 d. *out of* 269 li. 18 s. 10 d.

Place your numbers as in the Margin; then beginning with the leaſt denomination firſt, (which in this Example is Pence,) ſay, 8 *d.* from 10 *d.* and there remains 2 *d.* ſet the 2 *d.* under the line, and proceed to the next denomination, which is Shillings; ſaying, take 12 *s.* out of 18 *s.* and there remains 6 *s.* place 6 under the line, and go to the Pounds; ſaying 8 out of 9, and there remains 1; place 1 under the line, and ſay, 3 out of 6,

	li.	*s.*	*d.*
Lent	269	18	10
Paid	38	12	8
Reſts	231	6	2

and there remains 3; then (becauſe there are no more figures to be ſubtracted) ſay, nothing out of 2, and there remains 2, which ſet under the line; ſo is your Subtraction ended, and the remainder is 231 *li.* 6 *s.* 2 *d.*

Example 2. *Let it be required to ſubtract* 2628 li. 16 s. 10 d. *out of* 9320 li. 10 s. 7 d.

	li.	*s.*	*d.*
Lent	9320	10	07
Paid	2628	16	10
Reſts	6691	13	09

Place the numbers in order, and beginning with the Pence, ſay, 10 *d.* out of 7 *d.* I cannot, (therefore I muſt add 12 *d.* (which is one Shilling,) to 7 *d.* and it makes 19 *d.*) but 10 *d.* out of 19 *d.* and there remains 9 *d.* ſet the 9 *d.* under the line, and (becauſe I added 12 *d.* to 7 *d.*) I muſt therefore carry one to the place of Shillings; ſaying, 1 *s.* which I carried, and 16 *s.* is 17 *s.* then 17 *s.* from 10 *s.* you cannot take, therefore you muſt add 20 *s.* (which is one Pound,) to 10 *s.* and it makes 30 *s.* then 17 *s.* out of 30 *s.* and there remains 13 *s.* ſet 13 under the line, and carry one to the place of Pounds; ſaying, 1 which I carried and 8 is 9, take 9 out of 0 I cannot, but 9 out of 10, and there remains 1; ſet 1 under the line, and carry an unite to the next place; ſaying, 1 which I carried and 2 is 3, take 3 out of 2 I cannot, but 3 out of 12, and there remains 9; place 9 under the line, and carry 1 to the next place; ſaying, 1 which I carry and 6 is 7, take 7 out of 3 you cannot, but 7 out of 13, and there remains 6; place 6 under the line, and carry one to the next row; ſaying, 1 and 2 is 3, take 3 from 9, and there remains 6; place 6 under the line, ſo is your ſubtraction ended, and the remainder is 6691 *li.* 13 *s.* 9 *d.*

Example 3. *Suppoſe a Man had lent to another Man* 1000 li. *and that the borrower had paid thereof at one time* 127 li. *at another time* 430 li. 10 s. *and at a third payment* 50 li. *and the Creditour would know how much he hath received, and how much is owing of his Debtor.*

	li.	*s.*	*d.*
Money lent	1000	00	00
Paid at ſeveral times	127	00	00
	430	10	00
	50	00	00
Paid in all	607	10	00
Reſts to pay	392	10	00

Place the numbers as here you ſee; firſt the ſumm of money lent, and draw a line under it; then ſet the ſumms paid at ſeveral times one under another, and draw a line under them; then add all the ſumms which have been paid at ſeveral times together, which make 607 *li.* 10 *s.* which is the ſumm which the Debtor hath paid in all; then ſubtract this 607 *li.* 10 *s.* from 1000 *li.* and there will remain 392 *li.* 10 *s.* and ſo much is ſtill owing to the Creditour.

Other Examples for Practice.

	li.	*s.*	*d.*
Lent	2601	13	06
Paid	98	07	09
Reſts	2503	05	09

	li.	*s.*	*d.*
Owing in all	100	00	00
Paid in all	36	10	06
Reſts to pay	63	09	06

	li.	*s.*	*d.*	*q.*
Lent	3625	16	08	03
Paid at ſeveral times	100	00	00	00
	336	10	06	02
	039	12	09	02
	100	00	00	00
Paid in all	576	03	04	00
Reſts to pay	3049	13	04	03

The Proof *of* Subtraction.

The *Proof* of *Subtraction* is performed by *Addition*, for adding the number to be ſubtracted to the remainder, the ſumm of them muſt be equal to the number given, if you have truly wrought: As in the firſt Example of numbers of one denomination,

The number given is	986
The number to be ſubtracted is	234
The remainder is	752
Proof	986

Add the number to be ſubtracted 234, to the remainder 752, the ſumm of them is 986, equal to the number given.

Examples

Examples *for Practice proved.*

1. *Of* Money.

	li.	s.	d.
Lent	62	18	09
Paid	37	19	06
Refts	24	19	03
Proof	62	18	09

	li.	s.	d.
Borrowed	100	00	00
Paid	36	13	04
Due	63	06	08
Proof	100	00	00

2. *Of* Troy-weight.

	lb.	ou.	pw.	gr.
Bought of Silver	7	11	13	19
Sold	5	07	03	05
Unſold	2	04	10	14
Proof	7	11	13	19

3. *Of* Avoirdupois *great weight.*

	C.	q.	lb.	ou.
Bought	37	3	22	11
Sold	13	1	23	06
Reſts	24	2	27	05
Proof	37	3	22	11

4. *Of* Avoirdupois *little weight.*

	lb.	on.	dr.
Bought	84	12	13
Sold	26	08	11
Reſts	58	04	02
Proof	84	12	13

5. *Of* Time.

	D.	h.	min.
From	364	23	50
Take	76	09	22
Reſts	288	14	28
Proof	364	23	50

6. *Of* Motion.

	Sig.	Deg.	'	"	"'
From	6	24	12	16	28
Subtract	2	12	11	14	20
Reſts	4	12	01	02	08
Proof	6	24	12	16	28

	Sig.	Deg.	'	"	"'
From	8	16	14	12	21
Subtract	4	20	18	40	38
Reſts	3	25	55	31	43
Proof	8	16	14	12	21

§ III. *Queſtions performed by* Addition *and* Subtraction.

Queſtion 1. *What number is that which being added to* 376 *ſhall make* 1000?

Subtract 376 from 1000, the remainder is 624, the number ſought.

Queſtion 2. *What number of Pounds, Shillings, and Pence, muſt be added to* 36 li. 17 *s.* 3 d. *to make that ſumm up* 100 li.

Subtract 36 *li.* 17 *s.* 3 *d.* from 100 *li.* the remainder is 63 *li.* 2 *s.* 9 *d.* which added to 36 *li.* 17 *s.* 3 *d.* makes 100 *li.*

Queſtion 3. *In the year of our Lord* 1440, *the famous Art or Myſtery of Printing was invented, I would know how long it is ſince that time to this year of our Lord* 1690.

From 1690 ſubtract 1440, the remainder is 250, and ſo many years are expired ſince *Printing* was invented.

Queſtion 4. *An Army conſiſting of* 13721 *Horſe, and* 26850 *Foot; in an Engagement there were ſlain* 3760 *Horſe, and* 7523 *Foot; the Queſtion is, how many were ſlain in all, and how many Horſe, and how many Foot, eſcaped.*

From the 13721 Horſe that went out, ſubtract the 3760 that were ſlain, there remains 9961, and ſo many Horſe eſcaped; alſo, from the 26850 Foot which went out, ſubtract the 7523 which were ſlain, and there remains 19327, the number of Foot which eſcaped; and, by adding the 3760 Horſe which were ſlain, to the 7523 Foot that were ſlain, their total is 11283, and ſo many were ſlain in all.

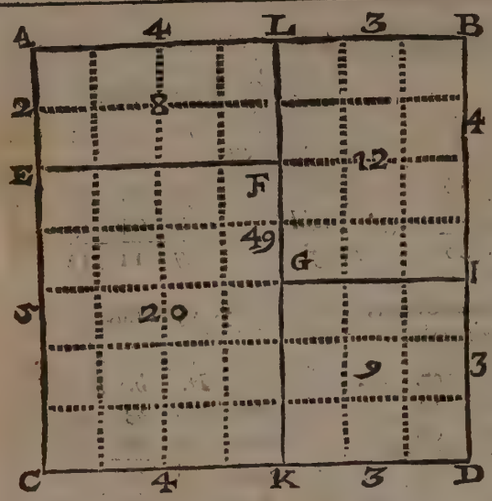
§ IV. ADDITION *and* SUBTRACTION *of Integers Geometrically demonſtrated.*

I. For *ADDITION.*

The Demonſtration of *Addition* of Integers depends upon this *Axiom, viz.*

The Whole is equal to all its Parts taken together.

And



And this is evident from this *Figure*, where the Square A B C D, each ſide whereof, as A B, B D, D C, and C A, contains ſeven, let them be *Feet*, *Yards*, *Perches*, *Acres*, *&c.* in length, then doth the whole Superficies A B C D, (as a Plain,) contain 49 *Squares* of the ſame Meaſure. Let the whole *Superficies* A B C D be divided into any two parts by the Line L K, as into A L K C, and L B D K; and again, let A L K C be divided into two other parts, *viz.* into A L E F, and E F C K; alſo, let L B D K be divided into two other parts L B G I, and G I D K, I ſay, that theſe Four Parts are all of them together equal to the Whole Square A B C D; for the ſide of the Square being 7, that multiplied in it ſelf, (or rather, in this caſe, ſeven ſevens added together,) makes for the *Product* or *Total* 49, and ſo many ſmall Squares are there contained in the Whole Square A B C D.

Now the *Parts* into which the Square A B C D is divided are,

A L F E	8	
E F C K	20	Little Squares.
L B I G	12	
G I D K	9	
	Their Summ 49	

Which is equal to the Number of Little Squares contained in the Whole Square A B C D, which was to be demonſtrated.

II. For *SUBTRACTION.*

This is but the Converſe of the Demonſtration of *Addition*, and depends upon this *Theorem.*

If from the Whole (of any diviſible thing) ſeveral equal or unequal Parts be taken one from the other ſucceſſively, till at laſt there remains nothing, thoſe Parts ſo ſubtracted, being added together, ſhall be equal to the Whole.

The Whole Square A B C D is 49

From which if you ſubtract the Part	A L E F 8	there will remain	41
	E F C K 20		21
	L B I G 12		9
	G I D K 9		9
	The Summ 49		Remains 0

Hence it is evident, that if from the *Whole* all the *Parts* are taken one from each others remainder, there at the laſt remains nothing, and therefore, All the *Parts* together are equal to the *Whole*, which was to be demonſtrated.

CHAP. V.

Of MULTIPLICATION.

§. I. *Definition.* MULTIPLICATION is that part of *Arithmetick* which teacheth how to increaſe one number by another, ſo that the number produced by their *Multiplication*, ſhall contain one of the numbers multiplied, ſo many times as there are *Unites* contained in the other. *Multiplication* may fitly be termed a Compendium of *Addition*, for that it performeth at one operation the ſame, which to effect by Addition would require many; for inſtance: If it were required to know how much

is

is 7 time 5, to perform this by Addition, I must set seven fives, or five sevens, one under another, and adding them together, I shall find that either of their Totals shall contain 35; but this by *Multiplication* is performed with far more brevity, as by *Examples* hereafter shall appear.

Before you enter upon the Practice of *Multiplication*, it is necessary to remember the Product produced by the multiplication of any one of the nine Digits, by any other of the same; as readily to know, that 4 times 5 is 20, 6 times 7 is 42, 2 times 9 is 18, 7 times 9 is 63, 8 times 9 is 72, *&c.* which this Table following will plainly declare, and must be perfectly learned by heart, before you attempt to multiply great numbers.

1	9	8	7	6	5	4	3	2	1
2	18	16	14	12	10	8	6	4	2
3	27	24	21	18	15	12	9	6	3
4	36	32	28	24	20	16	12	8	4
5	45	40	35	30	25	20	15	10	5
6	54	48	42	36	30	24	18	12	6
7	63	56	49	42	35	28	21	14	7
8	72	64	56	48	40	32	24	16	8
9	81	72	63	54	45	36	27	18	9

This Table is thus to be read: In the two outermost Rows or Colums, towards each hand, and also at the top of the Table, you have the nine *Digits* in bigger Figures than the rest; the Figures in the first Column beginning with 1, and so proceeding by 2, 3, 4, *&c.* to 9. those at the top of the Table beginning with 9 towards the left hand, and so backwards by 8, 7, 6, *&c.* to 1 at the right hand.

Now if by this Table you would know how much 8 times 7 is, find 8 among the great Figures at the head of the Table, and look down that *Row* or *Column*, till you come against 7 of the great Figures in either of the outermost *Columns*, against which you shall find 56, and so much is 8 times 7, or eight multiplied by 7.

In the same manner you may find, that 7 times 9 is 63, 5 times 6 is 30, 3 times 4 is 12, and so of any two of the nine Digits.

In *Multiplication* there are three Terms or Denominations commonly used, that is to say,

The *Multiplicand*, the *Multiplier*, and the *Product*.

The *Multiplicand* is the number to be multiplied.

The *Multiplier* is the number by which the *Multiplicand* is multiplied: and

The *Product* is the number which is produced by the multiplication of the *Multiplicand* and the *Multiplier* together.

Thus, if it were required to multiply 8 by 7, here 8 is the *Multiplicand*, 7 the *Multiplier*, and 56 is the *Product*, for 8 times 7, or 7 times 8 is 56.

In *Multiplication* it mattereth not which of the two numbers is made the *Multiplicand*, or which the *Multiplier*, for the *Product* produced by either, will be the same; but the usual way is to make the *Greater* number the *Multiplicand*, and the *Lesser* number the *Multiplier*: And to perform *Multiplication*, this is the *RULE*:

The numbers to be multiplied must be set one under another, viz. *the* Multiplicand (*or great number*) *above, and the* Multiplier (*or lesser number*) *below the last figure of the* Multiplier, *under the last figure of the* Multiplicand; *then draw a line under them, and* (having learned the preceding Table perfectly by heart) *multiply every* Digit *of the* Multiplier *into every* Digit *of the* Multiplicand, *setting the several Products under the line; then having finished your* Multiplication, *draw a line, and add all the Products together, and the summ of those Products is the general* Product *of the whole multiplication, as by the following* Examples *will appear:*

Example 1. *Let it be required to multiply* 736 *by* 7.

First, write down 736 the Multiplicand, and under it 7 the Multiplier, and under them draw a line: then multiply 7 into every Digit of the Multiplicand; saying, 7 times 6 is 42; place 2 under the line, under 7, and for the four tens keep 4 in mind: then say again, 7 times 3 is 21, and 4 which I kept in mind is 25; place 5 under the line and keep the two tens in mind: then say again, 7 times 7 is 49, and 2 which I kept in mind is 51; place 1 under the line, and the 5 tens kept in mind (because there are no more figures to be multiplied) set down under the line also; so is the work ended, and the Product of this multiplication is 5152.

736	*Multiplicand*
7	*Multiplier*
5152	*Product*

In the foregoing example the *Multiplier* consisted but of one *Digit*, we are now to shew how *Multiplication* is performed, when the Multiplier consists of more than one figure; therefore,

Example 2. *Let it be required to multiply* 5704 *by* 37.

Place

Place your numbers, and draw a line under them as you see in the Margin: Then begin your Multiplication in this manner; saying, 7 times 4 is 28; set 8 under the line, and keep two tens in mind: then say, 7 times nothing is nothing, but the two tens in the mind is 2; set 2 under the line: then say 7 times 7 is 49; set 9 under the line, and keep 4 in mind: then lastly, say, 7 times 5 is 35, and 4 in mind is 39; which being the last number to be multiplied, set down under the line; so is the multiplication of one of the Digits (namely 7) finished.

Multiplicand	5704
Multiplier	37
	39928
	17112
Product	211048

Then begin to multiply the second Digit; saying, 3 times 4 is 12; place 2 in the second line one place towards the left hand, and keep 1 in mind: then say, 3 times nothing is nothing, but 1 in mind is 1; set down 1 by the 2 in the second line: Thirdly, say, 3 times 7 is 21; place 1 in the second line, and keep the two tens in mind: Lastly, say, 3 times 5 is 15, and 2 is 17; which 17 (because there are no more figures to be multiplied) place in the second line also.

Having thus done, draw a line under them, and add these two lines together, as in common Addition of numbers of one denomination; saying, 8 is 8; place 8 under the line: then say, 2 and 2 is 4; place 4 under the line: then say, 1 and 9 is 10; place a cipher under the line, and carry one to the next place; saying, 1 and 1 is 2, and 9 is 11; place 1 under the line, and carry 1 to the next row; saying, 1 and 7 is 8, and 3 is 11; place 1 under the line, and carry 1 to the next place; saying, 1 which I carry and 1 is 2, place 2 under the line; and so is your Multiplication ended, and the product is 211048.

Example 3. *Let it be required to multiply* 57325 *by* 4032.

Multiplicand	57325
Multiplier	4032
	114650
	171975
	2293000
Product	231134400

Place the Multiplicand and the Multiplier one under another, and draw a line as before; then proceed to the multiplication as formerly; saying, First, 2 times 5 is 10; set down a Cypher, and keep 1 in mind: then 2 times 2 is 4, and 1 in mind is 5; place 5 under the line: then 2 times 3 is 6; set 6 under the line: then 2 times 7 is 14; set down 4 and keep one in mind: then 2 times 5 is 10, and 1 in mind is 11; which 11 (being the last) set down.

The Multiplication of one of the Digits being finished, proceed to the Multiplication of the next; saying, 3 times 5 is 15; set down 5 in the second line, a place more toward the left hand, and keep 1: then 3 times 2 is 6, and 1 kept is 7; set down 7: then 3 times 3 is 9, set down 9: then 3 times 7 is 21; set down 1 and keep 2 in mind: then 3 times 5 is 15, and 2 in mind is 17; which, being the last, set down also.

Two of the figures of the multiplier being finished, proceed to the third, which (in this example) being a cypher, you may wholly neglect, and proceed to the multiplication of the fourth figure; only remember to remove the Product of the fourth figure one place more to the left hand, or place one cypher in the place where the first figure should have stood, had it been a Figure, as in the example you may see; for the cypher, though it need not be written down, yet it must keep its place, and the figures following must be removed a place farther.

Then for the Multiplication of the fourth and last digit, say, 4 times 5 is 20; set down a cypher, (under 9,) and keep 2 in mind: then 4 times 2 is 8, and 2 in mind is 10; set down a cypher, and keep 1 in mind: then 4 times 3 is 12, and 1 is 13; set down 3 and keep 1: then 4 times 7 is 28, and 1 kept is 29; set down 9, and keep 2: then 4 times 5 is 20, and 2 is 22; which 22 (because the multiplication is ended) set down also.

Having thus multiplied all the Digits severally, draw a line under their Products, and add them all together, as in the former Example; so shall you find their general Product to be 231134400.

Other Examples for Practice.

73260	50762	6735041
45003	567	8602
219780	355334	13470082
366300..	304572	404102460
293040	253810	53880328
3296919780	128782054	57934822682

§ III. *The*

§ III. *The Reason of the Working of Multiplication.*

Multiply 426
by 327
2982
852
1278
139302

The Reason of this Operation.

				4	2	6	*Multiplicand.*
				3	2	7	*Multiplier.*
(1.) 7 times 6 is 42					4	2	*First* }
7 times 20 is 140				1	4	.	*Second* } *Product.*
7 times 400 is 2800			2	8	.	.	*Third* }
(2.) 20 times 6 is 120				1	2	.	*First* }
20 times 20 is 400				4	.	.	*Second* } *Product.*
20 times 400 is 8000			8	.	.	.	*Third* }
(1.) 300 times 6 is 1800			1	8	.	.	*First* }
300 times 20 is 6000			6	.	.	.	*Second* } *Product.*
300 times 400 is 120000	1	2	.	.	.	.	*Third* }
The total Summ of all the Products	1	3	9	3	0	2	*The General Product.*

The Proof of Multiplication.

The most certain Proof of *Multiplication* is by *Division*, but because *Division* is not yet known, I will here shew a near way by which *Multiplication* may be proved; for which this is the *RULE:*

Make a Cross; *then any Summs being multiplied, you may prove the truth of your work in this manner:* 1. *Cast away all the nines which you can find in the* Multiplicand; *what remaineth set on the right side of the* Cross. 2. *Cast away also the nines in the* Multiplier, *and what remains set on the left side of the* Cross. 3. *Multiply the figures on each side of the* Cross *together, and out of that* Product *cast away the nines also, setting the figure remaining over the* Cross: *Then,* 4. *Cast away all the nines in the* Product, *and if the figure remaining be the same with that which standeth over the* Cross, *then is your Multiplication truly performed, otherwise not.*

Example 1. Let it be required to prove this Summ in the Margin.

1. Cast away all the nines in the *Multiplicand*; saying, 4 and 3 is 7, and 2 is 9, which being rejected, there remains 4; which set on the right side of the Cross: Then,

2. Cast away all the nines in the *Multiplier*; saying, 2 and 3 is 5, which (being less then 9) set on the left side of the Cross: Then,

3. Multiply 4 by 5; saying, 4 times 5 is 20, from which cast all the nines, which are two, and there remains 2; place 2 over the Cross: And,

4. Cast away all the nines in the *Product*; saying, 2 and 5 is 7, and 4 is 11, cast away 9, and there remains 2; which exactly agrees with the figure over the *Cross*, and demonstrates that the Multiplication is truly performed.

4324 — 2
23 — 5 × 4
12972
8648 — 2
99452

§ IV. *Compendiums in Multiplication.*

1. If the *Multiplier* consist of Cyphers in the last place or places, you may omit the Multiplication of them, and place the former figures of the Multiplier under the Multiplicand: Thus, if it were required to multiply 3257 by 2600; place the numbers as you see in the Margin; then multiplying 3257 by 26, the Product will be 84682, to which if you add two Cyphers, (because there were two Cyphers in the Multiplier,) it will be 8468200, which is the true Product of the Multiplication.

3257
2600
19542
6514
8468200

2. If it be required to multiply any number by 10, 100, 1000, 10000, *&c.* you have no more to do, but to add ſo many Cyphers to the *Multiplicand* as there are Cyphers in the *Multiplier:* Thus, if you were to multiply 365 by 10, the Product will be 3650; or by 100, it would be 36500; or by 1000, it would be 365000; or by 10000, it would be 3650000.

§ V. *Queſtions performed by Multiplication only.*

Queſtion 1. *If a piece of Land be 236 Perches long, and 182 Perches broad, how many ſquare Perches are contained therein?*

Multiply 236 the length, by 182 the breadth, and the product is 42952, and ſo many ſquare Perches are contained in ſuch a ſquare piece of Land.

Queſtion 2. *In a year there are 365 days natural, and in every day 24 hours, how many hours are there in a year?*

Multiply 365, the number of days, by 24, the number of hours, the product is 8760; and ſo many hours are there in a year.

Queſtion 3. *From* London *to* Coventry *it is accounted 76 miles, how many yards therefore is it accounted from* London *to* Coventry?

Multiply 1760, (which are the number of yards contained in one mile,) by 79, the the product is 133760; and ſo many yards are between *London* and *Coventry.*

CHAP. VI.
Of DIVISION.

§ I. *Definition.* DIVISION is the juſt contrary to *Multiplication*; for that turns *Small* Denominations to *Greater*, as *Multiplication* turns *Greater* to *Smaller*; or (in whole Numbers, of which only we yet ſpeak) *Diviſion* is the asking how many times one Summ is contained in another, and the number which anſwereth to that queſtion is called the *Quotient.*

The Number containing, or which is to be divided, is called the *Dividend.*

The Number contained, or by which the *Dividend* is to be divided, is called the *Diviſor.*

And as often as the *Dividend* contains the *Diviſor*, ſo often doth the *Quotient* contain *Unity.*

The ways of performing Diviſion are divers, but I ſhall ſhew only Two, which of all others are the beſt; the one without cancelling any Figures, and the other by cancelling of them.

§ II. *The Firſt Way of Diviſion.*

This kind of Diviſion is very much uſed, and is in great requeſt with thoſe who have moſt occaſion to divide great numbers; the manner of working is made plain by the Examples and Rules following.

Example 1. *Let it be required to divide* 864 by 3.

Diviſor Dividend Quotient
3) 8 6 4 (
———

Set down your numbers as you ſee them placed in the Margin, *viz.* Firſt, ſet down 864, the *Dividend*; then on the left hand thereof ſet the *Diviſor* 3, with a crooked line between them; then on the right hand thereof make another crooked line, which muſt ſerve to ſet the figures of the *Quotient* in; ſo are your numbers placed in due order: then draw a line under the *Dividend*, and make a prick under the figure 8; this prick ſerves only to ſhew how far you have proceeded in your work, and muſt at every diviſion be removed a place farther, till at length you come to the laſt figure of the Dividend: your numbers being thus placed with a line under them, you are ready for the work, which muſt be performed according to the directions given in the following *RULE:*

Demand how often the Diviſor *may be had in the* Dividend, *and place that number in the* Quotient; *then multiply the* Diviſor *by the* Quotient, *and place the* Product *under the line; then ſubtract this* Product *from the* Dividend, *and ſet the* Remainder *under the* Product; *then make a prick under the next figure of the* Dividend, *and bring that figure down to the* Remainder, *and then proceed as before.*

Your

Your numbers being placed as is above directed, and as you ſee here in the Margin, begin your *Diviſion* thus:

1. Ask, How many times 3 (the *Diviſor*) can I have in 8, (the firſt figure in the *Dividend?*) the anſwer will be 2; ſet 2 in the *Quotient*, and multiply 2 by 3, (the *Diviſor*,) the *Product* will be 6; ſet 6 under the 8, and ſubtract it from 8, ſaying, 6 from 8, and there remains 2; ſet 2 under 6, and ſo is one figure of your dividend finiſhed.

2. Make a *Prick* or *Point* under 6, (the ſecond figure of your *Diviſor*,) and bring it down to 2, making it 26; then ask, how many times 3 can I have in 26? the anſwer will be 8 times; put 8 in the *Quotient*, and multiply 8, by the *Diviſor* 3, it makes 24; ſet 24 under 26, and ſubtract 24 from 26, and there will remain 2; which ſet under 24; ſo is the ſecond figure of your *Dividend* finiſhed.

3. Make a *Prick* under 4, (the laſt figure of the *Dividend*,) and bring it down to 2, making it 24; then ask, how many times 3 can I have in 24? the anſwer will be 8 times; put 8 in the *Quotient*, and multiply 8 by the *Diviſor* 3, the *Product* will be 24; which ſet under 24, and ſubtracting it from the 24 above, there will remain 0; ſo is your *Diviſion* ended, the *Quotient* being 288, and ſhews that 3 is contained in 864, 288 times.

Diviſor Dividend Quotient

3) 8 6 4 (2

6 *Product*

2

3) 8 6 4 (28

6 . . *Product*

2 6 .

2 4 *Product*

2 .

3) 8 6 4 (2 8 8

6 . . *Product*

2 6 .

2 4 . *Product*

2 4

2 4 *Product*

0 0

§ III. *The Proof of this Way of Diviſion.*

The beſt *Proof* of *Diviſion* is by *Multiplication*; for if you multiply 288, the *Quotient*, by 3, the *Diviſor*, the *Product* will be 864, equal to the *Dividend*.

But it may be otherwiſe proved by common *Addition*, according to this *RULE*:

Add all the Products *together, and the Summ of them (if you have wrought right) ſhall be equal to the* Dividend.

So in this Example,

The Product { 6 . . / 2 4 . / . 2 4 } added,

Their Summ is 8 6 4 equal to the Dividend.

Example 2. *Let it be required to divide* 43201 *by* 74.

Set the numbers in order as in the Margin: then, becauſe the firſt figure in the *Diviſor*, 7, is greater than 4, the firſt figure of the *Dividend*, you muſt make the firſt *Prick* under 2, the third figure of your *Dividend*: Then,

1. Ask, how many times 74 can I have in 432? or (more eaſily) thus, how many times 7 can I have in 43? 7 may be had 6 times in 43, and there will remain 1, which with the 2 makes 12, but then 6 times 4 makes 24, which ſhould be taken out of 12, but cannot, therefore you cannot have 6 times 74 in 432; wherefore ſay 5 times 7 in 43; ſet 5 in the *Quotient*, and by it multiply the *Diviſor* 74, the *Product* will be 370; which ſet under 432, and ſubtracting 370 from 432, there will remain 62; and ſo is the firſt work finiſhed.

2. Make a prick under 0 in the *Dividend*, and bring the 0 down to 62, making it 620; then ask how many times 7 can be had in 62, the anſwer will be 8 times; put 8 in the *Quotient*, and multiplying 74 by 8, the *Product* will be 592, which ſubtract from 620, and there will remain 28: ſo is the ſecond work ended.

Diviſor Dividend Quotient

7 4) 4 3 2 0 1 (5

3 7 0 . . *Product*

6 2

7 4) 4 3 2 0 1 (5 8

3 7 0 . . *Product*

6 2 0

5 9 2 *Product*

2 8

```
74)43201(583
...
370..  Product
.620.
.592.  Product
..281
..222  Product
  59   Remainder
```

3. Make a *Prick* under 1, (the laſt figure of the *Dividend*,) and bring it down to 28, making it 281; then ask, how many times 7 can I have in 28, the anſwer will be 3 times; put 3 in the *Quotient*, and by it multiply the *Diviſor* 74, the *Product* will be 222, which ſubtracted from 281, there will remain 59; and thus is your *Diviſion* ended, the *Quotient* being 583; which ſhews, that 74, the *Diviſor*, is contained in 43201, the *Dividend*, 583 times, and 59 over, as appears by the *Remainder*.

§ IV. *For Proof whereof,*

If you multiply 583 by 74, the *Product* will be 43142, to which if you add the *Remainder* 59, the *Summ* will be 43201, equal to the *Dividend*: Or,

```
The Product { 370..
            {  592.
            {   222   added
Their Summ    43142
The Remainder    59
Their Summ is 43201 equal to the Dividend.
```

Example 3. *Let it be required to divide* 162483 *by* 1321.

Your numbers being orderly placed, as in the Margin, begin your work in this manner:

```
1321)162483(123
...
1321..  Product
.3038.
.2642.  Product
..3963
..3963  Product
  0000
```

1. Ask, how many times 1321 can I have in 1624? ſay once; place one in the *Quotient*; by which 1 multiply the *Diviſor* 1321, beginning at the left hand; ſaying, once 1 is 1; place 1 under the line: then, once 2 is 2; ſet 2 under the line: then, once 3 is 3; place 3 under the line: laſtly, once 1 is one; place 1 under the line: then ſubtract this 1321 from 1624, and there will remain 303: To this 303 bring down the next figure in the *Dividend*, namely 8, (firſt making a prick under the 8,) ſo will that number be 3038; under which draw a line, and repeat the ſame work again; ſaying, how many times 1321 can I have in 3038? which may be had two times; place 2 in the *Quotient*, by which 2 multiply the *Diviſor* 1321; ſaying, 2 times 1 is 2; place 2 under the line: then 2 times 2 is 4; place 4 under the line: then, 2 times 3 is 6; place 6 under the line: laſtly, 2 times 1 is two; place 2 under the line; and ſubtract this 2642 from 3038, and there will remain 396: To this 396, bring down the next figure of the *Dividend*, which is 3, ſo is this number made 3963; under which draw a line, and repeat the work once again; ſaying, how many times 1321 can I have in 3963, which may be had 3 times, by which 3 multiply the *Diviſor* 1321; ſaying, 3 times 1 is 3; then, 3 times 2 is 6; then 3 times 3 is 9; and laſtly, 3 times 1 is 3; which place under the line, and ſubtract it from the line above, which in the example is the ſame number; therefore there remains nothing, and the work is ended; but if any *Remainder* had been, that ſhould have been ſet under the line, as by the former example appears.

§ V. *The Reaſon of the working of Diviſion.*

By the former Directions, *Let it be required to divide* 139302 *by* 327.

The Operation.

```
Diviſor  Dividend  Quotient
327)139302(426
...
1308
 850
 654
 1962
 1962
```

The

The Reason of this Operation.

	1	3	9	3	0	2	*Dividend.*
		3	2	7			*Divisor* advanced three Places; and is not now 327 but 32700.
I. *Quotient* 400	1	3	0	8			This *Product* is not 1308, but 130800, which is 4 times 32700; that is, 420 times 327, and is the *Quotient* in the Margin.
			8	5			That is, 1308 being deducted out of 1393, the Remainder is 8500; so is the First Operation ended
		8	5	0			*The Second Operation:* and this is not 850, but 8500
		3	2	7			Here the *Divisor* is advanced two Places, and is not 327, but 3270.
II. *Quotient* 20	6	5	4				That is 3270 doubled, which makes 6540, which in effect is 20 times 327, as by the *Quotient*.
		1	9	6			That is, 6540 deducted out of 8500; the Remainder is 1960; and so is the Second Operation ended.
	1	9	6	2			*The Third Operation.*
			3	2	7		Here 327 is brought down to its own value again.
III. *Quotient* 6	1	9	6	2			That is, 6 times 327 is equal to 1962, and so the work is ended.
	0	0	0	0			*Remainder.*

§ VI. *Other Examples for Practice.*

5624)793058(141
5624.. *Product*
23065.
22496. *Product*
..5698
..5624 *Product*
....74 *Remains*
Proof 793058

4325)763218(176
4325.. *Product*
33071.
30275. *Product*
.27968
.25950 *Product*
..2018 *Remains*
Proof 763218

In these Examples you may perceive, that the *Quotients* are 141 and 74 remaining, and 176 and 2018 remaining; so that their real *Quotients* are $141\frac{74}{5624}$, and $176\frac{2018}{4325}$.

§ VII. *A Second Way of Division.*

Another way of Division, not inferiour to the preceding, (which is also proved by Addition,) here followeth.

The manner of placing the figures is the same with the former; and for the performance of this Work, this is the *RULE:*

First write down the Dividend, *and on the left hand thereof the* Divisor, *with a crooked line between them; and on the right hand of the* Dividend *make another crooked line, wherein to place the figures of the* Quotient; *then draw a line under the* Dividend; *and also make a prick under that figure of the* Dividend, *under which the last figure of the* Divisor *will fall: This done, demand how often the* Divisor *may be found in those figures of the* Dividend, *and place that Digit in the* Quotient; *then by this Digit multiply the* Divisor, *and set the* Product *of this Multiplication directly under the* Dividend, *beginning at the place where you made the prick:* [Thus far as in the former way.] *Then subtract this* Product *from the figures of the* Dividend, *and place the* Remainder *over the* Dividend, *cancelling the figures of the* Dividend *as you proceed; so is the first figure of the* Quotient *finished: Then make a prick under the next figure of the* Dividend, *and demand how often the* Divisor *may be found in the last* Remainder, *and the other figure being added thereto, which place in the* Quotient, *and proceed in all respects as before, till you have pointed all the figures of the* Dividend.

Example. *Let it be required to divide* 763258 *by* 2345.

Place

2345)763258(

Place your numbers as you see in the Margin, and (because there are four figures in the *Divisor*, therefore) make a prick under the fourth figure of the *Dividend*, which is 2, and draw a line, then begin your Division in this manner; saying,

1. How many times 2345 can I have in 7632? (or, how many times 2 can I have in 7?) say 3 times; place 3 in the *Quotient*, by which 3 multiply the *Divisor*; saying, 3 times 5 is 15; place 5 under the prick; then 3 times 4 is 12, and 1 is 13; place 3 under the line: then, 3 times 3 is 9, and 1 is 10; place a cypher under the line: then, 3 times 2 is 6, and 1 is 7; place 7 under the line: then subtract 7035 from 7632; saying, 5 from 12, and there remains 7; place 7 over 2, (cancelling the 5 and the 2,) and bear one in mind: then, 1 and 3 is 4, out of 13, there remains 9; place 9 over 3, and cancel 3 and 3: then 1 which I carried from 6, and there remains 5; place 5 over 6, and cancel 0 and 6: lastly, 7 from 7, there remains 0; which you need not set down, but cancel the two sevens; then will the work stand as above, and the remainder will be 597.

597
2345)763258(3
7035

2. Make a prick under the next figure of the *Dividend*, namely 5; and say, how many times 2345 can I have in 5975? answer two times; place 2 in the *Quotient*, by which multiply the *Divisor*; saying, 2 times 5 is 10; place 0 under 5, and carry 1: then, 2 times 4 is 8, and 1 is 9; place 9 under 7: then, 2 times 3 is 6; place 6 under 9: lastly, 2 times 2 is 4; place 4 under 5; so is the product of this Multiplication 4690, which you must subtract from 5975; saying, 0 from 5, and there remains 5; place 5 over 5, and cancel 0 and 5: then, 9 out of 17, there remains 8; place 8 over 7, and cancel 9 and 7: then, 1 carried and 6 is 7, from 9 there remains 2; place 2 over 9, and cancel 6 and 9: lastly, 4 from 5, rests 1; place 1 over 5, and cancel 4 and 5; so have you finished your second figure, and your work will stand as in the Margin, and your remainder will be 1285.

128
5975
2345)763258(32
7035
4690

3. Make a prick under the next figure of your *Dividend*, (namely under 8,) and ask, how many times 2345 can I have in 12858? (or, how many times 2 can I have in 12?) say 5 times; place 5 in the *Quotient*, by which multiply the *Divisor*, saying, 5 times 5 is 25; place 5 under 8, and carry 2: then, 5 time 4 is 20, and 2 is 22; place 2 under 5, and carry 2: then, 5 times 3 is 15, and 2 is 17; place 7 under 8, and carry 1: then, 5 times 2 is 10, and 1 is 11; place 11 under 1 and 2; so is the product of this Multiplication 11725, to be subtracted from 12858; saying, 5 from 8 rests 3; place 3 over 8, and cancel 5 and 8: then, 2 from 5 rests 3; place 3 over 5, and cancel 2 and 5: then, 7 from 8 rests 1; place 1 over 8, and cancel 7 and 8: then 1 from 2 rests 1; place 1 over 2, and cancel 1 and 2: lastly, 1 from 1 rests nothing; so is your work ended, which you shall find to stand as in the Margin, the remainder being 1133.

11
1283
5975
2345)763258(325
7035
4690
1725

§ VIII. *A certain Rule, to know what Figure to put in the Quotient.*

In the practice of *Division*, there is nothing more difficult than in large Summs (especially if the first figures of the *Divisor* be either 1, 2, 3, or cyphers, and the last figures 7, 8 or 9) to know certainly what figure to put in the *Quotient*, and never to take one too *Great*, or too *Little*, but that which will justly serve, when you demand how often the *divisor* may be had in the *dividend*; for the certain finding whereof, and also to perform (with ease and certainty) the hardest, and most difficult *Summ* that can be proposed in *Division*, without the assistence of *Multiplication*, only by *Addition* and *Substraction*, not burthening the memory at all, (a little pains being taken before you begin your work,) do thus:

Suppose

Suppoſe you were to divide any Summ, as 1097909, by 309: First, ſet down the nine *Digits* 1, 2, 3, *&c.* one under another, and againſt the figure 1, ſet 309 your *Diviſor*, which doubled is 618, which ſet againſt 2, theſe added together make 927, which ſtands againſt 3: Add the *Diviſor* 309 to 927, it makes 1236, which is againſt 4; to this add the *Diviſor*, and it makes 1545, which ſtands under 5. And thus to every laſt number ſtill add the *Diviſor*, till you have gone through all the nine *Digits*, then will they be as in the Margin.

1	309
2	618
3	927
4	1236
5	1545
6	1854
7	2163
8	2472
9	2781

Having prepared this little Table, ſet your *Dividend* and *Diviſor* down, as in any of the ways of *Diviſion* pricking the *Dividend*, and drawing a line under it, as is before directed, and as you ſee here done.

Then laying your little Table (which may be called a Tariffa) before you, look in it for 1097, the four firſt figures of the *Dividend*, which you cannot exactly find there, but the neareſt number leſs (which you muſt always take when you cannot find the juſt number you look for) is 927, againſt which ſtands 3, ſet 3 in the *Quotient*, and ſubſtract 927, out of 1097, and there will remain 170, to which bring down 9, the next figure of your *Dividend*, and it is 1709: Look this number in your Tariffa, which you cannot find, but the next leſs is 1545, againſt which ſtands 5, ſet 5 in the *Quotient*, and ſubſtract 1545 from 1709, there will remain 164, to which bring down the next figure of your *Dividend*, (which here is a cypher,) making it 1640: Look this 1640 in the Tariffa, which you cannot find, but the next leſs is 1545, againſt which ſtands 5, ſet 5 in the *Quotient*, and ſubſtract 1545 out of 1640, and there will remain 95, to which bring down the laſt figure of your *Dividend* 9, making it 959: Look this number in the Table, or the next leſs, which is 927, againſt which ſtands 3, ſet 3 in the *Quotient*, and ſubſtract 927 from 959, the remainder is 32. So is your *diviſion* ended, and the *Quotient* is 3553 $\frac{32}{309}$. And with what eaſe and certainty this is effected, no *Multiplication* being uſed, I leave to the Reader to judge.

```
309)1097909(3553
   ....
  ------
  .927...   Product
  .1709..
  ------
  .1545..   Product
  ..1640.
  ------
  ..1545.   Product
  ....959
  ------
  ....927   Product
  .....32   Remainder
  -------
  1097909
```

§ IX. *The Proof of this ſecond way of* Diviſion.

This kind of Diviſion is alſo proved by Addition; for, *If you draw a line under the work, and add all the figures between the two lines together, (in order as they then ſtand) taking in the Remainder (if any be) the Total of this* addition *will be equal to the* Dividend, *if the work be true.*

```
        4
      xxx7
  542)76353(140
      5428
       xx6
  ---------
```

The Summ 75880
To which add 473 *the Remainder*
The Summ 76353 *equal to the Dividend*

```
        x766
         7449 4
  5678)x345678(413
       xx7xx84
         5673
          x78
  ---------
```

The Summ 2345014
The Remainder 664
The Summ 2345678 *equal to the Dividend*

§ X. *Queſtions*

§ X. *Questions performed by* Division *only.*

Question 1. *If a piece of Land lying in a long square, or* Parallelogram, *contain* 42952 *square* Perches, *and one of the sides thereof be* 236 Perches *long, how long must the other side be?*

Divide 42952 by 236, the Quotient will be 182, and so many Perches long must the other side be.

Question 2. *In a year there are* 8760 *hours, and in every natural day there are* 24 *hours, I demand how many days be there in a year?*

Divide 8760 by 24, the Quotient will be 365, and so many days be there in a year.

Question 3. *The distance from* London *to* Coventry *is* 133760 *yards, and in one mile there is contained* 1760 *yards, now I would know how many miles it is from* London *to* Coventry;

Divide 133760 by 1760, the Quotient will be 76, and so many miles it is from *London* to *Coventry.*

These Questions performed by *Division* only, are the converse of those that were performed by *Multiplication*, which I the rather make choice of, that the Reader might see how *Multiplication* and *Division* prove each other.

CHAP. VII.
Of REDUCTION.

Definition. REDUCTION is two fold; for the First turneth (or reduceth) *Great Denominations* into *smaller:* As *Pounds* into *Shillings*, *Pence*, or *Farthings*; which is performed by *Multiplication.*

The Second reduceth small Denominations into Greater; and that is performed by *Division,*

§ I. *The first kind of Reduction.*

Example 1. *Let it be asked how many Shillings, Pence, and Farthings, are contained in* 729 *l.* 11 *s.* 7 *d.* 3 *q.*

li.	*s.*	*d.*	*q.*
729	11	7	3
20			
14580			
11	*s.*		
14591	*Shillings*		
12			
29182			
14591			
175092			
7	*d.*		
175099	*Pence*		
4			
700396			
3	*q.*		
700399	*Farthings*		

First, a Shilling is contained in a Pound 20 times, therefore multiply 729 by 20, or (which is the same, but shorter) by 2, and put the 0 to the product, as in the Margine, this shews, that in 729 *l.* there are 14580 Shillings. To which add 11 *s.* it makes 14591 Shillings.

Secondly, Because one Peny is contained in one Shilling 12 times, multiply 14591 by 12, it produceth 175092, to which add the 7 pence, so the Summ will be 175099.

Thirdly, Because one Peny contains 4 Farthings, multiply 175099 by 4, the Product will be 700396, to which add the 3 Farthings, the Summ will be 700399; And so many Farthings are here in 729 *l.* 11 *s.* 7 *d.* 3 *q.*

Example 2. *Let it be asked how many Hogsheads, Gallons, and Pints, are contained in* 4 *Tuns,* 3 *Hogsheads,* 27 *Gallons, and* 3 *Pints.*

First, 1 *Tun* is equal to 4 *Hogsheads*, therefore 4 *Tun* is equal to 16 *Hogsheads*, to which add the 3 *Hogsheads*, so there is 19 entire *Hogsheads.*

Secondly, Because one *Hogshead* contains 63 *Gallons*, multiply 19 by 63, it produceth 1197 *Gallons*, to which add 27, it gives 1224 *Gallons.*

Thirdly, Because every *Gallon* contains 8 *Pints*, multiply 1224 by 8, it produceth 9792, to which add the 3 *Pints*, and it will be 9795; and so many *Pints* are contained in 4 *Tuns*, 3 *Hogsheads*, 27 *Gallons*, and 3 *Pints*, as appears by the work following.

Tun

Tuns. Hog. Gall. Pints.
4 — 3 — 27 — 3

4
16
3 *add*
19 *Hogsheads*
63
57
114
1197
27 *add*
1224 *Gallons*
8
9792
3 *add*
97915 *Pints*

After the same sort might Dry Measures be reduced; as *Quarters* to *Bushels*, *Pecks*, or *Gallons*; and likewise all *Weights*, and Outlandish *Coins*; of which the proportion of the greater to the lesser is (before) known or given.

§ II. *The Second Kind of* Reduction.

This reduces *Smaller Denominations* into *Greater*; as, *Farthings* into *Pounds*, *Shillings*, and *Pence*; *Pints* into *Gallons*, *Hogsheads*, and *Tuns*, *&c.* and is performed by *Division*: And for the illustration hereof, we will take the converse of the two foregoing *Examples*, which will be Proofs to each other: Therefore,

Example 1. *Let it be asked, In* 700399 Farthings, *how many* Pounds, Shillings, Pence, *and* Farthings *are there?*

First, Four *Farthings* make one *Penny*; therefore divide 700399 by 4, the Quotient will be 175099 *Pence*, and 3 *Farthings* remaining.

Secondly, Because 12 *Pence* make one *Shilling*, divide 175099 by 12, the Quotient will be 14591 *Shillings*, and 7 *Pence* remaining.

Thirdly, Because 20 *Shillings* make one *Pound*, divide 14591 by 20, the Quotient will be 729 *li.* and 11 *s.* remaining: Or, (which is easier done,) set down 14591 *s.* and cut off the last figure to the right hand, by a dash of the Pen, thus, 1459|1, then take the half of 1459, which will be 729, and 1 remaining; which 1 remaining is 10 *s.* and the 1 cut off is 1 *s.* so the *Pounds* are 729, and the *Shillings* 11: So that in 700399 *Farthings* there are contained 729 *li.* 11 *s.* 7 *d.* 3 *q.* as by the work appears.

Pence *Shillings* 1 *Shill.*

4)700399(175099 12)175099(14591 1459|1
729 *Pounds*

4
30
28
20
20
039
36
39
36
3 *Farthings*

12
55
48
79
60
199
108
19
12
7 *Pence*

Example 2. *In* 9795 Pints, *how many* Tuns, Hogsheads, Gallons, *and* Pints *are contained?*

First, Eight *Pints* make one *Gallon*; therefore divide 9795 by 8, the Quotient will be 1224 *Gallons*, and 3 *Pints* remaining.

Secondly, Because 63 *Gallons* make one *Hogshead*, divide 1224 by 63, the Quotient will be 19 *Hogsheads*, and 27 *Gallons* remaining.

Thirdly, Because 4 *Hogsheads* make one *Tun*, divide 19 by 4, the Quotient will be 4 *Tuns*, and 3 *Hogsheads* remaining: So that in 9795 *Pints* there are contained 4 *Tuns*, 3 *Hogsheads*, 27 *Gallons*, and 3 *Pints*, as by the Work following does plainly appear.

Pints	*Gallons*	*Hogsh.*	
8) 9795	(1224	63) 1224	(19
8		63	
17		594	
16		567	
19		27 *Gallons*	
16			
35			
32			
3 *Pints*			

4) 19	(4 *Tuns*
16	
3 *Hogsheads*	

Example 3. *Let it be asked how many* Pounds *are in* 109754 Pence?

Because a Pound contains a Shilling 20 times, and a Shilling contains a Penny 12 times, therefore if 109754 be divided first by 12, the Quotient shall be 9146 Shillings, and 2 Pence over; then if 9146 be divided by 20, the Quotient is 457 Pounds, and 6 Shillings remaining; so that 109754 Pence is equal to 457 *li.* 6 *s.* 2 *d.*

Or, if 109754 had been at first divided by 12 times 20, that is, 240, (which is the number of Pence contained in a Pound,) the Quotient had been 457 Pounds, and 67 Pence remaining; which is all one with the former, for 74 Pence is equal to 6 Shillings and 2 Pence. More instances are not needfull herein, because the thing of it self is very clear.

CHAP. VIII.

Of PROGRESSION.

Definition. PROGRESSION is also of two sorts: the first is of certain numbers in *Arithmetical Proportion* from 1; that is, such as differ equally, as 1, 2, 3, 4, 5, 6, where the *common difference* is 1, as is easily seen; or 1, 3, 5, 7, 9, 11, where the *common difference* is 2; or any other, as 1, 8, 15, 22, 29, 36, where the *common difference* is 7: this is called *Arithmetical Progression.*

Secondly, of certain numbers in *Geometrical Proportion* from 1; that is, such as increase by a *common Multiplication*, as 1, 2, 4, 8, 16, 32, where the *common Multiplier* is 2; that is, the first multiplied by 2, produceth the second; and the second multiplied by 2, produceth the third; and so on: or, as 3, 9, 27, 81, 234, where the *common Multiplier* is 3; this is called *Geometrical Progression.*

Both the *common Difference*, (in the first,) and the *common Multiplication*, (in the latter,) shall for shortness hereafter be called the *common excess.*

Now of this first sort, or *Arithmetical Progression*, the principal uses are these Six.

1. *If the* number of places *and* common excess *be given, to find the* last number.
2. *The* number of places *and the* last number *given, to find the* aggregate, *or* total summ *of all the numbers.*
3. *The* last number *and the* total summ *given, to find the* number of places.
4. *The* number of places *and* total summ *being given, to find the* last number.
5. *The* last number *and* number of places *given, to find the* common excess.
6. *The* last number *and* common excess *being given, to find the* number of places.

Examples of all these.

Example 1. The *Number of Places* and the *common excess* given, to find the *last* number, this is the *RULE*:

Multiply the number less by 1 *than the* number of places *by the* common excess, *and to the Product add the* first number, *the Summ is equal to the* last number.

Let

Let the number of places be 6, as 1, 7, 13, 19, 25, 31, and the common difference 6 alſo: Multiply 5, (which is leſs by 1 than the number of places,) by 6, the common exceſs, the Product is 30, to which the firſt number 1 being added, the Summ is 31, equal to the laſt number.

Places			
1	1		5
2	7		6
3	13	6	30
4	19		1
5	25		31
6	31		

Example 2. The *laſt number* and the *number of places* being given, to find the *total ſumm* of all the numbers, this is the *RULE:*

Add the firſt *and* laſt numbers *together, and multiply the ſumm by half the* number of places, *the product is equal to the* aggregate, *or* ſumm *of* all the numbers *added together.*

So if the firſt number be 1, and the laſt number 100, the firſt number added to the laſt gives 101, which multiplied by 50, (which is half the number of places,) produceth 5050, which is equal to all the hundred numbers added together.

And hereby may that vulgar Queſtion be anſwered, which is,

If a man take up 100 *Eggs, placed one yard from another, all in a right line, by one at once, and bring them back, one by one, to a Basket placed at his firſt ſtanding, how many yards doth he go backwards and forwards?*

It is ſhewed before that he goes forward 5050 yards, and he muſt needs come back juſt as much; that is, in all 10100 yards, which is 5 miles and three quarters, wanting 20 yards.

Or ſecondly, Suppoſe the number of places were 6, as theſe, 1, 9, 17, 25, 33, 41, whereof the common exceſs is 8; the firſt and laſt numbers added together give 42, which multiplied by 3, (half the number of places,) the Product is 126, which is the ſumm of them all.

Places			
1	1		1
2	9		41
3	17	8	42
4	25		3
5	33		126
6	41		
Total	126		

Example 3. The *laſt number* and the *total ſumm* being given, to find the *number of places*, this is the *RULE:*

Add the firſt *and* laſt *numbers together, and by the* ſumm *of them divide the total, the Quotient well be equal to half the* number of places.

So, the firſt and the laſt numbers being 1 and 41, and the total ſumm 126, if you add the firſt and laſt numbers together, their ſumm is 42; by which divide 126 the total ſumm, and the Quotient will be 3, the double whereof is 6, the number of places.

42)126(3
126
000

Example 4. The *total ſumm* and the *number of places* given, to find the *laſt number*, this is the *RULE:*

Divide the total *by half the* number of places, *and from the Quotient ſubtract* 1, *the reſt is the* laſt number.

As, let the numbers be 1, 3, 5, 7, 9, 11, 13, 15, or any other (in *Arithmetical Proportion*) whatſoever: the ſumm of theſe is 64, and the number of places 8, the half of it 4; now if 64 be divided by 4, the Quotient is 16; from which if 1 be taken, there remains 15 for the laſt number.

Places			
1	1		4)64(16
2	3		
3	5		
4	7	2	4
5	9		24
6	11		24
7	13		00
8	15		
Summ	64		

Example 5. The *laſt number* and the *number of places* given, to find the *common exceſs*, this is the *RULE:*

From the laſt number *take* 1, *and the remainder ſhall be the Dividend; then from the* number of places *alſo take* 1, *and make this latter remainder the Diviſor, and the Quotient of this Diviſion ſhall be the* common exceſs.

Let the numbers be 1, 4, 7, 10, 13, 16; from 16 take 1, and there remains 15, for the Dividend; then from 6 (which is the number of places) take 1 alſo, and there remains 5, for the Diviſor: now when 15 is divided by 5, the Quotient is 3; and 3 is alſo the common exceſs, or difference, between 1 and 4, or 4 and 7, *&c.*

Places			
1	1		5)15(3
2	4		
3	7	3	15
4	10		00
5	13		
6	16		

Example 6. Let the *laſt number* and the *common exceſs* be given, to find the *number of places*, this is the *RULE:*

From the laſt number *take* 1, *and divide the remainder by the* common exceſs; *then to the Quotient add* 1, *the ſumm is the* number of places.

As, let the numbers be 1, 5, 9, 13, 17, 21, 25, 29; from 29 take 1, remains 28, which divided by 4, (which is the exceſs,) the Quotient is 7, to which add 1, the ſumm is 8; which is the number of places, as in the Margin.

Places			
1	1		4)28(7
2	5		
3	9		28
4	13	4	00
5	17		
6	21		
7	25		
8	29		

§ II. *Of Geometrical Progreſſion.*

I ſhall not be ſo large in this as in the former, becauſe theſe things are of little uſe to the Arithmetician, except where a number is to be many times doubled, tripled, or the like; which cannot be ſo eaſily abridged here as in the other, becauſe there the laſt number ariſing of many *Additions* of the exceſs to 1, was eaſily found by one Multiplication; but here the laſt number being made by many *Multiplications* of the exceſs, is therefore many times harder than the other.

The varieties here ſhall be but two.

1. *The* common exceſs *and* number of places *being given, to find the* laſt number.
2. *The* exceſs *and* laſt number *being given, to find the* total ſumm.

For the Firſt of theſe:

Places	
1	1
2	2
3	4
4	8
5	16
6	32
7	64
8	128
9	256
10	512

Example 1. Let the numbers be 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, the exceſs 2, the places 10; find out the fifth number, (which is eaſily done, for any one may reckon ſo far by heart,) that is here 16; and multiply 16 by 16, it produceth 256, which is the ninth number: laſtly, multiply 256 by the exceſs 2, thence ariſeth 512, the number deſired.

So, if the places had been more, as 72; having found the ninth number 256, multiply it by 256, thence comes 65536 for the ſeventeenth number; which multiplied by the exceſs 2, gives 131072 for the eighteenth place, which multiplied by 131072, gives 17179869184, for the 35th place; and that multiplied again by the exceſs 2, gives 34359738368, for the 36th place; that multiplied by 34359738368, the product will be 1180591620717411327424, for the 71ſt place; which, laſtly, multiplied by the exceſs, gives 2361183241434822654848, for the 72d place; which is the laſt number in the Progreſſion required to be found.

Perhaps this may ſeem ſomewhat tedious, but where things cannot be performed without labour, the Reader muſt content himſelf with ſuch Rules as make it leſs; for it is certain, that this way is much ſhorter than to have multiplied ſtill by the exceſs 71 times, which elſe he muſt have done.

All this notwithſtanding, he is not bound to uſe the ſame numbers, much leſs in other Queſtions, where the number of places is not the ſame; but whereas I begin at 9, he may begin at 8, 10, or 12, or where he pleaſes, ſo as he remembers ſtill where he is; for this is general:

If the number belonging to any place whatſoever be multiplied by it ſelf, the Product ſhall be the number belonging to twice ſo many places wanting one place.

For the Second of theſe:

Example 2. By the *number of places* to find the *total ſumm* of all the numbers: For the performance hereof this is the *RULE:*

From the laſt number take the firſt, and divide the remainder by the exceſs wanting 1; *then multiply the Quotient by the exceſs, and to the Product add the firſt number; the ſumm of them is equal to the ſumm of all the numbers.*

So, if from the laſt number, or 72d place, be taken 1, the remainder is 2361183241——4348226547847, which ſhould be divided by 1, (that is the exceſs wanting 1, for the exceſs is but 2,) but becauſe 1 neither multiplies nor divides, that labour is ſaved; now, multiply this remain by the exceſs, the Product is 4722366482869645309696, to which adding the firſt number 1, by making the figure 6 next the right hand to be 7, you have the total ſumm of all the 72 numbers.

§ III. *A Queſtion reſolved by* Geometrical Progreſſion.

A Londoner, *ſojourning in a Countrey Market-Town, in Winter, made himſelf a new Friez Suit and Coat, on which were ſet ſix dozen of Buttons, of Silk and Silver; a Baker being in his Company, liked it ſo well, he would buy it of him; the Citizen conſented to let him have it, paying for the firſt Button a ſingle Barly-corn, for the ſecond* 2, *for the third* 4, *and ſo on, doubling to the laſt.*

The Bargain was liked on both parts for the preſent, but ſhortly after revoked; for it could not be performed, and no man can be holden to an impoſſibility.

But

But why this could not be performed may be judged, First, by inquiring into the worth of so much Barly in Money; and Secondly, the weight of it, and how it should be removed.

1. For the first, allowing 10000 corns to a pint, (which is more than enough,) then 5120000 corns make a quarter; and yet (for shortening the Division) we will allow 10000000 corns to a quarter; by which dividing the whole number of corns, (which is done by cutting off the first seven figures towards the right hand,) the Quotient will be 472236648286964, and so many whole quarters there are, omitting the remainder, as in this case inconsiderable.

Now allowing Barly were to be sold at 15 *d.* the Bushel, (which is cheap,) it is so many Angels; and therefore dividing it by 2, it is 2361183241434 82 pounds sterling; which is in words, *Two hundred thirty six millions of millions, one hundred and eighteen thousand, three hundred twenty four millions, one hundred forty three thousand, four hundred eighty two pounds*, which I take to be too much for any Tradesman to get or keep.

And reckoning Land for ever at twenty years purchase, if this summ of pounds be divided by 20, the Quotient is the yearly rent of 118059162071 74 pounds: and this divided again by 365, (the number of days in a year) the Quotient is 3234497591 8, that is, above *Thirty two thousand of millions* a day for ever. So great a vanity may be concluded on for want of a little premeditation.

2. Now secondly for the weight of it: If we put 8 bushels to weigh two hundred pounds weight, (for sure it doth weigh more,) then the whole number of quarters multiplied by 2 gives the weight of all to be 94447329657392 8 hundred weight, and if this be divided by 20, (which is but cutting off one figure towards the right hand, and dividing the rest by 2,) or, which is all one, cut off one figure from the number of quarters, the Quotient 4722366482869 6 is so many Tuns; and therefore it will require 4722366482 8 Ships of 1000 Tuns a-piece to carry it; and consequently, if every Nation in the World had above 10000 such Ships, yet there must be above *four millions* of such Nations; which I suppose are not to be found in this World.

And here I will leave this, having used this long example, (which though it require more labour, as all great examples do, yet the same skill will do it, as if the places had been fewer,) that the Reader being throughly exercised thereby, may the easier leap over others which are shorter.

CHAP. IX.

Of The GOLDEN RULE, or Rule of Three Direct.

Definition. THis is the most *usefull* and most *easie Rule* in *Arithmetick*, and deserves a *Golden Name*. It is when there are three numbers given, or known, to find a fourth in proportion with them.

But Four numbers are in proportion, and called *Proportional*, when as the first is to the third, so is the second to the fourth.

As, if there were given 3, 4, and 6, to find a fourth, which may be to 4 as 6 to 3, that is, double; and that fourth number is 8: and this is called *Proportion Direct*; and the Rule whereby it is done, *The Direct Rule.*

There is also another proportion called *Reciprocal*; which is, when as the first is to the third, so is the fourth to the second; as, 3, 4, 6, 2; this is called *The Reverse Rule.*

In *Direct Proportion* the Product of the two middle numbers multiplied together, is ever equal to the Product of the first and last multiplied together; which serves not only for a *Proof*, but a ground of the *Rule*, which *Rule* shall here follow, the *Reverse Rule* being deferred till we have done with this; and for the working of it, this is the *RULE:*

Multiply the Second Term, *(or* Number,*) by the Third, and divide the* Product *by the First, the* Quotient *shall be the Fourth Number desired.*

Example. Let the three numbers given be 2, 6, 3; multiply 6 by 3, the Product is 18; then divide 18 by 2, the Quotient is 9; which is the Fourth number in proportion with 2, 6, and 3.

For

For as 2 to 3, ſo 3 times 2, (which is 6,) is to 3 times 3, (which is 9.)

And ſo the Product 18 divided by 2, and the Quotient 9, cauſeth that the Product of 2 into 9 ſhall be alſo 18; and conſequently if 2 be the firſt of the four proportional numbers, and 6 and 3 the two middlemoſt, then 9 is the laſt.

Otherwiſe: *Divide the Second by the Frſt, and multiply the Third by the Quotient, the Product ſhall be the Fourth.*

So if one divide 6 by 2, the Quotient is 3; by which multiply 3, the Product is 9, for the fourth number, as before.

Other ways this Rule might be expreſſed, but where the firſt way is ſo ſhort and clear, there many other ways would rather trouble than help the perſon that ſhould uſe them.

In the firſt way, (which here we mean to uſe, and no other,) if the *firſt* number be 1, then the product of the *ſecond* and *third* gives the *fourth*, without any *Diviſion*; or, if the *ſecond* or *third* number be 1, then there needs no *Multiplication*, but dividing the greater of them by the firſt, the Quotient in whole numbers (for yet we ſpeak of them) is the *fourth* number, which was ſought.

Note 1. *To know when to uſe the* Direct Rule, *or the* Reverſe Rule, *Conſider, if* more *require* more, *or if* leſs *require ſtill* leſs, *then uſe the* Direct Rule: *But if more require* leſs, *or* leſs more, *then uſe the* Reverſe Rule: *This will be eaſily underſtood when we come to Examples.*

Note 2. *To know how to place the three numbers when they are confuſedly given: Remember, that two of them are always of one Denomination; as both* Pounds, *or both* Sheep, *or both* Yards, *or* Acres; *and the other number hath another Denomination; now know, that this ſingle number is ever the ſecond number in order; and one of the other two, namely, that which hath ſome relation to this* ſecond, *is the* firſt, *and the other is the* third *number, whoſe relation is ſought for in the fourth; whence it's plain, that the* ſecond *and* fourth *are alſo of the ſame Denomination.*

And having premiſed theſe things, let us now exemplifie the Rule in ſome Queſtions.

Queſtion 1. *If three yards of Cloth coſt* 4 li. *what ſhall* 21 *yards coſt?*

Y. li. Y.
If 3 coſt 4 what 21?
4
84

3)84(28
6
24
24

Set the numbers in order, as in the Example. If 3 yards coſt 4 *li.* what 21 yards? Here you ſee that the firſt number, and the third number, are both of one Denomination, *viz.* both *yards*; and the ſecond number is of another Denomination, namely, *pounds*; wherefore the *fourth* number, which is ſought for, muſt be alſo *pounds*; therefore multiplying (according to the Rule before given) the *ſecond* number by the *third*, and dividing the Product by the *firſt*, the Quotient ſhall anſwer the queſtion.

So, 21 multiplied by 4, (which is the third number multiplied by the ſecond) produceth 84; which divided by 3 the firſt number, the Quotient is 28 *li.* and ſo much ſhall 21 yards coſt: For 28 is to 4, as 21 to 3, ſeeing each contains either 7 times.

Queſtion 2. *If* 4 *men eat* 2 *Pecks of Corn in a Week, how many Pecks ſhall ſerve* 100 *men?*

Place your numbers as here you ſee; then multiply 100 by 2, (that is, the third number by the ſecond,) and the Product is 200; which divided by 4, the Quotient is 50, for the number of Pecks required.

Men Pecks Men
If 4 eat 2 what 100?
2
200

4)200(50 *Pecks.*
20
00

Queſtion 3. *If* 20 *Sheep coſt* 13 li. 13 s. 4 d. *what is that for every Sheep?*

Turn the Shilings and Pounds into Pence, thus,

s.	*li.*
13	240
12	13
26	720
13	24
156	3120

Multiply

Multiply 12 *s.* by 13, the Product is ——————————— 156 *Shill.*
And 13 *li.* by 240, (because 240 *d.* make one Pound,) the Product is—3120
To which add——————————————————— 4
It makes in all ——————————————————3280

Then the Question will be, *If* 20 *Sheep cost* 3280 *Pence, what shall* 1 *Sheep cost?*

Sheep *Pence*
If 20 cost 3280 what 1?

```
20)3280(164          12)164(13 Shillings
   20                   12
  128                   44
  120                   36
   80                    8 Pence
   80
   00
```

By the Rule before delivered, I should multiply the second number by the third, but in this Example, the third number being 1, it doth not multiply; I therefore divide 3280, the second number, by 20, the first number, and the Quotient, 164, is the Price of one Sheep in Pence; which divided by 12, the Quotient is 13 *s.* and 8 *d.* remaining; the price therefore of every Sheep is 13 *s.* 8 *d.*

Question 4. *How many* 10 *Inch Tiles will pave a Floor that contains* 16 *square yards?*

First, Remember there are 36 inches in one yard in length; which multiplied into 36 gives 1296, for the square inches contained in one square yard; multiply 1296 therefore by 16, thence comes 20736, the summ of all the 16 yards in inches.

Secondly, Seeing every Tile is 10 inches in length, and 10 in breadth, multiply 10 by 10, it produceth 100 for the square inches in one Tile. See the manner of the work.

```
  36        10
  36        10
 216       100
 108
1296
  16
7776
1296
20736
```

Then by the Golden Rule; *If* 100 *Inches require* 1 *Tile, what shall* 20736 *Inches require?*

Inches *Tile* *Inches*
If 100 require 1 what 20736?

207|36

Here because 1 doth neither multiply nor divide, (as hath been several times intimated,) therefore divide the third number 20736, by the first 100, which is done by cutting off (with a dash of the Pen, as above,) the two last figures of the Dividend, because the two last figures of the Divisor are Ciphers; and the other figure being 1, there can be no farther division; so the figures on the left side of the dash are the Quotient, which is 207; and those on the right side, *viz.* 36, the Remainder.

So it appears, that 207 is too little, and 208 too much to do the Work, the just number being $207\frac{36}{100}$, we shall not trouble the Reader with this till he know something of Fractions.

Question 5. *If* 100 li. *give* 6 li. *interest for one year, how much shall* 750 li. *give?*

Multiply 750 by 6, the Product is 4500; which divided by 100, (that is, the two last Ciphers cut off, for the Remainder,) the Quotient (that is, the figures on the right side of the dash,) is 45 *li.* for the thing required, as appears by the Work following:

li. li. li.
If 100 give 6 what 750?
6
4500

Question 6. *If* 750 li. *gives* 45 li. *interest for a year, what shall* 100 li. *give?*

Multiply 45 by 100, the Product is 4500, which divided by 750, the Quotient is 6 *li.* for the interest of 100 *li.* for a year.

li. li. li.
If 750 give 45 what 100?
100
4500

li.
750)4500(6
4500
0000

Many other Questions might be added, but the Rule is so plain, that it needs them not; and so general, that he who can resolve one, may as well resolve any other: and for that reason, and because in all the Rules which follow, this Rule will be constantly made use of, I will say no more of it here.

CHAP. X.

Of The GOLDEN RULE Reverse.

Definition. THe *Golden Rule Reverse* is just contrary to the *Golden Rule Direct*, the Proportion being reciprocal; for in the *Direct Rule*, as the first number is to the third, so is the second to the fourth; but in the *Reverse Rule*, as the first number is to the third, so is the fourth to the second; as will easily appear by some few Examples, wrought by the following *RULE:*

Multiply the First *term by the* Second, *and divide the* Product *by the* Third, *the* Quotient *is the number desired.*

Question 1. *If* 12 *Workmen doe any piece of work in* 8 *months, how many Workmen shall doe the same in* 2 *months?*

Here 12 is not the first number, though it be first named; but the three numbers placed in order stand thus, 8, 12, 2; for the middle term must always be of the same denomination with that which is required.

Now, Multiply 12 by 8, the Product is 96; which divided by 2, the Quotient is 48, which answers the Question; as in this Example:

Months *Men* *Months*
8 12 2
8
2)96(48
8
16
16

For if 8 months require 12 men, then (a fourth part of 8) 2 months shall require four times 12, that is 48 men.

For here *less* requires *more*; that is, *less time more hands*; and therefore it is wrought by the *Reverse Rule.*

Question 2. *How many Ells of Tapestry will serve to hang a Room* 3 *yards high,* 6 *yards long, and* 5 *yards broad; not regarding Doors, Windows, or Chimney, but as if there were no such?*

First, Multiply 6 by 3, the Product is 18, which doubled, (because there are two sides called *lengths*,) is 36 yards, for all the length.

Secondly, (for the same reason,) multiply 3 by twice 5, that is by 10, the Product is 30 yards, for all the breadth; which added to 36, gives 66 yards, equal to all the length and breadth in yards.

But

But now, becauſe an Ell, that is, a *Flemiſh* Ell, (for ſuch meaſure are Hangings ſold by,) is equal to 3 quarters of a yard, that is, their Ell is to our Yard as 3 to 4; ſay therefore, if 4 give 66, what 3? multiply 66 by 4, it produceth 264; then divide 264 by 3, the Quotient is 88: Again, multiply 88 by 4, and divide the Product (which is 352) by 3, the Quotient is 117, and 1 remaining, to which the Diviſor 3 being applied, the number juſtly anſwering the Queſtion is 117 Ells, and one third part of an Ell, as by the work appears.

If 4 give 66 what 3?

```
      4
3)264(88
 ..
---
24
24
---
00
```

```
      88
      4
3)352(117 1/3 Ells
 ...
---
3
05
3
22
21
1 Remains
```

Note 1. Becauſe here we had to deal with things which had equal length and breadth, that is, ſquare Yards, and ſquare Ells, therefore one Multiplication and Diviſion was not ſufficient to proportion this; but if inſtead of working by 4 and 3, it had been done by their ſquares, which are 16 and 9, it might have been performed at once, thus: Multiply 66 by 16, the Product is 1056, which divide by 9, the Quotient is 117 $\frac{1}{3}$, or $\frac{1}{3}$, as before; but I began not with this way, for I ſuppoſed my Reader ignorant of ſquares.

```
      66
      16
     396
     66
9)1056(117 1/3
   9
   15
    9
    66
    63
     3 Rem:
```

Note 2. It might alſo have been done by reducing all the terms into quarters of a Yard at the firſt, and after the number is found, reducing them again to Ells; but becauſe it is more proper to work thus till Fractions have been taught, I leave that, and proceed to another queſtion.

Queſtion 3. *If* 1 *Cloſe would graze* 21 *Horſes for* 6 *Weeks, then (ſuppoſing no waſte to be made) how many Horſes would it feed for* 7 *Weeks?*

Multiply 21 by 6, it produceth 126, which divided by 7, the Quotient is 18; at that rate therefore it would keep 18 Horſes for 7 Weeks.

```
W.      H.      W.
6 ------ 21 ------ 7
          6
      7)126(18 Horſes
       ..
      ---
       7
       56
      ---
       56
       00
```

Queſtion 4. *If* 1 *Cloſe will feed* 18 *Horſes for* 7 *Weeks, how long ſhall it feed* 63 *Horſes?*

Multiply (according to the Rule) 18 by 7, the Product is 126, which divide by 63, the Quotient is 2; therefore 2 Weeks it ſhall keep them, as appears by the Work.

```
H.      W.      H.
18 ------ 7 ------ 63
          7
      63(126(2 Weeks
       ..
      ---
       126
         0
```

The like way ſerves for Hay, Oats, or any other proviſion for Man or Beaſt; which may be of uſe in Gariſon, and ſuch like caſes, where ſcarcity may be feared, to proportion either the Mouths to the Meat, or the Meat to the Mouths.

Before I leave this *Rule*, (becauſe it comes not in ſo much uſe and practice as the *Direct Rule* doth, and therefore may be more apt to be forgotten,) I will, to exerciſe the Reader therein, propoſe the following Queſtions, giving the Anſwers of them, and leave the practice to the Reader, to find out of himſelf, the better to fix the Rule in his Memory.

Queſtion 1. *If* 12 *Men would raiſe a Frame in* 10 *Days, in how many Days would* 8 *Men raiſe the ſame?*

Here, becauſe the fewer Men would require the longer time, though the numbers be 12, 10, 8, yet you ſhall (by obſerving what hath been already delivered in this Rule,) find the fourth proportional (which is the number anſwering the Queſtion,) to be 15, and ſo many Men will doe the work in 8 days.

Queſtion 2. *If* 60 *Yards of Hangings of three quarters broad would hang a Room, how many Yards of half a Yard in breadth would ſerve to hang the ſame Room?*

Anſwer, Ninety Yards.

Queſtion 3. *If a Board being* 12 *Inches in breadth, do require* 12 *Inches in length to make a Foot ſquare; What number of Inches in length will make a Foot ſquare, when the breadth of the Board is* 16 *Inches?*

Anſwer, Nine Inches.

Queſtion 4. *If the baſe or end of any* Solid *(as a piece of* Timber *or* Stone,) *being* 144 *Inches, do require* 12 *inches in length of that piece to make a ſolid Foot; What number of Inches in length will make a ſolid Foot, when the ſquare at the end is* 216 *Inches?*

Anſwer, Eight Inches.

I will ſay no more of this Rule, neither will I treat of the *Double Rule of Three*, as a Rule by it ſelf, but come to the *Rule of Five Numbers*, which is an Abridgment of the other.

CHAP. XI.

Of The GOLDEN RULE Compound of Five Numbers.

Definition. THis *Rule* performeth at one Operation that which by the *Single Rule of Three* would require two; and ſo it is called the *Compound Golden Rule of Five Numbers.* To this *Rule* there belongeth always *Five Numbers*, the firſt Three contain a *Suppoſition*, the Two laſt a *Queſtion*, to which the *Sixth* Number found muſt be the *Anſwer*: Now any Queſtion in this Rule being propounded, to work the ſame this is the *RULE:*

Firſt, *Place the Five* Terms *or* Numbers *ſo, that the* Firſt *and the* Fourth *may be of one* Denomination, *the* Second *and the* Fifth *of another; and the* Anſwer *in the* Sixth *Number will be of the ſame* Denomination *with the* Third.

Secondly, *Multiply the* Firſt *and* Second *Numbers into each other, for a* Diviſor; *and the* Three laſt, *for a* Dividend; *ſo ſhall the* Quotient *anſwer the* Queſtion *demanded.*

Queſtion 1. *If a hundred pound weight (that is,* 112 *pound) carried* 126 *miles coſt* 14 *s. how much ſhall three quarters of a hundred (that is,* 84 *pound) coſt, being carried* 40 *miles?*

Firſt, Place your numbers according to the tenour of the Queſtion, and as you ſee done in the work following.

Your numbers being placed in order, reduce the 14 *s.* into pence, and it is 168 *d.* then multiply 168 by 84, the Product is 14112; which multiplied by 40, it produceth 564480, for the Dividend.

Then multiply 112 by 120, it produceth 13440 for the Diviſor.

Divide 564480 by 13440, the Quotient will be 42 pence, which is 3 *s.* 6 *d.* and anſwers the Queſtion.

If

lb. w. *Miles* *s.* *lb. w.* *Miles.*
If 112 —— 120 —— 14 —— 84 —— 40

120
2240
112
13440

12
28
14
168 *Pence*
84 *lb. weight*
627
1344
14112
40 *Miles*
13440)564480(42 *Pence*
··
53760
26880
26880

Here you ſee the *firſt* number and *fourth*, alſo the *ſecond* and *fifth*, and alſo the *third* and *ſixth* are of like Denomination and Nature.

Queſtion 2. *If* 100 li. *for* 6 *Months yield* 3 li. *intereſt, what ſhall* 625 li. *yield for* 36 *Months?*

Place them thus, 100, 6, 3, 6, 5, 36.

Multiply the three laſt, as before is ſhewed, the latter Product is 67500, for the *Dividend*; and the two firſt multiplied make 600, the *Diviſor*; then divide 67500 by 600, (or 675 by 6, which is all one,) the Quotient will be 112 whole pounds, and 300 (or 3) remaining; which, becauſe it is half the Diviſor, ſignifies the half of a pound, that is 10 *s.* So the anſwer to the queſtion is 112 *li.* 10 *s.*

li. *M.* *li.* *li.* *M.*
If 100 in 6 gain 3 what 625 in 36?

6
600

3
1875
36
11250
5625
6)67500(112 *li.* 10 *s.*
···00
6
07
6
15
12
3 *Rem.*

Which might have been given in one Denomination, namely, 2250 ſhillings, if before the work the Pounds had been turned into Shillings, by multiplying them by 20 as hath been ſhewed before.

But ſince moſt Queſtions, except ſuch as are ſtudied for the purpoſe, are apt to end in ſome Fraction, I ſhall next treat of *Fractions*.

Only firſt, having ſpoken of the *Double Rule of Three*, this may let you know, that all Queſtions which are wrought at once by the *Compound Rule of Five*, may be done at twice by the ſingle *Rule of Three*; and the doing of them ſo by two Operations, is called *The Double Rule:* But this will be better underſtood when *Fractions* are known, which we next intend to treat of.

CHAP. XII.
Of FRACTIONS.

§ I. *Definition.* THE word *Fraction* signifies a *breaking*, or *breach* of any entire thing into parts; and when a number is so broken, the parts (which must needs be every one less than the whole, and the whole is accounted but *One*, or *Unity*,) being less than Unity, are called Fractions (that is, Fragments or Pieces) of Unity. Now the Unite, or entire number, which is to be broken, may be any thing; as one *Pound*, in respect of which *Shillings*, and *Pence*, and *Farthings*, are *Fractions*; or, one *Shilling*, in respect of which *Pence* and *Parthings* are *Fractions*; or, one *Penny*, in respect of which *Farthings* are *Fractions*; and the like of *Weights* and *Measures*, or any other thing to be broken into parts.

In *Fractions*, we shall treat first of *Numeration*, then of *Multiplication* and *Division*, then of *Reduction*, and lastly, of *Addition* and *Subtraction*.

The reason of this order will soon be seen; for *Multiplication* and *Division* are here much easier than *Addition* and *Subtraction*, and therefore ought to be learned before them.

§ II. *NUMERATION of* Fractions.

Numeration is nothing else but the way of writing Fractions; and that this may be done, we must consider that any *Unity*, or *Number* representing an *Unite*, may be broken into two equal parts, and then each of those parts is called *one second*, or *half*; or it may be parted into three equal parts, and then each part is called *one third*, and two of them are called *two thirds*; and the like may be understood if it were parted into 4, 5, 6, 7, 8, 9, 20, 50, or 100, or how many parts soever.

Now to write these doe thus:

Write	One half	thus	$\frac{1}{2}$
	One third		$\frac{1}{3}$
	One fourth		$\frac{1}{4}$
	One fifth		$\frac{1}{5}$
	One sixth		$\frac{1}{6}$
	One seventh		$\frac{1}{7}$
	One eighth		$\frac{1}{8}$
	One ninth		$\frac{1}{9}$
	One tenth		$\frac{1}{10}$

In every one of these 10 *Fractions* the number below the line is called the *Denominator*, and shews into how many parts the *Unite* is broken; and the number above the line shews how many of those parts are taken, or contained in the *Fraction*, and is therefore called the *Numerator*: So in the Fraction $\frac{3}{5}$, the *Denominator* 5 shews the *Unite* to be broken into 5 parts; and the *Numerator* 3 signifies 3 of such parts to be contained in the *Fraction*, which Fraction therefore is called *three fifths*.

And here it is plain, that, as the *Numerator* is in proportion to the *Denominator*, so is the Fraction to 1, or Unity; for $\frac{3}{3}$, or $\frac{4}{4}$, or any the like, is equal to 1.

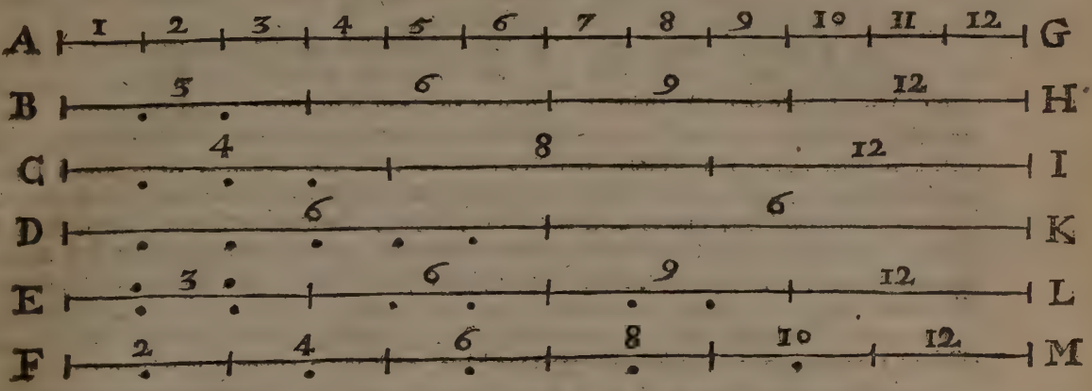
And therefore all Fractions are Quotients of lesser numbers divided by greater, as $\frac{4}{7}$ signifies 4 to be divided by 7; and as the Dividend 4 is to the Divisor 7, so is the Quotient $\frac{4}{7}$ to Uunity; and therefore this line of separation which is drawn between the *Dividend* and *Divisor* doth properly signifie *Division*.

Hitherto we have spoken only of such Fractions as are less than 1, and those are called *Proper Fractions*; but there are also $2\frac{1}{2}$, $3\frac{3}{4}$, $5\frac{1}{7}$, $6\frac{3}{5}$, and the like mixed Numbers, which so written signifie *two and an half*, *three and three quarters*, *five and a seventh*, *six and three fifths*. These, by multiplying the whole numbers by the Denominator, and to the Product adding the Numerators respectively, are turned to $\frac{5}{2}$, $\frac{15}{4}$, $\frac{36}{7}$, $\frac{33}{5}$ which are called *Improper Fractions*, because every one of them contains more than Unity.

These nevertheless may be *multiplied*, *divided*, *added*, or *subtracted*, in the same way as *Proper Fractions* are. And this shall serve for *Numeration of Fractions*.

§ III. *Fractions*

§ III. *Fractions Demonstrated.*



I. Of *Simple* (or *Proper*) Fractions.

A *Fraction* (as before it is defined) being a *broken number*, or part of any *Integer*, the *Denominator* shews into how many *equal parts* the *Integer* is to be divided; the *Numerator* denotes the number of those equal parts of the *Unite* (or *Integer*) to be *taken*.

For the farther illustration hereof, let the six Lines A G, B H, C I, D K, E L, and F M, all of one length, each contain a *Statute Foot*: Now if it were required to know the *Value* of this *Fraction* $\frac{1}{12}$, the *Foot* being the *Integer*; the Answer will be 1 *Inch*: for (according to the *Denominator*) the *Foot* is to be divided into 12 *parts*, and the *Numerator* being an *Unite*, the *Fraction* is one of those *equal parts* of the Line A G, containing 1 *Inch*: And $\frac{1}{4}$ part of a *Foot* is 3 *Inches*; for the *Foot* (according to this *Fraction*) is to be divided into *four* equal *parts*, as is the Line B H, and one of those divisions (which is 3 *Inches*) is to be taken: Also $\frac{1}{3}$ of a *Foot* is 4 *Inches*; 12 *Inches* being divided into *three* parts, as the Line C I, then one of those parts will be 4 *Inches*: And $\frac{1}{2}$ will be 6 *Inches*, as the Line D K: And $\frac{3}{4}$ will be 9 *Inches*, as by the Pricks under the Line E L doth evidently appear: And so in the other Line F M $\frac{5}{6}$ will be 10 *Inches*, as the Pricks under that Line do clearly demonstrate: And so of all other *Simple* (or *Proper*) *Fractions*, whose *Numerators* are always *less* than their *Denominators*.

II. Of *Fractions of Fractions.*

Such *Fractions* as we are now treating of are usually subscibed in this manner, $\frac{1}{3}$ of $\frac{1}{4}$: By the *Denominator* of the *Second Fraction* the *Integer* is to be divided into 4 *equal parts*; and according to the *Numerator*, three of them is to be taken; and by the *First Fraction* one third part of that is to be taken: As, in the former *Example*, $\frac{3}{4}$ of the Line E L 12, was 9 *Inches*, and that sub-divided into 3 parts, one of them will be 3 *Inches*, as by the divisions above the Line with 2 pricks does appear.

Again, $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{5}{6}$, are three *Compounded Fractions* given, whose value is required in the parts of a *Foot*.

The Line F M is divided by 2 *Inches* into 6 parts, 5 of those divisions is 10 *Inches*, $\frac{5}{6}$ of that is 2 *Inches*, and the half of that is 1 *Inch*, as doth evidently appear.

§ IV. *MULTIPLICATION of* Fractions.

For the working of *Multiplication* in *Fractions*, or Broken Numbers, this is the *RULE:*

Multiply all the Numerators *together, the last Product shall be the* Numerator *of the Product required: Likewise, Multiply all the* Denominators *together, the last Product shall be the* Denominator *of the Product sought.*

Example 1. If $\frac{3}{5}$ be to be multiplied by $\frac{4}{9}$, multiply the *Numerator* 3 by the *Numerator* 4, the Product is 12, for the *Numerator* of the new Product; also, multiplying the *Denominator* 5 by the *Denominator* 9, they produce 45, for the *Denominator* of the desired Product; so that Product which was required, is $\frac{12}{45}$.

$$\frac{3}{5} \times \frac{4}{9} = \frac{12}{45}$$

Example 2. If $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{9}$, and $\frac{2}{11}$, were to be multiplied all together; begin with the *Numerators*, saying, once 3 is 3, and 3 times 4 is 12, and 12 times 5 is 60, and 60 times

times 3 is 180, for the *Numerator*: then multiply the *Denominators*; saying, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 9 is 360, and 360 times 11 is 3960, for the new *Denominator*; so that the Product of all these is $\frac{180}{3960}$.

And thus it appears, that proper *Fractions* being less than 1, are still made less by multiplying; as here the Product $\frac{180}{3960}$, which (being reduced, as shall hereafter be shewn,) is $\frac{1}{22}$, is much less than $\frac{1}{11}$, which is the least Multiplier; and the reason hereof is plain; for seeing *Multiplication* is but the taking of a Number a certain number of times, if that number of times be more than 1, then the Number to be taken is increased by being taken more than once; but if the number of times be 1, it is not increased or diminished, but is still the same; lastly, if that number of times be less than 1, as $\frac{1}{2}$, the Number not being taken once, but half of once, produceth a Number less by half, that is, the half of the Number to be taken; and the like reason is of all others.

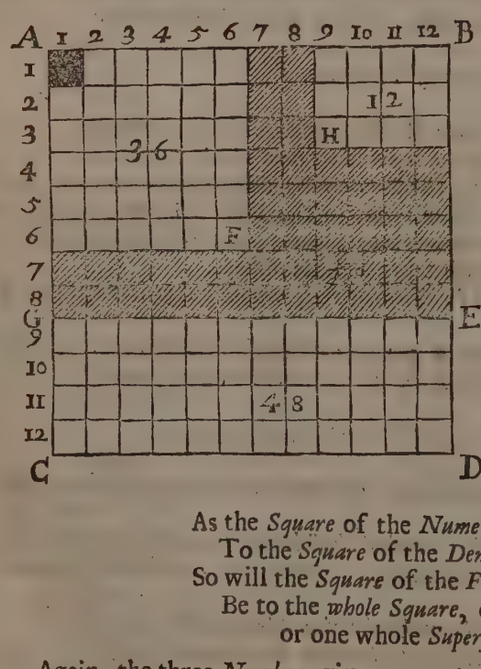
Example 3. Multiply the mixt numbers, $3\frac{1}{2}$, $4\frac{1}{3}$, and $5\frac{3}{4}$: First (as hath been shewn already) turn them to improper Fractions thus, first say 2 times 3 is 6, and 1 is 7; so the first is $\frac{7}{2}$: Secondly, 3 times 4 is 12, and 1 is 13; so the second is $\frac{13}{3}$: Lastly, 4 times 5 is 20, and 3 is 23; so the last is $\frac{23}{4}$. Now the Fractions to be multiplied are $\frac{7}{2}$, $\frac{13}{3}$, and $\frac{23}{4}$: First, For a new *Numerator*, say, 7 times 13 is 91, and 91 times 23 is 2093; then say, 2 times 3 is 6, and 6 times 4 is 24; so the new *Denominator* is 24; and the Product of all these Fractions is $\frac{2093}{24}$; that is, if real division be made, $87\frac{5}{24}$.

§ V. *Multiplication of Fractions demonstrated.*

I. Of *Proper Fractions.*

Multiplication of *Fractions* is contrary to *Multiplication* in *Whole Numbers*; for the *New Fractions* produced are always *less* than either of the terms given to be multiplied; yet in their operation they do make a *Square Figure* with this *Analogical Equality*, or *Proportion.*

As the *Square* made of the *Numerators*
Is to the *Square* of their *Denominators*,
So shall the *Superficial Square* of the *Fraction*
Be in proportion to the *Square* of the *Integer.*



In this *Geometrical Square* ABCD, every one of the Sides is divided into 12 equal parts, or Inches; so if AB 12 be multiplied by AC 12, the Product will be 144, the number of *Square Inches* in a *Superficial Foot*; as by the *Figure* doth plainly appear.

Now suppose two *Fractions* propounded to be multiplied, as $\frac{3}{4}$ of AB by $\frac{2}{3}$ of AC; 2 multiplied by 3 is 6, for the new *Numerator*; and 3 by 4 is 12, for the *Denominator*; so the *new Fraction* is $\frac{6}{12}$, or $\frac{1}{2}$, which is less than either of the terms given; for $\frac{3}{4}$ of AC is 9 Inches, and $\frac{2}{3}$ of AB is 8 Inches, which multiplied together produce 72 Square Inches, which is the half of 144, and so was the *new Fraction*, and their Proportion thus:

As the *Square* of the *Numerators* 6,
To the *Square* of the *Denominators* 12,
So will the *Square* of the *Fraction* 72,
Be to the *whole Square*, or Integer, 144,
or one whole *Superficial Foot.*

Again, the three *Numbers* given were 6, 12, and 72; and as 6 is the half of 12, so is 72 the half of 144; and consequently, 72 multiplied by 12, produceth 864, which divided by 6, the Quotient will be 144: So likewise in all the four Numbers, the Square of the two Means, *viz.* 12 and 72, will be 864, and so the two Extremes 144 and 6 make 864 also.

In

In the same manner $\frac{1}{2}$ multiplied by $\frac{1}{2}$ makes $\frac{1}{4}$; and so half the Line A B 6 Inches multiplied by half the Line A C 6 Inches, the Product will be 36 Inches, which is but $\frac{1}{4}$ part of a *Foot*, which contains 144 *Square Inches*.

And in the same manner and Proportion, $\frac{1}{12}$ of A B multiplied by $\frac{1}{12}$ of A C, will produce $\frac{1}{144}$; and so the *Little Square* made of the Fraction representing one Inch, is but 144 part of a Square Foot, which is the Integer.

II. Of *Fractions of Fractions.*

Admit that $\frac{1}{3}$ of $\frac{3}{4}$ were to be multiplied by $\frac{2}{5}$ of $\frac{5}{6}$: First reduce them, and you shall find $\frac{1}{3}$ of $\frac{3}{4}$ to be $\frac{3}{12}$, or $\frac{1}{4}$; and $\frac{2}{5}$ of $\frac{5}{6}$ to be $\frac{10}{30}$, or $\frac{1}{3}$; which multiplied by $\frac{1}{4}$ makes $\frac{1}{12}$, the true *Product* of the given *Fractions*; which being reduced is thus demonstrated: $\frac{1}{3}$ part of B A is 4 Inches, and $\frac{1}{4}$ part of B D is 3 Inches, which multiplied together, produce 12 *Square Inches*, which is but $\frac{1}{12}$ part of a Foot, or of the whole Square A D.

III. Of *Mixt*, or *Improper Fractions.*

If it be required to multiply $2\frac{1}{2}$ by $1\frac{3}{5}$: First reduce them to *Improper Fractions*, and and they will be $\frac{5}{2}$ and $\frac{8}{5}$, which being multiplied together produce $\frac{40}{10}$, or Four Integers. And here you are to note, that these last Products, if several *Denominations*, are but *Fractions* in respect of the *Square* made of the *whole Integer*, which is the *Denominator* to it, as $\frac{8}{12}$, or $\frac{2}{3}$: So A B $1\frac{3}{4}$ Inches multiplied by $\frac{2}{3}$ of A C a Foot, *viz.* A G 8 Inches, the Product will be 96, or $\frac{96}{144}$, which reduced into the least terms is $\frac{2}{3}$, as in the former *Demonstration* represented by the Parallelogram A B E G.

§ VI. *DIVISION of Fractions.*

Division, or to divide one Fraction by another, is but the cross multiplication of them; that is, the *Numerator* of the one by the *Denominator* of the other; and hereby the proportion of one Fraction to another is seen.

Example 1. Divide $\frac{3}{4}$ by $\frac{6}{8}$: To doe this, set them as in the Margin, and multiply as the Cross leads; saying, 3 times 8 is 24, which set over the Cross, for a new *Numerator*; and 6 times 4 is also 24, which set under the Cross for a new *Denominator*; so the Quotient is $\frac{24}{24}$, that is 1; which shews the Fractions to be equal one to another.

$$\begin{array}{c} 24 \\ \frac{3}{4} \times \frac{6}{8} \\ 24 \end{array}$$

Example 2. Divide $\frac{3}{5}$ by $\frac{4}{9}$: First set them as you see in the Margin; and say, 3 times 9 is 27, for a new *Numerator*; and 5 times 4 is 20, for the new *Denominator*; so the Quotient is $\frac{27}{20}$, and so many times is $\frac{4}{9}$ contained in $\frac{3}{5}$, that is, as 27 is to 20, so is $\frac{1}{5}$ to $\frac{4}{9}$; and so is $\frac{27}{20}$ to 1.

$$\begin{array}{c} 27 \\ \frac{3}{5} \times \frac{4}{9} \\ 20 \end{array}$$

In *Division* it is to be remembred, that the *Numerator* of the *Quotient* ever ariseth of the *Numerator* of the *Dividend*; and the *Denominator* of the *Quotient* comes of the *Denominator* of the *Dividend*, each being cross multiplied, as before: And remember always to set the *Dividend* on the left hand of the Cross.

If a Fraction be to be divided by a whole Number, multiply the *Denominator* by that *Number*, the Product gives the new *Denominator*, and the *Numerator* remains the same. So if $\frac{1}{4}$ be to be divided by 9, say, 9 times 4 is 36; so the Quotient is $\frac{1}{36}$.

$$\begin{array}{c} 1 \\ \frac{1}{4} \times 9 \\ 36 \end{array}$$

Or, If $\frac{1}{4}$ were to be multiplied by 9, the Product (by multiplying the Numerator by 9,) will be $\frac{9}{4}$, that is, $2\frac{1}{4}$.

Example 3. Divide $\frac{320}{8}$ by $\frac{45}{9}$, thus: say, 320 times 9 is 2880, for a *Numerator*; and 8 times 45 is 360, for a *Denominator*; so the Quotient is $\frac{2880}{360}$, or $\frac{8}{1}$.

$$\begin{array}{c} 2880 \\ \frac{320}{8} \times \frac{45}{9} \\ 360 \end{array}$$

For $\frac{320}{8}$ is equal to 40, and $\frac{45}{9}$ is equal to 5, but 40 contains 5 eight times.

And so in the second Example it may be proved, that as 27 to 20, so is $\frac{3}{5}$ to $\frac{4}{9}$: For first, multiply the two middle numbers together, then 20 times $\frac{3}{5}$ is $\frac{60}{5}$, that is 12.

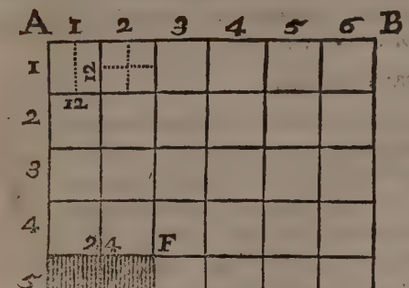
Secondly, multiply the first and last, and then 27 times $\frac{4}{9}$ is $\frac{108}{9}$, that is also 12.

Wherefore by what hath been said in the *Golden Rule*, the four numbers 27, 20, $\frac{3}{5}$, $\frac{4}{9}$, are proportional.

§ VII. *Division of Fractions demonstrated.*

In *Fractions* divided the *Quotient* will be greater than the *Terms* given; and the *Divisor* propounded may be *greater* or *less* than the *Dividend*, as will appear, they being in this Proportion:

As the *Divisor* of any *Fraction*
Is in proportion to the *Dividend*,
So will one *Integer*, or an *Unite*,
Be in proportion to the *Quotient*.



Let ABCD be a *Square* containing on every side Six Foot, and consequently the whole *Square* 36 square Feet.

Now admit that $\frac{1}{6}$ part of AB were to be *divided* by $\frac{1}{6}$ part of AC, which being *divided* one by the other, the new *Numerator* will be 6, and the new *Denominator* 6 also; and the new *Fraction* will be $\frac{6}{6}$, or *one Integer*; the *Square* of one side being 12 Inches, will be contained in 12 Inches once: So $\frac{1}{2}$ is in $\frac{1}{2}$, and $\frac{1}{4}$ in $\frac{1}{4}$ three times, demonstrated by the *first* and *second little Squares*: And $\frac{1}{3}$ of AB divided by $\frac{1}{3}$ of AC, will be likewise $\frac{3}{3}$, or one *Unite*, *viz.* EC by EF: For if the Lines AB and AC were equally divided into three parts, one of those parts must be 2 Foot, or 24 Inches; so 2 will be contained in 2 once, or 24 in 24 also once: and so $\frac{1}{3}$ divided by $\frac{1}{3}$ will be $\frac{3}{3}$, or One: And consequently in all *Fractions*, (as in *Integers*,) when the *Divisor* is equal to the *Dividend*, whether the *Fraction* be in *Numbers*, *Lines*, or *Geometrical Figures*, the *Quotient* is always an *Unite*.

In this Figure AB is a *Fathom*, or 6 Foot, which multiplied by AC 6, produceth the whole Square AD 36 square Feet.

Now in the first *Example* the half of it was to be divided by a fourth part of the same Square, 18 Feet by 9 Feet; the *Quotient* must be 2 Feet, that is, 2 of those *little Squares*; and this is according to the proportion stated, *viz.*

As the *Divisor* 9, is to the *Dividend* 18,
So is 1, unto 2.

Or, in the second *Example*,
As 18 is to 9
So is 1 to $\frac{1}{2}$.

§ VIII. *REDUCTION of Fractions.*

Reduction of *Fractions* is threefold.

1. *To reduce one Fraction (which is not already in the least) to its least Terms or Denomination.*

2. *To reduce many Fractions of divers Denominations, to one Denomination.*

3. *To reduce any Fraction from one Denomination (as near as may be) to any other Denomination desired.*

I. For the first of these, *To reduce a* Fraction *to its least Terms.* Divide both the *Numerator* and the *Denominator* by the greatest *Common Divisor* that you can think of, the two *Quotients* being placed respectively in a Fraction, that Fraction shall be equal to the former Fraction, and in lesser terms.

So (in the third Example of *Division*) to reduce $\frac{1880}{3360}$ to $\frac{8}{15}$, divide 1880 by 360, the *Quotient* is 8; then divide 360 by 360, the *Quotient* is 1; and the new Fraction $\frac{8}{15}$ is equal to the former Fraction $\frac{1880}{3360}$, and in less terms, as you may see: But to find the greatest *Common Divisor*, this is the *RULE*:

Divide the greater term *by the* lesser, [I mean by *terms*, the *Numerator* and *Denominator*,] *and by the Remainder (if any be) divide the* Divisor; *and if any thing still remains, by that divide the last* Divisor; *continuing this course till nothing remain greater than* Unity, *and that* Divisor *which is last of all, is the greatest* Common Measure *of both*

both terms, by which both being divided, and the Quotient *placed like a* Fraction, *that* Fraction *ſhall be equal to the former* Fraction, *and in the* leaſt terms.

Example. Reduce $\frac{148}{16}$ to the leaſt terms; firſt divide 148 by 16, the Quotient is 9, and 4 remains; again, divide 16 by 4, the Quotient is 4, and nothing remains; wherefore taking 4, (the laſt Diviſor,) for the greateſt common Diviſor, by it divide 148, the Quotient is 37; and by it alſo divide 16, the Quotient is 4; theſe two laſt Quotients placed orderly in a Fraction make $\frac{37}{4}$, which is equal to $\frac{148}{16}$, and in the leaſt terms; for no number greater than 1, will divide evenly both 37 and 4.

Other ways there are of leſſening Fractions, as dividing the terms (if they be even numbers) by 2, and the Quotients (if even) again by 2, or elſe by 3, or any other number that will divide them both evenly, that is, leave nothing remaining; but the former Rule being general and eaſie, ſhall ſerve for all.

II. Now ſecondly, *To reduce many* Denominations *to one* common Denominator: For the effecting whereof this is the *RULE:*

Multiply all the Denominators together, and the laſt Product ſhall be the Common Denominator to all the Fractions; then multiply every particular Numerator into all the Denominators except his own, and the laſt Product ſhall be Numerator to that Fraction.

Example. To reduce the Fractions $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{7}{8}, \frac{9}{10}$, into one Denomination; ſay, 2 times 4 is 8, and 8 times 5 is 40, and 40 times 8 is 320, and 320 times 10 is 3200; this laſt Product 3200 ſhall be the common Denominator: then to get Numérators for every one of them, as firſt, for the firſt, ſay, once 4 is 4, and 4 times 5 is 20, and 20 times 8 is 160, and 160 times 10 is 1600, for the firſt Numerator; ſo the firſt Fraction reduced is $\frac{1600}{3200}$: then for the ſecond Numerator, ſay, 3 times 2 is 6, and 6 times 5 is 30, and 30 times 8 is 240, and 240 times 10 is 2400; ſo the ſecond Fraction reduced is $\frac{2400}{3200}$: after the ſame manner may the other three be reduced to $\frac{2560}{3200}$, for the third; $\frac{2800}{3200}$, for the fourth; and $\frac{2880}{3200}$, for the laſt: theſe are ſeverally equal to the other; the firſt to the firſt, *&c.* as may be proved thus: Let the Unity be a pound *Sterling*, then,

	s.	*d.*
the $\frac{1}{2}$ of it is	10	0
and $\frac{3}{4}$ is	15	0
and $\frac{4}{5}$ is	16	0
and $\frac{7}{8}$ is	17	6
and $\frac{9}{10}$ is	18	0
In all	76	6

That is 3 whole Unites, and 16 *s.* 6 *d.* over: turn 16 *s.* 6 *d.* all to ſix-pences, it is 33, and becauſe 6 *d.* is the fortieth part of a Pound, therefore all the Fractions are equal to $3\frac{33}{40}$.

Now the new Fractions (which being of one Denomination) may be added like whole Numbers thus:

1600
2400
2560
2800
2880
12240

Which divided by the Denominator 3200, the Quotient is $3\frac{2640}{3200}$; now $\frac{2640}{3200}$ reduced to the leaſt terms, as hath been ſhewed how it may, will be $\frac{33}{40}$, ſo the ſumm of theſe alſo is $3\frac{33}{40}$; which is equal to the ſumm of the Fractions given to be reduced, and therefore they are equal in ſumm, and might be thus proved equal ſeverally; that is, the firſt propounded to the firſt reduced; divide the Numerator 1600 by the Numerator 1, the Quotient is 1600; alſo divide the Denominator 3200, by the Denominator 2, the Quotient is alſo 1600; and ſo may any of the reſt be proved equal by the equality of Quotients: But I leave it as plain enough already.

III. Thirdly, *Any Fraction being given, to change the Denomination to any other more requiſite, retaining ſtill (as near as may be) the ſame value: To doe which this is the RULE:*

Multiply the Numerator *given, by the* Denominator *required, and divide the* Product *by the* Denominator *given; the* Quotient *ſhall be the* Numerator *required.*

Example. Let the Fraction given be $\frac{7}{13}$ of a Pound *Sterling*, what is that in the twentieth parts or ſhillings? Multiply 7 by 20, the Product is 140; which divided by 13, the Quotient is $10\frac{10}{13}$, that is 10*s.* and $\frac{10}{13}$ of a Shilling; which may be brought to Pence thus: multiply 10 by 12, the Product is 120; which divided by 13, the Quotient is $9\frac{3}{13}$ *d.* and again, multiply 3 by 4, the Product is 12; which divide by 13, the Quotient is $\frac{12}{13}$ of a Farthing; ſo ſeven thirteenths of a Pound is 10*s.* 9*d.* and almoſt a Farthing.

But he who is reſolved to have it in the ſmalleſt coin, may doe it at the firſt work; for ſeeing a Farthing is the 960th part of a Pound, multiply 7 by 960, they produce 6720; which divided by 13, the Quotient is 516 Farthings, and $\frac{12}{13}$ of a Farthing; theſe Farthings may be turned to Shillings, dividing by 48, or to Pence by 4, as in *Reduction.*

This brief and plain Rule is of great uſe in *Arithmetick*, either for turning natural and ſurd Fractions into Decimals, or any other deſired *Denomination*, with ſuch facility and ſpeed as may be wiſhed.

Fractions of Fractions.

In Reduction of Fractions, ſome make another, or more parts, as *Fractions of Fractions* for one; that is, when there is a part of a *Fraction*, or a part of a part of a *Fraction*, *&c.* to be valued in one *Fraction*; for the doing of which obſerve this *RULE:*

Multiply all the Numerators *together, the laſt* Product *ſhall be the* Numerator *deſired; then multiply all the* Denominators, *and this laſt* Product *ſhall be the* Denominator *ſought.*

Example. Let the *Fractions of Fractions* propounded be $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$, for ſo they are uſually written, and let the *Numerators* be multiplied, ſaying, 4 times 3 is 12, and 12 times 1 is 12, the *Numerator* therefore required is 12; then for the *Denominator*, ſay, 5 times 4 is 20, and 20 times 2 is 40, the *Denominator* required; and $\frac{12}{40}$ is equal to $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{1}{2}$.

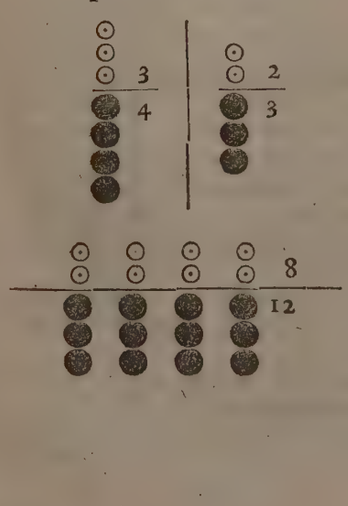
The Proof.

Let the Unite be 40*s.* one fifth of 40 is 8, and therefore $\frac{4}{5}$ is 32, of which one fourth is 8; and $\frac{3}{4}$ is 24, of which one half is 12; and therefore $\frac{12}{20}$ is the juſt ſumm of all the Fractions. This needs no farther exemplifying.

§ IX. *The Reaſon of the* Reducing *of* Fractions *from one* Denomination *to another, as alſo of* Reducing *them to the* leaſt Terms, *demonſtrated.*

The Reaſon of *tranſlating* or *reducing* of *Fractions* from one *Denomination* to another depends upon this *Theorem:*

If two Numbers *multiplying any* Number, *produce other* Numbers; *the* Numbers *produced of them ſhall be in the ſame* Proportion *that the* Numbers multiplied *are.* Eucl. El. Lib. 7. Theor. 16.



For the applying of this *Theorem* to the matter in hand take theſe two *Fractions* $\frac{2}{3}$ and $\frac{3}{4}$, as in this Figure.

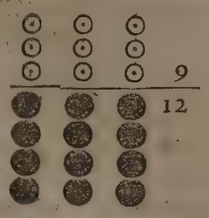
And firſt, I ſingle out $\frac{2}{3}$, and multiply them by 4, (the Denominator of the other Fraction,) they become $\frac{8}{12}$; for 4 times 2 is 8, and 4 times 3 is 12, as may be ſeen in the Margin.

And (by the *Theorem*) I ſay, that this *Multiplication* of the *Numerators* of the two *Fractions* 2 and 3, by a third *Number*, *viz.* [4] hath altered no *Proportion*; for the *Products* have the ſame *Proportion* one to the other, as the *Numerators* 2 and 3 have, by which they were *multiplied*; that is to ſay,

As 2 is to 3, ſo is 8 to 12.

Again,

Again, For the other *Fraction* $\frac{3}{4}$, I multiply theſe two (*viz.* 3 and 4, the *Numerator* and *Denominator* of the *Fraction* taken,) by a third *Number*, namely, by 3, (the *Denominator* of the other *Fraction*,) and the *Products* are 9 and 12, or $\frac{9}{12}$; as in this *Figure*. And this *Multiplication* hath not altered any *Proportion*, by the former reaſon: For,



As 3 is to 4, ſo is 9 to 12.

And by this means I have two *New Fractions* $\frac{8}{12}$ and $\frac{9}{12}$, which are the ſame in effect (or value) with the two given Fractions $\frac{2}{3}$ and $\frac{3}{4}$; and whereas they were before of *different Denominations*, they are now of the *ſame Denomination*.

And this is the true *Ground* and *Reaſon* of *Reducing* or *Tranſlating* of *Fractions* from one *Denomination* to another, and from *divers* and *different Denominations* to *one* and the *ſame Denomination*.

The way of Reducing *of* Fractions *to their Leaſt Terms demonſtrated.*

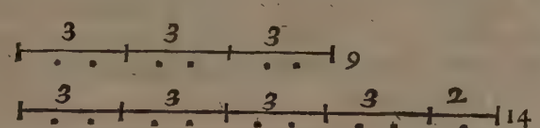
For the *Demonſtration* hereof it will be neceſſary to conſider ſome few *Definitions* and *Theorems* of *Euclide*, ſuch as theſe following:

I. *A* Prime Number *is that which can only be* meaſured *by* Unity.

That is to ſay, 2, 5, 7, 11, 13, *&c.* are *Prime Numbers*, becauſe neither of them can be divided into equal parts by any thing leſs than an *Unite*.

II. *Numbers* Prime *one to the other, are ſuch as* Unity *only doth* meaſure, *an* Unite *being their* Common Meaſure.

And ſuch Numbers are theſe, [9] and [14;] for theſe two Numbers cannot be *divided* into leſſer than *Unite* parts, ſo as that which meaſureth one may meaſure the other.



For though 3 will meaſure 9, that is, it may be found three times in 9, yet it will not meaſure 14; for after it hath been found as often as it can in 14, (that is, four times,) yet there will be two *Unites* left.

Again, although 2 will meaſure 14, that is, it will be found juſt 7 times in 14; yet it will not be found any certain number of times in 9, but there will be an odd *Unite* remaining.

III. *A* Compoſed Number *is that which ſome certain* Number *doth meaſure:*

So [15] is a *Compoſed Number* of 3 and 5, becauſe 3 multiplied by 5 maketh [15.] So is [20] a *Compoſed Number* of 4 and 5, becauſe 4 multiplied by 5 produceth [20.]

IV. Numbers compoſed *one to the other, are ſuch as ſome one* Number, *being a* Common Meaſure *to them both, doth* meaſure.

And thus are [8] and [12] *Compoſed Numbers* one to another; becauſe there is a certain *Number*, *viz.* [4] which being repeated, or added to it ſelf a certain number of times, *compoſes* both the *one* and the *other* of theſe *Numbers*; for [4] being twice repeated (or added) produceth the *Compoſed Number* [8,] and being thrice repeated it produceth the other *Compoſed Number* [12;] and this 4 is called the Common Meaſure.

Thus: Two Compoſed Numbers —— 8 and 12
Diviſors of theſe two —— 2 3
Their Common Meaſure —— 4

Again: Three Compoſed Numbers ——— 6 and 8 and 12
Diviſors of theſe Three ——— 3 4 6
Their Common Meaſure ——— 2

§X. *ADDITION of* Fractions.

To add many Fractions into one ſumm, conſider whether they be all of one Denomination or divers; if one, then add all the *Numerators* together into one ſumm, that ſumm is the new *Numerator*; and the *Denominator* in this caſe is not altered.

Example. Let the Fractions to be added be $\frac{2}{4}, \frac{4}{4}, \frac{5}{4}, \frac{1}{4}$: Add the Numerators together, ſaying, 2 and 4 is 6, and 5 is 11, and 1 is 12: So the ſumm of them all is $\frac{12}{4}$, that is, 3 Unites.

As, let the Unite be 20 *s.* whereof one fourth is 5 *s.* then $\frac{2}{4}$ is 10 *s.* and $\frac{4}{4}$ is 20 *s.* which added to 10 *s.* is 30 *s.* then, $\frac{5}{4}$ is 25 *s.*, which added to 30 *s.* gives 55 *s.* and laſtly, $\frac{1}{4}$ is 5 *s.* which added to 55 *s.* makes 60 *s.* that is, 3 times 20 *s.* which is 3 *li.* or 3 Unites.

320	But if the Fractions to be added be of divers Denominations; as,
360	let them be $\frac{2}{5}\frac{3}{8}$, $\frac{3}{4}, \frac{4}{5}, \frac{7}{8}$, then (by the *Reduction* aſore-going) they
384	muſt be turned all into Denomination, and then they will be $\frac{320}{480}$,
420	$\frac{360}{480}$, $\frac{384}{480}$, and $\frac{420}{480}$, and may be added like thoſe before, as in the
In all 1484	Margin: So the ſumm of all is $\frac{1484}{480}$, or $\frac{371}{120}$, that is $3\frac{11}{120}$; which if
	it be Money, and the Unite 1 *l.* it is then 3 *li.* 1 *s.* 10 *d.* as may be
	tried thus: $\frac{2}{5}$ of a Pound is 13 *s.* and 4 *d.* and $\frac{3}{4}$ is 15 *s.* and $\frac{4}{5}$ is 16 *s.*
	and $\frac{7}{8}$ is 17 *s.* 6 *d.* theſe all added together, make 3 *li.* 1*s.* 10 *d.*

§XI. *SUBTRACTION of* Fractions.

In *Subtraction* of one Fraction from another, if they be both of one Denomination, it is done by taking the *Numerator* of one, from the *Numerator* of the other, the Remainder is the new *Numerator*, and the *Denominator* the ſame as before.

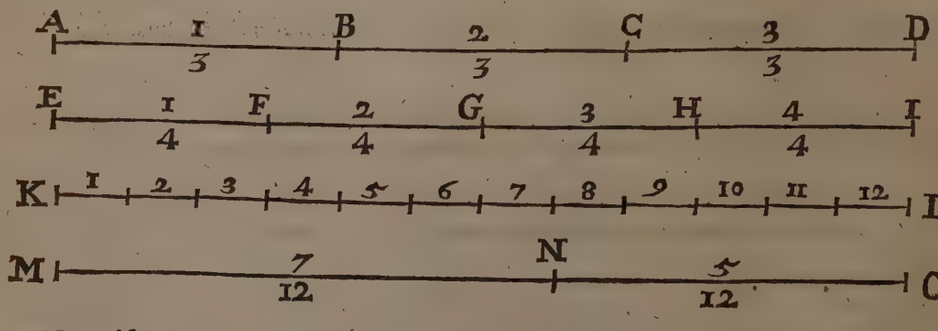
So if $\frac{3}{5}$ be ſubtracted from $\frac{8}{5}$, the Remainder is $\frac{5}{5}$; the like of all other.

But if they be not of one Denomination, they muſt firſt be reduced to be ſo; then that which is ſaid before is ſufficient.

§XII. Addition *and* Subtraction *of Fractions demonſtrated.*

I. Of *Addition.*

Admit the Line A D to be one Foot in length, divided into three equal parts, as A B, B C, C D; then is A B $\frac{1}{3}$ of a Foot, or 4 Inches, A C $\frac{2}{3}$ parts, or 8 Inches of the whole length A D 12 Inches.



Now, if A B were added to A C, that is $\frac{1}{3}$ to $\frac{2}{3}$, their ſumm would be $\frac{3}{3}$, or one Integer, as A D; for C D is equal to A B, which was added to A C.

Again, The Line E I is a Foot divided into Four equal parts, *viz.* E F, F G, G H, H I; every equal part containing 3 Inches; then will E F be $\frac{1}{4}$ of a Foot, or 3 Inches; which added to E G $\frac{2}{4}$, or 6 Inches, the ſumm is $\frac{3}{4}$, or 9 Inches, equal to E H; for G H is but 3 Inches, equall to E F, which was added to E G.

In like manner, If E F $\frac{1}{4}$ were added to E H $\frac{3}{4}$, the ſumm will be $\frac{4}{4}$, equal to the Integer E I.

But

But if Fractions have ſeveral Denominations, they muſt be reduced to one Denomination before they can be added: As, if A B $\frac{1}{3}$ of a Foot, were to be added to E F $\frac{1}{4}$ of a Foot, they muſt be firſt reduced, and then they will be $\frac{4}{12}$ and $\frac{3}{12}$, which added together make $\frac{7}{12}$ of a Foot, or 7 Inches; for the Integer divided into 12 equal parts, (according to the Denominator of the Fraction,) 7 of thoſe parts muſt be taken for the ſumm of the two Fractions: As, admit K L equal to A D, or E I, were divided into 12 equal parts, or Inches, and the Line M O equal to K L; then take A B 4 Inches, and E F 3 Inches, they will extend to 7 Inches in the Line K L, repreſented in M N, being $\frac{7}{12}$ of M O. And what is here ſaid of *Proper Fractions*, the like is to be underſtood of all other *Fractions*, whether *improper* or *mixt Numbers*.

II. Of *Subtraction*.

In the former Figure admit the Line A D to be a Foot, divided into 3 equal parts, from whence $\frac{1}{3}$ part, as A B, is to be ſubtracted; the Integer is $\frac{3}{3}$, from whence take $\frac{1}{3}$, the Remainder will be $\frac{2}{3}$, that is, B D or A C, and is the ſame in Number; for A B 4 Inches, ſubtracted from A D 12 Inches, the Remainder is 8 Inches, equal to A C or B D. Again, the Line E I is a Foot, divided into four equal parts, as E F, F G, G H, and H I; then is E F equal to $\frac{1}{4}$ of the whole Line E I, and E F ſubtracted from E H $\frac{3}{4}$, or 9 Inches, the Remainder will be E G $\frac{2}{4}$, or $\frac{1}{2}$ of a Foot, or 6 Inches: And ſo, if E F were to be ſubtracted from E I a Foot, that is $\frac{1}{4}$ from $\frac{4}{4}$, the Remainder will be $\frac{3}{4}$ or E H.

Thus when Fractions are of the ſame Denomination, but if they be of different Denominations, they muſt be reduced into one Denomination, and then ſubtract one Numerator from the other, as before.

For illuſtration hereof, M O is a Line of a Foot long, divided into two unequal parts in N; the part M N containing 7 Inches, or $\frac{7}{12}$ of a Foot, from which A B $\frac{1}{3}$ of a Foot, or 4 Inches is to be ſubtracted; theſe Fractions reduced will be $\frac{12}{36}$ and $\frac{21}{36}$; then ſubtract 12 from 21, and there will remain $\frac{9}{36}$, or (in its leaſt terms) to $\frac{1}{4}$: So if A B 4 Inches be ſubtracted from M N 7 Inches, the Remainder will be E F, equal to $\frac{1}{4}$ of a Foot, or 3 Inches.

In like manner M N $\frac{7}{12}$ ſubtracted from the Integer $\frac{12}{12}$, the Remainder will be $\frac{5}{12}$, equal to N O.

By what hath been here ſaid, it is apparent that *Addition* and *Subtraction* of *Fractions* do prove each other.

§ XII. *To work The GOLDEN RULE in* Fractions.

The *Golden Rule in Fractions* is the ſame as in whole Numbers, I will give you but one inſtance.

If $\frac{1}{4}$ of a yard of Tape coſt $\frac{1}{2}$ of a peny, what ſhall one Inch, that is, $\frac{1}{36}$ of a yard coſt?

Multiply the ſecond Fraction by the third, the Product is $\frac{1}{72}$; which divided by $\frac{1}{4}$, the Quotient is $\frac{4}{72}$ of a penny, for the price of $\frac{1}{36}$ of a yard.

Otherwiſe, ſeeing $\frac{1}{4}$ of a yard may be turned to 27 Inches, ſay, if 27 coſt $\frac{1}{2}$ what 1? Divide $\frac{1}{2}$ by 27, it makes $\frac{1}{54}$ for the anſwer; which is equal to $\frac{4}{72}$, and in the leaſt terms.

And whereſoever this may be done, to have the firſt and third Numbers of Fractions of one Denomination, the beſt way is to work with their Numerators, not regarding their Denominators at all: As, *If $\frac{3}{4}$ coſt $\frac{3}{4}$, what $\frac{7}{3}$?* Inſtead thereof write, *If 2 coſt $\frac{3}{4}$, what 7?* Multiply $\frac{3}{4}$ by 7, it produceth $\frac{21}{4}$; which divided by 2, the Quotient is $\frac{21}{8}$; and that is the anſwer in the leaſt terms.

And all this while it ſhould have been noted that the Fractions are ever written in a ſmaller figure than the whole Numbers.

CHAP.

CHAP. XIII.

Of The RULE of FELLOWSHIP.

Definition. THis Rule is uſefull for *Merchants*, and all ſuch as trade in *Companies*, with joint-ſtocks, and muſt ſhare a proportional part of the Gains, or Loſs, every one according to his ſtock which he laid in.

The Rule is two-fold, with *equal time*, or with *unequal time*; that which is with *equal time* is commonly called *The Rule of Fellowſhip without time*; of this we will firſt ſpeak, and for the working of it this is the *RULE*:

As the whole joint-ſtock is to all the Gain, or Loſs, ſo is each man's particular Stock to his part of the Gain, or Loſs.

Example 1. *Two Purchaſers* A. *and* B. *purchaſe* 700 li. *Land a-year for ever, (when Money is at* 8 per Cent.) *for* 14000 li. *of which* A. *paid* 8000 li. *and* B. 6000 li. *after* 5 *years (Money being fallen to* 6 per Cent.) *they ſell it for* 18700 li. *ſo there is gained* 4700 li. *how much muſt each have?*

Firſt for *A.* Say, if 14000 gain 4700, what 8000? Anſwer $2685\frac{10000}{14000}$.

Then for *B.* Say, if 14000 gain 4700, what 6000? Anſwer $2014\frac{4000}{14000}$. As by the following operation doth appear.

I. For *A.*

li. *li.* *li.*
If 14000 gain 4700 what 8000?
8
14000)37600000(2685$\frac{10}{14}$
28000
96000
84000
120000
112000
80000
70000
10000 *Remainder*

II. For *B.*

li. *li.* *li.*
If 14000 gain 4700 what 6000?
6
14000)28200000(2014$\frac{4}{14}$
28000
20000
14000
60000
56000
4000 *Remainder*

Here note, That this work might have been much abbreviated, if from each of the three numbers you had cut off two Ciphers towards the right hand, as hath been formerly ſhewed.

Now for the proof hereof, if you add the ſumm which *A.* gained——2685 $\frac{10000}{14000}$, or $\frac{10}{14}$,
To the ſumm which *B.* gained——2014 $\frac{4000}{14000}$, or $\frac{4}{14}$.
The ſumm of them is——47000,
Which is equal to the total gain.

And according to the proportion of theſe two Numbers, that is, as 8 to 6, or 4 to 3; ſo they ought to have parted the yearly Rent alſo, all the time they received it; that is, *A.* ought to have 400 *li.* and *B.* 300 *li.* yearly.

Example 2. A. B. *and* C. *join (their Moneys to make a Stock of* 25000 li. *of which* A. *laid in* 10000 li. B. 8000 li. *and* C. *put in* 7000 li. *with this (after a certain time in trading) they gained* 7500 li. *how muſt this be parted?*

Firſt for *A.* Say, if 25000 gain 7500, what 10000?

Or ſhorter, if 25 get $7\frac{1}{2}$, what 10?

Multiply $7\frac{1}{2}$ by 10, it produceth 75; which divided by 25, the Quotient is 3, that is, (reſtoring the three Ciphers,) 3000 *li.* for *A.*

Then for *B.* Say, if 2500 gain 7500, what 8000?

Or ſhorter, if 250 get 75, what 80?

Multiply and divide as the *Golden Rule* requires, to the Quotient reſtore the two Ciphers, then it will be 2400 *li.* for *B.*

Laſtly for *C.* Say, if 250 gain 75, what 70? Anſwer, 21; to which put the two Ciphers, it makes 2100 *li.* for *C.*

And theſe three 3000, 2400, and 2100, beeing added together, make 7500; and have that proportion as the particular Stocks had, and therefore the Work is right.

I. For

I. For *A*.
If 25 gain $7\frac{1}{2}$ what 10?

150
15 × 10 or $\frac{75}{1}$
2 × 1
2

25)75(3 or 3000
75
00

II. For *B*.
If 250 gain 75 what 80?

80
250)6000(2400
500
1000
1000

III. For *C*.
If 250 gain 75 what 70?

70
250)5250(2100
500
250
250
000

And if instead of gaining 7500 *li.* whereby every one is supposed to have his Stock, and a part of the Gains, they had lost 7500 *li.* then their particular Stocks had not been due to them, but so much as would be left after their proportional parts of the loss were abated.

Example 3. A. B. *and* C. *with a joint Stock of* 25000 li. *gain* 7500; *of which* A. *gets* 3000, B. 2400, C. 2100; *what was their Stock?*

This is but the Converse of the former; therefore say, if 7500 require 25000, what doth 3000 require? 10000 for *A*; and so work for the other two.

Many Examples are of little use (except to load the Reader's memory) where the Rule is so short and plain; I will therefore add no more to this part of the Rule, but immediately come to the *Rule of Fellowship with Time*.

CHAP. XIV.

Of The RULE of FELLOWSHIP with Time.

Definition. THis Rule is to be used when the times of the continuance of the particular Stocks are unequal, and differ; so that here the difference of Time, and also the difference of Stock being both to be considered, it can be done no better way than by taking the *Power* of them both to be the particular Stock; and all those *Powers* added, to be the whole Stock; (that which I call the *Power*, is the Product of the Money of every one multiplied by its time,) and then, this is the *RULE:*

As the summ of those Products is to the whole gain, so is each particular Product to its part of the gain.

Question 1. *Three Merchants* A. B. C. *make a Stock of* 10000 li. *of which* A. *lays in* 400 li. *for* 3 *Months*, B. 3000 li. *for* 6 *Months, and* C. 3000 li. *for* 8 *Months*; *with this they gain* 2000 li. *what is each Man's share?*

First, for *A*. Multiply 4000 by 3, it makes 12000; let that be accounted his particular Stock.

Secondly, for *B*. Multiply 3000 by 6, it makes 18000; his particular Stock.

Lastly, for *C*. Multiply 3000 by 8, it produceth 24000; for his Stock: add these, they make 54000 *li.* for the general Stock: Then say,

I. For *A*. If 54000 give 2000, what 12000? Answer 444 $\frac{24000}{54000}$, or $\frac{4}{9}$.

II. For *B*. If 54000 give 2000, what 18000? Answer, 666 $\frac{36000}{54000}$, or $\frac{6}{9}$.

III. For *C*. If 54000 give 2000, what 24000? Answer 888 $\frac{48000}{54000}$, or $\frac{8}{9}$.

The three Fractions may be reduced by dividing each Numerator and Denominator by 6000, and then the three shares will be 444 $\frac{4}{9}$, 666 $\frac{6}{9}$, 888 $\frac{8}{9}$; which all together make 2000, as they ought.

Question 2. *Three Farmers*, A. B. *and* C. *lay out* 1000 li. *to stock their Grounds with Cattel*, *of which* A. *put in* 200 li. *for* 6 *years*, B. *had* 300 li. *going for* 4 *years, and* C. 500 li. *for* 2 *years*; *at the end (by unseasonable times) there was lost* 200 li. *which made the remainder of their Stock but* 800 li. *what had each Man left?*

Multiply 200 by 6, it gives 1200; likewise, 300 by 4, it gives 1200; lastly, 500 by 2, the Product is 1000; all these added together make 3400, for the joint-stock.

Then

Then first for *A.* Say. if 3400 lose 200, what 1200? Answer, 70 $\frac{2000}{3400}$, for *A.* to which *B.* is equal, because the Power of his Stock is so.

Therefore for *C.* Say, if 3400 lose 200, what 1000? Answer, 58 $\frac{2833}{3400}$: So the three shares are 70 $\frac{20}{34}$, 70 $\frac{20}{34}$, and 58 $\frac{23}{34}$, equal to 200.

Now because *A.* put in 200 *li.* and lost 70 $\frac{20}{34}$, subtract the Loss from the Stock, and there remains 129 $\frac{14}{34}$.

And so doing for *B.* his remainder will be 229 $\frac{14}{34}$.

And for *C.* his remainder is 441 $\frac{6}{34}$.

Now these three remainders, 129 $\frac{14}{34}$, 229 $\frac{14}{34}$, and 141 $\frac{6}{34}$, make up 800 *li.* which was the whole remainder.

Question 3. A. *rents a Close for a year, to pay* 80 li. *he puts into it* 200 *Sheep; two Months after,* B. *puts* 40 *Sheep in; and five Months after that,* C. *puts in* 100 *Sheep; how much must every one pay of the Rent?*

Multiply 200 by 12, it produceth	2400
And 40 by 10, produceth	400
Lastly 100 by 5, (which is *C*'s time,) produceth	500
In all	3300

Then for *A.* Say, if 3300 pay 80, what 2400? Answer, 58 $\frac{600}{3300}$, or $\frac{6}{33}$.

Then for *B.* Say, if 3300 pay 80, what 400? Answer, 9 $\frac{2300}{3300}$, or $\frac{2}{33}$.

And for *C.* Say, if 3300 pay 80, what 500? Answer 9 $\frac{1500}{3300}$.

The whole numbers make 76, and the broken numbers make 4; in all 80.

Note. Whereas hitherto we have considered only difference of *time*, and *money*, it may be noted, that there may be difference of other kinds, as *persons* or *place*; but whatsoever they are, the Power of all is found, like these, by Multiplication; and are to be wrought like these, with so many uses of the *Golden Rule* as the Question requires. I will therefore add but one Question more, which is this:

Question 4. *One leaves a Legacy of* 900 li. *among four Kinsfolk,* A. B. C. D. *so as* B. *may have twice as much as* A. *and* C. *thrice as much as* B. *and* D. *as much and half as much as* C. *What is every one to have?*

Say, If *A.* be 1, *B.* is 2, *C.* 6, and *D.* 9; add these Numbers, 1, 2, 6, 9, together, they make 18; then say, If 18 require 900, what 1? Answer 50: So *A.* is to have 50 *li.* *B.* 100 *li.* *C.* 300 *li.* and *D.* 450 *li.* which are their just parts; and all together are equal to 900 *li.* and the work right.

CHAP. XV.
Of BARTER.

Definition. TO *barter* is to *exchange* one Commodity for another, the nature whereof will best appear by the resolving of some Questions.

Question 1. *Two Merchants barter; One hath Sugar at* 4 li. *the* C. *ready money, but in Barter he will have* 4 li. 13 s. 4 d. *the other hath French Wine at* 13 li. *the Hogshead ready money; at what price must he rate his Wine to equalize the others advance of his Sugar in Barter?*

Say, by the Rule of Three direct, If 4 *li.* in Barter require 13 *s.* 4 *d.* advance, what shall 13 *li.* in Barter require?

li.	*s.*	*d.*	*li.*
If 4	13	4	what 13?

12
30
13
160
13
480
160
2080

4)2080(520
20
08
8
00

12)520(43 *s.*
48
40
36
4 *d.*

Answer 2 *li.* 3 *s.* 4 *d.*

Question

Question 2. *Two Barter; one hath* 3 ½ C. *of Ginger at* 13 ½ d. per *Pound, the other hath Sugar at* 15 ½ d. per *Pound; How much Sugar must be delivered for the* 3 ½ C. *of Ginger.*

First, By the Rule of Three find what 3 ½ *C.* of Ginger comes to at 13 ½ *d. per* Pound; which will be found to be 22 *li.* 1 *s.* For, If 1 *li.* cost 13 ½ *d.* what 3 ½ *C.* cost? Answer, 22 *li.* 1 *s.*

Secondly, Say, If 15 ½ *d.* buy 1 *lb.* of Sugar, what shall 22 *li.* 1 *s.* buy?

Answer, 341 $\frac{13}{31}$ *lb.*

Question 3. *Two Barter; One hath Tobacco at* 14 d. per *Pound, which he will barter for Sugar at* 10 d. per *Pound, how much Tobacco must be given for* 8900 lb. *of Sugar?*

First, The 8900 *lb.* of Sugar at 10 *d. per* Pound, comes to 370 *li.* 16 *s.* 8 *d.*

Then, If 14 *d.* buy 1 *lb.* of Tobacco, what number of Pounds will 370 *li.* 16 *s.* 8 *d.* buy?

Answer, 6358 $\frac{1}{7}$ *lb.* and so many Pounds of Tobacco at 14 *d.* must be given for 8900 Pounds of Sugar at 10 *d.*

Question 4. *Two Barter; One hath broad Cloth at* 15 s. *the Yard, ready money; for which in Barter he will have* 16 s. 3 d. *the other hath Wool at* 2 s. 10 d. per *Pound, ready money; what price must his Wool be set at in Barter to equalize the advance which the other puts upon his Cloath?*

Say by the Rule of Three direct, If 15 *s.* ready money require 1 *s.* 3 *d.* in Barter, what shall 2 *s.* 10 *d.* ready money require? Answer, 2 *d.* 3 *q.* $\frac{1}{3}$.

So that he must rate his Wool at 2 *d.* 3 *q.* and $\frac{1}{3}$ of a Farthing *per* Pound.

The next Rule in order to be treated of is concerning *Interest*, both *simple* and *compound*, but of that (at large) in *Decimal Arithmetick*; and therefore I omit it here.

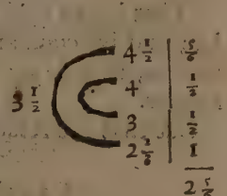
CHAP. XVI.

Of The RULE of ALLIGATION.

Definition. THis Rule hath its name from *binding*, *tying*, or *uniting* many particulars in one *Mass* or *Summ*; the nature of it will be understood in working some *Questions*, or *Examples.*

Question 1. *A Corn-master would mix* 4 *sorts of Grains together*; viz. Wheat *at* 4 s. *the Bushel*, Wheat *at* 2 s. 8 d. *the Bushel*, Rie *at* 3 s. *the Bushel, and* Barley *at* 4 s. 6 d. *the Bushel, so as to make* 15 *Quarters in all, to be sold at* 3 s. 6 d. *the Bushel; How much must he take of each?*

Place them as in the Margin, so as the greater and lesser may still be together, as 4 ½ with 2 ⅔, and 4 with 3; and place the price required by it self towards the left hand, as here you see 3 ½; then in a separated Column, note the difference between the price of a Bushel of every particular Grain, and a Bushel of that required; as the difference betwixt 4 ½ and 3 ½ is 1, which must not be placed against 4 ½, but against that number linked with 4 ½, that is, against 2 ⅔; and so must all the differences be ordered, as is easie to be seen in the Margin; then this is the *RULE:*



Multiply the whole Mass *to be made, by any particular* difference, *and divide the Product by the summ of all the* differences, *the Quotient shall be the just quantity of that* particular kind, *whose price standeth against the difference you wrought with.*

Example. First turn the Quarters into Bushels; saying, 8 times 15 is 120: then for the quantity of the first sort at 4 ½, multiply 120 by ⅚, the Product is 100; which divided by 2 ⅚, the Quotient is 35 $\frac{5}{17}$ Bushels of that sort at 4 *s.* 6 *d.* and working so for every of the other, they will be found to be thus:

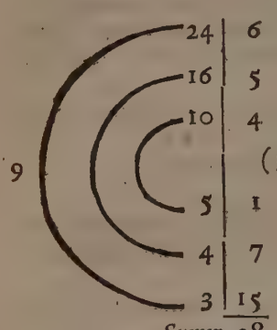
At 4 *s.* 6 *d.*	35 $\frac{5}{17}$	Bushels.
At 4 *s.*	21 $\frac{3}{17}$	
At 3 *s.* 6 *d.*	21 $\frac{3}{17}$	
At 2 *s.* 8 *d.*	42 $\frac{6}{17}$	
	120	

Now to prove this right; Firſt, multiply the whole Maſs 120 Buſhels by the deſired price $3\frac{1}{2}$ *s.* the ſumm is 420 *s.*

Then ſecondly, multiply $38\frac{1}{17}$ by $4\frac{1}{2}$, it is ——— $158\frac{14}{17}$,
And $21\frac{3}{17}$ by 4, it produces ——— $84\frac{12}{17}$,
And $21\frac{3}{17}$ by 3, it makes ——— $63\frac{9}{17}$,
And $42\frac{6}{17}$ by $2\frac{2}{3}$, is ——— $112\frac{16}{17}$.
In all ——— 420

And ſo much all ſhould be worth in Shillings, and therefore the Queſtion is rightly reſolved.

Queſtion 2. *One hath 6 ſorts of Fruits at ſeveral prices*, Dates *at* 2 s. Almonds *at* 1 s. 4 d. Corins *at* 10 d. Raiſins *at* 5 d. Prunes *at* 4 d. *and* Figs *at* 3 d. *the Pound*; *and would take of every ſort ſome, to make a mixed quantity of* 30 lb. *weight, to ſell one with another for* 9 d. *the Pound, how much muſt he take of each?*



6	
5	
4	(30
1	
7	
15	
Summ 38	

Having placed the Numbers, and their Differences, and the Summ of thoſe Differences, diſtinctly as hath been ſhewed before, and may be ſeen in the Margin; the Work is evermore like that in the former Queſtion; ſo 38 is the firſt number in the *Golden Rule*, 30 the ſecond, (which, that it may not be forgotten, may be ſet at the right ſide of the figure,) and every particular difference, as 6, 5, 4, *&c.* is the third in the Rule, to be repeated till all the Differences have been employed.

So 30 multiplied by 6, produceth 180; which divided by 38, the Quotient is $4\frac{28}{38}$ of a Pound weight, and ſo much muſt be taken of *Dates*, at 24 *d.*

Secondly, 5 times 30 is 150, which divided by 38, the Quotient is $3\frac{36}{38}$, for *Almonds*.

And working after the ſame manner with 4, 1, 7, 15, their reſpective quantities will be found to be theſe:

	lb.	38*th parts*
Dates	4,	28
Almonds	3,	36
Corins	3,	6
Raiſins	0,	30
Prunes	5,	20
Figgs	11,	32
In all	26,	152

That is, $26\frac{152}{38}$, and the reduction of the Fraction will make it 30, as it ought to be; and by comparing the prices of theſe particulars added, with the price of 30 *lb.* weight, at 9 *d. per* Pound weight, which makes 270 *d.* this may be proved like the former.

But that the Reader may be perfect in it, I will doe it here as followeth; ſaying,

		Denom.	*Numer.*
1.	24 times 4 is 96, 24 times 28 is 672,		96 672
2.	16 times 3 is 48, 16 times 36 is 576,		48 576
3.	10 times 3 is 30, 10 times 6 is 60,		30 060
4.	5 times 30 is 150,		00 150
5.	4 times 5 is 20, 4 times 20 is 80,		20 080
6.	3 times 11 is 33, 3 times 32 is 96,		33 096
	In all	227.	1634

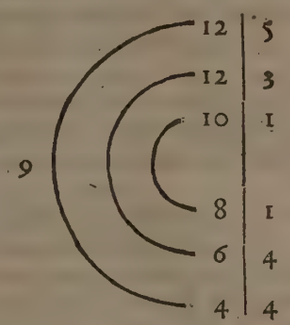
Now this 1634 being the ſumm of the Numerators of Fractions, whoſe common Denominator is 38, muſt be divided by 38, and the Quotiont will be 43; which added to the whole number 227, the ſumm is 270; and ſo much is 30 multiplied by 9, which ſhews the work to be right.

The

The Combination or Linking of Numbers may be varied at pleasure; as whereas above 24 was linked with 3, also 16 with 4, and 10 with 5; it might have been 24 with 5, and 16 with 4, and 10 with 3; or 24 with 4, and 16 with 3, and 10 with 5; of which diversity of linkings would follow diversity of solutions, but all true, as the Reader may easily prove by himself.

Likewise, if the Numbers to be linked were 3, 5, 7, or any odd Numbers, one of them may be linked to two severally, to make the work even.

Example 3. If the Numbers were 12, 10, 8, 6, and 4, and the mean, or common price required were 9; you might link them as you see here, taking 12 twice, or else you might take any other twice, as you shall like; and so the work will be every way right, though not the same, if the differences be rightly set off, and orderly used, as is taught before in the first Question.



Question 3. *A Goldsmith would mix* 3 *sorts of Silver*, A, B, C; A *is* 10 d. *weight better*, B 7 d. *weight better, and* C 4 d. *weight better, to make an Ingot of* 50 lb. *weight, which should be in fineness* 8 d. *weight better; How much must be taken of each?*

Set them, their Differences, and the Summ of their Differences, as in the Margin. Then,

First, 50 multiplied by 4, and divided by 9, is —— $\frac{200}{9}$

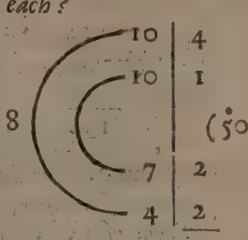
Secondly, 50 multiplied by 1, and divided by 9, is —— $\frac{50}{9}$

Thirdly, 50 multiplied by 2, and divided by 9, is —— $\frac{100}{9}$

Fourthly, The same again, —— $\frac{100}{9}$

The Summ, —— $\frac{450}{9}$

Which is equal to 50, the quantity required.



(50

Now the First Fraction $\frac{200}{9}$, multiplied by 10 (omitting the Denominator 9) is—2000

The Second also by 10, is —— 500

The third by 7, is —— 700

The Fourth by 4, is —— 400

The Summ —— 3600

That is, $\frac{3600}{9}$, which is equal to 400; and if the whole Ingot 50, be multiplied by the betterness required, namely by 8, they shall produce 400 also: So this is proved.

Note. In every Alligation, or linking of two Numbers, this is evident, *That if the Summ of the Numbers linked, be greater than the mean Number required, taken so many times as there are Numbers to be linked, the Question would be absurd, and the resolution thereof impossible.* And this shall serve for the *Rule* of *Alligation*.

CHAP. XVII.

Of The RULE of FALSE POSITION.

Definition. THis *Rule* serves to resolve such *Questions* as are not at the first propounding of them fit for the *Golden Rule*; and therefore in stead of the *True Number* sought for, you may suppose any Number, whether *Greater* or *Lesser* than the *True Number* it mattereth not; and make tryall with that *supposed Number*, according to the tenour of the Question, as if it were the *True Number*; and if it answer the Question without any *Errour*, you have hit upon the *Right Number*; but if it err,

you must guess again, and so by two *False Positions* you may find out the *True Number* desired.

This Rule is two-fold; *viz.* *Single* and *Double*.

I. The *Rule* of *Single Position* is to be used when at once, *viz.* by *One False Position* you have means to discover the true resolution of the Question.

II. The *Rule* of *Double Position* is when *Two False Positions* are required for the resolving of the Question.

This *Rule* will best be understood by the resolving of *Questions* in both kinds.

§ I. *In the Single Rule.*

For the resolving of *Questions* in this of *Single Position* this is the *RULE*: *Multiply the* supposed Number *by the* Number given, *and divide that Product by the result (or what arises) of the* supposed Number, *the Quotient shall be the* true Number *sought for.*

Question 1. *There was* 132 li. *to be divided among Four Persons*, A. B. C. D. *to be paid in such sort that* B. *was to have twice as much as* A. *and* C. *as much as* A. *and* B. *and* D. *was to have five times as much as* A. *How much must each Person's Share be?*

A. —— 2	Suppose *A.* to have 2 *li.* then *B.* must have as much more, that
B. —— 4	is, 4 *li.* and *C.* must have as much as *A.* and B. that is *6 li.* and
C. —— 6	*D.* must have 5 times as much as *A.* that is, 10 *li.* now all these
D. —— 10	added together make but 22 *li.* which should be 132 *li.* whereby
22	you may see that you have not supposed right, for the Share
	that *A.* is to have: But from this *Supposition* you may find
	the true Share of *A.* by the *Golden Rule*, thus:
A. —— 12	If 22 come of the *Supposition* 2, what will 132 come of?
B. —— 24	Multiply 132 by 2, the Product will be 264, which divided
C. —— 36	by 22, the Quotient will be 12 *li.* for *A.* then *B.* must have
D. —— 60	24, *C.* 36, and *D.* 60; all which added together do make up
132	132 *li.* equal to the whole Summ to be divided, as above-said,
	and proves the *Work* to be *true*.

Question 2. *One delivered into the hands of a* Trustee, *for a* Child's Portion, *a certain summ of Money, to be paid to the Party at the expiration of* Ten Years, *with the Profit of the same summ at* 6 li. per Cent. *simple Interest; and at the end of the* Ten Years, *the Party received* 450 li. *What was the* Summ *of Money that was put into the* Trustee's *hand?*

Suppose the *Summ* put in were 150 *li.* the *Use* of 150 *li.* in 10 Years is 90 *li.* which added to 150 *li.* the summ is 240 *li.* but the Party received 450 *li.* therefore the *Supposition* was not right; but from this *false Supposition* you may find the *true Number* by this following *Analogy*:

As 240 *li.* (the *supposed Summ*, with the *Interest* thereof for 10 Years,)
Is to 150 *li.* (the *supposed Summ*,)
So is 450 *li.* (the *Money* received,)
To 281 *li.* 5 *s.* (the *Money* delivered to the *Trustee*,)

So that the *Trustee* had 281 *li.* 5 *s.* put into his hands, to be disposed of according to the tenour of the *Question.*

§ II. *In the Double Rule.*

The *Rule* of *Double Position* is, when *Two False Positions* are required to solve the Question propounded.

For the resolving of Questions by the *Rule* of *Double Positions*, observe these two general *RULES*:

I. *Suppose any* Number, great, *or* small, *and make tryall of it, whether it resolve the Question without any* Errour; *if so, it is the* true Number *sought*; *if not, it is either too* great, *or too* little; *if it be too* great, *mark it with this Character* +, Plus, *or* More; *if it be too* little, *mark it thus* —, Minus, *or* Less. *Then suppose again, another* Number, *(it imports not whether it be nearer or farther off than the other* supposed Number *was,) and make tryall with this* Number *as you did with the former*; *and if it be not the* true Number, *then (as before) it is either* greater, *or* lesser, *and accordingly note it with* + Plus, *or* — Minus.

II. *Mul-*

II. *Multiply the* first Poſition *by the* ſecond Errour, *and the* ſecond Poſition *by the* firſt Errour, *and (if the* Errours *be both* +, *or both* —,) *ſubtract the* leſſer *from the* greater, *and keep the Remainder for a* Dividend, *and the Difference of the* Errours, *for the* Diviſor; *the* Quotient *of that Diviſion is the* true Number *which anſwers the* Queſtion.

But if the Errours *be one of them* +, *and the other* —, *the Summ of the* Products *muſt be the* Dividend, *and the Summ of the* Errours *the* Diviſor; according to this Diſtich:

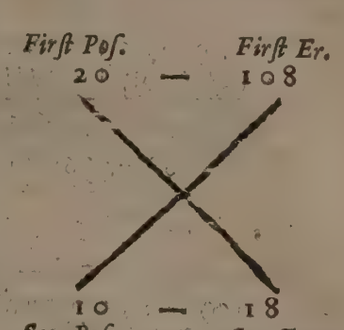
If Signs *be both* alike, Subtraction *make*;
But if of different kinds, *their* Summ *then take.*

Theſe two *Rules* ſhall be explained in the reſolving of ſome few *Queſtions.*

Queſtion 1. *A Man is to drive* 48 Turkies 40 *Miles*; *and for every* Turkie *that comes alive to the end of the Journey, he is to have* 3 *Pence*; *and for every one that dies by the way he is to pay* 6 *Pence*; *and at the end of the Journey, he received* 6 *Shillings, or* 72 *Pence*; *How many died by the way?*

1. Suppoſe there died by the way 20; for them he was to pay 10*s.* or 120*d.* and for 28 which lived he was to receive 84*d.* ſo he paid more than he received 36*d.* whereas he ſhould have got clear 72*d.* the ſumm of theſe two is 108; wherefore the firſt Errour is too *little* by 108, which ſet at the top of a Croſs.

2. Suppoſe there died by the way 10; for thoſe he was to pay 60*d.* and for the 38 which lived he was to receive 114*d.* the difference is 54, which ſhould be 72; ſo the ſecond Errour is 18 too *little*, which ſet at the bottom of the Croſs, as in the Margin.



Then for the ſecond part of the *Rule.*

Multiply 20 the *Firſt Poſition*, by 18 the *Second Errour*, and the Product will be 360; alſo, multiply 10 the *Second Poſition*, by 108 the *Firſt Errour*, and thereof will come 1080.

From 1080	*From* 108
Subſt. 360	*Subſt.* 18
Devidend 720	*Diviſor* 90

90)720(8
720

Now, becauſe the *Signs* were both alike, *viz.* both — *Minus*, ſubtract 360 (the Product of the *Firſt Poſition* by the *Second Errour*,) from 1080, (the Product of the *Second Poſition* by the *Firſt Errour*,) and the Remainder is 720 for a *Dividend*; alſo, ſubtract 18 (the *Second Errour*) from 108 (the *Firſt Errour*,) and the Remainder will be 90, for a *Diviſor*; by which divide 720, and the *Quotient* will be 8; and ſo many died by the way, and 40 came to the Journeys end alive: ſo,

For the 40 that came alive he received ——— 120*d.*
For the 8 that died by the way he paid ——— 48
The Difference — 72

Which was the number of *Pence* which he received.

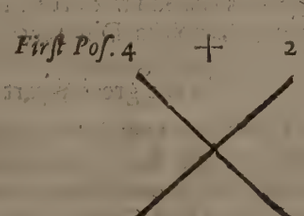
Queſtion 2. *If it were required to make up* 20*s.* *or a* Pound Sterling, *of* Shillings *and* Groats *only, and ſo as the number of* Groats *may be to the number of* Shillings, *as* 7 *to* 1; *How many* Shillings *muſt there be?*

First, Suppoſe the Shillings ——— 4
then the Groats muſt be equal to 16*s. viz.* — 48
but the Shillings taken 7 times are 28, } + 20
to which 48 ſhould be equal, but is }
Secondly, Suppoſe the Shillings — ——— 2
then the Groats (making 18*s.*) are ——— 54
which ſhould be equal to 7 times 2, but is + 40

Multiply 4 by 40, the Product is 160; then,
Multiply 2 by 20, the Product is 40; which taken from 160, reſts for the *Dividend*, — 120
And the Difference of Errours for the *Diviſor*, is ——— } 20

Laſtly, 120 divided by 20, the Quotient is 6.
The Number of Shillings therefore is — 6
And the Number of Groats is ——— 42

For as 7 is to 1, ſo is 6 times 7, which is 42, to 6 times 1, which is 6. So the work is done, and ſtands as in the Margin.

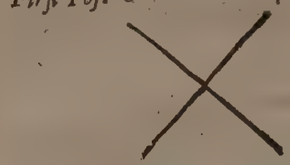


From 160	*From* 40
Subſt. 40	*Subſt.* 20
Dividend 120	*Diviſor* 20

20)120(6
120

Queſtion

Question 3. *If there be 4 several weights,* A. B. C. D. *of which* D. *is 24 ounces, and* C. *is double to* B. *and triple to* A. *and* D. *with twice* A. *is double to* C. *and quadruple to* B. *How much doth every one of these weigh?*



Sec. Pos. 4 + 4 Sec. Err.

16 32

To 16	To 4
Add 32	Add 4
Dividend 48	Divisor 8

8)48(6
48

First, Suppose *A.* to be——— 8
then *D.* with twice *A.* is 24, and 16, that is,—40
of which *C.* being the half is 20, and *B.* 10.
Now thrice *A.* is 24, to which *C.* should be equal, but is too little by——— — 4
Secondly, Let *A.* be supposed——— 4
then *D.* and twice *A.* is 32; and *C.* 16. and *B.* 8.
But thrice *A.* is 12. to which 16 should be equal, but is too much——— + 4
Then 8 multiplied by 4 gives 32, and 4 by 4 produces 16; both these Products gives 48, for the *Dividend*; and the summ of the Errours (because the first Sign is —, and the other +) gives 8 for the *Divisor*, and the Quotient will be 6; to which *A.* is equal, and twice *A.* with *D.* is 36, of which *C.* being half is 18, and *B.* is 9, and thrice *A.* is equal to *C.* namely 18, and all right. As in the Margin.

Whereas the first Errour is equal here to the second, it follows that the *Positions* were equally false; and therefore their *Difference*, which is 4, being parted into two equal parts, 2 and 2, if 2 be taken from 8, the remainder is the *true Number* 6; or if 2 be added to 4, (which was the second Position,) the summ will be also 6.

And farther, when the Errours be one +, and the other —, though they be not equal, yet then if the Difference between the Positions be parted into two parts, which are in proportion one to another as the two Errours are one to another respectively; then if the first part be taken from the first Position, if that be greater, or added to it, if it be the less, the same number required is thereby had.

As, Let the last Question be resumed,
And let the first Position for *A.* be——— 15
then the first Errour will be——— — 18
Then let the second Position be——— 3
and so the second Errour will be——— + 6
and the Difference of Position is——— 12

Which divided into two parts, 9 and 3, which have that proportion one to another as the Errours 18 and 6 have; then if the first part 9, be taken from the first Position 15, there remains the *true Number* 6: Or else, if the second part 3, be added to the second Position 3, thereby also is made the *true Number* 6.

The way of parting 12 (or any other Number) into two parts proportional with the Errours, is easily done by the *Golden Rule*, thus:

As the Summ of the Errours 24,
Is to the Difference of Position 12;
So is the greater Errour 18,
To the greater part required, namely 9.

Many other Questions may be wrought by this Rule; but I intend not to be large herein, because some of those Questions may be resolved without this Rule; I will therefore only instance in one Example more.

Question 4. *If there be a* Cistern *with* 4 Cocks, *which holds* 8 Barrels *of* Water, *and the first* Cock *will run it all out in 6 hours, the second in* 4, *the third in* 3, *and the last in* 2 *hours; In what time shall all of them run it out?*

If the first in 6 hours run——— 8
The second in the same time would run——— 12
The third ——— 16
The last ——— 24
In all ——— 60

Then say, If 60 require 6, what 8?

The Answer is $\frac{48}{60}$, that is, $\frac{4}{5}$ of an hour, or 48 minutes; in which time all the 4 Cocks together would run out all the 8 Barrels of Water.

CHAP.

CHAP. XVIII.

Of The EXTRACTION *of* ROOTS.

§I. *Of the* SQUARE ROOT.

Definition. A *SQUARE* is a *Geometrical Figure*, consisting of Four *equal Sides*, and as many *equal* (or *right*) *Angles*; of which, any one Side of that Figure is called the *Root*, and that *Root* multiplied in it self produceth a *Square Number*.

There are Three distinct *Species* or Kinds of *Square Numbers*, viz. *Single*, *Compound*, and *Irrational*.

I. *Single*, are such *Square Numbers* as are made by the Multiplication of any one of the *Nine Digits* in it self; as 5 multiplied in it self, (that is 5 by 5,) the Product is 25, a *Square Number*, of which 5 is the *Root*, and 25 the *Square*; also 7 times 7 is 49, which is a *Square Number*, and 7 is the *Root* of it.

So the *Root*, or *Side*,	1 2 3 4 5 6 7 8 9	multiplied in it self, produceth the *Square Number*	1 4 9 16 25 36 49 64 81	which is the *Square* thereof.

II. *Compound*, are such *Square Numbers* as are made by the Multiplication of more Figures than *One*; as 10 multiplied by 10, produceth 100; so is 100 a *Square Number*, and 10 the *Root* thereof; also 11 multiplied by 11, produceth 121, the *Square* of the *Root* 11; and so the *Square* of 12 will be 144, for 12 by 12 produceth 144, *&c.*

III. *Irrational*, are such Numbers whose *Roots* cannot be found in whole Numbers, but by some other artifice, (as will appear hereafter;) so 2, 3, 5, 6, 7, 8, *&c.* also 10, 12, 14, 18, *&c.* and 105, 224, and infinite others.

Now if a *Square Number* be given, to find the *Root* thereof, that is, to find out such a *Number*, which being multiplied in it self shall be equal to the *Number* given, is called the *Extraction* of the *Square Root*: And this *Extraction* of the *Square Root* is demonstrated by the Fourth *Theorem* of the Second Book of *Euclide*'s Elements, which saith thus:

THEOREM.

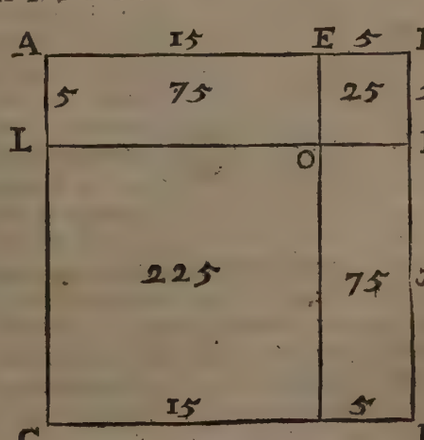
If a Right Line *be divided by chance, the* Squares *made of the* Parts, *together with the* Rectangle *made of the* Parts *twice, is equal to the* Square *of the* whole Line.

ILLUSTRATION.

Let the *Line* AB be divided by chance in the Point E, it is manifest that the *Square* of AB, that is to say, the *Square* ABCD, made of the whole Line AB, is equal to the *Square* of LO, which is equal to AE, and to the *Square* of ER, made of the part EB; and to the two *Rectangle Figures* AO and DO, which are made of the *Parts* AE and EB twice; according to the *Theorem*.

Now let the *Line* AB be supposed to be 20; and let it be divided into AE 15, and EB 5.

Then the *Square* of AB, which is made by multiplying the *Side* (or *Root*) 20 in it self, is equal to —— 400



The

The *Square* of A E 15, is made by multiplying 15 in it ſelf, which produces the *Square* O C. — 225
And the *Square* of B E is 5 in 5, *viz.* — 25
And the *Rectangle* A O is 15 in 5, *viz.* — 75
And the *Rectangle* D O is alſo 15 in 5, *viz.* — 75
The Summ of the *Parts* — 409
Equal to the *Square* of the *Whole Line* A B; all the *Parts* together equal to the *Whole*.

§ II. *The* GENESIS *for Extracting of the* Square Root.

Example. Let it be required to find the *Square Number*, whoſe Side is 57.

5	. 7	*Root*
25	. .	
7	0 .	
	49	
32	49	*Square*

1. Write down the *Root* 57, as in the Margin, with the interval of one Figure between the 5 and the 7, and draw a Line under them, as alſo two down-right Lines, the one next after 7, the other after 5, ſo that the Numbers to be found may be orderly placed for Addition: Then let the *Root* given be ſuppoſed to be divided into theſe two parts 50 and 7; then,

2. Multiply 5 in it ſelf, the Product is 25; which ſet under the Line, *Unites* under *Unites*.

3. Double 5, it makes 10, which multiply by 7, it makes 70; which ſet under 25, one place forward to the Right hand.

4. Multiply 7 in it ſelf, the Product is 49; which ſet under 7, *Unites* under *Unites*.

5. Add theſe three Numbers together, *viz.* 25 . . 70 . and 49, in the ſame order as they ſtand, the ſumm of them will be 3249, which is a *Square Number*, and the *Root* thereof is 57: which may be eaſily proved by multiplying 57 in it ſelf.

Other Examples for Practice.

4	.8	*Root*
16	. .	
6	4 .	
	64	
23	04	*Square*

9	.9	*Root*
81	. .	
16	2 .	
	81	
98	01	*Square*

7	.6	*Root*
49	. .	
8	4 .	
	36	
57	76	*Square*

§ III. *To Extract the* Square Root.

When a Number is given to have the *Square Root* thereof extracted, obſerve the Directions laid down in theſe few following Examples.

Example 1. *Let* 3249 *be a Number given, and let the* Square Root *thereof be required.*

Set the Number down, as in the Margin; and make a Prick over 9, the place of Unity, and another over 2, the ſecond figure from it to the left hand, obſerving the like order ſtill if there were more ſecondary Figures; then draw down-right Lines on the right ſide of each pricked figure, and put a Quotient-line, as in *Diviſion*, on the right ſide of all, and your Number is prepared for *Extraction*, which is performed in the following manner.

	.	.	
	32	49	(57
	25		
107)	7	49	*Reſolvend*
	7	49	*Product*
	0	00	

1. Here 32 (being the figures of the *firſt Period*,) not being a *Square Number*, find the neareſt *Square* among the *Digit Squares*, leſs than 32, which will be 25; ſet 25 under 32, and the *Root* thereof 5 put in the *Quotient*, and draw a Line under 25; then ſubtract 25 from 32, and ſet the Remainder 7 under the Line.

2. To this Remainder 7 bring down 49, (the two figures of the next *Period*,) ſo will the whole Number be 749, which you may call the *Reſolvend*; then on the left hand of 749 make a crooked Line for a *Quotient*, in which put the double of the *Quotient*, which is 10; then ask how many times 10 in 74, the anſwer will be 7; put 7 in the *Quotient* to 5, making it 57, and alſo in the other *Quotient*, making that 107; then multiply 107 by 7, (the laſt figure in the firſt *Quotient*,) and the *Product* will be 749, which ſet under 749, and ſubtracting it from 749 above, the Remainder will be nothing; which ſhews the Number 3249 to be a *Square Number*, and the *Root* thereof to be 57.

Example

Example 2. *Let it be required to find the* Square Root *of this Numbers* 27846729.

Set down your *Number* with *Points* over the proper *Figures*, and down-right Lines by the *pointed Figures*, with *Quotient Lines*, &c. as is done in the Margin: which done, begin your work in this manner.

	27 25	84	67	29	(5277
102)	2 2	84 04			*Resolvend* *Product*
1047)		80 73	67 29		*Resolvend* *Product*
10547)		7 7	38 38	29 29	*Resolvend* *Product*
		0	00	00	

1. The figures of the first *Period* being 27, the nearst *Square Number* thereto is 25, whose *Root* is 5; set 5 in the *Quotient*, and set 25 under 27, and under it draw a Line; and subtracting 25 from 27, the remainder will be 2; which set under 25.

2. To this remainder 2, bring down the two figures of the next *Period*, *viz.* 84, making it 284, for the first *Resolvend*.

3. Double the figure in the Quotient 5, it makes 10; which set in a Quotient on the left hand of 284, and ask how many times 10 in 28, the answer will be 2; set 2 in the Quotient, and also in the other Quotient by 10, making it 102.

4. Multiply 102 by 2, the last figure in the Quotient, and the Product will be 204; which set under 284; and subtracting 204 from 284, the remainder will be 80; which set under 204.

5. To this remainder 80, bring down the figures of the next *Period*, *viz.* 67, making the second *Resolvend* to be 8067.

6. Double the Quotient 52, and it makes 104; which set in a Quotient on the left hand of the *Resolvend* 8067; and ask how many times 104 can you have in 806, the answer will be 7 times; put 7 in the Quotient, and also in the other Quotient on the left hand, making it 1047.

7. Multiply the Quotient 1047 by the last figure in the Quotient 7, and the Product will be 7329; which set under 8067; and subtracting 7329 from the *Resolvend* 8067, there will remain 738; which set under the Line.

8. To this remainder 738 bring down the two figures of the next *Period*, *viz.* 29, making the third *Resolvend* to be 73829.

9. Double the Quotient 527, and it makes 1054; which set in a Quotient on the left hand of the *Resolvend* 73829, and ask how many times 1054 can you have in 7382, the answer will be 7 times; put 7 in the Quotient, and also in the other Quotient on the left hand of the *Resolvend*, making that Quotient to be 10547.

10. Multiply the Quotient 10547 by 7, the last figure in the Quotient, and the Product will be 73829; which subtract from the third *Resolvend* 73829, and the remainder will be nothing: which shews the given Number 27846729 to be a *Square Number*, and the *Root* thereof to be 5277. Which may be easily proved by multiplying 5277 in it self, for the Product of that Multiplication will be 27846729.

Other Examples for Practice.

	5 4	03	50	87	21	(22439
42)	1	03 84				*Resolvend* *Product*
444)		19 17	50 76			*Resolvend* *Product*
4484)		1 1	74 34	87 49		*Resolvend* *Product*
44869)			40 40	38 28	21 21	*Resolvend* *Product*
			00	00	00	*Remainder*

Another Example of a Number not Square.

	64 64	28	03	06	(8017
1601)		28 16	03 01		*Resolvend Product*
16027)		12 11	02 21	06 89	*Resolvend Product*
			80	17	*Remainder*

This Number 64280306 being not a Square Number, for that after the *Extraction* is ended, there remains 8017; which make the *Numerator* of a *Fraction*, and then double the *Root*, and add an *Unite* to it, and it will be 16035, which make the *Denominator* of the *Fraction*; and so will the near Root of 64280306 be $8017\frac{8017}{16035}$.

This is the usual way of finding the *Square Root* of a Number not Square; but see more of this in *Decimal Arithmetick*.

§ IV. *To find the* Square Root *of a* Vulgar Fraction, *which is commensurable.*

Before you attempt to extract the *Square Root* of a *Fraction*, you must first reduce the *Fraction* to its least Terms; for it may so fall out, that the *Fraction* in its given Terms may be incommensurable, but being reduced to its *least*, it may be commensurable; and then this is the *RULE*:

Extract the Square Root *of the Numerator of the* Fraction *given, and that shall be the Numerator of the* Root; *also, the* Square Root *of the Denominator of the given* Fraction, *shall be the Denominator of the* Root *of the given* Fraction.

Example. *Let it be required to find the* Square Root *of* $\frac{80}{245}$.

This Fraction being reduced to its least terms, (5 being the *common measure*,) will be $\frac{16}{49}$: now the *Square Root* of 16 is 4, the *Numerator* of the *Root*; and the *Square* of 49 is 7, for the *Denominator* of the *Root*: so that the *Square Root* of the Fraction $\frac{80}{245}$ (or $\frac{16}{49}$) is $\frac{4}{7}$.

Also, the *Square Root* of $\frac{25}{36}$ will be found to be $\frac{5}{6}$, and of $\frac{9}{81}$ will be found to be $\frac{1}{3}$, *&c.*

§ V. *To find the* Square Root *of a commensurable* Mixt Number.

For the effecting hereof this is the *RULE*:

Reduce the Mixt Number *into an* Improper Fraction, *and then the* Square Roots *of the* Numerator *and* Denominator *of the* Fraction *given, shall be the* Numerator *and* Denominator *of the* Square Root *thereof.*

Example. *Let it be required to* extract *the* Square Root *of* $34\frac{33}{64}$.

The *Mixt Number* given, *viz.* $34\frac{33}{64}$ reduced into an *Improper Fraction*, will be $\frac{2209}{64}$: this Fraction being in its *least terms*, the Square Root of 2209 the *Numerator*, is 47, for the *Numerator* of the *Root*; and the *Square Root* of the *Denominator* 64 is 8, for the *Denominator* of the *Square Root*: So that the *Square Root* of $\frac{2209}{64}$ is $\frac{47}{8}$, or which is equal to it $5\frac{7}{8}$.

Note. When the *Proper Fraction*, or *Mixt Number*, is incommensurable to its *Square Root*, prefix this Character before it, *viz.* $\surd$ or $\surd$q: So the *Square* of $7\frac{2}{3}$ will be thus expressed, $\surd$q $7\frac{2}{3}$; likewise the *Square Root* of $\frac{13}{19}$ must be thus expressed, $\surd$q $\frac{13}{16}$; for these, and such like, cannot be expressed by any *rational Numbers* whatsoever.

II. *Of the* CUBE ROOT.

§ I. *Of a* CUBE.

Definition. A *CUBE* is a *Geometrical Figure* of the *Third* Quantity, as a *Square* was of the *Second*; for as a *Square* consisteth of *Length* and *Breadth*, a *Cube* consisteth of *Length*, *Breadth*, and *Depth*; and so is composed of several *Superficies* added or connected together, and proceeds from Right Lines multiplied in themselves, constituting a *Superficies*, and that *Superficies* again multiplied in its *Side*, produceth a *Cube Number*, representing a *Body Solid*, consisting of six equal Sides.

Thus

Thus any Number multiplied in it ſelf, produceth a *Square*; and that *Square* multiplied again by its *Side*, produceth a *Cube*.

Figure I.

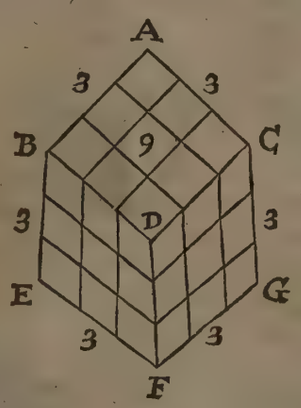
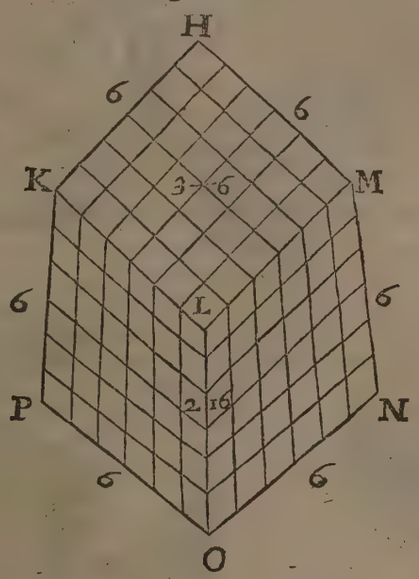


Figure II.



So 3 multiplied by 3 produceth the *Square* ABCD 9; and that *Square* ABCD 9 multiplied again in any of its Sides, AB, or BC 3, produceth 27, for the *Solid Cube* ABCDEFG; as in Figure I. *Fig.* I.

Where AB 3 multiplied in its ſelf, or any of the other Sides, AB or AC, produceth 9; and ſo many little *Squares* are contained in the *Square* ABCD.

Alſo, this *Square* ABCD, conſiſting of 9 little *Squares*, being multiplied in any of the other Sides, (as CG, BE, EF, or FG, each containing 3,) produceth 27; and ſo many ſmaller *Cubes* are contained in the *Cube* ABCDEFG.

Again, If the Side of the *Square* HKLM be 6, the *Square* will conſiſt of 36 *ſmaller Squares*; and thoſe 36 *Squares* multiplied again by the Side 6, will produce 216; and ſo many *ſmaller Cubes* are contained in the *greater Cube* HKPONM; as in Figure II. *Fig.* II.

And ſo of any other Number; as, let 12 be the Side of a *Square*, 12 multiplied by 12 will produce 144, for the *Square*; and 144 multiplied again by the Side 12 will produce 1728, for the *Cube*.

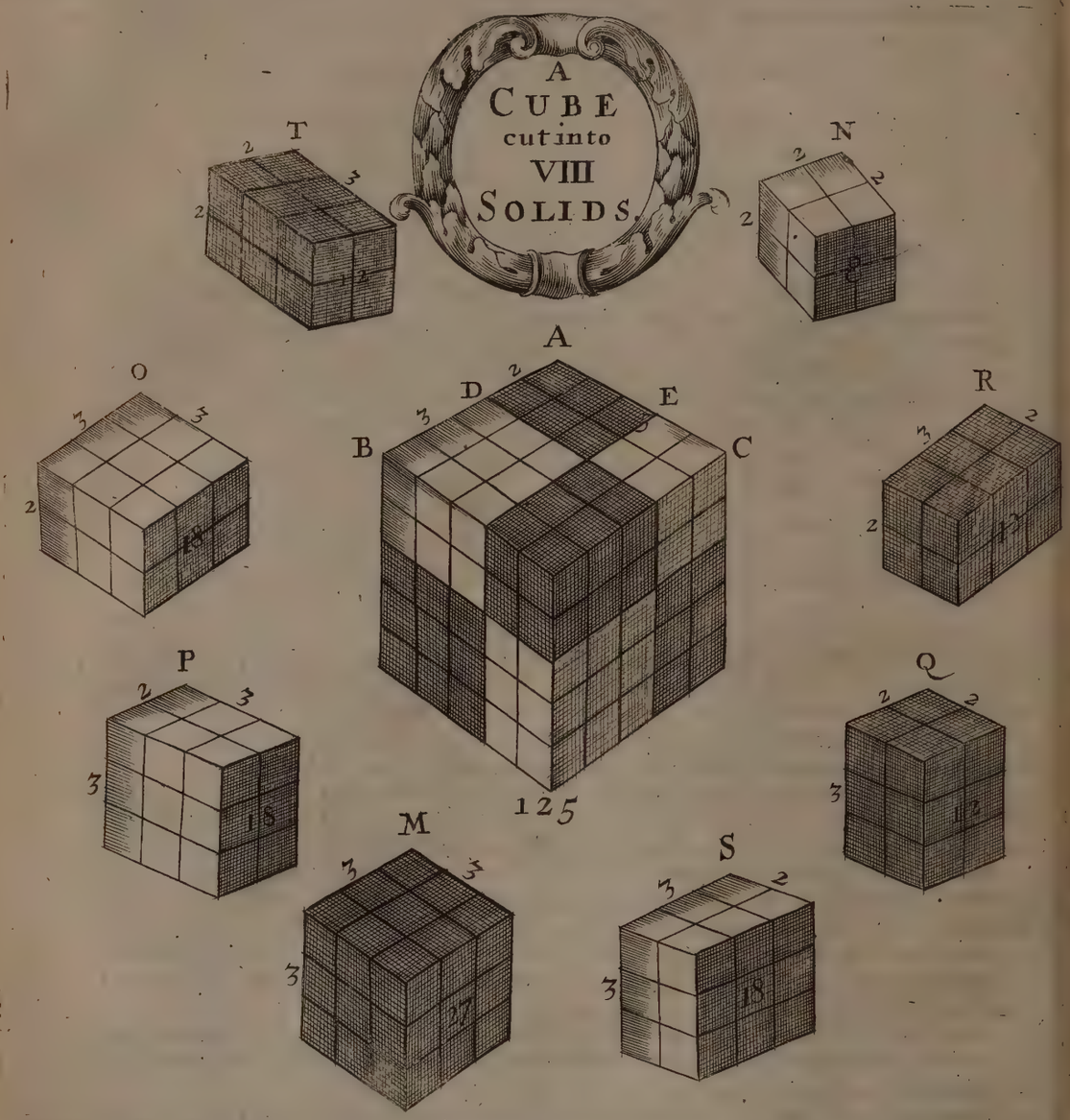
Wherefore to *Extract* the *Cube Root* is nothing elſe but to find out a Number, which being firſt multiplied in it ſelf, and then into the Product, produceth the given *Cube Number*.

Thus to *Extract* the *Cube Root* of 15625, is to find out the Number 25; which being firſt multiplied into it ſelf produceth 625, and that multiplied by 25, ſhall produce the given Number 15625.

Now becauſe this conſtruction of the *Cube* from a *ſingle Root*, contributes little towards the finding out that *Root* from a given *Cube Number*, therefore by the Ancients was found out that admirable Art of *cutting* or *dividing* of the *Root* into *Two Parts*; which they therefore called a *Binomial Root*; and from thoſe Two *Parts* they erected Eight *Solid Numbers*, whereof the *Greateſt* and the *Leaſt* are always pure *Cube Numbers*, made of thoſe two diſtinct *Parts* into which the *Root* is firſt divided; and of the other are made *Six Parallelopipedons*, by which is meant *Solid Numbers* made by multiplying the *Square* of one *Number* multiplied into another *Number*, in imitation of a *Geometrical Parallelopipedon*; which is *a* Solid Figure, *contained under* Six Quadrilatical Figures, *whereof thoſe which are oppoſite are Parrallel.*—Eucl. *El.* 11. *Def.* 30.

The Three *Greateſt Parallelopipedons* are equal one to the other, and each of them made by multiplying the *Square* of the *Greateſt Part* of the *Binomial Root* into the *Leſſer Part*.

The Three *Leſſer Parallelopipedons* are alſo equal one to the other, and each of them made by multiplying the *Square* of the *Leſſer Part* of the *Binomial Root* into the *Greater Part*.



Thus in the Figure, the *Whole Root* AB, or AC, is 5; and it is cut into *Two Parts*, AD 2, the *Leſſer*, and DB 3, the *Greater*.

The *Cube* of the *Greater Part* DB in DB in DB, that is 3 in 3 in 3, which makes — 27

And ſuch is the *Cube* noted with M.

The *Cube* of the *Leſſer Part* EC in EC in EC, that is, 2 in 2 in 2, which makes — 08

And ſuch is the *Cube* noted with N.

Each of the Three *Greater Parallelopipedons* is made of the *Square* of BD into AD, that is, of 3 in 3 in 2, and is — 18

The ſame again — 18

The ſame again — 18

Of theſe there are Three noted with O, P, and S.

Each of the Three *Leſſer Parallelopipedons* is made of the *Square* of EC into AD, that is 2 in 2 in 3, and is — 12

The ſame again — 12

The ſame again — 12

Of theſe alſo there are Three noted with Q, R, and T.

Then,

Then,

The Two *Cubes*, { M, the *Greater*, is ——— 27
N, the *Leſſer*, is ——— 08

The Three *Greater Parallelopipedons*, are { O, ——— 18
P, ——— 18
S, ——— 18

The Three *Leſſer Parallelopipedons*, are { Q, ——— 12
R, ——— 12
T, ——— 12

The total Summ of all theſe is ——— 125

And it is equal to the *Root* AB, 5 in 5 in 5; for 5 in 5 is 25, and 25 in 5 is 125; this being the whole *Cube*, and the others the *Parts* thereof: All the *Parts* equal the *Whole*. ——— 125

Now it is here to be underſtood that the *Unites* in this *Cube Number* 125, are ſo many ſmall *Cubes*, or *Unites*, (or ſo many *Dice*,) and,

27, 8 } of theſe being piled one upon the other equally, do make up the *Cube* — { M. N.

18, 18, 18 } of theſe being ſo orderly piled one upon the other, do make the *Parallelopipedon* { O. P. S.

12, 12, 12 } of theſe being ſo orderly piled one upon the other, do make the *Parallelopipedon* { Q. R. T.

125 The Summ.

After the ſame manner, let the *Root* be 25; (that is, by conceiving every one of the little *Squares* of the Side AB to be divided into 5 *Parts*, and ſo the Whole Side AB to be the *Root* 25,) and that to be made *Binomial*, by cutting it into two *Parts*, 20 and 5: Then.

The *Cube* of the *Greater Part* 20, is made of 20 in 20 in 20, that is, ——— 8000
The *Cube* of the *Leſſer Part* 5, is made of 5 in 5 in 5, that is, ——— 125
The *Greater Parallelopipedon* is made of the *Part* 20 in 20 in 5, that is 2000; of which there are Three, and they make ——— 6000
The *Leſſer Parallelopipedon* is made of the *Part* 5 in it ſelf, in the *Part* 20, that is 500; of which there are Three alſo, and they make ——— 1500
The Total ——— 15625

And that *Total Summ* of all the *Eight Solids*, is the Summ of the entire *Cube*, whoſe Side is 25.

Now, all that is here ſaid is, in effect, the ſame which *Ramus*, in his *Geometry*, *Lib.* 24. *de Cubo.* § 10. ſaith,

If a Right Line *be cut into two* Segments, *the* Cube *of the* Whole *ſhall be equal to the* Cubes *of the* Segments, *and thrice the double* Solids *made of the* Square *of the* Segments.

Theſe things here delivered being well underſtood, the *extracting* of the *Cube Root* of any Number will be eaſily conceived; and how it is to be effected ſhall hereafter be ſhewn.

§ II. *Of CUBE Numbers.*

Of *Cubical Numbers* there are three diſtinct Kinds or Species, *viz. Single*, *Compound*, and *Irrational.*

I. *Single.* Such are called *Single Cube Numbers* as are made of any one ſingle ſignificant figure, multiplied twice in it ſelf: As, 1 multiplies nothing, and ſo is both *Root* and *Cube*; but 2 times 2 is 4, and 2 times 4 is 8; thus 2 is the *Root*, and 8 is the *Cube*: alſo 3 times 3 is 9, and three times 9 is 27; here 3 is the *Root*, and 27 the *Cube*.

And

And so of all the Nine *Digit* Numbers, as in this Table. So,

The *Root*, or *Side*,	multiplied in it self, produceth the *Square Number*	and that multiplied again into the *Side*, produceth the *Cube Number*
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729

II. *Compound.* Such are called *Compound Cube Numbers*, whose *Roots* consist of more figures than *one*, as 12, the *Cube* whereof is 1728; the *Cube* of 22 is 10648, *&c.*

III. *Irrational.* Those are called *Irrational Cube Numbers*, whose exact *Cube Root* cannot be found out by any Artifice yet discovered, either in *Whole Numbers* or *Fractions*; such are the *Cube Roots* of 2, 4, 7, 10, and infinite others.

§III. *The* GENESIS *for extracting the* Cube Root.

Example. *Let it be required to find the* Cube Number *whose* Side *or* Root *is* 57.

5	..7
125	....
52	5..
7	35.
	343
185	193

1. Set down the *Root* 57, as in the Margin, with the interval of two figures between 5 and 7, and draw a Line under them, and also two downright Lines, the one next after 7, the other after 5, for the more orderly placing of the Numbers to be added; then let the *Root* given be supposed to be divided into these two parts, 500 and 7.

2. Set the *Cube* of 5, which is 125, under 5, (Unites under Unites, *&c.*) and the *Cube* of 7, which is 343, under 7, two places below the *Cube* of 25, (Unites under Unites.)

3. Triple the *Square* of 5, which is 25, and it makes 75; which multiply by 7, and it makes 525; which set under 125, one place forwarder to the right hand.

4. Triple 5, it makes 15; which multiply by 49, (the Square of 7,) it makes 735; which set under 525, one place forwarder to the right hand.

5. Add all these together in the same order as they stand, the Summ of them will be 185193; which is the *Cube Number* of which 57 is the *Root*: Which may be easily proved by multiplying 57 in it self, and that Product again by 57, the last Product will be 185193.

Other Examples for Practice.

4	..8
64	...
38	4..
7	68.
	512
110	592

9	..9
729	...
218	7..
21	87.
	729
970	299

7	..6
343	...
88	2..
7	56.
	216
438	976

§IV. *To Extract the* Cube Root.

When a Number is given to have the *Cube Root* thereof found, you must first write the Number down, then put a prick over the first figure towards the right hand, which is the place of Unites, and so over every third figure from that place of Unites; then by every pointed figure draw a downright Line, for the more orderly setting of the figures to be added and subtracted, and also a crooked Line for a *Quotient*, on the right hand; then is your *Number* fitted for *Extraction*; for the performance whereof observe the following directions.

Example 1. *Let it be required to find the* Cube Root *of the Compound* Cube Number 185193.

1. Set down the Number given 185193, and make a Prick over 3, the place of Unity; and, missing two places, make another Prick over 5; then make a crooked Line for a *Quotient*, and by 3 and 5 draw two down-right Lines, as you may see in the following work.

3. The

2. The three firſt figures of the given Number towards the left hand, 185, being the firſt *Period*, ſeek (by the foregoing Table) the neareſt *Cube Number* to it, being leſs, which you will find to be 125, whoſe *Cube Root* is 5; place the *Root* 5 in the *Quotient*, and the *Cube* thereof 125 under 185, drawing a Line under 125; and ſubtracting it from 185, there will remain 60; which ſet under the Line: And ſo is the firſt operation of your *Extraction* ended.

3. To the 60 which remained, bring down the three figures of the next *Period*, *viz.* 193, making the number 60193, which is called the *Reſolvend*, under which draw a Line, and then the Work will ſtand thus:

185	193	(5
125		
60	193	*Reſolvend*

4. Triple the Root in the Quotient 5, it makes 15; which ſet under the *Reſolvend*, in ſuch order, that the place of Unites in this Triple may ſtand under the place of Tens in the *Reſolvend*; ſo the Triple of the Root 5 being 15, ſet the Unite 5 under 9, the place of Tens in the *Reſolvend*: and then the Work will ſtand thus:

185	193	(5
125		
60	193	*Reſolvend*
	15	*Triple of the Root* 5

5. Triple the Square of the Root 5, and it makes 75, (for 5 times 5 is 25, and 3 times 25 is 75;) which place under the Triple of the Root, in ſuch order that the place of Unites in this may ſtand under the place of Tens in that: Then will the Work ſtand thus:

185	193	(5
125		
60	193	*Reſolvend*
	15	*Triple of the Root* 5
7	5	*Triple of the Square of the Root* 5

6. Draw a Line under the Triple of the Square of the Root, and add that and the Triple of the Root together, in the ſame order as they ſtand, ſo ſhall their Summ be 765, for a *Diviſor*; under which alſo draw a Line, and the Work will ſtand thus:

185	193	(5
125		
60	193	*Reſolvend*
	15	*Triple of the Root* 5
7	5	*Triple of the Square of the Root* 5
7	65	*Diviſor*

7. Draw a crooked Line on the left hand of the *Reſolvend*, in which place this *Diviſor* 765 laſt found, to which the whole *Reſolvend* (except the place of Unity) muſt be a ſuppoſed *Dividend*; that is, ask how many times 765 you can have in 6019, the anſwer will be 7 times, which place in the *Quotient*; and this 7 is the ſecond figure of the *Root*; and then the Work will ſtand thus:

	185	193	(57
	125		
765)	60	193	*Reſolvend*
		15	*Triple of the Root* 5
	7	5	*Triple of the Square of the Root* 5
	7	65	*Diviſor*

8. Cube

8. Cube 7, the laſt figure of the *Root*, and it is 343; which place under the *Reſolvend*, in ſuch order that Unites may ſtand under Unites, *&c.* ſo will 343, the *Cube* now found, ſtand under 193 of the *Reſolvend*; and the Work will ſtand thus:

	185 / 125	193	(57
765)	60	193	*Reſolvend*
		15	*Triple of the Root* 5
	7	5	*Triple of the Square of the Root* 5
	7	65	*Diviſor*
		343	*The Cube of* 7, *the laſt figure of the Root*

9. Multiply the Square of 7, the laſt figure of the *Root*, namely 49, by the *Triple Root* next under the *Reſolvend*, *viz.* by 15, the Product will be 735; which place under 343, the *Cube* laſt ſet down, in ſuch order that the place of Unites in this, may ſtand under the place of Tens in that; and then will the Work ſtand thus,

	185 / 125	193	(57
765)	60	193	*Reſolvend*
		15	*Triple of the Root* 5
	7	5	*Triple of the Square of the Root* 5
	7	65	*Diviſor*
		343	*The Cube of* 7, *the laſt figure of the Root*
	7	35	*The Square of* 7, *by the Triple Square of* 5

10. Multiply the Triple Square of the Root 5, *viz.* 75, by 7 the ſecond figure of the *Root*, the Product will be 525; which place under 735, the Number laſt found, in ſuch order that the place of Unites in this ſtand under the place of Tens in that; and then will the work ſtand thus:

	185 / 125	193	(57
765)	60	193	*Reſolvend*
		15	*Triple of the Root* 5
	7	5	*Triple of the Square of the Root* 5
	7	65	*Diviſor*
		343	*The Cube of* 7, *the laſt figure of the Root*
	7	35	*The Square of* 7, *by the Triple Square of* 5
	52	5	*The Triple Square of the Root* 5, *in the Root* 7

Laſtly, Draw a Line under the three laſt Numbers, and add them together in the ſame order as they ſtand, and their Summ will be 60193, which may be called the *Subtrahend*, becauſe it is always to be ſubtracted from the *Reſolvend*; and in this *Example* they are equal one to the other, for the *Remainder* is 00000; which ſhews the given Number to be a *Compound Cube Number*, whoſe *Cube Root* is 57; and the whole Work will ſtand thus.

	185 / 125	193	(57
765)	60	193	*Reſolvend*
		15	*Triple of the Root* 5
	7	5	*Triple of the Square of the Root* 5
	7	65	*Diviſor*
		343	*The Cube of* 7, *the laſt figure of the Root*
	7	35	*The Square of* 7, *by the Triple Square of* 5
	52	5	*The Triple Square of the Root* 5, *in the Root* 7
	60	193	*Subtrahend*
	00	000	*Remainder*

Now

Now if you compare this *Example* with the former *GENESI*, you ſhall find that the firſt Number 125, will be equal to the neareſt *Square* leſs than 185; alſo the ſecond Number there 525, is the ſame with the *Triple Square* of the *Root* 5, multiplied in the *Root* 7; again, the third Number there 735, is the ſame with the *Square* of the *Root* 7, multiplied in the *Triple Root* of 5; and laſtly, the Number 343 there, is the ſame with the *Cube* of the *Root* 7.

Example 2. *Let it be required to find the* Cube Root *of* 952561522 63.

1. Write down the Number, and point it, and draw a crooked Line for the Quotient, and downright Lines between every third figure from the left hand.

2. Find the neareſt *Cube* number, leſs than 95, the firſt *Period*; which will be 64, whoſe *Cube Root* is 4; ſet 4 in the Quotient, and ſet 64 the *Cube*, under 95, and draw a Line under it.

3. Subtract 64 from 95, the remainder is 31; ſet 31 under 64, and bring down to it the three figures of the next *Period*, 256; ſo will the number be 31256, for the firſt *Reſolvend*; under which draw a Line.

4. Multiply the *Root* 4 in the Quotient, by 3, that is, triple it, and it makes 12; ſet 12 under the *Reſolvend*, ſo as the place of Unites in 12 may ſtand under the place of Tens in the *Reſolvend*.

5. Triple the Square of the Root 4, *viz.* 16, it makes 48; which ſet under 12, ſo that the Unites place of this may ſtand under the place of Tens of the triple Root.

6. Draw a Line under theſe two Numbers, and add them together in the order they ſtand, and their Summ will be 492, for a *Diviſor*, under which draw a Line.

7. On the Left hand of the *Reſolvend* make a crooked Line for a Quotient, in which the foregoing *Diviſor* 492 is to be placed; and the *Reſolvend*, (all but the place of Unity,) namely, 3125 is to be a *Dividend*; and then ask how many times can you have 492 in 3125, the anſwer will be 5 times; ſet 5 in the Quotient.

8. Cube the laſt figure, or *Root*, in the Quotient, and it will be 125; which you muſt ſet under the firſt *Reſolvend*, Unites under Unites, *&c.* that is, 125 under 256.

9. Multiply the Square of 5, the laſt figure in the Quotient, which is 25, by the triple of the Root 4, namely 12 above found, and the Product will be 300; which ſet under 125, the Unites place of this under the place of Tens in that.

10. Multiply the triple Square of the Root 4 before found, *viz.* 48, by 5, the laſt figure in the Quotient, and the Product will be 240; which ſet under 300, the Unites place of this under the place of Tens in that.

11. Draw a Line, and add theſe three Numbers together in the ſame order as they ſtand, and their ſumm will be 27125; which you may call the *Subtrahend*, becauſe it is to be ſubtracted from the *Reſolvend*.

12. Draw a Line under the *Subtrahend*, and ſubtract it out of the *Reſolvend*, the remainder will be 4131; to which bring down 152, the three figures of the next *Period*, and it makes the Number 4131152, for a ſecond *Reſolvend*.

13. Triple the whole Quotient 45, it makes 135; which ſet under the ſecond *Reſolvend*, ſo that the Unites place of this may ſtand under the place of Tens in that.

14. Multiply the Square of the Quotient 45, which is 2025, by 3, the Product will be 6075; which ſet under 135, in ſuch order that the place of Unites in this may ſtand under the place of Tens in that.

15. Draw a Line under, and add theſe three laſt Numbers together, their ſumm will be 60885; which muſt be a ſecond *Diviſor*; under which draw a Line.

15. On the Left hand of the ſecond *Reſolvend* make a crooked Line for a Quotient, and in it put this new *Diviſor* 60885; to which the ſecond *Reſolvend* (all but the laſt figure towards the right hand) is to be a *Dividend*; then ask how many times 60885 can be had in 413115, the anſwer will be 6 times; ſet 6 in the Quotient, for the third figure of the *Root*.

17. Multiply the Square of 6, (the laſt figure in the Quotient,) *viz.* 36, by the triple of the Square of 45 before found, namely, by 135, and the Product will be 4860; which ſet under 216, the place of Unites in this under the place of Tens in that.

18. Multiply the triple Square of the Quotient 45, before found, (*viz.* 6075,) by 6, the laſt figure in the Quotient, the Product will be 36450; which ſet under 4860, ſo the Unites place in this may ſtand under the place of Tens in that.

19. Draw a Line, and add theſe three Numbers together in the ſame order as they ſtand, and the ſumm of them will be 3693816; which is a ſecond *Subtrahend*.

20. Draw a Line under it, and ſubtract this *Subtrahend* from the next *Reſolvend* above, that is, from 4131152, and the remainder will be 437336; to which bring down

263, the three figures of the next *Period*, and the whole ſumm will be 437336263, for a third *Reſolvend*; under which draw a Line: and,

21. Multiply the Quotient 456 by 3, the Product whereof will be 1368; which ſet under the *Reſolvend*, ſo that the Unites place of this Product may ſtand under the place of Tens in the *Reſolvend*.

22. Multiply the Quotient 456 in it ſelf, and that Product again by 3, and the laſt Product will be 623808; which ſet under 1368, the Unites place of this, under the place of Tens in that.

23. Draw a Line, and add theſe two Numbers together, their ſumm will be 6239448 for a third *Diviſor*; under which draw a Line.

24. On the Left hand of the third *Reſolvend* make a crooked Line for a Quotient, and in it put this third *Diviſor*, 6239448; to which the third *Reſolvend* (all but the laſt figure towards the Right hand) is to be a *Dividend:* Then ask how many times 6239448 can be had in 43733626; (or ſhorter, how many times 6 in 43, or 62 in 437,) the anſwer will be 7 times; ſet 7 in the Quotient, for the fourth and laſt figure of the *Root*.

25. Multiply the laſt figure in the Quotient 7, in it ſelf twice, that is, Cube 7, and it makes 343; which ſet under the *Reſolvend*, ſo that the Unites place of this, may ſtand under the Unites place of the third *Reſolvend*.

26. Multiply the laſt triple of the Quotient 1368 by 49, the Square of 7, the laſt figure in the Quotient, and the Product will be 67032; which ſet under 343, ſo that the Unites place in this may ſtand under the place of Tens in that.

27. Multiply the laſt triple Square of the Quotient 623808, by 7, the laſt figure of the Quotient, and the Product will be 4366656; which ſet under 67032, ſo that the Unites place of this may ſtand under the place of Tens in that.

28. Draw a Line, and add theſe three Numbers together in the ſame order as they ſtand, and their ſumm will be 437336263, for a third *Subtrahend*, which is to be ſubtracted from the third *Reſolvend*; which being equal thereunto, the remainder is nothing; which ſhews the Number given 952561522 63, to be a *Compound Cube Number*, and 4567 to be the *Root* thereof. For the better underſtanding hereof, view the *Synopſis* following:

A Synopſis of this laſt Extraction.

	95 64	256	152	263	(4567
492)	31	256			*Firſt Reſolvend*
	4	12 8			*Triple of the Quotient* 4 *Triple Square of the Quotient* 4
	4	92			*Diviſor*
	3 24	125 00 0			*Cube of the laſt Figure in the Quotient* 5 *Square of* 5 *by the Triple Square of* 4 *Triple Square of the Root* 4 *in the Root* 5
	27	125			*Subtrahend*
60885)	4	131	152		*Second Reſolvend*
		607	1 35 5		*Triple of the Quotient* 45 *Triple Square of the Quotient* 45
		608	85		*Diviſor*
	3	48 645	216 60 0		*Cube of* 6, *the laſt Figure in the Quotient* *Square of* 6, *by the Triple Square of* 45 *Triple Square of* 45 *in the laſt Figure* 6
	3	693	816		*Subtrahend*
6239448)		437	336	263	*Third Reſolvend*
		62	13 380	68 8	*Triple of the Quotient* 456 *Triple Square of the Quotient* 456
		62	394	48	*Diviſor*
		436	670 665	343 32 6	*Cube of* 7, *the laſt Figure in the Quotient* *Square of* 7, *by the Triple Square of* 456 *Triple Square of* 456 *in the laſt Fig. in the Quotient* 7
		437	336	263	*Subtrahend*
		000	000	000	*Remainder*

Note.

Note. When at any time in your Extraction you find a *Subtrahend* to be greater than the *Reſolvend* before-going, from whence it is always to be ſubtracted, your Work is erroneous, and muſt be amended by placing a leſſer figure in the Quotient.

§ V. *To Extract the* Cube Root *of a* Vulgar Fraction, *or* Mixt Number.

Before the Cube Root of a *Fraction*, or *Mixt Number* be extracted, it ought to be reduced to its leaſt terms; and then this is the *RULE:*

Extract the Cube Root *of the* Numerator *of the Fraction, and that ſhall be the* Numerator *of the* Cube Root; *alſo, the* Cube Root *of the* Denominator *of the Fraction, ſhall be the* Denominator *of the* Cube Root *of the Fraction.*

Example. *Let it be required to find the* Cube Root *of this* Proper Fraction.

Firſt, Neither the Numerator nor Denominator of this Fraction, as it now ſtands, are *Cube Numbers*; but being reduced to its leaſt terms, it becomes $\frac{8}{27}$, which are both ſo; and the *Cube Root* of 8 is 2, for the *Numerator* of the *Root*; and the *Cube Root* of 27 is 3, for the *Denominator* of the *Root:* So that the *Cube Root* of $\frac{48}{1944}$, or $\frac{8}{27}$, is $\frac{2}{3}$.

Example 2. *Let it be required to find the* Cube Root *of this* Mixt Number $20\frac{51}{64}$.

Reduce the *Mixt Number* $20\frac{51}{64}$ into an *Improper Fraction*, and it is $\frac{1331}{64}$, and ſeeing it is in its leaſt terms, it needs no *reducing*; then the *Cube Root* of the *Numerator* 1331 is 11; and the *Cube Root* of the *Denominator* 64 is 4: ſo that the *Cube Root* of the *Improper Fraction* $\frac{1331}{64}$ is $\frac{11}{4}$, or $2\frac{3}{4}$.

Note. When a *Vulgar Fraction* or *Mixt Number* is incommenſurable to its *Root*, ſuch are uſually thus expreſſed; *viz.* the *Cube Root* of $\frac{3}{4}$ thus, $\sqrt{c}\ \frac{3}{4}$; and the *Cube Root* of $2\frac{3}{4}$ thus, $\sqrt{c}\ 2\frac{3}{4}$, *&c.*

I might now proceed farther, to ſhew how to *extract* the *Biquadrate Root*, *&c.* but I leave that till I come to treat of *Artificial* or *Logarithmical Arithmetick*; wherein ſhall be ſhewed how, with eaſe and facility, to extract the *Root*, to the ſixth, ſeventh, or eighth Power.

§ VI. *Of finding the* Square *and* Cube Roots *of* Numbers *which are incommenſurable to their* Roots, *another way.*

The *Extraction* of the *Square* and *Cube Roots* hath been largely treated of before in this Chapter, but the *Examples* were of ſuch *Numbers* as were commenſurable to their *Roots*; that is to ſay, they were exact *Square* or *Cube Numbers:* all that I ſhall ſay of *Extracting* of *Roots* in this place, ſhall be to ſhew how to find the *Roots*, as near as may be, of ſuch *Numbers* as are *incommenſurable* to their *Roots*; wherein I ſhall be very brief, giving you only one *Example* of a *Square*, and another of a *Cube Number*, *incommenſurable.*

Example of the Square Root.

Let it be required to find the Square Root *of* 71958.

Having ſet down the Number, and pointed it, (according to the former directions,) and extracted the *Square Root* thereof; which you will find to be 268, and the *Remainder* to be 134: now ſeeing there is a *Remainder*, add to the *given Number* any even number of *Cyphers*, as two, four, ſix, or eight; (as in this Example ſix *Cyphers* are added, which make three places of *Decimal Parts* in the *Root*,) and then proceeding on in your *Extraction*, you will find the *Root* to be 268 *Integers*, and .249 *Parts.*

	7	19	58	00	00	00	(268.249	
	4							
46)	3	19						*Reſolvend*
	2	76						
528)		43	58					*Reſolvend*
		42	24					
5362)		1	34	00				*Reſolvend*
		1	07	24				
53644)			26	76	00			*Reſolvend*
			21	45	76			
536489)			5	30	24	00		*Reſolvend*
			4	82	84	01		
	Remainder			47	39	99	00	

By the Work you may see that there are three *Places* of *Decimal Parts* in the *Root*, the *Root* being 268 *Integers*, and 249 *Parts*, and yet there is a *Remainder* of 473999; which shews that the *Root* in the Quotient is too little; and if you should make the 249 *Parts* 250, it would be too much; and if you should proceed farther by adding two *Cyphers* more, the next figure in the Quotient will be 9, and the *Parts* would be 2499, and yet that would be too little; but as it is, it wanteth not the $\frac{1}{1000}$ part of an Unite; and if two *Cyphers* more were added, it would not want $\frac{1}{10000}$ part of an Unite; and the more *Cyphers* you add, you shall come the nearer the true *Root*, but never attain it justly.

Example of the Cube Root.

Let it be required to find the Cube Root *of this Number*, 23456789.

The Number being written down, pointed, and the *Cube Root* extracted, as is done in the Example following; the *Cube Root* will be found to be 286, and then there will be a *Remainder* of 63133; and to find the *Root* nearer, add thereto 3, 6, 9, or 12 *Cyphers*, whereby to attain more *Decimal Parts* in the *Root*; as in this Example, having added six *Cyphers* to the given *Number*, go on with your *Extraction* till you have two places of *Decimal Parts* in your *Root*; for it is 286 *Integers*, and 25 *Decimal Parts*; which is too little; for that work being ended, you will find a *Remainder* of 1732359375, to which if you add three *Cyphers* more, there would be three places of *Parts* in the *Root*; but this gives the *Root* to the $\frac{1}{1000}$ part of an Unite, and if another were added, it would give the *Root* to the $\frac{1}{10000}$ part of an Unite, *&c.* See the *Example.*

	23 8	456	789	000	000	(286.25
126)	15	456				*Resolvend*
	1	06 2				
	1	26				*Divisor*
	3 9	512 84 6				
	13	952				*Subtrahend*
23604)	1	504	789			*Resolvend*
		235	84 2			
		236	04			*Divisor*
	1	30 411	216 24 2			
	1	441	656			*Subtrahend*
2454738)		63	133	000		*Resolvend*
		24	538	8 58 8		
		24	547	38		*Divisor*
		49	34 077	32 6	8	
		49	111	928		*Subtrahend*
245739906)		14	021	072	000	*Resolvend*
		2	457	313	85 86 2	
		2	457	399	06	*Divisor*
		12	2 286	146 566	125 50 0	
		12	288	712	625	*Subtrahend*
		1	732	359	375	000 *Resolv. &c.*

Of

Of the following TABLE *of* SQUARE *and* CUBE ROOTS.

§ 1. *Any* Number (*not exceeding* 1000) *being given, to find the* Square *and* Cube *thereof.*

Let 446 be the *Number* given; find this Number in the first Column of the Table, (under the Title *Root,*) and against it, in the second Column, (under *Square,*) you shall find 198916, which is the *Square* of 446: and against the same Number in the third Column, (under *Cube,*) you shall find 88716536, which is the *Cube* of 446.

§ 2. *Any* Number (*not exceeding* 1000000) *being given, to find the* Square Root *thereof.*

Let 232324 be the *Number* given; look in the Column of *Squares*, till you find the given Number, the Number which stands against it on the left hand (under *Root*) is the *Square Root* thereof: so if you look for 232324 in the Column of *Squares*, against it you shall find 482, which is the *Square Root* of 232324.

§ 3. *Any* Number (*not exceeding* 1000000000) *being given, to find the* Cube Root *thereof.*

Let 700227072 be the *Number* given; look in the *Column* (under *Cube*) till you find the given Number, and right against it in the first Column (under *Root*) you shall find 888, which is the *Cube Root* of 700227072.

And in this manner you may find	25	to be the *Square Root* of	6756	and the *Cube Root* of	15625
	87		75569		648303
	112		125544		1404928
	276		761176		21024576
	844		7112336		601211584

§ 4. *To find the* Square Root *of a* Vulgar Fraction.

Find the *Square Root* of the *Fraction*'s *Numerator*, and that shall be the *Numerator* of the *Root*; also, find the *Square Root* of the *Fraction*'s *Denominator*, and that shall be the *Denominator* of the *Root*.

Let $\frac{49}{289}$ *be the given* Fraction.

The *Square Root* of 49 is 7, for the *Numerator*; and the *Square Root* of 289 is 17, for the *Denominator*: So that the *Square Root* of $\frac{49}{289}$ is $\frac{7}{17}$.

§ 5. *To find the* Square Root *of a* Mixt Number.

Reduce the *Mixt Number* into an *Improper Fraction*; then, the *Square Root* of the *Numerator* shall be the *Numerator* of the *Root*; and the *Square Root* of the *Denominator* shall be the *Denominator* of the *Root*.

Let the Mixt Number, *being reduced into an* Improper Fraction *be* $\frac{8281}{25}$.

The *Square Root* of 8281 is 91, for the *Numerator* of the *Root*; and the *Square Root* of 25 is 5, for the *Denominator* of the *Root*: So the *Square Root* of $\frac{8281}{25}$ is $\frac{91}{5}$, or $18\frac{1}{5}$.

§ 6. *To find the* Square *of a* Fraction.

The *Square* of the *Numerator* of the *Fraction* shall be the *Numerator* of the *Square*; and the *Square* of the *Denominator* of the *Fraction* shall be the *Denominator* of the *Square*.

Let the Fraction *given be* $\frac{657}{876}$.

The *Square* of 657 is 431649, for the *Numerator* of the *Square*; and the *Square* of 876 is 767376, for the *Denominator* of the *Square*: So that the *Square Root* of the *Fraction* $\frac{657}{876}$ is $\frac{431649}{767376}$.

§ 7. *To*

§ 7. *To find the* Square *of a* Mixt Number.

Let the Mixt Number *given be* $264\frac{2}{3}$, *which, reduced into an* Improper Fraction, *is* $\frac{794}{3}$.

The *Square* of 794 is 630436, and the *Square* of 3 is 9: So the *Square* of the *Mixt Number* $\frac{794}{3}$ is $\frac{630436}{9}$, which is $260145\frac{1}{3}$.

§ 8. *To find the* Cube *of a* Fraction.

The *Cube* of the *Numerator* of the *Fraction* shall be the *Numerator* of the *Cube*; and the *Cube* of the *Denominator* of the *Fraction* shall be the *Denominator* of the *Cube*.

Let the Cube *of the* Fraction $\frac{17}{236}$ *be required.*

The *Cube* of 17 is 4913, for the *Numerator*; and the *Cube* of 236 is 55696, for the *Denominator*: And so the *Cube* of $\frac{17}{236}$ is $\frac{4913}{55696}$.

§ 9. *To find the* Cube Root *of a* Fraction.

The *Cube Root* of the *Numerator* of the *Fraction*, shall be the *Numerator* of the *Root*; and the *Cube Root* of the *Denominator* of the *Fraction*, shall be the *Denominator* of the *Root*.

Let the Fraction *given be* $\frac{2744}{438976}$.

The *Cube Root* of 2744 is 14, which is the *Numerator* of the *Cube Root*; and the *Cube Root* of 438976 is 76, which must be the *Denominator* of the *Cube Root*: So that the *Cube Root* of the *Fraction* $\frac{2744}{438976}$ is $\frac{14}{76}$.

§ 10. *To find* (*nearly*) *the* Square Root *of a* Number *not* Square, *or incommensurable to its* Root.

If you cannot find the Number given in the *Table*, find the neareſt *Square Number* that is leſs, and note what is the *Root* ſtanding againſt it; which ſet down as the true *Root*: then double this *Root*, and add 1 unto it, ſo ſhall that be the *Denominator* of a *Fraction*; then ſubtract the Number in the *Table* from the given Number, and the *difference* ſhall be the *Numerator* to the former *Denominator*.

Let it be required to find the (*neareſt*) Square Root *of this Number* 33138.

In the *Table* you cannot find this *Number*; but the neareſt leſs than it in the *Table* is 33124, whoſe *Root* is 182, which *Root* ſet down as the *Root*; then double this *Root*, and add an *Unite* to it, and it will be 365; which muſt be a *Denominator*; Then, the *Difference* between the *Number* in the *Table* 33124, and 33138 the *Number given*, is 14, which muſt be the *Numerator*: And then will the *neareſt Square Root* to 33138 the given Number be $182\frac{14}{365}$.

§ 11. *To find* (*nearly*) *the* Cube Root *of a* Number *not exactly* Cubical.

If you cannot find the Number given in the *Table* of *Cubes*, take the neareſt Number to it which is leſs, and ſet down the *Root* of that Number; for the Integral part of the *Root* of the given *Number*: Then ſubtract the Number found in the *Table* from the Number given; for that difference is the *Numerator* of the *Fraction* of your *Root*. Then to find the *Denominator*, multiply the *Root* by 3, and adding 1 to the *Root*, multiply it by the *Triple* of the *Root*, this laſt *Product* (an *Unite* being added to it) ſhall be the *Denominator* of the *Fraction* of the *Root*.

Let it be required to find the (*neareſt*) Cube Root *of the* Number, 42893.

Looking into the Table of *Cubes* for the given Number, you cannot find it there, but the neareſt to it leſs, is 42877, whoſe *Cube Root* is 35; which ſet down: Then, the *Difference* between 42893 (the given Number) and 42877 (the Number in the *Table*) is 16, for the *Numerator* of the *Fraction* of the *Root*. Then to find the *Denominator*, multiply the *Root* 35 by 3, the *Product* will be 105; then add 1 to the *Root*, making it 36, by which multiply 105 and the Product will be 3780, to which add an Unite, and it will be 3781; and is the *Denominator* of the *Fraction* of the *Cube Root*: So that the neareſt *Cube Root* of 42893 is $35\frac{16}{3781}$.

Let this ſuffice for the Uſe of the Table.

A TA-

A TABLE OF Square and Cube ROOTS,

From 1 to 1000.

Root	Square	Cube	Root	Square	Cube	Root	Square	Cube
1	1	1	26	676	17576	51	2601	135651
2	4	8	27	729	19683	52	2704	140608
3	9	27	28	784	21972	53	2809	148877
4	16	64	29	841	24389	54	2916	157464
5	25	125	30	900	27000	55	3025	167375
6	36	216	31	961	29791	56	3136	175616
7	49	343	32	1024	32768	57	3249	185193
8	64	512	33	1089	35939	58	3364	195112
9	81	729	34	1156	39304	59	3481	205379
10	100	1000	35	1225	42825	60	3600	216000
11	121	1331	36	1296	46656	61	3721	216981
12	144	1728	37	1369	50653	62	3844	238328
13	169	2197	38	1444	54872	63	3969	250047
14	196	2744	39	1521	59319	64	4096	262244
15	225	3375	40	1600	64000	65	4225	274625
16	256	4096	41	1681	68921	66	4356	287496
17	289	4913	42	1764	70488	67	4489	300753
18	324	5832	43	1849	79507	68	4624	314432
19	361	6859	44	1936	85184	69	4771	329199
20	400	8000	45	2025	91125	70	4900	343000
21	441	9261	46	2116	97336	71	5041	357911
22	484	10648	47	2209	103823	72	5184	373348
23	529	12167	48	2304	110592	73	5329	389017
24	576	13824	49	2401	117649	74	5476	405224
25	625	15625	50	2500	125000	75	5625	411875

Root	Square	Cube
76	5776	438976
77	5929	456533
78	6084	474552
79	6241	493039
80	6400	512000
81	6561	531441
82	6724	550408
83	6889	571787
84	7056	592704
85	7225	614125
86	7396	636056
87	7569	658503
88	7744	681472
89	7921	705669
90	8100	729000
91	8281	753571
92	8464	778688
93	8649	804357
94	8836	830584
95	9025	857375
96	9216	884736
97	9409	915673
98	9604	941192
99	9801	970299
100	10000	1000000
101	10201	1030301
102	10404	1061208
103	10609	1092727
104	10816	1124856
105	11025	1157625
106	11236	1191016
107	11449	1225043
108	11664	1259712
109	11881	1295029
110	12100	1331000
111	12321	1367631
112	12544	1404928
113	12769	1442897
114	12996	1481544
115	13225	1520875
116	13456	1560896
117	13689	1601613
118	13924	1643032
119	14161	1685159
120	14400	1728000
121	14641	1771561
122	14884	1815848
123	15129	1860867
124	15376	1906624
125	15625	1953125
126	15876	2000376
127	16129	2048383
128	16384	2097172
129	16641	2146689
130	16900	2197000
131	17161	2248091
132	17424	2299968
133	17689	2352637
134	17956	2406104
135	18225	2460375
136	18496	2515856
137	18769	2570353
138	19044	2628072
139	19321	2685619
140	19600	2744000
141	19881	2803221
142	20164	2864288
143	20449	2924207
144	20736	2985984
145	21025	3027524
146	21316	3112136
147	21609	3176523
148	21904	3241792
149	22201	3307949
150	22500	3375000
151	22801	3442951
152	23104	3511808
153	23409	3581577
154	23716	3652264
155	24025	3723875
156	24336	3796416
157	24649	3869893
158	24964	3944312
159	25281	4019676
160	25600	4096000
161	25921	4173281
162	26244	4251528
163	16569	4330744
164	26896	4410944
165	27225	4492125
166	27556	4574296
167	27889	4657463
168	28224	4741632
169	28561	4826809
170	28900	4913000
171	29241	5000211
172	29584	5088448
173	29929	5177717
174	30276	5268024
175	30625	5359375
176	30976	5451776
177	31329	5545233
178	31684	5639752
179	32041	5735339
180	32400	5832000
181	32761	5929741
182	33124	6028568
183	33489	6128487
184	33856	6229504
185	34225	6331625
186	34596	6434856
187	34969	6539203
188	35344	6644672
189	35721	6751269
190	36100	6859000
191	36481	6967871
192	36864	7077888
193	37249	7189057
194	37636	7301384
195	38025	7414875
196	38416	7529536
197	38809	7645373
198	39204	7762392
199	39601	7880599
200	40000	8000000
201	40401	8120601
202	40804	8242408
203	41209	8365427
204	41616	8489664
205	42025	8615125
406	42436	8741816
207	42849	8869743
208	43264	8998912
209	43681	9129329
210	44100	9261000
211	44521	9393931
212	44944	9528128
213	45369	9663597
214	45796	9800344
215	46225	9938375
216	46656	10077696
217	47089	10218313
218	47524	10360232
219	47961	10503459
220	48400	10648000
221	48841	10793861
222	49284	10941048
223	49729	11089567
224	50176	11239424
225	50625	11390625
226	51076	11543176
227	51529	11697083
228	51984	11852452
229	52441	12008989
230	52900	12167000
231	53361	12326391
232	53824	12487168
233	54289	12649337
234	54756	12812904
235	55225	12977875
236	55696	13144256
237	56169	13312053
238	56644	13481272
239	57121	13651919
240	57600	13824000

Root	Square	Cube
241	58081	13997521
242	58564	14172448
243	59049	14348907
244	59536	14526784
245	60025	14705125
246	60516	14886936
247	61009	15069223
248	61504	15252992
249	62001	15438249
250	62500	15625000
251	63001	15813251
252	63504	16003008
253	64009	16194277
254	64516	16387064
255	65025	16581375
256	65536	16777216
257	66049	16974593
258	66564	17173512
259	67081	17473979
260	67600	17576000
261	68121	17779581
262	68644	17984728
263	69169	18191447
264	69696	18399744
265	70225	18609625
266	70756	18821096
267	71289	19034163
268	71824	19248832
269	72361	19465109
270	72900	19583000
271	73441	19902511
272	73984	20123648
273	74529	20346417
274	75076	20571024
275	75625	20796875
276	76176	21024576
277	76729	21253933
278	77284	21484952
279	77841	21717639
280	78400	21952000
281	78961	22188041
282	79524	22425768
283	80089	22665187
284	80656	22906304
285	81225	23149125
286	81796	23393656
287	82369	23539903
288	82944	23893872
289	83521	24137569
290	84100	24389000
291	84681	24642171
292	85264	24897088
293	85849	25153757
294	86436	25412184
295	87025	25672375
296	87616	25934336
297	88209	26198073
298	88804	26353592
299	89401	26730899
300	90000	27000000
301	90601	27270901
302	91204	27543608
303	91809	27818127
304	92416	28094464
305	93025	28372625
306	93636	28652616
307	94249	28934443
308	94864	29218112
309	95481	29503629
310	96100	29791000
311	96721	30080231
312	97344	30371328
313	97969	30664297
314	98596	30959144
315	99225	31255875
316	99856	31554496
317	100489	31855013
318	101124	32157432
319	101761	32461759
320	102400	32768000
321	103041	33076161
322	103684	33386248
323	104329	33698267
324	104976	34012224
325	105625	34328125
326	106976	34645976
327	106929	34965783
328	107584	35287552
329	108241	35611286
330	108900	35937000
331	109561	36264691
332	110224	36594368
333	110224	36926037
334	110889	37259704
335	111556	37595375
336	112225	37933056
337	112896	38272753
338	113569	38614472
339	114244	38958219
340	114921	39304000
341	115600	39651821
342	116281	40001688
343	116964	40353607
344	117649	40707584
345	118336	41063625
346	119025	41421730
347	119716	41781923
348	120409	42144192
349	121104	42508549
350	121801	42875000
351	122500	43243551
352	123201	43614208
353	123904	43986977
354	124609	44361864
355	125316	44738875
356	126736	45118016
357	127449	45499293
358	128164	45882712
359	128881	46268279
360	129600	46656000
361	130321	47045881
362	131044	47439928
363	131769	47832147
364	132496	48228544
365	133225	48627125
366	133956	49027896
367	134689	49430863
368	135424	49836032
369	136161	50243409
370	136900	50653000
371	137641	51064811
372	138384	51478848
373	139129	51895117
374	139876	52313624
375	140625	52734375
376	141376	53157376
377	142129	53582633
378	142884	54010152
379	143641	54439939
380	144400	54872000
381	145161	55306341
382	145924	55742968
383	146689	56181887
384	147456	56623104
385	148225	57066625
386	148996	57512456
387	149769	57960603
388	150544	58411072
389	151321	58863869
390	152100	59319000
391	152881	59776471
392	153664	60236288
393	154449	60698457
394	155236	61162984
395	156025	61629875
396	156816	62099136
397	157609	62570773
398	158404	63044792
399	159201	63521199
400	160000	64000000
401	160801	64481201
402	161604	64964808
403	162409	65450827
404	163216	65939264
405	164025	66430125
406	164836	66923416
407	165649	67419143
408	166464	67917312
409	167281	68417929
410	168100	68921000
411	168921	69426531
412	169744	69934528
413	170569	70444997
414	171396	70957944
415	172225	71473375
416	173056	71991296
417	173889	72511713
418	174724	73034632
419	175561	73560059
420	176400	74088000
421	177241	74618461
422	178084	75151448
423	178929	75686967
424	179776	76225024
425	180625	76765625

Root	Square	Cube
426	181476	77308776
427	182329	77854483
428	183184	78402752
429	184041	78953589
430	184900	79507000
431	185761	80062991
432	186624	80621568
433	187489	81182737
434	188356	81746504
435	189225	82312875
436	190096	82881856
437	190969	83453453
438	191844	84027672
439	192721	84604519
440	193600	85184000
441	194481	85766121
442	195364	86350888
443	196249	86938307
444	197136	87528384
445	198025	88121125
446	198916	88716536
447	199809	89314623
448	200704	89915392
449	201601	90518849
450	202500	91125000
451	203401	91733851
452	204304	92345408
453	205209	92959677
454	406116	93576664
455	207025	94196375
456	207936	94818816
457	208849	95443993
458	209764	96071912
459	210681	96702579
460	211600	97336000
461	212521	97972181
462	213444	98611128
463	214369	99252847
464	215296	99897344
465	216225	100544625
466	217156	101194696
467	218089	101847563
468	319024	102503232
469	219961	103161709
470	220900	103823000
471	221841	104487111
472	222784	105154048
473	223729	105823817
474	224676	106496424
475	225625	107171875
476	226576	107850176
477	227529	108531333
478	228484	109215352
479	229441	109902239
480	230400	110592000
481	231361	111284641
482	232324	111980168
483	233289	112678587
484	234256	113379904
485	235225	114084125
486	236196	114791256
487	237169	115501303
488	238144	116214272
489	239121	116930169
490	240100	117649000
491	241081	118370771
492	242064	119095488
493	243049	119823157
494	244036	120553784
495	245025	121287375
496	246016	122023936
497	247009	122763473
498	248004	123505992
499	249001	124251499
500	250000	125000000
501	251001	125751501
502	252004	126506008
503	253009	127263527
504	254016	128024064
505	255025	128787625
506	256036	129554216
507	257049	130323843
508	258064	131096512
509	259081	131872229
510	260100	132651000
511	261121	133432831
512	262144	134217728
513	263169	135005697
514	264196	135796744
515	265225	136590875
516	266256	137388096
517	267289	138188413
518	268324	138991832
519	269361	139798359
520	270400	140608000
521	271441	141420761
522	272484	142236648
523	273529	143055667
524	274576	143877824
525	275625	144703125
526	276676	145531576
527	277729	146363183
528	278784	147197952
529	279841	148035889
530	280900	148877000
531	281961	149721291
532	283024	150568768
533	284089	151419437
534	285156	152273304
535	286225	153130375
536	287296	153990656
537	288369	154854153
538	289444	155720872
539	290521	156590819
540	291600	157464000
541	292681	158340421
542	293764	159220088
543	294849	160103007
544	295936	160989184
545	297025	161878625
546	298116	162771336
547	299209	163667323
548	300304	164566592
549	301401	165469149
550	302500	166375000
551	303601	167284151
552	304704	168196608
553	305809	169112377
554	306916	170031464
555	308025	170953875
556	309136	171879616
557	310249	172808683
558	311364	173741112
559	312481	174676879
560	313600	175616000
561	314721	176558481
562	315844	177504328
563	316969	178453547
564	318096	179306144
565	319225	180262125
566	320356	181221496
567	321489	182154263
568	322624	183150432
569	323761	184220009
570	324900	185193000
571	326041	186169411
572	327184	187149248
573	328329	188132517
574	329476	189119224
575	330625	190109375
576	331776	191102976
577	332929	192100033
578	334084	193100552
579	335241	194104539
580	336400	195112000
581	337561	196122941
582	338724	197137368
583	339889	198155287
584	341056	199176704
585	342225	200201625
586	343396	201230056
587	344569	202262003
588	345744	203297472
589	346921	204336469
590	348100	205379000

Root	Square	Cube
591	349281	206425071
592	350464	207474688
593	351649	208527857
594	352836	209584584
595	354025	210644875
596	355216	211708736
597	356409	212776173
598	357604	213847192
599	358801	214921799
600	360000	216000000
601	361201	217081801
602	362404	218167208
603	363609	219256227
604	364816	220348864
605	366025	221445125
606	367236	222545016
607	368449	223648543
608	369664	224755712
609	370881	225866529
610	372100	226981000
611	373321	228099131
612	374544	229220928
613	375769	230346397
614	376996	231475544
615	378225	232608375
616	379456	233744896
617	380689	234885113
618	381924	236029032
619	383161	237176659
620	384400	238328000
621	385641	239483061
622	386884	240641848
623	388129	241804367
624	389376	242970624
625	390625	244140625
626	391876	245314376
627	393129	246491883
628	394384	247673152
629	395641	248858189
630	396900	250047000
631	398161	251239591
632	399424	252435968
633	400689	253636137
634	401959	254840104
635	403225	256047875
636	404496	257259456
637	405769	258474853
638	107044	259694072
639	408321	260917119
640	409600	262144000
641	410881	263374721
642	112164	264609288
643	413449	265847707
644	414736	267089984
645	416025	268336125
646	417316	269586136
647	418609	270840023
648	419904	272097792
649	421201	273359449
650	422500	274625000
651	423801	275894451
652	425104	277167808
653	426409	278445077
654	427716	279726264
655	429025	281011375
656	430336	282300416
657	431649	283593393
658	432964	284890312
659	434281	286191179
660	435600	287496000
661	436921	288804781
662	438244	290117528
663	439569	291434247
664	440896	292754944
665	442225	294079625
666	443556	295408296
667	444889	296740963
668	446224	298077632
669	447561	299418309
670	448900	300763000
671	450241	302111711
672	451584	303464448
673	452929	304821217
674	454276	306182014
675	455625	307546875
676	456976	308915776
677	458329	310288733
678	459684	311665752
679	461041	313046839
680	462400	314432000
681	463761	315821241
682	465124	317214568
683	466489	318611987
684	467856	320013504
685	469225	321419125
686	470596	322828856
687	471969	324242703
688	473344	325660672
689	474721	327082769
690	476100	328509000
691	477481	339939371
692	478864	331373888
693	480249	332812557
694	481636	334255384
695	483025	335702375
696	484416	337153536
697	485809	338608873
698	487204	340068392
699	488601	341532099
700	490000	343000000
701	491401	344472101
702	492804	345948408
703	494209	347428927
704	495616	348913664
705	497025	350402625
706	498436	351895816
707	499849	353393243
708	501264	354894912
709	502681	356400829
710	504100	357911000
711	505521	359425431
712	506944	360944128
713	508369	362467097
714	509796	363994344
715	511225	365525875
716	512656	367061696
717	514089	368601813
718	515524	370146232
719	516961	371694959
720	518400	373248000
721	519841	374805361
722	521284	376367048
723	522729	377933067
724	524176	379503424
725	525625	381078125
726	527076	382657176
727	528529	384240583
728	529984	385828352
729	531441	387420489
730	532900	389017000
731	534361	390617891
732	535824	392223168
733	537289	393832837
734	538756	395446904
735	540225	397065375
736	541696	398688256
737	543169	400315553
738	544644	401947272
739	546121	403583419
740	547600	405224000
741	549081	406869021
742	550564	408518488
743	552049	410172407
744	553536	411830784
745	555025	413493625
746	556516	415160936
747	558009	416832723
748	559504	418508992
749	561001	420189749
750	562500	421875000
751	564001	423564751
752	565504	425259008
753	567009	426957777
754	568516	428661064
755	570025	430368875

Root	Square	Cube
756	571536	432081216
757	573049	433798093
758	574564	435519512
759	576081	437245479
760	577600	438976000
761	579121	440711081
762	580644	442450728
763	582169	444194947
764	583696	445943744
765	585225	447697125
766	586756	449455096
767	588289	451217663
768	589824	452984832
769	591361	454756609
770	592900	456533000
771	594441	458314011
772	595984	460099648
773	597529	461889917
774	599076	463684824
775	600625	465484375
776	602176	467288576
777	603729	469097433
778	605284	470910952
779	606841	472729139
780	608400	474552000
781	609961	476379541
782	611524	478211768
783	613089	480048687
784	614656	481890304
785	616225	483736625
786	617796	485587656
787	619369	487443403
788	620944	489303872
789	622521	491169069
790	624100	493039000
791	625681	494913671
792	627264	496793088
793	628849	498677257
794	630436	500566184
795	632025	502459875
796	633616	504358336
797	635209	506261573
798	636804	508169592
799	638401	510082399
800	640000	512000000
801	641601	513922401
802	643204	515849608
803	644809	517781627
804	646416	519718464
805	648025	521660125
806	649636	523606616
807	651249	525557943
808	652864	527514112
809	654481	529475129
810	656100	531441000
811	657721	533411731
812	659344	535387328
813	660969	537367797
814	662596	539353144
815	664225	541343375
816	665856	543338496
817	667489	545338513
818	669124	547343432
819	670761	549353259
820	672400	551368000
821	674041	553387661
822	675684	555412248
823	677329	557441767
824	678976	559476224
825	680625	561515625
826	682276	563559976
827	683929	565609283
828	685584	567663552
829	687241	569722789
830	688900	571787000
831	690561	573856191
832	692224	575930368
833	693889	578009537
834	695556	580093704
835	697225	582182875
836	698896	584277056
837	700569	586376253
838	702244	588480472
839	703921	590589719
840	705600	592704000
841	707281	594823321
842	708964	596947688
843	710649	599077107
844	712336	601211584
845	214025	603351125
846	715716	605495736
847	717409	607645423
848	719104	609800192
849	720801	611960049
850	722500	614125000
851	724201	616295051
852	725904	618470208
853	727609	620650477
854	729316	622835864
855	731025	625026375
856	732736	627222016
857	734449	629422793
858	736164	631628712
859	737881	633839779
860	739600	636056000
861	741321	638277381
862	743044	640503928
863	744769	642735647
864	746496	644972544
865	748225	647214625
866	749956	649416896
867	751689	651714363
868	753424	653972032
869	755161	656234909
870	756900	658503000
871	758641	660776311
872	760384	663054848
873	762129	665338617
874	763876	667627624
875	765625	669921875
876	767376	672221376
877	769129	674526133
878	770884	676836152
879	772641	679151415
880	774400	681472000
881	776161	683797841
882	777924	686128968
883	779689	688465387
884	781456	690807104
885	783225	693154125
886	784996	695506456
887	786769	697864103
888	788544	700227072
889	790321	702595369
890	792100	704969000
891	793881	707347971
892	795664	709732288
893	797449	712121957
894	799236	714516984
895	801025	716917375
896	802816	719323136
897	804609	721734273
898	806404	724150792
899	808201	726572699
900	810000	729000000
901	811801	731432701
902	813604	733870808
903	815409	736314327
904	817216	738763264
905	819025	741217625
906	820836	743677416
907	822649	746142643
908	824464	748613312
909	826281	751089429
910	828100	753571000
911	829921	756058031
912	831744	758550528
913	833569	761048497
914	835396	763551944
915	837225	766060875
916	839056	768575296
917	840889	771095213
918	842724	773620632
919	844561	776151559
920	846400	778688000

Root	Square	Cube
921	848241	781229961
922	850084	783777448
923	851929	786330467
924	853776	788889024
925	855625	791453125
926	857476	794022776
927	859329	796597983
928	861184	799178752
929	863041	801765089
930	864900	804357000
931	766761	806954491
932	868624	809557568
933	870489	812166237
934	872356	814780504
935	874225	817400375
936	876096	820025856
937	877969	822656953
938	879844	825293672
939	881721	827936019
940	883600	830584000
941	885481	833237621
942	887364	835896888
943	889249	838561807
944	891136	841232384
945	893025	843908625
946	894916	846590536
947	896809	849278123
948	898704	851971392
949	900601	854670349
950	902509	857375000
951	904401	860085351
952	906304	862801408
953	908209	865523177
954	910116	868250664
955	912025	870983875
956	913936	873722816
957	915849	876467493
958	917764	879217912
959	919681	881974079
960	921600	884736000
961	923521	887503681
662	925444	890277218
963	927369	893056347
964	929296	895841344
965	931225	898632125
966	933156	901428696
967	935089	904231063
968	937024	907039232
969	938961	909853209
970	940900	912673000
971	942841	91549811
972	944784	918330048
973	946729	921167317
974	948676	924010424
975	950625	926859375
976	952576	929714176
977	954529	932574833
978	956484	935441352
979	958441	938313739
980	960400	941192000
981	962361	944076141
982	964324	946966168
983	966289	949862087
984	968256	952763904
985	970225	955671625
986	972196	958585256
987	974169	961504803
988	976144	964430272
989	978121	967361669
990	980100	970299000
991	982081	973242271
992	984064	976191488
993	986049	979146657
994	988036	982107784
995	990025	985074875
996	992016	988047936
997	994009	991026973
998	996004	994011992
999	998001	997002999
1000	1000000	1000000000

Some

Some USES of

SQUARE *and* CUBE ROOTS,

In the Solution of

Arithmetical and Geometrical Problems.

THE Demonſtration of the *Square* and *Cube Roots*, together with the manner how to *extract* the ſame, hath been taught already; and for the more eaſie performance thereof, I have added a *Table* of *Square* and *Cube Numbers*, with their *Roots*, from 1 to 1000: Some Uſes of the *Square* and *Cube Roots* I ſhall exhibit in the ſolution of the *Arithmetical* and *Geometrical Problems* following.

§I. *Of the* SQUARE ROOT.

PROBLEM I.

Admit the highth of the Wall *of a* Fort *or* Caſtle *to be* 30 *Foot, and the breadth of the* Trench *about the* Fort 40 *Foot; I demand of what length a* Scaling Ladder *ſhall be, juſtly to reach from the brow of the* Trench *to the top of the* Wall.

In the 47th Propoſition of the firſt Book of *Euclid*'s Elements, it is demonſtrated, that *the Square of the Hypotenuſal of all right angled plain Triangles is equal to to the Squares of the two other ſides*; therefore to reſolve this Problem, ſquare the highth of the *Wall*, which is 30, *facit* 900; alſo ſquare the breadth of the *Trench*, which is 40, *facit* 1600; theſe two added together make 2500, the *Square Root* whereof is 50; and ſo long muſt a *Scaling Ladder* be made to reach from the brow of the *Trench* to the top of the *Wall*.

PROBLEM II.

There be two Towns, as Chicheſter *and* York, *which lie North and South one from another, and their diſtance is* 220 *Miles; and* Exceſter *lieth directly Weſt from* Chicheſter, 120; *I deſire to know the diſtance of* York *from* Exceſter.

⊕ *Exceſter*

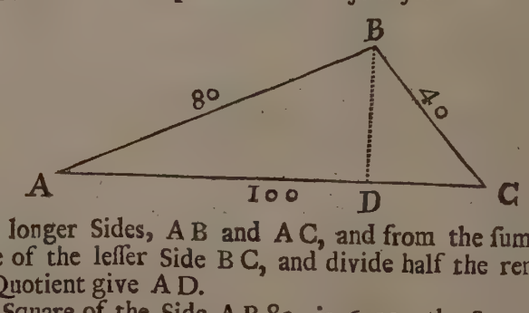
120

Chicheſter ⊕ 220 ⊕ *York*

Square 120, the diſtance of *Exceſter* and *Chicheſter*, it maketh 14400; likewiſe ſquare 220, the diſtance of *York* and *Chicheſter*, it maketh 48400; theſe two Numbers added together make 62800, whoſe Square Root extracted, (or found in the Table,) will be 250¾ near; and ſo many Miles is *Exceſter* diſtant from *York*.

PROBLEM III.

The three Sides *of an* oblique angled Triangle *ABC being given, to find the point in the* Baſe *thereof, where the* Perpendicular *BD ſhall fall.*



Square the two longer Sides, AB and AC, and from the ſumm of thoſe Squares ſubtract the Square of the leſſer Side BC, and divide half the remainder by the Side AC, ſo ſhall the Quotient give AD.

Example. The Square of the Side AB 80, is 6400; the Square of AC is 10000, their Summ is 16400: Then Square the Side BC 40, and it is 1600; which ſubtracted from 16400, the ſumm of the Squares of the two other Sides, the remainder is 14800, the half whereof is 7400; which divided by AC 100, the Quotient is 74, for AD; unto which point D, the perpendicular BD muſt be drawn.

PRO-

PROBLEM IV.

To find the Length *of the* Perpendicular *B D, in the former* Triangle *A B C.*

From the Square of A B 6400, ſubtract the Square of A D 5476, the difference is 942, whoſe Square Root $30\frac{42}{61}$, is the Length of the Perpendicular B D.

PROBLEM V.

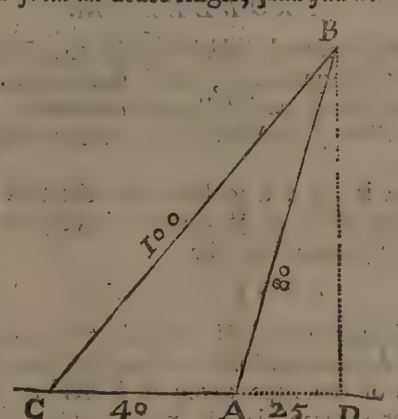
The Diametre *of a* Circle *being given, to find the* Area, *or* ſuperficial Content *thereof.*

The Proportion of the Square of a Circle's Diametre, is to its *Area* as 14 to 11; wherefore ſquare the Diametre of the given Circle, and multiply that by 11, and divide the Product by 14, the Quotient ſhall give the *Area*.

Example. Let a Circle's Diametre be 21 Inches, the Square thereof is 441; which multiplied by 11, the Product is 4851; which divided by 14, the Quotient will be $346\frac{1}{2}$, for the Circle's *Area*, whoſe Diametre is 21 Inches.

PROBLEM VI.

The three Sides *of an* oblique angled plain Triangle *being given, to find how far a* Perpendicular, *let fall from an* acute Angle, *ſhall fall without the* Triangle.



Let the Triangle be A B C, whoſe three Sides are A B 80, A C 40, and B C 100; and let it be required to find the Point D, or the length of A D.

From the Square of B C 10000, ſubtract the ſumm of the Squares B A 6400, and C A 1600, which is 8000, and the remainder is 2000, the half whereof 1000, divide by A C 40, the Quotient will be 25, for the diſtance A D.

PROBLEM VII.

To find the Length of the Perpendicular *B D of the former* Triangle.

From the Square of B C 10000, ſubtract the Square of the ſumm of C A and A D, (*viz.* 65,) 4225, the remainder is 5775; whoſe Square Root is $75\frac{45}{151}$, for the Length of the Perpendicular B D.

PROBLEM VIII.

If a Mountain *be one* Mile *(or* 5280 *Foot) high, how far may that* Mountain *be ſeen upon plain Ground, or at Sea?*

The Semidiametre of the Earth is found to be 3436 Miles, to which add one Mile for the highth of the Mountain, the ſumm of them is 3437, whoſe Square is 11812969, from which ſubtract the Square of 3436, the Semidiametre 11806096, the remainder will be 6873, whoſe Square Root is $82\frac{49}{171}$, that is, almoſt 83 Miles; and ſo far may that Mountain be ſeen at Sea.

PROBLEM IX.

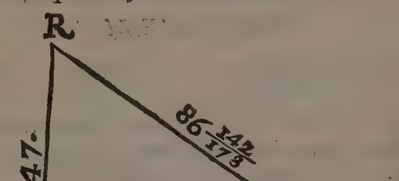
The Semidiametre *of the* Earth *being allowed to be* 3436 Miles, *and a* Mountain *at Sea is ſeen at* 83 *Miles diſtance; How high is that* Mountain?

This is but the converſe of the former *Problem*; for the Square of 3436 (the Semidiametre) is 11806096, the Square of 83 is 6889; their ſumm is 11812985, whoſe Square Root is $3437\frac{16}{2474}$, from which ſubtract 3436, (the Semidiametre,) there reſts $1\frac{16}{2474}$; that is 1 Mile, and a little more, for the highth of the Mountain.

PRO-

PROBLEM X.

Two Ships, *R and S, ſet ſail together from the* Port *P; the* Ship *S ſails from P North to S,* 73 *Leagues; and the Ship R ſails directly Weſt, from P to R,* 47 *Leagues; How far diſtant are theſe two* Ships *one from the other?*



The Square of P S 73 is 5329, and the Square of P R 2209, their ſumm is 7538 the Square Root whereof is 86 $\frac{142}{173}$; and ſo far are the two Ships R and S diſtant.

PROBLEM XI.

There is a Square Piece *of* Ground, *which contains* 144 Perches; *I would have another* Piece *of* Ground, *exactly* Square *alſo, which ſhould contain* 288 Perches; *How many* Perches *long was the Side of the* Wall *that contained* 144 Perches, *and how many* Perches *long muſt the Side of the* Wall *of that* Piece *which is to contain* 288 Perches *be?*

The Square Root of 144 is 12, and ſo long was the Side of the Wall of the leſſer Piece: The Square Root of 288 is 16 $\frac{12}{33}$, (which is very near 17;) and ſo long muſt the Side of the Wall of the greater Piece be.

PROBLEM XII.

If a Pipe *of* 3 *Inches Diametre will fill a* Veſſel *of Water in one hours time, of what Diametre muſt a* Pipe *be which will fill the ſame* Veſſel *in half an hour?*

Multiply the Diametre of the Pipe given in it ſelf twice, which added together make 18, the Square Root whereof is 4 $\frac{2}{5}$, for the Diametre of the Pipe that will fill the Veſſel in half an hour.

PROBLEM XIII.

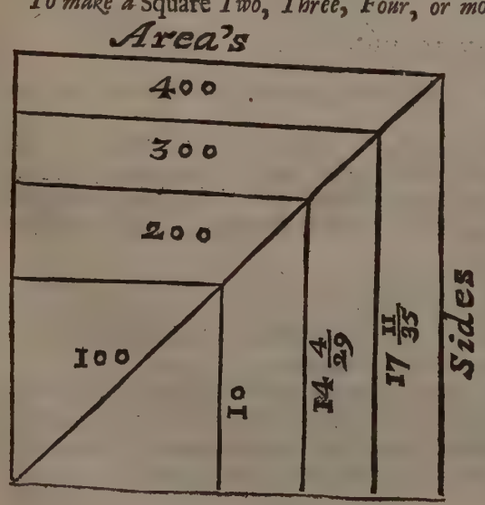
Between Two Numbers *given, to find a* Mean Proportional.

Multiply the two Numbers together, the Square Root of the Product ſhall be the Mean Proportional ſought.

Example. Let the Numbers be 48 and 12, their Product is 576, whoſe Square Root 24 is the Mean Proportional between 12 and 48.

PROBLEM XIV.

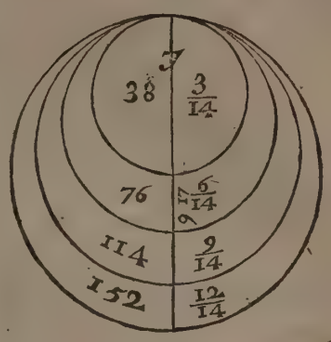
To make a Square *Two, Three, Four, or more times greater than another* Square *given.*



Let the Side of a given Square be 10, and let it be required to make three other Squares to contain double, triple, and quadruple the given Square.

The Side of the given Square being 10, the content thereof is 100, the Double whereof being 200, the Square Root whereof is 14 $\frac{4}{29}$, the Side of a Square containing 200; the Treble thereof being 300, the Square Root thereof is 17 $\frac{11}{35}$, for the Side of a Square containing 300; then Quadruple it, and it is 400, whoſe Square Root is 20, for the Side of a Square whoſe content is 400; and although the Side 20, be but double the Side 10, yet the content of the Square is four times as much, as in the Figure.

And



And what is here ſaid of Squares, the ſame may be done in Circles; for, having the Diametre of a Circle, you may find the *Area* by the fifth *Probl.* And ſo the Diametre of a Circle being 7 Inches, the *Area* thereof will be found to be $38\frac{3}{14}$: Then for a Circle that ſhall contain double as much, namely $76\frac{6}{14}$, ſquare the Diametre 7, and it is 49, and the double thereof is 98, the Square Root whereof is $9\frac{17}{18}$, for the Diametre of the Circle whoſe *Area* is $76\frac{6}{14}$: Then for the Diametre of a Circle which ſhall contain three times as much, *viz.* $114\frac{9}{14}$, multiply 49 by 3, the Product will be 147, whoſe Square Root is $12\frac{3}{25}$, for the Diametre of a Circle whoſe *Area* ſhall be 147: Then for a Circle that ſhall contain $152\frac{12}{14}$; multiply 49 by 4, the Product will be 196, the Square Root whereof is 14, for the Diametre of a Circle which contains four times as much as that whoſe Diametre was 7.

PROBLEM XV.

If one Fathom *of* Cable, *whoſe Compaſs about is* 10 *Inches, do weigh* 17 Pound, *how much ſhall one* Fathom *of* Rope *weigh, whoſe Compaſs or Circumference is* 8 *Inches?*

The Proportion to reſolve this *Problem* is,

As the Square of the Circumference of the Cable given,
Is to the Weight of one Fathom thereof;
So is the Square of the Compaſs of Rope required,
To the Weight of one Fathom thereof.

Thus the Square of 10 Inches is 100, and the Square of 8 is 64; wherefore multiply 64 by 17, the Product will be 1088; which divided by 100, the Quotient will be $10\frac{88}{100}$, for the Weight of one Fathom of that Cable whoſe Circumference is 8 Inches.

PROBLEM XVI.

If one Fathom *of a* Cable *whoſe Circumference is* 10 *Inches, do weigh* 17 lb. *of what Circumference muſt a* Cable *be that one* Fathom *thereof ſhall weigh* $10\frac{88}{100}$ *Pound?*

This is but the converſe of the foregoing *Problem*; for the Proportion is,

As the Weight of the Cable given, 17 Pound,
Is to the Weight of the Cable required $10\frac{88}{100}$;
So is the Square of the Circumference of the Cable given,
To the Square of the Circumference of the Cable required.

Then the Fractions being reduced into Improper Fractions they will be $\frac{17}{1}$, $\frac{1088}{100}$, and $\frac{100}{1}$: Then, as $\frac{17}{1}$ is to $\frac{1088}{100}$, ſo is $\frac{100}{1}$ to $\frac{108800}{1700}$; which Fraction reduced is 64, whoſe Square Root is 8, for the Circumference of the Cable whoſe Fathom ſhall weigh $10\frac{88}{100}$ Pounds.

PROBLEM XVII.

If a Company *of* Soldiers *were to be ordered in Battel array, in ſuch* Order *that the Diſtance between* Rank *and* File *ſhall be as* 8 *is to* 3; *that is,* 8 *Men in* Rank *for* 3 *in* File.

For the performance hereof this is the Proportion:

As the Term given for Men in File,
Is to the Term given for Men in Rank;
So will the Number of Men marſhalled be,
To the Square Root of the Men in Rank.

Example. Let the Number of Soldiers be 20184; which multiplied by 8, the Product will be 161472; which number divided by 3, the Quotient will be 53824, whoſe Square Root is 232, for the Number of Men to be in Rank.

Then for the Men to be in File, divide 20184 by 232, the Number of Men in Rank, and the Quotient will be 87, for the Number of Men to be in File.

PROBLEM XVIII.

How to marſhal *in* Battalia, *any number of* Soldiers, *when there is a double Proportion ſtated, as in reſpect of* Men *and* Ground, *both for* Rank *and* File.

For the Reſolving of this *Problem* this is the Proportion:

As the Product of the two Terms for the Rank,
Is to the Number of Soldiers to be Marshalled;
So is the Product of the two Terms for the File,
To a fourth Number,
Whose Square Root is the File.

Example. Let the Number of Soldiers to be marshalled be 41160, in this order; as for 3 in Rank 7 in File; and in respect of the Ground, as 2 is to 5: the Proportion of these Terms are 6 and 35; for 3 times 2 is 6, and 7 times 5 is 35;) Then say,

As 6 is to 35,
So is 41160 to 240100;

Whose Square Root will be 490, by which divide the Number of Soldiers 41160, the Quotient will be 84, for the Number of Men both in Rank and File. Or you may say,

As 35 is to 6,
So is 41160 to 7056;

Whose Square Root is 84, as before.

According to these Directions may an Army be drawn out into their several Regiments, and those again divided into several Squadrons, with their depth and proportion in Rank and File. These and many more *Problems*, both *Arithmetical*, *Geometrical*, and *Military*, may be performed by help of the Square Root, but let these few suffice.

§II. *Of the* CUBE ROOT.

Like Solids are in Proportion one to another, as are the Cubes of their homologous Sides, or Diametres; so that by help of the Cube and Cube Root, many *Geometrical Problems* may be resolved, of which (for a taste,) take these few following:

PROBLEM I.

There is a Cube *whose* Side *is* 6, *and its* Solidity *therefore is* 216; *I demand what shall be the* Side *of a* Cube *whose* Solidity *shall be double to the former.*

Double 216, it makes 432, whose Cube Root extracted will be $7\frac{89}{189}$, and that must be the Side of a Cube whose Solidity shall be 432.

PROBLEM II.

Having the Solid content *of a* Globe, Sphere, *or* Bullet, *to find the Side of a* Cube *whose* Solidity *shall be equal to that of the* Globe.

There is a Globe of Brass whose Solid content is 1331 Inches, and I would know the Side of a Cube which shall contain so many Cubical Inches: The Cube Root of 1331 is 11, and so many Inches must the Side of the Cube be.

PROBLEM III.

The Weight (*or* Solidity) *and the* Diametre *of a* Bullet *being given, to find the* Weight (*or* Solidity) *of another* Bullet (*of the same Metal*) *whose* Diametre *is also given.*

The Proportion is,

As the Cube of the Diametre of the Given Bullet,
Is to its Weight (or Solidity;)
So is the Cube of the Diametre of the other Bullet, (of the same Metal,)
To the Weight (or Solidity) thereof.

Thus, If an Iron Bullet, whose Diametre is 4 Inches, do weigh 9 Pound, what shall an Iron Bullet, whose Diametre is $6\frac{1}{4}$ Inches, weigh?

Multiply the Cube of $6\frac{1}{4}$, which is 244.140625, (the Diametre of the Bullet whose Weight is required,) by 9, (the Weight of the Bullet given,) the Product will be 2197.265625; which divided by 64, (the Weight of the given Bullet,) the Quotient will be $34\frac{11}{1024}$, and somewhat more, for the weight of the Bullet whose Diametre is $6\frac{1}{4}$ Inches.

PROBLEM IV.

The Sides *of two* Cubes, *or two* Bullets, *being given, to find the* Side *of one* Cube, *or* Bullet, *that shall be equal in* Solidity *to both the given ones.*

Let one of the Sides, or Diametres, be 6 Inches, the Side, or Diametre, of the other 8 Inches; and let the Side, or Diametre, of a Cube, or Bullet, equal to them both be required,

The

The Cube of 6 is 216, and the Cube of 8 is 512; these two added together make 728, whose Cube Root is 8 $\frac{216}{217}$, that is 9 Inches almost.

PROBLEM V.

The Diametre *of the* Concaves *of two* Guns *being known, and the quantity of* Powder *that will charge one of them, to find the quantity of* Powder *that will suffice to charge the other.*

The concave capacities of Guns are one to the other as are the Cubes of their Diametres.

Example. If a Cannon Royal, whose Diametre at the Bore is 8 Inches, require 26 *lb.* of Powder to load the same, what quantity of Powder shall suffice to load a Culverin, whose Diametre at the Bore is 5 Inches? The Proportion is,

As the Cube of 8 Inches, (the Diametre of the Cannon,) 512,
Is to 26 *lb.* the Powder that will load the same;
So is the Cube of 5 Inches, (the Diametre of the Culverin,) 125,
To 6 $\frac{13}{256}$ *lb.* the quantity of Powder that will load the Culverin whose Diametre is 5 Inches.

PROBLEM VI.

A Grenado Shell *being* 14 *Inches* Diametre, *and two Inches and an half substance of* Metall, *what is the Weight of the* Metall, *and the Content of the Concavity of the* Shell *in Cubical Inches?*

1. The Cube of 14, the Diametre of the Shell, is 2747; which multiplied by 11, the Product is 30184, and that divided by 21, the Quotient is 1437 $\frac{1}{3}$, which is the solid Inches in the whole, Metall and Concave both, as if it were a solid Bullet of 14 Inches Diametre.

2. For the Concavity, the thickness of the Metall being 2 $\frac{1}{2}$ Inches, the double thereof is 5 Inches, which subtracted from 14 Inches, there remains 9 Inches, for the Diametre of the Concave of the Shell: then, the Cube of 9 is 729; which multiplied by 11, produceth 8019; and that divided by 21, the Quotient will be 381 $\frac{3}{7}$, for the solid Inches contained in the Convexity of the Shell: The 381 being subtracted from 1437, (omitting the Fractions in both Numbers,) the remainder is 1056 Inches, the Cubical Inches of the Metall.

3. And because one Cubical Inch of cast Iron is by experience found to weigh 4 Ounces, multiply 1056 by 16, (the number of Ounces in one Pound,) the Product will be 16896 Ounces; which divided by 4, the Quotient will be 4224, and that number divided by 16, (the number of Ounces in one Pound,) the Quotient will be 264 Pounds, for the weight of the Grenado Shell.

PROBLEM VII.

By the Mold *and* Burthen *of one* Ship *being known, how to build another* Ship *of the same* Mold, *of any assigned* Burthen, *greater or lesser.*

Suppose a Ship of 100 Tun is found to be 44 Foot long by the Keel, 20 Foot broad upon the Mid-ship-beam, 9 Foot deep in the Hold; and did Rack it with the Stem forwards 13 Foot, and offward 7.

If (according to these Dimensions) you would build another Ship whose Burthen should be 200 Tun, the several dimensions of the Members may be found as followeth.

1. For the Keel, it being 44 Foot, the Cube thereof is 85184, double this number (because the Ship you are to build is to be double the Burthen of the other, *viz.* 200 Tun,) and it makes 170368, the Cube Root whereof is 55.437 Foot; which is 55 Foot, 4 Inches, $\frac{1}{3}$ of an Inch, for the length by the Keel.

2. For the breadth upon the Mid-ship-beam 20 Foot; the Cube of 20 is 8000, the double whereof is 16000, whose Cube Root is 25.199 Foot; that is, 25 Foot, 2 Inches, and $\frac{1}{3}$ of an Inch, for the breadth upon the Mid-ship-beam.

3. For the Depth in Hold 9 Foot; the Cube of 9 is 729, the double whereof is 1458, whose Cube Root is 11.339 Foot; that is 11 Foot, 4 Inches, for the depth in Hold.

4. For the Rack forward 13 Foot; the Cube of 13 is 2197, the double whereof is 4394, whose Cube Root is 16.379 Foot; that is, 16 Foot, 4 Inches, and an half, for the Rack forward.

4. For the Rack offwards 7 Foot; the Cube of 7 is 343, the double whereof is 686, whose Cube Root is 8.819 Foot; which is 8 Foot, 9 Inches, and $\frac{3}{4}$ of an Inch, for the Rack offwards.

This is the natural way of working of these and the like Proportions, but when you have many lengths to find, you may ease your self of extracting so many Cube Roots; for having found out one of them by the Cube Root, you may find out all the rest by the *Golden Rule* of *Proportion:* Thus, having found the Length by the Keel to be 55.437, and you would find the Length of the Mid-ship-beam proportionable to this, which in the Ship of 100 Tun was 20 Foot, say,

As 44, the Length by the Keel of the Ship of 100 Tuns,
Is to 55.437, the Length by the Keel of the Ship of 200 Tun;
So is 9 Foot, the Depth in the Hold of the Ship of 100 Tuns,
To 11.339, the Depth in the Hold of the Ship of 200 Tun.

And so of all the Members, as in this Synopsis.

1. For the Mid-ship-beam:

As 44 is to 55.437, so is 20 to 25.199.

2. For the Depth in Hold:

As 44 is to 55.437, so is 9 to 11.339.

3. For the Rack forward:

As 44 is to 55.437, so is 13 to 16.379.

4. For the Rack offward:

As 44 is to 55.437, so is 7 to 8.819.

Or thus, having the proportion of one Cube to another Cube, you may work by that, in this manner:

The Cube of		thereof is
1 being	1.000	
2 the double		1.260
3 the triple		1.442
4 the quadruple		1.557
5 the quintuple		1.710
6 the sextuple		1.817
7 the septuple		1.913
8 the octuple		2.000

And thus, by the foresaid supposition of a Ship of 100 Tuns being 44 Foot by the Keel, the Length of the Keel for a Ship of 200, 300, 400 Tun, may be found by these Proportions:

	Tun				*Feet*	
For a Ship of	200	Say, as 1.000 is to 44, so is	1.260	to	55.440	the Length by the Keel.
	300		1.442		63.448	
	400		1.557		69.828	
	500		1.710		75.240	
	600		1.817		79.948	
	700		1.913		84.172	
	800		2.000		88.000	

And from these few Proportions may be deduced many more, but these shall suffice to shew the use of the Cube Root in this particular.

The End of Natural, *or* Vulgar Arithmetick.

DECI-

DECIMAL ARITHMETICK.

PART II.

CHAP. I.

Of DECIMAL ARITHMETICK.

WHo was the first Inventor of this *Decimal Arithmetick*, or brought it first to light, is hard to determine; but that it hath been much improved since the Invention of *Logarithms*, all knowing Artists cannot but acknowledge, and that at this day it is arrived to the *Zenith* of its perfection. The excellency of this kind of *Arithmetick*, will best appear when it shall be applyed to *Mathematical Practices*; as in composing of *Tables* of *Interest* and *Rebate of Money*, in all sorts of *Measures*, whether *Superficial* or *Solid*; in which cases (and many others, as in the Subsequent *Tractates* will at large appear,) *Decimal Operations* do afford so great help, that many Ages before have not produced a more usefull Invention: And indeed this excellent kind of *Arithmetick* might be yet farther improved, if all our *Coins*, *Weights*, and *Measures*, (which are now divided into so many heterogeneous Denominations,) were divided, and sub-divided *Decimally*. For by this means all Calculations concerning *Trade* would be performed with much ease and pleasure; especially the Operations of *Multiplication* and *Division*: But these things last hinted are not in the power of any private Person to effect, but wholly in such as are Law-makers; and therefore I desist, and proceed to my designed Method of *Mathematical Institution*.

CHAP. II.

Of DECIMAL FRACTIONS.

Definition. A *Decimal Fraction*, is an Artificial way of setting down, and expressing of *Natural* or *Vulgar Fractions*, as *whole Numbers*; and whereas the *Denominators* of *Vulgar Fractions* are Diverse, the *Denominators* of *Decimal Fractions* are always *Certain*: For a *Decimal Fraction* hath always for its *Denominator* an *Unite*, with a *Cypher* or *Cyphers* annexed to it, and must therefore be either 10, 100, 1000, 10000, *&c.* And therefore in writing down of a *Decimal Fraction*, there is no necessity of writing down the *Denominator*, for by bare inspection it is certainly known, it consisting always of an *Unite*, with as many *Cyphers* annexed to it, as there are *Places* (or *Figures*) in the *Numerator*.

Example. This *Decimal Fraction* $\frac{25}{100}$ may be written thus, .25; its *Denominator* being known to be an *Unite* with *two Cyphers*, because there were two figures in the *Numerator*: In like manner $\frac{327}{1000}$ may be thus written, .327; $\frac{0372}{10000}$ thus, .0372; and so of any other.

2. And although *Cyphers* at the end of *Whole* (or *Integral*) *Numbers* do *increase* those Numbers in a decuple Proportion; yet they being annexed to the end of a *Decimal Fraction*, they do neither increase nor diminish the value thereof. So $\frac{3700}{10000}$ or .3700, is equivalent to $\frac{37}{100}$ or .37.— And $\frac{80}{100}$ or .80, is equal to $\frac{8}{10}$ or .8.——And although in *Whole* (or Integral) *Numbers*, Cyphers being prefixed before them, do neither increase nor diminish their *Value*; for 037 is but 37, and 004 is but 4, *&c.* yet *Cyphers* prefixed before a *Decimal Fraction*, do expres or diminish its *Value* in a *Decuple Proportion*: For .25 which is $\frac{25}{100}$, if you prefix a Cypher before it, it becometh $\frac{025}{1000}$ or .025, which

is

is $\frac{25}{1000}$: And by prefixing two Cyphers it is $\frac{0025}{10000}$, or .0025, which is $\frac{25}{10000}$, *&c.* And therefore when you are to write down a *Decimal Fraction*, whose *Denominator* hath more *Cyphers* in it than there are *Figures* (or Places) in the *Numerator*, they must be supplyed by prefixing so many *Cyphers* before the Figures of the *Numerator*. As suppose $\frac{29}{1000}$ were to be written down without its *Denominator*; here, because there are three *Cyphers* in the *Denominator*, and but two *Figures* in the *Numerator*, therefore prefix a *Cypher* before 29, and set it down thus .029

And so $\left\{\begin{matrix}\frac{8}{100}\\ \frac{2}{1000}\\ \frac{64}{10000}\end{matrix}\right\}$ must be thus written $\left\{\begin{matrix}.08\\ .002\\ .0064, \&c.\end{matrix}\right.$

3. The *Integers* and *Decimal Fractions* are distinguished each from other several ways, according to several Mens fancies; but the general received way (and indeed the easiest either to write or conceive) is by separating the *Integer* from the *Fraction*, by the interposition of a *Comma*, *Period*, or *Dash* of the Pen between them; or if there be no whole Number, then a *Comma* or *Point* before the *Fraction* is sufficient. And thus if you were to set down 763 $\frac{7626}{10000}$, it may thus be expressed 763.7626:——Or 5 $\frac{072}{1000}$ thus, 5.072:——Or $\frac{0076}{10000}$ thus, .0076, *&c.*

Some would express 763 $\frac{7626}{10000}$ thus $\left\{\begin{matrix}763|7626.\\ 763.7'.6''.2'''.6''''.\\ 763|7626, \&c.\end{matrix}\right.$

4. These Decimal Fractions are of all other the most natural, for all plain Numbers, as they are expressed by the nine Digits; 1, 2, 3, *&c.* are nothing else but Decimal Fractions: For let any Number be given, as 736, the last figure 6 is really $\frac{6}{10}$; and so are the last two figures 36 really $\frac{36}{100}$, and all three are a Decimal Fraction of $\frac{736}{1000}$. For as the Progression of these, and all other plain Numbers, is Decimal; that is, each figure on the left hand is ten times the value of the same Number placed in the next place on the Right. Thus in this Number [777,] the last 7 on the left hand is ten times the value of the 7 next to it on the Right Hand, and the 7 in the middle is ten times the last 7 on the Right, and but the tenth part of that 7 on the Left: And therefore, all or any of them may be either Fractions or Integers.

CHAP. III.

Of NOTATION of DECIMALS.

Definition. NOtation of *Decimals* is contrary to that of *Whole Numbers*; for, as in the whole Numbers, the value of figures are increased in a *Decimal Proportion*, by the continual addition of Cyphers towards the Right Hand; so (on the contrary) the values of the places of Decimal Fractions do decrease in the same Proportion. And whereas in *Whole Numbers Cyphers* towards the Left Hand are unnecessary; yet in *Decimal* Fractions they are absolutely necessary to discover the true Denominator.

Also Cyphers at the end (or towards the Right Hand) of *Decimal Fractions* are of no *Value*; for one single figure in a *Decimal Fraction* signifieth as much as the same figure doth with Cyphers after it, for .7 is equivolent to .70, .700, *&c.* for the *Denominators* of *Decimal Fractions* being an *Unite*, with *Cyphers* toward the Right Hand, (as hath been already declared,) therefore $\frac{70}{100}$ being reduced to its least Terms, will be $\frac{7}{10}$, and $\frac{600}{1000}$ will be $\frac{6}{10}$, and $\frac{370}{1000}$ will be $\frac{37}{100}$, *&c.* As in the following Table.

98765432 1	. 12345678
100000000	. 00000001
10000000	. 0000001
1000000	. 000001
100000	. 00001
10000	. 0001
A Thousand 1000	. 001 *Or* $\frac{1}{1000}$
An Hundred 100	. 01 *Or* $\frac{1}{100}$
Ten 10	. 1 *Or* $\frac{1}{10}$

CHAP.

CHAP. IV.

Of REDUCTION *of* DECIMALS.

Definition. REduction of *Decimals* is of two kinds; for *first* it teacheth how to *reduce Vulgar Fractions* into *Decimal Fractions*; and secondly, by *Reduction*, to know the *Value* of such a *Decimal Fraction* in known parts of the *Integer*. For the first of these this is the *Analogy*, or *Proportion*:

As the *Denominator* of the *Vulgar Fraction*, is to its *Numerator*;
So is an Unite, with so many Cyphers added as you would have your *Decimal Fraction* have Places,
To the *Decimal Fraction* required.

Thus, *If you would reduce this* Fraction $\frac{3}{4}$ *to a* Decimal *consisting of* 4 Places, say;
As 4 (the *Denominator* of the given Fraction) is to 3, (the *Numerator* of the same,)
So is 10000 (the *Denominator* of the *Decimal*, to .75, the *Decimal Fraction* required.
And the like of all others; and for the effecting hereof this is the *RULE*:

To the Numerator *of the* Fraction *given add so many* Cyphers *as you would have* Places *in your* Decimal Fraction; *then divide the* Numerator *of your given* Fraction (*the* Cyphers *being added*) *by the* Denominator *of the given* Fraction, *the* Quotient *shall be the* Decimal Fraction *required.*

Example 1. *Let it be required to reduce this* Vulgar Fraction $\frac{21}{112}$ *into a* Decimal Fraction *to consist of* 4 *Places.*

To the Numerator 21 add 4 Cyphers and it is 210000; which divide by the *Denominator* 112, and when the Division is ended you will find in the *Quotient* 1875, before which prefixing a *Point*, to distinguish it from a *Whole Number*, it is .1875 which *Decimal Fraction* is in Value equal to $\frac{21}{112}$.

Example 2. *Reduce* $\frac{3}{4}$ *into a* Decimal Fraction *of* 4 Places.

To the *Numerator* 3 add 4 Cyphers, and it is 30000; which divide by the *Denominator* 4, and the Quotient will be .7500 or .75; for *Cyphers* on the *Right* Hand of a *Decimal Fraction* do neither increase nor diminish the *Value* thereof, as is before declared.

Example 3. *Reduce this* Vulgar Fraction $\frac{3}{48}$ *into a* Decimal Fraction *of* 4 Places.

To the *Numerator* 3 add four *Cyphers*, and it will be 30000; which divided by the *Denominator* 48, the *Quotient* will be 625; and the Division ended, the Quotient consists but of three Places, whereas it should consist of four Places; to supply this defect you must prefix one *Cypher* before the significant figures of the Fraction towards the left Hand, and then it will be .0625, and will be the just value of $\frac{3}{48}$:

Example 4. *If this* Fraction $\frac{7}{896}$ *were to be reduced to a* Decimal Fraction *of seven Places.*

To the Numerator 7 add seven Cyphers, and it will be 70000000, which divided by the *Denominator* 896; at the end of the Division there will be in the Quotient 78125, which are but five places, whereas they should be 7, to supply which prefix two *Cyphers* before the Quotient towards the Left Hand, and it will be .0078125; which *Decimal Fraction* will be of equal value with $\frac{7}{896}$.

Example 5. *Let it be required to reduce* $\frac{2}{3}$ to *a* Decimal Fraction *of* 3 *Places.*

To the *Numerator* 2 add 3 Cyphers, and it is 2000, which divide by 3, the *Denominator*, and the Division ended, there will be in the Quotient .66, and 6 remaining: And so, if you should add never so many Cyphers to the Numerator, the Quotient will still be all sixes. And from hence observe this following *NOTE*:

That all Vulgar Fractions *cannot be* reduced *to* Decimal Fractions, *retaining exactly the same* Value, *although they will arrive extreamly near; and the more* Cyphers *are added to their* Numerators, *the nearer will they approach the truth, but never absolutely arrive unto it.*

Example 6. *If it were required to reduce this Fraction* $\frac{9}{11}$ *to a* Decimal *of* 4 Places.

Four *Cyphers* being added to the *Numerator* 9, and that *Product* divided by the *Denominator* 11 the *Quotient* will be .8181, which is not the exact *Value* of $\frac{9}{11}$; and yet it wanteth not $\frac{1}{10000}$ part of an *Unite* of the Truth: For if you should make it .8182, it will be somewhat more than the *Truth*; and if you should yet add 5 *Cyphers* to the *Numerator*, the *Quotient* then will be .81818 which is yet too little; yet it is nearer than the former, for it now wanteth not the $\frac{1}{100000}$; for if you make it 81819,

it

it will be too much, and if you should to the *Numerator* add an hundred *Cyphers*, the *Quotient* would still be 81, 81, 81, to the end: And let this suffice for the *First Part* of the *Rule*, to *reduce Vulgar Fractions* to *Decimal Fractions*; and now for the *Second Part*, which teacheth to find the *Value* of a *Decimal Fraction*, in the known parts of the *Integer*.

CHAP. V.

How to find the Value *of a* Decimal Fraction *in the known Parts of the* Integer.

TO find the *Value* of a *Decimal Fraction* in the known parts of the *Integer*, whether it be *Coin*, *Weight*, *Measure*, *Time*, &c. this is the *RULE*:

Multiply the Decimal Fraction *given, by the Number of known Parts of the next inferiour* Denomination *of the* Integer, *the* Product (*so many figures to the Right Hand being cut off, as there were* Places *in the given* Fraction) *shall shew the* Value *in that* Denomination; *and if there yet remain any* Decimal *Parts in the* Product, *multiply them by the next lower* Denomination, *cutting off as many figures as there were* Places *in the remaining* Product. &c.

Example 1. *Let* .86042 *be a* Decimal Fraction *given, representing a part of* 20*s. or of a* Pound *Sterling.*

	.86042
	20
Shillings	17.20840
	12
	41680
	20840
Pence	2.50080
	4
Farthings	2.00320

Multiply the Fraction .86042 by 20, (because there are 20*s.* in a *li.* Sterling,) and the Product will be 1720840, from which cut off 5 Places to the Right Hand, and it will be 17.20840; the 17 to the Left Hand are 17 *Shillings*, and the .20840 are the Fraction of one Shilling, or 12*d.* which therefore multiply by 12, and the Product will be 2.50080, the 2 towards the Left Hand are 2 *Pence*, and the 50080 are the Fraction of one *Peny*, or 4 *Farthings*; which therefore multiply by 4, and the Product will be 2.00320, the 2 towards the Left Hand are 2 *Farthings*, and the .00320 the Fraction part of a Farthing, which is the least Denomination of *English Coin*. And the *Decimal Fraction* reduced thereto is 17*s.* 2*d.* 2*q.* $\frac{32}{10000}$ parts of a Farthing.

Other Examples.

Fraction given	58326	*Fraction given*	.74083
	20		20
Shillings	11.66520	*Shillings*	14.8066 14.8166
	12		12 12
13304	3304		16132 296332
6652	6652		8066 148166
79824 *Pence*	6.9824		9.6792 1777992
319296	4		4 4
Farthings	3.9296	*Farthings*	2.7168 7111968
11*s.* 6*d.* 3 *q.* $\frac{9296}{10000}$. *i.e.* 1*d. Fere.*		14*s.* 9*d.* 2*q.* $\frac{7168}{10000}$.	

Example 2. *Let* .7315 *be a* Decimal Fraction *of a Standard* Foot *of* 12 Inches.

	.7315
	12
	14630
	7315
Inches	8.7780
	4
Quarters	3.112

Multiply the given Fraction by 12, the Inches in a *Foot*, the Product will be 8.778, (omitting the Cypher,) the 8 to the Left Hand are 8 *Inches*, and the .778 are parts of an *Inch*, which multiply by 4, and the Product will be 3.112; the 3 to the Left Hand are 3 quarters of an *Inch*, and the 112 are $\frac{2}{1000}$ parts of an *Inch*; so the Value of the *Decimal Fraction*, .7315 is 8 *Inches*, 3 *Quarters*, and $\frac{112}{1000}$ parts of a *Standard Foot.*

Example

Example 2. *Let* .7315 *be a* Decimal Fraction *of a Standard* Foot *of* 12 Inches.

Multiply the given Fraction by 12, the Inches in a Foot, the Product will be 8.778, (omitting the Cypher,) the 8 to the left hand are 8 Inches, and the .778 are parts of an Inch; which multiply by 4, and the Product will be 3.112, the 3 to the left hand, are 3 Quarters of an Inch, and the 112 are $\frac{112}{1000}$ parts of an Inch; so the value of the Decimal .7315 is 8 Inches, 3 Quarters, and $\frac{112}{1000}$ parts of an Inch of a Standard Foot.

	.7315
	12
	14630
	7315
Inches	8.7780
	4
Quarters	3.112

Example 3. *Let* .94363 *be a* Decimal Fraction *whose Value is required in* Avoirdupois Weight, *whose Hundred is* 112 lb.

1. Multiply the given Fraction .94363 by 4, (because there are 4 Quarters in one Hundred weight,) the Product will be 3.77452; the 3 to the left hand are 3 Quarters.

2. Multiply the remaining Fraction .77452 by 28, (because one Quarter contains 28 Pounds,) and the Product will be 21.68656; and the 21 to the left hand are 21 Pounds.

3. Multiply the remaining Fraction .68656 by 16, (because there are 16 Ounces in a Pound,) and the Product will be 10 Ounces $\frac{98496}{100000}$ parts of an Ounce, which is equivalent to one Ounce; and the Value of the given Fraction in *Avoirdupois Weight* will be 3 *Quarters*, 21 *lb.* 11 *ou. ferè.*

	.94363
	4
	3.77452
	28
	61961 6
	154904
Pounds	21.68656
	16
	411936
	68656
Ounces	10.98496

3 *Quar.* 21 *lb.* 11 *ou. ferè.*

Example 4. *Let* .74325 *be a* Decimal Fraction *of One Year, or* 365 *Days, and let the Value thereof be required in* Days, Hours, Minutes, *and* Seconds.

1. Multiply .74325 by 365, (the Days in a Year,) the Product is 271.28625; the 271 are Days, and the .28625 are parts of a Day.

2. Multiply .28625 by 24, (the Hours in a Day,) and the Product will be 6.87, that is, 6 Hours, and 87 parts of an Hour.

3. Multiply 87 by 60, (or 6,) the Product is 5.22; that is, 5 Minutes and 22 parts of a Minute.

4. Multiply 22 by 60, (or 6,) the Product will be 1.32; that is 1 Second $\frac{32}{100}$ parts of a Second: And so the Fraction reduced to the parts of Time is 271 Days, 6 Hours, 5 Minutes, 1 Second, and $\frac{32}{100}$ parts of a Second, as appear by the Work following.

	.74325
	365
	371625
	445950
	222975
Days	271.28625
	24
	114500
	57250
Hours	6.87000
	6
Minutes	5.22
	6
Seconds	1.32

Days.	*Hours.*	*Min.*	*Sec.*
271	6	5	1 $\frac{32}{100}$.

According to these *Rules* the following Tables of *Coin, Weights, Measures,* are made; and the like may be done for any Foreign *Coins, Weights,* or *Measures,* as any one shall have occasion.

CHAP. VI.

Teaching how to reduce the known Parts of Coins, Weights, Measures, &c. *into* Decimal Fractions.

I. Of *Coin.*

Example 1. *Let it be required to find a* Decimal Fraction *to* 5 *places, which shall repreſent* 7 Shillings *of a* Pound *Sterling, or* 20 Shillings.

Seven *Shillings is* $\frac{7}{20}$ of a *Pound*, therefore add 5 Cyphers to 7, and it makes 700000, which divided by 20, the Quotient is .35, for the *Decimal Fraction* repreſenting 7 *s.* In like manner, the *Decimal* of 3 *s.* will be found .15; of 19 *s.* .95, *&c.*

Example 2. *Let it be required to find the* Decimal Fraction *to* 5 *Places, belonging to* 7 *Pence.*

Seven Pence is $\frac{7}{12}$ of $\frac{1}{20}$ of a Pound, or 7 *d.* is $\frac{7}{240}$ of a Pound; to 7 add 5 Cyphers, and it is 700000; which divide by 240, the Quotient will be .02916, for the *Decimal Fraction* of 7 *Pence.*

Example 3. *Let the* Decimal *repreſenting* 13 s. 3 d. 3 q. *be required, to* 5 *Places.*

In 20 *Shillings* there are 960 *Farthings*, and in 13 *s.* 3 *d.* 3 *q.* there are 639 *Farthings* then is 13 *s.* 3 *d.* 3 *q.* the $\frac{639}{960}$ of a *Pound*, or 20 *s.* add 5 Cyphers to the *Numerator* of the Fraction, it will be 63900000; which divide by the *Numerator* 960, the Quotient will be .66562, which is the *Decimal* Fraction of 13 *s.* 3 *d.* 3 *q.*

II. *Of Weight.*

Example. *What is the* Decimal Fraction *which ſhall repreſent* 3 Quarters, 21 Pound, 10 Ounces *of* Avoirdupois Weight, *to* 5 *Places?*

In 112 *Pound* there are 1792 *Ounces*, and in 3 *Quar.* 21 *lb.* 10 *Ounces*, there are 1520 *Ounces*; therefore 3 *Quar.* 21 *lb.* 10 *Oun.* is the $\frac{1520}{1792}$ of a *Hundred:* To the *Numerator* 1690 add 5 Cyphers, and it will be 169000000; which divided by the *Denominator* 1792, the Quotient will be .94308; which is the *Decimal* of 3 *Quarters*, 21 *Pound*, and 10 *Ounces.*

III. *Of Meaſure.*

Example. *What is the* Decimal Fraction *anſwering to* 9 Inches, 3 Quarters, *the Foot being* 12 Inches, *to* 5 *Places?*

In one *Foot* there are 48 Quarters of Inches; and in 9 *Inches* 3 *Quarters*, 39 *Quarters*; ſo that 9 *Inches* 3 *Quarters* is $\frac{39}{48}$ of a *Foot:* Add 5 Cyphers to the *Numerator* 39, it is 3900000; which divide by 48, the *Denominator*, the Quotient will be .8125; which is the Decimal Fraction anſwering to 9 *Inches* 3 *Quarters:* And ſo of any other, as in the following Tables.

A Table

A Table of Englifh Coin, Two Shillings being the Integer.

d. q.		*d. q.*	
0 0		12 0	.05...
1	.00104	1	.05164
2	.00208	2	.05208
3	.00312	3	.05312
1 0	.00416	13 0	.05416
1	.00521	1	.05521
2	.00625	2	.05625
3	.00729	3	.05729
2 0	.00833	14 0	.05833
1	.00937	1	.05907
2	.01041	2	.06011
3	.01145	3	.06115
3 0	.01250	15 0	.0625.
1	.01354	1	.06354
2	.01458	2	.06458
3	.01562	3	.06562
4 0	.01666	16 0	.06666
1	.01770	1	.06777
2	.0187.	2	.0687.
3	.01979	3	.06979
5 0	.02083	17 0	.07083
1	.02187	1	.07187
2	.02292	2	.07292
3	.02396	3	.07396
6 0	.025..	18 0	.075..
1	.02604	1	.07604
2	.02708	2	.07708
3	.02813	3	.07813
7 0	.02917	19 0	.07917
1	.03021	1	.08021
2	.03125	2	.08125
3	.03229	3	.08229
8 0	.03333	20 0	.08333
1	.03437	1	.08437
2	.03542	2	.08542
3	.03646	3	.08646
9 0	.0375.	21 0	.08750
1	.03854	1	.08854
2	.03958	2	.08958
3	.04063	3	.09063
10 0	.04166	22 0	.09166
1	.04271	1	.09271
2	.04375	2	.09375
3	.04479	3	.09479
11 0	.04583	23 0	.09583
1	.04687	1	.09687
2	.04792	2	.09792
3	.04896	3	.09896

A Table of Foot Meafure 12 Inches being the Integer.

f. i.	
0 1	.02083
2	.04166
3	.06250
1 0	.08333
1	.10416
2	.12449
3	.14502
2 0	.16535
1	.18568
2	.20833
3	.22918
3 0	.25...
1	.27083
2	.29166
3	.31250
4 0	.33333
1	.35416
2	.375..
3	.39583
5 0	.41666
1	.4375.
2	.45833
3	.47918
6 0	.5....
1	.52083
2	.54166
3	.5625.
7 0	.58333
1	.60416
2	.62429
3	.64502
8 0	.66535
1	.68568
2	.70833
3	.72918
9 0	.75...
1	.77083
2	.79166
3	.8125.
10 0	.83333
1	.85416
2	.875..
3	.89583
11 0	.91666
1	.9375.
2	.95833
3	.97916

A Table of Avoirdupois great weight 28 Pound being the Integer.

lb.	
1	.00893
2	.01786
3	.02678
4	.03571
5	.04464
6	.05357
7	.0625.
8	.07143
9	.08036
10	.08928
11	.09812
12	.10714
13	.11607
14	.125..
15	.13393
16	.14286
17	.15179
18	.16071
19	.16964
20	.17857
21	.1875.
22	.19643
23	.20536
24	.21429
25	.22521
26	.23214
27	.24107

1 Quart.	.25
A half	.5
3 Quar.	.75

CHAP. VII.

Of ADDITION of DECIMALS.

ADdition of *Decimal Numbers* differeth nothing from the *Addition* of *Whole Numbers* of one *Denomination*, before taught, in which there is no difficulty: But in *Decimal Numbers* to be added, the chief care to be taken is in placing your *Integers* and *Fractions* in their due order; which you ſhall eaſily and certainly doe, by obſerving this general *RULE:*

Place your Integers *(or* Whole Numbers*) and* Decimal Parts *(or* Fractions*) one under another, ſo that the* Points *of ſeparation (which in* Decimal Numbers *do diſtinguiſh the* Integers *from the* Decimal Parts*) ſtand directly one under another; and when they are ſo ſeated, proceed to the* adding *of them together, as if they were* Whole Numbers *of one* Denomination.

Example 1. *Let it be required to add together theſe four* Decimal Mixt Numbers, *viz.* 36.1333, 19.0083, 31.8375, 6.1208, *being* Decimal Parts *of* Engliſh Coin; *the* Integers *being* Pounds, *and the* Decimal Parts *Fractions of a* Pound *ſ as* Shillings, Pence, *and* Farthings.

36,1333
19,0083
31,8375
6,1208
93,0999

Your *Decimal Numbers* being thus placed in due order, one under another, proceed to the adding of them together, as if they were Whole Numbers, and you ſhall find the ſumm or total of them to be 103.0999.

Now the 103 which ſtands towards the left hand, are 103 Pounds; and the 0999 which ſtand towards the right hand of the Point, is the Fraction-part of one Pound Sterling, the value whereof you may find (by the Rule taught in *Chap. V.*) to be two Shillings *ferè*; which ſhould be two Shillings exact, but it wanteth ſomewhat, *viz.* the $\frac{2}{1000}$ part of a Farthing, which is inſenſible; for if by the fore-mentioned Rule you ſeek the value of the Decimal Fraction .0999, you ſhall find it to be 1 Shilling, 11 Pence, 3 Farthings, and the $\frac{2}{1000}$ part of a farthing, which you may call in all 2 Shillings; for Decimal Numbers will ſeldom happen to give the exact value of Fractions, but will be either greater or leſſer than they ought to be; but in ſuch a ſumm as this, the thouſandth part of a farthing is not to be regarded.

Example 2. *Let it be required to add together in a Decimal way theſe ſumms following,* viz. 29 li. 18 s. 7 d. 3 q. 63 li. 11 s. 2 d. 1 q. 129 li. 4 s. 0 d. 2 q. *and* 3 li. 7 s. 10 d. 1 q.

First, for 29 *li.* 18 *s.* 7 *d.* 3 *q.* ſet down	29.93229
Secondly, for 63 *li.* 11 *s.* 2 *d.* 1 *q.* ſet down	63.55907
Thirdly, for 129 *li.* 4 *s.* 0 *d.* 2 *q.* ſet down	129.20208
Laſtly, for 3 *li.* 7 *s.* 10 *d.* 1 *q.* ſet down	3.39271
	226.08615

Your Decimal Numbers thus placed in order, add them together, as if they were Whole Numbers, and you ſhall find the ſumm of them to contain 226.08615.

Now the 226 which ſtands towards the left hand of the point, are 226 Pounds, and the other figures towards the right hand, *viz.* 08615, are the Fraction parts of a Pound ſterling; which if you reduce (as hath been already taught,) you ſhall find the value to be 1 Shilling, 8 Pence, three Farthings: So the whole ſumm is 226 *li.* 1 *s.* 8 *d.* 3 *q.*

And here note, that what hath been ſaid concerning *Money*, the ſame is alſo to be underſtood of *Weight*, *Meaſure*, *Time*, &c. as by the following Examples will appear.

Examples for Practice.

Example 1. In *Money.*	Example 2. In *Troy weight.*	Example 3. In *Avoird. little w.*	Example 4. In *Avoird. great w.*
135.8833	7.1625	12.7227	37.9442
95.5583	6.225	76.3594	9.3053
3.2875	3.5625	32.6256	33.6786
234.7291	16.9500	121.7077	80.9281
234 *li.* 14 *s.* 7 *d.*	16 *ou.* 9 *pw.* 0 *gr.*	121 *lb.* 11 *ou.* 5 *dr.*	80 *C.* 3 *q.* 19 *lb.* 15 *ou.*

CHAP. VIII.

SUBTRACTION of DECIMALS.

THE *Subtraction* of *Decimals* differeth nothing from the Subtracting of one whole Number from another, and the Decimal Numbers to be ſubtracted one from another, muſt be placed in the ſame order; as in *Addition* of Decimal Numbers; the practice of Subtraction ſhall be ſeen in the following Examples.

Example 1. *Let it be required to ſubtract* 31.8375 *from* 36.1333

Set your Numbers one under the other, as is before directed in Addition; and as is here done in the Margin: Then draw a Line under them, and ſubtracting the leſſer from the greater, you ſhall find the remainder to be 4.2958; the 4 on the left ſide of the Point, are four Pounds, and the .2958, which ſtandeth towards the right hand, is the fraction-part of a Pound, the value wherof being ſought, will be found to be 5*s.* 11*d.* So that if you ſubtract 31*li.* 16*s.* 9*d.* out of 36*li.* 2*s.* 8*d.* there will remain 4*li.* 5*s.* 11*d.*

	36.1333
	31.8375
	4.2958

But if divers ſumms be to be ſubtracted out of one greater ſumm, then you muſt firſt add all the ſeveral ſmaller ſumms together, and ſubtract the ſumm of them from the greater given ſumm; ſo ſhall the reſidue be the ſumm deſired.

Example for Practice.

Example 1. In *Money*		Example 2. In *Avoird.* great *w.*	
	2376.7432	*Bought*	103.7423
Paid at ſeveral times.	431.0072	*Sold*	37.9442
	100.0000	*Unſold*	66.7891
	72.6325		65 *C.* 39 *q.* 5 *lb.* 7 *ou.*
	90.1427		
Paid in all	693.7824		
Reſts in Money	1682.9608		
	1682 *li.* 19*s.* 2*d.* 2*q.*		

CHAP. IX.

MULTIPLICATION of DECIMALS.

Multiplication of Decimals differeth nothing at all from the *Multiplication* of whole Numbers; for making the greater number the *Multiplicand*, and the leſſer number the *Multiplier*, the number iſſuing from that Multiplication ſhall be called the *Product*.

Now in the Multiplication of Decimal Numbers one by another, if there be any Decimal Part or Fraction, either in the *Multiplicand* or *Multiplier*, or Fractions in both; ſo many Places as the Fractions contain, ſo many figures muſt be cut off from the *Product* towards the Right Hand, which ſhall be the Fraction of the *Product*, and the figures towards the Left Hand of the Point in the Product, ſhall be the *Integers* of the *Product*.

Example 1. *Let it be required to multiply* 34.2625 *by* 16.325.

Make 34.2625 your Multiplicand, 16.325 your Multiplier; then draw a line, and multiply theſe two numbers together, as if they were whole Numbers, and you ſhall find the Product of them to be 559.3353125. Now becauſe there are four figures in the Multiplicand, which are Fractions, namely, theſe four towards the Right Hand, *viz.* 2625; and there are alſo three figures in the Multiplier, which are Fractions, namely, theſe three towards the Right

Multiplicand	34.2625
Multiplier	16.325
	171.3125
	102.7875
	20.5575
	3.4262
Product	559.3353125

Hand,

Hand, *viz.* 325, that is in all seven figures, representing Fractions; therefore cut off from the *Product* the seven figures towards the Right Hand, by making of a Point, there to distinguish the whole Number from the Fraction: So is 559 the Integers or whole number, and 3353125 the Fraction of this Multiplication.

Multiplicand	5767.75
Multiplier	235
	2883875
	1730325
	1153550
Product	1355421.25

Example 2. If there be Fractions in the Multiplicand, and none in the Multiplier, yet the work is still the same, for you must cut off only so many figures from the Product, as there are Fractions either in the Multiplicand, Multiplier, or both: So if it were required to multiply 5767.75 by 235, you must first set down 5767.75 for your *Multiplicand*, and also set down 235 for your *Multiplier*; then multiplying them together, as if they were whole Numbers, you shall find the Product to be 1355421.25; and because there are only two Fraction figures, both which are in the *Multiplicand*, namely, the two last thereof .75, and none in the *Multiplier*; therefore cut off only two figures of the *Product*, namely, the two last, which are .25, so is the *Product* of this Multiplication 1355421.25.

.953	
.782	
1906	
7624	
6671	
.745246	

Example 3. If *Decimal Fractions* be to be multiplied by *Decimal Fractions*, you must then (as before,) multiply them as *Whole Numbers*, and from the *Product* cut off so many figures towards the Right Hand, as there are figures in the *Multiplicand* and *Multiplier*: So if it were required to multiply .953 by .782, you shall find their *Product* to be .745246, which being but six figures in all, cut them off, and that Fraction .745246, is the *Product* of the *Multiplication* of the two given *Fractions*.

.072	
.063	
2256	
4512	
.0047376	

Example 4. If the *Product* of any two *Decimal Fractions* being multiplied together, do not consist of so many places, as are required (by the former rules) to be cut off, you must then supply that defect by prefixing a *Cypher* or *Cyphers* before the *Product* towards the Left Hand: So if these *Decimal Fractions*, .063 and .0752 were to be multiplied, their product would be 47376. Now (by the former rules,) you should cut off *seven figures* of the *Product* towards the Right Hand, but this *Product* 47376 consisteth but of five figures; wherefore to make it seven figures, prefix two Cyphers before the *Product* on the Left Hand, making it .0047376, and that is the true *Product* produced by this *Multiplication*.

CHAP. X.

A Compendious way of Multiplication of Decimal Mixt Numbers.

A Great *Objection* that I have often heard made against *Decimal Multiplication*, hath been this; that when the *Mixt Numbers* to be multiplied together do consist of many *Decimal Parts*, the *Product* will be increased to 8, 9, 10, 12, or more *Decimal Places* to be cut off; whereas 2, 3, or 4 at the most, will serve the turn sufficiently, to perform the *Work* intended thereby. To this I answer, that in many *Cases* it may so fall out; but that *Objection* may be removed by this following *COMPENDIUM*: For the working of this *Compendium* this is the *RULE*:

Set down the bigger of the two Mixt Numbers, *and (though there be several* Places *of* Parts, *both in the* Multiplicand *and* Multiplier,*) set the place of* Unity *of the* Integer *of the* Multiplier, *under that figure of* Parts *in the* Multiplicand, *whose number of* Parts, *you would have in the* Product: *Then set all the figures of the* Multiplier, *the contrary way to what they were given; then by each figure in the* Multiplier, *multiply the figure over it in the* Multiplicand *(having regard to the figure going before it) and set down the* Product, *&c.* This will appear best by *Example.*

Example 1. *Let it be required to multiply* 34.2625 *by* 16.325, *so that there may be but two* places *of* Parts *in the* Product.

1. Set

1. Set down the bigger Number 34.2625 for the *Multiplicand*, and becauſe you would have but two *Places* of *Parts* in your *Product*, (for according to the former directions there would be 7 Places,) ſet the *Unites Place* of the *Integer* in the *Multiplier*, which is 6, under the ſecond place of Parts in the *Multiplicand*, which is 6 alſo, and tranſpoſe all the other figures of the *Multiplier*, as you ſee done in the Margin; for the multiplier given was 16.325, and here you ſee it is made 523.61, and 6, the place of Unity in the Multiplier, ſtands under 6 (the ſecond place of *Parts* in the *Multiplicand*:) The *Multiplicand* and *Multiplier* being placed in this order, begin your work as followeth, beginning at the Right Hand, ſaying, once 5 is 5, (for which becauſe it is 5, keep one in mind,) but ſet it not down, but ſay again, once 2 is 2, and one kept is 3; ſet 3 under 1, and go on with that Line, ſaying, once 6 is 6, once 2 is 2, once 4 is 4, and once 3 is 3, ſetting them down as you go; ſo ſhall the firſt Line or Product be 34263. Then go to the next figure at the *Multiplier* 6, and ſay, 6 times 2 is 12; for which bear one in mind, and ſay again, 6 times 6 is 36, and one in mind is 37; ſet 7 under 3, and go on with the reſt of the figures, ſaying, 6 times 2 is 12, and 3 in mind is 15; ſet down 5 and bear 1, and ſay, 6 times 4 is 24, and 1 is 25; ſet down 5 and carry 2, and ſay, 6 times 3 is 18, and 2 is 20, which being the laſt ſet down; ſo ſhall your ſecond Line or *Product* be 20557. Then go to the third figure of the *Multiplier* 3, and ſay, 3 times 6 is 18, for which (8 being above 15) bear 2 in mind, and ſay, 3 times 2 is 6, and 2 in mind is 8; ſet down 8 under 7, and ſay, 3 times 4 is 12; ſet down 2 and carry 1, then 3 times 3 is 9, and 1 is 10, which being the laſt ſet down: So is the third Line or Product 1028. Then for the fourth figure 2, ſay, 2 times 2 is 4, which being leſs than 5, bear none in mind, but go on, and ſay, 2 times 4 is 8; ſet down 8; and 3 times 2 is 6, which ſet down alſo: So is the fourth Line or *Product* 68. Then for the laſt figure of the *Multiplier* 5, ſay, 5 times 4 is 20, for which keep 2 in mind, and ſay, 5 times 3 is 15, and 2 in mind is 17, which ſet down, and ſo is your *Multiplication* ended; under which draw a Line, and add all the *Products* together in the ſame order as they ſtand in the Margin, and the general *Product* will be 55933, from which ſeparate two places to the Right Hand, (becauſe you were to have but two places of *Parts*,) and the *Product* will be 559 *Integers*, and .33 *Parts*. And thus may you doe to have what number of *Parts* you pleaſe in your general *Product*.

Multiplicand	34.2625
Multiplier	523.61
	34263
	20557
	1028
	68
	17
Product	559.33

Example 2. *Let it be required to multiply the ſame mixt Number* 34.2625, *by the ſame former mixt Number* 16.325; *ſo that there may be no parts in the* Product, *but all* Integers *only*.

In this Caſe, ſet the *Unites* place of the *Multiplier*, under the *Unites* place in the *Multiplicand*, (tranſpoſing all the figures of the *Multiplier*, as before,) as you ſee in the Margine.

The Numbers being thus placed, begin with the firſt figure of the multiplier 1, ſaying, once 6 is 6, for which bear 1 in mind, and ſay, once 2 is 2, and one in mind is 3; ſet down 3 under 1; then ſay, once 4 is 4, and once 3 is 3; ſet them down: So is your firſt *Product* 343. Then go to the ſecond figure 6, and ſay, 6 times 2 is 12, for which bear one, and ſay, 6 times 4 is 24, and one in mind is 25; ſet down 5 and bear 2 in mind; then 6 times 3 is 18, and 2 is 20, which ſet down; ſo is your ſecond *Product* 205: Then for the third 3, ſay, 3 times 4 is 12; for which bear 1 in mind, and ſay, 3 times 3 is 9, and 1 in mind is 10; which ſet down for your third Product. Laſtly, 2 times 3 is 6, for which (being above 5) ſet down 1 for the Product; add all together, the general Product is 559 *Integers*, free from all *Decimal Parts*.

Multiplicand	34.2625
Muliplier	523.61
	343
	205
	10
	1
	559.

This is an excellent *Compendium*, Examples whereof you ſhall find in the Conſtruction and uſe of the Tables of Compound Intereſt, *&c.* following at the end hereof; however take theſe few following ready done.

	72.2543
	2461.5
In this *Example*	38127
there ſhould have been	763
8 places of *Parts*, but	457
I needed but two.	30
	2
	393.79

	259879.890625
	8963711.1
In this *Example*	2598798 91
there ſhould have	2598798 9
been 13 places of	259870 1
Parts cut off, but	18192 59
three were ſufficient	7796 4
for the occaſion.	1559 2
	233 8
	20 7
	2903819.94

CHAP. XI.

Some Examples, *whereby the excellency of* Decimal Multiplication *of* Numbers, *of diverſe* Denominations *may appear to exceed that of* Vulgar Fractions, *in the like Operations.*

HAving in the foregoing *Section*, taken off the Vulgar *Exception* againſt *Decimal Multiplication*, I will here (in things of publick concern) exemplifie its Excellency: And I cannot better do it, than by ſhewing how to *multiply numbers of different Denominations* one by another, and rendring the *Reſult* or *Product* in the known *Parts* of the *Integer*: I will inſtance in two or three, and leave the underſtanding *Reader* to be judge of the *facility* of the one, and the *intricaſie* of the other.

Example 1. *Let it be required to multiply* 1 *li.* 19 *s.* 11 *d.* 3 *q.* *by* 1 *li.* 19 *s.* 11 *d.* 3 *q.* *and to give the* Reſult *or* Product *in the reſpective parts of the* Integer.

The *Decimal* anſwering to 1 *li.* 19 *s.* 11 *d.* 3 *q.* is ——— 1.99896

Which multiplyed in it ſelf (according to the *Rule* before given, for contracting of *Multiplication*) the *Product* will be 3.99581; that is 3 *li.* and the *Fraction* .99581 being reduced, is 19 *s.* 11 *d.* and that is the true *Product* of 1 *li.* 19 *s.* 11 *d.* 3 *q.* multiplied in it ſelf.

6.98991
199896
179906
17990
1598
179
11
3.99581

Example 2. *There are* ſeven Perſons *to receive* 325 *li.* 16 *s.* 9 *d.* *a piece, how much Money will pay them?*

The *Decimal* of 329 *li.* 16 *s.* 9 *d.* is 325.8375, which multiplied by 7, (the number of Perſons,) the *Product* will be 2280 *li.* 17 *s.* 3 *d.* and ſo much Money will pay them.

Example 3. *There is a* Board, *or* Plank, 15 *Foot*, 5 *Inches and an half* Long, *and* 3 *Foot*, 9 *Inches and a quarter* Broad; *how many* Square Feet *are there in that* Board *or* Plank?

The *Decimal* of 15 Foot, 5 Inches and an half, is 15.45833, which multiplied by 3.77083, the *Decimal* of 3 Foot, 9 Inches and a quarter, the Product will be 58.29073, which is the *Decimal* of 58 Foot, 3 Inches and an half almoſt; and ſo many Feet are there in the Board or Plank.

Example 4. *There is a piece of* Squared Stone *or* Timber, *which is* 26 *Foot*, 9 *Inches and an half long; it is* 2 *Foot*, 1 *Inch and* 3 *quarters broad, and* 1 *Foot*, 3 *Inches and an half deep; how many ſolid Feet be there in that* Piece.

The *Decimal* of 2 Foot, 1¾ Inches is 2.14502, the *Decimal* of 1 Foot, 8 ½ Inches is 1.29166, theſe multiplied together do produce 2.771, *&c.* then the *Decimal* of 26 Foot, 9½ Inches (the length) is 26.79166, which multiplied by 2.771, the *Product* will

will be 101.025, which reduced, is 101 Foot, and one quarter of an Inch; and so many *Solid Feet* be there in that *squared piece* of *Stone* or *Timber*, as appears by the work following:

2.145	2.771
292.1	976.2
2145	75420
429	22626
193	2640
4	339
2.771	101.025, *&c.*

CHAP XII.

Of DIVISION of DECIMALS.

Division of *Decimals* differeth nothing from that of *Whole Numbers*, the only difficulty is in knowing the true Value of the *Quotient*; that is, to know of what number of *Integers*, and what number of *Parts*: And the *Decimals* to be divided may be either,

1. A *Whole Number* by a *Whole Number.*
2. A *Whole Number* by a *Mixt Number.*
3. A *Whole Number* by a *Decimal Fraction.*
4. A *Mixt Number* by a *Whole Number.*
5. A *Mixt Number* by a *Mixt Number.*
6. A *Mixt Number* by a *Decimal Fraction.*
7. A *Decimal Fraction* by a *Whole Number.*
8. A *Decimal Fraction* by a *Mixt Number.*
9. A *Decimal Fraction* by a *Decimal Fraction.*

Before I proceed to the Practice of *Division* in *Decimals*, I shall lay down these Considerations following:

1. *If the* Divisor *consists of more* Places *than the* Dividend, *there must be a competent number of* Cyphers *added to the* Dividend, *to make it consist of as many at least (or more) than the* Divisor, *whether the* Dividend *be a* Whole Number, *or a* Mixt Number, *or a* Decimal Fraction.

2. *Consider whether there be as many* Decimal Parts *in the* Dividend, *as there are in the* Divisor; *if there be not, make them as many (or more) by annexing of* Cyphers.

3. *When there is an equal Number of* Decimal Parts *both in the* Dividend *and the* Divisor, *and a* Division *may be made; the* Quotient *will (always) be a* Whole Number, *without any* Decimal Parts.

4. *Consider, that as in* Multiplication *of* Decimal Fractions, *the* Product *doth always contain as many* Places *of* Decimal Parts, *as there were* Places *of* Decimal Parts *in the* Multiplicand *and the* Multiplier; *so in* Division *of* Decimals, *if the* Quotient *be multiplied by the* Divisor, *the* Product *shall be equal to the* Dividend; *for* Multiplication *and* Division *prove each other as hath been before intimated in the first Part.*

These Considerations duly minded, the value of the *Quotient* of any *Decimal Division*, may be certainly known by observing this following *RULE:*

When the work of Division *is ended, consider how many* Decimal Parts *here are in the* Dividend, *more than in the* Divisor; *for that excess is the number of* Decimal Parts, *which must be separated in the* Quotient *for* Decimal Parts. *But if there be not so many figures in the* Quotient, *as the said excess is, they must be supplied by prefixing of so many* Cyphers *in the* Quotient, *before the significant figures thereof towards the Left Hand, with a* Point *before them: And the* Quotient *shall then plainly discover its* Value.

I shall illustrate the foregoing Considerations, and this Rule, by giving *Examples* in all the Nine foregoing *Varieties.*

1. *To divide a Whole Number* 8976, *by a Whole Number* 23456.

Divisor.	*Dividend.*	*Quotient.*
23456)	8976.0000	(.3826, *&c.*
	. . . .	
	70368	
	193920	
	187648	
	62720	
	46912	
	158080	
	140736	
Remainder	17344	

In this Example the *Divisor* is greater than the *Dividend*, therefore add four *Cyphers* to the *Dividend*; the *Division* ended, you find the *Quotient* to be .3826, *&c. Parts*, and of such *Value* is the *Quotient*; and to prove this, multiply 23456, the *Divisor*, by 3826, the *Quotient*, the *Product* will be 89742656, to which if you add the *Remainder* 17344, the summ will be 8976.0000, equal to the *Dividend*.

And according to this *Rule*, 8976 divided by 54, the *Quotient* will be 166.22, *&c.*

Another Example of this Case:

Divide 48 by 43796

Divisor.	*Dividend.*	*Quotient.*
43796)	48.000000	(.1095, *&c.*
	. . . .	
	43796	
	420400	
	394164	
	262360	
	218980	
Remainder	43380	

In this Example the *Dividend* being less than the *Divisor*, add six Cyphers to the Dividend, because the *Quotient* should consist of six *Decimal Parts*, and the Division ended, you find but four *Decimal parts* there; therefore (by the Rule) prefix two Cyphers before the four significant figures in the *Quotient* to make them six, and then the *Quotient* is .001095 Parts, which is its Value.

2. *To divide a Whole Number* 586, *by a Mixt Number* 36.4865.

Divisor.	*Dividend.*	*Quotient.*
36.4865)	586.000000	(16.06, *&c.*
	. . . .	
	364865	
	2211350	
	2189190	
	2216000	
	2189190	
Remainder	26810	

In this Example although the *Dividend* be greater than the *Divisor*, yet there can be no Division made without adding of a competent number of *Cyphers* to the *Dividend*; as here you have added *Six*; and the Division ended, you find the *Quotient* to be 1606, and becauſe in the *Dividend* there are 6 *Decimal Parts*, and in the *Divisor* 4, the exceſs is 2; therefore cut off two *Places* in the *Quotient* towards the right hand, and the value of the *Quotient* will be 16.06, *&c.*

Another Example of this Case:

Divide the Whole Number 6, *by the Mixt Number* 27.423.

Divisor.	*Dividend.*	*Quotient.*
27.423)	6.000000	(218
	. . .	
	54846	
	51540	
	27423	
	241170	
	219384	
	21786	

In this Example, add 6 *Cyphers* to the *Dividend*, and it is 6.000000; which being divided by 27.423, and the Division ended, you find the *Quotient* to be 218. Now by the former *Rule*, there being 6 places of *Parts* in the *Dividend*, and but 3 in the *Divisor*, the exceſs is 3; ſo that 3 places of *Parts* muſt be cut off in the *Quotient*, which are all that are there; therefore the *Quotient* is .218 *Parts*, and if you had added more *Cyphers*, there would have been more *Parts* in the *Quotient*.

3. *To*

3. *To divide a Whole Number* 72, *by a* Decimal Fraction .0432.

In this Example (that Diviſion may be made) add 6 *Cyphers* to the *Dividend*, and the *Diviſion* ended, you find in the *Quotient* 166666; and by the *Rule* there being 6 places of *Parts* in the *Dividend*, and but 4 in the *Diviſor*, the exceſs is 2, therefore ſeparate 2 *Places* from the *Quotient* towards the Right Hand, for *Parts*; ſo is the *Quotient* 1666 *Integers*, and .66 *Parts*.

Diviſor.	*Dividend.*	*Quotient.*
.0432)	72.000000	(166666, *&c.*
	.....	
	432	
	2880	
	2592	
	2880	
	2592	
	2880	
	2592	
	2880	
	2592	
	2880	

Another Example of this Caſe:

Divide 968 by .4325.

.4325)	968.000000	(2238.1 *&c.*
	.....	
	8650	
	10300	
	8650	
	16500	
	12975	
	35250	
	34600	
	6500	
	4325	
	2175	

4. *To divide a Mixt Number* 58.271875, *by a Whole Number* 725.

In this Example the *Dividend* conſiſting of ſix places of *Parts*, and the *Diviſor* of none, you ſhould by the foregoing *Rule* ſeparate 6 places from the *Quotient*; but the Diviſion being ended, there are but 5 *Places* in the *Quotient*; wherefore to ſupply that defect, place a Cypher on the Left Hand of the *Quotient*, to make them ſix, as you ſee in the Example; and the *Quotient* is .080375 Parts.

Diviſor.	*Dividend.*	*Quotient.*
725)	58.271875	(.080375
	.....	
	5800	
	2718	
	2175	
	5437	
	5075	
	3625	
	3625	
	0000	

Another Example of this Case:

Divide 755.525 by 35.

In this Example also the *Dividend* consisting of four places of *Parts*, and the *Divisor* of none, four places must be cut off in the *Quotient* for *Parts*; then will the *Quotient* be 21 *Integers*, and .5864 *Parts*.

Divisor. *Dividend.* *Quotient.*

35) 755.5250 (21.5864

70

55

35

205

175

302

280

225

210

150

140

10

5. To divide a Mixt Number 885.69864, by a Mixt Number 72.361.

In this Example, the *Dividend* consisting of five places of *Parts*, and the *Divisor* but of three, the excess or difference of Places is *Two*, therefore separate two Places to the Right Hand out of the *Quotient*, and it is 12 *Integers*, and .24 *Parts*.

Divisor. *Dividend.* *Quotient.*

72.361) 885.69864 (12.24

72361

160288

144722

173666

144722

289444

289444

000000

Another Example of this Case:

Divide 723.61578888 by 885.69864.

Divide

885.69864) 723.61578888 (.817

708558912

150568768

88569864

61998904 8

619989048

000000000

In

In this Example, becauſe Diviſion may be made, add no *Cyphers* to the *Dividend*; and the Diviſion being ended, you find 8 *Decimal Parts* in the *Dividend*, and but 5 in the *Diviſor*, the exceſs or difference being 3; wherefore ſeparate three Figures in the *Quotient*, (which in this caſe are all,) for *Parts*, and the *Quotient* is .817 *Parts*.

6. *To divide a* Mixt Number 5328.625 *by a* Decimal Fraction .425.

In this Example, although the *Diviſor* may be had ſeveral times in the *Dividend*; yet, that you may have more places of *Parts* in the *Dividend* than in the *Diviſor*, add three *Cyphers* thereunto; and when the Diviſion is ended, you find 6 places of *Parts* in the *Dividend*, and but 3 in the *Diviſor*; therefore ſeparate 3 places of *Parts* on the Right Hand of the *Quotient*, and then the value of the *Quotient* will be 12537 *Integers*, and .941 *Parts*.

Diviſor. *Dividend.* *Quotient.*
425) 5328.625000 (12537.941

425
1078
850
2286
2125
1612
1275
3375
2975
4000
3825
1750
1700
500
425
75

Another Example of this Caſe:

Divide 54.07824 by .046

In this Example alſo, (where no *Cyphers* are added,) the *Diviſion* ended, the *Quotient* is 117561; and ſince there are 5 places of *Parts* in the *Dividend*, and but 3 in the *Diviſor*, ſeparate the exceſs of the Parts, which are two, on the Right Hand, and then the true value of the *Quotient* is 1175 *Integers*, and .61 *Parts*.

Diviſor. *Dividend.* *Quotient.*
.046) 54.07824 (1175.61

46
80
46
347
322
258
230
282
276
64
46
18

Example

Example 7. *To divide a* Decimal Fraction .06283 *by a* Whole Number 23.

Divisor.	*Dividend.*	*Quotient.*
23)	.06283	(.00273
	46	
	168	
	161	
	73	
	69	
	4	

In this Example there needs no adding of *Cyphers* to prepare the *Work* for Diviſion, but that may be done at Pleaſure, if need require; but ſeeing there is no occaſion for it in this *Caſe*, divide .06283 by 23, and the *Quotient* will be 237: Now the *Dividend* conſiſting of 5 places of *Parts*, and the *Diviſor* of none; there ſhould be 5 places of *Parts* in the *Quotient*, but there are but 3 figures there, therefore prefix two *Cyphers* before them, and the true Quotient will be .00273 Parts.

Another Example of this Caſe:

Divide .73 by 68803.

Diviſor.	*Dividend.*	*Quotient.*
68803)	.73000000	(.00001061
	68803	
	419700	
	412818	
	68820	
	68803	
	17	

In this Example there is a neceſſity of adding of *Cyphers* to the *Dividend*, and therefore add ſix; and then dividing .73000000 by 68803, the *Quotient* will be 1061; and there being eight places of *Parts* in the *Dividend*, there ſhould be as many in the *Quotient*; but there being but 4 figures there, prefix 4 Cyphers, and then the Quotient is .00001061 Parts.

Example 8. *To divide a* Decimal Fraction .846 *by a* Mixt Number 2.43.

In this Example there is a neceſſity of adding of Cyphers to the *Dividend*, wherefore add three, and then dividing, (as in whole Numbers,) the *Quotient* will be 3481; then there being ſix places of *Parts* in the *Dividend*, and but two in the *Diviſor*, the exceſs is 4, therefore there muſt be four places of *Parts* in the *Quotient*; wherefore before them prefix a Point, and the value of the *Quotient* is .3481 Parts.

Diviſor.	*Dividend.*	*Quotient.*
2.42)	.846000	(.3481
	729	
	1170	
	972	
	1980	
	1944	
	360	
	243	
	117	

Another Example of this Caſe:

Divide .7320065 by 321.56

Diviſor.	*Dividend.*	*Quotient.*
321.56)	.7320065	(.00228, *ferè.*
	64212	
	89886	
	64312	
	255745	
	225092	
	30653	

In this Example add no *Cyphers*, becauſe Diviſion may be made without them; and the Diviſion ended, you find in the *Quotient* 228, *ferè*: Then counting the Decimal Parts in the *Dividend*, you find them to be ſeven, and thoſe in the *Diviſor* two, the exceſs being five, therefore there muſt be five places of *Decimal Parts* in the *Quotient*; and to make them ſo, prefix 2 Cyphers before 228, and the value of the *Quotient* will be .00228, *ferè*.

Example

Example 9. *To divide a* Decimal Fraction .166592 *by a* Decimal Fraction 7358.

In this Example there was no need of adding of Cyphers, and ſo the Diviſion being ended, you find ſix places of *Parts* in the *Dividend*, and four in the *Diviſor*, the difference is 2, and ſo many figures you have in the *Quotient*; before which prefix a Point, and the value of the *Quotient* is .24 *Parts*.

Diviſor.	*Dividend.*	*Quotient.*
7358)	.176592	(.24
	14716	
	29432	
	29432	
	00000	

Another Example of this Caſe:

Divide .00046116 by .00732.

In this Example alſo there was no need of adding Cyphers, becauſe the Diviſion might be made without; and being ended, you find but two places of *Parts* in the *Quotient*, *viz.* 63, but the exceſs of places in the *Dividend* being more by three than thoſe in the *Diviſor*, prefix a Cypher before 63, and the value of the *Quotient* is .063 *Parts*.

Diviſor.	*Dividend.*	*Quotient.*
.00732)	.00046116	(.063
	4392	
	2196	
	2196	
	0000	

A third Example of this Caſe:

In this Example there is a neceſſity of adding of Cyphers to the *Dividend*.

Divide 4 by .325.

Diviſor.	*Dividend.*	*Quotient.*
.325)	.40000	(1.23
	325	
	750	
	650	
	1000	
	975	
	25	

CHAP. XIII.

Some Examples in Decimal Diviſion.

Example 1. *There is* 2280 *li.* 17 *s.* 3 *d. to be divided, ſo that each* Perſon *muſt have* 325 *li.* 16 *s.* 9 *d. How many* Perſons *muſt there be?*

THe *Decimal* of 2280 *li.* 17 *s.* 3 *d.* is 2280.8625, which divided by 325.8375, the *Decimal* of 325 *li.* 16 *s.* 9 *d.* the *Quotient* will be 7; and ſo many *Perſons* muſt there be.

325.8375

325.83375) 22808625 (7 *Persons*

22808625
00000000

Example 2. *There is a* Plank, *which contains* 58 *Foot*, $3\frac{1}{2}$ *Inches*, *and it is* 15 *Foot*, $5\frac{1}{2}$ *Inches* Long, *how* Broad *is that* Plank?

Divide 58.29073 (adding Cyphers) the *Decimal* of 58 Foot, $3\frac{1}{2}$ Inches, by 15.45833, the *Decimal* of 15 Foot, $5\frac{1}{2}$ Inches, the *Quotient* will be 3.771, *fere*; which reduced, is 3 Foot, $9\frac{1}{4}$ Inches, *fere*, for the breadth of the Plank.

The farther use of *Multiplication* and *Division* will more at large appear in the following *Chapters*, concerning the making of *Tables* of *Simple* and *Compound Interest* and *Rebate*; as also in *Annuities*, *Rents*, *Pensions*, &c. But more for the purposes here intended, in the measuring of *Geometrical Figures*, *Plain* and *Solid*; as *Board*, *Glass*, *Pavement*, *Stone*, *Timber*, &c. and also in the works of the several *Artificers* belonging to Building, as the *Carpenter*, *Bricklayer*, *Mason*, *Joyner*, *Painter*, *Plaisterer*, *Glasier*, *Pavier*, &c. of which at large in the following Books.

OF

OF INTEREST, BOTH Simple and Compound, WITH TABLES OF

Both kinds, ready Calculated, from the Rate of V. to XII. *per Cent. per Annum.*

Proœme.

USURY is derived particularly from Usura, ab usu æris; *or generally from the Use or Occupation of any thing, as* Money, *the worth or estimate of it upon some mutual Contract, wherein the Debtor allows the Cerditour a Loan in lieu of the Money or Goods borrowed: This in times past hath been at liberty, as Men could agree; but when the insatiable avariciousness of rich Misers extracted Extortion from the indigency of Borrowers, the corroding Use of Money was by provident Laws confined to a certain Rate, as to* 10 li. per Cent. per Annum, *afterwards to* 8 li. *and now of late Years to* 6 li. *that is, if a Creditour lends* 100 li. *for a Year, he may legally exact* 6 li. *profit for the Use of the* 100 li. *lent.*

Money put out to Use is divided into three parts, viz. *Principal, Time, and Interest: The first signifies the Summ or Value of the Money or Goods so lent; Time is the forbearance of it, as Days, Weeks Months, or Years; Interest is the profit that ariseth from the other two; and is derived from two compound Latine Words,* viz. inter *and* est, ab edo, *to eat or devour, as it is the property of Use to doe: They have these proportions; As the Principal to the Time, so is the Rate contracted for, and interest, to it self; and generally as commixt with one another.*

Interest resembles Janus *with two Faces, one looking upon the Time past, and the other on that to come. In this Track the Principal runs like a Snow-ball, rising upon an even Superficies, equally moving, but the increase unequal, although proportional to the Body, as it is magnified in the motion; and if continued, in time it will gather up all; and this is called* Forbearance: *And* Rebate, *or* Discompt *of Money is like the Track in which a Snow-ball is moved, and in its descent takes up all, untill it is stayed, leaving the Ground bare; from whence it takes a seeming original, where Time hath not arrived, but beholds it, as Seamen an Object, which seems little at a great distance, and increases (by the Optick Sense,) by insensible approaches.*

USE *or* INTEREST *hath in either Predicament two Species,* viz. *Simple and Compounded, the first is computed from the Principal and Time only, upon a certain Rate given or allowed, whether ascending or descending, as in Forbearance, or upon Discompt; which are thus explained:*

If 100 li. *be continued for Two Years, at* 6 li. per Cent. per Ann. *the Creditor at that term of time is to receive but* 112 li. *that is* 12 li. *for Simple Interest, in loan of the Money forborn: And here* 100 li. *is the Principal lent, the term of time two Years, the Use or Rate of Interest* 6 li. per Cent. per Ann. *and this is Simple Interest: But in Compound Interest, the first Payment attracts a proportional Use; as, admit in annual disbursements in the second Year there is an Use required, or imposed upon the* 6 li. *due, if continued; and therefore it is called* Interest upon Interest.

Discompt, or Rebate of Money, is upon a Legacy, or Summ of Money due to be paid at a time to come; yet satisfied or discharged with so much present Money, as immediately put forth at an Interest or Rate allowed, forborn untill the Legacy should have been due, returns again to its first Principal or Summ; the Money paid being computed at the same Rate of Interest for the Forbearance, as was the Discompt made. All which shall be illustrated by variety of Examples.

CHAP. I.
Of SIMPLE INTEREST.

WHat Simple Interest is, hath been already declared in the *Proœme*, and for the Arithmetical working of Questions of this nature this is the Proportion:

As the Principal and Time for which a Loan is allowed,
Is unto the Interest thereof;
So will any other Summ of Money to be borrowed,
Be to the Interest for the same time.

Example 1. *What is the Interest of* 145 li. *for a Year, at* 6 li. per Cent.

The Proportion is,

As 100 *li.* is to 6 *li.* so is 145 *li.* to 8.70 *li.*

The *RULE:*

Multiply 145 li. *by* 6 li. *the Product will be* 870; *which divided by* 100, *(by cutting off two figures,) the Quotient will be* 8.70 li. *which is* 8 li. 14 s. *for the Interest of* 145 li *for a Year.*

Example 2. *What is the Interest of* 250 li. *for* 5 *Months, at* 8 per Cent.

The Proportion is,

As 100 *li.* in 12 Months is to 8 *li.*
So is 250 *li.* in 5 Months, to 8 *li.* 6 *s.* 8 *d.*

The *RULE:*

Multiply 100 li. *by* 12 li. *the Product will be* 1200, *for a Divisor; then multiply* 250 li. *by* 8 li. *the Product will be* 2000, *and that multiplied by* 5 *Months, the Product will be* 10000 *for a Dividend; then divide* 10000 *(adding Cyphers if need be) by* 1200, *the Quotient will be* 8.33 li.

This Quotient 8.33 reduced is 8 *li.* 6 *s.* 8 *d.* for the Interest of 250 *li.* in 5 Months, at 8 *per Cent.* for a Year.

Example 3. *One lent* 650 li. *which was repayed at the end of* 6 *Months,* 3 *Weeks, and* 3 *Days; What came the Interest thereof to, at* 6 per Cent.

The Proportion is,

As 100 *li.* in 365 Days, is to 6 *li.*
So is 650 *li.* in 192 Days, to 20 *li.* 10 *s.* 3 *d.* 3 *q. fere.*

The *RULE:*

Multiply 365 *by* 100, *the Product will be* 36500, *for a Divisor; also multiply* 650 *by* 6, *and the Product of that by* 650, *the last Product will be* 748800; *then divide this Product (adding Cyphers) by the former Divisor* 36500, *and the Quotient will be* 20.515, *and answers the Question.*

Which

Which Quotient 20.515 reduced, is 20 *li.* 10 *s.* 3 *d.* 3 *q. ferè*: And so much doth the Interest of 650 *li.* amount unto at the end of 6 Months, 3 Weeks, and 3 Days, which is 192 Days.

Example 4. *What doth the Simple Interest of* 265 li. 13 s. 4 d. 1 q. *amount unto in a Year, at* 6 per Cent.

The Proportion is,

As 100 *li.* is to 6 *li.* in a Year,
So is 265 *li.* 13 *s.* 4 *d.* 1 *q.* to 15 *li.* 18 *s.* 9 *d.* 2 *q.* in a Year.

The *RULE*:

Reduce 265 li. 13 s. 4 d. 1 q. *into a Decimal Fraction, and it will be* 265.667 li. *this multiplied by* 6 li. *(the Rate of Interest,) produceth* 15.94002; *and that divided by* 100, *(by cutting off the two last Figures,) the Quotient will be* 15.940.

Which reduced is 15 *li.* 18 *s.* 9 *d.* 3 *q. ferè*, for the Interest of 265 *li.* 13 *s.* 4 *d.* 1 *q.* for a Year.

And thus may the Interest of any Summ of Money, for any time and any rate of Interest be attained; and by these few Rules the following Table is calculated, at the Rate of Interest now by Law established, *viz.* 6 *li. per Cent. per Annum.*

PROBLEM.

How to find the Interest due upon any Summ of Money, for any number of Days, and at any Rate of Interest.

To perform this Work this is a general *RULE*:

Multiply the Principal Summ by the Rate of Interest, and that Product by the Number of Days proposed; this last Product divided (always) by 36500, *(the number of Days in a Year, with two Cyphers added,) the Quotient will answer the Question.*

Example 1. *What will the Interest of* 752 li. *amount unto in* 232 *Days, at* 6 li. per Cent.

Multiply 752 *li.* (the principal Summ,) by 6 *li.* (the Rate of Interest,) the Product will be 4512; this multiplied by 232, (the number of Days,) produceth 1046784; and this Product (adding Cyphers, if need be,) divide by 36500, the Quotient will be 28.679 *ferè*, which Decimal Fraction reduced, is 28 *li.* 13 *s.* 7 *d.* and so much will the Interest of 752 *li.* amount unto in 232 days.

Example 2. *What will the Interest of* 80 li. 10 s. *amount unto in* 120 *Days, at* 9 li. per Cent.

The Principal	80.5
Multiplied by the Rate of Interest	9
The Product is	7245
Which multiplied by the nnmber of Days	120
The Product thereof is	869400

Divisor. *Dividend.* *Quotient.*

36500) 869400.000 (2.3819

Which Quotient reduced, is 2 *li.* 7 *s.* 8 *d.* for the Interest of 80 *li.* 10 *s.* in 120 Days.

Simple Intereſt, at VI. *per Cent.* for

	I Month.				II Months.				III Months.				IV Months.				V Months.				VI Months.			
Pounds	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*
1	0	0	1	1	0	0	2	2	0	0	3	2	0	0	4	3	0	0	6	0	0	0	7	1
2	0	0	2	2	0	0	4	3	0	0	7	1	0	0	9	2	0	1	0	0	0	1	2	2
3	0	0	3	2	0	0	7	1	0	0	10	3	0	1	2	2	0	1	6	0	0	1	9	2
4	0	0	4	3	0	0	9	2	0	1	2	1	0	1	7	1	0	2	0	0	0	2	4	3
5	0	0	6	0	0	1	0	0	0	1	6	0	0	2	0	0	0	2	6	0	0	3	0	0
6	0	0	7	1	0	1	2	2	0	1	9	2	0	2	4	3	0	3	0	0	0	3	7	1
7	0	0	8	2	0	1	4	3	0	2	1	1	0	2	9	2	0	3	6	0	0	4	2	2
8	0	0	9	2	0	1	7	1	0	2	4	3	0	3	2	2	0	4	0	0	0	4	9	2
9	0	0	10	3	0	1	9	2	0	2	8	2	0	3	7	1	0	4	6	0	0	5	4	3
10	0	1	0	0	0	2	0	0	0	3	0	0	0	4	0	0	0	5	0	0	0	6	0	0
11	0	1	1	1	0	2	2	2	0	3	3	2	0	4	4	3	0	5	6	0	0	6	7	1
12	0	1	2	2	0	2	4	3	0	3	7	1	0	4	9	2	0	6	0	0	0	7	2	2
13	0	1	3	2	0	2	7	1	0	3	10	3	0	5	2	2	0	6	6	0	0	7	9	2
14	0	1	4	3	0	2	9	2	0	4	2	2	0	5	7	1	0	7	0	0	0	8	4	3
15	0	1	6	0	0	3	0	0	0	4	6	0	0	6	0	0	0	7	6	0	0	9	0	0
16	0	1	7	1	0	3	2	2	0	4	9	2	0	6	4	3	0	8	0	0	0	9	7	1
17	0	1	8	2	0	3	4	3	0	5	1	1	0	6	9	2	0	8	6	0	0	10	2	2
18	0	1	9	2	0	3	7	1	0	5	4	3	0	7	2	2	0	9	0	0	0	10	9	2
19	0	1	10	3	0	3	9	2	0	5	8	2	0	7	7	1	0	9	6	0	0	11	4	3
20	0	2	0	0	0	4	0	0	0	6	0	0	0	8	0	0	0	10	0	0	0	12	0	0
30	0	3	0	0	0	6	0	0	0	9	0	0	0	12	0	0	0	15	0	0	0	18	0	0
40	0	4	0	0	0	8	0	0	0	12	0	0	0	16	0	0	1	0	0	0	1	4	0	0
50	0	5	0	0	0	10	0	0	0	15	0	0	1	0	0	0	1	5	0	0	1	10	0	0
60	0	6	0	0	0	12	0	0	0	18	0	0	1	4	0	0	1	10	0	0	1	16	0	0
70	0	7	0	0	0	14	0	0	1	1	0	0	1	8	0	0	1	15	0	0	2	2	0	0
80	0	8	0	0	0	16	0	0	1	4	0	0	1	12	0	0	2	0	0	0	2	8	0	0
90	0	9	0	0	0	18	0	0	1	7	0	0	1	16	0	0	2	5	0	0	2	14	0	0
100	0	10	0	0	1	0	0	0	1	10	0	0	2	0	0	0	2	10	0	0	3	0	0	0
200	1	0	0	0	2	0	0	0	3	0	0	0	4	0	0	0	5	0	0	0	6	0	0	0
300	1	10	0	0	3	0	0	0	4	10	0	0	6	0	0	0	7	10	0	0	9	0	0	0
400	2	0	0	0	4	0	0	0	6	0	0	0	8	0	0	0	10	0	0	0	12	0	0	0
500	2	10	0	0	5	0	0	0	7	10	0	0	10	0	0	0	12	10	0	0	15	0	0	0
600	3	0	0	0	6	0	0	0	9	0	0	0	12	0	0	0	15	0	0	0	18	0	0	0
700	3	10	0	0	7	0	0	0	10	10	0	0	14	0	0	0	17	10	0	0	21	0	0	0
800	4	0	0	0	8	0	0	0	12	0	0	0	16	0	0	0	20	0	0	0	24	0	0	0
900	4	10	0	0	9	0	0	0	13	10	0	0	18	0	0	0	22	10	0	0	27	0	0	0
1000	5	0	0	0	10	0	0	0	15	0	0	0	20	0	0	0	25	0	0	0	30	0	0	0
2000	10	0	0	0	20	0	0	0	30	0	0	0	40	0	0	0	50	0	0	0	60	0	0	0
3000	15	0	0	0	30	0	0	0	45	0	0	0	60	0	0	0	75	0	0	0	90	0	0	0
4000	20	0	0	0	40	0	0	0	60	0	0	0	80	0	0	0	100	0	0	0	120	0	0	0
5000	25	0	0	0	50	0	0	0	75	0	0	0	100	0	0	0	125	0	0	0	150	0	0	0
10000	50	0	0	0	100	0	0	0	150	0	0	0	200	0	0	0	250	0	0	0	300	0	0	0

Simple Intereſt at VI. *per Cent.* for

	VII Months.				VIII Months.				IX Months.				X Months.				XI Months.				A Year.			
Pounds	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q*	*li.*	*s.*	*d.*	*q*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*
1	0	0	8	2	0	0	9	2	0	0	10	3	0	1	0	0	0	1	1	1	0	1	2	2
2	0	1	4	3	0	1	7	1	0	1	9	2	0	2	0	0	0	2	2	2	0	2	4	3
3	0	2	1	1	0	2	4	3	0	2	8	2	0	3	0	0	0	3	3	2	0	3	7	1
4	0	2	9	2	0	3	2	2	0	3	7	1	0	4	0	0	0	4	4	3	0	4	9	2
5	0	3	6	0	0	4	0	0	0	4	6	0	0	5	0	0	0	5	6	0	0	6	0	0
6	0	4	2	2	0	4	9	2	0	5	4	3	0	6	0	0	0	6	7	1	0	7	2	2
7	0	4	10	3	0	5	7	1	0	6	3	2	0	7	0	0	0	7	8	2	0	8	4	3
8	0	5	7	1	0	6	4	3	0	7	2	2	0	8	0	0	0	8	9	2	0	9	7	1
9	0	6	3	2	0	7	2	2	0	8	1	1	0	9	0	0	0	9	10	3	0	10	9	2
10	0	7	0	0	0	8	0	0	0	9	0	0	0	10	0	0	0	11	0	0	0	12	0	0
11	0	7	8	2	0	8	9	2	0	9	10	3	0	11	0	0	0	12	1	1	0	13	2	2
12	0	8	4	3	0	9	7	1	0	10	9	2	0	12	0	0	0	13	2	2	0	14	4	3
13	0	9	1	1	0	10	4	3	0	11	8	2	0	13	0	0	0	14	3	2	0	15	7	1
14	0	9	9	2	0	11	2	2	0	12	7	1	0	14	0	0	0	15	4	3	0	16	9	2
15	0	10	6	0	0	12	0	0	0	13	6	0	0	15	0	0	0	16	6	0	0	18	0	0
16	0	11	2	2	0	12	9	2	0	14	4	3	0	16	0	0	0	17	7	1	0	19	2	2
17	0	11	10	3	0	13	7	1	0	15	3	2	0	17	0	0	0	18	8	2	1	0	4	3
18	0	12	7	1	0	14	4	3	0	16	2	2	0	18	0	0	0	19	9	2	1	1	7	1
19	0	13	3	2	0	15	2	2	0	17	1	1	0	19	0	0	1	0	10	3	1	2	9	2
20	0	14	0	0	0	16	0	0	0	18	0	0	1	0	0	0	1	2	0	0	1	4	0	0
30	1	1	0	0	1	4	0	0	1	7	0	0	1	10	0	0	1	13	0	0	1	16	0	0
40	1	8	0	0	1	12	0	0	1	16	0	0	2	0	0	0	2	4	0	0	2	8	0	0
50	1	15	0	0	2	0	0	0	2	5	0	0	2	10	0	0	2	15	0	0	3	0	0	0
60	2	2	0	0	2	8	0	0	2	14	0	0	3	0	0	0	3	6	0	0	3	12	0	0
70	2	9	0	0	2	16	0	0	3	3	0	0	3	10	0	0	3	17	0	0	4	4	0	0
80	2	16	0	0	3	4	0	0	3	12	0	0	4	0	0	0	4	8	0	0	4	16	0	0
90	3	3	0	0	3	21	0	0	4	1	0	0	4	10	0	0	4	19	0	0	5	8	0	0
100	3	10	0	0	4	0	0	0	4	10	0	0	5	0	0	0	5	10	0	0	6	0	0	0
200	7	0	0	0	8	0	0	0	9	0	0	0	10	0	0	0	11	0	0	0	12	0	0	0
300	10	10	0	0	12	0	0	0	13	10	0	0	15	0	0	0	16	10	0	0	18	0	0	0
400	14	0	0	0	16	0	0	0	18	0	0	0	20	0	0	0	22	0	0	0	24	0	0	0
500	17	10	0	0	20	0	0	0	22	10	0	0	25	0	0	0	27	10	0	0	30	0	0	0
600	21	0	0	0	24	0	0	0	27	0	0	0	30	0	0	0	33	0	0	0	36	0	0	0
700	24	10	0	0	28	0	0	0	31	10	0	0	35	0	0	0	38	10	0	0	42	0	0	0
800	28	0	0	0	32	0	0	0	36	0	0	0	40	0	0	0	44	0	0	0	48	0	0	0
900	31	10	0	0	36	0	0	0	40	10	0	0	45	0	0	0	49	10	0	0	54	0	0	0
1000	35	0	0	0	40	0	0	0	45	0	0	0	50	0	0	0	55	0	0	0	60	0	0	0
2000	70	0	0	0	80	0	0	0	90	0	0	0	100	0	0	0	110	0	0	0	120	0	0	0
3000	105	0	0	0	120	0	0	0	135	0	0	0	150	0	0	0	165	0	0	0	180	0	0	0
4000	140	0	0	0	160	0	0	0	180	0	0	0	200	0	0	0	220	0	0	0	240	0	0	0
5000	175	0	0	0	200	0	0	0	225	0	0	0	250	0	0	0	275	0	0	0	300	0	0	0
10000	350	0	0	0	400	0	0	0	450	0	0	0	500	0	0	0	550	0	0	0	600	0	0	0

CHAP. II.

Of Discompt or Rebate of Money.

FOR discompt or rebate of Money, this is the Proportion:
As 100 *li.* and its Interest, at a Years end,
Is to a 100 *li.*
So is a 100 *li.*
To the present worth of a 100 *li.*
For as 106 *li.* is to 100 *li.* so is 100 *li.* to 94.3396.

Divisor. *Dividend.* *Quotient.*
106) 10000000000 (94.3396

So that a 100 *li.* to be paid at a years end, is worth in present Money 94 *li.* 6 *s.* 9 *d.* 2 *q.* And this is the difference between Interest and Rebate; for you are not to rebate 6 *li.* of a 100 *li.* to be paid at the years end, but 5 *li.* 13 *s.* 2 *d.* 2 *q.* which is less then 6 *li.* by 6 *s.* 9 *d.* 2 *d.* Thus for a year, but if you would know the rebate of any other summ, and for any other time more or less than a year, then use this Proportion:

As 100 *li.* with the Interest thereof, for any time required,
Is to 100 *li.*
So is the debt due to be paid at the expiration of that time,
To the present worth thereof.

Example, *If a Legasie or other Summ of Money,* as 345 li. *be due to be paid at 6 Months end; what present Money will discharge the Debt?*
Say by the Rule of Proportion,
As 103 *li.* is to 100 *li.* so is 345 *li.* to 334 *li.* 19 *s.* 1 *d.*

Divisor. *Dividend.* *Quotient.*
103) 34500.000 (334.951

Which Quotient 334.951 reduced, is 334 *li.* 19 *s.* 0 *d.* 1 *q.* And so much is to be paid for the 345 *li.* abateing for the said six Months 10 *li.* 0 *s.* 11 *d.* 2 *q.* whereas the Interest thereof for the same time, is 10 *li.* 7 *s.* the difference being 6 *s.* 2 *q.* And according to this Proportion, is the following Table of Discompt or Rebate Calculated.

Discompt

Discompt, or Rebate, at VI. *per Cent.* for

	I Month.				II Months.				III Months.				IV Months.				V Months.				VI Months.			
Pound.	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*	*li.*	*s.*	*d.*	*q.*
1	0	19	10	3	0	19	9	2	1	19	8	2	0	19	7	1	0	19	6	1	0	19	5	0
2	1	19	9	2	1	19	7	1	1	19	5	0	1	19	2	2	1	19	0	1	1	18	10	0
3	2	19	8	2	2	19	4	3	2	19	1	1	2	18	9	3	2	18	6	2	2	18	3	0
4	3	19	7	1	3	19	2	2	3	18	9	3	3	18	5	1	3	18	0	2	3	17	8	0
5	4	19	6	0	4	19	0	0	4	18	6	1	4	18	0	2	4	17	6	3	4	17	1	0
6	5	19	4	3	5	18	9	3	5	18	2	3	5	17	7	3	5	17	0	3	5	16	6	0
7	6	19	3	3	6	18	7	1	6	17	11	1	6	17	3	0	6	16	7	0	6	15	11	0
8	7	19	2	2	7	18	5	0	7	17	7	2	7	16	10	1	7	16	1	1	7	15	4	0
9	8	19	1	1	8	18	2	2	8	17	4	0	8	16	5	3	8	15	7	1	8	14	9	0
10	9	19	0	0	9	18	0	1	9	17	0	2	9	16	1	0	9	15	1	2	9	14	2	0
11	10	18	11	0	10	17	9	3	10	16	9	0	10	15	8	1	10	14	7	2	10	13	7	0
12	11	18	9	3	11	17	7	2	11	16	5	2	11	15	3	2	11	14	1	3	11	13	0	0
13	12	18	8	2	12	17	5	0	12	16	1	3	12	14	10	3	12	13	8	0	12	12	5	1
14	13	18	7	1	13	17	2	3	13	15	10	2	13	14	6	0	13	13	2	0	13	11	10	1
15	14	18	6	0	14	17	0	2	14	15	6	3	14	14	1	2	14	12	8	1	14	11	3	1
16	15	18	5	0	15	16	10	0	15	15	3	1	15	13	8	3	15	12	2	1	15	10	8	1
17	16	18	3	3	16	16	7	3	16	14	11	3	16	13	4	0	16	11	8	2	16	10	1	1
18	17	18	2	2	17	16	5	2	17	14	8	1	17	12	11	1	17	8	2	2	17	9	6	1
19	18	18	1	1	18	16	3	0	18	14	4	2	18	12	6	2	18	10	8	3	18	8	11	1
20	19	18	0	0	19	16	0	2	19	14	1	0	19	12	2	0	19	10	3	0	19	8	4	1
30	29	17	0	1	29	14	0	3	29	11	1	2	29	8	2	3	29	5	4	2	29	2	6	1
40	39	16	0	1	39	12	1	0	38	8	2	1	39	4	3	3	39	0	5	3	38	16	8	2
50	49	15	0	1	49	10	1	1	45	5	2	2	49	0	4	3	48	15	7	1	48	10	10	2
60	59	14	0	1	59	8	1	2	59	2	2	1	58	16	5	3	58	10	8	3	58	5	0	2
70	69	13	0	2	69	6	1	3	68	19	3	3	68	12	6	2	68	5	10	1	67	19	2	3
80	79	12	0	2	79	4	2	0	78	16	4	1	78	8	7	2	78	0	11	3	77	13	4	3
90	89	11	0	2	89	2	2	1	88	13	4	3	84	4	8	2	87	16	1	1	87	7	6	3
100	99	10	0	2	99	0	2	2	98	10	5	1	98	0	9	2	97	11	2	3	97	1	9	0
200	199	0	1	1	198	0	4	3	197	0	10	2	196	1	6	3	195	2	5	1	194	3	6	0
300	298	10	1	3	297	0	7	1	295	11	4	0	294	2	4	1	292	13	8	0	291	5	3	0
400	398	0	2	2	396	0	9	2	394	1	9	1	392	3	1	3	390	4	10	2	388	7	9	0
500	497	10	3	0	495	0	11	3	492	12	2	2	490	3	11	0	487	16	1	1	485	8	8	3
600	597	0	3	2	594	1	2	1	591	2	8	0	588	4	8	2	585	7	3	3	582	10	5	3
700	696	10	4	1	693	1	4	3	689	13	1	1	686	5	5	3	682	18	6	2	679	12	2	3
800	796	0	4	3	792	1	7	0	788	3	6	2	784	6	3	1	780	9	9	0	776	13	11	3
900	895	10	5	1	891	1	9	2	886	13	11	3	882	7	0	3	878	0	11	3	873	15	8	3
1000	995	0	6	0	990	1	11	3	985	4	5	1	980	7	10	0	975	12	2	1	970	17	5	3
2000	1990	0	12	0	1980	3	11	2	1970	8	10	2	1660	15	8	1	1951	4	4	3	1941	14	11	2
3000	2985	1	6	0	2970	5	11	1	2955	13	3	3	2941	3	6	1	2926	16	7	0	2912	12	5	1
4000	3980	1	11	3	3960	7	11	0	3940	17	8	3	3971	11	4	2	3902	8	9	1	3883	9	10	3
5000	4975	2	5	3	4950	9	10	3	4926	2	2	0	4901	19	2	2	4878	0	11	3	4854	7	4	2
10000	9950	4	11	2	9900	19	9	2	9852	4	4	0	9803	18	5	1	9756	1	11	2	9708	14	9	0

Discompt, or Rebate, at VI. *per Cent.* for

Pounds	VII Months.				VIII Months.				IX Months.				X Months.				XI Months.				A Year.			
	li.	s.	d.	q.	li.	s.	d.	q.	li.	s.	d.	q.	li.	s.	d.	q.	li.	s.	d.	q.	li.	s.	d.	q.
1	0	19	4	0	0	19	2	3	0	19	1	3	0	19	0	2	0	18	11	2	0	18	10	2
2	1	18	7	3	1	18	5	2	1	18	3	1	1	18	1	1	1	17	11	0	1	17	8	3
3	2	17	11	3	2	17	8	1	2	17	5	0	2	17	1	3	2	16	10	2	2	16	7	1
4	3	17	3	2	3	16	11	0	3	16	6	3	3	16	2	1	3	15	10	0	3	15	5	3
5	4	16	7	2	4	16	1	3	4	15	8	1	4	15	2	3	4	14	9	2	4	14	4	0
6	5	15	11	1	5	15	4	2	5	14	10	0	5	14	3	2	5	13	9	0	5	13	2	2
7	6	15	3	1	6	14	7	2	6	13	11	3	6	13	4	0	6	12	8	2	6	12	1	0
8	7	14	7	0	7	13	10	1	7	13	1	1	7	12	4	2	7	11	8	0	7	10	11	1
9	8	13	11	0	8	13	1	0	8	12	3	0	8	11	5	1	8	10	7	2	8	9	9	3
10	9	13	2	3	9	12	3	3	9	11	4	3	9	10	5	3	9	9	7	0	9	8	8	1
11	10	12	6	3	10	11	6	2	10	10	6	1	10	9	6	1	10	8	6	1	10	7	6	2
12	11	11	10	2	11	10	9	1	11	9	8	0	11	8	6	3	11	7	5	3	11	6	5	0
13	12	11	2	2	12	10	0	0	12	8	9	3	12	7	7	2	12	6	5	1	12	5	3	2
14	13	10	6	1	13	9	2	3	13	7	11	1	13	6	8	0	13	5	4	3	13	4	1	3
15	14	9	10	1	14	8	5	2	14	7	1	0	14	5	8	2	14	4	4	1	14	3	0	1
16	15	9	2	1	15	7	8	1	15	6	2	3	15	4	9	1	15	3	3	3	15	1	10	3
17	16	8	6	0	16	6	11	0	16	5	4	1	16	3	9	3	16	2	3	1	16	0	9	0
18	17	7	10	0	17	6	1	3	17	4	6	0	17	2	10	1	17	1	2	3	16	19	7	2
19	18	7	1	3	18	5	4	2	18	3	7	3	18	1	10	3	18	0	2	1	17	18	6	0
20	19	6	5	3	19	4	7	2	19	2	9	1	19	0	11	3	18	19	1	3	18	17	4	1
30	28	19	8	2	28	16	11	0	28	14	2	0	28	11	5	1	28	8	8	3	28	6	0	2
40	38	12	11	1	38	9	2	3	38	5	6	2	38	1	10	3	37	18	3	2	37	14	8	2
50	48	6	2	1	48	1	6	2	47	6	11	1	47	12	4	2	47	7	10	2	47	3	4	3
60	57	19	7	0	57	13	10	1	57	8	4	0	57	2	10	1	56	17	5	1	56	12	1	0
70	67	12	8	0	67	6	1	3	66	19	8	2	66	13	4	0	66	7	0	1	66	0	9	0
80	77	5	10	3	76	18	5	2	76	11	1	1	76	3	9	3	75	16	7	0	75	9	5	1
90	86	19	1	2	86	10	9	1	86	2	5	3	85	14	3	2	85	6	2	0	84	18	1	1
100	96	12	4	2	96	3	1	0	95	13	10	2	95	4	9	1	94	15	8	3	94	6	9	2
200	193	4	8	3	192	6	1	3	191	7	9	0	190	9	6	1	189	11	5	3	188	13	7	0
300	289	17	1	1	288	9	2	3	287	1	7	2	285	14	3	2	284	7	2	2	283	0	4	2
400	386	9	5	2	384	12	3	3	382	15	6	0	380	19	0	2	379	2	11	1	377	7	2	0
500	483	1	10	0	480	15	4	2	478	9	4	2	476	3	9	3	473	18	8	0	471	13	11	2
600	579	14	2	2	576	18	5	2	574	3	3	0	571	8	6	3	568	14	5	0	566	0	9	0
700	676	6	6	3	673	1	6	2	669	17	1	2	666	13	4	0	663	10	1	3	660	7	6	2
800	772	18	11	1	769	4	7	2	765	11	0	0	761	18	1	1	758	5	10	2	754	14	4	0
900	869	11	3	3	865	7	8	1	861	4	10	2	857	2	10	1	853	1	7	1	849	1	1	2
1000	966	3	8	0	961	10	9	1	956	18	9	0	952	7	7	2	947	17	4	1	943	7	11	0
2000	1932	7	4	0	1933	1	6	2	1913	17	6	1	1604	15	2	3	1895	14	8	1	1886	15	10	1
3000	2898	11	0	1	2884	12	3	3	2870	16	3	1	2857	2	10	1	2843	12	0	2	2830	3	9	1
4000	3864	14	8	1	3846	3	1	0	3827	15	0	1	3809	10	5	3	3791	9	4	2	3773	11	8	2
5000	4830	18	4	1	4807	13	10	1	4784	13	9	1	4861	18	1	1	4739	6	8	3	4716	19	7	2
10000	9661	16	8	2	9615	7	8	1	9569	7	6	3	9523	16	2	1	9478	13	5	2	9433	19	3	1

CHAP.

CHAP. III.

Of Equation of Payments.

FOR the resolving of Questions belonging to this *Rule*, there are several Ways proposed by several Men, which (although they come near, yet they) do not exactly perform the Business intended (or aimed at.)

§1. *Equation of Payments* are of two kinds, *viz.* (1.) several *equal Payments* due at *equidistant times*: (2.) Of several *unequal Payments*, at several times not *equidistant*.

For the first of these, *viz.* for *Equation* of several *equal Payments* at *equidistant* times this is the general *RULE*:

From the whole Amount of the Annuity, *or monthly Payment, subtract the* Aggregate *of the several Payments, and the Remainder (if yearly) multiply by* 365, *or (if monthly) by* 30.416 *Days: Then divide that* Product *by the* Annual *or monthly Interest of the said Aggregate, and the Quotient will give the number of Days before the end (or Term) of the Annuity or monthly Payment.*

Example 1. *If* A *be to pay unto* B 100 li. per annum, *for five years; and they both agree that* A *shall pay to* B *the whole* 500 li. *at one intire payment, at* 6 per cent. *Now the Question is, to know at what time before the expiration of the whole five years,* A *must pay* B *this* 500 li.

According to the *RULE*:

From the whole amount of the Annuity in the five years	560
Subtract the Aggregate,	500
The Remainder is	60
Which multiplied by	365
The Product is	21900
That Product divided by 30, the five years Interest, the Quotient is	730

So that the 500 *li.* must be paid 730 days before the expiration of the five years: Divide 730 by 365 days, and the Quotient will be 2 years; so that *A* must pay unto *B* the 500 *li.* at the end of 3 years.

Example 2. A *is to pay unto* B 62 li. per annum *for four years, but they agree to pay the Aggregate of the several summs,* viz. 248 li. *at one intire payment, at* 6 per Cent. *How many days before the expiration of the four years must this payment be made?*

Look in the Table of 6 *per Cent.* for the Amount of 100 *li.* in 4 years, and you shall find it to be 436, this multiplied by 62, the Annual Payment; the Product is 27032, which divide by 100, and the Quotient is 270.32 *li.* for the Amount of 62 *li.* in 4 years, being thus prepared, go on according to the Rule; as followeth:

1. The Annual Payment for 4 years is	62.00
2. The Amount of that in 4 years is	270.32
3. The Aggregate, or four years payment is	248.00
4. The Annual Interest of the Aggregate is	14.88
Out of the whole Amount	270.32
Subtract the Aggregate	248.00
The Remainder is	22.32
Which Multiplied by 365 days	365
The Product is	8146.80

This Divide by 14.88 the Annual Interest,

Divisor. *Dividend.* *Quotient.*

14.88) 8146.80 (547.5

Which Quotient is 547 days and an half, and so many days before the expiration of the four Years, must the summ of 284 *li.* be paid at one intire payment; or which is the same, two years and 192 days and an half after the Agreement.

To prove this: Let 248 li. *be put out at Interest at* 6 per Cent. *for* 1 *Year, and* 172 *Days, and an half; I say if the Interest thereof in that time, do amount unto* 270.32 li. *the work is proved?*

Thus the Intereſt of one Pound for one Day is ——— .000164383
Which multiplied by ——— 547.5
The Product is ——— .089999625
Which Product again multiplied by 248, the Aggregate, the ſecond Product will be ——— 22.319999977680
To which add 248.
The Summ is 270.319999977680

Exactly proving the former Work.

Example 3. *For monthly Payments, A is to pay unto* B, 100 li. *a month, for* 5 *months; at what time muſt he pay it at one intire payment at* 6 per Cent.

The Intereſt of 1 *li. per* month is ——— .005
Therefore .005 being multiplied by 500 *li.* gives the monthly Intereſt of 500 *li. viz.* ——— 2.500
The full Amount of 5 monthly Payments of 100 *li.* is ——— 500.000
The double of the monthly Intereſt is ——— 5.000
The whole Amount ——— 505.000
From the whole Amount ——— 505.
Deduct the Aggregate ——— 500
The Remainder is ——— 005

Multiply 30.416 (the days in a compleat month) by 5, the Product will be 152.080 which divided by 2.500 (or the monthly Intereſt of 500 *li.*) the Quotient will be 60.832 days, that is 60 days, and about 20 hours, which is 2 compleat months before the 5 months; or three months after the day of agreement.

§ 2. For Equation of Payments unequal, and at times not equidiſtant, this is the general *RULE:*

Find firſt the true Amount of each of the Summs, from the firſt day of agreement, to the laſt day of Payment; (as ſuppoſing them to be forborn till the laſt:) Then out of that deduct the Aggregate of the reſpective Payments, and multiply the remainder (if annual) by 365, *or (if monthly) by* 30.4166. *and the Product divide by the Annual, or monthly Intereſt of the ſaid Aggregate, the Quotient is the number of Days from the laſt day of Payment, accounting backwards.*

Example 1. *If* A *be debtor to* B 500 li. *to be paid at ſeveral unequal Payments,* viz. 300 li. *at the end of four months,* 100 li. *at the end of ſix months, and* 100 li. *at the end of* 12 *months: Now it is agreed to diſcharge the Debt at one intire Payment. At what time muſt this Debt be paid, without damage to either party?*

The Operation according to the *RULE:*

Firſt the length of time from the beginning of the contract, to the laſt day of Payment thereof, is juſt twelve months.

1. Then in the firſt place 300 *li.* payable after 4 months, and being forborn to the end of 12 months; hath 8 months Intereſt to account for, *viz.* 12.000
2. In the ſecond place 100 *li.* payable after 6 months, and being forborn 12 months, hath 6 months Intereſt to account for; *viz.* ——— .3.000
3. To theſe two Intereſts, add the whole Debt, *viz.* ——— 500.000
The whole Amount is ——— 515.000
4. Then out of the whole Amount ——— 515.000
Deduct the whole Debt ——— 500.000
The Remainder is ——— 15.000

This done, the Proportion will be,
As 2 *li.* (the Intereſt of 500 *li.* for one month,)
Is to one month;
So is 15 *li.* (the Remainder,)
To 6 months.

And that reſolves the Queſtion, and is the time at which the Payment of the whole Debt is to be made.

As 2.5 *li.* is to 1 month, ſo is 15 *li.* to 6 months.

So that if the whole Legaſie be paid 6 months before 12 months be compleated, or (which is the ſame) at 6 months end, there will be no loſs or damage to either Party.

For

For proof of this,

1. The first 300 *li.* was due at 4 months end, and being continued 2 months longer; the Interest thereof for 2 months is 3 *li.* and the whole amount is — } *li.* 303
2. In the next place 100 *li.* paid at 6 months end is the time it was due, and is just — } 100
3. The other 100 *li.* paid 6 months before its due time, must take an abatement of 3 *li.* and therefore must be — } 97

The Total — 500

So that as in the first summ, there is an increase of 3 *li.* so in the last there is a decrease of 3 *li.* which set one against the other, the whole amount is 500 *li.* to be paid at 6 months end, and serves to prove the former Example.

Example 2. A *is to pay unto* B 100 li. per annum *for five years, and they agree that* A *shall buy it off at the end of any of the four years; for at the end of* 5 *years, nothing less then the whole* 560 li. *will satisfie the Debt?*

Thus the present worth of the whole amount 560 *li.* at the	First	years end, is	451.61,29
	Second		474.5762
	Third		500.0000
	Fourth		528.3018
	Fifth		560.0000

Which Numbers are found, by help of the Numbers in the third and fourth Column of the following Table (at 6 *per cent.*)

And the Proportion for finding them is,

As 100 *li.* (the amount of Annual Payment,)
Is to 94.33962 (the present worth of the first year;)
So is 560 *li.* (the whole amount in five years,)
To 528.3018 the present worth at the first year's end.

And so of the rest as followeth,

As	100	To	94.33962	So is 560 *li.* to	528.3018
	206		183.92856		500.0000
	318		269.49152		474.5762
	436		351.61290		451.6129
	560		430.76923		430.7690

A Table of the Amounts and Present Worths of an Annuity of 100 li. *for five Years, for seven several Rates of Interest.*

	Years.	The Amount of 100 *li.* in 1, 2, 3, 4, or 5 years.	Amounts of Annual payments at the end of 1, 2, 3, 4, or 5, years.	The present worth of the first year, 2d year, 3d year, 4th year, 5th year.	The particular present worths of the first, second, third, fourth, or fifth year.	The Amount of the Total present worth of all the five Payments, at the end of the 1, 2, 3, 4, or 5 years.
	1	101	100	99.00990	99.00990	490.57142
	2	102	201	197.05882	98.04892	495.42856
At I. *per Cent.*	3	103	303	294.17475	97.11593	500.28570
	4	104	406	390.38461	96.20986	505.14285
	5	105	510	485.71428	95.32967	509.99999
					485.71428	
	1	102	100	98.03921	98.03921	482.18181
	2	104	202	194.23076	96.19155	491.63635
At II. *per Cent.*	3	106	306	288.67924	94.44848	501.09089
	4	108	412	381.48148	92.80224	510.54543
	5	110	520	472.72727	91.24579	519.99999
					472.72727	
	1	103	100	97.08737	97.08737	474.69564
	2	106	203	191.50943	94.42206	488.52173
At III. *per Cent.*	3	109	309	283.48623	91.97680	502.34782
	4	112	418	373.21428	89.72805	516.17390
	5	115	530	460.86956	87.65528	529.99999
					460.86956	
	1	104	100	96.15384	96.15384	468.00000
	2	108	204	188.88888	92.73504	486.00000
At IV. *per Cent.*	3	112	312	278.57142	89.68254	504.00000
	4	116	424	365.51724	86.94582	522.00000
	5	120	540	450.00000	84.48276	540.00000
					450.00000	
	1	105	100	95.23809	95.23809	461.99999
	2	110	205	186.36863	91.12554	483.99999
At V. *per Cent.*	3	115	315	273.91304	87.54941	505.99999
	4	120	430	358.33333	84.42029	527.99999
	5	125	550	539.99999	81.66666	549.99999
					439.99999	
	1	106	100	94.33962	94.33962	456.61538
	2	112	206	183.92856	89.58894	482.46153
At VI. *per Cent.*	3	118	318	269.49152	85.56296	508.30769
	4	124	436	351.61290	82.12138	534.15384
	5	130	560	430.76923	79.15633	559.99999
					430.76923	
	1	110	100	90.90909	90.90909	440.00000
	2	120	210	175.00000	84.09090	480.00000
At X. *per Cent.*	3	130	330	253.84615	78.84615	520.00000
	4	140	460	328.57142	74.72527	560.00000
	5	150	600	400.00000	71.42858	600.00000
					399.99999	

This

This Table and the foregoing Examples of Equations of Payments, are such as are set down by Sir *S. Moreland* in his Doctrine of Simple Interest.

Here follow some other Rules for Equations of Payments, which (if not so exact yet) are performed with more ease, and although in great Summs, and long times of Payment, they may differ some small matter; yet small summs and short times being of more frequent use, and the difference being inconsiderable, I shall here insert the manner of making them,

§ III. *Of Equations, when the terms of Payment are equal.*

Example 1. A *owes to* B 400 li. *to be paid at 4 six months, that is* 100 li. *at 6 months,* 100 *li. at* 12 *months,* 100 li. *at* 18 *months; and the last* 100 li. *at* 24 *months: But it is agreed to pay all the money at one entire Payment. Now to know the true time of Payment is the thing required.*

Here in this Question, both the Summs to be paid, and the times of Payment are equal; and to resolve the Question this is the *RULE*:

Add one term of time of Payment to the terms of time given, and half that number of terms shall be the time of Payment?

In this Question the terms of Payment are four, and each of them six months, which are 24 months, to which add one term more, and they make 30 months; the half whereof is 15 months, at the termination of which time the Payment is to be made.

Example 2. *If* A *be to pay unto* B 100 li. *a year, for five years; at what term of time shall* A *pay unto* B *the whole* 500 li. *at one entire Payment?*

Here the terms of time are five, to which add one term, and they make six years; the half whereof is three years, at the end whereof *A* ought to pay unto *B* 500 *li.* at one entire Payment. And this agrees exactly with the first Example of the former way of working.

Example 3. A *is to pay unto* B 248 li. *at 4 equal Payments,* viz. 62 li. *at* 12 *months,* 62 li. *at* 24 *months,* 62 li. *at* 36 *months, and* 62 li. *at* 48 *months: At what time ought it to be paid at one entire Payment.*

Add 12 months to the 48 months, and they make 60 months, the half whereof is 30 months, at the end of which time *A* is to pay *B* the 248 *li.* at one entire Payment, which 30 months contain 547 days and an half; exactly agreeing with the foregoing second Example.

§ IV. *Of Equations of Payments, when the Summs of Money, or Times of Payment be different; as,*

Example 1. A *is to pay unto* B 400 li. *in this manner,* 100 li. *at* 3 *months,* 100 *li. at* 6 *months,* 100 *li. at* 12 *months, and* 100 li. *at* 24 *months. At what time is it payable at one intire payment?*

To resolve this and the like Questions, this is the *RULE*:

Multiply the several Summs by the times of their respective Payments, and add them all together, then divide that Summ by the whole Debt; and the Quotient shall give you the term of time, for the entire payment at once.

Thus, 100 *li.* multiplyed by	3	months, is	300
	6		600
	12		1200
	24		2400
		The Summ	3500

This Summ divided by 400 *li.* the whole Debt, giveth in the Quotient 11.25, or 11 months, and a quarter; at the expiration of which time, after the time of the contract, must the 400 *li.* be paid at once.

Example 2. A *is to pay unto* B *a Legasie of* 500 li. *in this manner,* 300 *li. at* 4 *months,* 100 li. *at* 6 *months, and* 100 li. *at* 12 *months: At what time must it be paid at once.*

According to the Rule before given:

300	multiplyed by	4	is 1200
100		6	600
100		12	1200
500		The Summ	3000

Which

Which Summ divided by 500; the Quotient will be 6, and at that time ought the 500 *li.* to be paid at once; and this agrees exactly with the ſame Queſtion, wrought by the preceeding Example 1.

And this reſolves the Queſtion, according to the Intereſt Table; for,

	li.				*li.*	*s.*	*d.*
The Intereſt of	300	in	4	months, is	6	0	0
	100		6		3	0	0
	100		12		6	0	0
				In all	15	0	0

And the Intereſt of 500 *li.* in 6 months is juſt 15 *li.*

But to reſolve this or the like Queſtion, more exactly, is by the Rebate of the Money, and not by the Intereſt: And then,

	li.				*li.*	*s.*	*d.*	*q.*
The Rebate of	300	for	4	months, is	294	2	4	1
	100		6		97	1	9	0
	100		12		94	6	9	2
					485	10	10	2
But the Rebate of 500 *li.* for 6 months is but					485	8	8	3
				The difference	000	2	1	3

And thus have I done with Simple Intereſt, and Rebate or Diſcompt of Money; as alſo with Equation of Payments: And now I ſhall proceed to Compound Intereſt, and Rebate.

CHAP. IV.

Of COMPOUND INTEREST.

AND of the Conſtruction and Uſe of Tables thereof ready computed, from 5 to 12 Pound, *per cent. per annum*, Compound Intereſt, and for any number of years under 31. With variety of Examples, to explain the uſes of them.

Table

TABLE I.

Shewing the Increaſe of One Pound (or 20s.) *yearly, being put forth at Intereſt upon Intereſt, at any of the Rates here under expreſſed:* Viz. *At*

Number of Years that the Money is put out for.	V. *Per Cent.*	VI. *Per Cent.*	VII. *Per Cent.*	VIII. *Per Cent.*	IX. *Per Cent.*	X. *Per Cent.*	XII. *Per Cent.*
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.12000
2	1.10250	1.12360	1.14490	1.16640	1.18810	1.21000	1.2544o
3	1.15762	1.19101	1.22504	1.25971	1.29502	1.33100	1.40492
4	1.21550	1.26247	1.31079	1.36048	1.41158	1.46410	1.57351
5	1.27628	1.33823	1.40255	1.46932	1.53862	1.61051	1.76234
6	1.34009	1.41851	1.50073	1.58087	1.67710	1.77156	1.97382
7	1.40710	1.50363	1.60578	1.71382	1.82803	1.94871	2.21068
8	1.47745	1.59384	1.71818	1.85093	1.99256	2.14358	2.47596
9	1.55132	1.68947	1.83845	1.99900	2.17189	2.35794	2.77307
10	1.62889	1.79084	1.96715	2.15892	2.36736	2.59374	3.10584
11	1.71033	1.89829	2.10485	2.33163	2.58042	2.85311	3.47854
12	1.79585	2.01219	2.25219	2.51817	2.81266	3.13842	3.89597
13	1.88564	2.13292	2.40984	2.71962	3.06580	3.45227	4.36349
14	1.97993	2.26090	2.57853	2.93719	3.34172	3.79749	4.88711
15	2.07892	2.39655	2.75907	3.17216	3.64248	4.17724	5.47536
16	2.18287	2.54035	2.95216	3.42594	3.97030	4.59497	6.13039
17	2.29201	2.69277	3.15881	3.70001	4.32763	5.05447	6.86604
18	2.40661	2.85433	3.37993	3.99601	4.71712	5.55991	7.68996
19	2.52695	3.02559	3.61652	4.31570	5.14166	6.11591	8.61276
20	2.65329	3.20713	3.86968	4.66095	5.60441	6.72749	9.64629
21	2.78596	3.39956	4.14056	5.03318	6.10880	7.40024	10.80384
22	2.92526	3.60353	4.43040	5.43654	6.65860	8.14027	12.10031
23	3.07152	3.81975	4.74053	5.87146	7.25787	8.95430	13.55234
24	3.22509	4.04893	5.07236	6.34118	7.91108	9.84973	15.17862
25	3.38635	4.29187	5.42743	6.84847	8.62308	10.83470	17.00006
26	3.55567	4.54938	5.80735	7.39635	9.39915	11.91817	19.04007
27	3.73345	4.82234	6.21386	7.98806	10.24508	13.10999	21.32488
28	3.92012	5.11168	6.64883	8.62710	11.16713	14.42099	23.88386
29	4.11613	5.41838	7.11425	9.31727	12.17218	15.86309	26.74993
30	4.32194	5.74349	7.61225	10.06265	13.26767	17.44940	29.95992

§ I. *The*

§ I. *The Fabrick or Conſtruction of* TABLE I.

The Proportion for the making of this Table, is this;

As 100 *li.*
Is to 100 *li.* with its increaſe for a year, (ſuppoſe 5 *li.*)
So is 1 *li.*
To 1 *li.* and its increaſe in a year:

For, as 100 *li.* is to 105 *li.* ſo is 1 *li.* to 1.05.

Here 105 being multiplied by 1, produceth only 105, which being divided by 100, the Quotient is 1.05, which is the increaſe of 1 *li.* for a year, and this 1.05 (with Cyphers) is the firſt Number in the Table of 5 *per Cent.*

Then for the ſecond Year,

As 100 *li.*
Is to 105 *li.* the increaſe of 100 *li.* in a year;
So is 1.05, that is, 1 *li.* and its increaſe
To 1.1025.

Which is the ſecond number in the Table, and is the increaſe of 1 *li.* in 2 years.

Then for the third Year,

As 100 *li.* to 105 *li.* ſo is 1.1025 *li.* to 1.1576225 *li.*

Multiply and divide, and the Quotient will be 1.157625, which is the third number in the Table; and is the increaſe of 1 *li.* in 3 years.

Then for the fourth Year,

As 100 *li.* to 105 *li.* ſo is 1.157625 to 1.2155062,

Which is the fourth Number in the Table, and the increaſe of 1 *li.* in 4 years.

For the fifth Number,

As 100 *li.* to 105 *li.* ſo is 1.2155062 to 1.2762815,

The fifth Number in the Table, and the increaſe of 1 *li.* in 5 years.

And ſo of all the reſt;

For as the Table of 5 *per Cent* is made, ſo are all the reſt of 6, 7, 8, *&c.* For,

	li.		
As 100 *li.* is to	106	So is 1 *li.* to	1.06000
	107		1.07000
	108		1.08000
	109		1.09000
	110		1.10000
	112		1.12000

The firſt Number in each Table.

And let this ſuffice for the Conſtruction of the Table; now followeth its Uſe.

The Uſe of *Table* I.

The Uſe of this Table (and of all the rates under this Head) will beſt appear by Examples.

Example 1. *If* 25 li. *be put out for* 7 *years, at Intereſt upon Intereſt at* 5 per Cent. *to what Summ will the ſaid* 25 li. *be augmented to, at the end of* 7 *years?*

Look in the Table (in the firſt Column to the Left Hand) for 7 years, againſt which (under 5 *per Cent.*) you ſhall find 1.4071; which is the increaſe of 1 *li.* in 7 years.

Then ſay by the *Rule* of *Three*,

If 1 li. *in* 7 *years will be augmented unto* 1.4071, *unto what will* 25 li. *be augmented in the ſame time?*

As 1 *li.* to 1.4071, ſo is 25 *li.* to 35.1775 *li.*

Multiply and divide, the Quotient will be 35.1775 *li.* or reduced (as before is taught,) 35 *li.* 3 *s.* 7 *d. ferè.*

And ſo much will 25 *li.* be augmented in 7 years, at 5 *li. per Cent.* Compound Intereſt.

Example 2. *If* 360 li. *be put out for* 16 *years, at* 8 li. per Cent. (*Compound Intereſt*,) *what will it be augmented unto, at the expiration of the* 16 *years?*

Look in the firſt Column of the Table for 16 years, and right againſt it, (under 8 *per Cent.*) you ſhall find this number 3.42594; and to ſo much will 1 *li.* be augmented in 16 years, at 8 *per Cent.*

Then

Then say by the *Rule* of *Three*,

If 1 li. *in* 16 *years will be increaſed to* 3.42594, *unto what will* 360 li. *be increaſed in that time?*

As 1 *li.* to 3.42594, ſo is 360 *li.* to 1233.33840.

Or reduced to 1233 *li.* 6 *s.* 9 *d.* 1 *q.* unto ſo much will 360 *li.* be increaſed, it being forborn 16 years.

And at 12 *li. per cent.* it would be augmented unto 2206 *li.* 18 *s.* 9 *d.* 3 *q.*

But if a Queſtion ſhould be offered, which exceeds the number of years in the Table, the Tables may be continued to as many years as you pleaſe, by the former directions; yet without that, the Tables themſelves as they are will doe the work. As if this Queſtion ſhould be propounded:

Example 3. *If* 327 li. *be put out at Intereſt upon Intereſt, at* 6 per cent. *for* 50 *years; unto what Summ will it be augmented at the expiration of that term?*

Look in the firſt Column of the Table (of 6 *per cent.*) for half the number of years, *viz.* 25. againſt which you ſhall find this number 4.29187, which number being multiplied in it ſelf, (by the inbreviate Decimal Multiplication,) the Product will be 18.42002; which is the number belonging to the 50*th* year: And the reaſon of this lieth here:

If 1 li. *being put out for* 25 *years, do increaſe to* 4.29187 li. *what ſhall* 4.29187 li. *increaſe to, in* 25 *years more? The anſwer will be* 18.42002.

According to the Rule given in Multiplication of Decimals, 4.29817 being multiplyed by 4.29187, ten figures muſt have been cut off from the Product; but by the *Compendium* in Multiplication of Decimals, you cut off only five parts, which are as many as are requiſite: And ſo you ſee that 1 *li.* being forborn 50 years (at 6 *per cent.*) will be increaſed to 18.42002 *li.* Then by the Rule of Three, ſay;

If 1 *li. give* 18.42002 *li.* what will 327 *li. Facit* 6023.34654.

So that 327 *li.* being forborn for 50 years, will amount unto 6023 *li.* 6 *s.* 11 *d.* 1 *q. ferè.*

S Table

TABLE II.

Shewing the Decreaſe of One Pound (or 20s.) *yearly: Or,* (*which is the ſame,*) *what One Pound due at the end of any number of Years,* (*not exceeding* 30,) *is worth in preſent Money, rebating Intereſt upon Intereſt, at any of the Rates here under expreſſed:* Viz. *At*

Number of Years that the Money becomes due.	V. *Per Cent.*	VI. *Per Cent.*	VII. *Per Cent.*	VIII. *Per Cent.*	IX. *Per Cent.*	X. *Per Cent.*	XII. *Per Cent.*
1	.952381	.943636	.934579	.925925	.917431	.909090	.892857
2	.907039	.889996	.873838	.857338	.841680	.826446	.797193
3	.863837	.839619	.816297	.793832	.772183	.751314	.711780
4	.822712	.792093	.763895	.735029	.708425	.683013	.635518
5	.783526	.747258	.712986	.680583	.649931	.620921	.567426
6	.746215	.704960	.666342	.630169	.596267	.564474	.506631
7	.710681	.665057	.622749	.583490	.547034	.513158	.452349
8	.676839	.627412	.582009	.540268	.501866	.466507	.403883
9	.644608	.591898	.543933	.500248	.460427	.424097	.360610
10	.613913	.558291	.508349	.463193	.422410	.385543	.321973
11	.584679	.526787	.475092	.428882	.387532	.350494	.287476
12	.556537	.496989	.444012	.397113	.355534	.318630	.256675
13	.530321	.468839	.414964	.367697	.326178	.289664	.229174
14	.505067	.442300	.387817	.340461	.299246	.263331	.204619
15	.481017	.417265	.362446	.315241	.274538	.239392	.182696
16	.458111	.393646	.338734	.291890	.251869	.217629	.163121
17	.436296	.371364	.316574	.270269	.231073	.197844	.145644
18	.415520	.350343	.295864	.250249	.211993	.179858	.130039
19	.395733	.330512	.276508	.231712	.194489	.163508	.116106
20	.376889	.311804	.258419	.214548	.178430	.148643	.103666
21	.358942	.294155	.241513	.198655	.163698	.135130	.092559
22	.341849	.277505	.225713	.183940	.150181	.122846	.082642
23	.325571	.261797	.210947	.170315	.137781	.111678	.073787
24	.310067	.246978	.197146	.157699	.126405	.101525	.065882
25	.295302	.232998	.184249	.146818	.115967	.092296	.058823
26	.281240	.219810	.172195	.135201	.106392	.083905	.052520
27	.263848	.207367	.160930	.125186	.097607	.076277	.046893
28	.255039	.193630	.159402	.115913	.089548	.069343	.041869
29	.242946	.184556	.140562	.107327	.082154	.063039	.037383
30	.231377	.174110	.131367	.099377	.075371	.057308	.033377

§ II. *The*

§. II. *The Fabrick or Conſtruction of* TABLE II.

The Proportion for the making of this Table is this ; (inſtancing in the Table of 5 *per Cent.*)

As 100 *li.* with its increaſe in a year, is to 100 *li.*
So is 1 *li.* to the decreaſe of 1 *li.* in a year.

Thus, As 105 *li.* is to 100 *li.* ſo is 1 *li.*

Here the third number in the Golden Rule being an Unite, there needs no Multiplicon, and therefore the middle Number muſt be divided by the firſt, (adding Cyphers to the middle Number,) and the Quotient will be the decreaſe of 1 *li.* in a year.

Diviſor.	*Dividend.*	*Quotient.*
As 105)	100.000000	(0.95238

Which Quotient 0.95238, is the firſt Number in the Table, the ſecond will be thus produced,

As 105 *li.* is to 100 *li.* ſo is .95238 *li.* to .90703 *li.*

Which Quotient .90703, is the ſecond Number in the Table: For the third number thus;

As 105 *li.* is to 100 *li.* ſo is .90703 00 *li.* to .86383 *li.*

Which Quotient .86383 is the third number in the Table and the decreaſe of 1 *li.* in 3 years.

And the ſame courſe is to be taken in the making of the Tables for the other Rates of Intereſt:

li.		*li.*	
106		.94339	
107		.93458	
As 108	is to 100 *li.*	.92592	The firſt Number
109	ſo is 1 *li.* to	.91743	in each Table.
110		.90909	
112		.89286	

The Uſe of *Table* II.

Theſe Tables ſhall alſo be made plain by Examples.

Example 1. *If* 546 li. *be due and payable* 11 *years hence, what is that Summ worth in ready Money, accounting Intereſt upon Intereſt at* 6 per Cent ?

In the firſt Column of the Table, look for 11 years, againſt which, under the title of 6 *per Cent.* you ſhall find 0.52678, which is the worth of 1 *li.* payable 11 years hence.

Then ſay by the *Rule* of *Three* :

If 1 li. *payable* 11 *years hence, will be worth but* 0.52678 li. *what will* 546 li. *payable* 11 *years hence, be worth in preſent Money ?*

If 1 *li.* decreaſe to .52678 *li.* what 546 *li. Anſwer*, to 287.62188.

Which reduced is 287 *li.* 12 *s.* 5 *d.* 1 *q.* and ſo much preſent money will diſcharge the debt of 546 *li.* due 11 years hence, at 6 *per Cent.*

Example 2. *At* 12 li. per Cent. *Compound Intereſt, what will* 700 li. *payable at* 21 *years end, be worth in preſent Money ?*

The number in the Table, ſtanding againſt 21 years under 12 *per Cent.* is 0.09256. and ſo much will 1 *li.* payable 21 years hence, be worth preſently : Then by the *Golden Rule*,

If 1 *li.* give but .09256 *li.* what will 700 *li. Anſwer*, 64.79200.

Which reduced is 64 *li.* 15 *s.* 10 *d.* and ſo much will ſatisfie for the payment of 700 *li.* 21 years hence.

But if in the Queſtion propounded, the years exceed thoſe in the Table, you may help your ſelf as in Table I.

Example 3. *What is* 500 li. *due* 40 *years hence, worth in preſent Money ?*

The number in the Table ſtanding againſt 20 years, (which is half the time) under 8 *per Cent.* is .214548, which multiplied in it ſelf, (according to the *Compendium* before taught in Multiplication of Decimals,) will produce in the Product .0460306, which is the decreaſe of 1 *li.* in 40 years. Then ſay by the *Rule* of *Three*,

If 1 *li.* decreaſe to .460306 *li.* what will 500 *li. Anſwer*, 23.01530 00.

And thus you ſee that 500 *li.* payable 40 years hence, will be worth in ready Money (at 8 *per Cent.*) but 23 *li.* 1 *d.*

TABLE III.

Shewing what One Pound (or 20s.) Annuity, to begin presently, and to continue any number of Years, (under 31,) is worth, in present Money, accounting Interest upon Interest, at any of the Rates here under expressed: Viz. *At*

Number of Years of the Annuity.	V. *Per Cent.*	VI. *Per Cent.*	VII. *Per Cent.*	VIII. *Per Cent.*	IX. *Per Cent.*	X. *Per Cent.*	XII. *Per Cent.*
1	0.95238	0.94339	0.93457	0.92592	0.91743	0.90909	0.89285
2	1.85941	1.83339	1.80801	1.78326	1.75911	1.73553	1.69005
3	2.72325	2.67301	2.62431	2.57709	2.53129	2.48685	2.40183
4	3.54595	3.46510	3.38721	3.31212	3.23971	3.16986	3.03734
5	4.32947	4.21236	4.10019	3.99270	3.88965	3.79078	3.60477
6	5.07569	4.91732	4.76653	4.62287	4.48591	4.35526	4.11140
7	5.78637	5.58238	5.38928	5.20636	5.03295	4.86841	4.56375
8	6.46321	6.20979	5.97129	5.74635	5.53481	5.33492	4.96763
9	7.10782	6.80169	6.51523	6.24688	5.99524	5.75901	5.32824
10	7.72173	7.36008	7.02358	6.71008	6.41765	6.14456	5.65022
11	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	5.93769
12	8.86325	8.38384	7.94268	7.53607	7.16072	6.81369	6.19437
13	9.39357	8.85268	8.35765	7.90377	7.48690	7.10335	6.42354
14	9.89864	9.29498	8.74546	8.24423	7.78614	7.36668	6.62816
15	10.37865	9.71224	9.10791	8.55947	8.06068	7.60608	6.81086
16	10.83776	10.10589	9.44664	8.85136	8.31255	7.82371	6.97398
17	11.27406	10.47725	9.76322	9.12163	8.54363	8.02155	7.11962
18	11.68958	10.82760	10.05908	9.37188	8.75562	8.20141	7.24966
19	12.08531	11.15811	10.33559	9.60359	8.95011	8.36492	7.36577
20	12.46220	11.46992	10.59401	9.81814	9.12854	8.51356	7.46944
21	12.82115	11.76407	10.83552	10.01680	9.29224	8.64869	7.56200
22	13.16300	12.04158	11.06124	10.20074	9.44242	8.77154	7.64464
23	13.48857	12.30337	11.27218	10.37105	9.58028	8.88322	7.71843
24	13.79864	12.55035	11.46933	10.52873	9.70661	8.98474	7.78431
25	14.09394	12.78335	11.65358	10.67477	9.82257	9.07704	7.84313
26	14.37518	13.00316	11.82577	10.80997	9.92897	9.16094	7.89505
27	14.64303	13.21053	11.98671	10.93516	10.02657	9.23722	7.94255
28	14.89812	13.40616	12.13711	11.05107	10.11612	9.30656	7.98442
29	15.14071	13.59071	12.27767	11.15840	10.19828	9.36960	8.02180
30	15.39244	13.76482	12.40904	11.25778	10.27365	9.43691	8.05518

§ III. *The*

§ III. *The Fabrick or Conſtruction of* TABLE III.

This Table is deduced from Table II. foregoing, the firſt number (in the Column of 5 *per Cent.*) being 0.95238, and this number alſo muſt be the firſt in this Table.

Then for the ſecond number,

Add the ſecond number in the former Table, 0.90703, to the firſt number thereof, 0.95238, and their Summ 1.85941 is the ſecond number in this Table.

Firſt Number of Table II.	0.95238
Second Number of Table II.	0.90703
Second Number of this Table.	1.85941

Then for the third Number,

To the ſecond Number of this Table add the third Number of the laſt Table, and that Summ ſhall be the third Number of this Table;

Second Number of Table III.	1.85941
Third Number of Table II.	0.86383
Third Number of this Table	2.72324

To this Summ 2.72324 add the fourth Number of the laſt Table 0.82271, and their Summ 3.54595 ſhall be the fourth Number of this Table.

And thus muſt you doe for the reſt of the Numbers of this Table of 5 *per Cent.* And the ſame Rule is to be obſerved in all the other Rates in this Table III.

The Uſe of *Table* III.

The Uſe of this (as of thoſe beforegoing) will beſt appear by Examples.

Example 1. *What is* 45 li. per Annum *for* 24 *years worth in preſent Money, at* 6 li. per Cent. *Compound Intereſt.*

Find 24 years in the firſt Column of the Table towards the left hand, and right againſt it (in the Column of 6 *per Cent.*) you ſhall find 12.55035, which is the value of 1 *li. per Annum* for 24 years; therefore 45 *li. per Annum* muſt be worth 45 times the ſame Summ,

Which is 564 *li.* 15 *s.* 3 *d.* 3 *q.* for the worth of 45 *li. per Annum* for 24 years.

Example 2. *What is* 320 li. per Annum *for* 21 *years, (at* 10 *li.* per Cent.) *worth in preſent Money?*

Look for 21 years in the firſt Column of the Table, and right againſt it (in the Column under 10 *per Cent.*) you ſhall find this number, 8.64869; which is the value, or worth of 1 *li.* for 21 years: Therefore 320 *li. per Annum* muſt be worth 320 times ſo much; that is, 2767.580, *&c.*

Which reduced, is 2767 *li.* 11 *s.* 7 *d.* 2 *q.* and ſo much is an Annuity of 320 *li. per Annum*, to continue 21 years, worth in preſent Money, accounting 10 *li. per Cent.* Compound Intereſt.

Example 3. *What is a Leaſe of Houſe or Land for* 21 *years,* 100 li. *being paid as a Fine, and an Annual Rent of* 30 li. *What Fine more muſt the Tenent give to bring down his Rent, from* 30 li. *to* 10 li. *a year, allowing* 6 per Cent. *Compound Intereſt.*

The difference between 30 *li.* and 10 *li.* is 20 *li.*

Look therefore for 21 years in the firſt Column of the Table, and againſt it (under 6 *per Cent.* you ſhall have this number, 11.76407, which is the worth of 1 *li.* for 21 years to come; multiply this Number by 20, and the Product ſhall give the preſent worth.

This Product being reduced, is 235 *li.* 5 *s.* 7 *d.* 2 *q.* and ſuch Fine muſt he pay to bring the Rent down from 30 *li.* to 10 *li. per Annum.*

Example 4. *A Landlord demands a Fine, or preſent Summ of Money, for a Leaſe of* 127 li. per Annum, *to continue* 7 *years? What is the Summ, allowing* 6 per Cent. *Compound Intereſt?*

Find 7 years in the firſt Column of this Table, and againſt it (under 6 *per Cent.*) you ſhall find the Number 5.58228, which multiply by the Annual Rent 127 *li.* the Product is the Anſwer to the Queſtion demanded.

Which Product 708.962 being reduced, gives 708 *li.* 19 *s.* 2 *d.* 3 *q.* and ſuch Fine muſt he pay.

Example 5. *What is the preſent worth of* 80 li. *Rent or Annuity, to continue* 25 *years, rebating at* 6 per Cent. per Annum *Compound Intereſt?*

Look

Look in the Table for 25 years, againſt which (and under 6 *per Cent.*) you have this Number 12.78335, which being multiplied by 80 *li.* (the Annuity or Rent propounded,) the Product will be 1022.66800; which reduced is 1022 *li.* 13 *s.* 4 *d.* 1 *q.* And to ſo much will 80 *li. per Annum* yearly payments, be rebated for 25 years.

Example 6. *One hath a Leaſe worth* 15 li. per Annum. *more than the Rent, and hath a Leaſe of it yet* 4 *years in being; the Tenent deſires to take another in Reverſion for* 21 *years at the ſame Rent; What muſt he pay for a Fine, Intereſt allowed at* 6 per Cent. *Compound Intereſt?*

The Decimal ſtanding againſt 4 years (under 6 *per Cent.*) is 3.46510, which multiplied by 15 *li.* the Product is 51.97650, which reduced is 51 *li.* 19 *s.* 6 *d.* 2 *q.* and ſo much the Leaſe of 4 years yet in being, is worth.

Now add the 4 years in being, to the 21 years, the term in Reverſion, the Summ is 25 years: Wherefore look in the Table for 25 years, and againſt it (under 6 *per Cent.*) you ſhall find 12.78335, which multiplied by 15 *li.* the Product will be 191.75025; which reduced, is 191 *li.* 15 *s.* the difference between this and the value of the 4 years in being, *viz.* 51 *li.* 19 *s.* 6 *d.* 2 *q.* is 139 *li.* 15 *s.* 5 *d.* 2 *q.* and ſuch Fine muſt he pay for a Leaſe of 21 years, to commence after the expiration of the 4 years in being.

Example 7. *A Tenent hath a Leaſe of* 21 *years, the preſent Rent of it is* 41 li. per Annum *for* 7 *years, and after that* 7 *years, he is to pay* 50 li. per Annum *for the other* 14 *years; what is the value of this Leaſe in preſent Money, Intereſt being diſcounted at* 6 per Cent?

Look in the Table for 21 years, againſt which (under 6 *per Cent.*) you ſhall find this Number, 11.76407, which multiply by 50 *li.* (the annual Rent of 21 years,) the Product will be 588.20350, which reduced is 588 *li.* 4 *s.* 0 *d.* 1 *q.* and this is the true value of the whole 21 years; but foraſmuch as the firſt 7 years pays but 41 *li. per Annum*, the difference is 9 *li.* then look again in the Table for 7 years, againſt which you have 5.58238, which multiplied by 9 *li.* (the difference of Rent,) the Product will be 50.24142, which reduced is 50 *li.* 4 *s.* 10 *d.* this Summ ſubtracted from the former, leaves 537 *li.* 19 *s.* 2 *d.* 1 *q.* the true value of the Leaſe required.

Example 8. *There is a Leaſe to be taken for* 21 *years, at* 30 li. per Annum, *and* 100 li. *Fine the Leſſee deſires the Rent to be brought down to* 10 li. per Annum, *and is willing to give a Fine, proportionable to the Rent abated; what muſt that Fine be?*

The annual Rent abated is 20 *li.* for 21 years, look in the Table for 21 years, againſt it is 11.76407; this multiplyed by 20 *li.* (the Rent abated) the Product is 235.28140; which reduced, is 235 *li.* 5 *s.* 7 *d.* 2 *q.* the value of 20 *li. per Annum* for 21 years, to which add the former Fine, 100 *li.* the Summ will be 335 *li.* 5 *s.* 7 *d.* 2 *q.* the true Summ to be paid for the Fine.

TABLE

TABLE IV.

Shewing what One Pound (or 20s.) Annuity, to begin presently, and to continue any number of Years, (under 31,) is worth, to be paid for all together at the end of the Term of Years for which it was to continue, at any of the Rates of Compound Interest here under expressed: Viz. *At*

Number of Years of the Forbearance of the Annuity.	V. *Per Cent.*	VI. *Per Cent.*	VII. *Per Cent.*	VIII. *Per Cent.*	IX. *Per Cent.*	X. *Per Cent.*	XII. *Per Cent.*
1	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
2	2.05000	2.06000	2.07000	2.08000	2.09000	2.10000	2.12000
3	3.15250	3.18360	3.21490	3.24640	3.27819	3.31000	3.37440
4	4.31012	4.37461	4.43994	4.50611	4.57312	4.64100	4.77932
5	5.52563	5.63709	5.75073	5.86660	5.98471	6.10510	6.35284
6	6.80191	6.97531	7.15329	7.33592	7.52333	7.71561	8.11518
7	8.14200	8.39383	8.65402	8.92280	9.20043	9.48717	10.08901
8	9.64910	9.89746	10.25980	10.63662	11.02847	11.43588	12.29969
9	11.02656	11.49131	11.97798	12.48755	13.02103	13.57947	14.77565
10	12.57789	13.18079	13.81644	14.48656	15.19292	15.93742	17.54873
11	14.20678	14.97164	15.78359	16.64548	17.56029	18.53116	20.65458
12	15.91712	16.86994	17.88845	17.97712	20.14071	21.38428	24.13313
13	17.71298	18.88213	20.14064	21.49529	22.95338	24.52271	28.02910
14	19.59863	21.01506	22.55048	24.21492	26.01918	27.97198	32.39260
15	21.57856	23.27596	25.12902	27.15211	29.36091	31.77248	37.27971
16	23.65749	25.67252	27.88805	30.32428	33.00339	35.94972	42.75328
17	25.84036	28.21287	30.84021	33.75022	36.97370	40.54470	48.88367
18	28.13238	30.90565	33.99903	37.45024	41.30133	45.59917	55.74971
19	30.53900	33.75999	37.37896	41.44626	46.01845	51.15909	63.43968
20	33.06595	36.78559	40.99549	45.76196	51.16011	57.27499	72.05244
21	35.71925	39.99272	44.86517	50.42292	56.76453	64.00249	81.69873
22	38.50521	43.89228	49.00573	55.45675	62.87333	71.40274	92.50258
23	41.43047	46.99582	53.43614	60.89329	69.53193	79.54302	104.60289
24	44.50199	50.81557	58.17667	66.76475	76.78981	88.49732	118.15524
25	47.72709	54.86451	63.24903	73.10593	84.70089	98.34705	133.33387
26	51.11345	59.15638	68.67646	79.95441	93.32397	109.18176	150.33393
27	54.66912	63.70576	74.48382	87.35076	100.72313	121.09994	169.37400
28	58.40258	68.52810	80.69769	95.33882	112.96821	134.20993	190.69888
29	62.32271	73.63979	87.34652	103.96593	124.13535	148.63092	214.58275
30	66.42884	79.05818	94.46078	112.28321	136.30753	164.49402	241.33268

§ IV. *The*

§ IV. *The Fabrick or Construction of* TABLE IV.

This Table is to be gathered out of Table I. which shews the increase of one Pound, (or 20 s.) yearly, reckoning Interest upon Interest at the several Rates in that Table expressed.

One pound Annuity, to be paid for at the end of one year is ——— 1.00000

And that is the first Number in this Table, to which if you add the first Number in Table I. which is 1.05000, the Summ will be 2.05000; and this is the second Number in this Table.

To this add 1.10250, (the second Number in Table I.) and the Summ is 3.15250; and that is the third Number in this Table.

To this add 1.15762, (the third Number in Table I.) and the Summ is 4.31012; and that is the fourth Number in this Table.

So that the first Number of Table I. added to the first Number in this Table, maketh the second Number of this Table.

The second number of Table I. added to the second Number of this Table, maketh the third Number of this Table.

Also the third Number of Table I. added to the third Number of this Table, maketh the fourth Number of this Table, *&c.*

And as this Column of 5 *per Cent.* is made, so must all the other Columns be produced, according to their respective Rates.

The Use of *Table* IV.

Example 1. *What will an Annuity of* 25 li. per Annum. *payable yearly, be augmented unto in* 9 *years; the purchase Money being all that time forborn at* 6 per Cent. *Compound Interest?*

Look in the Table for 9 years in the first Column, and against it (under 6 *per Cent.*) you shall find 11.49131, which shews what one Pound Annuity will amount unto in that time. Wherefore by the Golden Rule say,

As 1 li. Is to 11.49131, in 9 years, So is 25 li. To 287.2828 in 9 years.

Which is 287 li. 5 s. 8 d. *ferè*; and so much will an Annuity of 25 li. *per Annum* be augmented unto in 9 years, the Purchase Money being forborn all the time.

Example 2. *A Gentleman put his Son out Apprentice for* 7 *years, and at the same time letteth an Annuity of* 57 li. per Annum. *for* 7 *years, forbearing the Money all the time, to raise a stock for his Son, against his Apprenticeship be expired: Unto what will this* 57 li. per Annum *be augmented at the* 7 *years end, at* 8 per Cent. *Compound Interest?*

Look in the first Column of the Table for 7 years, and right against it, you shall find (in the Column under 8 *per Cent.*) 8.92280 which is the Amount of 1 *li.* forborn 7 years. Thus by the Golden Rule say,

As 1 li. is to 8.9228, So is 57 li. to 508.5996, Which is 508 li. 12 s. *ferè*; and so much will the Annuity be augmented unto in 7 years for the Son's stock.

Example 3. *If an Annuity of* 60 li. per Annum. *be forborn* 7 *years, How much will it amount unto, when that time expires?*

Look for 7 years in the first Column, and against it (in the Column under 6 *per Cent.*) you shall find 8.39384, which multiplied by 60 *li.* The Product is 503.63040, and answers the Question.

Which reduced, is 503 *li.* 12 *s.* 7 *d.* 1 *q.* the value of the 60 *li.* Annuity forborn 7 years.

Example 4. A *did owe unto* B 186 li. *and upon Covenant to pay unto the said* B *a Rent of* 20 li. 13 s. 4 d. per Annum, *untill the Debt should be discharged: But after this contract, they mutually agree to respite the payment untill the last were due, with this proviso, to pay it all in then; allowing Interest upon Interest, the forbearance at* 6 per Cent.

First say by the *Rule* of *Three*,

As 20.666, the Decimal of 20 *li.* 13 *s.* 4 *d.* Is to 1 year; So is 186 *li.* To 9 years.

Which Rent is to be respited during the aforesaid Term.

Now if you look in the Table against 9 years, and under 6 *per Cent.* you shall find 11.49131, which being multiplyed by 20.666, the Decimal of 20 *li.* 13 *s.* 4 *d.* the Product will be 237.478, which answers the Question.

Which Product being reduced is 237 *li.* 9 *s.* 6 *d.* 3 *q.* and for so much must *A.* be responsible to *B.* at 9 years end.

Example

Example 5. A *was to pay unto* B *at the end of* 5 *years* 200 *li. for the ſatisfaction of which Debt* A *was content to make* B *a Leaſe of* 35 li. *a year for the ſame time. Which of theſe two gained by this contract, Intereſt being allowed at* 6 per Cent.

Look in the Table for 5 years, againſt which (under 6 *per Cent.*) ſtands this Number 5.63709. This multiplyed by 35 (the Annual Rent) produceth 197.29815, which being reduced, is 197 *li.* 5 *s.* 11 *d.* 3 *q.* which being ſubtracted from 200 *li.* the remainder will be 2 *li.* 14 *s.* 0 *d.* 1 *q.*

And ſo much did *A* gain by the contract made with *B.*

T TABLE

TABLE V.

Shewing what Annuity to continue any number of Years, (under 31,) One Pound (or 20s.) will purchase: Or, (which is the same in effect,) what yearly Payment is equal to a Summ of Money due at present, at any of the Rates of Compound Interest here under expressed: Viz. *At*

Number of Years of the Annuity.	V. *Per Cent.*	VI. *Per Cent.*	VII. *Per Cent.*	VIII. *Per Cent.*	IX. *Per Cent.*	X. *Per Cent.*	XII. *Per Cent.*
1	1.05000	1.06000	1.07000	1.08000	1.09000	1.10000	1.12000
2	0.53780	0.54543	0.55309	0.56076	0.56846	0.57619	0.59169
3	0.36721	0.37411	0.38105	0.38803	0.39505	0.40211	0.41635
4	0.28209	0.28859	0.29519	0.30192	0.30866	0.31547	0.32923
5	0.23097	0.23739	0.24389	0.25045	0.25709	0.36379	0.27741
6	0.19701	0.20336	0.20979	0.21631	0.22291	0.22960	0.24322
7	0.17281	0.17913	0.18555	0.19207	0.19869	0.20545	0.21911
8	0.15472	0.16103	0.16476	0.17401	0.18067	0.18744	0.20130
9	0.14069	0.14702	0.15348	0.16007	0.16679	0.17364	0.18768
10	0.12950	0.13586	0.14237	0.14902	0.15582	0.16274	0.17698
11	0.12038	0.12679	0.13335	0.14007	0.14694	0.15396	0.16841
12	0.11222	0.11927	0.12590	0.13269	0.13965	0.14676	0.16143
13	0.10465	0.11296	0.11965	0.12652	0.13356	0.14077	0.15567
14	0.10102	0.10758	0.11434	0.12129	0.12843	0.13574	0.15087
15	0.09634	0.10296	0.10979	0.11682	0.12405	0.13147	0.14682
16	0.09226	0.09895	0.10585	0.11298	0.12029	0.12781	0.14339
17	0.08869	0.09444	0.10242	0.10962	0.11704	0.12466	0.14045
18	0.08554	0.09235	0.09941	0.10670	0.11421	0.12192	0.13793
19	0.08274	0.08962	0.09675	0.10412	0.11173	0.11954	0.13576
20	0.08024	0.08718	0.09439	0.10184	0.10954	0.11745	0.13388
21	0.07799	0.08500	0.09228	0.09983	0.10761	0.11562	0.13224
22	0.07597	0.08304	0.09040	0.09803	0.10590	0.11400	0.13081
23	0.07413	0.08127	0.08871	0.09642	0.10438	0.11257	0.12995
24	0.07247	0.07967	0.08718	0.09497	0.10302	0.11126	0.12846
25	0.07097	0.07822	0.08581	0.09367	0.10180	0.11016	0.12750
26	0.06956	0.07690	0.08456	0.09250	0.10071	0.10915	0.12665
27	0.06829	0.07569	0.08342	0.09144	0.09973	0.10825	0.12509
28	0.06712	0.07459	0.08239	0.09048	0.09885	0.10745	0.12524
29	0.06604	0.07357	0.08144	0.08961	0.09805	0.10672	0.12454
30	0.06506	0.07264	0.08058	0.08882	0.09733	0.10607	0.11414

§ V. *The Fabrick or Conſtruction of* TABLE V.

This Table is deduced from Table III. and the Numbers in this Table are from thence thus found:

To 1 *li.* add a competent number of Cyphers, ſo many as that you may have ſix figures in your Quotient: This 1 *li.* with Cyphers muſt always be your *Dividend*, and the numbers in Table III. always your *Diviſor*.

So if you divide { First / Second / Third / Fourth / Fifth } 1 *li.* with Cyphers, by the { First / Second / Third / Fourth / Fifth } Number in Table III. the *Quotient* will be the { First / Second / Third / Fourth / Fifth } Number in this Table V.

Example, *In the Column of* 5 per Cent.

To 1 *li.* add ten or more Cyphers, and ſet that down as a Dividend; then take the firſt Number of Table III. (under 5 *per Cent.*) which is .95238, and ſet that down as a Diviſor, and by it divide 1.0000000000, the Quotient will be 1.05000; which is the firſt Number in this Table under 5 *per Cent.*

Diviſor.	*Dividend.*	*Quotient.*
.95238	1.0000000000	1.05000

For the Second Number,

Divide 1 *li.* with Cyphers by 1.85941, the Second Number in Table III. the Quotient .53780 will be the Second Number in this Table.

Diviſor.	*Dividend.*	*Quotient.*
1.85941	1.0000000000	.53780

So likewiſe for the Third Number.

One Pound with Cyphers divided by .27232, will give in the Quotient .36721 *ferè*, for the Third Number in this Table.

Diviſor.	*Dividend.*	*Quotient.*
.27232	1.0000000000	.36721

And ſo for the reſt of this and the other Rates.

The Uſe of *Table* V.

The beſt Precedent for the Uſe of this Table (as of all the reſt) will be by Examples.

Example 1. *What Annuity to continue* 25 *years, will* 450 li. *purchaſe, that I may have* 8 li. per Cent. *Compound Intereſt, for my Money?*

Look in the firſt Column of this Table for 25 years, againſt which (and under 8 *per Cent.*) you ſhall find this Number .09368, which is the Annuity which 1 *li.* will purchaſe for 25 years. Then ſay by the Rule of Three,

If 1 *li.* will purchaſe 0.09368. what 450? Anſwer 42.15600, or 42 *li.* 3 *s.* 1 *d.*

Example 2. *One hath* 325 li. *lying by him, with which he would purchaſe an Annuity for* 9 *years, and would have* 10 li. per Cent. *for his Money; What Annuity for that time will that Summ purchaſe?*

Againſt 9 years (under 10 *per Cent.*) there ſtands this Number 0.17364, the Annuity that 1 *li.* will purchaſe for that time: Then,

If 1 *li.* purchaſe 0.17364, what 325? Anſwer 56.43300, or 56 *li.* 8 *s.* 8 *d. ferè.*

And ſuch Annuity will 325 *li.* purchaſe for 9 years, at 10 *li. per Cent.*

Example 3. *One oweth* 365 li. *which he cannot preſently pay, but agrees to pay the ſame ſumm with Compound Intereſt at* 6 per Cent. *in* 9 *years time, by equal yearly payments; How much Money muſt he pay yearly?*

Look in the Table for 9 years in the firſt Column, and againſt it (under 6 *per Cent.*) you ſhall find 0.14702. Then ſay by the Rule of Proportion,

As 1 *li.* is to 0.14702, ſo is 365 to 53.72800.

Which reduced is 53 *li.* 14 *s.* 6 *d.* 3 *q.* and ſo much muſt he pay annually to diſcharge his Debt of 365 *li.* in 9 years time.

Example 4. *What annual Payment will diſcharge* 450 li. *due* 5 *years hence, accounting Intereſt upon Intereſt, at* 6 per Cent.

1. Seek by Table II. what 450 *li.* due at the end of 5 years is worth in ready money; and you will find it to be .74726 *li.* Then say by the Golden Rule,

As 1 *li.* is to .74726, so is 450 to 336.26700 *li.* ready Money.

2. Look for 5 years in the first Column of this Table, and against it (under 6 *per Cent.*) you shall find this Number 0.23739, and such an Annuity will 1 *li.* purchase for 5 years: Then say again by the Rule of Three,

As 1 *li.* to 0.23739, so 336.267 to 79.82642313.

Which reduced, is 79 *li.* 16 *s.* 6 *d.* 2 *q.* and such annual Payment will discharge the Debt of 450 *li.* in 5 years.

Example 5. *What yearly Payment will satisfie* 340 li. *due at* 7 *years end, reckoning Interest upon Interest at* 8 per Cent?

1. Find out by Table II. what 340 *li.* due 7 years hence is worth in present Money, and you shall find it to be 198.3866.

2. Look in this Table under 8 *per Cent.* what one Pound will purchase for 7 years, which you will find to be .19207: Then say by the Rule of Three,

As 1 *li.* to .19207, so 198.3866 to 38.104114262.

Which is 38 *li.* 6 *s.* 1 *d.* and such annual Payment will discharge a Debt of 340 *li.* due at 7 years end.

And the Answer to this Question may be proved by Table III. for if you look into that Table, (under 8 *per Cent.*) against the seventh year, you shall find the value of one Pound Annuity at the end of 7 years to be 8.9228; which known, then say by the Rule of Three,

As 1 *li.* to 8.9228, so 38.104 to 339.994; or 339 *li.* 19 *s.* 10 *d.* 3 *q.*

Which should have been 340 *li.* the difference arising only in the Unites place of the Fraction .104, which makes the Summ too little by 1 $\frac{1}{2}$ *d.* whereas if it had been .105 it would have been 3 *q.* too much.

Example 6. *What Annuity, Rent, or Pension, will* 250 li. *present Money purchase, for a Lease of* 7 *years, Interest allowed at* 6 per Cent. per Annum?

Look for the seventh year in the Table, and right against it (under 6 *per Cent.*) you shall find this Number .17913, which multiplied by 250 *li.* the Product will be 44.78250; which reduced, is 44 *li.* 15 *s.* 7 *d.* 3 *q.* and such an Annuity for 7 years will 250 *li.* purchase.

Example 7. *One purchased a Lease for* 25 *years at* 10 li. per Annum, *and paid a Fine of* 150 li. *How much was the annual Rent of this Lease valued at, Money being at* 6 per Cent?

Look in the Table for 25 years, against which (under 6 *per Cent.*) stands .07822, which multiplied by 150 *li.* the Product will be 11.73300; which reduced, is 11 *li.* 14 *s.* 8 *d.* and such an Annuity will 150 *li.* purchase for 25 years.

Example 8. *There is a Lease of* 25 *years to come, set at* 10 li. *Rent* per Annum, *and* 150 li. *Fine demanded; the Tenent is willing to give* 100 li. *Fine, and an advance of Rent during the time for the* 50 li. *abated of the Fine; What must be the annual Rent, at* 6 li. per Cent?

There is no essential difference betwixt this and the last Example; for the Fine being diminished, the annual Rent must be increased; therefore take the difference between 150 *li.* (the Fine demanded,) and 100 *li.* (the Fine offered,) which is 50 *li.* then the Decimal against 25 years .07822, being multiplied by 50, the Product will be 3.91100; which being reduced, is 3 *li.* 18 *s.* 2 *d.* 3 *q.* and such must be the advance of Rent for the 50 *li.* abated in Fine; which added to the fixed Rent 10 *li.* is 13 *li.* 18 *s.* 2. 3 *q.* and such must be the annual Rent.

Example 9. *A Person giving over Trade, commits the same, with his Goods in the Shop, and Lease of his House for his term, all being valued (or appraised) at* 1658 li. *and this Money the Person giving over is willing to take of him to whom he resigned in* 7 *years, by equal annual Payments, at* 6 per Cent. per Annum; *What annual Rent will discharge the Debt of* 1685 li.

This annual Rent will be easily discovered by this fifth Table; for against the seventh year you shall find this Number .17913, which multiplied by 1658, produceth 296.999; which may very well be called 297 *li.* and such annual Payment must be paid for 7 years, for the Lease and Wares in the Shop.

Example 10. *A Tenent took a Lease of a House and Land for the term of* 21 *years, at* 160 li. *Fine, and* 16 li. *Rent* per Annum, *at* 7 *years end, the Lessee was resolved to put it off; What annual Rent or Annuity must he set the Tenent at to withdraw his former Fine, or reserving the same Rent, propose another importional for the years to come, at* 6 per Cent.

First,

First, For the impoſing a proportional Rent, you muſt (by Table V.) find what 160 *li.* will purchaſe for 21 years: The Decimal ſtanding againſt 21 years is .08500, which multiplied by 160, the Product is 13.60000; which reduced is 13 *li.* 12 *s.* and ſuch an annual Penſion 160 *li.* will purchaſe for 21 years: this added to 16 *li. per Annum*, (the Rent of the Tenement,) the ſumm is 29 *li.* 12 *s.* and ſo the Tenent putting off the Farm at that Rent for the remaining 14 years, does exactly ſave himſelf.

But ſecondly, To diſcover what Fine muſt be impoſed, the old Rent reſerved, and yet a proportional part for the firſt Fine.

The term of years remaining are 14, whoſe Decimal in Table III. is 9.29498, which being multiplied, by the Decimal of 13 *li.* 12 *s.* before found, (*viz.* 13.6,) the Product will be 126.411; which reduced, is 126 *li.* 8 *s.* 2 *d.* 3 *q.* and ſuch Fine muſt be paid.

§ VI. *Queſtions differing from the former, inſtanced in the particular Examples of the Five foregoing* Tables *reſpectively, which may often offer themſelves, or others of the like nature.*

In the ſolution of the following Queſtions I ſhall be but brief, becauſe they are in effect (due conſideration had of the nature of the Queſtion propounded,) the ſame as thoſe before exemplified; wherefore to inſtance in ſome few:

Queſtion 1. *What is a Leaſe of* 40 li. per Annum *for* 28 *years worth in preſent Money, when as no Rent is to be received for the ſame till after the firſt ſeven years, at* 8 li. per Cent. per Annum?

This Queſtion is reſolvable by Table III. where the Number ſtanding againſt the 28th year (under 8 *per Cent.*) is 11.05108, the Number againſt the ſeventh year is 5.20637, the difference of theſe two Numbers is 5.84471; which multiplied by the Annuity 40 *li.* the Product is 233.78840; which reduced, is 233 *li.* 15 *s.* 9 *d.* 1 *q.* and ſo much is the Leaſe worth at 8 *per Cent.*

Queſtion 2. *What is* 55 li. per Annum *for* 7 *years in reverſion after* 10 *years, worth in preſent Money, at* 8 per Cent. *Compound Intereſt?*

In Table III. the Number againſt the tenth year 6.71008, ſubtracted from the Number (or Decimal) ſtanding againſt the ſeventeenth year 9.12163, the Remainder is 2.41155; which multiplied by the Annuity 55, the Product is 132.63525; which reduced, is 132 *li.* 12 *s.* 8 *d.* 2 *q.* and ſo much is 55 *li. per Annum*, for ſeven years in reverſion after 10 years worth.

Queſtion 3. *What is* 100 li. per Annum *for* 21 *years, and* 120 li. per Annum *for* 7 *years, after the* 21 *years are expired, worth in preſent Money at* 10 li. per Cent?

Look in Table III. and againſt 21 years, under 10 *per Cent.* you ſhall find this Number 8.64869; which multiplied by 100, (by adding two Cyphers,) the Product is 864.86900, that is 864 *li.* 17 *s.* 3 *d.* 2 *q.* and that is the value of 100 *li.* for 21 years.

Then to anſwer the ſecond part of the Queſtion, namely, *To find the worth of* 120 li. per Annum, *for* 7 *years in Reverſion, after* 21 *years:* Look in the foreſaid Table III. for the Number ſtanding againſt 28 years, which is 9.30656; and alſo for the Number ſtanding againſt the 21 years, which is 8.64869; ſubtract the latter from the former, the Remainder is .65787; which multiplied by 120 *li.* the Product is 78.94440; which reduced, is 78 *li.* 18 *s.* 10 *d.* 3 *q.* and ſo much is the 120 *li. per Annum* for 7 years in Reverſion after 21 years worth; and the ſumm of theſe two Products added together anſwers the whole Queſtion, making 943 *li.* 16 *s.* 2 *d.* 1 *q.*

Queſtion 4. *If* 200 li. *Fine be demanded for a Leaſe for* 21 *years, What is a Leaſe of the ſame Premiſſes worth for* 15 *years, the Rate of Intereſt being* 10 per Cent?

Look in Table III. for 21 years, againſt which (under 10 *per Cent.*) ſtands this Number 8.64869, which muſt be the firſt Number in the Rule of Proportion; alſo the Number in the ſame Table ſtanding againſt 15 years, *viz.* 7.60608, muſt be the third Number, and 200 *li.* the mean (or middle) Number: then multiply and divide according to the Rule, and the Quotient will anſwer the Queſtion.

As 8.64869 to 200, ſo 7.60608 to 175.89 *ferè.*

Which Quotient being reduced is 175 *li.* 18 *s. ferè.* and ſo much is the Leaſe worth for 15 years.

Queſtion 5. *If a Leaſe for* 7 *years be worth* 45 li. *what ſhall a Leaſe for* 21 *years (of the ſame Premiſſes) be worth at* 8 per Cent.

Look in the foregoing Table III. for the Number ſtanding againſt 21 years, and 7 years which are 10.0168, and 5.20636; then ſtate your Queſtion thus:

As 5.20636 to 45 *li.* ſo 10.0168 to 86.5789 *li.* Or 86 *li.* 11 *s.* 6 *d.* 3 *q.*

§ VII. *Shewing*

§ VII. *Shewing how the Five foregoing* Tables *may be made ſerviceable to reſolve Queſtions of Leaſes, Annuities, &c. which are for longer time than thoſe* Tables *are calculated for.*

I ſhall inſtance in two or three Queſtions of that kind wrought by the Table of 6 *per Cent.* only.

Queſtion 1. *What is a Leaſe or Annuity of* 160 li. per Annum *worth for* 60 *years, at* 6 per Cent. *Compound Intereſt?*

Look in the Table for 30 years, (which is the half of 60 years,) and the Number ſtanding againſt it is 5.743491; which Number multiplied by it ſelf, produceth 32 9870, which would have been the Number in the Table for the value of 1 *li.* in 60 years, had the Table been continued ſo far.

The Number for 60 years,	32.9870
An Unite ſubtracted,	1.
There remains	31.9870
Two Cyphers added to it,	31.987000
Divided by 6, (the Rate of Intereſt,) the Quotient will be	5.331167
This multiplied by the Annuity 160, produceth	852.986720
This Product (Cyphers being added) divided by the Number in the Table for the 60th year, the Quotient is	2585.827

Which being reduced, is 2585 *li.* 16 *s.* 6 *d.* 2 *q.* and ſo much is a Leaſe or Annuity of 160 *li. per Annum* worth for 60 years.

Queſtion 2. *What is* 75 li. per Annum *worth for* 50 *years at* 6 per Cent?

The Number in the Table ſtanding againſt 25 years, (which is the half of 50 years,) is 4.29187, which multiplied in it ſelf produceth 18.42012, which is the Tabular Number for 50 years, or the value of 1 *li.* Annuity for 50 years.

The Number for 50 years,	18.420112
An Unite ſubtracted and two Cyphers added, is	17.42011200
One ſixth part for the Rate of Intereſt, is	2.90335534
Which multiplied by the Annuity 75 *li.* the Product is	217.75150550
This divided by the Tabular Number for 50 years, the Quotient is	1182.139

Which being reduced is 1182 *li.* 2 *s.* 9 *d.* 2 *q.* and that is the value of an Annuity of 75 *li. per Annum* for 50 years.

Queſtion 3. *What is* 20 li. per Annum *for* 40 *years worth, to be paid for all together at the end of the term of* 40 *years, at* 6 per Cent.

The Number in the Table againſt 20 years, (the half of 40,) is	3.207135
Which multiplied in it ſelf produceth the Number for 40 years,	10.285426
One Unite ſubtracted, and two Cyphers added, makes	9.28542600
One ſixth part (for the Rate) is	1.54757100
Which multiplied by the Annuity 20 *li.* the Product will be	3095.142000

Which reduced, is 3095 *li.* 3 *s. ferè*, for the value of the Annuity to be paid all together at the expiration of the 40 years.

And thus have I done with theſe Five Tables, and the particular Uſes of them, although many more Queſtions than theſe which I have inſtanced in may be reſolved by them: and indeed, I have been the briefer in them, becauſe by Table I. all Queſtions of the like nature may be reſolved without the other four; and for that reaſon I have added a Table of the ſame kind for the Rate of Intereſt now by Law eſtabliſhed, namely, 6 *li. per Cent. per Annum*, from 1 to 30 years, not only for whole years, but for halves, quarters, months, and days; and theſe Tables I ſhall exemplifie in the ſolution of Queſtions of divers kinds.

A TABLE *Shewing the Increase of One Pound, (or 20 Shillings,) at Six Pound* per Cent. per Annum *Compound Interest: Calculated to Years, Halves and Quarters of Years; and to 365 Days.*

Y.	*Q.*		*Y.*	*Q.*		*Y.*	*Q.*	
0	0	1.000000	10	0	1.790847	20	0	3.207135
	1	1.014674		1	1.817126		1	3.254196
	2	1.029563		2	1.843790		2	3.301948
	3	1.044670		3	1.870846		3	3.350400
1	0	1.060000	11	0	1.898298	21	0	3.399564
	1	1.075550		1	1.926154		1	3.449448
	2	1.091336		2	1.954418		2	3.500065
	3	1.107351		3	1.983096		3	3.551424
2	0	1.123600	12	0	2.012196	22	0	3.603537
	1	1.140087		1	2.041723		1	3.656415
	2	1.156817		2	2.071683		2	3.710069
	3	1.173792		3	2.102082		3	3.764509
3	0	1.191016	13	0	2.132928	23	0	3.819749
	1	1.208493		1	2.164226		1	3.875800
	2	1.226226		2	2.195984		2	3.932673
	3	1.244219		3	2.228207		3	4.990380
4	0	1.263477	14	0	2.260904	24	0	4.048934
	1	1.281002		1	2.294080		1	4.108348
	2	1.299799		2	2.327743		2	4.168633
	3	1.318872		3	2.361900		3	4.229803
5	0	1.338225	15	0	2.396558	25	0	4.291870
	1	1.357862		1	2.431725		1	4.354849
	2	1.377787		2	2.467407		2	4.418751
	3	1.398005		3	2.503614		3	4.483591
6	0	1.418319	16	0	2.540351	26	0	4.549383
	1	1.439334		1	2.577628		1	4.616139
	2	1.460455		2	2.615452		2	4.683876
	3	1.481885		3	2.653831		3	4.752607
7	0	1.503630	17	0	2.692773	27	0	4.822346
	1	1.525694		1	2.732286		1	4.893108
	2	1.548082		2	2.772379		2	4.964909
	3	1.570798		3	2.813061		3	5.037763
8	0	1.593848	18	0	2.854339	28	0	5.111686
	1	1.617236		1	2.896223		1	5.186695
	2	1.640967		2	2.938722		2	5.262803
	3	1.665046		3	2.981844		3	5.340029
9	0	1.689479	19	0	3.025599	29	0	5.418388
	1	1.714270		1	3.069996		1	5.497896
	2	1.739425		2	3.115045		2	5.578571
	3	1.764949		3	3.160755		3	5.660431
10	0	1.790847	20	0	3.207135	30	0	5.743491

Month		*Month*		*Month*	
1	1.004867	5	1.024575	9	1.044470
2	1.009753	6	1.029562	10	1.049755
3	1.014673	7	1.034574	11	1.054865
4	1.019612	8	1.039610	12	1.060000

The Increase of One Pound for any number of Dayes under 366.

Da.		Da.		Da.		Da.	
1	1.000180	56	1.008980	111	1.017878	166	1.026855
2	1.000319	57	1.009141	112	1.018040	167	1.027018
3	1.000479	58	1.009302	113	1.018203	168	1.027182
4	1.000639	59	1.009463	114	1.018365	169	1.027346
5	1.000798	60	1.009624	115	1.018528	170	1.027510
6	1.000958	61	1.009786	116	1.018691	171	1.027675
7	1.001118	62	1.009947	117	1.018853	172	1.027839
8	1.001278	63	1.010108	118	1.019016	173	1.028003
9	1.001438	64	1.010269	119	1.019179	174	1.028167
10	1.001598	65	1.010431	120	1.019341	175	1.028331
11	1.001757	66	1.010592	121	1.019504	176	1.028495
12	1.001917	67	1.010753	122	1.019667	177	1.028659
13	1.002077	68	1.010915	123	1.019830	178	1.028824
14	1.002237	69	1.011076	124	1.019992	179	1.028988
15	1.002397	70	1.011237	125	1.020155	180	1.029152
16	1.002557	71	1.011398	126	1.020318	181	1.029316
17	1.002717	72	1.011560	127	1.020481	182	1.029481
18	1.002878	73	1.011722	128	1.020644	183	1.029645
19	1.003038	74	1.011883	129	1.020807	184	1.029809
20	1.003198	75	1.012045	130	1.020970	185	1.029974
21	1.003358	76	1.012207	131	1.021133	186	1.030138
22	1.003518	77	1.012368	132	1.021296	187	1.030302
23	1.003678	78	1.012530	133	1.021459	188	1.030467
24	1.003839	79	1.012691	134	1.021622	189	1.030632
25	1.003998	80	1.012853	135	1.021785	190	1.030796
26	1.004159	81	1.013015	136	1.021948	191	1.030961
27	1.004320	82	1.013177	137	1.022112	192	1.031126
28	1.004480	83	1.013338	138	1.022275	193	1.031290
29	1.004640	84	1.013500	139	1.022438	194	1.031455
30	1.004801	85	1.013662	140	1.022601	195	1.031619
31	1.004961	86	1.013824	141	1.022765	196	1.031784
32	1.005121	87	1.013986	142	1.022928	197	1.031949
33	1.005282	88	1.014147	143	1.023091	198	1.032114
34	1.005442	89	1.014309	144	1.023254	199	1.032278
35	1.005603	90	1.014471	145	1.023418	200	1.032443
36	1.005764	91	1.014633	146	1.023581	201	1.032608
37	1.005924	92	1.014795	147	1.023745	202	1.032773
38	1.006085	93	1.014957	148	1.023908	203	1.032938
39	1.006245	94	1.015119	149	1.024072	204	1.033103
40	1.006406	95	1.015281	150	1.024235	205	1.033268
41	1.006567	96	1.015443	151	1.024399	206	1.033433
42	1.006727	97	1.015605	152	1.024562	207	1.033598
43	1.006888	98	1.015768	153	1.024726	208	1.033763
44	1.007049	99	1.015930	154	1.024889	209	1.033928
45	1.007209	100	1.016093	155	1.025053	210	1.034093
46	1.007370	101	1.016254	156	1.025217	211	1.034258
77	1.007531	102	1.016417	157	1.025380	212	1.034423
48	1.007692	103	1.016579	158	1.025544	213	1.034588
49	1.007853	104	1.016741	159	1.025708	214	1.034753
50	1.008014	105	1.116903	160	1.025871	215	1.034919
51	1.008175	106	1.017066	161	1.026035	216	1.035084
52	1.008336	107	1.017228	162	1.026199	217	1.035249
53	1.008497	108	1.017391	163	1.026363	218	1.035414
54	1.008658	109	1.017553	164	1.026527	219	1.035580
55	1.008818	110	1.017715	165	1.026691	220	1.035745

The Increase of One Pound for any number of Days under 366.

Da.		Da.		Da.		Da.	
221	1.035910	261	1.042546	301	1.049225	341	1.055946
222	1.036076	262	1.042713	302	1.049393	342	1.056115
223	1.036241	263	1.042879	303	1.049560	343	1.056284
224	1.036407	264	1.043046	304	1.049728	344	1.056452
225	1.036572	265	1.043212	305	1.049895	345	1.056621
226	1.036737	266	1.043379	306	1.050063	346	1.056790
227	1.036903	267	1.043545	307	1.050230	347	1.056958
228	1.037069	268	1.043712	308	1.050398	348	1.057127
229	1.037234	269	1.043879	309	1.050566	349	1.057296
230	1.037400	270	1.044045	310	1.050734	350	1.057465
231	1.037565	271	1.044212	311	1.050901	351	1.057633
232	1.037731	272	1.044379	312	1.051069	352	1.057802
233	1.037897	273	1.044545	313	1.051237	353	1.057971
234	1.038062	274	1.044712	314	1.051405	354	1.058140
235	1.038228	275	1.044879	315	1.051573	355	1.058309
236	1.038394	276	1.045046	316	1.051741	356	1.058478
237	1.038560	277	1.045213	317	1.051908	357	1.058647
238	1.038725	278	1.045380	318	1.052076	358	1.058816
239	1.038891	279	1.045546	319	1.052244	359	1.058985
240	1.039057	280	1.045713	320	1.052412	360	1.059154
241	1.039223	281	1.045880	321	1.052580	361	1.059323
242	1.039389	282	1.046047	322	1.052748	362	1.059492
243	1.039555	283	1.046214	323	1.052916	363	1.059661
244	1.039721	284	1.046381	324	1.053084	364	1.059830
245	1.039887	285	1.046548	325	1.053253	365	1.060000
246	1.040053	286	1.046715	326	1.053421		
247	1.040219	287	1.046883	327	1.053589		
248	1.040385	288	1.047050	328	1.053757		
249	1.040551	289	1.047217	329	1.053925		
250	1.040717	290	1.047384	330	1.054094		
251	1.040883	291	1.047551	331	1.054262		
252	1.041050	292	1.047719	332	1.054430		
253	1.041216	293	1.047886	333	1.054599		
254	1.041382	294	1.048053	334	1.054767		
255	1.041548	295	1.048220	335	1.054935		
256	1.041715	296	1.048388	336	1.055104		
257	1.041881	297	1.048555	337	1.055272		
258	1.042047	298	1.048723	338	1.055441		
259	1.042214	299	1.048890	339	1.055609		
260	1.042380	300	1.049057	340	1.055778		

§ VIII. *The Use of the foregoing* Table *of* Compound Intereſt, *at* 6 per Cent. *for Years, Quarters, Months; exemplified in the ſolution of ſeveral Queſtions of divers kinds.*

Queſtion 1. *What will* 136 li. 15 *s.* 6 *d. be augmented unto, being forborn* 20 *years at* 6 per Cent. *Compound Intereſt?*

The 136 *li.* 15 *s.* 6 *d.* muſt be reduced into a Decimal Fraction, (by the Rule for that purpoſe before given in the Reduction of Vulgar Fractions into Decimals,) and it will be 136.775.

This number being multiplied by 3.207135, (the number in the Table ſtanding againſt 20 years,) the Product will be 438.654: And this Product being reduced will be 438 *li.* 13 *s.* 1 *d.* 1 *q.* And ſo much will 136 *li.* 15 *s.* 6 *d.* be augmented unto in 20 years.

Queſtion 2. *What will* 250 li. *being forborn 9 years and an half, be augmented unto?*

Look in the Table for 9 years 2 quarters, in the left hand Column of the Table, and againſt it you ſhall find this Number 1.739425, which is the amount or increaſe of 1 *li.* in nine years and an half: Then,

As 1 *li.* is to 1.739425, ſo is 250 *li.* to 434.856250.

Which reduced, is 434 *li.* 17 *s.* 1 *d.* 2 *q.* And to ſo much will 250 *li.* be increaſed in 9years.

Queſtion 3. *What is* 240 li. *due to be paid ſeven years and a quarter hence, worth in preſent Money?*

Look for 7 years and a quarter in the firſt Column of the Table, and againſt it you have this number 1.525694. Then unto 240 add a competent number of Cyphers as 9, and then divide that number by 1.525694, the Quotient will be 157.305; which being reduced, is 157 *li.* 6 *s.* 1 *d.* 1 *q.* and that Summ will diſcharge the Debt of 240 *li.* payable 7 years and a quarter hence.

Queſtion 4. *What is* 40 li. per Annum *for* 10 *years worth in preſent Money?*

1. Seek the number in the Table ſtanding againſt 10 years, which is 1.790847, from which ſubtract one Integer, then will the remainder be .790847. which multiplied by 100 makes it .79084700.

2. Divide this number by the Rate of Intereſt (which is 6,) and the Quotient will be 13180782.

3. Multiply this Quotient by the Annuity, which is 40 *li.* the Product is 527231280

4. Divide this Product (adding Cyphers to it) by the (whole) tenth number in the Table, *viz.* by 1.790847, and the Quotient will be 294.403, which reduced, is 294 *li.* 8 *s.* 0 *d.* 3 *q.* and ſo much is 40 *li.* per Annum, worth in ready Money.

Queſtion 5. *What is* 232 li 10 *s.* per Annum, *for* 15 *years and an half, worth in preſent Money?*

1. Look in the Table for the 15 th. year and an half, where you ſhall find this number 2.467407, from which ſubtract one Integer, and it will be 1.467407; which multiply by 100, by adding two Cyphers to it, and it will be 1.46740700: Which divide by the Rate of Intereſt (namely by 6,) and the Quotient will be 24456783.

2. Multiply this Quotient by the Annuity, *viz.* by 232 *li.* 10 *s.* and the Product will be 56862020475.

3. Divide this Product (adding Cyphers to it) by the number againſt 15 years and an half, *viz.* 2.467407, and that Quotient will anſwer the Queſtion.

Which Quotient 2304.546 being reduced, is 2304 *li.* 10 *s.* 11 *d.* 1 *q.* and ſo much is an Annuity of 232 *li.* 10 *s. per Annum*, to continue 15 years and an half, worth in preſent money, at 6 *per Cent. per Annum*, Compound Intereſt.

Queſtion 6. *What is an Annuity of* 35 li. per Annum *worth for* 7 *years, the purchaſe money being forborn till the end of that term of* 7 *years at* 6 per Cent. *Compound Intereſt?*

1. Seek the number in the Table againſt the 7th year, which is 1.50363, from which ſubtract one Integer, and it is .50363, to which add two Cyphers, making it .5036300.

2. Divide this number by the Intereſt (*viz.* 6,) and the Quotient will be 839383.

3. Multiply this Quotient by the Annual Rent of the Annuity, (*viz.* 35,) and the Product will be 293.78405; which Product reduced, is 293 *li.* 15 *s.* 8 *d.* 1 *q.* And ſo much is the Annuity worth to be paid for at the end of 7 years.

Queſtion

Queſtion 7. *What Annuity to continue* 21 *years will* 500 li. *purchaſe, at* 6 per Cent. *Compound Intereſt?*

1. Seek the number ſtanding againſt the 12 years, which is 3.399563, from which ſubtract one Integer, and it will be 2.399563, and to it add two Cyphers, making it 2.39956300.

2. Divide this number by 6, (the Rate of Intereſt,) and the Quotient will be 39.992716, which Quotient with Cyphers added, muſt be divided by 3.399563, (the number ſtanding againſt 21 years in the Table, and the Quotient will be 11.764075.

3. Divide the purchaſe money 500 *li.* (adding Cyphers) by this Quotient, and the Quotient of this laſt Diviſion will be 42.502. which reduced, is 42 *li.* 10 *s.* 0 *d.* 2 *q.* and ſuch an Annuity will 500 *li.* purchaſe for for 21 years.

Queſtion 8. *What yearly payment to begin preſently, is equal to* 600 li. *due* 7 *years hence?*

1. Seek the number ſtanding againſt the 7th. year which is 1.503630, from it ſubtract one Intiger, and it is .503630; add two Cyphers to it, and it is .50363000: Divide this laſt number by the Rate of Intereſt *(viz.6.)* and the Quotient is 8393836.

This done, divide the 600 *li.* (adding Cyphers thereto) by the firſt Quotient 83938, (for the two laſt figures may be omitted, to ſhorten the Diviſion,) and this laſt Quotient will be 71.481, which being reduced, is 71 *li.* 9 *s.* 7 *d.* 2 *q.* So that 71 *li.* 9 *s.* 2 *d.* 3 *q.* being paid Annually will diſcharge 600 *li.* in 7 years.

Queſtion 9. *What will* 234 li. 15 s. 6 *d.* 2 *q. amount unto, being forborn* 35 *years and an half, at* 6 per Cent. *Compound Intereſt?*

Firſt you muſt reduce the 234 *li.* 15 *s.* 6 *d.* 2 *q.* into a Decimal Fraction, and it will be 234.777; multiply this by the number ſtanding againſt the 25 years and an half, which number is 4.418751, and the Product will be 937.420; which being reduced, is 1037 *li.* 8 *s.* 5 *d.* and ſo much will 234 *li.* 15 *s.* 6 *d.* 2 *q.* amount unto in 25 years.

Queſtion 10. *What will* 378 *li. amount unto, it being forborn* 7 *months, at* 6 per Cent?

The number in the Table anſwering to 7 months, is 1.034574; which multiplied by 378 *li.* the Product will be 391.068; which being reduced, is 391 *li.* 1 *s.* 4 *d.* 2 *q.* for the amount of 378 *li.* in 7 months.

Queſtion 11. *What will* 420 li. *amount unto, it being forborn* 104 *days, at* 6 per Cent?

The number in the Table ſtanding againſt 104 Days, is 1.016741; which multiply by 420, the Product will be 427.031220; which being reduced, is 427 *li.* 7 *d.* 2 *q.* and ſo much will 420 *li.* be augmented to in 104 Days.

Queſtion 12. *If* 210 li. *be forborn* 3 *years,* 3 *months, and* 24 *days; what will the amount thereof be in the whole, at the end of the Term?*

Seek the Decimal for 3 years and a quarter, which is 1.208493; alſo look for the Decimal for 24 days, which (without the Index,) is .003839; theſe two added together make 1.212332; which multiply by 210 *li.* the Product will be 254589720; which being reduced, is 254 *li.* 11 *s.* 9 *d.* 2 *q.* and ſo much will 210 *li.* be increaſed to in 3 years, 3 months, and 24 days.

§ IX. *The Uſe of the following* Tables *of the preſent worth of one Pound due after any number of years, under* 31; *in Tears, Quarters, Months and Days.*

The Uſe of theſe Tables will be beſt exemplified in reſolving of ſome Queſtions.

Queſtion 1. *If* 438 li. 13 s. 1 d. 1 q. *be due and payable* 20 *years hence, how much is the preſent worth thereof?*

The given ſumm being reduced into a Decimal, will be 438.6552; which being multiplied by .3118047, (the number ſtanding againſt the 20th. year in the Table.) The Product will be 136.772; which reduced, is 136 *li.* 15 *s.* 5 *d.* 2 *q.* and ſo much preſent money will ſatisfie a Debt of 438 *li.* 13 *s.* 1 *d.* 1 *q.* due and payable 20 years hence.

Queſtion 2. A *is to pay unto* B *at the end of* 6 *months* 500 li. *but do agree (upon preſent pay) to diſcount for the ſame, after the rate of* 6 per Cent. *what preſent money will diſcharge the Debt?*

Look in the Table for 6 months, againſt which ſtands this number .9712858; which multiply by 500, the Product will be 485.6429000; which reduced, is 485 *li.* 12 *s.* 10 *d.* 2 *q.* and ſo much preſent money will diſcharge the Debt of 500 *li.*

Queſtion 3. A *hath a Leaſe in reverſion, which at the expiration of* 7 *years, was valued worth* 1200 li. *which Leaſe* B *would purchaſe for preſent pay, rebating at* 6 per Cent. per Annum, *Compound Intereſt.*

There is no essential difference between this and the last preceeding Question: For it is no more than to find the value of 1200 *li.* not due till 7 annual revolutions be expired. Look therefore in the Table for 7 years, against which stands this number .6650571; and this number multiplied by 1200 *li.* the Product will be 798.0685200; which reduced, is 798 *li.* 1 *s.* 4 *d.* 2 *q.* and so much is the Lease in reversion worth.

Question 4. A *is to pay unto* B *a Legasie of* 1800 li. *at three several equal payments*, viz. 600 li. *at the end of* 6 *months*, 600 li. *more at the term of a year, and the last* 600 li. *at the end of* 18 *months.* B *desires the Legasie at one intire and present payment, which* A *consents to, rebating after the rate of* 6 per Cent. *Compound Interest; what summ of money will discharge the Legasie?*

First, Look in the Table for the Decimal of 6 months, which is .9712858, which multiplied by 600, (the first payment,) the Product will be 582.7714808; which reduced, is 582 *li.* 15 *s.* 5 *d.* 1 *q.*

Secondly, the Decimal for a year, or 12 months, is .9433962, which multiplied by 600 *li.* the Product will be 566.0377200; and that reduced, is 566 *li.* 0 *s.* 9 *d.*

Thirdly, The number for the last 6 months may be thus found:

The Decimal for a year is ——— .9433962
The Decimal for 2 years is ——— .8899964
These two multiplied together do produce ——— 839618666416

The Square Root of which number is .916307, which number multiplied by 600 *li.* the Product will be .549.784200; which reduced, is 549 *li.* 15 *s.* 8 *d.* 2 *q.*

These Three Products added together make .698.5934; which reduced, is 1698 *li.* 11 *s.* 10 *d.* 3 *q.* and so much will discharge the Legasie at one present payment.

Question 5. *What is the present worth of* 43 li. 9 s. 6 d. *due after the expiration of* 7 *months, worth in present money?*

This summ being reduced to a Decimal, is 43.45, and this multiplied by .966581, produceth 41.996; which reduced, is 41 *li.* 19 *s.* 11 *d.* 1 *q.* for the present worth.

Question 6. *What is the present worth of* 100 li. *after* 216 *days?*

The number standing against 216 days, is .9661053; this number being multiplied by 100 *li.* (which is done by adding of two Cyphers,) the Product will be .96.6105300; or reduced, 96 *li.* 12 *s.* 2 *d.* 2 *q.* the present worth.

A TA-

A TABLE *Shewing the Increaſe of One Pound, (or 20 Shillings,) due after any number of years under* 31. *at Six Pound* per Cent. per Annum *Compound Intereſt: Calculated for Years, Halves, Quarters, Months and Days.*

Y.	Q.	Preſent worth.
0	0	.0000000
	1	.9855383
	2	.9712858
	3	.9572394
1	0	.9433962
	1	.9297531
	2	.9163074
	3	.9030560
2	0	.8899964
	1	.8771256
	2	.8644409
	3	.8519397
3	0	.8396193
	1	.8274770
	2	.8155103
	3	.8037167
4	0	.7920936
	1	.7806387
	2	.7693493
	3	.7582233
5	0	.7472581
	1	.7364516
	2	.7258013
	3	.7153050
6	0	.7049605
	1	.6947656
	2	.6847182
	3	.6748160
7	0	.6650571
	1	.6554393
	2	.6459606
	3	.6366189
8	0	.6274123
	1	.6183389
	2	.6093967
	3	.6005839
9	0	.5918984
	1	.5833386
	2	.5749026
	3	.5665885
10	0	.5583947

Y.	Q.	Preſent worth.
10	0	.5583947
	1	.5503194
	2	.5423609
	3	.5345175
11	0	.5267875
	1	.5191693
	2	.5116612
	3	.5042618
12	0	.4969693
	1	.4897823
	2	.4826993
	3	.4757187
13	0	.4688390
	1	.4620588
	2	.4553767
	3	.4487912
14	0	.4423009
	1	.4359045
	2	.4296006
	3	.4233879
15	0	.4172650
	1	.4112307
	2	.4052836
	3	.3994226
16	0	.3936463
	1	.3879535
	2	.3823430
	3	.3768137
17	0	.3713644
	1	.3659939
	2	.3607010
	3	.3554847
18	0	.3503438
	1	.3452772
	2	.3402839
	3	.3353629
19	0	.3305130
	1	.3257332
	2	.3210226
	3	.3163801
20	0	.3118047

Y.	Q.	Preſent worth.
20	0	.3118047
	1	.3072955
	2	.3028515
	3	.2984718
21	0	.2941554
	1	.2899014
	2	.2857089
	3	.2815771
22	0	.2775051
	1	.2734919
	2	.2695367
	3	.2656388
23	0	.2617972
	1	.2580112
	2	.2542799
	3	.2506026
24	0	.2469785
	1	.2434068
	2	.2398867
	3	.2364176
25	0	.2329986
	1	.2296291
	2	.2263082
	3	.2230355
26	0	.2198100
	1	.2166312
	2	.2134983
	3	.2104108
27	0	.2073679
	1	.2043690
	2	.2014135
	3	.1985008
28	0	.1956301
	1	.1928010
	2	.1900128
	3	.1872648
29	0	.1845567
	1	.1818877
	2	.1792573
	3	.1766649
30	0	.1741101

Month	Preſent worth.
1	.9951560
2	.9903355
3	.9855383
4	.9807644

Month	Preſent worth.
5	.9760136
6	.9712858
7	.9665810
8	.9618988

Month	Preſent worth.
9	.9572394
10	.9528026
11	.9479884
12	.9433961

The Present worth of One Pound for any number of Days under 366.

Da.		Da.		Da.		Da.	
1	.9998404	56	.9911000	111	.9824359	166	.9738478
2	.9996808	57	.9909418	112	.9822791	167	.9736922
3	.9995212	58	.9907836	113	.9821223	168	.9735368
4	.9993616	59	.9906254	114	.9819656	169	.9733814
5	.9992021	60	.9904673	115	.9818088	170	.9732260
6	.9990426	61	.9903092	116	.9816521	171	.9730707
7	.9988831	62	.9901511	117	.9814954	172	.9729154
8	.9987237	63	.9899930	118	.9813387	173	.9727600
9	.9985643	64	.9898350	119	.9811821	174	.9726047
10	.9984048	65	.9896769	120	.9810254	175	.9724495
11	.9982455	66	.9895190	121	.9808688	176	.9722942
12	.9980861	67	.9893611	122	.9807123	177	.9721390
13	.9979268	68	.9892031	123	.9805557	178	.9719839
14	.9977675	69	.9890452	124	.9803992	179	.9718287
15	.9976083	70	.9888874	125	.9802427	180	.9716736
16	.9974490	71	.9887297	126	.9800862	181	.9715185
17	.9972898	72	.9883418	127	.9799298	182	.9713634
18	.9972306	73	.9884139	128	.9797733	183	.9712084
19	.9969714	74	.9882561	129	.9796169	184	.9710534
20	.9968123	75	.9880983	130	.9794606	185	.9808983
21	.9966532	76	.9879406	131	.9793042	186	.9707433
22	.9964941	77	.9877829	132	.9791479	187	.9705884
23	.9963350	78	.9876252	133	.9789916	188	.9704334
24	.9961759	79	.9874676	134	.9788353	189	.9702785
25	.9960169	80	.9873100	135	.9786792	190	.9701236
26	.9958579	81	.9871523	136	.9785228	191	.9699688
27	.9956990	82	.9869948	137	.9783666	192	.9698140
28	.9955400	83	.9868372	138	.9782105	193	.9696591
29	.9953810	84	.9866797	139	.9780543	194	.9695044
30	.9952222	85	.9865222	140	.9778982	195	.9693496
31	.9950633	86	.9863647	141	.9777421	196	.9691949
32	.9949045	87	.9862073	142	.9775860	197	.9690400
33	.9947457	88	.9860498	143	.9774300	198	.9688954
34	.9945869	89	.9858924	144	.9772739	199	.9687308
35	.9944282	90	.9857350	145	.9771179	200	.9685762
36	.9942694	91	.9855777	146	.9769620	201	.9684216
37	.9941107	92	.9854204	147	.9768060	202	.9682670
38	.9939520	93	.9852631	148	.9766500	203	.9681124
39	.9937934	94	.9851058	149	.9764942	204	.9679579
40	.9936347	95	.9849486	150	.9763383	205	.9678033
41	.9934760	96	.9847913	151	.9761824	206	.9676489
42	.9933175	97	.9846341	152	.9760266	207	.9674944
43	.9931590	98	.9844770	153	.9758708	208	.9673400
44	.9930004	99	.9843198	154	.9757150	209	.9671855
45	.9928419	100	.9841627	155	.9755593	210	.9670311
46	.9926834	101	.9840056	156	.9754036	211	.9668768
77	.9925250	102	.9838485	157	.9752479	212	.9667224
48	.9924665	103	.9836914	158	.9750922	213	.9665681
49	.9923081	104	.9835344	159	.9749366	214	.9664138
50	.9921497	105	.9833774	160	.9747809	215	.9662596
51	.9919914	106	.9832204	161	.9746253	216	.9661053
52	.9917330	107	.9830635	162	.9744697	217	.9659511
53	.9915747	108	.9829066	163	.9743143	218	.9657969
54	.9914165	109	.9827497	164	.9741587	219	.9656428
55	.9912582	110	.9825928	165	.9740032	220	.9654886

The Present worth of One Pound for any number of Days under 366.

Da.		Da.		Da.		Da.	
221	·9653345	261	·9591799	301	·9530843	341	·9470177
222	·9651804	262	·9590369	302	·9529322	342	·9468665
223	·9650263	263	·9588837	303	·9527800	343	·9467154
224	·9648723	264	·9587306	304	·9526280	344	·9465642
225	·9647183	265	·9585775	305	·9524759	345	·9464131
226	·9645643	266	·9584245	306	·9523239	346	·9462621
227	·9644103	267	·9582715	307	·9521719	347	·9561110
228	·9642563	268	·9581185	308	·9520199	348	·9459600
229	·9641024	269	·9579656	309	·9518679	349	·9458090
230	·9639485	270	·9578127	310	·9517160	350	·9456580
231	·9637946	271	·9576598	311	·9515640	351	·9455071
232	·9636408	272	·9575069	312	·9514121	352	·9453561
233	·9634870	273	·9573541	313	·9512603	353	·9452052
234	·9633332	274	·9572013	314	·9911084	354	·9450543
235	·9631794	275	·9570485	315	·9509566	355	·9449035
236	·9630256	276	·9568957	316	·9508048	356	·9447520
237	·9628719	277	·9567430	317	·9506530	357	·9446018
238	·9627182	278	·9565902	318	·9505013	358	·9444511
239	·9625645	279	·9564375	319	·9503495	359	·9443003
240	·9624109	280	·9562849	320	·9501978	360	·9441495
241	·9622573	281	·9561322	321	·9500462	361	·9439988
242	·9621037	282	·9559796	322	·9498945	362	·9438481
243	·9619500	283	·9558270	323	·9497429	363	·9436975
244	·9617963	284	·9556744	324	·9495913	364	·9435468
245	·9616430	285	·9555219	325	·9494397	365	·9433962
246	·9614895	286	·9553693	326	·9492881		
247	·9613360	287	·9552168	327	·9491366		
248	·9611825	288	·9550644	328	·9489851		
249	·9610291	289	·9549119	329	·9488336		
250	·9608757	290	·9547595	330	·9486822		
251	·9607223	291	·9546071	331	·9485307		
252	·9605690	292	·9544547	332	·9483793		
253	·9604157	293	·9543023	333	·9482279		
254	·9602623	294	·9541500	334	·9480766		
255	·9601091	295	·9539977	335	·9479251		
256	·9599558	296	·9538454	336	·9477739		
257	·9598026	297	·9536932	337	·9476226		
258	·9596494	298	·9535409	338	·9474713		
259	·9594962	299	·9533887	339	·9473201		
260	·9593430	300	·9532365	340	·9471689		

ARTI-

ARTIFICIAL ARITHMETICK, BOTH LOGARITHMETICAL AND LOGISTICAL.

PART III.

CHAP. I.
Of LOGARITHMS.

BEfore I come to ſhew how to work the Ordinary and Extraordinary Rules of Arithmetick by Logarithms, whereby Multiplication is performed by Addition, Diviſion by Subtraction, the Square Root extracted by Bipartition, the Cube Root by Tripartition, the Biquadrate Root by Quadrupartition, *&c.* it will be neceſſary, firſt, to give you the Definition and Nature; and ſecondly, the *Geneſis*, Fabrick, or Conſtruction of theſe artificial Numbers, called Logarithms.

§ I. *Of the* Definition *or Nature of* Logarithms.

What Arithmetical and Geometrical Proportion is, ſee in Vulgar Arithmetick, Part I. Chap. 8. *and in the Theorem of the ſecond Book following.*

Logarithms are borrowed Numbers, which differ among themſelves by Arithmetical Proportion; as the Numbers which borrow them, do differ by Geometrical Proportion. Thus in the firſt Column of the following Table, the Numbers in Geometrical Proportion are 1, 2, 4, 8, 16, *&c.* Now to theſe you may aſſume any other ſeries of Numbers, for their borrowed Numbers or Logarithms; ſuch as are in four other Columns, noted at the heads of them with A. B. C. D. and at the bottoms of them with L. for Logarithms (or borrowed Numbers) to them, in the firſt Column; provided, that the borrowed Numbers or Logarithms ſo aſſigned, do differ among themſelves in Arithmetical Proportion; as the Numbers in the firſt Column do by Geometrical Proportion.

	A	B	C	D
Geometrical Proportionals.				
1	1	5	5	35
2	2	6	8	32
4	3	7	11	29
8	4	8	14	26
16	5	9	17	23
32	6	10	20	20
64	7	11	23	17
128	8	12	26	14
256	9	13	29	11
512	10	14	32	8
1024	11	15	35	5
	L	L	L	L
	Arithmetical Proportionals			

Thus in the Column C, if you will appropriate 5 to be the Logarithm of 1, 8 of 2, 11 of 4, 14 will be the Logarithm of 8, and 35 the Logarithm 1024, and ſo on; for the Numbers in the Column C, *viz.* 5, 8, 11, 14, *&c.* do differ among themſelves in Arithmetical Proportion, (the common difference being 3,) as 1, 2, 4, 8, *&c.* (The Proportional Numbers unto which they are aſſigned,) do differ by Geometrical Proportion: And what is here ſaid of the Numbers in the Column C, is to be underſtood of thoſe in the three other Columns A, B, and D; or of any other Numbers which you ſhall aſſign for Logarithms, to any Rank of Numbers which are in Geometrical Proportion. And theſe Logarithms, or borrowed Numbers, you may propound to decreaſe downwards, as thoſe in the Column D, as well as upwards, as thoſe in the Column, A, B, and C. And from hence it will follow, That,

Any four Numbers Geometrically Proportional being given, the ſumm of the Logarithms *of the two mean (or middle Numbers) is equal to the Summ of the two extreme Numbers.*

So

So the four Geometrical Proportional Numbers, 2, 8, 16, and 64 in the first Columns, whose Logarithms in the Column C are 8, 14, 17, 23, I say, that the Summ of the Logarithms of the two means, *viz.* 14 and 17, which is 31, is equal to the Summ of the Logarithms of the two extreams, *viz.* 8 and 23, which is also 31, and for the Geometrical means, 8 and 16, being multiplied together, produce 128; and the two extreams 2 and 64, multiplied together, produce 128; also from whence it appears, that Logarithms perform that work by Addition, which common Numbers doth by Multiplication.

And from hence again it will follow, that if three Numbers are given, and a fourth Proportional Number to them be required; it may be found by Addition and Subtraction of the Logarithms of those Numbers: For, as in natural Numbers; if instead of multiplying the two means together, and dividing of the Product by the given Extream, the Quotient shall produce the fourth Proportional; so I say, that if you add the Logarithms of the two Extreams together, and from their Summ subtract the Logarithm of the given Extream; the remainder shall be the Logarithm of the fourth Proportional Number required:

Example. *Let the three Proportional Numbers be* 2, 16, *and* 64, *to which a fourth Proportional Number is required.*

If (as in natural Numbers) you multiply the two means 16 and 64, the Product will be 1024; which divided by 2, the given Extream, the Quotient will be 512; the fourth Proportional: But if (instead hereof)

To the Logarithm of 16, which is	17
You add the Logarithm of 64, which is	23
The Summ of them is	40
From which subtract the Logarithm of 2, which is	8
The Remainder is	32

Which is the Logarithm of 512, as in the foregoing Table.

§ II. *Of the* Genesis, Fabrick *and Tabular* Construction *of the* Logarithms.

The Construction of Logarithms consists in the framing of a Table of Logarithms, that is, a Table to contain the Logarithms of all Numbers, from 1, to 1000, 10000, 100000; or to any farther extent.

To frame such a Table, you must propound to your self a convenient Rank of Numbers, Geometrically Proportional from 1; and then assign unto them that kind of Logarithm, which may be aptest for use in Arithmetical Operations: And such are those you find in this Table; in which, in the Column A you have a Rank of Numbers, Geometrically Proportional from 1, and over against each of them in the Column B his respective Logarithm.

A	B
1	0.00000
10	1.00000
100	2.00000
1000	3.00000
10000	4.00000
100000	5.00000
1000000	6.00000
10000000	7.00000
100000000	8.00000
1000000000	9.00000
10000000000	10.00000
Proportional Numbers.	*Their Logar.*

So the Logarithm of 1, is 0.00000; the Logarithm of 10, is 1.00000; the Logarithm of 100, is 2.00000; *&c.* And this kind of Logarithm was that which was agreed upon at a conference held between the Inventor and Propagator of them, at the first; namely, *John Neper*, Baron of *Merchiston* in *Scotland*, and Mr. *Henry Briggs*, then Professor of Geometry in *Oxford*.

§ III. *What the* Characteristick (*or Index*) *of a* Logarithm *is.*

The first figure of each Logarithm, which is separated from the rest by a Point or Prick, is called the Characteristick, or *Indices* of the Logarithm.

So the Characteristick of the Logarithm of 1 is 0, of 10 1. of 100 2. of 1000. 3, *&c.* And these Characteristicks of the Logarithms increase by Unities; for the Characteristick of the Logarithm of 1 being 0, the Characteristick of Logarithm of 10 is 1, *&c.* also the Characteristick of the Logarithm of anysNumber comprehended betwixt any two of the Proportionals in the Table, differs not from the Characteristick of the Logarithm of the first of those two Proportional; as for Example.

The Characteristick of all Numbers between	1	and	10	is 0.
	10		100	1.
	100		1000	2.
	1000		10000	3.
	10000		100000	4.
	100000		1000000	5.

So that the Characteristick of any Logarithm is one Unite less than the Number of Places in the Geometrical Proportional, to which it is a Logarithm.

As in this Table you may observe, that 2, 4, and 8, (being numbers comprehended between 1 and 10;) their Logarithms have for their Characteristicks, 0.

C	D
1	0.00000
2	0.30102
4	0.60205
8	0.90308
16	1.20411
32	1.50514
64	1.80617
128	2.10720
256	2.40823
512	2.70926
1024	3.01029
The Geometrical Power.	*Their Logarithms. Propor.*

Also,

16, 32, 64 } Being Numbers between 10, and 100; have for their Characteristick. } 1. 1. 1.

Again,

128, 256, 512 } Being Numbers between 100, and 1000; have for their Characteristick, } 2. 2. 2.

And Lastly, 1024 being a number between 1000 and 10000, hath for its Characteristick 3, being one Unite less than the Number of Places in the Geometrical Proportional 1024, to which 3.01029 is a Logarithm: And the like is to be understood of all the Logarithms, comprehended between those Proportional Numbers of the former Table, noted with A and B.

The Logarithms of this kind, ought all of them to consist of an equal Number of Places for you may propose them to consist of either six Places, as those in the two foregoing Tables, A B and C D; or of seven, eight, ten, or fifteen Places, (as those in Mr. *Briggs*'s,) or as many as you please. But when you have determined of how many Places your Table shall consist, you must not alter your first resolution; as to make the Logarithm of one Number to consist of six places, and another of seven or eight.

§ IV. *Of the Arithmetical Complement of a* Logarithm.

The Arithmetical Complement of a Logarithm, is the remainder of the other part thereof, (besides the Characteristick) being deducted out of the Logarithm of 10.

So the Logarithm of 10, in the Table A B being 1.00000; if you subtract .60205 (the rest of the Logarithm of 4, besides the Characteristick in the Table C D,) the remainder is .39795; which is the Arithmetical Complement of the Logarithm of 4. In like manner .79589 is the Arithmetical Complement of 1.20411, the Logarithm of 16; and so of any other.

§ V. *Of the nature of* Logarithms, *in Multiplication, Division, and Proportional Numbers.*

Putting the Logarithm of 1 to be 0.00000 in Multiplication, the Summ of the Logarithms of the Multiplicand, and of the Multiplicator, is equal to the Logarithm of the Product: So if 64 were to be multiplied by 16, the Product will be 1024.

For as 1 is to 16; so is 64 to 1024.

So in the foregoing Table C D, the Logarithm of 16 is 1.20411; and the Logarithm of 64 is 1.80617, these two Logarithms added together, make 3.01028 which is the Logarithm of 1024; as in the Table C D: For the want of over-plus of an Unite or two in the last figure of the Logarithm produced, causeth no error in the work.

In Division the Summ of the Logarithm of the Divisor and Quotient is equal to the Logarithm of the Dividend; for, as the Divisor is to 1, so is the Dividend to the Quotient. Let it then be required to divide 1024, by 64; I say, that if from 3.01029 (the Logarithm of 1024, the Dividend,) you shall subtract 1.80617, (the Logarithm of 64, the Divisor,) the remainder 1.20412 shall be the Logarithm of 16 the Quotient, as by the Table C D, and the converse of the Rule last foregoing, is evident.

In

In any continued Rank of Numbers Geometrically Proportional from 1, the Logarithm of any one of them being divided by the Denomination of the Power which it poſſeſſeth in the ſame Rank; the Quotient ſhall give you the Logarithm of the Root of that Power.

Thus in the Rank of Proportional Numbers in the Table C D, where 2 being the Root or I. Power, 4 the Square or II. Power, 8 the Cube or III. Power, 16 the Biquadrate or IV. Power, 32 the V. Power, *&c.* I ſay, that if the Logarithm, of 4, 8, 16, 32, or of any of the other following Proportionals in that Rank, be divided by the Number of the Denomination the Power claimeth in that Rank, the Quotient ſhall be the Logarithm of 2, the Root.

Example: Let it be required to find the Root of the third Power in the Table. The third Power (or Cube) is 8, whoſe Logarithm is 0.90308; this divided by 3, (the Power) giveth 0.30102, for the Logarithm of the Root of the third Power, which is 2. Alſo, if it were required to find the Root of the tenth Power, which is 1024, whoſe Logarithm is 3.01029, which divide by 10, (the number of the Power,) and before it prefix a Cypher, (as its proper Characteriſtick,) the Quotient will be 0.301029, the Logarithm of 2, the Root, as before.

In any continued Rank of Numbers Geometrically proportional from 1, the Logarithm of the Root being multiplied by the Denomination of the Power, the Product ſhall be the Logarithm of the ſame Power.

Example. In the foregoing Table C D.

0.30102 the Log. of the Root 2, being multiplied by	2	produceth	0.60205	the Log. of the	II.
	3		0.90308		III.
	4		1.20411		IV.
	5		1.50514		V.
	6		1.80617		VI.
	7		2.10720		VII.

Power.

§ VI. *How to find the* Logarithms *of* Mean Numbers.

In the foregoing Table noted with *A* and *B*, the Logarithm of

1	being put to be	0.0000
10		1.0000
100		2.0000
1000		3.0000
10000		4.0000

In the next place it will be requiſite to find out the Logarithms of the mean (or intermediate) Numbers, ſituate amongſt (or between) thoſe Proportionals.

As of	2	3	4	which are mean Numbers between	1	and	10
	11	12	13		10		100
	101	102	103		100		1000

Which how to perform ſhall be taught by the following Rules and Examples.

The RULE.

Making choice of one of the Proportional Numbers in the Table A B, *you ſhall by a continued Extraction of the Square Root, create a Rank of continual mean Proportionals, betwixt that number and* 1; *in ſuch ſort, that the continual mean which comes neareſt to* 1, *may be a mixt number leſs than* 2, *and ſo near* 1, *that it may have ſix Cyphers before the ſignificant figures of the Numerator?*

Out of the Table A B take 10, the ſecond Proportional in that Table, and annex unto it a competent number of Cyphers (as 24.) Then extract the Square Root of that Number, which you find to be 3.162277660168; again, annex unto this Root 12 Cyphers, and working by that intire Number ſo ordered, as if it were a whole Number, extract the Root thereof, which you find to be 1.778279410038; and ſo proceeding ſucceſſively by a continued extraction, you produce 24 continual means betwixt 10 and 1, which you write down in the firſt Column of the following Table; in which you may obſerve the three laſt Numbers, marked with the Letters, G H L, namely theſe following,

1.000000548979
1.000000274489
1.000000137244

to be each of them mixt Numbers leſs than 2, and greater than 1; and alſo to have 6 Cyphers before the ſignificant figure of each of them, according to the tenure of the Rule laſt delivered.

	Continual Means.	*Their Logarithms.*	
	10.000000000000	1.000000000000	
A	3.162277660168	0.500000000000	D
B	1.778279410038	0.250000000000	
C	1.333521432163	0.125000000000	
	1.154781984689	0.062500000000	
	1.074607828321	0.031250000000	
	1.036632928437	0.015625000000	
	1.018151721718	0.007812500000	
	1.009035044841	0.003906250000	
	1.004507364254	0.001953125000	
	1.002251148292	0.000976562500	
	1.001124941399	0.000488281250	
	1.000562312602	0.000244140625	
	1.000281116787	0.000122070012	5
	1.000140548516	0.000061035150	7
	1.000070271789	0.000030517578	1
	1.000035135277	0.000015258789	0
	1.000017567484	0.000007629394	5
	1.000008783703	0.000003814697	2
	1.000004391842	0.000001907348	6
	1.000002195918	0.000000953674	3
	1.000001097958	0.000000476837	1
G	1.000000548979	0.000000238418	5 N
H	1.000000274489	0.000000119209	2 K
L	1.000000137244	0.000000059604	6 M

And here you are to take notice, that although you are directed to extract ſo many continual Means, that the laſt ſhould have but ſix figures before the ſignificant figures of the Numerator; yet you are alſo to underſtand, that this is meant (and is only neceſſary,) when the Table of Logarithms you intend to make, ſhall conſiſt only of ſix Figures or Places, as thoſe in the preceding Table A B and C D do. So that when you intend the number of Places in your Table of Logarithms, to conſiſt of eight, ten, twelve, or ſixteen, or any greater number of Places; you muſt produce ſo many continual Means, till the laſt of them may have as many Cyphers before the ſignificant figures of his Numerator, as the Logarithms of your intended Table ſhall have Places.

Having thus produced a ſufficient company of continual Means, annex unto them their proper Logarithms; by halving firſt the Logarithm of the number taken, and then ſucceſſively the Logarithms of the reſt.

As for Example 1.000000000000 being put for the Logarithm of 10, the number taken, the Logarithm 0.500000 *&c.* (marked with the Letter D, in the ſecond Column of this laſt Table,) which is the half of 10.000, *&c.* is the Logarithm of the Number A, (the Square Root of 10.) In like manner 0.25000 *&c.* being half of 0.5000 *&c.* is the Logarithm of the Number B, and 0.115000, *&c.* the Logarithm of the Number C, and ſo of all the reſt in their order; ſo that at the laſt, you have in the firſt Column of the Table 24 continual Means between 10 and 1, as is aforeſaid: And in the other Column, you have to each of thoſe continual Means his reſpective Logarithm.

When a number leſs than 2, and greater than 1 comes ſo near to 1, that it hath ſix Cyphers placed before the ſignicant figures of the Numerator; the firſt ſix ſignificant figures of the Numerator of ſuch a Number, and the firſt ſix ſignificant figures of the Numerator of his Square Root, leſſen themſelves like their Logarithms, that is, by halves.

This Rule is illuſtrated from the laſt foregoing Table, where in the ſecond Column of the Table, the number N being the Logarithm of the number G, ſay;

As the Logarithm K is half the Logarithm N,

So is 274489, (the firſt ſix ſignificant figures of the Numerator of the Number H,) are half 548979; the firſt ſix ſignificant figures of the Numerator of the number G.

And

And from hence it will follow, that any two Numbers of this kind being given, their Logarithms, and the ſignificant figures of their Numerators are Proportional.

Example: If the Numbers G and H be given, ſay,

As 548979, the ſignificant figures of the Numerator of the Number G,
Are to 274489, the ſignificant figures of the Numerator of the Number H;
So is 2384185 the Logarithm of the Number G,
To 1192092, the Logarithm of the Number H.

In the ſame manner, G and L being given;

As 548979, the Mean G,
Is to 137244, the Mean L;
So is 2384185, the Logarithm of the Number G,
To 596046, the Logarithm of the Number L.

And this Rule holds true in any other Numbers of this kind, though it be not one of the continual Means betwixt 10 and 1; for the ſignificant figures of the Numerator of any ſuch Number, bear the ſame proportion to his proper Logarithm, that the ſignificant figures of any of the Numbers marked by the Letters G H or L, bear to his.

By what hath beeen hitherto delivered, it is manifeſt that a number of this kind being given, the Logarithm thereof may be obtained as followeth:

The Proportion to find the Logarithms of Numbers.

As the ſignificant figures of the Numerator of any one of the Numbers (ſigned in the firſt Column of the laſt foregoing Table; by the Letters G H and L)
Are to his reſpective Logarithm;
So are the ſignificant figures of the Numerator of the Number given,
To the Logarithm of the ſame Number.

Example: Let the number 1.000000102130 be given, and the Logarithm thereof demanded; then ſay,

As 548979, the ſignificant figures of the Numerator of the Number G,
Are to 2384185, the Logarithm of the ſame Number G,
So are 102130, the ſignificant figures of the Numerator of the Number given;
To 443545, the Logarithm of the given Number.

Before which if you prefix eight Cyphers, that it may have as many places as the Logarithms of the laſt Table, (namely 14,) the true and entire Logarithm of 1.000000102130, the number given, is 0.000000443545; which is alſo the Logarithm demanded.

§ VII. *A general Rule to find the* Logarithm *of any Number Propounded.*

The firſt thing to be done, is to ſearch out ſo many continual Means, betwixt the ſame Number and 1, till the continual Mean that cometh neareſt 1, hath ſix Cyphers placed before the ſignificant figures of his Numerator. Again, this being done, you are (in the next place) to find the Logarithm of that continual Mean. And laſtly, by often doubling and re-doubling the Logarithm ſo found, (according to the Number of the continual Means produced) in concluſion you ſhall fall upon the Logarithm of the Number given.

Example: The number 2 being given, let the Logarithm thereof be required. Firſt, in imitation of that which is before taught, *viz.* to find the Logarithms of mean Numbers; you produce ſo many continual Means betwixt 2 and 1, till that which comes neareſt 1, hath ſix Cyphers before the ſignificant figures of the Numerator; which after twenty continual Extractions, you find to be 1.000000661036.

This continual Mean being thus found by the foregoing Rule, you find the Logarithm of the ſame, which is 0.000000287084 2; this Logarithm being doubled, will produce the Logarithm of the Square Proportional next, above 1.000000661036. And ſo doubling the Logarithm of each continual Mean, ſucceſſively one after another, according to the number of the Extractions, (namely 20 times,) at laſt you happen upon this Logarithm 0.301029602099 2, which is the Logarithm of 2, the Number given.

The whole Fabrick of the Work is evidently expreſſed, in this enſuing Table.

Continual Means.	*Their Logarithms.*
2.000000000000	0.301029960 20992
1.414213562373	0.150514801 0496
1.189207115002	0.075257400 5248
1.090507732665	0.037628700 2624
1.044273782432	0.018814350 1312
1.021897148656	0.009407175 0656
1.010889286052	0.004703587 5328
1.005429901113	0.002351793 7664
1.002711275050	0.001175896 8832
1.001354719892	0.000587948 4416
1.000677130693	0.000293974 2208
1.000338508052	0.000146987 1104
1.000169239705	0.000073493 5552
1.000084616274	0.000036746 7776
1.000042307241	0.000018373 3888
1.000021153396	0.000009186 6944
1.000010576642	0.000004593 3472
1.000005288307	0.000002296 6736
1.000002644150	0.000001148 3368
1.000001322074	0.000000574 1684
1.000000661036	0.000000287 0842

Having found this Logarithm of 2, which you see consists of 13 Places (besides the Index or Characteristick,) and that the intended Logarithms (such as are in the foregoing Table C D,) do consist but of five places besides the Characteristick or Index; you take only the Characteristick and the first five figures following it, (respecting the rest as superfluous;) and then at last the proper Logarithm of 2, the given number will be 0.30102, as in the fore-cited Table C D: And thus, as the Logarithm of 2 is found out, so may the Logarithm of any other number whatsoever be known; notwithstanding, the Logarithms of some few of the prime Numbers being by this means once discovered, the Logarithms of many other derivative numbers may be found out, without the trouble of such continued extraction of the Square Root; as is before intimated, and as this following Rule will direct.

§ VIII. *An easie way to find the Logarithms of Derivative Numbers.*

The *RULE.*

Having found the Logarithm of 2, *the Logarithm of* 5 *will be easily found; for, dividing* 10 *by* 2, *the Quotient will be* 5; *for (by the fourth Section hereof) the summ of the Logarithms of the Divisor and Quotient, is equal to the Logarithm of the Dividend: Therefore if you subtract* 0.30102, *(the Logarithm of* 2 *the Divisor,) from* 100000, *(the Logarithm of* 10 *the Dividend,) the Remainder will be* 0.69898; *which is the Logarithm of* 5 *(the Quotient:) For,*

The Logarithm of the Divisor 2 is —0.30102
The Logarithm of the Quotient 5 is —0.69898
The Logarithm of 10 the Dividend —1.00000

And again, (besides the Logarithm of 5,) with the like facility, you may find the Logarithms of any other Number, that may be made by the Multiplication or Division of these three Numbers; 5, 2, and 10, *viz.* Multiplication 4, 8, 16, 32, 64, *&c.* of the Numbers 25, 125, 625, *&c.* of the Numbers 20, 50, 100, 200, *&c.* by observing the Directions of the fourth Section hereof.

According to this method, the Table of Logarithms at the end of this Book is framed, and are carryed on to seven Places beside the Characteristick; in which Table you may observe, that the Column intituled *Num.* contains all Numbers from 1 successively to 10000, as they stand one after another in their natural Order: And in the other Column (signed at the top by *Logarithm,*) you have just against each Number his respective Logarithm.

The

The Table of Artificial Signs and Tangents at the end of this Book also, are the Logarithms of the natural Signs and Tangents, and are carryed on to seven Places also, beside the *Index*, or Characteristick, answerable to the Places of the Logarithms, for the orderly using of them together.

CHAP. II.

The general Use of the Table of Logarithms.

THe Fabrick and Tabular Construction of the Logarithms, being already largely treated of, and from thence the Table prepared, as at the end of the Book, which contains the Logarithms of all Numbers, naturally increasing from One to Ten-Thousand; I come now to shew the Uses thereof, which are principally two, *viz.*

1. *An Absolute Number, a Mixt Number, a Fraction (either Vulgar or Decimal) being proposed, to find the Logarithm thereunto belonging.*

2. *A Logarithm being given, to find the Absolute Number, (whether Whole, Broken, or Mixt,) unto which that Logarithm belongeth.*

Under these two Heads, there are several Cases and Considerations, all which shall be cleared by Examples, in these following Sections.

§ I. *How to find the Logarithm of any Absolute Number, under* 10000.

Seek the Number given in the first Column of the Table (under the word *Number*,) and right against it, in the Column next to the right hand (under *Logarithms*,) you shall find the Logarithm thereof.

Example. *Let* 3572 *be the Absolute Number given, and let the Logarithm thereof be required.*

Look for 3572 in the first Column, and right against it you shall find 3.5529114, which is the Logarithm of 3572. In like manner the Logarithm of 789 will be found to be 2.8970770; the Logarithm of 62 will be found 1.7923917; and the Logarithm of 7 will be 0.8450989; as is here seen.

The Logarithm of	7	is 0.8450980
	62	1.7923917
	789	2.8970770
	3572	3.5529114

And by this you may perceive, that if the Absolute Number given do consist of,

1	Figures, the Characteristick or *Index* thereof is,	0.
2		1.
3		2.
4		3.

Always an Unite less than the Number of Places in the given Number, as hath been before declared.

§ II. *How to find the* Logarithm *of any Absolute Number which consists of five Places.*

Although the Table at the end of this Book extends but to four Places, *viz.* to 9989, yet the Logarithm of any Number consisting of five, six, or seven Places, may be found thereby.

Example. *Let it be required to find the Logarithm of* 58476.

First find the Logarithm of 5847, (the four first figures of the given Number,) which is (without the Characteristick) .7669331; and because the given Number consists of 5 Places, the Characteristick must be 4, which place before .7669331 thus, 4.7669331: Now to find the part Proportional to be added to this Logarithm for the 6 which remains, subtract the Logarithm of 5848, *viz.* .7670074, from the Logarithm 5847, *viz.* .7669331; and you find the Difference to be 743. And then by the Rule of Proportion say,

As

As 10 (for the one figure remaining above the four, *viz.* 6.)
Is to 6, (the last figure of the Number given,)
So is 743, (the Difference between the Logarithm of 5847 and 5848,)
To 446.

Which 446, being added to 4.7669331, it makes 4.7669777; for the whole Logarithm of 58476.

§ III. *An Absolute Number, consisting of six Places, being propounded, to find the Logarithm thereof.*

Example. *Let* 864993, *be the Number given, whose Logarithm is required.*

First seek the Logarithm of 8649, (the four first figures) which is (without the *Index*) .9369659, the Logarithm of 8650 is .9370161; and their Difference is 502: Then say by Proportion,

As 100 (for the two figures remaining of the four, *viz.* 33,)
Is to 33 (the two last figures of the Number given;)
So is 501 (the Difference of the Logarithm of 8649, and 8650,)
To 166, *ferè*.

Which 166 being added to .9369659, (the Logarithm of 8649) makes it .9369825; before which, if you prefix 5 (for the Characteristick or *Index*,) it is 5.9369825, for the true Logarithm of 864933.

§ IV. *An* Absolute Number *consisting of seven Places being given, to find the Logarithm thereof.*

Example. *Let the Logarithm of this Number* 5423758 *be demanded.*

The Logarithm of 5423 is	.7342396
The Logarithm of 5424 is	.7343197
Their difference,	801

This done, say,

As 1000, (for the three last Figures, *viz.* 758,)
Is to 758, (the three last Figures,)
So is 801, (the difference of the Logarithms,)
To 607.

Whice added to .7342396, the Logarithm of 5423, it makes .7343003, before which if you prefix 6, for the Characteristick, (because the given Number consisted of seven Places,) the entire Logarithm of 5423758 will be 6.7343003.

§ V. *A Vulgar Fraction being given, to find the Logarithm thereof.*

The *RULE.*

Subtract the Logarithm of the Numerator from the Logarithm of the Denominator, the Remainder shall be the Logarithm of the Fraction.

Example. *Let the Logarithm of* $\frac{3}{4}$ *be required.*

The Logarithm of 3 is 0.4771212, the Logarithm of 4 is 0.6020600, from which the Logarithm of 3 being subtracted, there remains 0.1249388, the Logarithm of $\frac{3}{4}$. And so of any other, as in these Examples following, $\frac{125}{12357}$ and $\frac{125}{123572}$.

$\frac{125}{12357}$ { Logarithm of the Numerator, —2.0969100
Logarithm of the Denominator, —3.0993353
—1.0024253, the Logarithm of $\frac{125}{12357}$.

$\frac{125}{12572}$ { Logarithm of the Numerator, —2.0969100
Logarithm of the Denominator, —4.0994944
—2.0024944, the Logarithm of $\frac{125}{123572}$.

And here note, That the Logarithm of a Fraction is always defective; that is, the Value thereof is less than 0, or nothing; for the Logarithm of 1 being put 0.0000000, the Logarithm of $\frac{1}{2}$, which is less than 1, must needs be less than nothing; and by how much a Fraction approaches nearer to 1, by so much less is the quantity of his Logarithm, *& è contra*; because, as the Logarithms of Numbers that are greater than 1, increase *à nihilo ad infinitum* above 1, so the Logarithms of Numbers less than 1, increase

increaſe likwiſe *à nihilo ad infinitum* under 13 wherefore when you meet with ſuch a Logarithm, (to diſtinguiſh it from a perfect Logarithm,) prefix before it this mark —, as in the preceding and following Examples.

If theſe Decimal Fractions were propounded $\frac{25}{100}$ and $\frac{5}{100}$, (which may be written without their Denominators thus, .25 and .05, their Logarithms are —0.6020600 and —0.3010300.

The Logarithm of .25 is 1.3979400
The Logarithm of 100 is 2.0000000
—0.6020600, the Logarithm of $\frac{25}{100}$.

The Logarithm of .05 is 0.6989700
The Logarithm of 100 is 2.0000000
—1.3010300, the Logarithm of $\frac{5}{100}$.

When the Fraction propounded is a Decimal, the Logarithm thereof may be more eaſily found, thus:

The *RULE.*

Find the Logarithm of the Decimal given, (without the Characteriſtick,) as if it were a Whole Number; then take the Complement Arithmetical of that Logarithm, and place before it its proper Characteriſtick, which muſt conſiſt of ſo many Unites as there are Cyphers before the Decimal Fraction,) that Number ſo ordered ſhall be the Logarithm of the Decimal propoſed.

Example: *Let the Logarithm of this Decimal* .125 *be required.*

The Logarithm of .125 (beſides the Characteriſtick) is .0969100, which ſubtracted from the Logarithm of 10, which is 1.0000000, the Remainder will be .9030900; before which if you place the proper Characteriſtick 0, (becauſe the Decimal given had no Cyphers prefixed before it,) the entire Logarithm of .125 will be —0.9030900: So likewiſe the Logarithm of .0125 will be found to be —1.9030900, and the Logarithm of .00125 to be —2.9030900.

§ VI. *How to find the Logarithm of a Mixt Number.*

The *RULE.*

Reduce the Mixt Number into an Improper Fraction; then ſubtract the Logarithm of the Denominator out of the Logarithm of the Numerator, the Remainder is the Logarithm of the Mixt Number.

Example: *Let the Logarithm of* 125 $\frac{3}{4}$ *be demanded.*

This Mixt Number being reduced into an Improper Fraction will be $\frac{503}{4}$: The Logarithm of 503 the Numerator is 2.7015680, the Logarithm of the Denominator 4 is 0.6020600; this ſubtracted from 2.7015680, the remainder is 2.0995080, the Logarithm of the Mixt Number 125 $\frac{3}{4}$.

But if this Mixt Number were given in a Decimal Mixt Number, it would be 125.75, and then you may find the Logarithm thereof by this *RULE:*

Seek the Logarithm of the Number given, without the Characteriſtick, and place before it the proper Characteriſtick belonging to the whole part thereof, and that ſhall be the Logarithm of the Mixt Number.

Example: This Mixt Decimal 125.75 taken as a Whole Number, the Logarithm thereof (by § 2 hereof) will be (without the Characteriſtick,) .0995080; before which if you prefix 2, for the proper Characteriſtick for the Whole Number 125, the perfect Logarithm for the Mixt Decimal will be 2.0995080, as it was before when taken as an Improper Fraction.

Hitherto we have treated upon the firſt part of this Chapter, namely, to find the Logarithm of any Number, whole or broken; I proceed now to the ſecond part, which is to find out what Abſolute Number belongs to any given Logarithm, and whether that Number be a Whole or a Mixt Number.

§ VII. *A Perfect Logarithm being given to find what Abſolute Number it belongs unto.*

In the Table at the end of this Book the Abſolute Numbers which ſtand in the firſt Column proceed by natural ſucceſſion from 1 to 10000, by 1, 2, 3, *&c.* and againſt

every Number in the first Column, under [*Numb.*] you have his Logarithm in the second, under [*Logarithm.*]

Example: *Let* 2.4771212 *be a Logarithm given, and the Absolute Number thereunto belonging is required.*

Seek in the Table under [*Logarithm*] till you find 2.4771212, which when you have found look in the adjoyning Column towards the Left hand, and there (against it) you shall find 300, which is the absolute Number, unto which the Logarithm 2.4771212 belongeth. And so you shall find that the Absolute Number belonging to——

The Logarithm	0.8450980	is	7
	1.7923917		62
	2.8970770		789
	3.5529114		3572

And so of any other, so far as the Table at the end of this Book extendeth; that is, from One to Ten Thousand.

This Example, with the above Logarithms and their Absolute Numbers adjoyned, may suffice for such Absolute Numbers as the Table is capable to receive: But if a Logarithm shall be proposed without the verge of the Table, that is, any Logarithm whose Characteristick is above 3, as 4, 5, 6, *&c.* then some other means must be used to find the Absolute Number belonging to that given Logorithm.

Example 1. *Let this Logarithm* 4.7669777 *be given, and let the absolute Number thereunto belonging be required.*

You see by the Characteristick 4, that the absolute Number thereunto belonging must consist of five places, (whereas these Tables go but to four,) notwithstanding seek the neareſt Logarithm in the Table, which is less, and you will find it to be 3.7669331, which is the Logarithm of 5847; also seek the next Logarithm in the Table greater, which you will find to be 3.7670074, which is the Logarithm of 5848; these two Logarithms thus found set down in this order.

Logarithm given,	4.7669777	446 Diff.	5847
Next less,	3.7669331		
Next greater,	3.7670074	743 Diff.	5848

Then subtract the lesser Logarithm from the given Logarithm, and the difference will be 446, which is the Logarithm of 5847; also, take the difference between the lesser Logarithm and the greater, and you will find it to be 743: Then say by the Rule of Proportion:

As 743, the greater difference,
Is to 446, the lesser difference,
So is 10, for the one figure wanting in the Absolute Number to make it up five places.
To 6.

Which 6 is to be set on the Right hand after the lesser Number, *viz.* 5847, making it to be 58476; and that is the Absolute Number belonging to the given Logarithm 4.7669777.

Example 2. *Let the Absolute Number belonging to this Logarithm* 5.9369826 *be required.*

By the Characteristick 5 you see that the Absolute Number must consist of six places, wherefore (as before) find in the Table the neareſt greater, and the neareſt lesser Logarithm, finding their differences, and then work as before, only in the Proportion use 100 in this, for the 10 in the former Example, because this wants two places to be supplied, whereas the other wanted but one place.

Logarithm given,	5.9369826	167 Diff.	8649
Next lesser,	5.9367659		
Next greater,	3.9370161	502 Diff.	8650

The differences being thus taken, say,

As 502, the greater difference,
Is to 167, the lesser difference,
So is 100, for the two figures to be added,
To 33.

Which 33 is to be set to the lesser Number 8649, and it will make it 864933; which is the Absolute Number belonging to the given Logarithm 5.9369826.

§ VIII. *A Defective Logarithm being given, to find the respective Number (or Decimal Fraction) unto which it belongs.*

A Defective Logarithm is the Logarithm of a Number less than 1; now to find the correspondent Number of such a Logarithm, this is the *RULE:*

Take the Complement which the Logarithm given (besides the Characteristick) wants of the Logarithm of 10, *and seek what Number answers unto it; this done, place before those Figures for every Unite of the Characteristick of the Logarithm given a Cypher, and at last before all prefix a Point, those Figures so ordered is the Number you look for, which is always (in this Case) a Decimal Fraction.*

Example. *I demand the Number unto which this Logarithm* —0.9030900 *doth belong.*

The Arithmetical Complement of this Logarithm to the Logarithm of 10 (besides the Characteristick) is .0969100, and the Number against it is 125; now because the Characteristick of the Logarithm given was 0, prefix no Cypher before 125, but only a Point, to signifie it a Decimal Fraction; so the correspondent Number answering to the Logarithm given will be found to be .125, or $\frac{125}{1000}$.

But if the Logarithm given had been 1.9030900, its correspondent Number would have been .0125, because the Characteristick of the given Logarithm was 1.

So likewise the correspondent Number belonging to this Logarithm —1.301030, whose Complement is .69897; the Number standing against which Logarithm is 5, before which, because the given Logarithm had 1 for its Characteristick, place one Cypher, and before all a Point, thus .05, and that is the Correspondent Number answering to the Defective Logarithm given, —1.301030.

And in like manner the Number belonging to this Logarithm —2.602060 will be found (by the former Rule) to be .0025, *&c.*

CHAP. III.

MULTIPLICATION by LOGARITHMS.

BY the Construction of the Logarithms it doth plainly appear, that what Common Arithmetick (whether *Vulgar* or *Decimal*) performs by Multiplication and Division, this Artificial or Logarithmetical Arithmetick performs by Addition and Subtraction: wherefore passing by the two last mentioned Rules we begin this our Artificial Arithmetick with Multiplication.

In Multiplication by the Logarithms there are several Cases, as these following, *viz.*

To multiply
1. A Whole Number by a Whole Number.
2. A Whole Number by a Decimal Fraction.
3. A Whole Number and a Decimal Fraction, by a Whole Number and a Decimal Fraction.
4. A Decimal Fraction by a Decimal Fraction.
5. A Perfect Logarithm by a Defective Logarithm.

RULE I.

When the Logarithms given to be multiplied be both of one kind, that is, both perfect, *or both* defective *Logarithms, add them them together, their Summ is the Logarithm of the Product; and is always of the same kind with the Logarithms of the given Numbers, that is, perfect if they be perfect, or defective if they be defective.*

RULE II.

When the Logarithms of the Numbers given are of divers kinds, that is, one perfect, *and the other* defective, *subtract the lesser out of the greater, the Remainder is the Logarithm of the Product required; which Logarithm (in this Case) is always of the same kind with the greater Logarithm of the Numbers given.*

§I. *To multiply a Whole Number by Whole Number.*

Multiply 86 by 28.

Multiplicand	86	*its Logarithm*	1.9344984
Multiplier	28	*its Logarithm*	1.4471580
The Product	2408	*its Logarithm*	3.3816564

Multiply 376 by 24.

Multiplicand	376	*its Logarithm*	2.5751878
Multiplier	24	*its Logarithm*	1.3802112
The Product	9024	*its Logarithm*	3.9553990

Multiply 3712 by 16.

Multiplicand	3712	*its Logarithm*	4.5696079
Multiplier	16	*its Logarithm*	1.2041200
The Product	59392	*its Logarithm*	5.7737279

§II. *To multiply a Whole Number by a Decimal Fraction.*

Multiply 132 by 18.25.

Multiplicand	132	*its Logarithm*	2.1205739
Multiplier	18.25	*its Logarithm*	1.2612629
The Product	24.09	*its Logarithm*	3.3818368

Multiply 72 by 83.07.

Multiplicand	83.07	*its Logarithm*	1.9194442
Multiplier	72	*its Logarithm*	1.8573325
The Product	5981.01	*its Logarithm*	3.7767767

§III. *To multiply a Whole Number and a Decimal Fraction by a Whole Number and a Decimal Fraction.*

Multiply 17.268 by 4.5.

Multiplicand	17.268	*its Logarithm*	1.2372420
Multiplier	4.5	*its Logarithm*	0.6532125
The Product	77.706	*its Logarithm*	1.8904545

Multiply 22.2 by 7.05.

Multiplicand	22.2	*its Logarithm*	1.3463530
Multiplier	7.05	*its Logarithm*	0.8481891
The Product	156.51	*its Logarithm*	2.1945421

§IV. *To multiply a Decimal Fraction by a Decimal Fraction.*

Multiply .75 by .125.

Multiplicand	.75	*Log. Comp. Arith.*	—0.1249388
Multiplier	.125	*Log. Comp. Arith.*	—0.9030900
			—1.0280288
The Product	.09375	*Log. Comp. Arith.*	—0.9719712

Multiply .025 by .42.

Multiplicand	.025	*Log. Comp. Arith.*	—0.6020600
Multiplier	.42	*Log. Comp. Arith.*	—0.3768504
			—0.9789104
The Product	.01050	*Log. Comp. Arith.*	—0.0210896

§V. *To*

§ V. *To multiply a Perfect Logarithm, by a Defective Logarithm.*

Multiply 25 by .125.

Multiplicand	25.	*Log.*	1.3979400
Multiplier	.125	*Log. Com. Arith.*	—0.9030890
The Product	3.125	*Log.*	0.4949510

Multiply 25 by .00125.

Multiplicand	25.	*Log.*	1.3979400
Multiplier	.00125	*Log. Com. Arith.*	—2.9030900
			—1.5051500
The Product	.03125	*Log.*	.4948500

CHAP. IV.

DIVISION by LOGARITHMS.

IN Diviſion there are the ſame variety of Caſes as in Multiplication; and therefore, before I proceed to Examples, I ſhall lay down the Rules following.

RULE I.

In Diviſion by Logarithms, when the Dividend is greater than the Diviſor, the Logarithm of the Quotient is always a Perfect Logarithm; but if the Diviſor be the greater, then the Logarithm of the Quotient is a Defective Logarithm.

For (in Diviſion) after the Logarithm of the Quotient is diſcovered, there may yet remain ſome doubt, whether that Logarithm be a Perfect or Defective Logarithm: Therefore I have here prefixed this Rule, to the end this difficulty may be removed; for if you conſider the three Terms given in every Diviſion, you ſhall there find this Proportion:

As the Diviſor is to the Dividend,
So is 1 to the Quotient.

according to the obſervation before made, in the fifth Section of the firſt Chapter hereof; I ſay therefore, when the Dividend (which is the ſecond Term) is greater than the Diviſor (which is the firſt Term,) the Quotient (which is the fourth Term) muſt needs be greater than 1, (which is the third Term;) and therefore (in this Caſe) the Logarithm of the Quotient is always Perfect, *i. e.* the Logarithm of a Whole, or of a Mixt Number: On the contrary, when the Dividend is leſs than the Diviſor, the Quotient is leſs than 1. And therefore, in this Caſe, the Logarithm of the Quotient is always a Defective; *viz.* the Logarithm of a Fraction: This being premiſed, I ſhall proceed to Examples, obſerving this,

RULE II.

When the Logarithms of the numbers given be both of one kind, ſubtract the leſſer out of the greater; the reſidue or remainder is the Logarithm of the Quotient.

§ I. *To divide a Whole Number by a Whole Number, or a Defective by a Defective.*

1. Divide 94325 by 3947.

Dividend.	94325	*Logarithm*	4.9746268
Diviſor.	3947	*Logarithm*	3.5962671
Quotient.	23.898	*Logarithm*	1.3783597

2. Divide .3947 by .0753.

Dividend	.3947	*Log. Com. Arith.*	—0.4037329
Diviſor	.0573	*Log. Com. Arith.*	—1.2418454
Quotient	6.8883	*Log.*	0.8381135

In

In these two Examples you may observe that 1.3783597, and 0.8381135 the Logarithms of the two Quotients, are both of them Perfect, because the whole Numbers 94325, being the Dividend in the first Example, is greater than 3947 his Divisor. And the Fraction .3947, being likewise Dividend in the second Example, is greater than the Fraction .0573 the Divisor; according to the Rule before premised.

1. Divide 3947 by 94325.

Dividend	3947	*Logarithm*	3.5962671
Divisor	94325	*Logarithm*	4.9746268
Quotient	.041845	*Logarithm*	—1.3783597

2. Divide .0573 by .3947.

Dividend	.0573	*Log. Com. Arith.*	—1.2418454
Divisor	.3917	*Log. Com. Arith.*	—0.4037329
			—0.8381125
Quotient	.14517	*Log.*	.1618875

In these two Examples the Dividends being less than the Divisors, both the Logarithms found for the Quotients are Defective.

For here ——1.3783597, being the Logarithm found in the first Example, is (is by the seventh Section of *Chap.* 2.) the Logarithm of a Decimal .041845, and 0.8381125 being the Logarithm found in the second Example, is (by the same seventh Section,) the Logarithm of the Decimal .14517.

§ II. *To divide a perfect* Logarithm *by a Defective, and the contrary.*

RULE III.

When the Logarithms of the numbers given are of divers kinds, add them together; their Summ is the Logarithm of the Quotient.

Example. 3947 being given to be divided by .14517, the Quotient is 27188; for the Logarithm of 3947 is 3.596267, and the Logarithm of .14517 is 0.838112: These Logarithms being added together their Summ is 4.434379; which being a Perfect Logarithm (by the 7th Section before cited,) is the Logarithm of 27188, the Quotient required. But if .14517 were propounded to be divided by 3947, the Quotient would be found to be .000036781, by the Rule above. *Example.*

1 Divide 3947, by .14517.

Dividend	3947	*Log.*	3.596267
Divisor	.14517	*Log. Com. Arith.*	—8.838112
Quotient	27188	*Log.*	4.434379

2. Divide .14517 by 3947.

Dividend	.14517	*Log. Com. Arith.*	—0.838112
Divisor	3947.	*Log.*	3.596267
Quotient	.000036781	*Log.*	4.434379

In like manner 6.8883 being given to be divided by .3947, the Quotient will be 17.452; for the Logarithm of 6.8883 (by the 6th Section of *Chap.* 1.) is 0.838112, and the Logarithm of .3947 is —0.403733; these Logarithms being added together, their Summ is —1.241845, which is the Logarithm of 17.452 for the Quotient saught. But if .3947 had been to be divided by 6.8883, the Quotient would be .0573.

See these two Examples.

Dividend	6.8883	*Log.*	0.838112
Divisor	.3947	*Log. Com. Arith.*	—0.403733
Quotient	17.452	*Log.*	1.236845
Dividend	.3945	*Log. Com. Arith.*	—0.403733
Divisor	6.8883	*Log.*	0.838112
			—1.241845
Quotient			.758155

I have insisted the longer upon Multiplication and Division, (as being necessary to be well understood,) because I intend more brevity in the Rules following.

CHAP.

CHAP. V.

EXTRACTION of ROOTS by LOGARITHMS.

§ I. *A Number being given to find the Square Root thereof.*

The *RULE.*

Seek the Logarithm of the Number given, half that Logarithm is the Logarithm of the Square Root of that given Number.

Example 1. *Let* 56644 *be a Number given, whose Square Root is required.*

The Number given 56644 ——— Logarithm ——— 4.7531539
The half thereof ——— 2.3765769

Which is the Logarithm of 238, and that is the Square Root of 56644.

Example 2. *Let* 12572 *be given, and the Square Root thereof be required.*

The Logarithm of 12572 is ——— 4.0994044
The half thereof is ——— 2.0497022

Which is the Logarithm of 112.12, the Square Root sought.

§ II. *A Square Root being given, to find the Square Number belonging to that Root.*

The *RULE.*

Find the Logarithm of the Root given, which Logarithm multiply by 2, *the Product shall be the Logarithm of the Square Number required.*

Example: *Let* 238 *be a Square Root given, and let the Square Number thereof be required.*

The Logarithm of 238, is ——— 2.3765770
2
Which multiplied by 2 produceth ——— 4.7531540

Which is the Logarithm of 56644 the Square Number, of which 238 is the Root.

§ III. *A Number being given to find the Cube Root thereof.*

The *RULE.*

Seek the Logarithm of the given Number, one third part thereof is the Logarithm of the Cube Root of that Number.

Example 1. *Let* 97336 *be a Number given, and let the Cube Root thereof be required.*

The Logarithm of 97336, is ——— 4.9884073
One third part of which is ——— 1.6628024

Which is the Logarithm of 46, and that is the Cube Root of 97336.

Example 2. *Let* 79304 *be a Number given, and let the Cube Root thereof be required.*

The Logarithm of 79304, is ——— 4.8992951
One third part thereof is ——— 1.6330983

Which is the Logarithm of 42.963, and that is the Cube Root of the given Number 79304.

§ IV. *A Cube Root being given, to find the Cube Number thereunto belonging.*

The *RULE.*

Seek the Logarithm of the Root given, which being multiplied by 3, *will produce the Logarithm of the Cube Number sought.*

Example: *Let* 46 *be given, and let the Cube Root thereof be required.*

The Logarithm of 46 is ——— 1.6628024
3
That multiplied by 3 produces ——— 4.9884072

Which is the Logarithm of 97336, and that is the Cube Number of which 46 is the Root.

§ V. *The*

§ V. *A quick and easie way to extract any Root, out of the highest Powers and vastest Numbers, by the help of Logarithms.*

That the faculty of extracting all sorts of Roots, is necessary to an *Algebrist*; and that no method for extraction is so easie as that by Logarithms are Truths, as I take it, beyond all possibility of denial.

The only thing that allays their Excellency is, that the largest *Cannons* extant, profess not by any seekings, to give us the Logarithm of any Number to above 40 Millions. And this provision is so scanty, that without some farther invention, it will not furnish us with Logarithms for the Cube of 343: The Sur-solid of 34, the second Sur-solid of 13, or the third Sur-solid of 5; and I know no other that hath made it his business to assist us in this strait. But, by the help of the Rule following, any Root may be extracted, though the Number given exceed 40 Thousand Millions, of Millions, of Millions, 40 Thousand times taken: For this Number 4722366482869645213696, is much larger than so; yet by this Rule, any Root demanded may be truly and speedily extracted out of it.

The *RULE*.

Seek the Logarithm of seven, or (if possible,) eight of the leading figures of the Number given, and prefix to that Logarithm an Indices *or* Characteristick, *answerable to the Number of Places, in the whole given Number; (that is, one Unite less than the Number of its Places,) that Logarithm being divided by the* Index, *or* Exponent *of the Power, whose Root you seek, will give you the Logarithm of the Root sought.*

Example: I desired the Root of the ninth Power, (called ordinarily the Cubed Cube, though others call it otherways; I find the Logarithm of the seven leading Figures, 4722366 to be 67415963, and the *Index* to be prefixed 21, (because the whole Number consists of 22 Places.) This intire Logarithm (*Index* and all) being divided by 9, (the *Index* or *Exponant* of the Cubed Cube,) the Quotient will be 2.40823995: The Logarithm of 256, as here (by the work) is seen.

Indices 21 — *The seven leading Figures* 6741 5963

The Power 9) 21.67415963 (2.40823995

18
36
36
007
74
72
21
18
35
27
89
81
86
81
53
45
8

The Proof 21.67415963 = *Given Number.*

Now if any list to try it by *Multiplication*, he will find that 256 Cubified, makes 16777216; and that again Cubified, produceth the first great Number; *viz.* 4722366482869645213696.

But if you would find the Square Root out of this vast Number, you cannot go to it immediately, for the Logarithm 21.67415963 being halfed, gives 10.83707981

for

for the Logarithm of the Square Root: And this having 10 for its *Index*, ſhews that the Number to which it anſwereth, muſt needs conſiſt of 11 Places, which no *Canon* of Logarithms can come near. But this ſeeming difficulty is ſoon removed; for, if you ſeek the Root of any higher Power compounded of ſuch Power, as in this caſe, the ſquared Square, the Squared Cube, or Squaredly Squared Square; the Root primilary intended, is by *Multiplication* ſoon produced.

As for Example; if you divide the Logarithm 21.67415963 by 4, the *Index* of the Squared Square, it gives 5.41853990 the Logarithm of 262144; and that being the Squared Square's Root, (or the Square Root of the Square Root,) I Square, or multiply it by it ſelf, and it gives 68719476736, the Square Root of 4722366482869645621 3696; or if the ſaid Logarithm 21.67415963, had been divided by 6, (the *Index* of the Squared Cube,) the Quotient would have been 3.61235993, the Logarithm of 4096; and that multiplied Cubically (becauſe the Squared Cube Root is but the Cubick Root of the Square Root, it will give us again 68719476736 for the Square Root, as before. And by this kind of working you will find as followeth:

The Root	Square	789456762		28097.26898176
	Cube	9664382		213.00458
	Sur-ſolid	847288609443	is	243
	2 Sur-ſolid	4586471424		24
	7 Sur-ſolid	8388608		2

I would have done theſe at large, but for taking up too much room. One doubt however, I thought good to clear. Some may be unſatisfied how the Logarithm having 21 for its *Index*, and yet taking in only 7 of the leading figures, could be any thing commenſurate to the whole Number of 22 Places. But the truth is, that the 15 figures left out, do not bear ſo great a Proportion to the ſeven taken in, (conſidering their Places,) as a ſingle farthing to 10000 Pounds; which to any ſmatterer in the *Mathematicks* is thus demonſtrated.

1. If a ſeparating Line (or Point,) were made after the ſeven leading figures, making the Unites Place at the ſecond 6, and turning all the fifteen following into Decimal Fractions, there would be the ſame Proportion between the ſeven leaders, and followers that now there is: As in theſe two Numbers 35 and 3.5, the 5 primes in the latter bear the ſame Proportion to 3, that 5 in the former bears to 30, *viz.* both a ſixth part.

2. It is evident, that if a *Separatrix* were placed thus 4722366.4828696452 13696; and the number 4722366 counted *Pounds*, all the Fractions will ſcarce reach 9 *Shillings*, 8 *Pence*, or 464 *Farthings*.

3. If you divide 4722366 by 464, the Quotient will give you for every *Farthing* in the Fractions 10177 *Pounds*, 10 *Shillings*, 3 *Pence* in the Integers: So that if all theſe fifteen figures had been changed into Cyphers, (or other figures at Pleaſure,) due Places and *Index* being obſerved; there would have been no obſervable difference. And for this Particular let this ſuffice.

CHAP. VI.

PROPORTIONS wrought by LOGARITHMS.

§ I. *Three Numbers being given to find a Fourth, in a direct Proportion to them.*

LET the Three Numbers given be 24, 12, and 864: Then, as 24 is to 12; ſo is 864 to 432; to find which by the Logarithms this is the *RULE*:

Add the Logarithms of the ſecond and third Terms together, and from the Summ of them ſubtract the Logarithm of the firſt Term, the Remainder ſhall be the Logarithm of the fourth Term ſought.

As 24 the Firſt Term	Logarithm	1.3802112
Is to 12 the Second Term	Logarithm	1.0791812
So is 864 the Third Term	Logarithm	2.9365137
	The Summ	4.0156949
To 432 the fourth Proportional ſought	Logarithm	2.6354837

Note, Here it is to be obſerved, that the Rule here delivered for finding a fourth Proportional Number, holds only true, when the Logarithms of the Numbers given, are all Perfect, or all Defective: For when ſome be Perfect, and ſome Defective, the Operation will vary.

And this does plainly appear by what hath been already delivered in the ſeveral Caſes, in Multiplication and Diviſion by Logarithms in the two Preceding Chapters; and this Conſideration likewiſe ought to be had in all the following Rules wrought by Logarithms. This I thought good to intimate in this Place, becauſe I take no farther notice of it in the Problems following. Only take two or three Examples more of this kind; for the work of this Chapter is to perform that which is done by that Rule which is commonly called

The Golden RULE.

Example 1. *If* 7 *yards of Cloth coſt* 6.5 Pounds, *what ſhall* 30 *yards coſt?*

As 7 Yards	Logarithm	0.8450980
Is to 6.5 Pounds	Logarithm	1.8129133
So is 30 Yards	Logarithm	1.4771212
		2.2900345
To 278.571 Pounds, *i. e.* 278 *li.* 11 *s.* 5 *d.* 1 *q.*		2.4449365

This is the general and moſt conſentaneous way of working this Rule of Proportion, becauſe it uſeth Addition inſtead of Multiplication, and Diviſion inſtead of Subtraction: But the work may yet be ſomewhat Abbreviated, and performed by Addition only, obſerving this *RULE:*

If inſtead of the Logarithm of the firſt Term (which is to be ſubtracted) you take its Complement Arithmetical, and add it to the Logarithms of the two oher Terms, the Total (cuttitg off the firſt figure to the left hand) ſhall be the Logarithm of the fourth Proportional Term ſought,

As in the former Example.

As 7 Yards		9.1549020
Is to 6.5 Pounds,	Log. Com. Arith.	1.8129133
So is 30 Yards		1.4771212
To 278.571		*x*2.4449365

Which reduced, is 278 *li.* 11 *s.* 5 *d.*

Likewiſe, if two or more Logarithms be to be ſubtracted, you may add the Complements Arithmetical of every of them; deducting from the whole Summ ſo many Unites as there were Complements added.

Take two or three Examples more of this Rule.

As 5.25 is to 13.5; ſo is 85.75 to 212.8 *ferè.*

As 5.25	Log. Com. Arith.	7.2798407
Is to 13.5	Log.	2.1303337
So is 82.75	Log.	3.9177680
To 212.8 *ferè.*	Log. Summ	*x*3.3278424

Again,

As 24 is to is to 3, So is 228 to 28.5.

As 24	Log. Com. Arith.	8.6197888
Is to 3	Log.	0.4771212
So is 228	Log.	2.3579348
To 68.5	Log. Summ	*x*1.4548448

§ II. *Having three Numbers given to find a fourth in an Inverſe or Reciprocal Proportion to them.*

This is to work the reverſe or backward Rule of Three by Logarithms, and to perform it, this is the *RULE:*

Add the Logarithms of the firſt and ſecond Terms together, and from the Summ of them ſubtract the Logarithm of the third Term; the Remainder ſhall be the Logarithm of the fourth Term. Or,

Add the Complement Arithmetical of the third Term, to the Logarithms of the firſt and ſecond, and the Summ ſhall be the Logarithm of the fourth Term?

Example

Example. *Let the Question be, if 375 Men will build a Wall about a Town in 72 Days, how many Dayes will 133 Men be a building of the same Wall.*

As 375 Men	Log.	2.5740313
Are to 72 Days	Log.	1.8573325
	The Summ	4.4313638
So are 133 Men	Log.	2.1238516
To 203 Days	Log.	2.3075122

And so many Days will 133 Men be a building of the like Wall.

But if instead of the Logarithm of 133, you take the Complement Arithmetical, the work will stand thus:

As 375 Men	Log.	2.5740313
To 72 Days	Log.	1.8573325
So are 133 Men	Log. Com. Arith.	7.8761484
To 203 Days		12.3075122

§ III. *Having three Numbers given, to find a fourth in a duplicate Proportion to them.*

This Rule is to find a Proportion between Superficies and Superficies of the same kind, as the two former were between Lines and Lines: And to work this by Logarithms, this is the *RULE*:

Double the Difference of the Logarithms, which belong to the two Terms that have the same Denomination; then (if the first Term be lesser than the second) add that Difference (being doubled) to the Logarithm of the other Term, that Summ shall be the Logarithm of the fourth Term sought.

Thus the Superficial content of a Circle, whose Diametre is 14 Inches, being 154 Square Inches; the content of another Circle, whose Diametre is 28 Inches, will be found to be 616 Square Inches.

The Diametre 14	Log.	1.1461280
The Diametre 28	Log.	1.4471580
Difference of the Logarithms		.3010300
Difference doubled to be added		.6020600
The Content given 154	Log.	2.1875007
The Content required 616	Log.	2.7895607

But if the first Term be greater than the second, subtract the Difference doubled, from the Logarithm of the other Term, and the Remainder will be the Logarithm of the fourth Term sought.

Example.

The Diametre 28	Log.	1.4471580
The Diametre 14	Log.	1.1461280
Difference		.3010300
Difference doubled, to be subtracted		.6020600
The Content given, 616	Log.	2.7895607
The Content required, 154	Log.	2.1875607

§ IV. *Three Numbers being given, to find a fourth in a triplicated Proportion to them.*

This Rule is to find a Proportion between like Solids, as the preceding was between like Superficies, and to work this by Logarithms, this is the *RULE*:

Triple the Difference of the Logarithms of the two Terms that have the same Denomination: Then (if the first Term be lesser than the second) add that Difference (tripled) to the Logarithm of the other Term; so shall the Summ be the Logarithm of the fourth Proportional required.

Example: If a *Bullet* whose Diametre is 4 Inches do weigh 9 Pound, What shall another *Bullet* (of the same Mettle) weigh, whose Diametre is 8 Inches?

Diametre 4 Inches ——— Log. ——— 0.6020600
Diametre 8 Inches ——— Log. ——— 0.9030900
The Difference ——— .3010300
The Difference tripled to be added ——— .9030900
Weight given 9 Pound ——— Log. ——— 0.9542425
Weight required 72 Pound ——— Log. ——— 1.8573325

But if the first Term be greater than the second, subtract the Difference (tripled) from the Logarithm of the other Term; so shall the Difference be the fourth Proportional sought.

Example 1.

Diametre 8 ——— Log. ——— 0.9030900
Diametre 4 ——— Log. ——— 0.6020600
The Difference ——— .3010300
The Difference tripled subtracted ——— .9030900
Weight given 72 ——— Log. ——— 1.8573325
Weight required 9 ——— Log. ——— 0.9542425

§ V. *Between two Numbers given, to find a mean Proportional.*

A mean Proportional Number between two Numbers given, is such a Number, which multiplied in it self, the Square (or Product) thereof shall be equal to the Rectangle (or Product) of the two given Numbers multiplied one by the other. And to find such a Number by Logarithms, this is the *RULE*:

Add the Logarithms of the two given Numbers together, and divide the Summ of them by 2, so shall you have the Logarithm of the mean Proportional required.

The Numbers given, { 8 ——— Log. ——— 0.9030900
32 ——— Log. ——— 1.5051500
Their Summ 2.4082400
The half Summ 16 the Mean Proportion ——— Log. ——— 1.2041200

Example 2.

The Numbers given, { 1028 ——— Log. ——— 3.0102999
124 ——— Log. ——— 2.1072099
Their Summ 5.1175098
The half Summ, the Logarithm of 362.05 ——— 2.5587549

§ VI. *Between two Numbers given, to find a Mean Proportional.*

The *RULE.*

Add the Logarithms of the two given Numbers together, and take one third part thereof, and add it to the Logarithm of the least of the Numbers given; and that is the Logarithm of the lesser Mean. Also to this Logarithm add the same third part, and it shall give you the Logarithm of the other Mean Proportional required.

The Numbers given, { 8 ——— Log. ——— 0.9030900
64 ——— Log. ——— 1.8061800
Difference ——— .9030900
A third part of the Difference to be added ——— .3010300
The lesser Mean Proportional 16 ——— Log. ——— 1.2041200
The greater Mean Proportional 32 ——— Log. ——— 1.5051500

CHAP.

CHAP. VII.

ANATOCISM, or Compound Intereſt, by LOGARITHMS.

§ I. *A Summ of Money being forborn for a certain time, to find how much it will be increaſed to at the expiration of that time, accounting Intereſt upon Intereſt, at any rate propounded.*

The RULE.

From the Logarithm of 100 li. *with the rate of Intereſt propoſed, ſubtract the Logarithm of* 100 li. *then multiply the Difference by the Time propoſed, and to that ſumm add the Logarithm of the Principal, and that ſumm ſhall be the Logarithm of the Increaſe (both Principal and Intereſt) in that Time.*

Example 1. *If* 2735 li. *be forborn for* 11 *years, What will it be augmented unto in that Time, counting Intereſt upon Intereſt at* 8 per Cent. per Annum *for a year?*

108 *li.* The Principal and Intereſt of 100 *li.* for a year, the Logarith.	2.0334237
100 *li.* The Logarithm	2.0000000
Difference of theſe two Logarithms,	334237
11 Years, the Time given,	11
The Product of the Difference being multiplied by the Time	.3676607
The Logarithm of 2735 *li.* the Principal,	3.4369573
The ſumm of theſe two Logarithms,	3.8046180

Which is the Logarithm of 6377.1 *li.* that is 6377 *li.* and 2 *s. ferè*; and ſo much will 2738 *li.* be augmented unto, being forborn 11 years at 8 *per Cent.*

Example 2. *If* 50 li. 16 s. *be forborn for* 7 *years, Unto what Summ will it be increaſed, at* 8 li. per Cent?

108 *li.* The Principal and Intereſt of 100 *li.* for a year,	2.0334237
100 *li.* the Logarithm	2.0000000
The Difference,	334237
7 Years, the given Time,	7
The Product of the Difference multiplied by the Time,	.2339659
50 *li.* 16 *s.* (or 50. 8 *li.*) the Logarithm of the Principal,	1.7058637
The Logarithm of 87.061.	1.9398296

Which is 87 *li.* 1 *s.* 2 *d.* 3 *q. ferè*; and to ſo much will 50 *li.* 16 *s.* be increaſed, being forborn 7 years.

§ II. *A Summ of Money becoming due at a certain time to come, to find what it is worth in preſent Money.*

The RULE.

The Rule is the ſame with that in the foregoing Section; only, you muſt (in this caſe) ſubtract the Product of the Difference of the two firſt Logarithms multiplied by the Time out of the Logarithm of the Principal, and the Remainder will be the Logarithm of the preſent Money.

Example. *What is* 87 li. 1 s. 2 d. 3 q. *due to be paid* 7 *years hence worth in preſent Money, rebating at* 8 per Cent. *Compound Intereſt?*

108 *li.* The Principal and Intereſt of 100 *li.* for a year,	2.0334237
100 *li.* its Logarithm,	2.0000000
The Difference,	334237
7 Years the Time given,	7
Product of the Difference multiplied by the Time,	.2339659
The Money due 7 years hence, 87.061,	1.9398296
The Diff. the Logarithm of 50.8 *li.*	1.7058637

So that 50.8 *li.* (or 50 *li.* 16 *s.*) preſent Money will diſcharge a Debt of 87 *li.* 1 *s.* 2 *d.* 3 *q.* due 7 years hence.

§ III. *A*

§ III. *A Principal Summ of Money, and the Summ of the Principal and Intereſt together, with the Time, being given, to find the Rate of Intereſt.*

The RULE.

From the Logarithm of the Principal and Intereſt together ſubtract the Logarithm of the Principal, and divide their Difference by the Time; then add that Quotient to the Logarithm of 100; *ſo ſhall that ſumm be the Logarithm of* 100 li. *and the Intereſt of* 100 li. *for a year.*

Example. A *puts into the hands of* B 230 li. *with condition to receive in lieu thereof* 850 li. *at the expiration of* 21 *years, and in the interim to receive no Intereſt; the Queſtion is, At what rate of Intereſt doth* B *hold this Money of* A?

850 *li.* Principal and Intereſt, ———	Log. ——	2.9294189
230 *li.* The Principal only, ———	Log. ——	2.3617278
Their Difference, ———		5676911
Which divided by 21, the Time, the Quotient is ———		270329
100 *li.* The Logarithm, added, ———		2.0000000
The ſumm is the Logarithm of 106.42 *li.* ———		2.0270329

Which is 106 *li.* 8 *s.* 0 *d.* 2 *q.* ſo that *B* did hold this 230 *li.* of *A* for 21 years, at the rate of 6 *li.* 8 *s.* 0 *d.* 2 *q. per Cent.* Compound Intereſt.

§ IV. *An Annuity or yearly Rent being forborn a certain number of years; to find what the Arrearages thereof will amount unto, according to any Rate propounded.*

The RULE.

Diſcover the Principal that anſwers to the Rent or Annuity in Queſtion; then find unto what Summ that Principal will be augmented, (according to the given Rate,) at the end of the Term propounded; this done, ſubtract the ſame Principal out of the Summ, and the Remainder is the Summ of the Arrearages.

Example: *A Rent or Annuity of* 12 li. *a year, being forborn* 16 *year; what will the Arrearages thereof amount unto, they being conceived to increſe (as they grow due,) after the Rate of* 8 per Cent.

Firſt, To find the Principal that anſwers to 12 *li.* ſay thus, by the Rule of Proportion;

As 8 *li.* Is to 100 *li.*
So is 12 *li.* To 150 *li.*

Which 150 *li.* is the reſpective Principal of 12 *li.*

Secondly, (By the firſt Section hereof) you may find that 150 *li.* being forborn 16 years, will amount unto 513.89 *li.* that is 513 *li.* 17 *s.* 9 *d.* therefore deduct 150, (the correſpondent Principal of the Annuity given,) out of this 513 *li.* 17 *s.* 9 *d.* the Remainder will be 363 *li.* 17 *s.* 9 *d.* and that is the ſumm of the Arrearages required.

§ V. *A Tearly Rent or Annuity being propounded, to find what it is worth in preſent Money.*

The RULE.

You muſt firſt find what the Arrearages thereof will amount unto at the end of the term propounded, and then what thoſe Arrearages are worth in preſent Money, which ſhall likewiſe be the required Price or Value of the Rent or Annuity propounded.

Example 1. *What may a Man adventure to give for a Leaſe of* 12 li. per Annum, *that hath yet* 16 *years in being, and have* 8 li. per Cent. *for his Money?*

You may find (by the laſt Section) that the Arrearages of 12 *li. per Annum* being forborn 16 years, will amount unto 363 *li.* 17 *s.* 9 *d.* and you may find likewiſe (by the ſecond Section hereof) that the ſame ſumm, 363 *li.* 17 *s.* 9 *d.* is worth in preſent Money 106 *li.* 4 *s.* 2 *d.* wherefore you may conclude, that the value of the Leaſe propounded (at the rate of 8 *li. per Cent.*) is worth 106 *li.* 4 *s.* 2 *d.*

When the Term of the Annuity begins not preſently, but after certain years to come, find what the Arrearages forborn for all the Time are worth in preſent Money.

Example 2. *Thus in the Example laſt propoſed; If the Annuity of* 16 *years were not to begin till after the expiration of* 5 *years.*

In

In this Case you are to enquire what the Arrearages (*viz.* 363 *li.* 17 *s.* 9 *d.*) being forborn 21 years, are worth in present Money, which you shall find (by § 2. hereof) to be 72 *li.* 5 *s.* 9 *d.* and that will be the value of the Annuity required.

§ VI. *A Summ of Money being propounded to find what Annuity (to continue any Number of Years, and according to any Rate of Interest given,) that Summ of Money will buy.*

The RULE.

Take an Annuity at pleasure, and find the value of that Annuity in present Money; which done, work by the Rule of Proportion, as followeth, viz.

As the Value found,
Is to the Annuity taken,
So is the Summ given,
To the Annuity required.

Example: *What Annuity (to continue 16 years) will* 1205 li. *deserve, so that the Purchaser may have* 8 li. per Cent. *for his Money laid out.*

First, (at adventure) take 12 *li. per Annum* to continue 16 years, and you will find the value thereof in present Money (by the fifth Section of this Chapter) to be 106 *li.* 4 *s.* 2 *d.* or 106.21; then say,

If 106.21 *li.*
Will purchase 12 *li. per Annum* for 16 years,
What will 1205 *li.* purchase for the like time?
The Answer will be 171.39 *li.*

Which reduced is 171 *li.* 7 *s.* 9 *d.* so that 1205 *li.* will purchase an Annuity of 171 *li.* 7 *s.* 9. *per Annum* for 16 years, after the Rate of 8 *li. per Cent. per Annum*, Interest upon Interest.

CHAP. VIII.

MILITARY ORDERS by LOGARITHMS.

§ I. *Any Number of Souldiers being given, to find what Number of Men must be placed in Rank and File, to make of these Souldiers a Square Battail, that is, to have an equal Number both in Rank and File.*

The RULE.

HAlf the Logarithm of the number of Souldiers given, will be the Logarithm of the number of Men that must be placed both in Rank and File, to make a square Battail.

Example: *Let there be* 533 *Souldiers to be disposed of in a square Battail; How many must be placed in Rank and File?*

The Loghrithm of 533 is ——— 2.726727 2
The half whereof is ——— 1.363 3636

Which is the Logarithm of 23 neareſt; and so many must be placed in Rank, and as many in File, to make a square Battail, which will consist but of 529 Men, which is less than the given Number by 4 Men, which may be disposed of for Scouts, Centinels, Pioneers, or other Offices; for in Military Affairs Fractions are not to be regarded.

§ II. *Any Number of Souldiers being propounded, to order them into a double Battail of Men, which may have twice as many in Rank as in File.*

The RULE.

Take half the Logarithm of half the number of Souldiers given; the Number answering to that Logarithm shall be the Number of Souldiers in File, and that number doubled, will be the number to be in Rank.

Example: *If* 1342 *Souldiers were thus to be disposed of, How many must there be in File, and how many in Rank?*

Half

Half the number of Souldiers given is 671, whose Logarithm is ——— 2.8267225
The half whereof is ——— 1.4133612

Which is the Logarithm of (neareſt) 26; and ſo many muſt be placed in File, and 52 in Front; for 52 multiplied by 26, produceth 1352, which is 10 more than the Number given, and thoſe muſt be ſupplied, Or elſe if you place but 25 in File, and 50 in Rank, then there will be an overplus of 92 Men, which muſt be diſpoſed of as before.

§ III. *Any Number of Souldiers given, to order them into a Quadruple Battail, that ſhall have four times as many in Rank as in File.*

The RULE.

Half the Logarithm of one quarter of the number of Souldiers given, ſhall be the Logarithm of the number of Souldiers which ſhall be in File, and that number multiplied by 4, ſhall be the number in Rank.

Example: *Let 2048 Souldiers be given to be thus diſpoſed of; How many muſt be placed in File, and how many in Rank?*

One quarter of 2048 is 512, whoſe Logarithm is ——— 2.7092699
The half Logarithm is ——— 1.3546849

Which is the Logarithm of (neareſt) 22, and ſo many Souldiers muſt be in File, and four times as many, which is 88, muſt be in Rank; for 88 multiplied by 22, the Product is 1936, which wants of the number given 112 Men, which muſt otherwiſe be diſpoſed of.

§ IV. *Any number of Souldiers being given, to make Three ſquare Battails of them, that each Battail ſhall have a like number of Men in Rank and File.*

The RULE.

Half the Logarithm of one third part of the given number of Souldiers ſhall be the Logarithm of the number of Souldiers which muſt be in Rank and File.

Example: *Let 768 Souldiers be thus to be diſpoſed of; How many muſt be in Rank and File?*

The third part of 768 is 256, whoſe Logarithm is ——— 2.4082400
Half thereof is ——— 1.2041200

Which is the Logarithm of 16, and ſo many muſt be both in Rank and File in each Battail; for 16 multiplied by 16 produceth 256, and ſo many Souldiers will there be in each Battail; and three times 256 is 768, equal to the number of Souldiers given.

§ V. *Any number of Souldiers being given, how to order them in Rank and File, according to the Reaſon of any two given Numbers.*

The RULE.

To the Logarithm of the given number of Souldiers add the Logarithm of the proportional Number given for the Rank; then, if out of their Aggregate you ſubtract the Logarithm of the proportional Number given for the File, half the Remainder is the Logarithm of the Number of Men to be placed in Rank; which Logarithm being ſubtracted out of the Logarithm of the whole Number of Souldiers, the Remainder will be the Logarithm of the Number of Souldiers to be placed in File.

Example: *Let 2500 Souldiers be given to be thus ordered, ſo that the Number of Men placed in File may bear ſuch proportion to thoſe in Rank as 5 bears to 12.*

Firſt, Say by the Rule of Proportion,

As 5 (for the Rank,)
Is to 12 (for the File,)
So is 2500 (the whole Number of Souldiers given,)
To 6000, whoſe Square Root is 77, (neareſt,) for the number in Rank.

See the Work.

The Summ of the Logarithms of 2500 the number of Souldiers, and 12 the porportional numb. in File, (*viz.* 3.3979400 and 1.0791812,) is } 4.4771212
0.6989700, Log. of 5, the proportional number in Rank ſubtracted, reſts 3.7781512
The half of which is the Logarithm of 77, the number to be in Rank.

And this Log. ſubtracted from the Log. of 2500, there remains ——— 1.5088644
Which is the Logarithm of 32; and ſo many Men muſt be placed in File.

§ VI. *Any*

§ VI. *Any Number of Souldiers being given, together with their Diſtance in Rank and File, how to order thoſe Souldiers into a ſquare Battail of Ground.*

The RULE.

Add the Logarithm of the Diſtance in Rank to the Logarithm of the Number of Souldiers, and from their Summ ſubtract the Logarithm of their Diſtance in File; half that Remainder is the Logarithm of the Number of Men to be placed in File: And this Logarithm ſubtracted out of the Logarithm of the whole Number of Souldiers, the Remainder ſhall be the Logarithm of the Number to be placed in Rank.

Example: 2500 *Men are to be ſo marſhalled into a ſquare Battail of Ground, ſo that their Diſtance in File may be* 7 *Foot, and in Rank* 3 *Foot, and the Ground they ſtand upon may be juſt ſquare.*

The Proportion will be,

As 7 (the Diſtance in File,)
Is to 3 (the Diſtance in Rank,)
So is 2500 (the whole Number of Souldiers,)
To 1071, whoſe Square Root (neareſt) is 32; the Number of Men to be placed in File.

Now if from the Logarithm of 2500, you ſubtract the Logarithm of the Number of Men in File, the Remainder ſhall be the Logarithm of the Men in Rank, namely 76 (neareſt.)

See the Work.

The Logarithm of 2500, the whole Number of Men, is	3.3979400
The Logarithm of the Diſtance in Rank 3 Foot, is	0.4771212
Which added together, their ſumm is	3.8750612
From which ſubtract the Logarithm of 7 Foot, the Diſtance in File,	0.8450980
The Remainder is	3.0299632
Half this Logarithm is	1.5149816

Which is the Logarithm of 32, (neareſt,) for the Number of Men in File.

Again, the ſame Logarithm of 32,	1.5149816
Subtracted from the Logarithm of 2500, there will remain,	1.8829584

Which is the Logarithm of 76, (neareſt;) and ſo many Men muſt be placed in Rank.

Note. This Order is termed a Square Battail, as well as that in the firſt Section of this Chapter, and yet there is much difference between them; for in a Square Battail of Men the Number of Rank and File are equal, although the Ground whereon they ſtand be not juſt ſquare: On the contrary, In a ſquare Battail of Ground it is neceſſary that the Place whereon they ſtand ſhould be juſt Square, although the Number of men in Rank be not equal to thoſe in File; and therefore the French (as well as we) do call that Order, *Bataillon quarre d'hommes*, a ſquare Battail of Men; and this Order they call *Bataillon quarre de Terrain*, a ſquare Battail of Ground.

§ VII. *Any Number of Souldiers being propoſed, together with the Diſtance in Rank and File, to order them in Rank and File, according to the Reaſon of any two Numbers given.*

The RULE.

Add the Logarithm of the whole Number of Souldiers and the Logarithms of the proportional Numbers in Rank and File together; from that Summ ſubtract the Logarithm of the diſtance in Rank, and of the proportional Number given for the File; this done, half that Remainder ſhall be the Number of Men to be placed in Rank: Which Logarithm being ſubtracted out of the Logarithm of the whole Number of Souldiers, that Remainder ſhall be the Number of Men to be placed in File.

Example: *There are* 12750 *Souldiers ſo to be ordered, that the Diſtance in File may be* 7 *Foot, and the Diſtance in Rank* 3 *Foot; ſo that the length of the Ground for the Rank may bear the ſame proportion to the length of the Ground for the File that* 12 *bears to* 5.

See the Work.

12750, (the Number of Souldiers) — Log.	4.1055102
12, (the proportional Number for Rank) — Log.	1.0791812
7, (the Diſtance in File,) — Log.	0.8450980
The Summ of theſe Three,	6.0297894

3, (the distance in Rank,)——————————Log. 0.4771212
5, (the Proportional Number for Rank,)——————————Log. 0.6989700
The Summ of these two, to be subtracted from the Summ of the other three——————————} 1.1760912
The Remainder is——————————4.8536982
The half whereof——————————2.4268491

Which is the Logarithm of 276, the Number of Men to be placed in Rank.

And the same Logarithm of 276 being subtracted from the Logarithm of the whole Number of Souldiers, there will remain 16786611, which is the Logarithm of 47. And so many Souldiers must be placed in File.

CHAP. IX.

Of LOGISTICAL ARITHMETICK;

SHEWING

The Description, Construction, and Use of the Table of Logistical Logarithms, in Multiplication, Division, and the Rule of Proportion, in Astronomical Fractions, Sexagenary.

THE Table of Logistical Logarithms, which I now come to shew the Description, Construction, and Use of, is placed at the end of the Book, next after the ten Chiliads of Logarithms; and is of excellent Use, in finding of the part Proportional in Astronomical Tables Sexagenary, as shall appear hereafter.

§ I. *A Description of the* TABLE.

The Table consisteth of 12 Pages, and in each Page are 13 Columns; at the head of each Page, you have (in every second Column) all Numbers from 1 to 60; (the Number of Minutes of Motion in one Degree, in large figures, and under them you have the Number of Seconds which are contained in so many Minutes as are over that Column: As, in the fourth Column of the first Page of the Table, under Motion 1, you have 60; and so many seconds are in one Minute: Also at the Head of the 12th Column of the first Page of the Table, you have Motion 5, and under it 300; which are the Number of Seconds in 5 Minutes.

And so under { 11 / 36 / 51 } Minutes you have { 660 / 2160 / 3060 } Seconds, and so of any other.

Then in the first Column of each Page of the Table towards the left hand, you have Seconds of Motions, beginning at 1 Second, and proceeding downwards to 60 Seconds, or one Minute of Motion at the bottom.

Again, for Time:

At the head of each Page of the Table you have (in every second Column) all Numbers from 1 to 24 (the Number of Hours in a Natural Day) in larger figures, and under the Hour in the same Column, you have the Minutes of each Hour, from 1 Minute to 60 Minutes; so that 24 Hours of Time will be at the end of the Table, where 60 Minutes of motion also is: The same Logistical Logarithm, *viz.* 1000000 serving for both 60 Minutes of Motion, and 24 Hours of Time.

§ II. *To find the Logistical Logarithm belonging to any Number of Minutes, and Seconds of Motion of Time.*

1. For Motion, one Degree, or 60 Minutes being the Integer.

Example 1. *Let it be required to find the Logistical Logarithm belonging to* 5 *Minutes* 25 *Seconds of Motion.*

Look in the Head of the Table for Motion 5, (which you will find in the 12th Column of the first Page of the Table, and look for 25 Seconds in the first Column of the

the Table towards the left hand, and under 5 at the Head, and 25 in the Side, you ſhall find this Number 895559, which is the Logiſtical Logarithm anſwering to 5 Minutes, 25 Seconds of Motion.

Example 2. *Let it be required to find the Logiſtical Logarithm belonging to 23 Minutes, 35 Seconds of Motion.*

Seek 23 Minutes of Motion in the Head of the Table, among the great figures, and 35 Seconds in the firſt Column of that Page towards the left hand, and right againſt 35; and under Motion 23, you ſhall find this Number 959445, which is the Logiſtical Logarithm of 23 Minutes, 35 Seconds of Motion.

2. For Time, 24 Hours or a natural Day being the Integer.

Example 3. *Let it be required to find the Logiſtical Logarithm, belonging to two Hours, and 10 Minutes of Time.*

Look for two Hours in the Head of the Table, *viz.* H 2, among the great figures, (which you ſhall find in the Head of the laſt Column of the firſt Page of the Title,) and 10 Minutes in the ſame Column under 2 Hours, againſt which towards the left hand you ſhall find 895559; which is the Logiſtical Logarithm belonging to 2 Hours, and 10 Minutes, and is the ſame with the Logiſtical Logarithm of 5 Minutes, 25 Seconds, found as in the firſt Example.

Example 4. *Let it be required to find the Logiſtical Logarithm, belonging to 9 Hours, 26 Minutes of Time.*

Look for 9 Hours in the Head of the Table, and for 26 Minutes in the ſame Column under H 9, againſt which 26 Minutes towards the left Hand you ſhall find this Number 959445, which is the Logiſtical Logarithm belonging to 9 Hours, 26 Minutes of Time; and is the ſame with the Logiſtical Logarithm of 23 Minutes, 25 Seconds of Motion before found, as in the ſecond Example.

	M. *S.*		*Minutes* *S.*		*Logiſ. Log.*
And in this manner you	8—18		480—18		914039
may find the Logiſtical	12—21	or	1260—21	to be	955125
Logarithm anſwering to	24—43		2640—43		987232

Alſo,

	H. *M.*		
	2—35		903196
The Logiſt. Log. anſwering to	9—32	will be found to be	959893
	20—17		992693

And for that in the Uſe of this Table of Logiſtical Logarithms, ſometimes we may have occaſion to uſe the Logiſtical Logarithms of Parts, greater than a Degree, or 60 Minutes, there is therefore added another Page, extending the Table to 12 Minutes of Motion farther, namely, to 1 Degree, 12 Minutes of Motion; by which laſt Page of the Table, you may find the Logiſtical Logarithm belonging to 1 Degree, 4 Minutes, 40 Seconds to be .1003254, and that to 1 Degree, 9 Minutes, 45 Seconds to be 1006538, *&c.* Let this ſuffice for the Diſcription of this Table. I ſhall now ſhew,

§ III. *The Conſtruction of the* TABLE.

The Table of Logiſtical Logarithms is deduced from the Table of Logarithms of Abſolute Numbers at the end of this Book; and for the Compoſition of it, this is the

RULE.

To the Arithmetical Complement of the Logarithm of 3600, *(which are the Seconds in one Hour, or* 60 *Minutes;) add the Logarithm of the Abſolute Number at the Head of each Column of the Table; and that Summ ſhall be the Logiſtical Logarithm anſwerable to that Number; and if you add any Number of Seconds, found in the firſt Column of the Table, to the Number at the Head of any Column, the Logarithm of that Number added to the Complement Arithmetical of* 3600, *the Summ ſhall be the Logiſtical Logarithm anſwering to thoſe Minutes and Seconds.*

Example 1. *Let it be required to find the Logiſtical Logarithm anſwering to 7 Minutes, or 420 Seconds.*

The Logarithm of 3600 is 355360, its Complement Arithmetical	6·44370
To which add the Logarithm of 420, which is	262325
The Logiſtical Logarithm of 7 Minutes, or 240 Seconds, is	906695

Example 2. *What is the Logiſtical Logarithm, anſwering to* 21 *Minutes, or* 1260 *Minutes,* 12 *Seconds?*

The Complement Arithmetical of the Logarithm of 3600 is	644370
To which add the Logarithm of 1272, which is	310448
The Logiſtical Logarithm anſwering to 1272 Seconds	954818

§ III. *The Uſe of the Table of Logiſtical Logarithms, in Multiplication, Diviſion, and finding the Part Proportional in Aſtronomical Fractions.*

1. In Multiplication and Diviſion.

Multiplication is performed by Addition; for the Logiſtical Logarithms of the two given Numbers being added together, their Summ is the Product of their Multiplication.

Becauſe in the Uſe of this Table, a ſimple Multiplication or Diviſion is ſeldom required, but as it concerns the Rule of Proportion; therefore for the moſt part, a Tacit Addition or Subtraction of an Unite, which is 1.000000, muſt be uſed; which, for that it hath the like Uſe, we will call the *Radius*.

Example 1. *If* 1 *Degree give* 4′ 26″, *what will* 16′ 00″ *give?*

By Multiplication,

The Logiſtical Logarithm of 4′ 26″	886694
16′ 00″ (added)	942597
The Logiſtical Logarithm of 1′ 13″ the Product	1829291

And ſo the Anſwer is 1′ 13″.

Example 2. *If* 24′ 50″ *give* 1 *Degree, what will* 4′ 10″ *give?*

By Diviſion,

The Logiſtical Logarithm of 4′ 10″ (*Radius* added)	1.884164
24′ 50″ (ſubtracted)	961688
The Logiſtical Logarithm of 10′ 9″ the Quotient	922476

The like may be done for Time, ſuppoſing a Day, or 24 Hours, to be the Integer; and then you muſt for the Minutes uſe thoſe Columns of the Table which have Hours at the Head of them. But if you will ſuppoſe one Hour to be the Integer, then for the Minutes you may uſe the firſt Column towards the left hand.

Example 3. *If the* Sun *in one Day move* 59′ 37″, *how much doth he move in* 21 *Hours,* 39 *Minutes?*

The Logiſtical Logarithm of 21h 39′	995524
59′ 37″ (added)	999721
The Logiſtical Logarithm (abating *Radius*) of 53′ 46″	1995245

And ſo much doth he move in 21 Hours, 39 Minutes.

Example 4. *If the* Sun *in* 21 *Hours,* 39 *Minutes move* 53′ 46″, *how much doth he move in one Day, or* 24 *Hours?*

The Logiſtical Logarithm of 21h 39′ (ſubtract)	995524
— 53′ 46″ (*Radius* add.)	1.995245
The Logiſtical Logarithm of 59′ 37″ Quotient	999721

And ſo much doth he move in 24 Hours.

By theſe Tables alſo in the Uſe of *Ephemerides*, or Aſtronomical Tables, for finding the *Planets* Motions at any time, and for computing the Time of the *Aſpects*, by uſing Degrees and Minutes, as if they were Hours and Minutes; by the Golden Rule,

Example 5. *If a* Planet *move* 1 Degree, 2′ 13″ *in* 60′ (*or* 24 *Hours,*) *in how much time will it move in* 36′ 24″?

The Logiſtical Logarithm of 1 Degree 2′ 13″ (ſubtract)	1001575
60′ or 24 Hours (*Radius* add.)	1.
36′ 42″	1978652
	977077

The Logiſtical Logarithm of 35′ 24″ or 14 Hours, 9′.

And in ſo much time will it move in 35 24″ or in 14 Hours, 9 Minutes of Time.

Example 6. *If* 1 *Degree* 7′ 9″ *give* 58′ 12″, *what ſhall* 57′ 34″ *give?*

The Logiſtical Logarithm of 58′ 12″	998677
57′ 34″	998202
Their Summ	1996879
1 Degree 7′ 9″ (ſubtracted)	1004890
49′ 53″ the Anſwer	991989

In

In this and the like Examples, where neither 60 Minutes or 24 Hours are ingredient in the Queſtion, the work may be more eaſily performed by Addition only; as thus,

The Logiſtical Logarithm of	10° 7′ 9″ Comp. Arith. is		795109
	58′ 12″ is		998677
	57′ 34″ is		998202
	49′ 53″ the Anſwer		2991988

By theſe Tables alſo may be found the *Conjuctions* and other *Aſpects* of the *Planets*; as in this Example following.

Example 7. *Suppoſe the* Sun *and* Moon *upon the* 26th *and* 27th *days of* March *to be both of them in* Aries, *and let the time of their true Conjunction be required.*

	d.	′			
Day 26, the *Moon* is in	6	16	♈	and the *Sun* in 16	6 ♈
Day 27, the *Moon* is in	18	57		17	5
The Diurnal Motion of ☾	12	41		of the *Sun*	05 9
The Diurnal Motion of ☉	0	59			
Difference of their Diurnal Motions.	11	42			

	d.	′	
The place of the ☾ the 26 Day	6	16	♈
The place of the ☉ the 26 Day	16	06	♈
Diſtance of the ☾ from the ☉	9	50	

Being thus prepared, have recourſe to the Table.

The Logiſtical Logarithm of	the diſtance of the ☾ from the ☉ 9° 50′		1.961349
	the difference of Diurnal Motion 11	42	968797
The Logiſtical Logarithm of 2 h. 10′			992452

Wherefore the true time of the Conjunction of the Luminaries was *March*, 26 day, 20 hours, 10 minutes, that is the 27 day, 10 minutes after 8 in the morning.

The end of Logarithmetical *and* Logiſtical Arithmetick.

INSTRUMENTAL ARITHMETICK,

PART IV.

The First Way By DECIMAL SCALES.

CHAP. I.

How by Decimal Scales, by inspection only, to Reduce Vulgar Fractions to Decimals; and the contrary.

THE Arithmetick which we now come to treat of, and which I call *Instrumental Arithmetick*, is not any new kind of Arithmetick, but is indeed the same with *Decimal Arithmetick* before taught; only, whereas in *Decimal Arithmetick* there were certain Tables made of Money, Weight, and Measure, by help of which the Decimal of any Fraction of Money, Weight, and Measure, might be set down (as) in Whole Numbers, here in this *Instrumental Part*, I have contrived certain Scales of Money, Weight, Measure, Time, *&c.* equally divided into the several Denominations, into which the Weights and Measures for which they are contrived, may be equally divided. Unto all which Scales there is joyned a Scale of 100, 1000, or 10000, equal parts, according to the length of the Scale; so that by inspection only, you may readily and exactly, without Addition, set down the Decimal Fraction of any part of Money, Weight, or Measure, with great Celerity and Exactness, if the Scale be well divided, and be but of a reasonable length.

Now if the Scales which I have chiefly made choice of in this work, as being of most use with English Men, (though other Scales may be made for the Coyns, Weights, or Measures of any other Country as well, and upon the same Ground,) are chiefly these, *viz.*

Figure I.

1. English Coyn, two Shillings.
2. Troy Weight, two Penny Weight.
3. Averdupois great Weight, 28 *lb.*
4. Averdupois little Weight, 16 Ounces.
5. Liquid Measures, 36 Gallons.
6. Dry Measure, 8 Bushels.
7. Long Measure, one Ell or Yard.
8. Foot Measure, 12 Inches.
9. Time and Motion, one Hour, or one Minute, *&c.*

} Being made the Integer.

Unto every of these Scales, is joyned another Scale of 100 or 1000 equal parts, these Scales are made to face one another, so that if you look upon any Division in the one, you shall also discern plainly what Division or part of a Division answereth thereunto in the other.

These Scales being thus disposed, as they may easily be upon any Ruler of Silver, Brass, or Wood; but best of all upon a Square Ruler, made in form of a *Parallelopipedon*, will

will by inspection only give you any Decimal Fraction required without Addition, or (on the contrary) reduce the Fraction into the known parts of the Integer, by inspection also, without Subtraction.

CHAP. II.

NUMERATION from the SCALES.

THE Scales here to be described are in Number eight, as hath been already shewed, and as by the figure of them appears. Now Numeration upon a Scale, is to find upon what part of the Scale any Number upon the same Scale will fall.

We will begin with the first, and so proceed till we have given an Example in every one.

1. The first Scale is of English Money, and is divided into 24 equal parts, which represent 24 pence or 2 Shillings; these parts are numbred with Arithmetical figures, from the beginning thereof, by 1, 2, 3, 4, 5, *&c.* to 24, each Division representing one Peny, and the whole 24 Divisions represent 24 Pence, or 2 Shillings: So that where the figure 1 standeth, that part of the Scale representeth one Peny; where the figure 2 standeth, it representeth two Pence; where the figure 18 standeth, it representeth 18 Pence, or one Shilling six Pence; and so of any other figure of the same Scale.

Then, because there are four Farthings contained in a Peny, each of these Pence (or Divisions) is sub-divided into 4 other equal parts by short lines, every one of these representing one Farthing; so is the whole Scale divided in all into 96 equal parts, which are the Number of Farthings contained in two Shillings. Thus if you look into the Scale of Money for 8 Pence 3 Farthings, you shall find it at the Letter *a*; also if you would find in the Scale the place of 18 *d.* half-penny, you shall find it at *c*, and thus you may find the place of any Number of Pence and Farthings, under two Shillings upon the Scale.

Unto this Scale of Money, (as to all the rest of the Scales) there is joyned another Scale, which I shall always hereafter call the Scale of 1000, the use of which Scale is this: When you have found any Number of Pence or Farthings upon the Scale of Money, you shall find upon the Scale of 1000, what parts of a Thousand is the Decimal of those Pence and Farthings: Thus when in the Scale of Money you find at the Letter *a* 8 Pence 3 Farthings, if you cast your Eye directly cross to the Scale of 1000, you shall find .364 to stand directly against 8 *d.* 3 *q.* which 364 is the Decimal of 8 *d.* 3 *q.* Also, if you find upon the Scale of Money 18 *d.* half-penny, which is at the Letter *c*, you shall find against it in the Scale of 1000, this Number .771, which is the Decimal of 18 *d.* half-peny. And in this manner may the Decimal of any Number of Pence or Farthings under two Shillings, be most easily and exactly obtained.

Now on the contrary, suppose a Decimal Fraction were given, representing some part of English Coyn, if you look in the Scale of 1000 for your Number given, right against it in the Scale of Money, you shall find what Number of Pence and Farthings is represented thereby. As for Example: Suppose .364 were a Decimal given, and it were required to find what part of Coyn it doth represent; look in the Scale of 1000 for the Number .364, and right against it you shall find 8 Pence, 3 Farthings. Also if .771 were a Decimal given, if you look in the Scale of 1000 for .771, you shall find against that Number 18 Pence 2 Farthings. And thus of any other.

By what hath been already said, it may be easily discerned of what exceeding expedition these Scales thus disposed are of.

I have been very tedious in shewing the use of these Scales to find the Fraction parts of Money, but the reason is, because I intend to be the briefer in the rest, for Weight and Measure; the manner of working (when the Division of the Scale is known) being the same in all respects without the least alteration.

2. The second Scale is of Troy-Weight, two-Peny-Weight being the Integer, which Scale is divided into 48 equal Parts or Divisions, each of which Divisions contains one Grain, and are numbered by Arithmetical figures at every three Grains, by 3, 6, 9, 12, *&c.* to 24; and at the place where 24 should stand, there standeth P W, which signifieth one Peny Weight, or 24 Grains: this P W standeth in the middle of the Line.

Line. Then is the ſame Scale continued farther by Arithmetical figures, 3, 6, 9 12, *&c.* as before, to 24, and there is written PW again, repreſenting two-Peny Weight, or 48 Grains.

The Scale being thus divided, it is eaſie to find the place where any Number of Grains under 48 ſhall be upon the Scale; as for Example: If it be required to find where 8 Peny-Weight ſhall fall, look upon the Scale of Troy-Weight, from the beginning thereof, and count the figures 3 and 6; then count alſo two of the ſmaller Diviſions, and that makes 8 Grains, which you ſhall find to ſtand at the Letter *d*, which is the place of 8 Grains; alſo if upon the Scale you would find the place of 1 Peny-Weight, 10 Grains, you ſhall find it at the Letter *e*, and ſo of any other Number of Grains under 48, or two-Peny-Weight.

But if you had a Decimal given, and would know what Number of Grains it repreſenteth, if you ſeek your Decimal given in the Scale of 1000, right againſt it in the Scale of Troy-Weight, you ſhall find the Number of Grains repreſented thereby.

3. The third Scale is of Averdupois great Weight, 28 Pounds, or one quarter of an Hundred being the Integer, this Scale is numbred by 1, 2, 3, 4, *&c.* to 28, which 28 repreſenteth 28 *li.* or a quarter of an Hundred, and each of thoſe is ſub-divided into four ſmall parts, each repreſenting one quarter of a Pound.

Now if you would know what is the Decimal of any number of Pounds or Quarters under 28, if you ſeek the number of Pounds in the Scale of Averdupois great Weight, right againſt it in the Scale of 1000 you ſhall find the Decimal thereof; and ſo the Decimal of 8 Pound and an half, will be found at the Letter *g*, and right againſt it .304, which is the Decimal of 8 Pound and an half.

4. The fourth Scale is of Averdupois little Weight, 16 Ounces or one Pound being the Integer: This Scale is firſt divided into 16 equal parts, and numbred by 1, 2, 3, 4, *&c.* to 16, each Diviſion repreſenting one Ounce. Then again, each of theſe Ounces is ſub-divided into 8 other ſmaller parts or Diviſions, each of which Diviſions repreſenteth two Drachms; but if your Scale be large enough, you may have each Ounce divided into 16 equal Parts or Diviſions, each Diviſion repreſenting one Drachm.

Now to find the Decimal belonging to 6 Ounces and 6 Drachms, look for 6 Ounces and 6 Drachms in the Scale, and you ſhall find it at the Letter *h*, and right againſt it .398, which is the Decimal of 6 Ounces and 6 Drachms.

5. The fifth Scale is of Dry Meaſures, one Quarter or 8 Buſhels being the Integer: This Scale is firſt divided into 8 equal parts, and numbred by 1, 2, 3, *&c.* to 8; each of which Diviſions repreſenteth a Buſhel, and each of thoſe parts is again ſub-divided, firſt into 4 equal Parts or Diviſions, each repreſenting one Peck; and then thoſe again ſub-divided into 4 other ſmaller Parts, repreſenting Quarters, Halves, and Three Quarters of a Peck.

So to find the Decimal belonging to 5 Buſhels, 2 Pecks, and half a Peck, if you look into the Scale of dry Meaſures, you ſhall find 5 Buſhels, 2 Pecks and an half, to ſtand at the Letter *k*, and right againſt it .702, which is the Decimal anſwering to 5 Buſhels, 2 Pecks, and an half.

6. The ſixth Scale is of Liquid Meaſures, the Integer being 36 Gallons or one Barrel, this Scale is divided firſt into 36 equal parts or Diviſions, and numbred by 5, 10, 15, *&c.* to 36, then every of theſe Diviſions is again ſub-divided into 4 other ſmall Diviſions, each repreſenting a Quart.

So to find a Decimal repreſenting 10 Gallons and 2 Quarts, or 4 Pints, if you ſeek in the Scale for Liquid Meaſures for 10 Gallons, 2 Quarts, you ſhall find it at the Letter *m*, againſt which you ſhall find .292, which is the Decimal of 10 Gallons 4 Pints.

7. The ſeventh Scale is of long Meaſure, the Integer being Yards or Ells: This Scale is divided into 4 equal Parts, and numbred by 1, 2, 3, 4, repreſenting 1 quarter, 2 quarters, 3 quarters, or 4 quarters of a Yard or Ell; theſe are again ſub divided, firſt into 4 other equal parts, repreſenting Nails.

Thus if it be required to find the Decimal belonging to 1 quarter and 3 Nails, if you ſeek this in the Scale, you ſhall find it ſtand at the Letter *o*, againſt which ſtands .437, which is the Decimal anſwering to 1 quarter, and 3 Nails of a Yard or Ell.

8. The eighth and laſt Scale is of repreſentative inches, the whole Scale being divided into 12 equal parts, and numbred by 1, 2, 3, *&c.* to 12; and thoſe parts are again ſub-divided into halves, quarters, and half quarters, as Carpenters Rules are uſually divided.

Unto

Unto this Scale (as unto all the other) there is joyned a Scale of 1000; this Scale will readily discover what is the Decimal belonging to any number of Inches, halves, or quarters, and the use is the same with the Scales before mentioned.

Thus I have given you a brief Description of these Scales, and the uses of them, and do now suppose my Reader to be perfectly acquainted with the way of numbring or counting upon them; wherefore I intend only to give you a question or two in the most usual Rules of Arithmetick, and so conclude.

CHAP. III.

ADDITION from the SCALES.

WHat Addition is, and the manner of working it hath been already taught, both in the first and second Parts, we will now come to an Example, which let be in Addition of English Coin, and let the Summs to be added be 36 *li.* 8 *s.* 8 *d.* 29 *li.* 0 *s.* 2 *d.* 31 *li.* 16 *s.* 9 *d.* and 6 *li.* 2 *s.* 5 *d.*

First, set down 36 *li.* 29 *li.* 31 *li.* and 6 *li.* one under another, in such order as you see in the following Example, drawing a line by the side of them as you see there done, and also a Line under them.

This done, seeing that your first Number to be set down to 36 *li.* is 8 *s.* 8 *d.* you must for the 8 *s.* (because two shillings, which we called a Decade, or the tenth part of a Pound, is made the Integer, in the Scale of Money) set down 4, which is done by Memory, and after it make a Point; then your next Number to be set by 29 *li.* being 0 *s.* 2 *d.* for the 0 *s.* set down a Cypher; thirdly, for your Number to be set by 31 *li.* being 16 *s.* 9 *d.* for the 16 *s.* set down 8 Decades, with a Point after it, and lastly, the number to be set by 6 *li.* being 2 *s.* 5 *d.* for the 2 *s.* set down 1 Decade with a Point after it.

Then take your Scale in Hand, and seeing your first number of pence are 8 *d.* look in your Scale of Money for 8 *d.* and against it in the Scale of 1000, you shall find 333, which set by 36 *li.* 8 *s.* behind the Point, then your next number of pence being 2 *d.* look in your Scale for 2 *d.* and against it in the Scale of 1000, you shall find 083, which set to 29 *li.* 0 *s.* behind the Point. Then your third number of pence being 9 *d.* look in your Scale for 9 *d.* and against it in the Scale of 1000, you shall find 376, which set to 31 *li.* 16 *s.* and lastly, your last number of pence being 5 *d.* look in your Scale for 5 *d.* and against it you shall find 208, which set to 6 *li.* 2 *s.* and then will your whole work stand, as here you see.

36	4.333
29	0.083
31	8.376
6	1.208

Your Summs being thus set down, which is done with more facility than you can imagine, till you make trial and be something perfect therein, you must then add all the numbers together, as in Addition of Decimals, and you shall find the Summ of them to be 103|4.000: Now to know this in Money, is as easie as it was to set the several Summs down, for the figures 103, which stand behind the down right line, are 103 *li.* and the figure 4 which stands between the down right line and Point, are 4 Decades or 8 *s.* and being the rest to the right hand are all Cyphers, they signifie neither Pence nor farthings, so is the total of this Addition 103 *li.* 8 *s.* 0 *d.*

li.	
36	4.333
29	0.083
31	8.376
6	1.208
103	4.000

That the manner of working may appear more plain, I will give you another short Example as difficult as I can invent, which I performed by a Scale of Wood but of 8 Inches long. Let the Summs to be added together be these following:

li.	*s.*	*d.*	*q.*
332	17	4	1
159	6	8	1
217	5	3	3
709	9	4	1

1. Set down your several Summs of Pounds one under another as before, and draw a line by the side of them, and another under them; so will they stand as in the following Example.

2. Your ſumms of Pounds being thus orderly placed and lines drawn, repair to your Scale; and ſeeing your firſt number of Shillings, Pence, and Farthings, is 17 *s.* 4 *d.* 1 *q.* for your 17 *s.* ſet down 8 Decades, which is 16 *s.* with a Point after it, then will there reſt to be ſet down 1 *s.* 4 *d.* 1 *q.* or 16 *d.* 1 *q.* which if you ſeek in your Scale of Money, you ſhall find to ſtand againſt it in the Scale of 1000 this number.677, which is the Decimal of 1 *s.* 4 *d.* 1 *q.*

3. Your ſecond number of Shillings, Pence, and Farthings, is 6 *s.* 8 *d.* 1 *q.* for your 6 *s.* ſet down 3 Decades, which is 6 *s.* and then there will remain 8 *d.* 1 *q.* which if you ſeek in your Scale of Money, you ſhall find to ſtand againſt it in the Scale of 1000 this number .344; which is the Decimal of 8 *d.* 1 *q.*

4. Your third number of Shillings, Pence, and Farthings is 5 *s.* 3 *d.* 3 *q.* for your 5 Shillings ſet down 2 Decades, which is 4 ſhillings with a Point after it; then will there reſt to be ſet down 1 *s.* 3 *d.* 3 *q.* or 15 *d.* 3 *q.* which if you ſeek in your Scale of Money, you ſhall find to ſtand againſt it in the Scale of 1000 this number .656, which is the Decimal of 15 *d.* 3 *q.* 1 *s.* 3 *d.* 3 *q.* and the three ſumms to be added together will ſtand as here you ſee.

332	8.677
159	3.344
217	2.656
709	4.677

Theſe ſumms being added together according to the Rule for Addition of Decimals, you ſhall find the ſumm of them to be 7094.677 now to know what this is in Money, take notice that the 709 which ſtands to the left hand of the down right line are 709 Pounds, and the figure 4 which ſtands between the down right line and the Point, are 4 Decades or 8 *s.* but (becauſe the firſt figure next after the Point is above 5, *viz.* 6,) you muſt add 1 *s.* to the 4 Decades, making them 9 *s.* then will there remain 177, wherefore if you look in the Scale of 1000 for 177, you ſhall find againſt it in the Scale of Money 4 *d.* 1 *q.* So is the whole ſumm of this Addition 709 *l.* 9 *s.* 4 *d.* 1 *q.* as by the preceding work doth appear.

Here Note, that when you had ſet down your 709 *l.* 4 Decades or 8 *s.* there remained beyond the Point 677, which if you had ſought in your Scale of 1000, you would have found againſt it in the Scale of Money 15 *d.* 1 *q.* or 1 *s.* 3 *d.* 1 *q.* (which is all one) as before; for it appeareth plainly by the Scale, that 500 in the line of 1000 is equal to one ſhilling.

I might proceed farther in giving you Examples in Weight, and Meaſure, but being before acquainted with Decimal Arithmetick, and with Numeration upon the Scales, you cannot be deficient in the applying of the other Scales of Weight, and Meaſure to the ſame purpoſe for which they were contrived, I having ſo largely exemplified the uſe of the Scale of Money.

CHAP. IV.

SUBTRACTION from the SCALES.

SUbtraction is the taking one or more ſmaller ſumms out of one greater; I ſhall only give you an Example or two, as I have taken the numbers from a Scale.

Example: *Delivered to a Gold-ſmith of old Plate* 297 *Ounces,* 13 *Penny Weight,* 19 *Grains.*

Received of the ſame Goldſmith firſt 165 *Ounces,* 11 *Penny Weight, and* 7 *Grains; and after that received of the ſame Goldſmith* 32 *Ounces,* 19 *Penny Weight, and* 32 *Grains; what Plate remains in the Gold-ſmith's Hands?*

Delivered	297	6.896
Received firſt	165	5.646
Received more	32	9.979
Received in all	198	5.625
Reſts	99	1.271

Take your Numbers out of the Scale of Troy weight, and ſet them down as here you ſee.

Then add the ſeveral weights of Plate received together, and they make 198|5.625, or 198 Ounces, 11 Penny weight, 6 Grains; which if you ſubtract from 297|6.896, or 257 Ounces, 13 Penny weight, 19 Grains, which was the quantity of Plate delivered, there will remain 99|1.271, or 99 Ounces, 2 Penny weight, 13 Grains, and ſo much Plate is ſtill in the Gold-ſmith's Hand. And let thus much ſuffice for Subtraction; and as for Multiplication and Diviſion, *&c.* by Scales, they ſhall be taught in the ſecond and third Parts following.

INSTRU-

INSTRUMENTAL ARITHMETICK.

The Second Way, by

RHABDOLOGIA,

Or

Numbering Rods, vulgarly called *Neper's Bones.*

Proœme.

These Rods were the contrivance of the Honourable John Neper, *Baron of* Merchiston *in* Scotland; *the first Inventer of those never enough to be admired Artificial Numbers, called Logarithms: In the Calculation of which (as may partly appear by what is said concerning the Construction of them, in the third part of this first Book,) the Multiplications, Divisions, and Extraction of Roots being very Tedious and Laborious; he found out these Compendious and certain Rods, now corruptly called* Neper's Bones; *for that himself in his Description and Fabrick of them saith, they may be made of* Silver, Ivory, *or* Box, *and he gives this Definition of them,*

Rhabdologia est Ars Computandi per Virgulas Numeratrices; *i. e. Rhabdologia is the Art of counting by Numbering Rods.*

10

CHAP. I.

Of the Fabrick of these RODS.

THey may be made of *Silver*, *Ivory*, *Brass*, *Box*, &c. or delineated upon fine Card Paste-board; they are made flat, each Rod having two Sides or Faces, and contains in length about 2 Inches and $\frac{1}{10}$, in breadth one quarter of an Inch, and in thickness one twelfth part of an Inch.

FIGURE II. *Fig.* II.

One set of these Rods consisteth of five pieces, and therefore hath ten Faces or Sides. Upon the first of these five pieces you have a Cypher at the Head, and 9 at the bottom thereof: Upon the second of them you have 1 at the Head, and 8 at the Bottom: Upon the third 2 at the Head, and 7 at the bottom: Upon the fourth 3 at top, and 6 at bottom; and upon the fifth 4 at top, and 5 at the bottom. Every of the two figures at top and bottom together make 9; as 0 and 9 is 9, 1 and 8, 2 and 7, 3 and 6, 4 and 5. And here observe, that the figures 98765, which stand at the bottom of the Scheme, stand with their heels upwards in this manner, 68495; and so do all the other figures under them, till you come to the double Line which is in the middle of the Scheme, noted with *A* and *B*, at which Line, if the Scheme were cut ino two pieces, and folded or pasted on the back-side of the other half, so that the 9 at the

bottom were placed upon the Cypher at the top, and ſo 8 upon 1, 7 upon 2, 6 upon 3, and five upon 4, and then the Scheme cut again into five little ſlippets by the down-right Lines; theſe five ſlippets would exactly repreſent one ſet of theſe Rods, for on one ſide of one of theſe pieces, you ſhould have a Cypher, and 9 on the other: Upon the next 1 and 8, upon another 2 and 7, upon another 3 and 6, and upon the other 5 and 4; both the Figures on either ſide making 9, as before was deſcribed.

Theſe five ſlippets do now contain the whole Multiplication Table of *Pythagoras*, before inſerted in Multiplication of Vulgar Arithmetick; but ſo few are not of ſufficient uſe, for there can be but four figures of one kind, which in all caſes is not ſufficient: Therefore as theſe Rods are made now a-days, they do commonly make ſix ſets of them; that is, 30 pieces, which contain 60 faces; and theſe will be of good uſe, and there will ſeldom be found a want.

Concerning a Caſe for theſe Rods.

For the orderly keeping and ready finding of theſe Rods, I have often (for my ſelf and others,) had a Box made of Wallnut-tree, or Pear-tree, with five partitions in it, each partition to hold five or ſix ſets of theſe Rods, or more if more were required; every of theſe partitions being figured on the ſide thereof next the eye, with ſuch figures as the Rods in ſuch a partition had at the top; ſo that the Party that was to uſe them could take them as readily out of his partition, as a Printer can take his Letters out of their reſpective Boxes to make any word.

Fig. IV. FIGURE IV.

In this Box there is alſo convenient room made for one other Rod, double in breadth to theſe here deſcribed, but of the ſame length and thickneſs; upon the one ſide whereof there is a Table or Plate uſefull in the extracting of the Square Root, and on the other ſide another for the extracting of the Cube Root, which in the Figure thereof is noted with *Square*, *Cube*; of which I ſhall forbear to ſay any thing till I come to ſhew you how to extract the Square and Cube Roots by them and the Rods.

Of a Board with a Frame, upon which to lay your Rods when any Operation is to be wrought by them, which I call a TABELLET.

Fig. III. FIGURE III.

In the uſing of theſe Rods care is to be had firſt of the orderly laying of them, and ſecondly for the keeping of them in that poſition till your Work be ended: For the effecting whereof both neatly and certainly, there is a little Table or Frame contrived, containing in breadth $\frac{1}{20}$ of an Inch more than the length of the Rods, and in length at pleaſure, but it may well be about once and an half the length of the breadth.

It ought to be made of a thin piece of Pear or Wallnut-tree, or of ſuch matter as your Box or Caſe is made of; and it may very commodiouſly be contrived to be put into the Box, as I ever had them made to do, for that I found it inconvenient to carry it looſe.

Upon the Superficies of this Board, cloſe to one of the edges thereof, muſt be glued or otherwiſe faſtened with Pins, a ſmall piece of the ſame matter, and alſo of the ſame length, breadth, and thickneſs of one of your Rods; which muſt be divided into nine equal parts, and Lines drawn croſs the piece; ſo will there be nine Squares, in which you muſt grave or ſtamp the nine Digits, beginning with 1 at the top, and ſo deſcending by 2, 3, 4, to 9, at the bottom thereof: And it were neceſſary that theſe figures (as alſo thoſe which are at the head of every of your Rods) were graven or ſtamped of ſomething a bigger figure than the other figures of your Rods are.

Under the end of this ledge beginning at the figures, and ſo continuing the whole length of the Board, muſt another ledge of the ſame matter and thickneſs be glued or pinned, and then is your Tabellet finiſhed. It is called a *Tabellet*, for that when the Rods are laid thereon for any operation to be wrought by them, we uſually ſay, the Rods are *Tabellated*.

Being thus prepared with Rods and Tabellet, you are ready for the work intended by them, as in Figure III. is evident.

To

To what Use these Rods generally serve.

The chief Uses which these small Rods serve unto, are all manner of Multiplications and Divisions, and Extraction of Roots, Square or Cube; all which are so easily and expeditiously performed, and that by Addition and Subtraction only; for here is no charge of the memory, and you shall always take your Quotient-figure in Division certain, neither too great nor too little. But ceasing to say more of their Properties, I will now come to shew their Use.

CHAP. II.

How to apply, or lay down, any Number by the Rods.

HAving already given you a large Description of these Rods, I will now shew the Use of them, which will best appear by the Propositions following:

PROP. I.

Any Number being given, how to tabulate, or lay it down by the Rods.

FIGURE III. *Fig.* III.

Let it be required to tabulate, or lay down, this Number 3496.

First, From among your sets of Rods (or out of your Case) take one which hath the figure 3 at the top thereof, and lay it on your Tabellet close to the edge thereof: then,

Secondly, Take another Rod from your Case, which hath the figure 4 at the top of it, and lay that also upon your Tabellet, close by the side of the other.

Thirdly, Take another Rod, which hath the figure 9 at the top of it, and lay that upon your Tabellet, close by the other two.

And lastly, Take a fourth Rod, having the figure 6 at the head thereof, and lay that also upon your Tabellet close by the rest.

These four Rods thus taken and laid upon the Tabellet, you shall see in the uppermost row (which standeth against the figure 1 on the side of your Tabellet) these four figures, 3, 4, 9, 6, that is 3496, equal to your given number; in the second row (against the figure 2 of your Tabellet) you shall find the double thereof; in the third (against the figure 3) you shall find the triple thereof; in the fourth the quadruple; in the fifth, the quintuple; and so on to the ninth and last, in which you shall find the noncuple of the number given.

PROP. II.

How these Rods will appear when tabulated, and being tabulated, how to read the Multiplication of that Number when tabulated, by any of the Nine Digits.

The Four Rods being tabulated, they will appear exactly as they are represented in *Figure* III. which Figure lively represents the four Rods lying upon the Tabellet, which mind well, for upon the true tabulating and right reading of the Rods so tabulated, depends the whole work.

The Rods thus tabulated, as you see them in the Figure, do appear in the form of a Glass-window, every Pane thereof representing a *Rhomboyades*, or Diamond-form: In the reading of the Figures which are in these several *Rhomboyades*, observe these few Directions following, which will illustrate the whole business intended, and therefore especially to be minded.

NOTE,

I. *That the Figures upon the Rods are to be read from the right hand towards the left.*

II. *That in every* Rhomboyades *or Diamond, there is either One Figure, or Two Figures, but never more than Two.*

III. *If there be but one Figure in a Rhombus, then that Figure is the Figure to be set down alone, (be it either a Figure or a Cypher,) but if there be two Figures in a* Rhomboyades, *(as for the most part there is,) then add those two Figures together, and set down their summ in one Figure.*

IV. *But if the summ of the two Figures in one* Rhomboyades *or Diamond do exceed Ten, then you must set down the overplus above Ten, and keep one in mind, which one you must carry to the next* Rhomboyades.

V. *That*

V. *That the first towards your right hand, and the last towards your left hand are but half* Rhomboyades *or Diamonds, and never have in them more than one Figure, but all between them are whole ones, and for the most part have two Figures in them, or one Figure and a Cypher.*

VI. *If in either* Rhomboyades, *or half* Rhomboyades, *you find no Figures but Cyphers, you must not neglect, but set them down as if they were Figures.*

These Rules being rightly understood, all that follows will be familiar and easie, and these I shall explain by the Example following.

Example 1. For the illustration of the preceding Rules, I will make use of those Rods which were before tabulated; therefore have recourse to *Figure* III. where this Number 3496 is tabulated.

The Figures at the top of the four Rods are those which signifie the former given number 3496, and this number stands against the figure 1 on the right side of the Tabellet; then I say that the figures in the next row, standing against the figure 2 of the Tabellet are double thereunto, which is thus proved.

Repair to the Rods as they lie upon the Tabellet, and in that row which lieth against the figure 2, you shall find in the first half *Rhomboyades* towards your right hand, (where by *Rule* I. you must begin,) the figure 2; wherefore set down with your Pen upon Paper the figure 2: In the next *Rhomboyades* in the same row, you shall find 8 and 1, which added make 9; set down 9 on the left hand of 2: In the next *Rhombus* you shall find 8 and 1 again, which is 9 also; set down 9 on the left hand of the other: And in the last *Rhomboyades* you shall find only 6, wherefore set down 6 on the left hand of 9; so have you in all 6992, which is double to 3496.

Again, the figures in the row which stand against the figure 3 in the Tabellet are triple to 3496; for in the first half *Rhomboyades* towards your right hand, you have 8; set down 8: In the next *Rhomboyades* you have 7 and 1, which is 8; set down 8 again; In the next you have 2 and 2, which is 4; set down 4: In the next *Rhomb.* you have 9 and 1, which makes 10; set down 0 and carry 1; but it is the last *Rhomboyades*, and (because there is never another to carry the 1 unto,) you must therefore set it down, so have you this number 10488, which is triple to 3496: And so of all the other Digits.

Thus have I given an Example shewing how the Numbers upon the Rods are to be read and written down; in the delivery of this Example, I have made the following work so plain and easie, that the meanest capacity may by what is here delivered, read or write down any number that can be tabulated: And that this Chapter may be throughly understood, I will give you the products of 7009078, multiplied by all the nine Digits, which are as followeth, *viz.*

7009078 being multiplied by	produceth
2	14018156
3	21027234
4	28036312
5	35045390
6	42054468
7	49063546
8	56072624
9	63081702

Thus have I sufficiently described these Rods, and the manner of numbering upon them; and now I think it time to apply them to that use for which they were intended, namely, the more difficult parts of Arithmetick, as *Multiplication*, *Division*, and *Extraction of Roots*.

CHAP. III.

MULTIPLICATION by the RODS.

IN multiplying by the Rods, you are to consider (as in Vulgar Arithmetick,) three Terms, Things, or Numbers, *viz.*

1. The *Multiplicand*, which is the Number to be multiplied.
2. The *Multiplier*, which is the Number by which the *Multiplicand* is multiplied.

3. The

3. The *Product*, which is the Summ produced by the multiplying of the two former together.

And here note, that the Product doth contain the Multiplicand ſo many times as there be Unites in the Multiplier.

Thus for the Definition of Multiplication; now for the working thereof by the Rods, for which this is the *RULE:*

Firſt, Set down upon your Paper the Multiplicand, *and orderly under it the* Multiplier; *(it matters not greatly which of the two given Numbers be made Multiplicand or Multiplier, but it it uſual and beſt to make the greateſt Multiplicand, and the leſſer Multiplier,) then draw a Line with your Pen under them; and having tabulated your Multiplicand, (or greater Number,) look what Numbers in your Rods ſtand againſt the firſt figure towards your right hand, and that Number which you ſhall find upon your Rods ſtanding againſt that firſt figure found in your Tabellet, ſet down under your Line which you formerly drew under your Multiplicand and Multiplier; and having ſo done with the firſt figure of your Multiplier, doe ſo with the reſt, ſetting them down one under another, removing every figure one place more towards the left hand than that which went before it, as is done in common Multiplication, and as you ſee in the following Examples.*

Example 1. *Let it be required to multiply* 3496 *by* 489.

1. Set down your given Numbers 3496 and 489, one under another, and draw your Line under them, as here you ſee done.

2. Your Multiplicand 3496 being tabulated, and 9 being the firſt figure to the right hand in your Multiplier, look upon your Rods what ſumm there ſtands againſt 9, in the ſide of your Tabellet, and you ſhall find 31464, which is the Product of 3496 multiplied by 9; wherefore ſet down this Number under your Line, as in the Margin.

3496	*Multiplicand.*
489	*Multiplier.*
31464	
27968	
13984	
1709544	*Product.*

3. Look what ſumm upon the Rods ſtands againſt 8, which is the ſecond figure of your Multiplier, and you ſhall find 27968; ſet this Number under the former, moving it one place forward towards the left hand.

4. Look what ſumm upon the Rods ſtands againſt 4, which is the third figure in your Multiplier, and you ſhall find 13984; which ſet down under the other, one place more to the left hand.

Laſtly, Under theſe three ſumms draw a Line, and add the three ſumms together, and they make 1709544, which is the Product of 3496 multiplied by 489; and this 1709544 the Product, contains 3496 the Multiplier, 489 times.

Practiſe well this firſt Example, and compare it with the Rods as they are tabulated in Figure IV. I will give you one or two more Examples.

Example 2. *Let it be required to multiply the ſame ſumm* 3496 *by* 261.

Set the Numbers down as is here done; then look upon the Rods for the Product of 3649 by 1, and you ſhall find it to be the ſame; wherefore ſet down 3496 under the Line: then look upon the Rods for the Product of 3496 by 6, and you ſhall find it to be 20976; which ſet down under the other Number, one place more toward the left hand: Again, look in the Rods for the Product of 3496 multiplied by 2, and you ſhall find it to be 6992, which ſet down under the other two.

3496
261
3496
20976
6992
912456

Laſtly, Draw a Line under them, and add the three Numbers together in order as they ſtand, and the ſumm of them will be 912456, which is the Product of 3496 multiplied by 261.

Example 3. *Let it be required to multiply the ſame Number* 3496 *by* 520.

Set down your Numbers as you ſee in the Margin: Then becauſe the firſt figure in your Multiplier towards the right hand is a Cypher, wholly omit it, and multiply 3496 by 52 only, ſo ſhall you find the Product of 3496 by 2, to be 6991; which ſet down: alſo the Product by 5 will be 17480; which ſet down under the other, one place farther: then draw a Line, and add theſe two ſumms together, and they make 181792; to which if you add a Cypher for the Cypher which you omitted in your Multiplier, the ſumm will be 1817920, which is the Product of 3496 by 520.

3496
520
6992
17480
1817920

Example

Example 4. *Let it be required to multiply the ſame* 3496 *by* 7003.

```
  3496
  7003
 10488
24472..
24482488
```

Set down your numbers as before, and as you ſee here done: Then having tabulated 3496, ſee what the Product thereof is upon the Rods, being multiplied by 3, the firſt figure in your Multiplier, and you ſhall find it to be 10488; which ſet down under the Line: Then the two next places of your Multiplier being Cyphers, make two pricks under the former Number, one under 8, the other under 4, as you ſee in the Example; or inſtead of 2 pricks you may make two Cyphers: Then look in the Rods for the Product of 3496 by 7, and you ſhall find it to be 24472; which ſet under the other ſumm, beginning your number at the fourth place, or beyond the two Pricks or Cyphers: Laſtly, draw a Line and add theſe two together, and their ſumm is 24482488; which is the Product of 3496 multiplied by 7003.

Thus have you four Examples in Multiplication, in which are included all the varieties that may at any time happen in that Rule, *viz.* two where the Multiplier conſiſted all of figures, as in the firſt and ſecond Examples they did; another where the latter place of the Multiplier conſiſted of a Cypher, as in this laſt Example, where Cyphers were intermixed with the figures.

CHAP. IV.

DIVISION by the RODS.

AS in Multiplication, ſo alſo in Diviſion there are three Numbers, Terms, or Things, *viz.*

1. The *Dividend*, or Number to be divided.
2. The *Diviſor*, or Number by which the Dividend is divided.
3. The *Quotient*, which is the Number iſſuing from the Dividend's being divided by the Diviſor: And this Quotient doth always conſiſt of ſo many Unites as the Diviſor is times contained in the Dividend.

Thus much for the Definition of Diviſion; now let us come to the practice of it by the Rods, to perform which this is the

RULE:

Tabulate the Diviſor, (which is always the leſſer Number of the two given,) and ſet down the Dividend, and ſet the Diviſor on the left hand, and draw a crooked Line on the right hand for your Quotient, as in common Arithmetick: Then look upon your tabulated Rods (always) for the Number leſs than the Number in the firſt figures of your Dividend, and what figure ſtands againſt that number on the edge of your Tabellet muſt be the figure you muſt put in your Quotient, and that Number you muſt always ſubtract from the figures of your Dividend, and to the Remainder add another figure, ſo proceeding from figure to figure till your Diviſion be wholly ended.

Example: *Let it be required to divide* 1709544 *by* 3496.

Having tabulated 3496, ſet down your Dividend, your Diviſor on the left hand thereof, and a crooked Line for the Quotient on the right hand thereof, as by the Rule preceding you were directed, and as you ſee done in the Example adjoyning.

And becauſe at your firſt ſetting down of your Diviſor 3496, it would reach (if it were ſet under your Dividend 1709544,) as far as the figure 5; therefore under the figure 5 make a prick, to intimate how far you are gone on in your work; and under this prick draw a Line quite under your Dividend; then is your ſumm ſet down ready for work, and will appear as here you ſee:

```
3496) 1709544 (
      ---------
```

Your ſumm thus prepared, ask how often can you have 3496 in 17095, look in your tabulated Rods for 17095, which you cannot there find, but the neareſt number thereunto amongſt the Rods, which is leſs than 17095, (for you muſt always take a leſs) is 13984, which number ſtands againſt the figure 4 in the Tabellet; wherefore ſet 4 in your Quotient, and 13984 under the Line, and ſubtract 13984 from 17095, and

and there will remain 3111; ſo is the firſt part of your Diviſion ended, and your work will ſtand thus:

3111
3496)1709544(4
13984

Then make another prick under 4, the next figure of your Dividend, ſo will the remaining number be 31114: Then look upon your Rods for the number 31114, (or the neareſt leſs than it,) and the neareſt leſs you ſhall find to be 27968, which ſtands againſt 8 in your Tabellet; put 8 in your Quotient, and ſet 27968 under 31114, and ſubtract 27968 from 31114, ſo will there remain 3146, which ſet over head; ſo is the ſecond part of your Diviſion ended, and your work will appear thus:

3146
3111
3496)1709544(48
13984
27968

Laſtly, Make another Prick under the next figure of your Dividend, which is 4 alſo, making the remaining number to be 31464; ſeek among your tabulated Rods for this number, or the neareſt leſs, but looking you ſhall find the very number, againſt which ſtands on your Tabellet the figure 9; ſet 9 in the Quotient, and the number 31464 under the Line, and ſubtract it from 31464, the Remainder, which ſtands above the Line, and nothing remains; and ſeeing there is never another figure in your Dividend, your Diviſion is ended, and your Work will ſtand thus: and 3496 is contained in 1709544, 489 times.

00000
3146..
3111..
Diviſor. 3496)1709544(489 *Quotient.*
13984
27968
31464

Another Example, and by another way of Diviſion.

Let it be required to divide 912456 *by* 3496.

Set down your Dividend and Diviſor, draw a crooked Line for your Quotient, and alſo make a Prick under the fourth figure of your Dividend, and draw a Line under it; ſo is your ſumm prepared to be divided, and will ſtand thus:

3496) 912456 (
———

Then your Diviſor being tabulated, look amongſt your Rods for the neareſt number to 9124, which is leſs, and you ſhall find it to be 6992, againſt which ſtands on your Tabellet the figure 2; ſet 2 in the Quotient, and this number under the Line, and ſubtract it from 9124, and there will remain 2132: to which number add the next figure of your Dividend, namely 5, and it makes 21325, under which number draw a Line, then will your Summ ſtand thus:

3496)912456(2
6992
21325

Then among your Rods ſeek the neareſt number to 21325, and you ſhall find 20976 to be the neareſt number leſs, againſt which in your Tabellet ſtands 6; ſet 6 in the Quotient, and 20976 under the Line, ſubtracting it from 21325, which when you have done, there will remain 349; to this 349 add the next figure in your Dividend, which

is 6, your laſt figure, and it makes 3496; under which draw a Line, and your Work will ſtand as here you ſee:

```
3496) 912456 (26
      ...
      ----
      6992
      21325
      -----
      20976
       3496
       ----
```

This done, look amongſt your Rods for the neareſt Number to 3496, and you ſhall find the exact Number at the top of the Rods, againſt which ſtands the Figure 1 on the Tabellet, ſet 1 in the Quotient, and ſubtract 3496 from 3496, the Remainder is nothing; and ſo is your Diviſion ended, the Work ſtanding thus; and 3496 the Diviſor, is contained in 912456 the Dividend, 261 times.

```
3496) 912456 (261
      ...
      ----
      6992
      21325
      -----
      20976
       3496
       ----
       3496
       0000
```

A Third Example ready wrought by the laſt and beſt way of Diviſion.

I will only ſet it down ready wrought, leaving the practice of it to your ſelf.

Let it be required to divide 73020506 *by* 3496.

```
3496) 73020506 (20886 3050/3496
      ......
      -------
      6992
      31005
      -----
      27968
       30370
       -----
       27968
        24026
        -----
        20976
         3050
```

This Summ thus divided, produceth in the Quotient 20886, and 3050 remaining; ſo that the Quotient, with Fraction and all, is 20886 $\frac{3050}{3496}$; which ſhews, that 3496 the Diviſor, is contained in 73020506 the Dividend, 20886 times, and 3050 remaining.

This Example well practiſed, together with them beforegoing, are ſufficient inſtruction for any Student whatever, and he that can perform theſe, need not deſpair of the moſt difficult that can be propoſed. And ſo I conclude Diviſion.

CHAP. V.

Of the Extraction of ROOTS.

THe Extraction of Roots, which is the difficulteſt part of Multiplication, is expeditiouſly and certainly performed by the Rods, for the eaſie and expedite performance of which, there are two Rods on purpoſe, one for the Square, the other for the Cube Root, of which I will ſpeak; firſt, of their Fabrick; ſecondly, of their Uſe.

Of

Of the Fabrick of the Rods for extracting of Roots.

Of the same matter, and of the same length and thickness of your other Rods, let there be made another Rod, but three times the breadth of the former, the Inscription on the one side serving to extract the Square, and that on the other side for the Cube Root, each of which are divided into three Rows or Columns.

That which serveth for the Square Root, hath in the top or uppermost Square between the Diagonal thereof, these figures 0-1, in the second 0-4, in the third 0-9, in the fourth 1-6, in the fifth 2-5, in the sixth 3-6, in the seventh 4-9, in the eight 6-4, and in the ninth or lowermost 8-1, which are the Square Numbers belonging to the nine Digits.

In the second Column of the same Rod, in the first Square is inscribed 2, in the second 4, in the third 6, in the fourth 8, in the fifth 10, in the sixth 12, in the seventh 14, in the eighth 16, and in the ninth 18.

In the last or third Column, there are the nine Digits orderly descending, namely, 1, 2, 3, 4, 5, 6, 7, 8, 9. This Rod thus made, is fitted for the Square Root.

That which serveth for the Cube Root, hath in the top or uppermost Square of the first Column towards the Left Hand, between the Diagonal thereof, these figures 0-01, in the second 0-08, in the third 0-27, in the fourth 0-64, in the fifth 1-25, in the sixth 2-16, in the seventh 3-43, in the eighth 5-12, and in the ninth 7-29, which are Cube Numbers orderly descending.— The second Column of this Rod contains these Square Numbers, 1, 4, 9, 16, 25, 36, 49, 64, 81, orderly descending.—— The third and last Column of this Rod hath in it the nine Digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, orderly descending also.

This Rod, thus prepared and Inscribed, is fit for extracting of the Square and Cube Roots, that which serveth for the Square Root having the word *Square* written by the side, that for the Cube Root, hath *Cube* written by the side.

Thus having given you the Fabrick and Inscription of these Rods, I will now shew their Use.

FIGURE V.

Fig. V.

§ I. *The Extraction of the Square Root.*

To extract the Square Root of any Number, you must first prepare it; that is, set down the Number on a Paper; then under the first and lowest figure next the Right Hand, make a point with your Pen, and under the third figure make another, under the fifth another Point, and so forth; under every second figure of the Number make a Point, always leaving between each point one figure unpointed, according to the ordinary Rule by the Pen; this being done, you shall see how many figures will be in the Root, for so many Points as you have, so many figures shall you bring into the Quotient for the Square Root of the Number given; next draw a Quotient-Line, as in Division, and your Number is ready prepared for Operation, and will stand as in the Example following, where the Number given is 119025, and the Root Square thereof is required: This Number being set, and pointed as afore is shewed, you may perceive that the Root thereof will be of three figures, because there be three points under the Number given, the two figures belonging to the highest Point next the Left Hand are 11. the two figures belonging to the second point are 90, and the two figures belonging to the third point are 25. and the figures for the Root answerable to those several Points, are to be found by the Rods, as followeth.

```
1 1 9 0 2 5 ( 3
. . . .
-------
  9
```

Take the Tabellet, the Rods, and *Lamina*, and lay them before you; and first place the *Lamina* in the Frame next to the Left Hand Ledge, with that side upwards, whereupon are the inscriptions belonging to the Square Root, and marked at the top with the Letter S, then consider what is the greatest Square Number in 11, the figures belonging to the first point; the *Lamina* presently sheweth you that the greatest Square Number in 11 is 9, and his Root is 3, for 3 times 3 is 9; therefore put 3 in the Quotient for the first figure of the Root: Then set 9 under the 11, and subtract it there-from, and there will remain 2; this 2 set over 11, and cancel the 11, as you use to doe in Division, and so you have gained the first Figure of the Root, and the Work will stand as in this Example.

```
  2
1 1 9 0 2 5 ( 3
. . . .
-------
  9
```

Having the first figure of the Root, to get the second, and so all the rest in order, you must proceed in this manner: Double the Root found, which place or tabulate upon the Rods between the *Lamina* and the ledge of the Frame, as in this Example; the duple of 3, the Root found, is 6, therefore place a Rod that hath in his top or upper Square 6, between the Ledge and the *Lamina*; then look upon your Number given, what figures or Number it is that belongeth unto the second point, which you shall see will be 290 in this our Example; then turn your Eye to the *Lamina* and Rod now tabulated, and search thereupon what Number will (being less yet) come nearest unto 290, the Number out of which the second figure of the Root is to be found, and there you may soon see it is the Number 256, which of any Number upon the Rods, less than 290, cometh nearest thereunto; for the next greater Number upon the Rods, above 256, is 325, which is greater than 290, and therefore cannot be taken out of it; but 256 is the only Number to work withall, against which, on the Ledge and *Lamina*, is the figure 4, this 4 must you put in the Quotient for the second figure of the Root, and then subtract 256 out of that 290, and there will rest 34; this 34 set over the 290 in its due place, and cancel the 290, and the Work will stand as in the Example. If you please you may write your Numbers to be subtracted under the Number of which Subtraction is to be made, as I have done here in this Example, for instruction sake; or you may omit that, if you please, being you have them before you upon the Rods.

```
 234
xxgø25 (34
 · · ·
 ---
  9
  256
```

And now for the third figure of the Root, look upon the Number given, and there you shall see that the Remainder 34, with the two figures 25 still uncancelled, belonging to the third point, being all joyned together, make 3425, out of which the third figure of the Root is to be extracted: To find out what this third figure shall be, double the Root found already, which is thus done very readily; take forth a Rod that on his top Square hath 8 the duple of 4, the figure last found, and this Rod put into the Frame between the *Lamina* and the Rod that is already tabulated; this being done, you have no more to doe, but to look over the Rods and *Lamina*, for such a Number as will come neareſt unto the Number 3425, that belongeth to the third point, and is less than it, and the figure that you see against that Number so found, put in the Quotient for the third figure of the Root; thus looking upon the Rods you shall at the first sight find the very Number it self 3425, that belongeth to the third point, in the fifth line of Squares, against the figure 5, therefore put 5 in the Quotient for the third figure of the Root; and if you subtract 3425, the Number now found, from 3425, the Number belonging to the third point, there will be no Remainder; so is the Work done, and the Number given, 119025, is a perfect Square Number, and the Square Root thereof is 345, which was the thing required to be found.

```
 234
xxgøz5 (345
 · · ·
 ---
  9
  256
   3425
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Now if you multiply this 345 by it self, it produceth 119025, the first given Number; which proveth the Work to be truly wrought: For note this evermore, that for proof of the Extracting the Square Root, you must multiply the Root found by it self, (to the Product adding the remain, if any be,) and the total will produce the first Number given, if the Work be truly wrought; otherwise not.

§ II. *The Extraction of the Cube Root by the Rods.*

To extract the Cube Root of any Number, you are first to write down the Number given, whose Root is required, and make a point under the lowest figure next to the Right Hand, and another point under the fourth figure, and so under every third figure, omitting between every two points two figures unpointed; and then so many points as you have under your Number, so many Figures you shall have in your Root. Next draw a Quotient-Line, as in the Extraction of the Square Root; this being done, the Number is prepared for Extraction; then to find the Cube Root this is the

RULE:

First, seek what is the greatest Cube Number, in the Number standing above, or belonging to the highest point next the Left Hand, which the Lamina *will shew upon his left side of that Face for the Cube Root, and in the Column, upon the right side of the same Face, is the Root thereof; and when either by your Memory or* Lamina, *you have found the greatest Cube Root in that Number belonging to that first point, then*

(as

(as in Extraction of the Square Root) subtract the greatest Cube Number to that first Point belonging, or that in the Number to the said first Point belonging can be found, from the said Number, &c. then cancel it, and set the Remainder over head thereof, as before in the Extraction of the Square Root, and put the respective Digit-number found in the Quotient, for the first figure of the Root. Now to find the second figure of the Root, (and so of all the rest how many soever they be,) you must always Triple the Root found, and that Triple multiply again by the Root found, and that last Product tabulate upon the Rods on the left hand of the Lamina, *as before in the Extraction of the Square Root you did the double of the Root found; then look upon those tabulated Rods and* Lamina *together, and what Number you can find upon them that will come neareſt to the Number belonging to the Number next following, and less than it, which Number is the Divisor, and the figure on the right hand of the* Lamina *is the the figure of the Root answerable to that point, and that figure put in the Quotient for the respective figure of the Root, belonging and answerable to that Point: But note always that you take the Divisor so often and no oftener, but that you may yet also take another Number from the Number belonging to that Point, which other Number is the Square of the Digit new found, multiplied into the former Triple; the Product add to the Divisor, with this Proviso, that you place this new Product one place higher toward the left hand than is the Divisor; that is to say, set the lowest place of that new Product under the second place of the Divisor; the total of this Addition subtract from the Number belonging to the Point in action, cancelling the said Number, and the Remain set over the head thereof, as you use to doe in Division, and Extraction of the Square Root; and so proceed to the next Point, if you have any more: But for the the better understanding hereof take the following Example.*

Example, *Let* 110592 *be a Number given, and the Cubick Root thereof required; this Cubick Root is thus found:*

First, Prepare your Number, that is to say, write it down, and make a Point under 2, the first figure thereof next the right hand, and another point under 0, the fourth figure thereof, leaving two figures between unpointed; then draw a Quotient-line, and then the Number will stand ready prepared, as in the Margin, with two points, whereby it appeareth that the Root will consist of two figures, which are to be found out according to the former *Rule*: And first observe that 110 is the number belonging to the first Point; and upon the *Lamina* you may also observe that the greatest Cube Number in that 110 is 64, and its Cube Root 4; therefore put 4 in the Quotient, for the first figure of the Root; and then subtract 64 from 110, and there will remain 46; this 46 set over the 110, and cancel 110, and the work of the first point is done. And here you may now observe, that the 46 remaining, with the other figures 592, make 46592; which is the Number belonging to the second Point, and whereout the second figure is to be found.

```
  46
 xx0592 (4
 ·  ·
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  64
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To obtain this second figure, proceed in this manner: Triple 4, the Root found, and it is 12; and that triple 12 multiply by the Root found 4, and the Product is 48; this Product 48 tabulate upon the Rods on the left hand of the *Lamina*, between it and the Ledge; then view over these Rods and *Lamina* thus tabulated, and see what Number there will come neareſt unto 46592, the Number belonging to the second Point, and be less than it, and you shall find it to be that in the ninth Line, 43929, and its respective figure for the Root is 9: Now square this 9, and it is 81; this 81 multiply by the former Triple 12, and it yieldeth 972; this 972 add unto 43929, the Number found on the Rods, (being set in addition one place higher than is ordinary, as was before shewed,) and the total will be 53949; which if you compare with 46592, the Number belonging to the point in action, you shall see it is too great to be taken out of it; whereby it appeareth that you must not take 9 for your Root: for by your general Rule you must take the Divisor no oftner, but that you may also take the Product made by the square of that new Digit-number multiplied into the first Triple, out of that Number belonging to the second Point; therefore take a less Number upon the Rods, as the Number 38912 in the eighth Line, which will surely serve the turn; wherefore put 8 in the Quotient for the second figure of the Root sought for. Now write (if you will) this Number 38912, just under 46592, the Number belonging to the said second point; then according to the Rule, square this Digit 8 new found, and it giveth 64; this 64 multiply by the former triple 12,

```
  48
 xx0592 (48
 ·  ·
 ----
  64
 38912
  768
 ----
 46592
```

and

and it produceth 768, which write down under the former Number 38912, ſetting the loweſt figure 8 of this new Product directly under 1, the ſecond figure of 38912, and its ſecond figure 6 under the third figure 9, and ſo of the other in order; then add theſe two Numbers together, and the total will be 46592, which is equal to that Number above: therefore if ſubtraction be made, there will nothing remain, and ſo the work is ended; whereby you may conclude that 110592, the Number given, is a right Cubick Number, and that 48 is the Cube Root thereof, which was the thing required to be found.

Now if you would at any time prove your work, whether you have wrought right or not; multiply the Root found cubickly, and add the Remainder (when any is) to that Product, and if the total be the firſt Number given, then the work is truly wrought, or elſe not, as here in this Example: If you multiply the Root found 48 by 48, it is 2304, and this 2304, multiply again by 48, and it produceth 110592, the Number firſt given; and therefore conclude the work is truly wrought.

INSTRU-

INSTRUMENTAL ARITHMETICK.

The Third Way,

By Logarithmetical or Proportional SCALES.

Proœme.

THE Logarithmetical or Proportional Scales here mentioned, are no other than the Artificial Numbers or Logarithms, (whose Nature and Construction is treated of in the first Chapter of Artificial or Logarithmetical Arithmetick;) and these Proportional Lines (for so I shall often hereafter call them) here to be described, are no other than those Artificial Numbers (or Logarithms) transferred from these Tables to Scales put upon a streight Ruler, made of Brass, Wood, or Ivory, and of any length whatsoever; (but two Foot is a competent Length, and most usefull,) and the manner how to transfer these Numbers from the Tables to the Scales is so well known to all such as make Mathematical Instruments, that I shall say nothing concerning their Construction in this place, but proceed.

CHAP. I.

How the several SCALES are to be disposed upon the Ruler.

FIGURE V. *Fig.* V.

THE Proportional or Logarithmetical Scales upon the Ruler are in number Four, besides Equal Parts, Chords, and other Scales of frequent Use: The four Logarithmetical or Proportional Lines are as followeth,

I. The first and uppermost, is one single Scale containing almost the whole length of the Ruler, and is divided, first into Ten unequal parts, and those again subdivided into Ten other unequal parts, and those again into Ten, so often as the quantity will admit.

II. Next unto this Scale and facing of it, are three other Scales of the same kind, all of equal length, and all three of them together equal to the former single Scale; and this is called the *Triple Scale:* Upon which is also the word *Cube*, because it serveth to extract the Cube Root.

III. The Third Scale is a Scale of the same kind with the first, having the figure 1 in the middle thereof, and broken off at either end of the Ruler, at the Number 31 and 62 hundred parts: and this is called the *Broken Scale.*

IV. Underneath this Broken Scale (and facing of it) is a double Scale of the same nature with all the former, each half whereof is the just half of the first (or large) Scale:

Scale: which Scale hath 1 at the beginning thereof, and is numbered unequally, by 1, 2, 3, *&c.* to 10 in the middle; but that 10 in the middle is numbered only by a figure of 1; and from thence the Scale is divided again into ten other of the ſame unequal parts, by 1, 2, 3, *&c.* to 10, at the end thereof. Upon the lowermoſt half of this Scale is written *Square Even*, and upon the uppermoſt half *Square Odd:* This Scale ſerveth to extract the Square Root, and ſheweth when the Number whoſe Square Root is to be extracted conſiſts of Even or Odd figures or places, upon which half of the Scale the Number is to be counted.

Theſe four Scales thus diſpoſed may moſt conveniently be put upon one ſide of a Two-foot Ruler, together with Inch and Foot Meaſure, as in Figure V. and ſo it will be a moſt convenient Rule for all kind of Arithmetical and Metrical Operations; as ſhall at large be taught in this and the following Parts. And on the other ſide of the Ruler there may be other artificial Scales placed, for Aſtronomical, Geographical, Horological, and Nautical Purpoſes, whoſe Deſcription and Uſe ſhall be ſhewn in their due place; but here I ſhall only ſhew the Uſe of the Four above-mentioned Scales in Arithmetick.

CHAP. II.

NUMERATION upon the SCALES.

THE Four Scales (as hath been already declared,) are all of them of one and the ſame nature or kind, and Numeration upon one of them being underſtood, Numeration upon any of the other will be underſtood alſo; and therefore I ſhall teach Numeration upon the firſt or *Single Scale* only, and what is ſaid of that, the ſame is to be underſtood of all the reſt.

1. This Scale hath theſe Words [*Single Scale*] at the beginning of it, and then the figures 1, 2, 3, 4, *&c.* to 10 at the end of the Scale.

2. The diſtance between the figures 1 and 2, is alſo divided into 10 other unequal Parts, but no figures ſet to them, but ſmall diviſions; the Diviſion at five of thoſe parts being a longer Line than the reſt, for the more eaſie numbering upon it.

3. Every one of theſe Ten unequal Diviſions between 1 and 2 is again divided into Ten other ſmaller Diviſions, but not numbered by any figures, but by ſmaller Diviſions than the former, as may be better ſeen in the Figure of the Scale at the end of the Tractate, than many words can inform you.

4. What hath been here ſaid concerning the Diviſions between the figures 1 and 2, the ſame is to be underſtood of all the other parts; for as the diſtance between 1 and 2, is divided firſt into Ten unequal parts, and thoſe again into Ten other unequal parts; ſo alſo is the diſtance between 2 and 3, between 3 and 4, *&c.* till between 9 and 10: But this is to be underſtood, that as the ſpace between 1 and 2 is greater than that between 2 and 3, therefore the ſub-diviſions of Tens will be alſo leſſer; and ſo of all the reſt, as between 3 and 4, the ſpace is firſt divided into ten parts, and each of theſe but into five parts, each part ſignifying two parts: Alſo the ſpace between 6 and 7 is firſt divided into ten parts, and each of thoſe into two parts, each of which ſignifies five parts; and ſo the ſame between 9 and 10: and ſo the ſub-diviſions of the Scale will be more or leſs according as the Scale is in Length.

Numeration upon the Scale.

1. The figures 1, 2, 3, *&c.* to 10, upon the Scales do ſometimes ſignifie themſelves ſimply, ſometimes ten times themſelves, ſometimes 100 times themſelves.

Firſt, As when the figure 1 ſignifies only 1, then the ten grand Diviſions between 1 and 2 are 1.1, that is 1 and $\frac{1}{10}$; 1.2, that is 1 and $\frac{2}{10}$; 1.3, $1\frac{3}{10}$; 1.4, $1\frac{4}{10}$, *&c.*

Alſo, 2.1 is $2\frac{1}{10}$; 2.2 is $2\frac{2}{10}$, *&c.* And every one of the ten ſmaller ſub-diviſions ſignifies hundred parts between 1 and 2; as 1.01 is $1\frac{1}{100}$; 1.02 is $1\frac{2}{100}$; alſo 2.04 is $2\frac{4}{100}$; 7.08 is $7\frac{8}{100}$, *&c.*

Secondly, When the figure 1 ſignifies 10, then 2 is 20, 3 is 30, *&c.* and 10 is 100; and then the firſt grand diviſion after every figure is 10.1, 20.1, 30.1, *&c.* that is, $10\frac{1}{10}$, $20\frac{1}{10}$, $30\frac{1}{10}$, *&c.* The ſecond grand diviſion is $10\frac{2}{10}$, $20\frac{2}{10}$, *&c.* and one of the ſmaller ſub-diviſions is $10\frac{1}{100}$, $20\frac{1}{100}$, $30\frac{1}{100}$, *&c.*

Thirdly,

Thirdly, When the figure 1 signifies 100, then 2 is 200, 3 is 300, *&c.* and 10 at the end is 1000. And then, the first grand Division after each figure is 110, 210, 310, *&c.* And every of the smaller sub-divisions is $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, *&c.* And beyond these Numbers I think no Man will expect to work by Instrument; and therefore let this suffice for Numeration upon the Scales: For what is said concerning this single Scale, the same is to be understood of all the other, as the Double, Triple, and broken Scales.

CHAP. III.

MULTIPLICATION by the SCALES.

IN Multiplication there are three Terms, *viz.* the Multiplicand, the Multiplier, and the Product. And to work Multiplication upon the Scales, this is the

RULE:

The extent of the Compasses from 1 to the Multiplicand, shall reach the same way from the Multiplier to the Product: Or, The extent from 1 to the Multiplier, shall reach the same way from the Multiplicand to the Product.

The Proportion being,

As 1 to the { Multiplicand, / Multiplier, } so is the { Multiplier, / Multiplicand, } to the Product.

The following Examples will make this plain.

Example 1. *Let it be required to multiply* 9 *by* 7.

Extend the Compasses (upon the single Scale) from 1 to 9, (the Multiplicand,) the same extent will reach from 7 (the Multiplier) to 63, (the Product.)

Or, The extent of the Compasses between 1 and 7, will reach from 9 to 63 the Product, as before.

Example 2. *Let* 21 *be multiplied by* 3.

Extend the Compasses from 1 (at the beginning of the double Scale) to 3, the same extent shall reach from 21 to 63, the Product.

Or, Extend the Compasses from 1 (in the middle of the double Scale, (now called 10,) upwards to 21, the same extent will reach from 3 to 63, the Product, as before.

Or again by the single Scale, the extent from 1 to 3, will reach from 21 to 63, the Product.

Example 3. *Let it be required to multiply* 4.3 *by* 8.

Extend the Compasses from 1 at the beginning of the double Scale to 4.3, the same extent will reach from 8 (in the lower part of the Scale) to 34.4, the Product.

Or, The extent of the Compasses from 1 at the beginning of the double Scale to 8, will reach from 4.3 to 34.4, the Product.

Example 4. *Let* 5.25 *be to be multiplied by* 4.75.

Extend the Compasses from 1 at the beginning of the double Scale to 4.75, the same extent will reach the same way from 5.25, to 24.93, the Product.

Or, The extent from 1 to 5.25, will reach from 4.75, to 24.93, as before.

Example 5. *Let* 36.3 *be to be multiplied by* 23.5.

Extend the Compasses from 1 to 23.5, the same extent will reach from 36.3 to 853.05.

CHAP. IV.

DIVISION by the SCALES.

IN Division there are three Terms, *viz.* the Dividend, the Divisor, and the Quotient: And to work Division upon the Scales this is the

RULE:

The extent of the Compasses from the Divisor to 1, shall reach from the Dividend (counted the same way) to the Quotient: Or, the extent from the Divisor to the Dividend, shall reach the same way from 1 to the Quotient.

The Proportion being,
As the Diviſor is to the Dividend, ſo is 1 to the Quotient.
Or, As the Diviſor is to 1, ſo is the Dividend to the Quotient.
Examples will make this plain,

Example 1. *Let it be required to divide* 63 *by* 7.

Extend the Compaſſes from 7 the Diviſor (in the lower part of the double Scale) downwards to 1, the ſame extent will reach from 63, the Dividend, downwards to 9, the Quotient.

Or, The extent from 7, the Diviſor, upwards to 63, ſhall reach from 1, at the beginning of the Scale, upwards to 9, the Quotient.

Example 2. *Let it be required to divide* 63 *by* 3.

Extend the Compaſſes from 3, the Diviſor, downwards to 1, the ſame extent will reach from 63, the Dividend, downwards to 21, the Quotient.

Or, The extent from 3, the Diviſor, upon the ſingle Scale, upwards to 63 the Dividend, the ſame extent will reach upwards from 1 to 21, the Quotient, as before.

Example 3. *Let it be required to divide* 34.4 *by* 4.3.

Extend The Compaſſes from 4.3, the Diviſor, in the lower part of the double Scale, upwards to 34.4, the ſame extent will reach from 1 at the beginning to 8, the Quotient.

Or, the extent from 4.3, in the lower part of the double Scale, downwards to 1, the ſame extent will reach from 34.4 downwards to 8, the Quotient, as before.

Example 4. *Let it be required to divide* 24.93 *by* 5.25.

Extend the Compaſſes from 5.25, the Diviſor, in the lower part of the double Scale upwards to 24.93, the Dividend, the ſame extent will reach from 1 at the beginning to 4.75, the Quotient.

Or, The extent from 5.25 downwards to 1 at the beginning, will reach downwards from 24.93, the Dividend, to 4.75 the Quotient, as before.

Example 5. *Let it be required to devide* 853.05 *by* 36.3.

Extend the Compaſſes from 36.3, the Diviſor, (counted in the upper part of the double Scale) upwards to 853.05, the ſame extent will reach from 1 in the middle upwards to 23.5, the Quotient.

And let theſe Examples ſerve for Diviſion, they being the converſe to thoſe in Multiplication; and therefore ſerve to prove each other.

CHAP. V.

The Rule of Three (or Proportion) Direct and Reverſe by the Scales.

1. *Of the Direct Rule.*

THE Direct Rule of Three, teacheth how by having three Numbers given, to find a fourth in Proportion to them; and for the working of it by the Scales this is the

RULE:

Extend the Compaſſes from the firſt Number to the Second, the ſame extent will reach from the third to the fourth: Or, the extent from the firſt Number to the third, will reach from the ſecond to the fourth.

The Proportion being
As the firſt is to the ſecond, ſo is the third to the fourth.
Or, As the firſt is to the third, ſo is the ſecond to the fourth.
Examples to explain this,

Queſtion 1. *If* 4 *Shillings buy* 7 *Yards or Pounds of any Commodity, how many Yards or Pounds will* 12 *Shillings buy?*

Here the Terms given are 4, 7, and 12, of which the the firſt and third, *viz.* 4 and 12, are of the ſame Denomination, *viz.* both Shillings; and the ſecond Term, *viz.* 7, is of another Denomination, *viz.* Yards or Pounds; and of the ſame Denomination with that muſt the fourth Term which is required be.

Wherefore

Wherefore extend the Compaſſes from 4 (the firſt Term,) to 7 (the ſecond Term,) the ſame extent will reach from 12 (the third Term,) to 21 which is the fourth Number (or Term) required; and ſo many Yards or Pounds of any Commodity will 12 Shillings buy.

Or, The extent from 4 to 12, will reach from 7 to 21, as before.

Queſtion 2. *If the length of the ſhadow of any upright Building (as a Steeple, &c.) be 76 Foot, and at the ſame time the length of the ſhadow of a walking-ſtaff (or the like) being 3 Foot long, caſt its ſhadow 7 Foot: How high is that Building, as a Steple, or the like?*

The Proportion is,

As 7 Foot, the length of the ſhadow of the Staff,
Is to 3 Foot, the length of the Staff;
So is 76 Foot, the length of the ſhadow of the Building,
To 32.57, that is 32 Foot and 7 Inches for the heighth of the Building.

Wherefore extend the Compaſſes from 7 downwards to 3, the ſame extent will reach from 76 downwards, to 32.57 the heighth required.

Or, The extent from 7 upwards to 76, will reach from 3 upwards to 32.57, as before.

Queſtion 3. *If 5 fathom of Rope or Cable do weigh 22 Pound, what will 42 fathom of the ſame Cable weigh?*

Extend the Compaſſes from 5 to 22, the ſame extent will reach from 42 to 184.8, and ſo many pound will 42 fathom of the ſame Cable weigh.

Or, The extent from 5 to 42, will reach from 22 to 184.8, as before.

2. Of the Reverſe Rule.

As in the Direct Rule, ſo much as the third Term is greater than the firſt, ſo much the fourth is greater than the ſecond; ſo contrarily in this Rule Reverſe, by how much the third Term is leſs than the firſt, by ſo much is the fourth Term leſs than the ſecond. For,

As 2 is to 4, ſo is 6 to 12 Direct.
Contrarily, As 12 is to 6, ſo is 4 to 2 Reverſe.

And for the working of this Reverſe, or backward Rule of Three, upon the Scales, this is the

RULE:

Extend the Compaſſes from one Term to the other of like Denomination, the ſame extent being laid the contrary way from the other Term of the other like Denomination, ſhall reach to the Number which will anſwer the Queſtion.

Examples of this Rule.

Queſtion 1. *If 80 Men will doe a piece of Work in 12 Days, in how many Days will 40 Men doe the ſame piece of Work?*

It is apparent that fewer Men (as 40) muſt have longer time than more Men (*viz.* 80.) therefore the fourth Term muſt be greater than the ſecond; wherefore,

Extend the Compaſſes from 80 downwards to 40, (the Terms of like Denomination, *viz.* Men,) the ſame extent will reach the contrary way, *viz.* upwards, from 12 (the Number of the other Denomination, *viz.* Days) to 24, the Number of Days that 40 Men will doe the Work in.

Queſtion 2. *If a Board, being 12 Inches broad, requires 12 Inches in length to make a Square Foot; how many Inches in length will make a Foot Square, when the Board is 18 Inches broad?*

Extend the Compaſſes from 12 upwards, to 18 (the two Terms of breadth,) the the ſame extent will reach from 12, (the middle Term of length,) downwards to 8 Inches, (the fourth Term of length;) for the greater breadth, the leſſer length. So again,

If 18 Inches in breadth require 8 Inches in length to make a Foot, what length will 7 Inches in breadth require?

Extend the Compaſſes from 18, downwards to 7 (the two Terms of breadth,) the ſame extent will reach from 8, (the Inches of length,) upwards to 20.57 Inches; for the length to make a Square Foot, at ſeven Inches broad.

CHAP. VI.

Some other Uses of the Scales in Proportions of ſeveral kinds.

§ I. *Two Numbers being given to find a third, fourth, fifth,* &c. *in continual Proportion Geometrical.*

TO perform this upon the Scales, this is the

RULE:

The extent of the Compaſſes from one of the given Numbers to the other, ſhall reach (the ſame way) from that ſecond to a third, and from that third to a fourth, &c.

Example 1. *Let the Numbers given be* 1 *and* 2.

Extend the Compaſſes from 1 (at the beginning of the double Scale) to 2, the ſame extent will reach from 2 to 4, from 4 to 8, from 8 to 16, from 16 to 32, from 32 to 64, from 64 beyond the Line; wherefore (keeping the Compaſſes ſtill at the ſame extent,) ſet one Foot in, 64 (counted in the lower part of the double Scale,) and the other will reach from 64 to 128, and from 128 to 256, from 256 to 512, *&c. ad infinitum.*

§ II. *Between two Numbers given, to find a Mean Proportional Number.*

A Mean Proportional is ſuch a Number, which being multiplied in it ſelf, the Product ſhall be equal to the Product of the two given Numbers multiplied by each other; and to find ſuch a Number by the Scales, this is the

RULE:

The extent of the Compaſſes from one of the given Numbers to the other, (taken upon the double Scale,) that diſtance applyed to the Leſſer of the given Numbers upon the broken Scale, ſhall reach the ſame way to the Mean Proportional Number ſought.

Example. *Let the two given Numbers be* 8 *and* 18.

Extend the Compaſſes from 8 to 18 upon the double Scale, the ſame extent will reach from 8 upon the broken Scale to 12; ſo is 12 a Mean Proportional between 8 and 18; for 18 multiplied by 8, produceth 144, and ſo doth 12 by 12 produce 144.

A Practical Example of this Rule.

Queſtion. *There is a piece of Land which is* 32 *Rod long, and* 8 *Rod broad, which contains* 256 *Square Rods; how long muſt the ſide of a Square piece of Land be which ſhall contain the like Number of Square Rods?*

Extend the Compaſſes from 8 to 32 upon the double Scale, that extent applied to the broken Scale, will reach from 8 to 16; and ſo many Rod long muſt the ſide of the Square piece of Land be; for 32 multiplied by 8 produceth 256, and ſo much doth 16 by 16 produce.

§ III. *Between two Numbers given, to find two Mean Proportional Numbers.*

Example. *Let the two Numbers given be* 4 *and* 32.

Extend the Compaſſes from 4 to 32 upon the Triple Scale, the ſame extent will reach from 4 (the leſſer Number,) upon either the ſingle or broken Scale to 8, for the firſt Mean, and from 8 to 16 the other Mean.

For the Product of 4 by 32 is 128, and ſo is the Product of 16 by 8 alſo 128.

A Practical Example of this Rule.

Queſtion. *There is an Iron Shot or Bullet, whoſe Diametre is* 2.78 *Inches, and it weigheth* 3 *pound; what ſhall the Diametre of an Iron Shot be which weigheth* 71 *pound?*

Extend the Compaſſes upon the Triple Scale from 3 to 71, the ſame diſtance applied from 2.78 on the ſingle Scale, will reach to 8, the Diametre of the Shot required.

CHAP.

CHAP. VII.

To find the Square Root of any Number by the Scales, both with and without Compasses.

THis Work may be performed diverse ways, and by several of the Scales, and as they are here disposed both with and without Compasses; and the manner how to find the Square Root will best appear by Examples.

Example 1. *Let it be required to find the Square Root of* 36.

1. *By the single Scale and Compasses.*

Extend the Compasses from 10 at the end of the Scale downwards, (because the Number of figures in the given Number are even, *viz.* two,) to 36, half that distance will reach from 10 downwards to 6; which is the Square Root of 36.

Example 2. *Let the Square Root of* 256 *be required.*

Extend the Compasses from 1 at the beginning of the Scale, upwards to 256, (because the Number of figures in the given Number are odd, *viz.* three,) half that distance will reach from 1 at the beginning of the Scale, upwards to 16, which is the Square Root of 256.

2. *By the double Scale and Compasses.*

Extend the Compasses from 1 in the middle, downwards to 36, half that distance will reach from 1 in the middle downwards to 6, which is the Square Root of 36.

Also extend the Compasses from 1 in the middle, upwards to 256, half that distance will reach from 1 in the middle to 16, the Square Root of 256.

3 *By the double Scale, and the broken Scale joyntly.*

Extend the Compasses from 1 in the middle of the double Scale, upwards to 36, the same extent will reach from 1 at the beginning of the double Scale, to 6 in the broken Scale, which is the Square Root of 36.

Also extend the Compasses from 1 at the beginning of the double Scale, to 256 upon the same Scale; the same extent will reach from 1 in the broken Scale, to 16 in the same Scale, which is the Square Root of 256.

4. *By the double and broken Scale joyntly without Compasses.*

You are first to consider whether the Number of figures in your given Number (being Integers or whole Numbers) be even or odd; if they be even, as 2, 4, 6, 8, 10, *&c.* or odd; as 3, 6, 9, 11, *&c.* If the Number of places be even, count the Number in the lower part of the double Scale; but if the Number of figures be odd, count the Number in the upper part of the double Scale; in either of which Cases, right against your given Number found in the double Scale, you shall find the Square Root thereof in the broken Scale.

So if you count 36 in the lower part of the double Scale, right against it in the broken Scale you shall find 6, which is the Square Root of 36.

Also, if you count 256 in the upper part of the double Scale, right against it in the broken Scale stands 16, which is the Square Root of 256.

Example 3. *Let* 5276 *(a number consisting of even places) be given, and let the Square thereof be required.*

Count the given Number 5276 in the lower part of the double Scale, and right against it in the broken Scale you shall find 72.65; which is the neareſt Square Root of 5276.

Example 4. *Let* 72796 *(being a Number of odd places) be given, and the Square Root thereof is required.*

Count 72796 in the upper part of the double Scale, and right against it in the broken Scale, you shall find 296; which is the near Square Root of 72796.

A Root being given to find the Square thereof by the Scales, both with and without Compasses.

1. *By the single Scale with Compasses.*

Example 1. *Let* 6 *be a Root given, and let the Square Root thereof be required.*

Extend the Compasses from 1 to 6, and that extent will reach from 6 to 36, the Square thereof.

Example

Example 2. *Let 16 be a Root given, and let the Square thereof be required.*

Extend the Compaſſes from 1 to 16, the ſame will reach from 16 to 256, the Square thereof.

2. By the double Scale and Compaſſes.

This is done in all reſpects as the former, for the extent from 1 in the middle to 6, will reach from 6 in the lower part, to 36 in the upper part, for the Square of 6.

Alſo, The extent from 1 at the beginning to 16, will reach from 16 in the upper (or lower) parts, to 256 the Square of 16.

3. By the double Scale and broken Scale without Compaſſes.

Example 3. *Let 6 be a Root given, whoſe Square is required.*

Count 6 upon the broken Scale, and right againſt it in the double Scale you ſhall find 36, the Square thereof.

Alſo, Count 16 upon the broken Scale, and right againſt it in the double Scale, you ſhall find 256 for the Square of 16.

4. Other ways by the ſame Scales.

1. When the given Root conſiſts of one ſingle figure, ſeek that figure in the broken Scale, and againſt it in the double Scale you ſhall have its Square; ſo 2 being the Root given, that found in the broken Scale againſt it ſtands 4, the Square thereof: Alſo againſt the Root 8 in the broken Scale, ſtands 64, the Square thereof in the double Scale.

2. When the Root given conſiſts of two figures, ſeek it in the broken Scale as before, and againſt it in the double Scale is the Square thereof; ſo 20 being the Root, 400 will be the Square thereof, by adding of two Cyphers to 4, for this reaſon, *viz.* Look by how many figures or Cyphers you increaſe the Root more than on the Scale is expreſſed, twice ſo many figures or Cyphers muſt be added to the Square: For when 6 upon the broken Scale is called 60, (increaſed by one Cypher,) then 36 muſt be called 3600, (increaſed by two Cyphers; alſo when 7 is called 700, (increaſed by two Cyphers,) then 49 the Square thereof muſt be called 490000, (increaſed by 4 Cyphers.)

3. When the Root conſiſts of three figures, ſeek it (as before) in the broken Line, and againſt it in the double Line you ſhall have the Square thereof; ſo 146 being the Root, which found in the broken Scale, againſt it is 21316, by adding of four figures to the 2 upon the Scale, ſtanding againſt 146: Alſo the Square of the Root 450, will be found (by the foregoing reaſon) to be 202500, *&c.* But to more places than theſe, I preſume none will attempt to approach by any Inſtrument how large ſoever.

CHAP. VIII.

To find the Cube Root of any Number by the Scales, both with and without Compaſſes.

THis Work may be performed ſeveral ways, and by ſeveral of the Scales, as by the Examples following will appear.

Example 1. *Let 216 be a Number given, whoſe Cube Root is required.*

1. By the ſingle Scale and Compaſſes.

Extend the Compaſſes from 10 at the end of the Scale downwards to 216, upon the ſingle Scale, one third part of that diſtance will reach from 10 downwards to 6, which is the Cube Root of 216.

Again, Let the Cube Root of 1728 be required.

The extent of the Compaſſes between 1 and 1728 upon the ſingle Scale, being divided into three equal parts, that diſtance will reach from 1 to 12, which is the Cube Root of 1728.

2. By the ſingle Scale, and the triple Scale, which face each other without Compaſſes.

When any Number is given, and the Cube Root thereof is required, you muſt firſt conſider whether your given Number conſiſts of 1, 4, 7, or 10 places, of 2, 5, or 8 places; or of 3, 6, or 9 places.

If

If it consist of { 1, 4, 7, or 10, / 2, 5, or 8, / 3, 6, or 9, } places, it must be counted upon the { First, / Second, / Third, } part of the Triple Scale.

For the Triple Scale, as it is divided into three parts, so also is it numbered by three distinct Denominations; the first whereof is numbered by 1, 2, 3, *&c.* to 10 at the end; the second part by 10, 20, 30, *&c.* to 100 at the end thereof; the third part by 100, 200, 300, *&c.* to 1000 at the end thereof.

But the number of figures in the Root, is according to the Number of points that the Number given is capable of by extraction by the Pen, the Number being always pointed; at the first, fourth, seventh, and 10 figure, and so forwards, as in this Number following.

5 2 7 9 1 6 7 3 2 5

And such a Number as this is, will have four Integers in the Root of it.

Examples of these three Varieties.

1. Let 8 be the Number given; this counted upon the first part of the Triple Scale, against it on the single Scale stands 2, the Cube Root of 8.

2. Let 27 be the Number given; this counted upon the second part of the Triple Scale, against it on the single Scale stands 3, which is the Cube Root of 27.

3. Let 512 be the Number given; this counted upon the third part of the Triple Scale, against it on the single Scale you will find 8, for the Cube Root of 512.

Thus for Numbers whose Root consists of one single Digit; now for Numbers whose Root will consist of two figures.

1. Let the Number given be 6859; count this Number in the first part of the Triple Scale, against it you shall find 19 in the single Scale, which is the Cube Root of 6859.

2. Let 91125 be the Number given; which being counted upon the second part of the Triple Scale, against it in the single Scale stands 45, which is the Cube Root thereof.

3. Let 474552 be the given Number, this counted upon the third part of the Triple Scale, will point out 78 in the single Scale, which is the Cube Root thereof.

And thus for Numbers whose Roots consist of two figures: Now for Numbers whose Roots consist of three figures.

So if the Number given were { 1 2 5 9 7 1 2 / 1 1 6 9 7 0 8 3 / 2 7 2 0 9 7 7 9 2 } which counted on the { First, / Second, / Third, } part of the tripleScale, will give { 108 / 227 / 648 } for the Cube Root.

To find the Cube Number belonging to any Cube Root given.

The Root being found upon the single Line, directly against it upon the Triple Line is the Cube Number answerable to the Root given.

But when the Root is increased by one figure, the Cube Number must be increased by three figures; when the Root is increased by two figures, the Cube Number must be increased by six figures, *&c.*

Example: The Cube of 2 is 8. the Cube of 4 is 64, the Cube of 9 is 729, directly as the figures upon the Scales give it.

But the Cube of 20 (where the Root 2 is increased by one figure) is 8000, (the Number upon the Scale increased by three figures,) and the Cube of 80 is 512000, being three figures more than 512, the Number upon the Scale.

Again, The Cube of 150 being two figures more than, $1\frac{1}{1}$ (or 1.50,) is increased by six-figures more than $3\frac{375}{1000}$, the Cube of 1.50, *viz.* 3375000.

The Cube of a whole Number, and a Fraction is found as any whole Number.

Example: The Cube Root of $38\frac{6}{10}$ is $3\frac{13}{100}$, *&c.* But to pretend to Fractions of greater Numbers by the Scales, will but frustrate your expectation: but a Cube Root in Integers, may be given to four places, by a large Scale. And let this suffice for the Square and Cube Root by inspection.

CHAP.

CHAP. IX.

Of the Reduction of Vulgar Fractions to Decimal Parts.

THE Proportion for effecting of this Work is,

As the Denominator of the Vulgar Fraction,
Is to the Numerator of the ſame Fraction;
So is 1, (or Unity,)
To the Numerator of a Decimal Fraction (or part,) whoſe Denominator is an Unite, with ſo many Cyphers as there are figures in the Numerator.

Example 1. *Let it be required to reduce* $\frac{3}{4}$ *to a Decimal Part.*

1. *By the ſingle Scale.*

Set one Foot of the Compaſſes upon 4, the Denominator of the given Fraction, and extend the other downwards to 3, the Numerator of the ſame Fraction, the ſame extent will reach from 10, (or 100,) at the end of the Scale, to .75 which is the Decimal Fraction anſwering to $\frac{3}{4}$: For as $\frac{3}{4}$ of a pound ſterling, or 20 *s.* is 15 *s.* or $\frac{3}{4}$ of a Foot is 9 Inches, ſo .75 is $\frac{3}{4}$ of a Foot, the Foot being divided into 100 parts.

2. *By the double Scale.*

Example 2. *What Decimal Part ſhall be equal to* $\frac{7}{21}$?

Extend the Compaſſes from 21 downwards to 7, the ſame will reach from 1 in the middle of the Scale downwards to .33, which is the decimal part anſwering to $\frac{7}{21}$.

Example 3. *What Decimal part ſhall be equal to* $\frac{26}{54}$?

Extend the Compaſſes from 54 downwards to 26, the ſame extent will reach downwards from 10 (or 1000) at the end of the Scale, to .482 almoſt, which is the Decimal part equal to $\frac{26}{54}$.

CHAP. X.

Of Simple Intereſt by the SCALES.

THis Work will be beſt illuſtrated by propoſing and anſwering ſome Queſtions hereunto belonging.

Queſtion 1. *If* 100 li. *in* 12 *months will gain* 6 li. *Intereſt, what will* 326 li. *gain in the ſame time?*

As 100 is to 6, ſo is 236 to 19.56.

By the double Scale.

Extend the Compaſſes from 1 (in the middle) downwards to 6, the ſame extent will reach from 326 downwards to 19.56 *li.* which reduced, as is taught before in Decimal Arithmetick, is 19 *li.* 11 *s.* 2 *d.* 2 *q.* and ſo much will 326 *li.* amount unto in a year.

Queſtion 2. *What will the Intereſt of* 175 li. 18 *s.* 11 *d.* (*or* 175.95,) *at* 6 li. per Cent. *for a year amount unto?*

Extend the Compaſſes from 1 in the middle downwards to 6, the ſame extent will reach from 175.95 to 10.554; which reduced, is 10 *li.* 11 *s.* $2\frac{1}{2}$ *d.*

Queſtion 3. *What will the Intereſt of* 784 li. *for* 9 *months amount unto, at* 6 li. *in the* 100 li. *for a year?*

This will require two Operations, and therefore two Proportions.

1. As 100 is to 6, ſo is 784 to 47.04.
2. As 12 M. is to 47.04, ſo is 9 M. to 35.28.

1. Extend the Compaſſes from 1 in the middle downwards to 6, the ſame extent will reach the ſame way, from 784 to 47.04, which is the Intereſt of 784 *li.* for a year.

2. Extend

2. Extend the Compaſſes from 12 months to 47.04, the ſame extent will reach the ſame way from 9 Months to 35.28; which reduced, is 35 *li.* 5 *s.* 7 *d.* 1 *q.* the Intereſt of 784 *li.* for 9 months.

Queſtion 4. *If I receive* 60 li. 8 s. 4 d. *(or* 60.162,*) for the Intereſt of* 755 li. 5 s. *(or* 755.25,*) for a year; What Rate of Intereſt have I for my Money?*

The Proportion is,

As 755.25 is to 60.162, ſo is 100 *li.* to 8 *li.*

Extend the Compaſſes from 755.25 downwards to 60.162, the ſame extent will reach the ſame way from 100 to 8, which was the Rate of Intereſt required.

CHAP. XI.

Of ANATOCISM, or Compound Intereſt, by the SCALES.

THis Rule alſo will beſt appear by reſolving of Queſtions thereunto belonging; of which I ſhall inſtance theſe few following:

Queſtion 1. *Unto what Summ of Money will* 143 li. 10 *s. (or* 143.5,*) be increaſed in* 10 *years, accounting Intereſt upon Intereſt, at* 6 per Cent?

The Proportion is,

As 100 *li.* is to 106 *li.* ſo is 143.5 *li.* to 257 *li.*

Extend the Compaſſes from 100 to 106, upon the Single Scale, the ſame extent being ſet unto 143.5, and turned over from thence upon the ſame Scale, will (at the tenth remove) reſt upon 257 *li.* and unto ſo much will 143 *li.* 10 *s.* be increaſed in ten years.

Queſtion 2. *What is* 275 li. *to be paid at the end of ten years worth in preſent Money, diſcounting Intereſt upon Intereſt, at* 6 per Cent?

This is the Converſe of the former Queſtion; wherefore extend the Compaſſes from 100 to 106, as before; then ſet one point upon 257, and turn that diſtance downwards upon the Scale ten times, and at the tenth remove, the Compaſs point will reſt upon 143.5 *li.* and ſo much is 257 *li.* due at ten years end, worth in preſent Money.

Queſtion 3. *What is a yearly Rent or Annuity of* 10 li. *a year forborn four years, worth at* 8 li. per Cent. *Compound Intereſt?*

The Proportion is,

As 8 *li.* is to 100 *li.* ſo is 10 *li.* to 125 *li.*

Extend the Compaſſes from 8 to 100, the ſame extent will reach the ſame way from 10 to 125; then (by the firſt Queſtion) you may find that 125 *li.* forborn four years at 8 *per Cent.* will be worth 170 *li.* from which ſumm ſubtract 125 *li.* and there will remain 45 *li.* and ſo much will the Rent or Annuity, with the forbearance, of four years, be worth in preſent Money at 8 *per Cent.*

Queſtion 4. *What is a Leaſe or Annuity of* 12 li. *a year to continue* 16 *years, worth in ready Money, at* 10 li. *in the* 100, *Compound Intereſt?*

The Proportion is,

As 10 *li.* is to 100 *li.* Principal, ſo is 12 *li.* to 120 *li.* Principal.

Extend the Compaſſes from 10 (upon the Triple Scale) to 100, the ſame extent will reach from 12 unto 120: Then find by the foregoing Queſtion, what 120 *li.* being forborn 16 years, will be worth preſently, which will amount unto 551 *li.* Principal and Intereſt; from which take 120 *li.* the Principal, and there will remain 431 *li.* the Arrears: Then by the ſecond Queſtion, find what 431 *li.* due after the expiration of 16 years, is worth in preſent Money at 10 *per Cent.* the anſwer will be 93 *li.* 14 *s.* and ſo much is the Leaſe or Annuity of 12 *li. per Annum* for 16 years.

Queſtion 5. *What Annuity to continue* 16 *years will* 500 li. *purchaſe, the Purchaſer to have* 10 li. per Cent. *profit for his Money?*

By the laſt Queſtion it was found that 93 *li.* 14 *s.* (or 93.7 *li.*) will purchaſe 12 *li.* a year for 16 years, at 10 *li. per Cent.* Which known,

The Proportion is,

As 93.7 *li.* is to 12, ſo is 500 *li.* to 64 *li.*

Extend the Compaſſes from 93.7 to 12; the ſame extent will reach (the ſame way) from 500 to 64 *li.* and ſuch an Annuity for 16 years will 500 *li.* purchaſe, and the Purchaſer have 10 *li. per Cent.* Compound Intereſt for his Money.

Many more Queſtions of this nature might be propoſed and wrought by the Scales, as alſo Queſtions in all the other Rules of Arithmetick, but let theſe ſuffice; and for the farther Uſe of theſe Scales in other kinds of Menſurations, *&c.* they ſhall be ſhewed in their due places.

The End of the Fourth Part, and of the Firſt Book.

CURSUS MATHEMATICUS.

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Cursus Mathematicus.

BOOK II.

OF GEOMETRY PLAIN.

PART I.

Proœme.

Of the Antiquity, Progress, and Benefit, of *Geometry.*

GEOMETRY, *which* Philo *the Jew calleth the Princess and Mother of all Sciences, hath properly its Name from* Measuring the Earth; *and is called of the* Greeks Γεωμετρία, *of* γῆ, i. e. Terra, *and* μετρέω, i. e. metior; *and therefore* Architas, *a famous Geometrician, by* Horace *was called* Mensorem.

This Science had its first original in Egypt; *for when* Nilus *with its Inundation drowned and confounded the Limits and Boundings of their Lands, certain of the most ingenious Inhabitants (necessity constraining them) found out some Rules of Geometry, by the benefit whereof (after the fall of the Water) every one had his portion of Ground allotted to him again. Out of* Egypt Thales *brought it into* Greece, *where it received great perfection, and almost extended to its present maturity. The Utility and Benefit of this most excellent Art, is of such importance, that without it we can scarcely live, much less preserve our selves from danger, or hinder the irruption of foreign Enemies; for by the Rules hereof the Geodetian may measure, plot, and give you the true Symetry, Situation, and superficial Content of your Lands; search out all manner of Highths, Depths, and Distances, whether they be accessible or inaccessible: The Military Architect may hereby fabricate his Fort: The General, or Major, embattail his Army: The Quarter-master order and set out his Quarters, and incamp his Soldiers: The Engineer plant his Cannon, convey his Mine to any assigned place under the Ground, to doe execution upon his Enemy to their greatest disadvantage. Neither doth the benefit of this Science terminate in these terrestrial practices upon the superficies of the Earth only, but it extendeth to the Mensuration of the Celestial Bodies also, discovering the Magnitudes, Motions, Appearances, and Distances of the Sun, Moon, Planets, and fixed Stars; and solveth the* Phænomena *of that heroick* Hypothesis *of* Copernicus, *of the Stability of the Sun, and Motion of the Earth; and confuteth (by demonstration) the absurd Conjectures of the Ancients concerning its Species. And yet farther, by help hereof, the complete Sciographer can cause the Sun to trace out his way upon the Earth; and by the Shadow of an* Axis, *to point out unto us those Atomes of Time into which our artificial Day is artificially divided. In fine, how assistant it is to all persons in general, as well as particular, those that are proficients in Opticks, Trochilike Perspective, Geography, &c. do well know.*

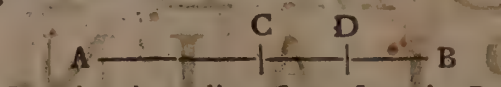
SECT. I.

Geometrical DEFINITIONS.

I. *A Point is that which hath no Part.*

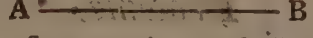
THat is, it hath no Parts into which it ſhould be divided: So that a Point is the leaſt thing that by Mind or Underſtanding can be imagined or conceived, than which there can be nothing leſs; as the Point or Prick ſigned by A. A .

II. *A Line is a Length without Breadth.*



There pertain to Quantity three dimenſions, Length, Breadth, and Depth, (or Thickneſs,) of which a Line is the firſt, and hath Length only, without Breadth or Thickneſs; as the Line A B, which may be divided into parts, either equally in the point C, or unequally in the point D.

III. *The Ends or Limits of a Line are Points.*



For a Line hath its beginning from a Point, and likewiſe endeth in a Point; as the Points A and B, are only the ends of the Line A B, and are no parts thereof.

IV. *A Right Line is that which lieth equally between its Points.*

Figure I. As the Line A B, which lieth ſtreight and equally between the Points A and B, and not bending or ſwerving on either ſide, as the Lines A G B, and A D B do; ſo that a Right Line is the ſhorteſt diſtance that can be drawn between Point and Point.

V. *A Superficies is that which hath only Length and Breadth.*

Figure II. A Superficies is the ſecond kind of Quantity, and is capable of two dimenſions, *viz.* Length and Breadth; as the Superficies A B C D, whoſe Length is from A to B, or from C to D; and its breadth from A to C, or from B to D.

VI. *The Extremes (or Limits) of a Superficies are Lines.*

As the Ends, Bounds, or Limits of a Line are Points, ſo the Bounds or Limits of a Superficies are Lines; as the Superficies A B C D is bounded or limited by the four Lines A C and B D limiting the Breadth, and A B and C D the Length thereof.

VII. *A Plain Superficies is that which lieth equally between its Lines.*

Figure III. As the Plain Superficies A B C D lieth equally between the two Lines A B and C D, between A C and B D; and as a Right Line lieth equally between its Points, ſo a Plain Superficies lieth equally between its Lines.

VIII. *A Plain Angle is an Inclination (or bowing) of two Right Lines the one to the other, and the one touching the other, and not being directly joyned together.*

Fig. III As the two Lines A B and B C incline the one to the other, and touch each other in the Point B; in which Point, by reaſon of the Inclination of the ſaid two Lines, is made the Angle A B C: But if the two Lines which incline one to the other, do when they meet make one ſtreight Line, then do they make no Angle at all; as the Lines D E and E F incline one to the other, and touch each other in the Point E, and yet they make no Angle.

IX. *And if the Lines which contain the Angle be Right Lines; then it is called a Right-lined Angle.*

As the Angle A B C is a Right-lined Angle, becauſe it is contained of the two Right Lines A B and C B.

And

And here note, That an Angle (for the moſt part) is denoted by three Letters, of which the middlemoſt Letter repreſents the Angular Point; as in the Angle A B C, the Letter B denotes the Angular Point. Figure III.

A Crooked-lined Angle is that which is contained of Crooked Lines, as F G H.

A Mixt Angle is that which is contained of one Right Line and one Crooked Line, as K L M, and may be variouſly figured.

Of Angles there are three kinds, *viz.* Right, Acute, and Obtuſe, whoſe Definitions follow.

X. *When a Right Line ſtanding upon a Right Line maketh the Angles on either ſide equal, then either of thoſe Angles is a Right Angle; and the Right Line which ſtandeth erected is called a Perpendicular Line to that upon which it ſtandeth.*

As upon the Right Line C D, ſuppoſe there do ſtand another Right Line A B, in ſuch ſort that it maketh the Angles A B C and A B D, on either ſide of the Line A B, equal, then are each of thoſe Angles A B C and A B D right Angles; and the Line A B, which ſtandeth erected upon the Line C D (without inclining on either ſide) is a Perpendicular. Figure IV.

XI. *An Obtuſe Angle is that which is greater than a Right Angle.*

As the Angle C B E is an Obtuſe Angle, it being greater than the Right Angle A B C, by the quantity of the Angle A B E. Figure V.

XII. *An Acute Angle is that which is leſs than a Right Angle.*

As the Angle E B D is an Acute Angle, it being leſs than the Right Angle A B D, by the quantity of the Angle A B E.

XIII. *A Limit, or Term, is the End of any thing.*

Foraſmuch as there is no Quantity (or Magnitude) of which Geometry entreateth, but it hath Bounds or Limits; as Points are the Bounds or Limits of Lines; Lines of Superficies; and Superficies of Solids, (or Bodies.)

XIV. *A Figure is that which is contained under one Limit or Term, or many.*

As the Figure A is contained under one Line or Limit, B under two, C under three, D under four, E under ſix, *&c.* which are their reſpective Bounds or Limits. Figure VI.

XV. *A Circle is a Plain Figure, contained under one Line, which is called a Circumference, (or Peripherie,) unto which all right Lines, drawn from one Point within the Figure, and falling upon the Circumference thereof, are equal the one to the other.*

As the Figure B C D, contained under the one crooked Line B C D, is a Circle, whoſe Circumference is B C D, and in the middle thereof is a Point A, from which all the right Lines A B, A C, A D, drawn from that Point to the Circumference B C D, are equal. Figure VII.

XVI. *And that Point A is called the Centre of the Circle.*

XVII. *A Diametre of a Circle is a Right Line, which, drawn by the Centre thereof, and ending at the Circumference on either ſide, divideth the Circle into Two equal parts.*

As the Line B A C is a Diametre, becauſe it paſſeth from the Point B of the Circumference on the one ſide, to the Point C on the other ſide; and paſſeth alſo by the Point A, which is the Centre of the Circle; and moreover it divideth the Circle into two equal parts, namely B D C above, and B E C below. Figure VIII.

XVIII. *A Semicircle is a Figure which is contained under the Diametre, and under that part of the Circumference which is cut off by the Diametre.*

As, B A C is a Semicircle, and alſo B D C.

XIX. *A Section, or portion of a Circle, is a Figure which is contained under a Right Line and a part of the Circumference, greater or leſſer than a Semicircle.*

Figure VIII. A right Light drawn from one ſide of the Circumference of a Circle, divideth the Circle into two unequal parts or Sections; as the Line F G divideth the Circle DBFEGC into two unequal Sections, namely, into the Section FBDCG, greater than a Semicircle, and into the Section F E G, leſſer than a Semicircle.

The Semidiametre of a Circle is half the Diametre, or it is any Right Line drawn from the Centre to the Circumference of the Circle, as the Lines A B, AC, A D, in Fig. 7. are all of them Semidiametres; and it is often times called the Radius *of the Circle.*

XX. *Right Lined Figures are ſuch as are contained under Right Lines.*

Of which ſome are contained under Three, ſome under Four, Five, Six, or more Right Lines; but no Right Lined Figure can conſiſt of leſs than three Right Lines.

XXI. *Three ſided (or Trilateral) Figures are ſuch as are contained under Three Right Lines.*

Figure IX. As the Figure A B C is a Figure of three Sides, for that it is contained within the limits or bounds of the three Right Lines AB, BC, and C A: And a Figure of three Sides is called a Triangle.

XXII. *Four ſided (or Quadrilateral) Figures, are ſuch as are contained under Four Right Lines.*

As the Figure E F G H, is a Four ſided (or Quadrilateral) Figure, for that it is contained, limitted, or bounded, under the four Right Lines D E, E G, G F, and F D.

XXIII. *Many ſided Figures are ſuch as have more Sides than Four.*

As theſe Figures, H, K, L, and infinite others.

And here it is to be noted, That every Right Lined Figure hath as many Angles as it hath Sides, and taketh its Denomination as well from its number of Angles as of Sides; as a Three ſided Figure from its Three Sides, and a Triangle from its Three Angles; alſo a Four ſided Figure is called a Quadrilateral, from its Four ſides; and a Quadrangular Figure, from its four Angles, &c.

XXIV. *Of Trilateral Figures, (or Triangles.) An Equilateral Triangle is that which hath Three Equal Sides.*

Figure X. Triangles have their differences partly of their Sides, and partly of their Angles: As touching the difference of their Sides, there are three kinds; For, 1. Either all the Three Sides are equal; and then it is called an Equilateral Triangle, as is the Figure A: Or, 2. Two only are equal, and the third unequal; and then it is called an *Iſoſceles*, as is the Figure B: Or, 3. All the Three Sides are unequal; and then it is called *Scalenum*, as is the Figure C.

XXV. *An* Orthigonium *(or Right Angled Triangle) is a Triangle which hath one Right Angle.*

Figure XI. As there are Three kinds of Triangles by reaſon of the diverſity of Sides, ſo there are Three kinds alſo by reaſon of the diverſity of Angles: For every Triangle containeth either 1. One Right Angle and two Acute Angles, as the Figure D; and then it is called *Orthogonium*, the Angle at E being a Right Angle, and the two other Angles at F and G being Acute: Or, 2. The Triangle hath one Obtuſe Angle and two Acute, as is the Figure H; and then it is called *Ambligonium*, the Angle at K being Obtuſe, and the other two at L and M Acute: Or, 3. The Triangle hath all the Three Angles Acute, as is the Figure R, all whoſe Angles, at N, O, and P, are Acute, and ſuch a Triangle is called *Oxigonium.*

XXVI. *Of Four ſided Figures (or Quadrilaterals) a Quadrate or Square, is that whoſe Sides are equal, and whoſe Angles are right.*

Figure XII. As Triangles have their difference and variety, by reaſon of their Sides and Angles, ſo have Quadrilateral Figures alſo.

And

And First, Of four ſided Figures. If all the Sides be equal, and the Angles right Angles, as the Figure O, whoſe Sides A B, B C, C D, and D A, are all equal, and the ſeveral Angles at A, B, C, and D, are Right Angles, then ſuch a Figure is called a Quadrate or Square. Figure XII.

Secondly, A four ſided Figure having one Side longer, or Square-like, as the Figure P, having the Sides E F and G H longer than the Sides E G, and F H, and all the Angles at E, F, G, and H, right Angles, then ſuch a Figure is called a long Square, or a Rectangled Parallelogram.

Thirdly, If the Quadrilateral Figure have four equal Sides, but not right Angles, as the Figure Q, whoſe four Sides K L, L M, M N, and N K, are all equal, but the Angles at K and M are obtuſe, and the Angles at L and N are acute Angles, ſuch a Figure is called a *Rhombus*, or Diamond Figure.

Fourthly, If the Quadrilateral Figure have its two oppoſite Sides, and oppoſite Angles equal each to its oppoſite, as the Figure R, whoſe oppoſite Sides S T and V X are equal, and the other two Sides S V and T X alſo equal, and alſo the Angle at S equal to that at X, acute; and the Angle at T equal to that at V, obtuſe; ſuch a Figure is called *Rhomboyades*, or Diamond-like Figure.

XXVII. *All other Quadrilaterals, or Four Sided Figures, (beſides thoſe laſt mentioned,) which have neither equal Sides, nor equal Angles, are called* Trapezia, *or Tables.*

As are the Figures W and Z.

XXVIII. *Parallel (or equidiſtant) Right Lines are ſuch, which being in the ſame Superficies, and infinitely produced on both Sides, would never meet.*

A ——————— B
C ——————— D

And ſuch are theſe two Lines A B and C D.

XXIX. *A Parallelogram is a Quadrilateral Figure, whoſe oppoſite Sides are parallel, or equidiſtant.*

And ſuch are the Figures E F G H, and S T V X; of which that which is noted with P is called a right angled Parallelogram, in reſpect of its right Angles, the other noted with R being oblique.

And here note alſo, That a Parallelogram although it conſiſts of four Sides, as that noted with P, whoſe Sides are E F, F H, G H, and G E, it is repreſented by two of the oppoſite angular Points; ſo is that Parallelogram E F G H, called the Parallelogram E H, or F G; and the Parallelogram S T V X, (noted with R,) is called the Parallelogram T V or S X.

XXX. *In every Parallelogram, when a Diametre and two Lines are drawn parallel to the two Sides, cutting the Diametre in the ſame Point, ſo that the Parallelogram be divided into four Parallelograms, thoſe two through which the Diametre paſſeth not, are called Supplements, or Complements; and the other two, through which the Diametre paſſeth, are called the Parallelograms ſtanding about the Diametre; and the two Supplements, and any one of the Parallelograms which ſtand about the Diametre, are called a* Gnomon.

In the Parallelogram A B C D, when the Diametre C B is drawn, and two Lines G H and E F parallel to the two Sides A B and B D, cutting one the other in the Diametre in the Point K, then is the Parallelogram A B C D divided into four Parallelograms, *viz.* into the two Parallelograms A K and K D, which are the two Complements, or Supplements; and into the two Parallelograms E H and G F, the Parallelograms about the Diametre; and the Parallelograms A K and K D, together with the Parallelogram E H, is called a *Gnomon*, and is joyned by the circular Line O P R. Figure XIII.

XXXI. *Equal Circles, are ſuch whoſe Diametres are equal, or whoſe Lines drawn from the Centres are equal.*

The Circles A and B are equal, as their Diametres C D and E F are equal: Or, they are Figure XIV.

are equal, becauſe their Semidiametres (or Lines B C and A E, drawn from their Centres,) are equal.

XXXII. *A Right Line is ſaid to touch a Circle, which toucheth the Circumference or Peripherie of the Circle, and being produced cutteth it not.*

Figure XV. As the right Line E F, drawn from the Point E, paſſeth by a Point of the Circle at G, to the Point F, only touching the Circle G H, but cutteth it not; and ſuch a Line is alſo called a Tangent Line.

XXXIII. *A Sector of a Circle is (an Angle being ſet at the Centre of the Circle) a Figure contained under the two Right Lines which make that Angle, and the part of the Circumference received by them.*

Figure VII. The Angle at the Centre A, which is made by the two Lines B A and A D, and the part of the Circumference B D contained between them; that is, the Figure B A D is a Sector of the Circle B C D; alſo the Figures D A C and B A C, are all Sectors of the ſame Circle.

XXXIV. *A Right lined Figure is ſaid to be inſcribed in a Right lined Figure, when every of the Angles of the Figure inſcribed do touch every of the Sides of the Figure in which it is inſcribed.*

Figure XVI. As the Triangle L O M, is inſcribed in the Triangle K N P, for that the ſeveral Angles of the Triangle L O M, do touch the Sides K N, N P, and P K, of the Triangle K N P.

XXXV. *A Right lined Figure is ſaid to be circumſcribed about a Right lined Figure, when every one of the Sides of the circumſcribed Figure toucheth every one of the Angles of the Figure about which it is circumſcribed.*

So the Triangle K N P is ſaid to be circumſcribed about the Triangle L O M, becauſe the Sides thereof K N, K P, and N O, do touch the Angles L, M, and P, of the Triangle L O M, about which the Triangle K N P is circumſcribed.

And note, That what is here ſaid of Triangles, the like is to be underſtood of all other Figures: As the five ſided Figure Q R S T V is inſcribed in the five ſided Figure A B C D E, becauſe the Angles of the Inſcribed Figure, Q R S T and V, do touch every of the Sides A B, B C, C D, D E, and E A, of the circumſcribed Figure A B C D E.

XXXVI. *A Right lined Figure is ſaid to be inſcribed in a Circle, when every one of the Angles of the Figure inſcribed do touch the Circumference of the Circle.*

Figure XVII. So the Triangle G H K, and the Quadrilateral Figure G L M N, are both of them inſcribed within the Circle G L H M K N.

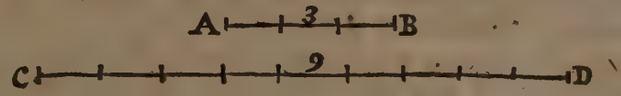
XXXVII. *A Circle is ſaid to be inſcribed in a Right lined Figure, when the Circumference of the Circle toucheth every one of the Sides of the Figure within which it is inſcribed.*

Figure XVIII. So the Circle A B C D is inſcribed within the Triangle O P Q, becauſe the Circumference of the Circle toucheth all the Sides thereof; as the Side O P in B, O Q in C, and P Q in D; and it alſo toucheth the Sides of the Square R S T V; within both which Figures the Circle A B C D is inſcribed.

XXXVIII. *A Part is a leſs Magnitude in reſpect of a greater, when the leſs meaſureth the greater.*

A leſs Quantity is ſaid to meaſure a greater, when the leſs being taken from the greater, as many times as it may, there ſhall nothing remain. Let the Line A B contain 3, and the Line C D 9; then doth the Line A B meaſure the Line C D; For if A B 3, be taken as often as it can from C D 9, (which will be three times,) there will nothing remain; therefore A B, the leſs Magnitude, meaſureth the greater C D.

But

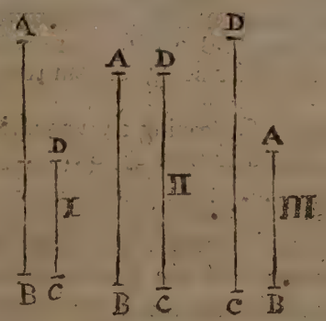


But any Quantity may be said to be a less, in comparison of a greater Quantity, whether the less do exactly measure the greater or not.

Suppose the Line AB to be 17, and let it be divided in the point C, namely into AC 12, and CB 5: Now either of these Lines is a Part of the whole AB, but neither of them do measure it; for if CB 5 be taken 3 times, it maketh but 15, so there will want two parts of 17; if it be taken 4 times, it will make 20, which is three parts more than 17: Also for the other part AC 12, if that be taken once from AB, it will be but 12, and so will want 5 of 17; and if it be taken twice, it will be 24, which is 7 more than 17; so that neither of those parts will measure exactly the whole, and yet either of them may be said to be a less Magnitude in respect of the whole AB, though neither of them do exactly measure it.

XXXIX. *Multiplex is a greater Magnitude in respect of a lesser, when the lesser measureth the greater.*

This is evident from the first part of the last Definition, where AB 3 measureth CD 9, and therefore 9 is Multiplex to 3: So in others, as 12 is Multiplex to 3; for 12 is the greater Magnitude, and 3 may be taken 4 times from it, for 4 times 3 is 12, *&c.*

XL. *Proportion* (*or* Ratio) *is the mutual habitude, or respect, of two Magnitudes of the same kind according to Quantity.*

That is, of one Number to another Number, one Line to another Line, one Figure to another Figure of the same kind: But Quantities of divers kinds cannot be compared together; as a Figure to a Line, nor a Solid to a Superficies, *&c.* for that they are not of one kind. Now if two Quantities be compared together, the one Quantity is to the other either equal, greater, or lesser. An Example of two Lines, AB and CD, compared the one to the other, according to some certain respect of greatness, lessness, or equality: Then let AB be greater than CD, and contain it twice; now this habitude or respect of AB to CD is called Proportion, (or *Ratio*,) as in the first two Lines. Likewise, if AB be equal to CD, as in the second two Lines. Or, if AB be less than CD, as in the third two Lines. It is likewise the same in Numbers; for,

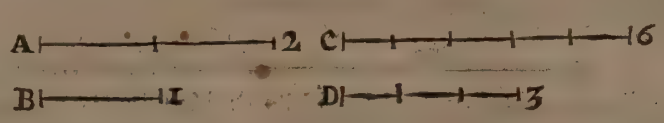
Compare { 5 to 5 / 6 to 3 / 4 to 8 } it is { Equal to Equal. / Greater to Less. / Less to Greater. }

And so of any other Quantities of the same kind. And the Quantities thus compared together are generally called the Terms of the Proportion or *Ratio*; and of the two Terms that which is compared is called the Antecedent, whether it be equal, greater, or lesser; and the second, namely that to which the comparison is made, is called the Consequent, whether it be equal, greater, or less.

So if { AB / CD / 12 } be compared to { CD / AB / 32 } then { AB / CD / 12 } is Antecedent, and { CD / AB / 32 } Consequent.

XLI. *Proportion is a Similitude of* Ratio's.

That which is here termed Proportion, is more rightly called Proportionality, or Analogy; for Proportion commonly denotes no more than the *Ratio* between two Magnitudes.

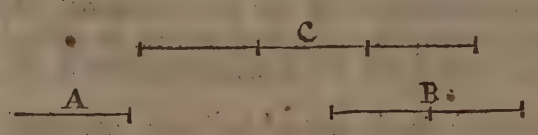


As if you would compare the *Ratio* of the Line A, which is 2, to the Line B, which is 1; and the Line C, which contains 6, to the Line D containing 3, either Proportion is *Dupla*, and may be called double *Ratio*.

So in Numbers, 9 to 3, and 21 to 7, either of which are triple; for 3 times 3 is 9, and 3 times 7 is 21; and this is called triple *Ratio*.

And here note, That Proportions compared together, are said to be like; but Magnitudes so compared are said to be equal one to the other.

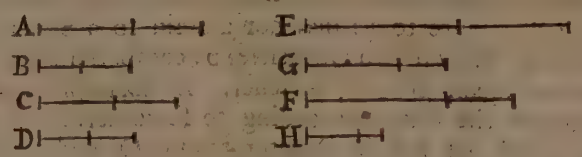
XLII. *Those Magnitudes or Numbers are said to have a Proportion* (*or* Ratio) *between them, which being multiplied may exceed one the other.*



Let the Line A be multiplied by what Number soever, as by 2, so will it be the Line B; by 3, and it will be the Line C; or any other Number, as 2 and an half, 7 and a quarter, *&c.* between which Lines A, B, and C, there is a Proportion: For the one remaining unmultiplied, and the other being certain times multiplied, shall be greater than it.

So 3 to 24 hath a Proportion; for leaving 24 unmultiplied, and multipling 3 by 9, you shall produce 27, which is greater than 24, *&c.*

XLIII. *Magnitudes are said to be in the same* Ratio, *the first to the second, and the third to the fourth; when the Equimultiplices of the first and the third, being compared with the Equimultiplices of the second and fourth, according to any Multiplication, either* (*together*) *exceed one the other, are equal one to the other, or are less one than the other.*



Let A B C D be four Magnitudes: Now if you take the equimultiplices of A and C, the first and the third, (that is, if you multiply A and C by the same Number,) as let the Multiplex of A be E, and the Equimultiplex of C be F: Also, if you multiply B and D, the second and fourth, by any one Number, whether it be by that by which you multiplied A and C, or by any greater or lesser Number, as let the Multiplex of B be G, and the Equimultiplex of D be H: Now if the Equimultiplices of A and C, be both greater than those of B and D; that is, if the Multiplex of A, be greater than the Multiplex of B, and the Multiplex of C, greater than the Multiplex of D; or if they be both less than they, or both equal to them, then are the Magnitudes A B, and C D, in one and the same Proportion.

In Numbers, 8 to 6 hath a *Ratio*, also 4 to 3 hath a *Ratio*; now to see whether they be in the same *Ratio*, set them in order as in the Example; then multiply 8 and 4, the first and third Terms, by 3, and they make 24 and 12; also multiply 6 and 3, the

the ſecond and fourth Terms of the *Ratio*, by the ſame, and they make 18 and 9. Now you ſee that the triple of 8, the firſt Term, namely, 24, exceeds the triple of 6, the ſecond Term, 18; and the triple of 4, the third Term, 12, exceeds the triple of 3, the fourth Term, 9; wherefore the Numbers 8 and 6, and 4 and 3, are in the ſame *Ratio*; becauſe (by the firſt part of the Definition) the Equimultiplices of 8 and 4, (the firſt and third) do both of them exceed the Equimultiplices of the ſecond and fourth.

	24		12
First	8	*Third*	4
Second	6	*Fourth*	3
	18		9

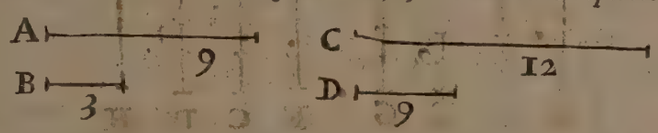
Again, Take the ſame Numbers 8 and 4, and multiply them by 3; ſo ſhall you have 24 for the firſt, and 12 for the third, as before: Then for 6 and 3, multiply them by 4, and they produce 24 and 12. Now here you ſee that the Equimultiplices of 8 and 4, the firſt and third, are equal to the Equimultiplices of 6 and 3, the ſecond and fourth; wherefore (by the ſecond part of the Definition,) the Numbers given are in the ſame *Ratio*.

	24		12
First	8	*Third*	4
Second	6	*Fourth*	3
	24		12

Again, Take the ſame Numbers 8, 6, 4, 3, and multiply 8 and 4, the firſt and third, by 2, whereof will come 16 and 8: Alſo multiply 6 and 3, the ſecond and fourth, by 3, whereof will come 18 and 9. Now here you ſee, that the Equimultiplices of 8 and 4, the firſt and third Terms, are both of them leſs than 18 and 9, the Equimultiplices of 6 and 3, the ſecond and fourth Terms: And ſo (by the third part of the Definition) the Numbers propoſed are in the ſame *Ratio*.

	16		8
First	8	*Third*	4
Second	6	*Fourth*	3
	18		9

XLIV. *Magnitudes that have the ſame* Ratio, *are called Proportional.*



If the Line A have the ſame Proportion to the Line B, as the Line C hath to the Line D; then are the four Magnitudes, A, B, C, D, called Proportional.

Alſo in Numbers, 9 hath the ſame *Ratio* to 3, as 12 hath to 4; ſo theſe Numbers 9, 3, 12, 4, are Proportional: And this likeneſs of *Ratio*, is called Proportionality.

This Proportionality is of two kinds, either Continual or Diſcontinual.

Continual Proportionality is, when the Magnitudes ſet are ſo joyned together, that the

Second, Fourth, } which is conſequent to the } { First, Third, } is Antecedent to the { Third. Fifth, &c.

So that every quantity in this Proportionality, is both Antecedent and Conſequent, except the

First, Laſt, } which is only { Antecedent Conſequent } to that which { followeth. went before.

An Example in theſe Numbers 16, 8, 4, 2, 1; for what Proportion 16 hath to 8, 8 hath to 4, 4 hath to 2, and 2 hath to 1, for they are all in double *Ratio*, or Proportion. And,

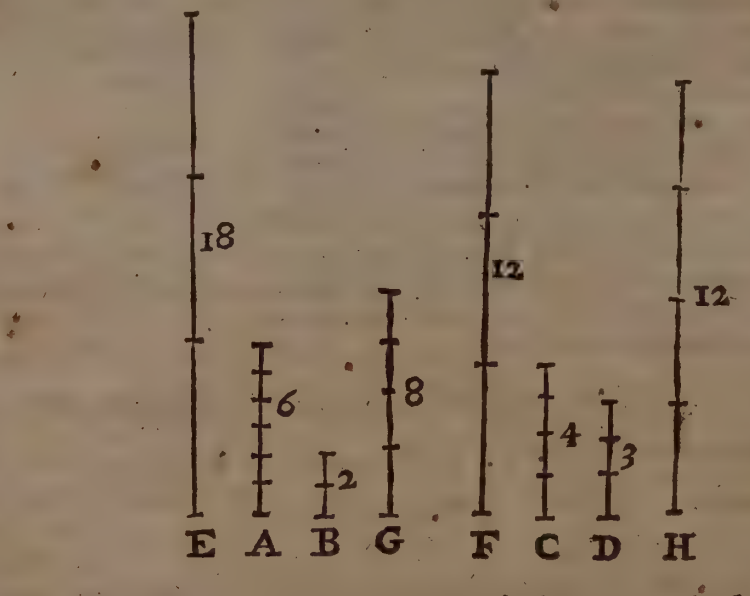
16, 8, 4, 2, 1 } the { First, Second, Third, Fourth, Fifth } is Antecedent to 8, 4, 2, 1, None } the { Second, Third, Fourth, Fifth, } and Conſequent to None, 0, 16, 8, 4, 2 } the { First. Second. Third. Fourth.

Diſcon-

Diſcontinual Proportionality is when the Magnitudes are not continually ſet as before they were, but have their Terms diſtinct: As the firſt is Antecedent to the ſecond, ſo is the third Antecedent to the fourth.

In Numbers thus: As 8 is to 4, ſo is 6 to 3; for either Proportion is duple, and hath its own Antecedent and Conſequent diſtinct; 8 and 6 being Antecedents, and 4 and 3 being Conſequents: And by reaſon of this diſcontinuance in the Proportionality, the quantities compared may be of divers kinds, as Superficies to Superficies, Body to Body, as in Lines and Numbers.

XLV. *When of Equimultiplices the Multiplex of the firſt Magnitude exeeedeth that of the ſecond, but the Multiplex of the third exceedeth not the Multiplex of the fourth; then hath the firſt to the ſecond a greater Proportion, than the third hath to the fourth.*



Let there be four quantities, A, B, C, D, of which let A the firſt contain 6, B the ſecond contain 2, C the third 4, and D the fourth 3. Now take the Equimultiplices of A and C the firſt and third, which let be E and F; ſo that as Multiplex E is to A, ſo let Multiplex F be to C, *viz.* triple; ſo have you 18 for the Multiplex of A, and 12 for the Multiplex of C: Likewiſe take the Equimultiplices of B and D, the ſecond and fourth Terms, and multiply them by ſome one Number, *viz.* by 4; ſo have you for the Multiplex of B the ſecond, the Line G 8, and for D the fourth, the Line H 12. Now becauſe the Line E 18, Multiplex of the firſt, exceedeth the Line G 8, Multiplex of the ſecond; and the Line F 12, Multiplex of the third, exceedeth not the Line H 12, Multiplex of the fourth, (for they are equal;) the Proportion of A to B, the firſt to the ſecond, is greater than the Proportion of C to D, the third to the fourth.

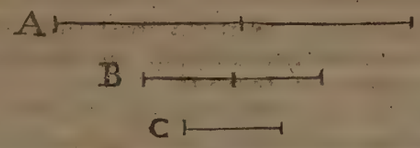
	12		14
Firſt	11	*Third*	7
Second	2	*Fourth*	3
	12		18

So likewiſe in Numbers, take 11 to 2, and 7 to 3, multiply 11 and 7 (the firſt and third) by 2, ſo ſhall you have 22 for the Multiplex of the firſt, and 14 for the Multiplex of the third; and multiplying 2 and 3, (the ſecond and fourth) by 6, you ſhall have 12 for Multiplex of the ſecond, and 18 for Multiplex of the fourth. Here you ſee that 22, the Multiplex of firſt, exceeds 12, the Multiplex of the ſecond; but 14, the Multiplex of the third, exceeds not 18, the Multiplex of the fourth: Wherefore the Proportion of 11 to 2, is greater than the Proportion of 7 to 3.

XLVI. *Proportionality conſiſteth of three Terms at leaſt.*

In Proportionality there muſt of neceſſity be two Proportions, and each hath two Terms, his Antecedent and Conſequent; and therefore in Proportionality there are four Terms, but ſometimes one Term ſupplies the room of two, for in reſpect to the firſt it is Conſequent, and in reſpect to that which followeth, it is Antecedent; therefore three Terms at leaſt may ſuffice in Proportionality, the which are in Power four, and occupy the Room of four.

In



In these three Lines A B C, let us suppose that A hath to B that Proportion which B hath to C; then are these three Quantities, A B C, set in the least Number of Proportionality.

Likewise in Numbers, { As 8 to 4, so 4 to 2 } &c.
{ As 9 to 6, so 6 to 4 }

XLVII. *When three Magnitudes are Proportional, the first shall have a Duplicate* Ratio *to the third of that it hath to the second; but when four Magnitudes are Proportional, the first shall have a Triplicate* Ratio *to the fourth of what it had to the second: And so always in order, one more as the Proportion shall be extended.*

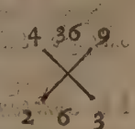
This Definition is to be understood in Continual Proportionality, as if A, B, C, be three Magnitudes Proportional; then shall the *Ratio* of A, the first, to the *Ratio* of C, the third, be double to the *Ratio* which is between A and B, the first and second; that is, the *Ratio* of A to B taken twice, shall make the *Ratio* of A to C.

For the easier understanding of this, it is necessary to know how Proportions may be added one to another; for effecting whereof, this is the

RULE:

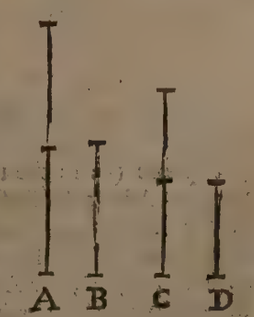
Multiply the Antecedent of the one Proportion by the Antecedent of the other, the Number produced shall be Antecedent of the Proportion which containeth them both. Likewise multiply the Consequent of one Proportion, by the Consequent of the other Proportion; and the Number arising shall be Consequent of the Proportion, which shall contain them both.

Example: If you would add the Proportion which is between 4 and 2, (which is *Dupla*,) to the Proportion which is between 9 and 3, (which is *Tripla*,) multiply 4 (the Antecedent of the first Proportion) by 9, (the Antecedent of the second,) and thereof will come 36, the Antecedent of the Proportion you seek for: Also multiply 2, the Consequent of the first Proportion, by 3, the Consequent of the second, and thereof will come 6, which 6 shall be Consequent to the former Antecedent 36; and so shall the Proportion which is between 36 and 6, (namely *Sextupla*,) contain in it the two Proportions given, namely, *Dupla* and *Tripla*. And by this means are they added together, and brought into one.



XLVIII. *Magnitudes of like* Ratio *are said to be Antecedents to Antecedents, and Consequents to Consequents.*

These Magnitudes are said to be of like *Ratio*, whose Antecedents have like respect to their Consequents, and whose Consequents the like respect to their Antecedents; so putting four Magnitudes A, B, C, D, if A Antecedent to B, be double to B, and C, Antecedent to D, be also double to D, then have the two Antecedents like respect to their Consequents. Also, if B the Consequent be half of A, and also D the Consequent be half of C; then the two Consequents B and D, receive of their Antecedents A and C, like habitude or respect; and by this Definition, the Magnitudes A, B, C, D, are of like *Ratio*, or Proportion.



Also

Antecedents.
9 . 6
3 . 2
Conſequents.

Alſo in Numbers, 9, 3, 6, 2; becauſe 9 the Antecedent, is Triple to 3 his Conſequent, and the Antecedent 6, is alſo Triple to his Conſequent 2, the two Antecedents 9 and 6 have like reſpect to their Conſequents; and becauſe that 3 the Conſequent, is one third of his Antecedent 9, and 2 the Conſequent, one third part of his Antecedent 6, the two Conſequents 3 and 2, receive alſo a like reſpect to their Antecedents: And therefore are Numbers of like *Ratio* or Proportion.

XLIX. *Alternate Proportion (or by Permutation) is, when Antecedent is compared to Antecedent, and Conſequent to Conſequent.*

The underſtanding of this Definition and of the ſix following, depend upon the laſt Definition, and do uſe it for a general Suppoſition, namely, to have four quantities in Proportion. Suppoſe four Magnitudes A, B, C, D, (ſuch as in the figure of the laſt Definition) to be in Proportion: As A to B, ſo C to D; now if you compare A the Antecedent of the firſt Proportion, to C the Antecedent of the ſecond, (as to his Conſequent;) and likewiſe if you compare B, the Conſequent of the firſt Proportion, as an Antecedent to D the ſecond, (as to his Conſequent;) then ſhall you have the Magnitudes in this ſort, *viz.* As A to C, Antecedent to Antecedent; ſo B to D, Conſequent to Conſequent: And this is called Permutation, or Alternate Proportion.

Dupla *Dupla*
12 . 6 8 . 4
Seſquialtera

In Numbers thus,
As 12 to 6, ſo 8 to 4.
Antecedent to Conſequent, either is *Dupla.*
Wherefore, (by Permutation of Proportion,)
As 12 is to 8, ſo is 6 to 4.
Conſequent to Conſequent, for either are *Seſquilatera.*

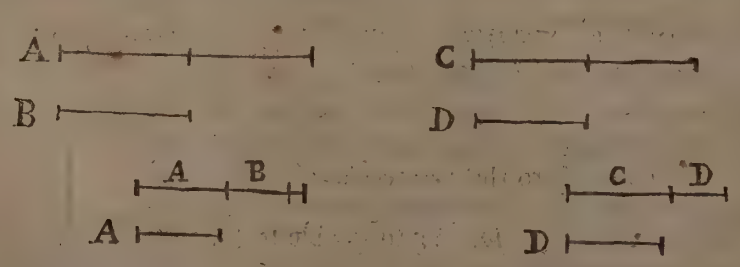
L. *Inverſe Proportion is, when the Conſequent is taken as the Antecedent, and ſo is compared to the Antecedent, as to the Conſequent.*

Suppoſe, as before, in the two laſt Definitions, four Magnitudes A, B, C, D, in Proportion, As A to B, ſo C to D, if you refer B, the Conſequent of the firſt Proportion, as Antecedent to A, the Antecedent of the firſt, as to his Conſequent; and likewiſe, if you refer D, the Conſequent of the ſecond Proportion, as an Antecedent to C, the Antecedent of the ſecond Proportion, as to his Conſequent, then ſhall you have the Magnitudes in this Order: As B to A, Conſequent to Antecedent, ſo D to C, Conſequent to Antecedent. And this is called Inverſe (or Converſe) Proportion.

Triple *Triple*
9 . 3 6 . 2
Subtriple *Subtriple*

So alſo in Numbers,
9 to 3, as 6 to 2.
Either is Triple; wherefore comparing 3 to 9, the Conſequent of the firſt to his Antecedent 9; and alſo 2, the Conſequent of the ſecond, to 6 his Antecedent: By this Converſe Proportion it cometh to paſs, that as 3 to 9, ſo is 2 to 6; either of which are Subtriple.

LI. *Compound Proportion is when the Antecedent and Conſequent taken together as one, are compared to the Conſequent it ſelf.*



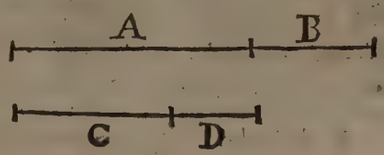
Suppoſe (as in the three laſt Definitions) four Magnitudes, A, B, C, D, in Proportion, as A is to B, ſo C is to D. If you add A and B, the Antecedent and Conſequent of the firſt Proportion, together, compare them, ſo added, as one Antecedent to B, the Conſequent of the firſt Proportion, as to his Conſequent; and likewiſe, if you add together C and D, the Antecedent and Conſequent of the ſecond Proportion, and, ſo added, compare them as one Antecedent to D, the Conſequent of the ſecond Proportion, as

as to his Conſequent, then ſhall you have the Magnitudes in this Order: As AB to B, ſo is CD to D; for either of them is *Tripla*: And this is called compoſed (or compound) Proportion.

In Numbers, as 8 to 4, ſo 6 to 3. Now 8 and 4, the Antecedent and Conſequent of the firſt Proportion, added together make 12, which 12, as Antecedent, compare to 4, the Conſequent of the firſt Proportion, as to his Conſequent: Likewiſe add together 6 and 3, the Antecedent and Conſequent of the ſecond Proportion, and they make 9, which 9, as Antecedent, compare to 3, the Conſequent of the ſecond Proportion, as to his Conſequent; ſo ſhall you have (by Compoſition of Proportion,) as 12 to 4, ſo is 9 to 3; for either of them is *Tripla*.

8	4	6	3
12	4	9	3

LII. *Divided Proportion is when the Exceſs, wherein the Antecedent exceedeth the Conſequent, is compared to the Conſequent.*



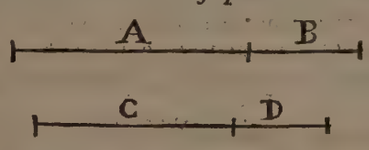
This Definition is the Converſe of the former; for in that was uſed Compoſition, in this is uſed Diviſion.

Suppoſe four Magnitudes in Proportion, AB the firſt, B the ſecond, CD the third, and D the fourth; then as AB to B, ſo is CD to D, AB the Antecedent of the firſt Proportion, exceedeth B the Conſequent of the ſame, by the Magnitude A; wherefore A is the Exceſs of the Antecedent AB, above the Conſequent B. So likewiſe CD, the Antecedent of the ſecond Proportion, exceeds D, the Conſequent of the ſame by the Magnitude C; wherefore C is the Exceſs of the Antecedent CD, above the Conſequent D. Now if you compare A, the Exceſs of AB, the firſt Antecedent, above the Conſequent B, as Antecedent to B, the Conſequent, as to his Conſequent: Alſo if you compare D, the Exceſs of CD, the ſecond Antecedent CD, above the Conſequent D, the Antecedent to D, the Conſequent, as to his Conſequent; then ſhall your Magnitudes be in this Order: As A is to B, ſo is C to D; which is called Diviſion of Proportion.

So in Numbers, as 9 to 6, ſo 12 to 8, either Proportion is *Seſquialtera*: The Exceſs of 9, the Antecedent of the firſt Proportion, above 6, the Conſequent of the ſame, is 3; the Exceſs of 12, the Antecedent of the ſecond Proportion, exceedeth 8, the Conſequent thereof, by 4. Then if you compare 3, the Exceſs of 9, the firſt Antecedent above the Conſequent, as Antecedent to 6, the Conſequent as to his Conquent: And alſo, if you compare 4, the Exceſs of 12, the ſecond Antecedent above the Conſequent, as Antecedent to 8, the Conſequent as to his Conſequent, you ſhall have the Numbers in this Order, by Diviſion of Proportion: As 3 to 6, ſo 4 to 8; for either Proportion is *Subdupla*.

9	6	12	8
3	6	4	8

LIII. *Converſe Proportion is when the Antecedent is compared to the Exceſs, wherein the Antecedent exceeds the Conſequent.*

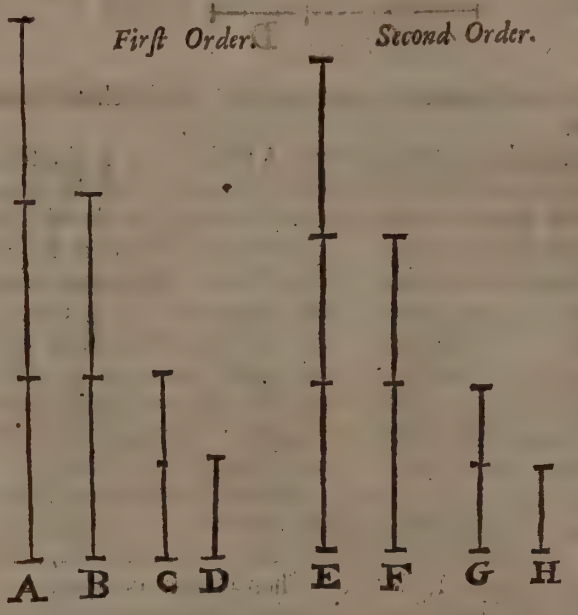


As in the former, ſo in this, ſuppoſe four Magnitudes in Proportion, AB the firſt, B the ſecond, CD the third, and D the fourth, then will the Order be; As AB to B, ſo CD to D. AB the Antecedent of the firſt, exceedeth B, the Conſequent of the ſame, by the Magnitude A; wherefore A is the exceſs of the Antecedent AB, above the Conſequent B; ſo alſo C is the exceſs of CD, the Antecedent of the ſecond, above D the Conſequent of the ſame. Now if you refer AB, the Antecedent of the firſt, as Antecedent to A, the exceſs thereof above the Conſequent B, as to his Conſequent: Alſo, if you compare CD, the Antecedent of the ſecond, as Antecedent to C, the exceſs thereof above the Conſequent D, as to his Conſequent; then ſhall your Magnitudes come to this Order, As AB to A, ſo CD to C. And this is called Converſe Proportion.

Likewiſe in Numbers; As 9 to 6, ſo 12 to 8: The exceſs of 9, the Antecedent of the firſt, above 6, the Conſequent of the ſame, is 3; the exceſs of 12, the Antecedent of the ſecond, above 8, the Conſequent of the ſame, is 4. Now compare the Antecedent of the firſt, 9, as Antecedent to 3, the exceſs thereof above 6, the Conſequent as to his Conſequent; likewiſe compare 12, the Antecedent of the ſecond, as Antecedent to 4, the exceſs thereof above 8, the Conſequent as to his Conſequent; ſo ſhall (according to converſe Proportion,) the Numbers ſtand thus, As 9 to 3, ſo 12 to 4; for either *Ratio* is Triple.

9 6 12 8
9 3 12 4

LIV. *Proportion of Equality is where there are taken more Magnitudes than two in one Order, and alſo as many Magnitudes in another Order, comparing two to two, being in the ſame* Ratio, *it cometh to paſs, that as, in the firſt Order of Magnitudes, the firſt is to the laſt, ſo, in the ſecond Order of Magnitudes, is the firſt to the laſt. Or otherwiſe, it is a Compariſon of the Extremes together, the Mean Magnitudes being taken away.*



If there be taken (in ſome determinate Order) any Number of Magnitudes, for inſtance four, A, B, C, D, and in the ſame Order four more, E, F, G, H; then take the equal Proportions by two and two, As A to B, ſo E to F; As B to C, ſo F to G; and as C to D, ſo G to H.

Now if A, the firſt Magnitude of the firſt Order, be to D, the laſt Magnitude of the ſame Order,

As E, the firſt Magnitude of the ſecond Order, is to H, the laſt Magnitude of the ſecond Order; then is it called Proportion of Equality: As in *Definition* 38.

Likewiſe, if you leave out the mean Magnitudes in either Order, as B C in the firſt Order, and F G in the ſecond Order; and compare the Extreams of each Order together, then (by this Definition,) as A is to D, in the firſt Order, ſo ſhall E be unto H, in the ſecond Order.

In Numbers: Let there be two Orders of Numbers, 27, 9, 12, 24, 15, in the firſt, and 9, 3, 4, 8, 5, in the ſecond Order; then ſee that all the Proportions taken by two and two be like, as between

27, 9, 12, 24 and 9, 12, 24, 15 Numbers of the firſt Order, and 9, 3, 4, 8 and 3, 4, 8, 5 Numbers of the ſecond Order, *viz.*

- *Tripla.*
- *Subſeſquitertia.*
- *Subdupla.*
- *Supertripartiens quinta.*

Wherefore (by this Definition) leaving out all the mean Numbers of each ſide, you may compare only the Extreams, and conclude, that

As

As 27, the first Term in the first Order,
Is to 15, the last Term in the same Order;
So is 9, the first Term in the second Order,
To 5, the last Term in the second Order.

For the Proportion of each is *Superquadripartiens quintas.*

And here it is to be minded, that there is no necessity that all the Proportions in each row of Numbers be set in like Order, as before; but it shall be sufficient that the Proportions be the same, and in equal Numbers in each row, whether in the same or in contrary Order. As in these Numbers 12, 6, 2, in the first row, and 24, 8, 4, in the second; for

12	24
6	8
2	4

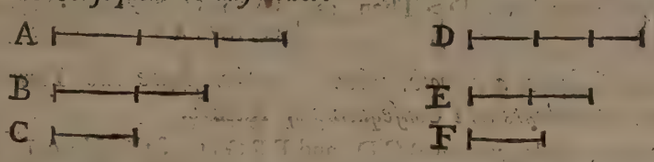
As 12 to 6 (the first to the second of the first row,)
So is 8 to 4 (the second to the third of the second row,)
Either is in Duple Proportion: And,
As 6 to 2 (the second to the third in the first row,)
So is 24 to 8 (the first to the second in the second row.)

Where the Numbers are not placed in the same Order, or Course; yet by equality of Proportion, (leaving out the means 6 and 8,)

12	24
2	4

As 12 to 2 (the first and last of the first row,)
So is 24 to 4, (the first and last of the second row;)
And so of others, howsoever placed.

LV. *Ordinate Proportionality is, when as the Antecedent is to the Consequent, so is the Antecedent to the Consequent; and as the Consequent is to any other, so is the Consequent to any other.*



For the explanation of this Definition is required two Orders of Magnitudes: Suppose in the first Order that the Antecedent A, to his Consequent B, have the same Proportion that the Antecedent D, hath to its Consequent E in the second Order; and make the Consequent B Antecedent to some other quantity as C; also make the Consequent E Antecedent to another quantity, as F; so that there be the same *Ratio* of B to C, as of E to F. And this disposition of Proportion, is called Ordinate Proportionality.

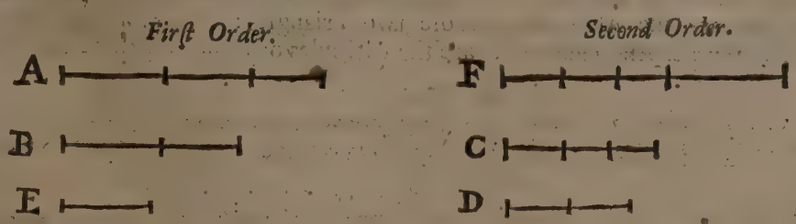
In Numbers, as these, 18, 9, 3, and 6, 3, 1.

18	6
9	3
3	1

As 18 to 9 (Antecedent to Consequent,)
So is 6 to 3 (Antecedent to Consequent.)
Either is in Duplicate Proportion; Then,
As 9 (the Consequent,
Is to another Number, *(viz.* 3.)
So is the Consequent 3,
To another, namely, to Unity.

And this ordinate Proportionality may be extended as far as you list; according to the preceding Definition.

LVI. *Inordinate Proportionality is, when as the Antecedent is to the Consequent, so is the Antecedent to the Consequent; and as the Consequent is to another, so is another to the Antecedent.*



This Definition also requires two Orders of Magnitudes: Suppose (in the first Order) the Antecedent A be to the Consequent B, as the Antecedent C (in the second Order)

is to the Conſequent D; and let B (the Conſequent of the firſt) be to ſome other, (namely, to E,) as ſome other (namely F) is to the Antecedent C of the ſecond; this kind of Proportionality is called Perturbate, or Inordinate.

In Numbers,

As 9 to 6 (Antecedent to Conſequent,)	9	3
So 3 to 2 (Antecedent to Conſequent,)	6	2
	3	6

Either Proportion it *Seſquialtera*: And,

As 6 (the Conſequent of the firſt,)
Is to another, (namely to 3;)
So is another, (namely 6,)
To 3, (the Antecedent of the ſecond Proportion.)

For either is Dupla, or double Proportion.

LVII. *Like right lined figures are ſuch whoſe Angles are equal one to the other, and whoſe ſides about the equal Angles are Proportional.*

Figure XIX. In the two Triangles A B C, and D E F; if the Angle at A be equal to the Angle at D, and the Angle at B equal to the Angle at E, and the Angle at C equal to the Angle at F: And alſo, if the ſide A B have the ſame Proportion to the ſide B C, as the ſide D E hath to the ſide E F: And alſo, the ſide B C to C A, as the ſide E F to F D; and the ſide C A to A B, as F D to D E; then are thoſe two Triangles ſaid to be like: And ſo of other figures.

Hence follows this Proportionality.

If { B, A, C } be equal to { E, D, F } Then, as { A B to B C, B A to A C, B C to C A } So is { D E to E F, D E to D T, E F to F D }

LVIII. *Reciprocal figures are thoſe, when in either figure the Terms of Proportion are Antecedents and Conſequents of* Ratio's.

Figure XX. As in the two Parallelograms A B C D, and E F G H, if the ſide A B (an Antecedent of the firſt figure) have to the ſide E F (a conſequent in the ſecond figure) mutually the ſame Proportion (or *Ratio*) as the ſide A G hath to the ſide A C, (an Antecedent in the ſecond, to a Conſequent in the firſt,) then are thoſe two figures in reciprocal *Ratio*, or Proportion.

Some Authors put this Definition in theſe words; *Reciprocal Figures are, when the ſides of either be mutually Proportional.*

LIX. *A right Line is ſaid to be divided by extreme and mean* Ratio, *or Proportion, when the whole is to the greater part, as the greater part is to the leſſer.*

If the Line A B be ſo divided in the Point C, that the whole Line A B have the ſame Proportion (or *Ratio*) to the greater part A C, as the ſame greater part A C, hath to the leſſer part C B; then is that Line A B divided by extreme and mean *Ratio* in the Point C, (A B being to A C in the ſame *Ratio*, as A C is to C B.)

And this Diviſion of a Line is by ſome Authors thus defined. *A Line divided by Proportion, having one Mean and two Extremes.*

LX. *The Altitude of any figure is a Perpendicular Line, drawn from the top of the figure to the Baſe of the ſame figure.*

Figure XXI. As the Altitude or heighth of the Triangles, A B C and D E F, the Perpendiculars A G and D H are the Altitudes of thoſe two Triangles: And in all the other figures, as K L M, the Perpendiculars O P are the Altitudes of thoſe reſpective figures.

POSTU-

POSTULATES, or grantable Requests.

I. *From any Point to any Point to draw a Right Line.*

A ——————— B

As from the Point A to the Point B, who will deny, but easily grant, that a Right Line may be drawn? for two Points (howsoever they be set) are imagined to be in one and the same plain Superficies; wherefore from the one to the other there is some shortest distance, which is a Right Line.

II. *To produce a Right Line, finite, streight forth continually.*

As to draw in length continually the Line A B, who will not grant? for there is no Magnitude so great, but that there may be a greater; nor any so little, but there may be a less: And a Line being a draught from one Point to another, therefore from the Point B (which is the end of the Line A B) may be drawn a Line to some other Point, as to C, and from that to another, and so infinitely.

III. *Upon any Centre, and at any distance, to describe a Circle.*

As upon the Centre A, and upon the space A B, you may describe the Circle B C; and upon the same Centre, upon the distance A D, you may describe the Circle D E: Or, upon the same Centre A, at the distance A F, you may describe the Circle F G; and so infinitely. Figure XXII.

IV. *All right Angles are equal one to the other.*

Forasmuch as a right Angle is made by the falling of one right Line perpendicularly upon another, therefore no one right Angle can be greater than another; neither do the length or shortness of the Lines alter the greatness of the Angle. For the right Angle A B C, though it be made of Lines longer than the Angle D E F, yet is that Angle no greater than the other; for if you set the Point E just upon the Point B, then shall the Line E D fall upon the Line A B, and the Line E F evenly upon the Line B C; and so shall the Angle D E F be equal to the Angle A B C, for that the Lines which cause them are of like Inclination. Figure XXIII.

V. *When a right Line, falling upon two right Lines, does make on one and the same side the two inward Angles less than the two right Angles; then shall those two right Lines (being produced) meet on that part on which the two Angles are less than two right Angles.*

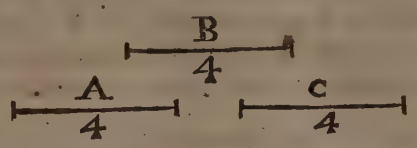
As if the right Line A B fall upon two right Lines, C D and E F, so that it make the two inward Angles on the one side, namely, D H I and H I F, less than two right Angles; those two Lines, C D and E F, being extended on that side, shall at length concurr in the point D. Contrariwise, the same Lines drawn in length on the other side, where the two inward Angles, A H C and A I E, are greater than two right Angles, shall never meet, but being extended on that side, the longer they are drawn out, the farther they shall be distant one from the other. Figure XXIV.

VI. *Two right Lines include not a Superficies.*

If the Lines A B and A C, being right Lines, should include a Superficies, they must of necessity be joyned together at both their ends, and the Superficies must be between them. Let them be joyned together on one side in the Point A, and imagine the Point B to be drawn to the Point C; then shall the Line A B fall upon the Line A C, and cover it, and so, being all one with it, can never include a Superficies. Figure XXV.

AXIOMS, Or things manifeſt to Senſe, and acknowledged by all Men.

I. *Things equal to one and the ſame thing, are alſo equal the one to the other.*



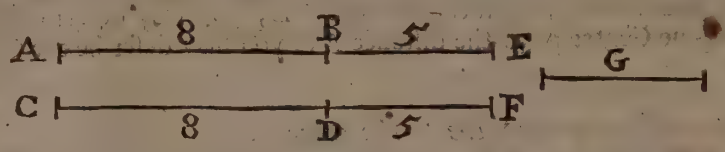
As if the Line A be equal to the Line B, and if the Line C be alſo equal to the Line B; then of neceſſity the Lines A and C ſhall be equal the one to the other. So it is to be underſtood in Numbers, Superficies, Angles, and in all other things (of one kind) that may be compared together.

In Numbers,

If A be equal to 4
And B be equal to 4
Then C is equal to B 4
} Then B 4 is equal to either A or C, or both.

And ſo all three, B, A, and C, are all equal one to the other.

II. *If to equal things you add equal things, the wholes ſhall be equal.*



As if the Line A B, be equal to the Line C D, and to the Line A B, be added the Line B E, (equal to the Line G;) and to the Line C D, be added the Line D F, (equal alſo to the Line G;) then are two equal Lines, B E and D F, added to two equal Lines A B and C D; then (by the *Axiom*) muſt the whole Line A E be equal to the whole Line C F: And ſo of Numbers, and other quantities.

In Numbers,

If to A B ——— 8
You add B E ——— 5
The Summ is ——— 13 equal to A E.

Again,

If to C D ——— 8
You add D F ——— 5
The Summ is ——— 13 equal to C F; ſo that A E and C F are equal.

III. *From equal things if you take away equal things, the things remaining will be equal.*

As, if from the two equal Lines, A E and (F, you take away two Lines, E B and F C, (both of them equal to the Line G,) the remaining Lines, A B and C D, will be equal.

In Numbers,

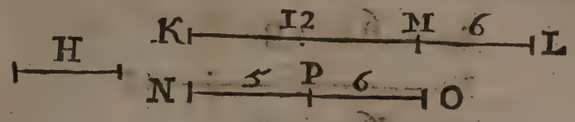
If from A E ——— 13
You take B E ——— 5
There remains ——— 8 equal to A B.

Again,

If from C F ——— 13
You take D F ——— 5
There remains ——— 8 equal to C D; ſo that A B and C D are equal.

IV. *If*

IV. *If to Unequal things you add Equal things, the Wholes will be unequal.*



As if the Lines K M and N P, being unequal, (for K M is longer then N P,) if to either of them you add two Lines, L M and O P, (either of them equal to the Line H;) I say, the whole Lines, K L and N O, shall also be unequal; and K L longer than N O.

In Numbers,

If to K M ——— 12
You add M L ——— 6
Their Summ is ——— 18 equal to K L.

Again,

If to N P ——— 5
You add P O ——— 6
Their Summ is ——— 11 equal to N O; so that K L and N O are unequal.

V. *If from Unequal things you take away Equal things, the things Remaining will be Unequal.*

As if the Lines K L and N O being unequal, you take away two Lines, M L and P O, (each of them equal to the Line H;) I say, the remaining Lines, K M and N P, shall also be unequal.

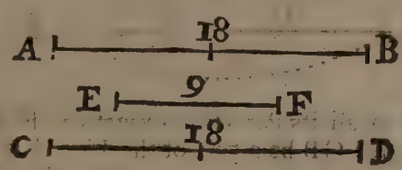
In Numbers,

If from K L ——— 18
You take M L ——— 6
The Remainder is ——— 12 equal to K M.

Again,

If from N O ——— 11
You take P O ——— 6
The Remainder is ——— 5 equal to N P; so that K M and N P are unequal.

VI. *Things which are Double to one and the same thing, are Equal the one to the other.*



If the Line A B be double to the Line E F, and if the Line C D be also double to the Line E F; then may you conclude from this *Axiom*, that the Lines A B and C D are equal the one to the other. And this is true in all quantities, and not only when they are double, but when Treble, Quadruple, or in whatsoever Proportion it be of the greater inequality.

In Numbers,

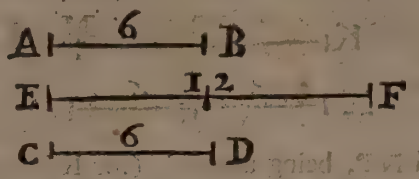
From the whole A B ——— 18
Take the half E F ——— 9
The Remainder is ——— 9 equal to A G or G B.

Again,

From the whole C D ——— 18
Take the half E F ——— 9
The Remainder is ——— 9 equal to C H or H D; so that A B and C D are equal.

VII. *Things*

VII. *Things which are half of one and the ſame thing, (or of things equal,) are equal the one to the other.*



If the Line A B be half of the Line E F, and if the Line C D be alſo half of the Line E F, then may you conclude by this *Axiom*, that the Lines A B and C D, are equal one to the other: Conceive the ſame when quantities are either Triple, Quadruple, or in any Proportion.

In Numbers,

If from E F —— 12
You take A B —— 6
The Remainder is —— 6, equal to half E F.

Again,

If from E F —— 12
You take C D —— 6
The Remainder is —— 6, equal to half E F; ſo that A B and C D are equal.

VIII. *Things which agree together, are equal the one to the other.*

Figure XXVI.

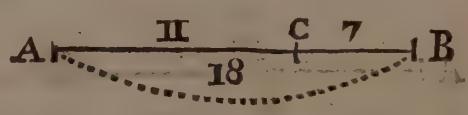
Things are ſaid to agree together, when they are applied the one to the other, or being ſet one upon the other, the one exceedeth not the other in any part.

As if theſe two Triangles, A B C and D E F, were applied the one to the other, and the Triangle A B C were ſet upon the Triangle D E F; if then the Angle A do juſtly agree with the Angle D, the Angle B with the Angle E, and the Angle C with the Angle F: And alſo the Line A B do juſtly fall upon the Line D E, and the Line A C upon the Line E F, and A C upon D F; ſo that there is a juſt agreement in all the Parts. Then may you conclude the two Triangles to be equal.

The Converſe of this *Axiom* is true in right Lines and Angles, but not in figures unleſs they be like.

Moreover Magnitudes are ſaid to agree, when the parts of the one, being applied to the parts of the other; they fill up an eqnal (or the ſame) ſpace.

IX. *Every Whole is greater than its Part.*

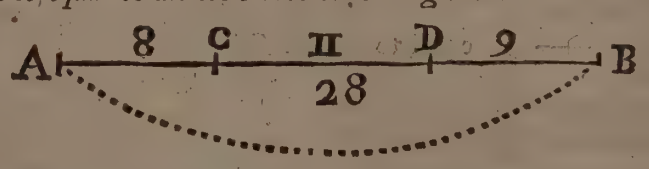


As the whole is equal to all its Parts taken together, ſo is it greater than any one Part thereof. As if the Line C B be a part of the Line A B, then by this *Axiom* you may conclude, that the whole Line A B, is greater than the part thereof C B: And this is general in all things.

In Numbers,

The whole Line A B is —— 18
The Part A C is —— 11
The Part C B is —— 7
Their Summ —— 18 equal to A B the whole.

X. *Every Whole is equal to all its Parts taken together.*



If the whole A B, be divided into three (or any other Number of) unequal Parts, as into A C, C D, and D B: I ſay, that all the Parts, A C, C D, and D B are equal to the whole Line A B.

In

In Numbers,

If the whole Line A B, be ——— 28

Let the Part { A C, be ——— 8 / C D, be ——— 11 / D B, be ——— 9 }

The Summ of the Part is ——— 28 equal to the whole A B.

And this is also true in Superficies,

Suppose in this figure E, F, G, H, the length of the Parallelogram, E F, were divided into three unequal Parts E K, E L, and L F; and the side, or breadth E G, were also divided into other unequal Parts, E M, M N, and N G: If Lines be drawn from K, and L, Parallel to E G, and F H; and also other Lines through the Points M, and N, Parallel to the sides E G, and F H. The Parallelogram E, F, G, H, shall by those Parallel Lines, be divided into nine unequal Parallelograms, *viz.* into the Parallelogram, O, P, Q, R, S, T, V, X, and Z, all which together are equal to the whole Parallelogram, E, F, G, H. Figure XXVII.

In Numbers, let the length E F be 20, and the breadth E G 16, and let the length E F be divided into E K 7, K L 8, and L F 5, all which together make 20.

Also, let the breadth E G, containing 16, be divided into E M 5, M N 7, and N G 4, making in all 16.

I say now, that the length of the Parallelogram E F 20, being multiplied by the breadth, E G will contain 320. Whether Perches, Yards, Feet, Inches, or what Measure soever. Now,

The Parallelogram	O, P, Q, R, S, T, U, X, Z,	is compounded of the Numbers,	7 8 5 7 7 7 7 8 5	and	5 5 5 7 8 5 4 4 4	which multiplied together, produce	35 40 25 49 56 35 28 32 20	for O, P, Q, R, S, T, U, X, Z,

In all 320

Equal to E F 20, multiplied by E G 16; and therefore all the Parts together are equal to the whole, according to the *Axiom*.

Of PROPOSITIONS, &c.

HAving laid down the Principles of *Geometry*, consisting of *Definitions*, *Postulates*, (or grantable Truths,) *Axioms*, (or Sentences proved) next follow the Propositions.

Propositions are of two kinds:

The one is called a *Theorem*.

The other a *Problem*.

I. A *Theorem* is a Proposition which searcheth out, and demonstrateth the Property or Passion of some figure; wherein is only speculation and contemplation of Mind, without Operation or acting of any thing. So the first Proposition following, which is, *If two Angles of two Triangles be equal the one to the other, the sides subtended under the equal Angles, shall also be equal one to the other*, which is a *Theorem*: For it is required only to prove, and make plain by reason and demonstration, that the sides of the one Triangle subtending the equal Angles of the other Triangle, shall be equal one to the other, without working or doing of any thing; and every *Theorem*, after the demonstration is cleared, concludes either with, *which is impossible*, Or, *which is contrary to the Hypothesis*, Or, (and that for the most part) *which was to be demonstrated, or proved*.

II. A *Problem* is a *Propoſition* which requireth ſome action or doing, as to make ſome figure, to divide ſome figure, to add or ſubtract one figure to or from another, to inſcribe one figure within another, to circumſcribe one figure about another, to divide Lines proportionally equally, &c. as the firſt *Problem* in this Book is, *To divide a Right Line into two equal Parts, and at Right Angles:* This is a *Problem*, for it requires ſomething to be done, namely, *To divide a Right Line into two equal Parts, at Right Angles.* And at the cloſe of every *Problem* is ſaid, *Which was required to be done.*

III. A *Corollary* is a *Conſectary* or ſome conſequent Truth, gained from a preceding Demonſtration.

IV. A *Lemma* is a Demonſtration of ſome premiſe, whereby the proof of the thing in Hand becomes ſhorter.

V. A *Scholium* is a Gloſs or compendious Expoſition of a preceding Propoſition.

SECT.

SECT. II.

Geometrical THEOREMS.

I. *If two Angles,* ABC *and* ACB, *of a Triangle* ABC, *be equal the one to the other, the Sides* AB *and* AC, *ſubtended under the equal Angles, ſhall alſo be equal one to the other.*

Conſtruction. IF the Sides be not equal, let one be bigger than the other: ſuppoſe BA greater than CA, make BD equal to CA, and draw the Line CD. Figure I.

Demonſtration. In the Triangles DBC and ACB, becauſe BD is equal to CA, and the Side BC is common, and the Angle DBC equal to ACB, the Triangles DBC and ABC ſhall be equal the one to the other, *viz.* a Part equal to the Whole. *Which is impoſſible.* Eucl. El. L. 1. P. 6.

Corollary.

Hence it follows, That every equiangular Triangle is alſo equilateral.

II. *When a Right Line* AB, *ſtanding upon a Right Line* CD, *maketh the Angles* ABC *and* ABD, *it maketh either two Right Angles, or two Angles equal to two Right Angles.*

Conſtruction. IF the Angles ABC and ABD be equal, then they make two Right Angles; if unequal, then from the point B, let there be erected a Perpendicular BE. Figure II.

Demonſtration. Becauſe the Angle ABC is greater than a Right Angle, by the Angle ABE; and the Angle ABD leſſer than a Right Angle, by the Angle ABE; therefore ſhall the Angles ABC and ABD be equal to two Right Angles: for BE being perpendicular to CD, the Angles EBC and EBD are Right Angles; but ABC is more than a Right Angle by the Angle EBA, and ABD ſo much leſs; therefore the two Angles ABC and ABD are equal to two Right Angles. *Which was to be demonſtrated.* Eucl. El. Lib. 1. P. 13.

Corollaries.

1. Hence, If one Angle ABD be right, the other ABC is alſo right; if one be acute, the other is obtuſe; and ſo on the contrary.
2. If more right Lines than one ſtand upon the ſame right Line, at the ſame point; all the Angles they make ſhall be equal to two Right Angles.
3. Two Right Lines cutting each other, make Angles equal to four Right Angles.
4. All the Angles made about one Point, make four Right Angles. *Corol.* 2.

III. *If to any Right Line* AB, *and a Point therein* B, *two Right Lines, not drawn from the ſame Side, do make the Angles* ABC *and* ABD, *on each Side equal to two Right Angles, the Lines* CB *and* BD *ſhall make one ſtreight Line.*

Conſtruction. IF you deny it, let CB and BD make one Right Line. Figure III.

Demonſtration. If ſo, then ſhall the Angles ABC and ABE be equal to two Right Angles; and equal to ABC and ABD. *Which is abſurd.* Eucl. El. L. 1. P. 14.

IV. *If two Right Lines,* AB *and* CD, *cut through one another, then are the Angles which are oppoſite,* viz. CEB *and* AED, *equal one to the other.*

Demonſtration. FOR the Angles AEC and CEB are equal to two Right Angles; and they are alſo equal to AED and DEB: therefore CEB is equal to AED. *Which was to be demonſtrated.* Eucl. El. L. 1. P. 15. Figure IV.

V. *The greater Side* AC *of every Triangle* ABC *ſubtends the greateſt Angle* ABC.

Conſtruction. FRom AC take away AD equal to AB, and joyn BD.

Demonſtration. Then is the Angle ADB equal to ABD: But ADB is greater than C; therefore ABD is greater than C: Therefore the whole Angle ABC is greater than Figure V.

than C. After the ſame manner may be proved A B C to be greater than A. *Which was to be demonſtrated.* Eucl. El. L. 1. P. 18.

VI. *In every Triangle* A B C, *under the greateſt Angle* A, *is ſubtended the greateſt Side* B C.

Figure VI. *Demonſtration.* IF A B be ſuppoſed equal to B C, then will the Angle at A be equal to the Angle at C; which is contrary to the *Hypotheſis:* And if A B be greater than B C, then ſhall the Angle at C be greater than the Angle at A; which is alſo againſt the *Hypotheſis:* Wherefore rather, B C greater than A B, and after the ſame manner B C greater than A C. *Which was to be demonſtrated.* Eucl. El. L. 1. P. 19.

VII. *If a Right Line* E F, *falling upon two Right Lines,* A B *and* C D, *ſhall make the outward Angle* A G E *of the one Line, equal to* C H G, *the inward and oppoſite Angle of the other on the ſame Side; or make the inward Angles on the ſame Side* A G H, *and* C H G, *equal to two Right Angles, then are the Right Lines* A B *and* C D *Parallels.*

Figure VII. *Demonſtration.* FIrſt, Becauſe the Angle A G E is equal to the Angle C H G, therefore are the Angles B G H and C H G alternate Angles, and equal; and therefore A B and C D are parallel.

Secondly, Becauſe the Angles A G H and C H G are equal to two Right Angles, for they are equal to A G H and B G H; thence is C H G equal to B G H, and therefore are A B and C D parallel. *Which was to be demonſtrated.* Eucl. El. L. 1. P. 28.

VIII. *Of any Triangle* A B C, *one Side* B C *being drawn out, the outer Angle* A C D *ſhall be equal to the two inward and oppoſite Angles* A *and* B; *and the three inward Angles of the Triangle* A C B, C A B, *and* A B C, *ſhall be equal to two Right Angles.*

Figure VIII. *Conſtruction.* FRom C draw the Line C E, parallel to A B.

Demonſtration. Then I ſay, that the Angle B A C is equal to the Angle A C E, and the Angle A B C is equal to the Angle E C D; therefore the Angles B A C and A B C together, are equal to the Angles A C E and E C D, that is, equal to the whole Angle A C D: For the Angles A C D and A C B together, are equal to two Right Angles; therefore the Angles A B C, B A C, and A C B together, are equal to two Right Angles. *Which was to be demonſtrated.* Eucl. El. L. 1. P. 32.

Corollaries:

1. The three Angles of any Triangle taken together, are equal to the three Angles of any other Triangle, taken together. From whence it follows,
2. That if in one Triangle, two Angles (taken ſeverally, or together) be equal to two Angles of another Triangle (taken ſeverally, or together) Then is the remaining Angle of the one equal to the remaining Angle of the other. In like manner, if two Triangles have one Angle of the one, equal to one Angle of the other, then is the Summ of the remaining Angles of the one Triangle, equal to the Summ of the remaining Angles of the other.
3. If one Angle of a Triangle be Right, the other two are equal to a Right: Likewiſe that Angle in a Triangle which is equal to the other two, is a Right Angle.
4. When in an Iſcoſceles Triangle the Angles made by the equal Sides are Right, the other two upon the Baſe, are each of them half a Right Angle.
5. One Angle of an Equilateral Triangle makes two third parts of a Right Angle: for $\frac{1}{3}$ of two Right Angles is $\frac{2}{3}$ of one.

Scholium.

By help of this Threorem you may know how many Right Angles, the inward and outward Angles of a Right-lined Figure make; as may appear by the two next following Theorems.

IX. *All*

IX. *All the Angles of a Right-lined Figure together, make twice as many Right Angles, abating Four, as there are Sides of the Figure.*

FRom any Point within the Figure let Right Lines be drawn to all the Angles of the Figure, which shall resolve the Figure into so many Triangles as there are Sides of the Figure: Wherefore, whereas every Triangle affords two Right Angles, all the Triangles taken together, will make up twice as many Right Angles as there are Sides; but the Angles about the Point within the Figure make up four Right Angles: Therefore, If from the Angles of all the Triangles you take away the Angles which are about the said Point, the remaining Angles (which make up the Angles of the Figure) will make twice as many Right Angles (abating Four) as there are Sides of the Figure. *Which was to be demonstrated.* Figure IX.

Corollary.

Hence all Right-lined Figures of the same Species, have the Summs of their Angles equal.

X. *All the outward Angles of any Right-lined Figure, taken together, make up Four Right Angles.*

FOR all the several inward Angles of a Figure, with the several outward Angles of of the same, make two Right Angles; therefore all the inward Angles, together with all the outward, make twice as many Right Angles as there are Sides of the Figure: But, (as it was now shewed,) all the inward Angles, with four Right Angles, make twice as many Right Angles as there are Sides of the Figure; therefore the outward Angles are equal to four Right Angles. *Which was to be demonstrated.*

Corollary.

All Right-lined Figures, of whatsoever Species, have the Summ of their outward Angles equal.

XI. *In every Parallelogram* ABCD, *the Complements* DG *and* GB, *of those Parallelograms* HE *and* FI, *which stand about the Diametre* AC, *are equal one to the other.*

Demonstration. FOrasmuch as the Triangle ACD is equal to the Triangle ACB, and the Triangle AGH to the Triangle AGE; and also the Triangle GCF to the Triangle GCI; therefore the Parallelogram DG is equal to the Parallelogram BG. *Which was to be demonstrated.* Eucl. El. L. 1. P. 43. Figure X.

XII. *In Right-angled Triangles* BAC, *the Square* BE, *which is made of the Side* BC, *that subtendeth the Right Angle* BAC, *is equal to both the Squares* BG *and* CH, *which are made of the Sides* AB *and* AC, *which contain the Right Angle.*

Construction. JOin or draw the Lines AD and AE, and by the Point A draw the Line AM, parallel to EC.

Demonstration. Becaus the Angle DBC is equal to the Angle FBA, add to them the Angle ABC, common to them both; then is the Angle ABD equal to FBC: Moreover, AB is equal to FB, and BD to BC; therefore is the Triangle ABD equal to the Triangle FBC: But the Parallelogram BM, is equal to the Triangle ABD; and the Parallelogram BG, equal to the Triangle FBC; (for GAC is one right Line by the 14 *P.* 1. *Lib. Eucl.*) Therefore is the Parallelogram BM equal to the Square BG; and by the same way of argument, the Parallelogram CM is equal to the Square CH; therefore is the whole Square BE equal to both the Squares BG and BH. *Which was to be demonstrated.* Eucl. El. L. 1. P. 47. Figure XI.

Scholium.

This most excellent and usefull Theorem, hath deserved the Title of *Pythagoras* his Theorem, becaus he was the Inventor of it; and for that by help of it, the Addition and Subtraction of Squares, Circles, *&c.* and many of the Problems in this Book are by it demonstrated.

The foregoing Theorem illustrated in Numbers.

Let the Side AB of the Triangle ABC be 28, the Side AC 21, and the Side BC 35; I say that the Square BE, made of the Side BC 35, is equal to the Squares BG and CH, made of the Sides BA 28, and AC 21.

The

The Multiplication of the Side A C 21, in it self.

21
21
21
42
441 The content of the Square C H.

The Multiplication of the Side A B 28, in it self.

28
28
224
56
784 The content of the Square B G.

The Multiplication of the Side B C 35, in it self.

35
35
175
105
1225 The content of the Square B E.

CH 441
BG 784
1225 The Summ of the two Squares B G and C H, equal to the Square B E.

XIII. *If the Square made upon one Side* B C *of a Triangle, be equal to the Squares made of the other two Sides of the Triangle* A B *and* A C; *then the Angle* B A C, *comprehended under those other two Sides of the Triangle* A B *and* A C, *is a right Adgle.*

Figure XII. *Construction.* DRaw to the Point A, in the Line C A, a perpendicular Line D A, making A D equal to A B, and join C D.

Demonstration. Now forasmuch as the Line D A is equal to A B, the Square made of the Line D A, is equal to the Square of made the Line A B: Put the Square of the Line A C common to them both: Wherefore, the Squares of the Lines D A and A C, are equal to the Squares of the Lines B A and A C: But (by the foregoing Theorem) the Square of the Line D C is equal to the Squares of D A and A C, for that the Angle D A C is a Right Angle; and the Square of B C is (by supposition) equal to the Squares of A B and A C: Wherefore the Square of D C is equal to the Square of B C, and the Side D C, equal to the Side B C. And forasmuch as A B is equal to A D, and A C is common to them both; therefore these two Sides D A and A C, are equal to B A and C A, the one to the other, and the Base C D is equal to the Base B C; wherefore the Angle D A C is equal to B A C: But the Angle D A C is a Right Angle, wherefore also B A C is a Right Angle. *Which was to be demonstrated.* Eucl. El. L. 1. P. 48.

XIV. *In obtuse angled Triangles* A B C, *the Square that is made of the side* A C, *subtending the obtuse Angle* A B C, *is greater than the Squares of the sides* B C, *and* A B, *that contain the obtuse Angle* A B C, *by a double rectangle figure* D M, *composed of one of the sides* C B, *which are about the obtuse Angle* A B C, *on which side (being produced), the Perpendicular* A D *falls, and under the Line* B D, *taken without the Triangle, from the Point on which the Perpendicular* A D *falls, to the obtuse Angle* A B C.

XIII. *Construction.* FIrst upon the sides A C, A B, and C B, describe the Squares C G, A L, and B R, and draw out the Side C B: Then from the Point A, let fall a Perpendicular to the side C B, extended in D, and continue it to S.

Demonstration: I say, that the Square C G subtending the obtuse Angle B, is equal to the Squares A L, and B R, by twice the rectangle D M.

For

For the Squares of { AC / CD and AD, / CB, BD, AD, and twice the Paral. DM / CB, AB, and twice the Paral. DM } are all equal.

Scholium.

Hence the sides of any obtuse angled Triangle ABC being known, the Segment BD intercepted between the Perpendicular AD, and the obtuse Angle ABC, as also the Perpendicular it self AD, shall be easily found out.

Illustration in Numbers.

Let AC be 20, AB 14, and CB 10; then is the Square of AC 400, of AB 196, of CB 100: Take the Square of AB 196, and the Square of CB 100, in all 296, out of 400, the Square of AC, and there will remain 104, for twice the rectangle figure BDMS, the half whereof is 52 for BDMS. Divide this 52 by CB 10, and thence will come $5\frac{1}{5}$, for the side of the rectangle BD; and so having AB and BD, the Perpendicular AD, may be found by the *Theorem* last before going, 47 *P. Eucl.*

XV. *In acute angled Triangles* ABC, *the Square made of the side* AB, *subtending the acute Angle* ABC, *is less then the Squares made of the sides* AC *and* BC, *comprehending the acute Angle* ACB, *by a double rectangle contained under one of the sides* BC, *which are about the acute Angle* ABC, *on which the Perpendicular* AD *falls, and under the Line* CD, *taken within the Triangle, from the Perpendicular* AD, *to the acute Angle* ACB.

Construction. UPon the sides AB, AC, and CB, describe the respective Squares AL, AF, and BG, and from the Point A, let fall the Perpendicular AH, cutting the side BC in the Point D. Figure XIV.

Demonstration: I say that the Squares of the sides AC and BC, comprehending the acute Angle ACB, are greater than the Square of AB, opposite to the acute Angle ABC, by twice the rectangle DG, for

The Squares of { AC and BC / AD, DC and BC / AD, BD, and twice the Paral. GD, / AB and twice the Paral. DG. } are all equal.

Corollary.

Hence the sides of any acute angled Triangle, ABC, being known, you may find out the segment DC, intercepted between the Perpendicular AD, and the acute Angle ACB; as also the Perpendicular it self AD.

Illustration in Numbers.

Let AB be 13, AC 15, and BC 14; then will,

The Square of { AB be 169. / AC be 225. / BC be 196. }

Take AB 169, from AC and BC 225 and 196, that is from 421, and there will remain 252, for twice the Parallelogram DG; wherefore DG will be 126, which divide by BC 14, and thereof will come 9 for DC; from whence it follows, that AD is equal to the Root of 225, want the Square of CD 9 which is 81, that is 144; whose Square Root is 12 for the Perpendicular AD.

XVI. *In a Circle* EABC, *equal right Lines* AC *and* BD *are equally distant from the Centre* E, *and right Lines* AC *and* BD, *which are equally distant from the Centre, are equal among themselves.*

Construction. FRom the Centre E, draw the Perpendiculars EF and EG, which will bisect the Lines AC and BD; joyn also EA and EB. Figure XV.

Demonstration: I say 1. AC is equal to BD, therefore AF is equal to BG; but also EA is equal to EB; therefore the Square of FE, is equal to the Square EA, wanting the Square AF; equal to the Square of BG; and therefore EF and EG are equal.

2. EF

2. E F is equal to E G, therefore the Square of A F is equal to the Squnre E A, wanting the Square of E F, equal to the Square of E B, wanting the Square E G: And therefore A F is equal to B G; and consequently, A C to B D. *Which was to be Demonstrated.* Eucl. El. Lib. 3. P. 14.

Figure XVI. XVII. *In a Circle* D A B C, *the Angle* B D C, *at the Centre, is double to the Angle* B A C, *at the Circumference; when the same Arch of the Circle* B C, *is the Base of the Angles.*

Construction. DRaw the Diametre A D E.

Demonstration: Then the outward Angle B D E, is equal to the Angles D A B and D B A, both which Angles are equal to the Angle D A B twice. Likewise the Angle E D C is equal to twice D A C; therefore in the first Case, the whole Angle B D C is equal to twice B A C; and in the third Case, the remaining Angle B D C is equal to the Angle B A C twice. *Which was to be Demonstrated.* Eucl. El. Lib. 3. Pr. 20.

Figure XVII. XVIII. *The Angles* A D C *and* A B C *of a* Quadrilateral *Figure,* A B C D, *described in a Circle, which are opposite one to the other, are equal to two right Angles.*

Construction. Draw the two right Lines A C and B D.

Demonstration: I say that the three Angles of the Triangle A B C, *viz.* B A C, A B C, and B C A, are equal to two right Angles: But B D A is equal to B C A, and B D C is equal to B A C: Therefore A B C and A D C, are equal to two right Angles. *Which was to be Demonstrated.* Eucl. El. Lib. 3. P. 22.

Corollary.

1. Hence, if one side A B of a *Quadrilateral*, described in a Circle, be produced, the external Angle E B C, is equal to the internal Angle A D C; which is opposite to that A B C, which is adjacent to E B C.

2. A Circle cannot be described about a *Rhombus* or *Rhomboyades*, because their opposite Angles are greater or lesser than two right Angles.

Scholium.

If in a *Quadrilateral* figure A B C D, the Angles A and C, or B and D (which are opposites,) be equal to two right Angles, then may a Circle be described about that Equilateral figure.

XIX. *In a Circle the Angle* A B C, *which is in the Semicircle, is a right Angle; but the Angle which is in the greater Segment* B A C, *is less than a right Angle, and the Angle which is in the lesser Segment* B F C, *is greater than a right Angle. Moreover, the Angle of the greater Segment is greater than a right Angle, and the Angle of the lesser Segment is less than a right Angle.*

Figure XVIII. *Construction.* FRom the Centre D, draw a Line D B.

Demonstration: Because D B is equal to D A, therefore is the Angle D A B, equal to the Angle A B D, and the Angle D C B equal to the Angle D B C. Therefore the Angle A B C, is equal to the two Angles C A B, and B C A, equal to E B C the outer Angles; so that A B C and E B C are right Angles; which was to be demonstrated: therefore B A C is an Acute Angle, which was also to be demonstrated. And farthermore, whereas B A C and B F C are equal to two right Angles, therefore B F C is an obtuse Angle. Lastly, the Angle contained under the right Line B C, and the Arch B A C, is greater than the right Angle A B C: But the Angle made by the right Line C B, and the Periphery of the lesser Segment B F C, is less than the right Angle A B C. *Which was to be demonstrated.* Eucl. El. Lib. 3. P. 31.

Scholium.

In a right angled Triangle A B C, if the Hypotenuse (or subtended Line) A C, be bisected in D, a Circle drawn from the Centre D, through the Point A, shall also pass by the Point B.

XX. *If*

XX. *If a right Line* A B *touch a Circle, and from the Point of contact be drawn a Line* C E, *cutting the Circle; the Angles* E C B *and* E C A *which it makes with the Tangent Line, are equal to thoſe Angles* E D C *and* E F C, *which are made in the Alternate Segments of the Circle.*

Demonſtration. LET C D, the ſide of the Angle E D C, be Perpendicular to A B, therefore is C D the Diametre, and therefore, alſo the Angle C E D is a right Angle, it being in the Semicercle D E C. Therefore the Angles C D E and D C E are equal to a right Angle; that is, they are equal to E C B, and D C E: Therefore the Angle C D E is equal to E C B. *Which was to be demonſtrated.* Eucl. El. Lib. 3. P. 32. Figure XIX.

Now whereas the Angles E C B and E C A are equal to two right Angles, namely, to D and F, from both of theſe take away E C B and D, (which are equal,) then remains B C A equal to F. *Which was to be demonſtrated.* Eucl. El. Lib. 3. P. 32.

XXI. *If in a Circle* C A D B, *two right Lines* A B *and* D C *cut each other, the Rectangle comprehended under the Segments* A E *and* E B *of the one, ſhall be equal to the Rectangle comprehended under the Segments* E C *and* E D *of the other.*

Conſtruction. JOin C A and B D.

Demonſtration. Foraſmuch as the Angles C E A and D E B are equal, and alſo C and B (upon the ſame Arch A D) are equal; thence are the Triangles C E A and B E D equiangular: Wherefore C E is to E A, as E B is to E D; and conſequently, the Rectangle Figure made of C E and E D, ſhall be equal to the Rectangle Figure made of A E and E B. *Which was to be demonſtrated.* Eucl. El. Lib. 3. P. 35. Figure XX.

XXII. *If any Point* D *be taken without a Circle* E B C A, *and from that Point two right Lines* D A *and* D B *fall upon the Circle, whereof one* D A *cuts the Circle, and the other* D B *only touches it; the Rectangle comprehended under the whole Line* D A *that cuts the Circle, and under* D C *that part which is taken from the given Point* D, *to the convex of the Periphery, ſhall be equal to the Square made of the Tangent Line* D B.

Conſtruction. DRaw the two right Lines A B and B C. Figure XXI.

Demonſtration. Foraſmuch as the Angle at A, and the Angle D B C are equal, and the Angle at D common to them both; thence are the Triangles B D C and A B D equiangular: Wherefore A D is in Proportion to D B, ſo is D B to D C. And conſequently, the Rectangle Figure made of A D and B C ſhall be equal to the Square made of D B. *Which was to be demonſtrated.* Eucl. El. Lib. 3. P. 36.

Corollaries.

1. Hence, if from any point A, taken without a Circle, there be ſeveral Lines as A B, A C drawn which cut the Circle, the Rectangles comprehended under the whole Lines A B, A C, and the outward Parts A E, A F, are equal between themſelves.

For if the Tangent A D be drawn, then is the Rectangle made of C A and A F, equal to the Square of A D: Alſo the Rectangle made of A B and A E, ſhall be likewiſe equal to the Square made of A D.

2. It appears alſo from hence, That if two Lines, A B and A C, drawn from the Point A without the Circle, do touch the Circle, thoſe two Lines are equal one to the other.

For if A E be drawn cutting the Circle, then is the Square of A B equal to the Rectangle made of E A and A F; and alſo to the Square of A C.

3. It is alſo evident, That from a Point A, taken without a Circle, there can be drawn but two Lines, A B and A C, that ſhall touch the Circle.

For if a third Line A D be ſaid to touch the Circle, then is A D equal to A B, and alſo to A C; *which is abſur'd.*

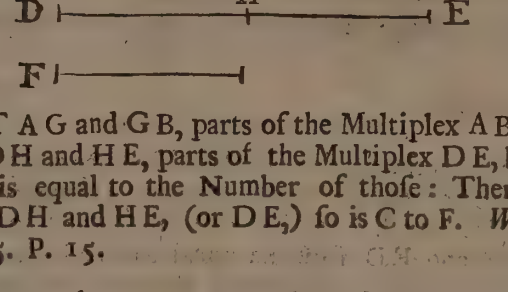
4. And on the contrary, it is plain, That if two equal right Lines, A B and B C, fall from any Point A upon the Convex Periphery of a Circle, and if that one of these equal Lines A B touch the Circle; then the other Line A C touches the Circle also.

For, (if possible,) let not A C, but some other Line, A D, touch the Circle; therefore is A D equal to A B or C D; *which is also absurd.*

5. It is also demonstrable, That either of the Lines A B or A C, are a Mean Proportional Line, between A E and A F.

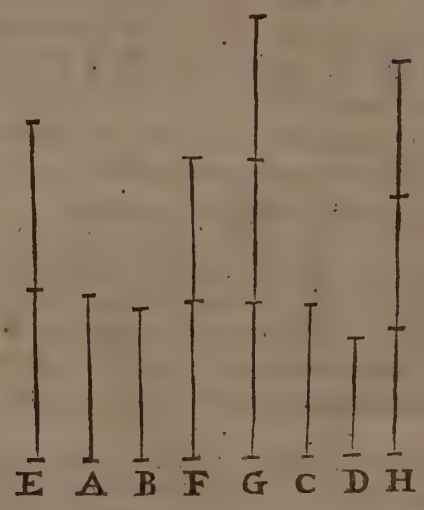
For it is already demonstrated, That the Square made of A C or C B is equal to the Rectangle Figure made of A E and A F.

XXIII. *Parts,* C *and* F, *are in the same* Ratio *with their like Multiplices,* A B *and* D E; *if taken correspondently,* (*as* A B *is to* D E, *so is* C *to* F.)



Demonstration. LET A G and G B, parts of the Multiplex A B, be equal to C; and let D H and H E, parts of the Multiplex D E, be equal to F; the Number of these Parts is equal to the Number of those: Therefore, whereas G A and G B (or A B) is to D H and H E, (or D E,) so is C to F. *Which was to be demonstrated.* Eucl. El. Lib. 5. P. 15.

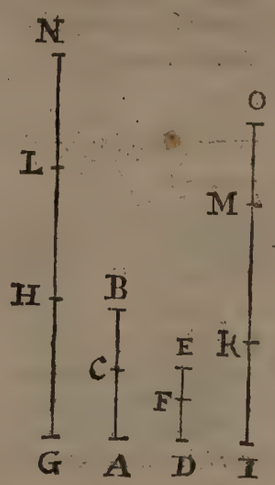
XXIV. *If four Magnitudes* A, B, C, *and* D, *be Proportional, they also shall be alternately Proportional;* (*as* A *to* C, *so* B *to* D.)



Demonstration. TAke E and F, Equimultiplices of C and D: Therefore E is to F as A to B, and A is to B as C is to D, and C is to D as G is to H. Wherefore, if E be greater, equal, or less than G; then likewise is F greater, equal, or less than H: Therefore A is to C, as B is to D. *Which was to be demonstrated.* Eucl. El. Lib. 5. P. 16.

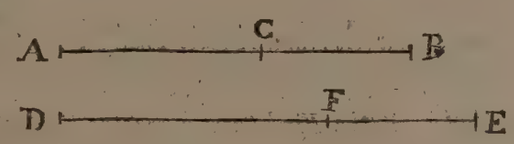
XXV. *In*

XXV. *If Magnitudes compounded be Proportional,* (*as* A B *to* C B, *ſo is* D E *to* F E,) *they ſhall be Proportional alſo when divided*; (A C *to* C B; *as* D F *to* F E.)



Demonſtration. TAke G H, H L, I K, K M, in Order; the Equimultiplices of A C, C B, D F, and F E; and alſo L N, M O, the Equimultiplices of C B, and F E. The whole G L is as Multiplex to the whole A B, as one G H, of one A C; that is, as I K of D F: Or, as the whole I M, of the whole D E. Alſo, H N, (or H L, and L N,) is as Multiplex of C B, as K O, (or K M, and M O,) is of F E: Therefore, whereas by the Hypotheſis, A B is to B C, as D E to E F. If G L be greater, equal or leſs than H N; then likewiſe, will I M be greater, equal, or leſs than K O. Take from thoſe H L and K M that are equal, and if the Remainder G H be greater, equal or leſs than L N, then will I K be greater, equal, or leſs than M O. Whence, A C is to C B, as D F is to F E. *Which was to be demonſtrated.* Eucl. El. Lib. 5. P. 17.

XXVI. *If the whole* A B, *be to the whole* D E, *as the part taken away* A C, *is to the part taken away* D F; *then ſhall the reſidue* C B, *be to the reſidue* F E, *as the whole* A B *is to the whole* D E.



Demonſtration. BEcauſe A B is to D E, as A C is to D F; therefore by permutation, A B is to B C, as D E is to E F, and thence by Diviſion: A C is to C B, as D F to F E. Wherefore again by permutation, as A C is to D F, as C B is to F E; that is, as A B is to B E, ſo is C B to F E. *Which was to be demonſtrated.* Eucl. El. Lib. 5. P. 19.

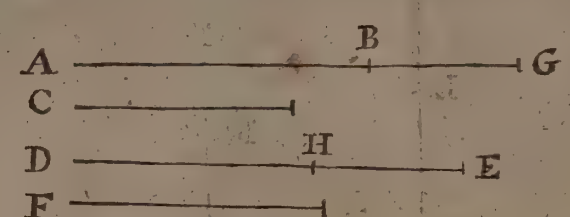
Corollary.

Hence if like Proportionals be ſubtracted from like Proportionals; the reſidues ſhall be Proportional.

Hence is Converſe *Ratio* demonſtrated.

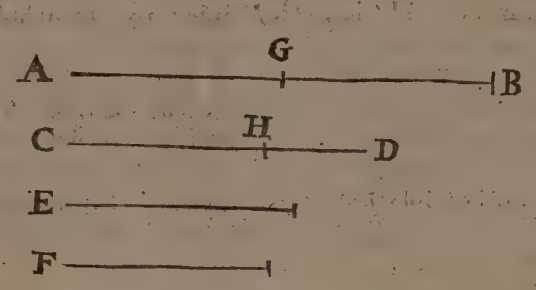
Let A B be to C B, as D E to F E: I ſay that A B is to A C, as D E is to D F; for by permutation, A B is to D E, as C B is to F E: Therefore A B is to D E, as A C to D F; whence again by permutation, A B is to A C, as D E is to D F. *Which was to be demonſtrated.*

XXVII. *If the First Magnitude* A B, *have the same* Ratio *to the Second* C, *which the Third* D E *hath to the Fourth* F, *and if the Fifth* B G *have the same* Ratio *to the Second* C, *which the Sixth* E H *hath to the Fourth* F: *Then shall the First, compounded with the Fifth,* (A G,) *have the same* Ratio *to the Second* C, *which the Third, compounded with the Sixth,* (D H,) *hath to the Fourth* F.



Demonstration. FOR because A B is to C, as D E is to F; and by the *Hypothesis* and inversion, C is to B G, as F to E H: Therefore, by equality, A B is to B G, as D E to F H; whence by compounding, A B is to B G, as D H to E H. Also, B G is to C, as E H is to F. Therefore again by equality, A G is to C, as D H is to F. *Which was to be demonstrated.* Eucl. El. Lib. 5. Pr. 24.

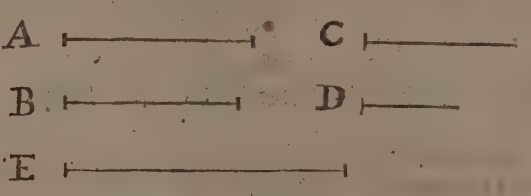
XXVIII. *If Four Magnitudes be Proportional,* (*as* A B *to* C D, *so* E *to* F,) *the greatest* A B, *and the least* F, *shall be greater than the rest,* C D *and* E.



Construction. MAke the Magnitude A G equal to E, and C H equal to F.

Demonstration: Because A B is to C D, as E to F, and E is to F, as A G is to C H; thence is A B to C D, as G B is to H D: But A B is greater than C D; wherefore G B is greater than H D: But A G and F are equal to E and C H; therefore A G and F and G B, are greater than E and C H and H D. That is, A B and F together, are greater than E and C D together. *Which was to be demonstrated.* Eucl. El. Lib. 5. Pr. 25.

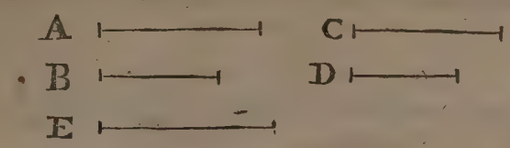
XXIX. *Of Four Magnitudes, if the First have a greater Proportion to the Second, than the Third to the Fourth; then contrariwise, by Conversion, the Second shall have a less Proportion to the First, than the Fourth to the Third.*



Demonstration. LET the *Ratio* of A to B be greater then that of C to D: I say, that the *Ratio* of B to A, is less than that of D to C: For conceive the *Ratio* of C to D to be equal to the *Ratio* of E to B; therefore the *Ratio* of A to B is greater than the *Ratio* of E to B: Whence A is greater than E; therefore the *Ratio* of B to A is less than that of B to E, or of D to C. *Which was to be demonstrated.*

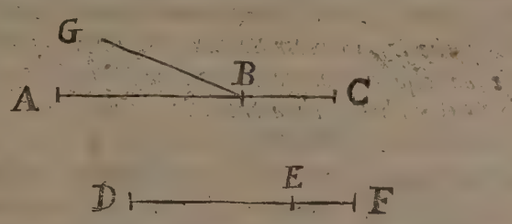
XXX. *Of*

XXX. *Of Four Magnitudes, if the First have a greater Proportion to the Second, than the Third hath to the Fourth; then, alternately, the First shall have a greater Proportion to the Third, than the Second hath to the Fourth.*



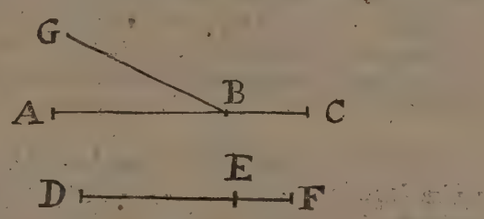
Demonstration. LET the *Ratio* of A to B be greater than that of C to D; then I say that the *Ratio* of A to C shall be greater than that of B to D: For conceive the *Ratio* of E to B to be equal to that of C to D; therefore A is greater than E, and therefore the *Ratio* of A to C is greater than that of E to C, or of B to D. *Which was to be demonstrated.*

XXXI. *Of Four Magnitudes, if the First have a greater Proportion to the Second than the Third to the Fourth; then the First, compounded with the Second, shall have a greater Proportion to the Second, than the Third, compounded with the Fourth, hath to the Fourth.*



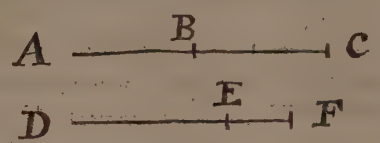
Demonstration. LET the *Ratio* of AB to BC be greater than that of DE to EF: I say that the *Ratio* of AC to BC is greater than that of DE to EF; for conceive the *Ratio* of GB to be equal to that of DE to EF; therefore is AB greater than GB: add BC to each Part; then will AC be greater than GC, and therefore the *Ratio* of AC to BC is greater than that of GC to BC; that is, of DF to FE. *Which was to be demonstrated.*

XXXII. *Of Four Quantities, If the First, compounded with the Second, have a greater Proportion to the Second, than the Third, compounded with the Fourth, hath to the Fourth; then, by Division, the First shall have a greater Proportion to the Second, than the Third hath to the Fourth.*



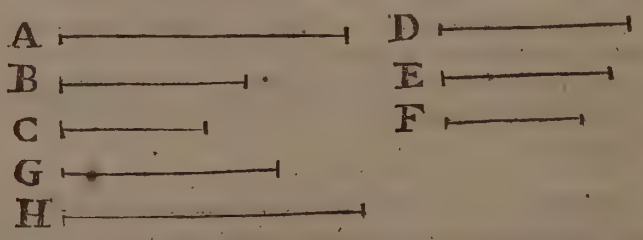
Demonstration. LET the *Ratio* of AB to BC be greater than that of DE to EF; then I say, that the *Ratio* of AB to BC is greater than that of DE to EF; for conceive the *Ratio* of GC to BC, to be equal to that of DE to EF, therefore AC is greater than GC: Take away BC, which is common, and there will remain AB greater than BG: Therefore, the *Ratio* of AB to BC is greater than that of GB to BC, or that of DE to DF. *Which was to be demonstrated.*

XXXIII. *Of Four Quantities, If the First, compounded with the Second, have a greater Proportion to the Second, than the Third, compounded with the Fourth, hath to the Fourth; then, by converse* Ratio, *shall the First, compounded with the Second, have a lesser* Ratio *to the First, than the Third, compounded with the Fourth, shall have to the Third.*



Demonstration. LET the *Ratio* of A to BC be greater than that of DF to EF: Then I say that the *Ratio* of AC to AB is greater than that of DF to DE; for because that the *Ratio* of AC to that of BC, is greater than that of DF to EF: Therefore, by Division, the *Ratio* of AB to BC, shall be greater than that of DE to EF; by conversion therefore, the *Ratio* of BC to that of AB shall be lesser than the *Ratio* of EF to that of DE; and therefore, by composition, the *Ratio* of AC to that of AB, shall be lesser than that of DF to DE. *Which was to be demonstrated.*

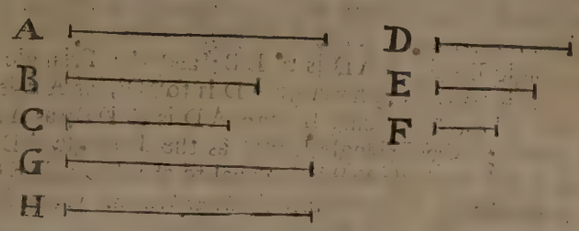
XXXIV. *If there be Three Magnitudes* A, B, *and* C, *and other Three also* D, E, *and* F, *equal to them in number; and if there be a greater Proportion of the First of the former to the Second, than there is of the First of the last to their Second,* (A *to* B *greater than* D *to* E:) *And there be also a greater Proportion of the Second of the First Magnitude to the Third, than there is of the Second of the last Magnitudes to their Third,* (B *to* C *greater than* E *to* F;) *then by equality also, shall the* Ratio *of the First of the former Magnitudes to the Third be greater than the* Ratio *of the First of the latter Magnitudes to the Third,* (A *to* C *greater than* D *to* F.)



Demonstration. CONceive the *Ratio* of G to C, to be equal to the *Ratio* of E to F; therefore is B greater than G; and therefore the *Ratio* of A to B, is greater than that of A to G: Again, conceive the *Ratio* of H to G, to be equal to that of D to F; therefore the *Ratio* of H to G, shall be lesser than that of A to B; therefore, much more shall the *Ratio* of H to G, be lesser then that of A to G, wherefore A is greater than H, and consequently, the *Ratio* of A to C, greater than that of H to C, or D to F. *Which was to be demonstrated.*

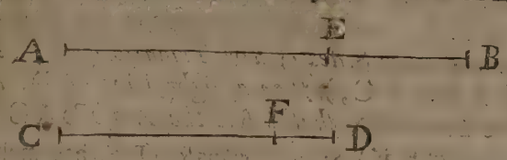
XXXV. *If there be Three Magnitudes* A, B, C, *and others* D, E, F, *equal to them in Number, and there be a greater Proportion of the First of the former Magnitudes to the Second, than there is of the Second of the latter to the Third,* (*the* Ratio *of* A *to* B *greater than the* Ratio *of* E *to* F;) *and also the* Ratio *of the Second of the former to the Third, be greater than the* Ratio *of the First of the latter to the Second,* (*the* Ratio *of* B *to* C *greater than that of* D *to* E;) *then by equality also, shall the Proportion of the First of the former to the Third, be greater than that of the First of the Latter to the Third,* (*the* Ratio *of* A *to* C *greater than that of* D *to* E.)

Demon-



Demonstration. THE Demonstration of this Theorem is altogether like that of the precedent Theorem.

XXXVI. *If the Proportion of the Whole* AB *to the Whole* CD, *be greater than the Proportion of the Part taken away* AE, *to the Part taken away* CF; *then shall also the* Ratio *of the Remainder* EB *to the Remainder* FD, *be greater than that of the Whole* AB *to the Whole* CD.



Demonstration. BEcause that the *Ratio* of AB to CD is greater than the *Ratio* of AE to CF; therefore by permutation, the *Ratio* of AB to AE, is greater than that of CD to CF: Therefore by converse *Ratio*, the *Ratio* of AB to EB, is lesser than that of CD to FD: and by permutation again, the *Ratio* of AB to CD, is lesser than that of EB to FD. *Which was to be demonstrated.*

XXXVII. *Triangles,* ABC *and* ACD, *and Parallelograms,* BCAE *and* CDFA, *which have the same highth, are in proportion one to the other, as are their Bases* BC *and* CD.

Construction. TAke as many as you please, BG and GH, equal to BC, and also DK equal to CD, and joyn AG, AH, and AK, Figure XXII.

Demonstration. I say the Triangles ACB, ABG, and AGH are equal; and also the Triangle ACD equal to the Triangle ADK: Therefore the Triangle AHC, is as Multiplex of the Triangle ABC, as the Base HC is of the Base BC; and the Triangle ACK, as Multiplex of the Triangle ACD, as the Base CK is of CD. But if HC be greater, equal, or less than CK, then likewise is the Triangle AHC, greater, equal, or less than the Triangle ACK; and therefore BC is to CD, as the Triangle ABC is to the Triangle ACD; and as the Triangles ABC, and ACD are in Proportion, so are the Parallelograms CE and CF. *Which was to be demonstrated.* Eucl. El. L. 6. P. 1.

Scholium.

Hence Triangles, ABC and DEF, and Parallelograms, AGBC and DEFH, whose Bases BC and EF are equal, are in such Proportion as their Altitude.

Construction. MAke IL equal to CB, and KM equal to EF, and joyn IA, LG, MD, and MH. Figure XXIII.

Demonstration. Then it is evident, that the Triangle ABC is to the Triangle DEF, as ALI is to DKM; and ALI is to DKM, as AL is to DK; and AL to DK, as the Parallelogram AGBC is to the Parallelogram DEFH. *Which was to be demonstrated.*

XXXVIII. *If to one side* BC *of a Triangle* ABC, *be drawn a Parallel right Line* DE, *the same shall cut the sides of the Triangle Proportionally;* (*as* AD *to* DB, *so is* AE *to* EC;) *and if the sides of the Triangle be Proportionally cut,* (*as* AD *to* BD, *so is* AE *to* EC;) *then a right Line,* DE, *joyned at the Sections* D *and* E, *shall be Parallel to the remaining side of the Triangle* BC.

Construction. DRaw a Line DE Parallel to the side BC of the Triangle ABC: Also draw the right Lines CD and BE. Figure XXIV.

Demonstration. For the first part of the *Theorem*, because the Triangle DEB is equal

equal to the Triangle DEC; therefore ſhall the Triangle ADE be in proportion to the Triangle DBE, as the Triangle ADE is to the Triangle ECD: But the Triangle ADE is to the Triangle DBE, as AD is to DB; and the Triangle ADE is to the Triangle DEC, as AE is to EC: Therefore AD is to DB, as AE is to EC.

For the Second Part of the Theorem: Becauſe AD is to DB, as AE is to EC, that is, the Triangle ADE is to the Triangle DBE, as the Triangle ADE is to the Triangle ECD; therefore the Triangle DBE is equal to the Triangle ECD: And therefore DE and BC are Parallels. *Which was to be demonſtrated.* Eucl. El. L. 6. P. 2.

Scholium.

If there be drawn many Parallels to one Side of any Triangle, then all the Segments of the Sides ſhall be proportional.

XXXIX. *If an Angle* BAC *of a Triangle* BAC, *be biſected, and the right Line* DA, *that biſects the Angle, cuts the Baſe alſo; then ſhall the Segments of the Baſe have the ſame* Ratio (*or Porportion*) *that the other Sides of the Triangle have:* (*that is, as* BD *to* DC, *ſo is* AB *to* AC.) *And if the Segments of the Baſe have the ſame* Ratio, *that the other Sides of the Triangle have;* (*that is, as* BD *to* DC, *ſo* AB *to* AC;) *then a right Line,* AD, *drawn from the top* A, *to the Section* D, *ſhall biſect the Angle* BAC *of the Triangle.*

Figure XXV. *Conſtruction.* PRoduce the Side BA of the Triangle ABC, and make AE equal to AC, and join C and E.

Demonſtration. For the Firſt Part of the Theorem: Becauſe AE is equal to AC, therefore the Angle ACE is equal to the Angle AEC, and alſo to half the Angle BAC, which is equal to DAC; therefore DA and CE are parallel: Wherefore BA is to BE (or AC) as BD is to DC.

For the Second Part of the Theorem: Becauſe BA is to AC, (or AE,) as BD is to DC; therefore DA and CE are Parallels; and therefore is the Angle BAD equal to the Angle BEC, and the Angle DAC to the Angle ACE, (or AEC:) Therefore the Angle BAD is equal to the Angle DAC: Wherefore the Angle BAC is biſected. *Which was to be demonſtrated.* Eucl. El. L. 6. P. 3.

XL. *Of Equiangular Triangles,* ABC, *and* DCE, *the Sides which are about the equal Angles are proportional:* (AB *is to* BC, *as* DC *is to* CE, *&c.*) *And the Side* AB *is to* DC, *as* AC *to* DE, *&c. which are ſubtendent under the equal Angles* ACB, *and* DEC, *&c. are homologous, or of like* Ratio, *or Proportion.*

Figure XXVI. *Conſtruction.* SET the Side BC of the Triangle ABC in a direct Line to the Side CE of the Triangle DCE, and produce the Sides BA and ED till they meet in F.

Demonſtration. Becauſe the Angle ABC is equal to the Angle DCE, therefore BF and CD are parallel: Alſo, becauſe the Angle BCA is equal to the Angle CED, therefore are CA and EF parallel: Therefore the Figure CAFD is a Parallelogram; and alſo AF and CD are equal, and AC and FD alſo equal. Whence it is evident, That AB is to AF, (or CD,) as BC is to CE: By permutation therefore, AB is to BC, as DC is to CE: Alſo, as BC is to CE, ſo is FD or (AC) to DE; and thence by permutation, as BC is to AC, ſo is CE to DE: Wherefore alſo by equality, as AB is to AC, ſo is DC to DE: Therefore, of Equiangular Triangles, *&c. Which was to be demonſtrated.* Eucl. El. L. 6. P. 4.

Corollary.

Hence as AB to DC, ſo is BC to CE.
And as BC is to CE, ſo is AC to DE.

Scholium.

Hence if in a Triangle BFE there be drawn AC, parallel to one Side thereof FE, the Triangle BAC ſhall be like to the whole Triangle FBE.

XLI. *If two Triangles,* ABC *and* DEF; *have one Angle* ABC, *equal to one Angle* DEF, *and the ſides about the equal Angles* ABC *and* DEF *Proportional,* (*as* AB *to* BC, *ſo* DE *to* DF;) *then thoſe Triangles* ABC *and*

and DEF, *are Equiangular, and have thoſe Angles Equal, under which are ſubtended the* Homologous *Sides.*

Conſtruction. AT the ſide EF, make the Angle FEG equal to the Angle ABC, and the Angle EFG equal to the Angle ACB, then will the Angle EGF be equal to the Angle BAC. Figure XXVI.

Demonſtration. Therefore GE is to EF, as AB is to BC; and AB is to BC, as DE is to DF; and therefore, DE is equal to GE. But the Angle DEF is equal to the Angle ABC, and alſo to GEF; therefore the Angles at D, G, and A, are all equal, and conſequently, the Angle EFD, equal to the Angle ACB. *Which was to be Demonſtrated.* Eucl. El. Lib. 6. P. 6.

XLII. *If two Triangles,* ABC *and* DEF, *have one Angle* A, *equal to one Angle* D, *and the ſides about the other Angles* ABC *and* DEF, *Proportional, (as* AB *to* BC, *ſo* DE *to* DF,) *and if they have both of the remaining Angles* C *and* F, *either leſs or not leſs than a right Angle, then ſhall the Triangles* ABC *and* DEF, *be Equiangular, and have thoſe Angles equal, about which the Proportional ſides are.*

Demonſtration. FOR if it can be, let the Angle ABC, be greater than the Angle E, and make the Angle ABG equal to E: Now whereas the Angle A is equal to D, thence is the Angle AGB equal to the Angle F: Therefore AB is to BG, as DE is to DF; and therefore BG ſhould be equal to BC, and the Angle BGC equal to BCG; therefore BGC (or C) is leſs than a right Angle, and conſequently, AGB (or F) is greater than a right Angle. Therefore the Angles C and F are not of the ſame kind or ſpecies; *which is againſt the* Hypotheſis. Eucl. El. Lib. 6. P. 7. Figure XXVIII.

XLIII. *If in a right angled Triangle* ABC, *from the right Angle* BAC *there be drawn a Perpendicular* AD *to the Baſe* BC; *then the Triangles about the Perpendicular,* ADB *and* ACD, *are alike, both to the whole Triangle* ABC, *and alſo one to the other.*

Demonſtration. FOR becauſe BAC and ADB are right Angles, and ſo equal, and the Angle B common, the Triangles BAC and ADB are like; and by the ſame Argument BAC and ADC are like: Whence alſo ADB and ADC will be like. *Which was to be Demonſtrated.* Eucl. El. Lib. 6. P. 8. Figure XXIX.

Corollary.

Firſt, As BD to DA, ſo DA to DC.
Secondly, As BC to AC, ſo AC to DC. And,
Thirdly, As CB to BA, ſo BA to BD.

XLIV. *Equal Parallelograms, having one Angle* ABC, *equal to one* EBG, *have the ſides* BD *and* BF, *which are about the equal Angles, reciprocal, (as* AB *to* BG, *ſo is* EB *to* BC;) *and thoſe Parallelograms* BD *and* BF, *which have one Angle* ABC, *equal to one* EBG, *and the ſides which are about the equal Angles reciprocal, are equal.*

Conſtruction. FOR, let the ſides AB and BG about the equal Angles, make one right Line; wherefore EB, and BC ſhall doe the ſame. Let DC and FG be produced till they meet in H. Figure XXX.

Demonſtration. As to the firſt part of the *Theorem,* AB is to BG, as BD to BH.—BD is to BH, as BF to BH.—BF is to BH, as BE is to BC.

As to the ſecond Part of the *Theorem,* BD is to BH, as AB is to BG; AB is to BG, as BE is to BC; BE is to BC, as BF is to BH: Therefore the Parallelograms BD and BF are equal. *Which was to be Demonſtrated.* Eucl. El. Lib. 6. P. 14.

XLV. *If four right Lines be Proportional, (as* AD *to* FG, *ſo* EF *to* CB,) *the Rectangle* AC, *comprehended under the Extremes* AD *and* CB, *is equal to the Rectangle* EG, *comprehended under the Means* FG *and* EF. *And if the Rectangle* AC, *comprehended under the Extremes* AD *and* CB, *be equal to the Rectangle* EG, *comprehended under the Means*

FG *and* EF, *then are the four right Lines Proportional.* (*As* AB *to* FG, *so is* EF *to* CB.)

Figure XXXI. *Demonstration.* FOR the first part of the *Theorem*, the Angles B and F are right, and consequently equal: and by the *Hypothesis* AD is to FG, as EF to CB: Therefore the Rectangle AC is equal to the Rectangle EG.

Then for the second part of the *Theorem*, AC is equal to EG, and the Angle B equal to the Angle F: Therefore, As AD to FG, so is EF to BC. *Which was to be Demonstrated.* Eucl. El. Lib. 6. P. 16.

XLVI. *Like Triangles,* ABC *and* DEF, *are in like duplicate* Ratio *of their* Homologous *sides,* BC *and* EF.

Figure XXXII. *Construction.* LET there be made BC to EF, as EF to BG, and let AG be drawn.

Demonstration. Because that AB is to DE, as BC to EF; and as BC to EF, so is EF to BG; and the Angle B equal to the Angle E; therefore the Triangle ABG is equal to the Triangle DEF: But the Triangle ABC is to the Triangle ABG, as BC is to BG; and the *Ratio* of BC to BG, is equal to the *Ratio* of BC to EF twice: Therefore the *Ratio* of ABC to ABG, that is, the *Ratio* of ABC to DEF, is equal to the *Ratio* of BC to EF twice. *Which was to be Demonstrated.* Eucl. El. Lib. 6. P. 19.

Corollary.

Hence, if three right Lines (BC, EF, and BG) be Proportional; then, as the First is to the Third, so is a Triangle made upon the first AC, to a Triangle like, and a like described upon the second EF: Or, so is a Triangle described upon the second EF to a Triangle like, and a like described upon the third.

XLVII. *In every Parallelogram* ABCD, *the Parallelograms* EG *and* HF, *which are about the Diametre* AC, *are like to the whole; and also one to the other.*

Figure XXXIII. *Demonstration.* FOrasmuch as the Parallelograms EG and HF, have each of them one Angle common to the whole, therefore they are Equiangular to the whole, and also one to the other; also both the Triangles ABC, AEI, and IHC, and the Triangles ADC, AGI, and IFC, are mutually Equiangular: Therefore AE is to EI, as AB to BC; and AE to AI, as AB to AC; and AI to AG, as AC to AD. Therefore by equality, AE is to AG, as AB is to AD; therefore the Parallelograms EG and BD are like: And after the same manner, are HF and BD like also. *Which was to be proved.* Eucl. El. Lib. 6. P. 24.

XLVIII. *In right Angled Triangles* BAC, *any figure,* BF, *described upon the side* BC, *subtending the right Angle* BAC, *is equal to the figures* BG *and* AL, *described upon the sides* AB *and* AC, *containing the right Angle; like, and alike situate to the former* BF.

Figure XXXIV. *Construction.* FRom the right Angle BAC let fall a Perpendidular AD.

Demonstration: Because that DA is to CA, as CA is to CB; therefore AL is to BF, as DC to DB, also, because DB is to BA, as AB to BC; therefore BG is to BF, as DB to BC: Therefore the Parallelograms AL and BG together, shall be equal to BF. *Which was to be Demonstrated.* Eucl. El. Lib. 6. P. 31.

Or thus, BG is to BF, as the Square of BA is to the Square of BC; and AL is to BF, as the Square of AC is to the Square of BC; therefore the Parallelograms BG and AL is to the Parallelogram BF, as the Squares of BA and CA is to the Square of BC: Therefore, whereas the Squares of AB and CA together, are equal to the Square of CB, thence are the Parallelograms BG and AL together, equal to the Parallelogram BF. *Which was to be Demonstrated.*

Corollary.

From this Theorem is produced the Addition and Subtraction of like Figures, by the same method that is used in Adding and Subtracting of Squares.

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PARALLEL

BETWEEN

Arithmetick and Geometry,

Being the Second Book of

EUCLIDE'S ELEMENTS

Geometrically and Arithmetically Demonstrated.

I. *If there be two right Lines* A *and* B C, *and if one of them* B C, *be divided into Parts, how many soever, as* B D, D E, *and* E C; *the rectangle figure* B H, *comprehended under the two right Lines* A *and* B C, *is equal to all the rectangle figures comprehended under the Line* A *undivided, and under every one of the Parts of the other Line* B C.

Construction. FRom the Point B raise the Perpendicular Line B F, and make the Line B G equal to the Line A, and from G draw a Line G M, parallel to B C, and from the Points D, E, and C, draw the Lines D K, E L, and C H, parallel to B G. Figure I.

Demonstration. I say the Parallelogram B H, is equal to the Parallelograms B K, D L, and E H; but the Parallelogram B H, is equal to that which is contained under the Lines A and B C, (for it is comprehended under the Lines B G, and B C, and the Line G B is equal to A.) And the Parallelogram B K, is equal to that which is contained under the Lines A and B D, (for it is comprehended under the Line G B, and B D, and B G is equal to A.) And the Parallelogram D L, is equal to that which is contained under the Lines A and D E, (for the Line D K, equal to B G, is equal to A.) And Likewise, the Parallelogram E H, is equal to that which is contained under the Lines A and E C: Wherefore that which is comprehended under the Lines A and B C, is equal to that which is comprehended under A and B D, and unto that under A and D E, and unto that under A and E C. *If therefore there be two right Lines, and if one of them be divided into parts, how many soever, the rectangle Figure, comprehended under the two right Lines, is equal to the rectangle Figures comprehended under the Line undivided, and under every one of the Parts of the other Line.* Which was to be demonstrated.

Illustration in Numbers.

Take two Numbers, the one undivided (as 74,) the other divided into what Parts and how many soever, as 37, divided into 20, 10, 5, and 2, (which together make the whole 37. Then if you multiply the Number undivided, *viz.* 74 into all the parts of the Number divided, as into 20, 10, 5, and 2, you shall have these Products, 1480, 740, 370, and 148, which added together make 2738. And the like Number will also be produced, if you multiply the two first given Numbers one into the other, *viz.* 74 undivided, and 37 divided.

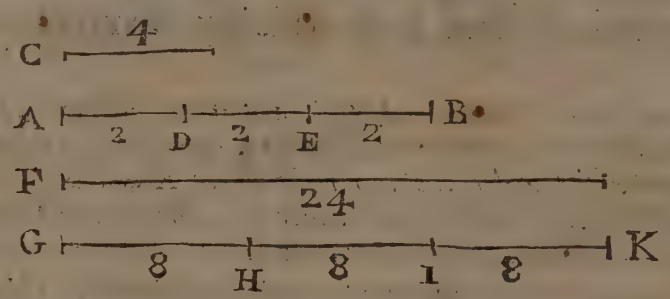
From this *Theorem* was the Compendious way of Multiplication now in Use derived by breaking one of the whole Numbers given, into its respective Parts.

These first Ten Propositions of this Second Book of *Euclide*'s Elements, are all of them demonstrated in Numbers by one *Barlaam*, a *Greek* Authour, whose Demonstrations shall follow after every *Theorem*; but first take these Principles of his.

1. *A Number is said to multiply another Number, when the Number multiplied is so often times added to it self, as there be Unites in the Number which multiplieth; whereby is produced a certain Number, which the Number multiplied measureth by the Unites which are in the Number which multiplieth.*

2. *And the Number produced of that,* Multiplication, *is called a* Plain, *or* Superficial Number.
3. *A* Square Number *is that which is produced of the Multiplication of any Number into it ſelf.*
4. *Every Leſs Number compared to a Greater, is ſaid to be a Part of the Greater, whether the Leſs meaſure the Greater or not.*
5. *Numbers which one and the ſelf ſame Number meaſure equally, that is, by one and the ſelf ſame Number, are equal the one to the other.*
6. *Numbers that are Equimultiplices to one and the ſelf ſame Number, that is, which contain one and the ſame Number equally, and alike, are equal the one to the other.*

Propoſition I. *Two Numbers being given, if one of them be divided into any Numbers, how many ſoever, the Plain or Superficial Number, which is produced of the Multiplication of the two Numbers firſt given, the one into the other, ſhall be equal to all the Superficial Numbers, which are produced of the Multiplication of the Number not divided into every part of the Number divided.*



Suppoſe that there be two Numbers A B and C, and divide the Number A B into certain other Numbers, how many ſoever, as unto A D, D E, and E B; then I ſay, the Superficial Number which is produced of the Multiplication of the Number C, into the Numbers A D, and of C into D E, and of C into E B; for let F be the Superficial Number produced of the Multiplication of the Number C, into the Numbers A B; and let G H be the Superficial Number produced of the Multiplication of the Number C, into A D; and let H I be produced of the Multiplication of C into D E: And finally of the Multiplication of C into E B, let there be produced the Number I K. Now foraſmuch as A B multiplying the Number C, produceth the Number F; therefore the Number C, meaſureth the Number F by the Unites, which are in the Number A B: And by the ſame reaſon may be proved, that the Number C doth alſo meaſure the Number G H by the Unites which are in the Number A D, and that it doth meaſure the Number H I by the Unites which are in the Number D E; and finally, that it meaſureth the Number I K by the Unites which are in the Number E B. Wherefore, the Number C meaſureth the whole Number G K by the Unites which are in the Num- A B; but it before meaſured the Number F, by the Unites which are in the Number A B; wherefore either of theſe Numbers F and G K is Equimultiplex to the Number C. But Numbers which are Equimultiplices to one and the ſelf ſame Number, are equal the one to the other, (by the ſixth *Definition*:) Wherefore the Number F is equal to the Number G K. But the Number F is the Superficial Number produced of the Multiplication of the Number C, into the Number A B; and the Number G K is compoſed of the Superficial Numbers produced of the Multiplication of the Number C, not devided into every one of the Numbers A D, D E, and E B: If therefore there be two Numbers given, and the one of them be divided, *&c. Which was required to be proved.*

II. *If a right Line* A B *be divided by chance in* C, *the Rectangle Figures* A F *and* C E, *which are comprehended under the Whole* A B, *and every one of the Parts,* A C *and* C B, *are equal to the Square made of the Whole* A B.

Figure II.

Conſtruction. UPon the Line A B, deſcribe a Square A B C D, and by the point C draw a Line Parallel to either of the ſides A D or B E, as C F.

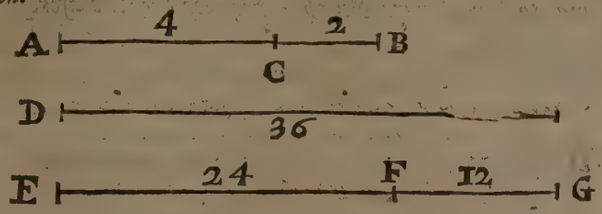
Demonſtration: Now I ſay, the Parallelogram A E, is equal to the Parallelograms A F

A F and C E: But A E is the Square made of A B, and A F is the Rectangled Parallelogram comprehended under the Lines B A and A C, for it is comprehended under the Lines A D and A C, but the Line A D is equal to the Line A B. Likewise, the Parallelogram C E is equal to that which is contained under the Lines A B and C B, for the Line E B is equal to the Line A B: Wherefore that which is contained under B A and A C, together with that which is contained under the Lines A B and C B, is equal to the Square made of the Line A B. Therefore, *If a right Line be divided by chance, &c.* Which was required to be demonstrated.

Illustration in Numbers.

Take any Number as 11, and divide it into two parts, as 7 and 4, first multiply 11 into 7, and then into 4, and thereof will come these Products 77 and 44, which added together make 121; which is equal to the Square Number of 11 multiplied in it self.

Proposition II. *If a Number given be divided into two other Numbers, the Superficial Numbers which are produced by the Multiplication of the Whole into either Part added together, are equal to the Square Number made of the Whole Number given.*



Suppose that the Number given be A B, and let it be divided into two other Numbers A C and C B: Then I say, that the two Superficial Numbers, which are produced of the Multiplication of A B into A C, and of A B into B C; these two Superficial Numbers added together shall be equal to the Square Number produced of the Multiplication of the Number A B into its self. For let the Number A B, multiplying it self, produce the Number D; let the Number A C also, multiplying the Number A B, produce the Number E F: Again, let the Number B C multiplying the same Number A B, produce the Number F G: now forasmuch as the Number A C, multiplying the Number A B, produces the Number E F; therefore the Number A B measureth the Number E F by the Unites which are in A C. Again, forasmuch as the Number C B multiplied the Number A B, and produced the Number F G; therefore the Number A B measureth the Number F G by the Unites which are in the Number C B; but the same Number A B before measured the Number E F by the Unites which are in the Number A C; wherefore the Number A B measureth the whole Number E G by the Unites which are in A B; and forasmuch as the Number A B, multiplying its self, produced the Number D; therefore the Number A B measureth the Number D, by the Unites which are in its self; wherefore it measureth either of these Numbers; namely, the Number D, and the Number E G, by the Unites which are in it self. Wherefore how Multiplex the Number D is to the Number A B, so Multiplex is the Number E G to the same Number A B: But Numbers which are Equimultiplices to one and the self same Number, are equal one to the other; wherefore the Number D is equal to the Number E G. And the Number D is the Square Number made of the Number A B, and the Number E G is composed of the two Superficial Numbers, produced of A B into B C, and of B A into A C: Wherefore the Square Number produced of the Number A B, is equal to the Superficial Numbers, produced of the Number A B, into the Number B C, and of A B into A C added together. *If therefore*, &c. Which was to be proved.

III. *If a right Line* A B *be divided by chance in* C, *the rectangled Figure* A E, *comprehended under the whole Line* A B, *and one of the Parts* B C, *is equal to the rectangled Figure* C F, *comprehended under the Parts* A C *and* A B, *and unto the Square* B D, *which is made of the aforesaid Part* B C.

Construction. DEscribe upon the Line B C a Square C D E B, and extend E D unto F; and by the Point A draw a Line Parallel unto either of the Lines C D or B E, as A F. Figure III.

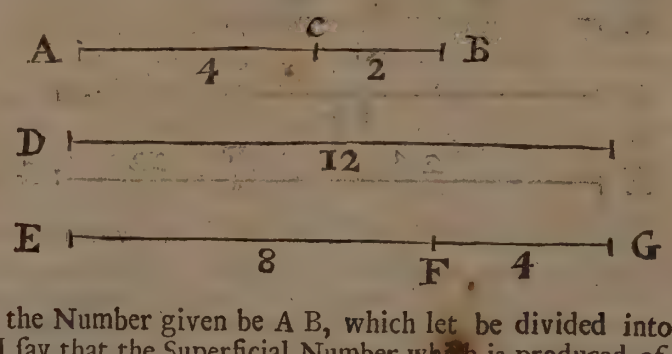
Demon-

Demonstration. Now the Parallelogram A E, is equal unto the Parallelograms A D and C E; and A E is the rectangled figure comprehended under A B and B C; for it is comprehended under the Lines A B and B E, which Line B E is equal to the Line B C; and the Parallelogram A D, is equal to that which is contained under the Lines A C and C B, for the Line D C, is equal unto C B, and D B is the Square which is made of the Line C B. Wherefore the rectangled figure, comprehended under the Lines A B and B C, is equal to the rectangled figure comprehended under the Lines A C and C B; and also unto the Square which is made of the Line B C. *If therefore a right Line* &c. Which was to be Demonstrated.

Illustration in Numbers.

Suppose the Number 14, to be divided into two Parts 8 and 6; the whole Number 14 multiplied into 8, one of his Parts, produceth 112: The Parts 8 and 6 multiplied together, produce 48, which added to the Square of 8, (which is 64,) the Amount is 112, equal to the former.

Proposition III. *If a Number given be divided into two Numbers, the Superficial Number, which is produced of the Multiplication of the Whole into one of the Parts, is equal to the Superficial Number which is produced of the Parts one into the other, and to the Square Number produced of the aforesaid Part.*



Suppose that the Number given be A B, which let be divided into two Numbers, A C and C B: I say that the Superficial Number which is produced of the Multiplication of the Number A B into the Number B C, is equal to the Superficial Number which is produced of the Multiplication of the Number A C into the Number C B, and to the Square Number produced of the Number C B. For let the Number A B, multiplying the Number C B, produce the Number D; and let the Number A C, multiplying the Number C B, produce the Number E F; and finally let C B multiplying its self produce F G: Now forasmuch as the Number A B, multiplying the Number C B, produces the Number D; therefore the Number C B, measureth the Number D by the Unites which are in the Number A B. Again, forasmuch as the Number A C, multiplied the Number C B, and produced the Number E F; therefore the Number C B, measureth the Number E F, by the Unites which are in A C. Again, forasmuch as the Number C B multiplied it self, and produced the Number F G; therefore the Number C B, measureth the Number F G, by the Unites which are in it self. But, as we have before proved, the self same Number C B measureth also the Number E F, by the Unites which are in the Number A C; wherefore the Number C B measureth the whole Number E G, by the Unites which are in the Number A B, and it also measureth the Number D by the Unites which are in the Number A B: Wherefore the Number C B equally measureth either Number, namely, the Number D, and the Number E G; but those Numbers whom one and the self same Number measureth equally, are equal the one to the other, wherefore the Number D is equal to the Number E G. But the Number D is a Superficial Number, produced of the Multiplication of the Number A B, into the Number B C; and the Number E G is the Superficial Number produced of the Multiplication of the Number A C into the Number C B, and of the Square of the Number C B: Wherefore the Superficial Number produced of the Multiplication of the Number A B into the Number C B, is equal to the Superficial Number produced of the Number A C into the Number B C, and to the Square of the Number C B. Therefore &c. *Which was required to be proved.*

IV. *If a right Line* A B *be divided by chance in* C, *the Square* A E *which is made of the whole Line* A B, *is equal to the Squares* C K *and* H F, *made of the Parts* A C *and* C B, *and unto the two Rectangle Figures* A G *and* G E, *comprehended under the Parts* A C *and* C B.

Con-

Construction. UPon the Line A B describe a Square, A B D E, and draw a Line from B to D; and by the Point C draw a Line Parallel unto either of the Lines A D or B E, as C F, cutting the Diametre D B in G; and by the Point G draw a Line Parallel to A B or D E, as H K. Figure IV.

Demonstration. Forasmuch as the Line C F is a Parallel unto A D, and upon them falleth a right Line B D, therefore the outward Angle C G B, is equal unto the inward and opposite Angle A D B; but the Angle A D B, is equal unto the Angle A B D, for the side B A is equal to the side A D: Wherefore the Angle C G B is equal to the Angle G B C; wherefore the side B C is equal to the side C G: But C B is equal to G K, and C G to K B; wherefore G K is equal to K B. Wherefore the figure C G K B consisteth of four equal sides: I say also, that it is a rectangle figure; for forasmuch as C G is Parallel to B K, and upon them falleth a right Line C B; therefore the Angles K B C, and G C B, are equal unto two right Angles. But the Angle K B C is a right Angle; wherefore the Angle B C G is also a right Angle: Wherefore the Angles opposite unto them, namely, C G K, and G K B, are also right Angles, Wherefore C G K B is a rectangle figure, and it was before proved that the sides are equal; wherefore it is a Square, and is described upon the Line B C.—And by the same reason also H F is a Square, and is described upon the Line H G, that is, upon the Line A C; wherefore the Squares H F and C K are made of the Lines A C and C B.—And forasmuch as the Parallelogram A G is equal to the Parellelogram G E, and A G is that which is contained under A C and C B, for C G is equal to C B, and the Parallelogram G E is equal to that which is contained under A C and C B; wherefore A G and G E are equal to that which is comprehended under A C and B twice. And the Squares H F and C K, are made of the Lines A C and C B; wherefore these four rectangle figures, H F, C K, A G, and G E, are equal unto the Squares which are made of the Lines A C and C B, and to the rectangle figure which is comprehended under the Lines A C and C B twice. But the rectangle figures H F, C K, A G, and G E, are the whole rectangle figure A D E B, which is the Square made of the Line A B; wherefore the Square which is made of the Line A B is equal to the Squares which are made of the Lines A C and C B, and unto the two rectangle figures A G and G E, which are comprehended under the Lines A C and B C. *If therefore a right Line*, &c. Which was to be Demonstrated.

Corollary.

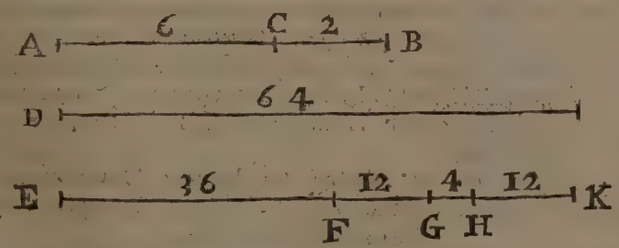
Hence it is manifest, that the Parallelograms which consist about the Diametres of a Square are Squares.

Upon this *Theorem* is grounded the Extraction of the Square Root, and many other things in *Algebra* are by it Demonstrated.

Illustration in Numbers.

Suppose a Number, as 17, to be divided into two Parts 9 and 8; the whole Number multiplied in it self produceth 289. The Square Numbers of 9 and 8 are 81 and 64; the Numbers produced by the parts 9 and 8, multiplied in themselves twice, are 72 and 72, which two Numbers added to the two Square Numbers of 9 and 8. *viz.* 81 and 64 doe make the Number 289, equal to the Number made by the Multiplication of the whole Number 17, into its self.

Proposition IV. *If a Number given be divided into two Numbers, the Square Number of the Whole is equal to the Square Number of the Parts, and to the Superficial Number which is produced of the Multiplication of the Parts, the one into the other twice.*



Suppose that the Number given be A B, which let be divided into two Numbers, A C and C B; then I say that the Square Numbers of the whole Number A B, is equal to the Squares of the Parts, that is, to the Squares of the Numbers A C and C B, and to the Superficial Number produced of the Multiplication of the Numbers A C and C B, the

the one into the other twice. Let the Square Number produced of the Multiplication of the whole Number A B into it ſelf be D, and let A C multiplied into itſelf produce E F, and C B multiplied into itſelf produce G H: And, finally, of the Multiplication of the Numbers A C and C B, the one into the other twice, let there be produced either of theſe Superficial Numbers F G and H K: Now foraſmuch as the Number A C, multiplying itſelf, produced the Number E F; therefore the Number A C meaſureth the Number E F by the Unites that are in itſelf. And foraſmuch as the Number C B, multiplied the Number C A, and produced the Number F G; therefore the Number A C, meaſureth the Number F G by the Unites which are in the Number C B; but it before alſo meaſured the Number E F by the Unites which are in itſelf: Wherefore the Number A B, multiplying the Number A C, produceth the Number E G; and therefore the Number E G is the Superficial Number produced of the Multiplication of the Number A B, into the Number A C. And by the ſame Reaſon may we prove, that the Number G K is the Superficial Number produced of the Multiplication of the Number A B into the Number B C. Farther, the Number D is the Square of the Number A B; but if a Number be divided into two Numbers, the Square of the whole Number is equal to the two Superficial Numbers, which are produced of the Multiplication of the whole into either of the Parts, (by the ſecond *Theorem*;) wherefore the Square Number D is equal to the Superficial Number E K. But the Number E K is compoſed of the Squares of the Numbers A C and B C, and of the Superficial Number which is produced of the Multiplication of the Number A C and C B, the one into the other twice; and the Number D is the Square of the whole Number A B: Wherefore the Square Number produced of the Multiplication of the Number A B into it ſelf, is equal to the Square Numbers of the Parts, that is, to the Square Numbers of the Numbers A C and B C, and to the Superficial Number produced of the Multiplication of the Numbers A C and B C, the one into the other twice. *If therefore &c.* which was to be proved.

V. *If a right Line* A B, *be divided into two equal Parts in* C, *and into two unequal Parts in* D; *the rectangle figure* A H, *comprehended under the unequal Parts of the Whole* A D *and* D B, *together with the Squares of* C D, *which is between the Section, is equal to the Square of the half Line* A C *or* C B.

Figure V. *Conſtruction.* UPon C B deſcribe a Square, as C E F B, and draw a Line from B to E; and by the Point D, draw a Line Parallel to each of the Lines C E or B F, cutting the Diametre B E in H, as the Line D G: And again by the Point H draw a Line Parallel to the Lines A B and E F, as K O, equal to A B: And again by the Point A draw a Line Parallel to C L and B O, as A K.

Demonſtration. Foraſmuch as the Supplement C H is equal to the Supplement H F, put the figure D O common to them both: Wherefore the whole figure C O is equal to the whole figure D F: But the figure C O is equal to the figure A L; for the Line A C is equal to the Line C B; wherefore the figure A L alſo is equal to the figure D F.—Put the figure C H common to them both; wherefore the whole figure A H is equal to the figures D L and D F; but A H is equal to that which is contained under the Lines A D and D B, for D H is equal to D B, and the figures D F and D L are the *Gnomon* M N X: Wherefore the *Gnomon* M N X, is equal to that which is contained under A D and D B.—Put the figure L G common to them both, which is equal to the Square which is made of C D; wherefore the *Gnomon* M N X, and the figure L G, are equal to the rectangle figure comprehended under A D and D B, and unto the Square which is made of C D.—But the *Gnomon* M N X, and the figure L G, are the whole Square C E F B, which is made of B C: Wherefore the rectangle figure comprehended under A D and D B, together with the Square which is made of C D, is equal to the Square which is made of C B. *If therefore a right Line*, &c. Which was to be Demonſtrated.

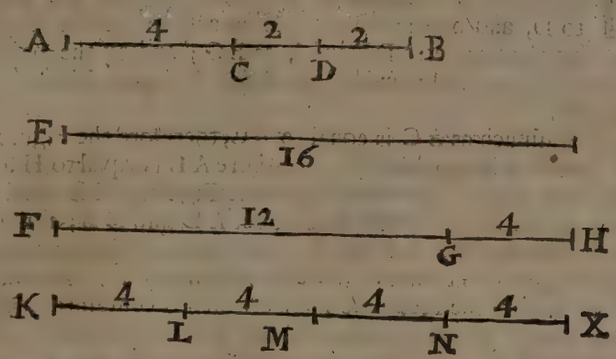
This *Theorem* is of great uſe in *Algebra*, for by it is Demonſtrated that Equation wherein the greateſt and leaſt Characters (or Numbers) are equal to the middle.

Illuſtration in Numbers.

Take any Number, as 20, and divide it into two equal Parts 10 and 10, and alſo into two unequal Parts 13 and 7, and take the difference of the half, and one of the unequal Parts, which is 3: Then multiply the unequal Parts 13 and 7 into each other, and they produce 91; take alſo the Square of 3, which is 9, and add it to 91, which makes

makes 100. Then multiply 10, half of the whole Number, into itſelf, and it makes 100, which is equal to the Number before produced by the Multiplication of the unequal Parts, the one into the other, and of the difference into itſelf, which is alſo 100.

Propoſition. V. *If an even Number be divided into two equal Parts, and again alſo into two unequal Parts, the Superficial Number which is produced of the Multiplication of the unequal Parts one into the other, together with the Square of the Number ſet between the Parts, is equal to the Square of half the Number.*



Suppoſe that A B be an even Number, which let be divided into two equal Numbers A C and C B; and into two unequal Numbers, A D and D B: Then I ſay that the Square Number which is produced of the Multiplication of the half Number C B into itſelf, is equal to the Superficial Number produced of the Multiplication of the unequal Numbers, A D and D B, the one into the other, and to the Square Number produced of the Number G D, which is ſet between the ſaid unequal Parts. Let the Square Number produced of the Multiplication of the half Number C B, into itſelf be E, and let the Superficial Number produced of the Multiplication of the unequal Numbers A D and D B, the one into the other, be F G; and let the Square of the Number D C, which is ſet between the Parts, be G H. Now foraſmuch as the Number B C, is divided into the Numbers B D and D C; therefore the Square of the Number B C, that is the Number E, is equal to the Squares of the Number B D and D C, and to the Superficial Number which is compoſed of the Multiplication of the Numbers B D and D C, the one into the other twice, (by the fourth Propoſition of this Book.) Let the Square of the Number B D be the Number K L, and let N X be the Square of the Number D C. And finally of the Multiplication of the Numbers B D and D C, the one into the other twice, let be produced either of theſe Numbers, L M, or M N. Wherefore the whole Number K X, is equal to the Number E. And foraſmuch as the Number B D, multiplying itſelf, produceth the Number K L; therefore it meaſureth it by the Unites which are in it ſelf. Moreover, foraſmuch as the Number C D multiplying the Number B D, produceth the Number L M; therefore alſo B D meaſureth L M, by the Unites which are in the Number C D, but it before meaſured the Number K L, by the Unites which are in itſelf; wherefore the Number B D, meaſureth the whole Number K M, by the Unites which are in C B; but the Number C B, is equal to the Number C A; wherefore the Number D B, meaſureth the Number K M, by the Unites which are in C A. Again, foraſmuch as the Number C D, multiplying the Number D B, produceth the Number M N; therefore the Number B D, meaſureth the Number M N, by the Unites which are in the Number A C; wherefore the Number B D meaſureth the whole Number K N, by the Unites which are in the Number A D; wherefore the Number F G, is equal to the Number K N: For Numbers which are Equimultiplices to one and the ſelf ſame Number, are equal the one to the other; but the Number G H is equal to the Number N X, for either of them is ſuppoſed to be the Square of the Number C D; wherefore the whole Number K X is equal to the whole Number F H; but the Number K X is equal to the Number E; wherefore alſo the Number F H is equal to the Number E, and the Number F H is the Superficial Number produced of the Multiplication of the Numbers A D and D B, the one into the other, together with the Square of the Number D C; and the Number E is the Square of the Number C B; wherefore the Superficial Number produced of the Multiplication of the unequal Parts A D and

D B, the one into the other, together with the Square of the Number D C, which is set between those unequal Parts, is equal to the Square of the Number C B, which is the half of the whole Number A B. *Therefore*, &c. which was to be proved.

VI. *If a right Line* A B *be divided into two equal Parts in* C, *and if unto it be added another right Line* D B, *directly, the rectangle Figure contained under the whole Line, with that which is added,* A D, *together with the Square which is made of the half* B C, *is equal to the Square which is made of the half Line* A C, *and of that which is added,* B D, *as of one Line.*

Figure VI. *Conſtruction.* UPon the Line C D deſcribe a Square C E F D, and draw a Line from E to D, and by the Point B draw a Line Parallel to either of the Lines E C or D F, cutting the Diametre D E in H, as B G; and by the point H draw a Line Parallel to A D and E F, as K M; and by the Point A draw a Line Parallel to the Lines C L, or D M, as A K.

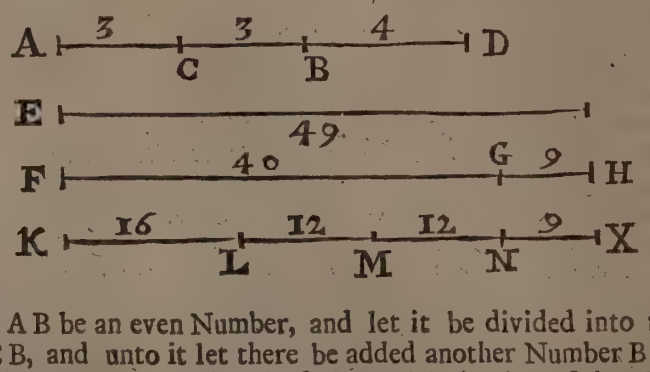
Demonſtration. Foraſmuch as A C is equal to B C, therefore the Figure A L, is equal to the Figure C H; but C H is equal to H F, wherefore A L is equal to H F: Put the Figure G M common to them both; wherefore the whole Figure A M is equal to the *Gnomon* N X O; but A M is that which is contained under A D and D B; for D M is equal unto D B, wherefore the *Gnomon* N X O is equal to the rectangle Figure contained under A D and D B. Put the Figure L G common to them both, which is equal to the Square which is made of C B; wherefore the rectangle Figure which is contained under A D and A B, together with the Square which is made of C B, is equal to the *Gnomon* N X O, and unto L G. But the *Gnomon* N X O and L G, are the whole Square C E F D, which is made of C D; wherefore the rectangle Figure contained under A D and D B, together with the Square which is made of B C, is equal to the Square which is made of C D. *If therefore a right Line be divided*, &c. Which was to be Demonſtrated.

By this *Theorem* in *Algebra* is demonſtrated that Equation wherein the two leſſer Numbers be equal to the Number of the greateſt Denomination.

Illuſtration in Numbers.

Take any even Number as 18, and add unto it any other Number, as 3, which make 21; multiply 21 into the Number added, namely 3, and they make 63. Take alſo half of the even Number, namely 9, and multiply it into itſelf, it produceth 81, which added to 63, (the Number produced of the whole even Number, and the Number added into the Number added,) and they make 144. Then add 9, the half of the whole even Number, unto 3, the Number added, and they make 12; which multiplied in itſelf, produceth 144, equal to the Number compoſed of the Multiplication of the whole Number, and the Number added into the Number added, and of the Square of the Number added, which is alſo 144.

Propoſition VI. *If an even Number be divided into two equal Numbers, and unto it be added ſome other Number; the Superficial Number which is made of the Multiplication of the Number compoſed of the whole Number, and the Number added into the Number added, together with the Square of the half Number, is equal to the Square of the Number compoſed of the half, and the Number added.*



Suppoſe that A B be an even Number, and let it be divided into two equal Numbers A C and C B, and unto it let there be added another Number B D: Then I ſay that the Superficial Number produced of the Multiplication of the Number A D into the Number B D, is equal to the Square of the Number C D; for let the Square of the Number C D, be the Number E: And let the Superficial Number produced of the Multiplication

tiplication of the Number A D into the Number D B be the Number F G: And, finally, let the Square Number of C B be the Number G H; and forasmuch as the Square of the Number C D is (by the fourth Propoſition) equal to the Squares of the Number D B and B C, together with the Superficial Number which is produced of the Multiplication of the Number D B and B C, the one into the other twice. Let the Square of the Number B D be the Number K L, and let the Superficial Numbers produced of the Multiplication of the Numbers B D and B C, the one into the other twice, be either of theſe Numbers L M or M N; and, finally, let the Square of the Number B C be the Number N X: Wherefore the whole Number K X, ſhall be equal to the Square of the Number C D, but the Square of the Number C D is the Number E; wherefore the Number K X, is equal to the Number E. And forasmuch as the Number B D, multiplying it ſelf, produced the Number K L; therefore the Number B D meaſureth the Number K L by the Unites which are in itſelf: But it alſo meaſureth the Number L M by the Unites which are in the Number C B; wherefore the Number D B meaſureth the whole Number K M by the Unites which are in the Number C D. The Number D B alſo meaſureth the Number M N, by the Unites which are in the Number C B, and the Number C B is equal to the Number C A by Suppoſition. Wherefore the Number B D meaſureth the whole Number K N by the Unites which are in the Number A D; but the Number D B doth alſo meaſure the Number F G by the Unites which are in the Number A D: For by Suppoſition, the Number F G is the Superficial Number produced of the Multiplication of the Numbers A D and B D, the one into the other: Wherefore the Number F G is equal to the Number K N; but the Number H G is equal to the Number N X, for either of them is the Square Number of the Number C B; wherefore the whole Number F H, is equal to the Number K X, and the Number K X is proved to be equal to the Number E; Wherefore the Number F H ſhall alſo be equal to the Number E; and the Number F H is the Superficial Number, produced of the Multiplication of the Numbers A D and B D, the one into the other, together with the Square of the Number C B, and the Number E is the Square of the Number C D. Wherefore the Superficial Number produced of the Multiplication of the Number A D and D B, the one into the other, together with the Square of the Number C B, is equal to the Square of the Number C D. *If therefore &c.* which was to be proved.

VII. *If a right Line* A B *be divided by chance in* C, *the Square which is made of the Whole* A B, *together with the Square which is made of one of the Parts* B C, *is equal to the rectangle Figure* A F, *which is contained under the Whole* A B, *&c. the ſaid Part* B C *twice, and to the Square which is made of the other Part* A C.

Conſtruction. UPon the Line A B deſcribe a Square A D E B, and draw the Diametre D B; then through the point C draw the Line C N Parallel to A D and B E, cutting the Diametre in G: Alſo through G, draw the Line H F Parallel to A B. Figure VII.

Demonſtration. Forasmuch as the Figure A G is equal to the Figure G E, put the Figure C F common to them both; wherefore the whole Figure A F, is equal to the whole Figure C E: Wherefore the Figures A F & C E are double to the Figure A F; but the Figures A F and C E are the *Gnomon* K L M, and the Square C F; wherefore the *Gnomon* K L M, and the Square C F is double to the Figure A F; but the double of A F is that which is contained under A B and B C twice, for B F is equal to B C: Wherefore the *Gnomon* K L M, and the Square C F are equal unto the rectangle Figure contained under A B and B C twice.—Put the Figures D G common to them both, which is the Square made of A C; wherefore the *Gnomon* K L M, and the Squares B G and D G, are equal unto the rectangle Figure, which is contained under A B and B C twice, and unto the Square which is made of A C: But the *Knomon* K L M, and the Squares B G and D G, are the whole Square B A D E, and the part or Square C F, which Squares are made of the Lines A B and B C; therefore the Squares which are made of A B and B C, are equal unto the rectangle Figure which is contained under A B and B C twice, and alſo unto the Square of A C. *If therefore a right Line*, &c. Which was to be Demonſtrated.

Corollary.

The Squares of two unequal Lines do exceed the rectangle Figure contained under the ſaid two Lines by the Square of the Exceſs, by which the greater Line exceedeth the leſs.

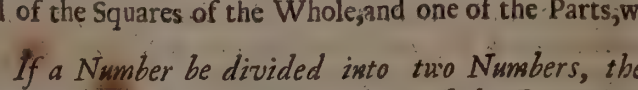
For if the Line A B be the greater, and the Line B C the leſs, it is manifeſt that the Squares of A B and B C, are equal to the rectangle Figure contained under the Lines A B and B C twice; and alſo the Square of the Line A C, whereby the Line A B exceedeth the Line B C.

By this *Theorem* was found out the Extraction of Square Roots in irrational Numbers, and divers other uſefull things.

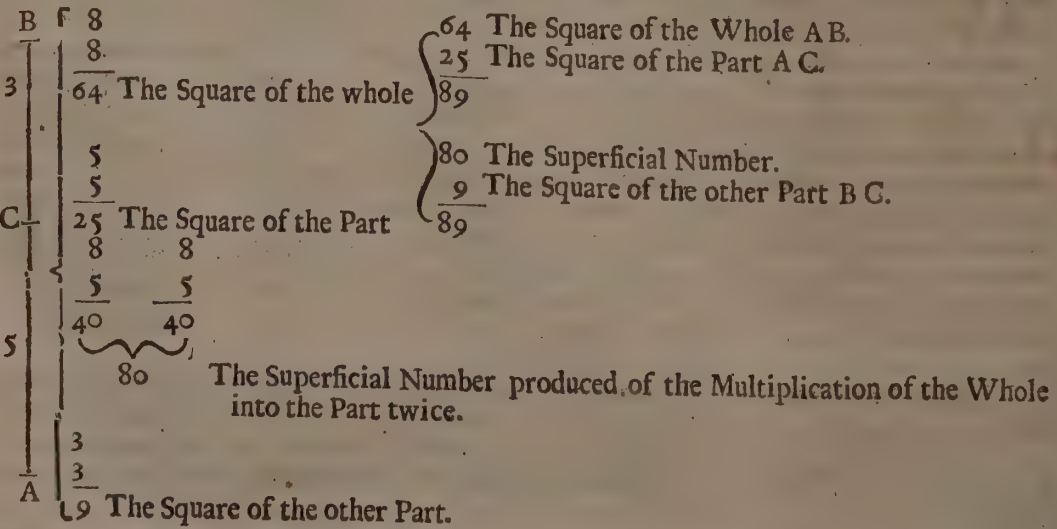
Illuſtration in Numbers.

Take any Number, as 13, and divide it into two parts, as 4 and 9; take the Square of 13 which is 169; take alſo the Square of 4 which is 16; add theſe two Squares together, and they make 185. Then multiply the whole Number 13 into 4 (the aforeſaid part) twice, and you ſhall produce 52 and 52; take alſo the Square of the other part, (that is of 9,) which is 81, and add it to the Products of 13 into 4 twice, that is, into 52 twice, and thoſe three Numbers added together ſhall make 185; which is equal to the Number compoſed of the Squares of the Whole, and one of the Parts, which is alſo 185.

Propoſition VII. *If a Number be divided into two Numbers, the Square of the Whole Number, together with the Square of one of the Parts, is equal to the Superficial Number produced of the Multiplication of the Whole Number into the foreſaid Part twice, together with the Square of the other Part.*



Suppoſe that the Number A B be divided into the Numbers A C and C B, then I ſay that the Square Numbers of the Numbers B A and C A, are equal to the Superficial Number produced of the Multiplication of the Number B A, into the Number A C twice, together with the Square of the Number B C: For, foraſmuch as (by the fourth of this Book) the Square of the Number A B, is equal to the Squares of the Numbers A C and B C, and to the Superficial Number produced of the Multiplication of the Numbers B C and C A, the one into the other twice; add the Square of the Number A C common to them both. Wherefore the Square of the Number A B, together with the Square of the Number A C, is equal to two Squares of the Number A C, and to one Square of the Number C B, and alſo to the Superficial Number produced of the Multiplication of the Number B C and A C, the one into the other twice: And foraſmuch as the Superficial Number produced of the Multiplication of the Numbers B A and C A, the one into the other once, is equal to the Superficial Number produced of the Multiplication of B C into A C once, and to the Square of the Number C A, (by the third of this Book;) therefore the Number produced of the Multiplication of B A into A C twice, is equal to the Number produced of the Multiplication of B C into A C twice; and alſo to the two Squares of the Number C A, add the Square Number of B C common to them both. Wherefore two Squares of the Number A C, and one Square of the Number B C, together with the Superficial Number produced of the Multiplication of B C unto A C twice, are equal to the Superficial Number produced of the Multiplication of the Number B A unto the Number A C twice, together with the Square of the Number C B: Wherefore the Square of the Number A B, together with the Square of the Number A C, is equal to the Superficial Number produced of the Multiplication of the Number B A unto A C twice, together wih the Square of the Number C B. *If therefore* &c. Which was required to be Demonſtrated.



VIII. If

VIII. *If a right Line* A B *be divided by chance in* C, *the rectangled Figure comprehended under the Whole* A B, *and one of the Parts* B C, *four times together, with the Square which is made of the other Part* A C, *is equal to the Square which is made of the Whole* A B, *and of the aforesaid Part* B C, *as of one Line.*

Construction. Extend the Line A B unto D, making B D equal to B C, and upon A D describe a Square A E F D; then draw the Diametre E D, and from the point of Division C, and the Point B, draw the Lines C H and B L, parallel to A E and D F, cutting the Diametre in the Points K and P, through which points K and P draw the right Lines M N and X O, parallel to A D and E F; and constitute the *Gnomon* S T U. Figure VIII.

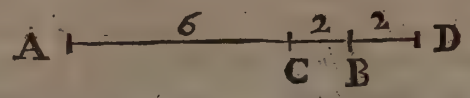
Demonstration. Forasmuch as C B is equal to B D, but C B is equal to G K, and likewise B D is equal to K N; wherefore G K also is equal to K N, and (by the same reason) P R is equal to R O. And forasmuch as B C is equal to B D, and G K to K N, therefore the Figure C K is equal to the Figure D K, and the Figure G R to the Figure R N; but the Figure C K is equal to the figure R N, for they are the Supplements of the Parallelogram C O; wherefore the Figure D K also is equal to the Figure N R; wherefore these Figures D K, C K, G R, and R N, are equal the one to the other: Wherefore those Figures are Quadruple to the Figure C K. Again, forasmuch as C B is equal to B D; but B D is equal to B K, that is, unto C G; and C B is equal unto G K, that is unto G P; therefore C G is equal to G P. And forasmuch as C G is equal to G P, and P R unto R O; therefore the Figure A G is equal to the Figure M P, and the figure P L is equal to the Figure R F; but the Figure M P is equal to the Figure P L, for they are the Supplements of the Parallelogram M L: Wherefore the Figure also A G is equal to the Figure R F; wherefore these four Figures A G, M P, P L, and R F, are quadruple to the Figure R F: And it is proved, that these four Figures C K, K D, G R, and R N, are quadruple to the Figure C K; wherefore, the eight Figures which contain the *Gnomon* S T U, are quadruple to the Figure A R: And forasmuch as the Figure A K is that which is contained under the Lines A B and B D, for the Line B K is equal to the Line B D; therefore that which is contained under the Lines A B and B D four times, is quadruple unto the Figure A K. And it is proved that the *Gnomon* S T U, is quadruple to A K; wherefore that which is contained under the Lines A B and B D four times, is equal unto the *Gnomon* S T U. Put the Figure X H, which is equal to the Square made of A C, common unto them both: Wherefore the Rectangle Figure comprehended under the Lines A B and B D four times, together with the Square which is made of the Line A C, is equal to the *Gnomon* S T U, and unto the Figure X H. But the *Gnomon* S T U, and the Figure X H, are the whole Square A E F D, which is made of A D; wherefore that which is contained under the Lines A B and B D four times, together with the Square which is made of the Line A C, is equal to the Square which is made of A D: But B D is equal to B C; wherefore the Rectangle Figure contained four times under A B and B C, together with the Square which is made of A C, is equal to the Square which is made of A D; that is, unto that which is made of A B and B C as one Line. *If therefore a right Line*, &c. Which was to be Demonstrated.

Illustration in Numbers.

Take any Number as 17, and divide it into two parts 6 and 11, and multiply 17 into 6 (one of the parts) four times, and you shall produce 102, 102, 102, 102: Take also the Square of 11, (the other part,) which is 121, and add it unto the four Numbers produced of the Whole 17, multiplied into the Part 6 four times, and you shall have 529. Then add the whole Number 17 unto the part 6, and it makes 23, which multiplied in itself produceth 529, equal to the Number composed of the Whole 17, into one of the Parts 6 four times, and of the Square of the other Part, which makes 529 also.

Proposition VIII. *If a Number be divided into two Numbers, the Superficial Number produced of the Multiplication of the Whole into one of the Parts four times, together with the Square of the other Part, is equal to the Square of the Number composed of the Whole Number, and the aforesaid Part, as one Number.*

Suppose



Suppose that the Number A B be divided into two Numbers A C and C B, then I say that the Superficial Number produced of the Multiplication of the Number A B into the Number C B four times, together with the Square of the Number A C, is equal to the Square of the Number composed of the Numbers A B and C B; for unto the Number B C, let the Number B D be equal. Now forasmuch as the Square of the Number A D is equal to the Squares of the Numbers A B and B D, and to the Superficial Number produced of the Multiplication of the Numbers A B and D B, the one into the other twice, (by the fourth part of this Book;) and the Number B D is equal to B C. therefore the Square of the Number A D is equal to the Squares of the Numbers A B and B C, and to the Superficial Number produced of the Multiplication of the Numbers A B and B C, the one into the other twice. But the Squares of the Numbers A B and B C, are equal to the Superficial Number produced of the Multiplication of the Numbers A B and B C, the one into the other twice, and to the Square of A C, (by the former Proposition;) wherefore the Square of the Number A D, is equal to the Superficial Number produced of the Multiplication of the Numbers A B and B C, the one into the other four times, and to the Square of the Number A C. But the Square of the Number A D, is the Square of the Number composed of the Numbers A B and B C; for the Number B D is equal to the Number B C. Wherefore the Square of the Number composed of the Numbers A B and B C, is equal to the Superficial Number produced of the Multiplication of the Number A B and B C, the one into the other four times, and to the Square of the Number A C. *Therefore*, &c.

8 8 8 8
7 2 2 2
16 16 16 16

64 The Superficial Number produced of the Multiplication of the Number A B and B C, the one into the other four times.

6
6
36 The Square of A C.

10
10
100 The Squares of the Number composed of A B and B C.

64
36
100 The Superficial Number produced of the Multiplication made four times, and the Square Number of A C.

IX. *If a Right Line* A B *be divided into Two equal Parts in* C, *and into Two unequal Parts in* D; *the Squares which are made of the unequal Parts of the Whole* A D *and* D B, *are double to the Squares which are made of the half Line* A C, *and of that Line* C D, *that is between the Sections.*

Figure IX. *Construction.* FRom the Point C erect a Perpendicular to the Line A B, as C E, making C E equal to A C, or C B; and draw the right Lines A E and B E; and by the Point D draw a Line parallel to C E, as D F; and by the Point F draw F G; parallel to A B; and lastly, draw a right Line from A to F.

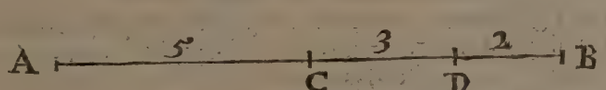
Demonstration. Forasmuch as A C is equal unto C E, therefore the Angle E A C is equal to the Angle C E A; and forasmuch as the Angle at C is a right Angle, therefore the Angles remaining, E A C and C E A together, are equal to one right Angle; wherefore each of these Angles C E A, and C A E, are the half of a right Angle; and by the same reason also, each of the Angles C E B and E B C are the halves of a right Angle; where-

wherefore the whole Angle AEB is a right Angle. And forasmuch as the Angle GEF is the half of a right Angle; but EGF is a right Angle, for it is equal to the inward and oppoſite Angle, that is, it is equal unto ECB; wherefore the remaining Angle EFG is the half of a right Angle; wherefore the Angle GEF is equal to the Angle GFE; wherefore alſo the Side EG is equal to the Side FG. Again, forasmuch as the Angle at the Point B is the half of a right Angle, but the Angle FDB is a right Angle, for it is equal to the inward and oppoſite Angle ECB; wherefore the Angle remaining, BFD, is half of a right Angle; wherefore the Angle at the Point B is equal to the Angle DFB; wherefore the Side DF is equal to the Side DB: And forasmuch as AC is equal unto CE, therefore the Square which is made of AC is equal to the Square which is made of CE; wherefore the Squares which are made of CA and CE are double to the Square which is made of AC: But the Square which is made of AE is equal to the Squares made of CA and CE; for the Angle ACE is a right Angle; wherefore the Square of AE is double to the Square of AC. Again, forasmuch as EG is equal unto GF, the Square therefore which is made of EG is equal to the Square which is made of GF; wherefore the Squares which are made of EG and GF, are double to the Square which is made of GF; but the Square which is made of EF is equal to the Squares which are made of GE and GF; wherefore the Square which is made of EF, is double to the Square which is made of GF: But GF is equal unto CD; wherefore the Square which is made of EF, is double to the Square which is made of CD; and the Square which is made of AE, is double to the Square which is made of AC; wherefore the Squares which are made of AE and EF, are double to the Squares which are made of AC and CB; but the Square which is made of AF is equal to the Squares which are made of AE and EF, the Angle AEF being a right Angle: Wherefore the Square which is made of AF is double to the Squares which are made of AC and CD; but the Squares which are made of AD and DF are equal to the Square which is made of AF; for the Angle at the Point D is a right Angle; wherefore the Squares which are made of AD and DF are double to the Squares which are made of AC and CD: But DF is equal to BD; wherefore the Squares which are made of AD and DB are double to the Squares which are made of AC and CD. *If therefore a Right Line*, &c. Which was to be demonſtrated.

Illuſtration in Numbers.

Take any Even Number, as 12, and divide it firſt Equally into 6 and 6, and then unequally into 8 and 4, and take the difference of the half to one of the unequal parts, which is 2: Then take the Square Numbers of the unequal Parts 8 and 4, which are 64 and 16, which added together make 80: Then take the Squares of the half 6, and the difference 2, and they are 36 and 4, which added together make 40; unto which Number the Number compoſed of the Squares of the unequal parts (which is 80) is double.

Propoſition IX. *If a Number be divided into two Equal Numbers, and again into two Unequal Numbers; the Square Numbers of the Unequal Numbers are double to the Square which is made of the Multiplication of the half Number into it ſelf, together with the Square which is made of the Number ſet between them.*



For let the Number AB, being an even Number, be divided into two equal Numbers AC and CB, and into two unequal Numbers AD and DB; then I ſay that the Square Numbers of AD and DB, are double to the Squares which are made of the Multiplication of the Numbers of AC and CD: For, forasmuch as the Number AB is an even Number, and is divided alſo into two equal Numbers, AC and CB, and afterward into two unequal Numbers AD and BD; therefore the Superficial Number produced of the Multiplication of the Numbers AD and BD the one into the other, together with the Square of the Number DC, is equal to the Square of the Number AC, (by the fifth Propoſition;) wherefore the Superficial Number produced of the Multiplication of the Numbers AD and BD the one into the other twice, together with the two Squares of the Number CD, is double to the Square of the Number AC. Forasmuch as alſo the Number AB is divided into two equal Numbers AC and CB; therefore the Square Number of AB is quadruple to the Square Number

Number produced of the Multiplication of the Number A C into itſelf, (by the fourth Propoſition.) Moreover, foraſmuch as the Superficial Number produced of the Multiplication of the Numbers A D and B D, the one into the other twice, together with the two Squares of the Number D C, is double to the Square Number of C A: And foraſmuch as there are two Numbers, of which the one is quadruple to one and the ſelf ſame Number, and the other is double to the ſame Number: Therefore that Number which is quadruple, ſhall be double to that Number which is double; wherefore the Square of the Number A B is double to the Number produced of the Multiplication of the Numbers A D and D B, the one into the other twice, together with the two Squares of the Number D C; wherefore the Number which is produced of the Multiplication of the Numbers A D and D B, the one into the other twice, is leſs than half of the Square Number of A B by the two Squares of the Number D C. And foraſmuch as the Number produced of the Multiplication of the Numbers A D and D B the one into the other twice, together with the Number compoſed of the Squares of the Numbers A D and D B, is (by the fourth Propoſition) equal to the Square of the Number A B; therefore the Number compoſed of the Squares of the Numbers A D and D B, is greater than the half of the Square Number of A B, by the two Squares of the Number D C. And the Square of the Number A B is quadruple to the Square of the Number A C; wherefore the Number compoſed of the Squares of the Number A D and D B, is greater than the double of the Square of the Number A C, by the two Squares of the Number D C; wherefore the ſaid Number is double to the Squares of the Number A C and C D. *Therefore* &c. Which was to be Demonſtrated.

X. *If a right Line* A B *be divided into two equal Parts in* C, *and unto it be added another right Line directly,* B D; *the Square which is made of the Whole, and that which is added as one Line* A D, *together with the Square of the Line which is added to* B D: *Theſe two Squares ſhall be double to the Square which is made of the half Line* A C, *and to the Square which is made of the other half, with the Line added* C D.

Figure V. *Conſtruction.* FRom the Point C, in the middle of the Line A B, erect the Perpendicular C E, and make C E equal to A C or B C; then draw the Lines A E and E B, and through the point E draw a Line Parallel; to C D, as E F and by the Point D draw a Line Parallel to C E, as D F, and produce the Lines E B and F D, till they concurr in the point G; and then draw the Line A G.

Demonſtration. Forſmuch as the Line A C is equal to the Line E C, the Angle alſo A E C is equal to the Angle E A C, and the Angle at C is a right Angle; wherefore each of the Angles E A C and C E A is the half of a right Angle: And by the ſame reaſon, each of theſe Angles C E B and E B C is the half of a right Angle, and the Whole Angle A E B is a right Angle. And foraſmuch as the Angle E B C is the half of a right Angle; therefore the Angle D B G is the half of a right Angle. But the Angle B D G is a right Angle, for it is equal to the Angle D C E, they being alternate Angles; wherefore the Angle remaing D G B is the half of a right Angle; wherefore the Angle D G B is equal to the Angle D B G; wherefore the ſide B D is equal to the ſide D G: Again, foraſmuch as the Angle E G F is the half of a right Angle, and the Angle at F is a right Angle, (for it is equal to the oppoſite Angle E C D;) wherefore the Angle remaining F E G is half of a right Angle; wherefore the Angle E G F is equal to the Angle F E G, and F E is equal to F G. And foraſmuch as E C is equal to C A, the Square alſo which is made of E C is equal to the Square made of C A; wherefore the Squares which are made of C E and C A, are double to the Square which is made of C A: But the Square which is made of E A is equal unto the Squares made of A C and C E; and the Square which is made of A E, is double to the Square made of A C. Again, foraſmuch as G F is equal to E F, the Square alſo which is made of F G, ſhall be equal to the Square made of E F; wherefore the Squares which are made of G F and E F are double to the Square which is made of E F: But the Square which is made of E G is equal to the Squares made of E F and G F; therefore the Square which is made of E G, is double to the Square which is made of E F. But E F is equal to C D; wherefore the Square which is made of E G is double to the Square which is made of C D. And it is proved, that the Square which is made of A E is double to the Square which is made of A C; wherefore the Squares which are made of A E and E G are double to the Squares which are made of A C and C D. But the Square which is made of A G is equal to the Squares made of A E and E G; wherefore the Square

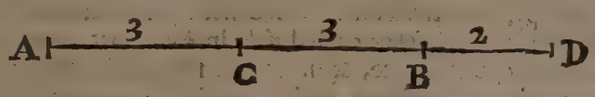
which

which is made of A G is double to the Squares made of A C and C D. But unto the Square which is made of A G, the Squares that are made of A D and D G are equal; wherefore the Squares which are made of A D and D G, are double to the Squares made of A C and D G. But D G is equal to D B; wherefore the Squares which are made of A D and D B, are double to the Squares which are made of A C and D C. *If therefore a right Line*, &c. Which was to be Demonstrated.

Illustration in Numbers.

Take any even Number, as 18, and take the half of it which is 9, and unto 18 the Whole add any Number, as 3, which makes 21. Take the Square Number of 21, (the whole Number 18, and the 3 added,) which is 441: Take also the Square of 3 (the Number added) which is 9, these two Squares added together make 450: Then add the half Number 9 to the Number added 3, and they make 12: Also take the Square of 9, (the half Number,) and the Square of 12, (the half Number and the Number added,) which Squares are 81 and 144; which two Squares added together make 225, unto which Number the aforesaid Number 450 is double.

Proposition X. *If an even Number be divided into two equal Numbers, and unto it be added any other Number; the Square Number of the Whole Number, composed of the Whole Number and of that which is added, and the Square Number of the Number added: These two Square Numbers added together, are double to the Square Numbers of the half Number, and of the Square of the half Number and the Number added, as one Number.*



Suppose that the Number AB, being an even Number, be divided into two equal Numbers A C and C B; and unto it let be added another Number, B D: Then I say, that the Square Numbers of the Numbers A D and B D, are double to the Square Numbers of A C and C D: For forasmuch as the Number A D, is divided into the Numbers A B and B D; therefore the Square Numbers of the Numbers A D and D B, are equal to the Superficial Number produced of the Multiplication of the Numbers A D and D B, the one into the other twice, together with the Square of the Number A B, (by the 7th Proposition.) But the Square of the Number A B, is equal to four Squares of either of the Numbers A C or C B, (for A C is equal to C B;) wherefore also the Squares of the Numbers A D and D B, are equal to the Superficial Number produced of the Multiplication of the Numbers A D and D B, the one into the other twice, and to four Squares of the Number A C or B C: And forasmuch as the Superficial Number produced of the Multiplication of the Numbers A D and D B, the one into the other, together with the Square of the Number C B, is equal to the Square of the Number C D, (by the 6th Proposition;) therefore the Number produced of the Multiplication of the Numbers A D and D B, the one into the other twice, together with the two Squares of the Number C B, is equal to two Squares of the Number C D. Wherefore the Squares of A D and D B are equal to the two Squares of the Number C D, and to two Squares of the Number A C: Wherefore they are double to the Squares of the Numbers A C and C D; and the Square of the Number A D, is the Square of the Whole, and of the Number added; and the Square of D B is the Square of the Number added; the Square also of the Number C D, is the Square of the Number composed of the half, and of the Number added. *Therefore*, &c. Which was required to be proved.

8 — 2
8 — 2
64 The Square of A D — 4 The Square of D B.

5
5
25 The Square of C D, namely of the Number composed of the half, and of the Number added.

3 — 25 — 64 — 68 — 2
3 — 9 — 4 — 34
9 The Square of the half A C — 34 — 68

SECT. III.
Geometrical PROBLEMS.

I. *How to divide a Right Line into Two equal Parts, and at Right Angles.*

Figure I. LET the right Line given be AB. Open the Compasses to any convenient distance greater than half the length of the given Line AB; then setting one foot of the Compasses in the Point A, with the other describe the occult (or obscure) Arch *aa*; then (the Compasses being still open to the same distance) set one foot in the Point B, and with the other describe the occult Arch *bb*, crossing (or cutting) the former Arch *aa* in the Points C and D: A right Line being drawn through the Points C and D, shall divide the given Line AB into two equal Parts in the Point E, and also at right Angles. *Which was required to be done.*

II. *How to divide a Right Line into two equal Parts, but not at Right Angles.*

Figure II. Let the right Line given to be divided be FG. It may prove inconvenient in some cases to open the Compasses to a greater distance than half the length of the given Line, as in the foregoing Problem is required; to prevent which inconvenience, open the Compasses to any small (but competent) distance, with which distance set one foot of the Compasses in the point F, and with the other describe the occult Arch *cc*; also set one foot of the Compasses in the Point G, and with the other describe the Arch *dd*; then apply a Ruler to the two Arches *cc* and *dd*, in such sort that it may only touch the said Arches, and not cut the same, then by the side of the Ruler draw the right Line HK, which will divide the given right Line FG into two equal parts in the Point I. *Which was*, &c.

This way is not so artificial, nor (indeed) so Geometrical as the former.

III. *Upon a Right Line, and from a Point therein given, to erect a Perpendicular.*

Figure III. Let the Line given be LM, and let the Point from whence the Perpendicular is to be erected be the Point Z. Open the Compasses to any small distance, and setting one foot in the given point Z, with the other make the two marks N and O, on either side of the given Point Z, and in the given Line LM; then open the Compasses to any other convenient distance (greater than the former,) and setting one foot in the Point N, with the other describe the occult Arch *cc*; and (the Compasses resting at the same distance) set one foot in the Point O, and with the other describe the occult Arch *ff*, crossing the former Arch *cc* in P: A right Line drawn from the given Point Z through the Point P, as PZ, shall be perpendicular to the given Line LM. *Which was*, &c.

IV. *How to erect a Perpendicular from the End of a right Line given, with once opening of the Compasses.*

Figure IV. Let the given Line be QR, and let R be the Point from which the Perpendicular is to be erected. Open the Compasses to any convenient small distance, as RS; then setting one foot of the Compasses in R, with the other foot describe the occult Arch S*hg*; this done (and the Compasses remaining still at the same distance,) set one foot in S, and with the other make the marks *h* and *g*, in the obscure Arch S*hg*. Again, Set one foot of the Compasses in *h*, and with the other describe the occult Arch *lg*, and setting one foot in *g*, describe the occult Arch *hk*, crossing the former Arch *lg* in T; a right Line drawn from the given Point R, throught T, shall be a Perpendicular to the given Line QR. *Which was*, &c.

V. *How (at once opening of the Compasses) to erect a Perpendicular upon the end (or any other Point) of a Right Line given.*

Figure V. Let VW be the Line given, and V the Point from whence the Perpendicular is to be erected. Open the Compasses to any convenient distance, and setting one foot in the given Point V, pitch the other down at all adventures in the Point *m*; and keeping one foot in *m*, turn the other about till it cross the given Line VW in *n*; and also one foot still resting in *m*, with the other describe the occult Arch *oo*; lay a Ruler from

from *m* to *n*, and it will cut the Arch *o o* in X: A right Line drawn from the given Point V, through X, shall be perpendicular to the given Line V W. *Which was*, &c.

VI. *How from a Point above to let fall a Line which shall be perpendicular to a Line given.*

Let the Point given be C, from which it is required to draw a right Line, which shall be perpendicular to the given Line A B. Open the Compasses to any distance greater than C E, and setting one foot in the given point C, with the other make the marks *p* and *q* in the given Line A B; this done set one foot of the Compasses (they being opened to any convenient distance) in *q*, and with the other describe the occult Arch *r r*; and one foot being placed in *p*, with the other describe the obscure Arch *s s*, crossing the former Arch in D: A Ruler laid from D to the given Point C, shall cut the given Line A B in E; and a Line drawn from C to E shall be perpendicular to the given Line A B. *Which was*, &c. Figure VI.

VII. *How to perform the same another way, when the Point above falls near the end of the given Line, or when there is not competent room below the given Line to draw the former Arches* r r *and* s s.

Let H be the Point given, from which Point let it be required to draw a right Line, which shall be perpendicular to the given right Line T G. Draw an obscure right Line from H, the Point given, to G, the end of the Line given, as the obscure Line H G, which divide into two equal parts in *u*; upon *u*, as a centre, with the distance *u* H, or *u* G, describe an obscure Semicircle G K H, cutting the given right Line in K: a right Line drawn from the given Point H to the Point K, shall be perpendicular to the given Line T G. *Which was*, &c. Figure VII.

VIII. *Between two Points given to interpose two other Points, which shall be in the same right Line with the two given Points.*

Let the two given Points be L and M. Open the Compasses to any distance greater than half the distance between the two given Points, and one foot being placed in L, with the other describe the occult Arch *a a*, also, (the Compasses unaltered,) set one foot in the Point M, and with the other describe the occult Arch *b b*, cutting the former Arch in the Points *c* and *d*: Again, open the Compasses to any distance greater than half *c d*, and setting one foot in *d*, with the other describe the occult Arch *e e*; and placing one foot of the same distance in *c*, with the other describe the occult Arch *f f*, cutting the former Arch in the Points N and O, so shall the two Points N and O be interposed between the two given Points L and M, and in the same right Line with them. *Which was*, &c. Figure VIII.

This Problem may be of use, when two Points are proposed at a distance longer than the length of your Ruler.

IX. *A right Line being given, to draw another right Line which shall be parallel thereunto, and at any distance assigned.*

Let the given Line be P Q, and it is required to draw another right Line parallel thereunto, at the distance of the length of the given Line R. Take in your Compasses the length of the given Line R, and setting one foot in the given Line P Q towards the ends thereof, with the other describe the two occult Arches *g g* and *h h*: A Ruler so laid that it may only touch the tops of the said two Arches, and not cut them, and a right Line drawn by the side thereof, as S T, shall be parallel to the given Line P Q, and at the distance of the given Line R. *Which was*, &c. Figure IX.

X. *How to draw a right Line which shall be parallel to a right Line given, and shall also pass through a Point assigned.*

Let the Line given be V W, and let X be the Point given, through which a right Line is to be drawn parallel to the given Line V W. From the given Point X draw an obscure Line to any part of the given Line V W, as the occult Line X *l*; then setting one foot of the Compasses in *l*, extend the other to X, and draw the occult Arch X *k*; and (the Compasses unaltered) set one foot in X, and with the other describe the occult Arch *l m*; then take in your Compasses the distance between the given Point X Figure X.

and *k*, and set that distance upon the Arch *l m*, from *l* to Y: A right Line drawn from the given Point X through Y, shall be parallel to the given Line V W, and shall also pass through the assigned Point X. *Which was*, &c.

XI. *How to make an Angle equal to an Angle given.*

Figure XI. Let A B C be the Angle given, unto which it is required to make another Angle equal. First draw a right Line D E, of any length; then opening your Compasses to any convenient distance, set one foot in D, and with the other describe the occult Arch *p q*; this done, (and the Compasses unaltered,) set one foot in the given Angular Point B, and with the other describe the occult Arch *n o*; then take the distance between *n* and *o*, and set it upon the obscure Arch *p q*, from *p* to F, and draw the right Line D F: So shall the Angle F D E be equal to the given Angle A B C. *Which was*, &c.

Note, *That an Angle is always signed by Three Letters, the middlemost whereof expresseth the Angular Point, as here* B *and* D.

XII. *How to divide an Angle into Two equal Parts.*

Figure XII. Let the Angle given be G R H. The Compasses being opened to any convenient distance, set one foot in the Angular Point R, and with the other describe the obscure Arch *r s*, cutting the two sides containing the Angle in *r* and *s*; then the Compasses opened to the same (or any other convenient) distance, set one foot in *r*, and with the other describe the occult Arch *t t*; also set one foot in *s*, and with the other describe the occult Arch *v v*, crossing the former obscure Arch *t t* in L: A Line drawn from the given Angular Point R through L, shall divide the given Angle G R H into two equal Parts. *Which was*, &c.

XIII. *A Right Line being given, how to divide the same into any number of equal Parts.*

Figure XIII. Let M N be a right Line given, and let it be required to divide the same into four equal Parts. From the end N, draw a right Line N O, making any Angle; also, through the Point M draw the right Line M P, parallel to O N, so shall the two Angles O N M and N M P be equal; this done, open your Compasses to any small distance, and setting one foot in N, run that distance three times (which is one less than the Line is to be divided into parts,) upon the Line N O, at 1, 2, 3; also run the same three times upon the Line M P, at 1, 2, 3; lastly, if you draw the occult Lines 1 3, 2 2, and 3 1, each to its opposite, they shall divide the given right Line M N into four equal parts in the Points *x y z*. *Which was*, &c.

XIV. *How to make a Triangle, whose three Sides shall each of them be equal to a right Line given.*

Figure XIV. Let the given Line be the Line Q, and let it be required to make a Triangle, each of whose three Sides shall be of equal length thereto. Take in your Compasses the length of the Line Q, unto which make the Line R S equal; then (the Compasses being still open to the length of the Line Q) set one foot in the Point R, and with the other describe the occult Arch *a a*; also, set one foot in the Point S, and with the other describe the occult Arch *b b*, crossing the former Arch in the Point T; lastly, if you draw the right Lines T R and T S, you shall have constituted the Triangle R S T, each of whose Sides are equal to the given Line Q. *Which was*, &c.

Such a Triangle as this is called Equilateral, for that the Three Sides are all equal.

XV. *How to make a Triangle whose three Sides shall be equal to three given right Lines.*

Figure XV. Let the three given right Lines be V, W, and X, of which a Triangle is to be made whose Sides shall be equal to those three right Lines. First, Take the longest Line X in your Compasses, and make the Line Y Z equal thereunto; then take the Line W in your Compasses, and setting one foot in Z, with the other describe the occult Arch *c c*; also, take in your Compasses the given Line V, and setting one foot in Y, with the other describe the Arch *d d*, cutting the former Arch in A: Now if you draw the right Lines A Y and A Z, you shall have constituted the Triangle A Y Z, whose three Sides are equal to the three given Lines V, W, X. *Which was*, &c.

Note,

Note, *That the performance of this Problem will be defective, if the two shorter of the three given Lines being added together, be not longer then the longest given Line.*

XVI. *How to make a Geometrical Square, whose sides shall be equal to a right Line given.*

Let the given Line be B, to which the sides of the Geometrical Square are to be equal. Take the Line B in your Compasses, equal unto which make the Line C D: Then (by the 4th or 5th Problem) upon the point D erect the Perpendicular D E, making D E equal to C D: Then taking the given Line B in your Compasses, set one foot in the Point E, and with the other describe the occult Arch *f f*, and removing the Compasses to C, set one foot in C, and with the other describe the Arch *e e*, cutting the former Arch in F. Lastly, draw the right Lines F E and F C, so shall you have constituted the Geometrical Square C D E F, each of whose sides are equal to the given right Line B. *Which was*, &c. Figure XVI.

XVII. *How to make a Parallelogram or long Square, whose Length and Breadth shall be equal to two given right Lines.*

Let the two given Lines be G and H. Take the longest Line H in your Compasses, unto which make K L equal: Then (by the 4th or 5th Problem) upon the Point L, erect the Perpendicular L M, making the Line L M equal to the given Line G. Then take the Line H in your Compasses, and setting one foot in M, with the other describe the obscure Arch *g g*: Also take the Line G in your Compasses, and setting one foot in K, with the other describe the Arch *h h*, crossing the former Arch in the Point N. Lastly, draw the right Lines N M and N K, and so you shall have constituted the Parallelogram or long Square K L M N, whose length K L is equal to the given Line H, and breadth L M equal to the given Line G. *Which was*, &c. Figure XVII.

XVIII. *How to make a* Rhombus *whose side shall be equal to a given right Line.*

Let the given Line be O, unto which the side of the *Rhombus* is to be equal. Take the Line O in your Compasses, unto which make P Q equal: Also the length of the Line O being taken in the Compasses, set one foot in Q, and with the other describe the occult Arch *m m*: Then (the Compasses being still open at the distance of the given Line O) set one foot in the point P, and with the other cross the occult Arch *m m* in R, and setting one foot of the Compasses in R, cross the former obscure Arch *m m*, again in the Point S. Lastly draw the right Lines P R, R S, and S Q, and they will constitute the *Rhombus* P Q R S, whose sides are all equal to the given Line O. *Which was*, &c. Figure XVIII.

XIX. *How to make a* Rhomboyades, *whose longer and shorter sides shall be equal to two given right Lines.*

Let the two Lines given, to which the sides of the *Rhomboyades* are to be equal, be T and V: Take the Line V in your Compasses, unto which make W X equal: Also take the Line T in your Compasses and set one foot in W, and prick the other down at all adventure as at Y, (if the Angle of the *Rhomboyades* be not limited;) then with the length of the Line V, set one foot of the Compasses in Y, and with the other describe the occult Arch *o o*: Also with the length of the Line T, set one foot in X, and with the other describe the Arch *n n*, crossing the former Arch in Z. Lastly, draw the right Lines Y Z and Z X, and you shall have constituted the *Rhomboyades* W X Y Z, whose longer sides, Y Z and W X, shall be equal to the given Line V, and whose shorter sides, Y W and Z X, equal to the given Line T. Figure XIX.

XX. *How to make a* Rhomboyades *whose two sides shall be equal to two right Lines given, and whose acute Angle shall be also equal to an Angle given.*

Let the two given Lines to which the two sides of the *Rhomboyades* are to be equal, be A and B, and the given Angle let be the Angle C: Take the Line A in your Compasses and make D E equal thereto; then upon the point D, (by the 11th *Problem*,) make an Angle equal to the given Angle C, and let the containing side thereof D F, be made equal to the given Line B: Then with the length of the given Line A, set one foot of the Figure XX.

the Compasses in F, and with the other describe the occult Arch *p p*: Also with the length of the given Line B set one foot in the point E, and with the other describe the obscure Arch *q q*, crossing the former Arch in G. Lastly, draw the right Lines F G and G E, and they shall enclose the *Rhomboyades* D E F G, whose two shorter sides, D F and E D, shall be equal to the given B; and the two longer sides D E and F G are equal to the given Line A; and the acute Angles, F D E and F G E, either of them equal to the given Angle C. *Which was*, &c.

XXI. *How to make a* Trapezia *whose four sides shall be equal to any (possible) four right Lines given, and one of its Angles equal to an Angle given.*

Figure XXI. Let the four given Lines to which the four sides of the *Trapezia* are to be equal be the four right Lines H, I, K, and L; and let the given Angle (to which one of the Angles of the *Trapezia* is to be equal) be the Angle M. Take the Line L in your Compasses, to which make N O equal: Then upon the point O (by the 11th Proposition) make an Angle equal to the given Angle M, and make the other containing side of that Angle, O P, equal to any of the other given Lines, (as here it is made equal to the given Line I:) Then take in your Compasses the given Line H, and setting one foot in P, with the other describe the occult Arch *r r*, and taking in your Compasses the Line K set one foot in N, and with the other describe the Arch *s s*, crossing the former in the Point Q. Lastly, draw the right Lines N Q and Q P, and you shall have constituted a *Trapezia*, whose four sides shall be equal to the four given Lines H, I, K, and L, and one of whose Angles (namely, the Angle P, O, N,) equal to the given Angle M. *Which was*, &c.

XXII. *A Circle being given to find the Centre.*

Figure XXII. Let the Circle given be T S V and the Centre thereof is required. Draw a Line within the Circle in any part thereof, so that it may touch the Circumference thereof in two Points, as the Line T V: Then open the Compasses to any convenient distance, greater than half T V, and set one foot in T, and with the other describe the obscure Arch *v v*. Also with the same distance of the Compasses, set one foot in V, and with the other describe the occult Arch *t t*, crossing the former Arch in the points S and R, by which Points draw the right Line R S: Again, set one foot of the Compasses in R, and extend the other to Z, and upon R, describe the obscure Arch *x x*, and with the same distance upon Z, describe the occult Arch *y y*, crossing the former obscure Arch in the Points W and X, through which Points draw the right Lines W X, crossing the right Line R S in A; so shall A be the Centre of the Circle. *Which was*, &c.

XXIII. *How to find the Centre of a Circle, whose Circumference shall pass through any points given, which three Points do not lie in a right Line.*

Figure XXIII. Let the three points through which the Circumference of a Circle is to pass be the points B, C, and D. Place one foot of the Compasses in one of the given Points B, and extend the other to another of the given Points, as to C, and upon B as a Centre, describe the obscure Arch *a a*. Then (the Compasses remaining still at the same distance,) set one foot in C, and with the other cross the former obscure Arch in the Points *b* and *c*, and through those points draw the right Line *b c* at length: Again, (the Compasses unalter'd,) set one foot in the third given point D, and with the other cross the former obscure Arch *a a*, in the points *e* and *f*, and draw the right Line *e f*, crossing the other right Line *b c*, in the point E; so shall E be the Centre, upon which if a Circle be described, at the distance of any of the given points, it shall pass through all three of them. *Which was*, &c.

XXIV. *A Circle being given to find the length of a Line, which shall divide the Circumference of that Circle into any number of equal Parts, not exceeding Ten.*

Figure XXIV. Let the Circle given be A F C G.

First, draw the Diametre A C, and (by the 1st Problem) divide the same into two equal Parts at right Angles in E, and draw the right Line F G, which shall divide the Circumference into two equal Parts.

Secondly,

Secondly, Take in your Compasses the length of the Line E F or E G, and set it from A to B, and from A to D, and draw the right Line B D, which Line shall divide the Circumference into three equal Parts.

Thirdly, A right Line drawn between any two next intersections of the *Semidiametres*, with the Circumference, as the Lines A F or G C, shall divide the Circumference into four equal Parts.

Fourthly, Set one foot of the Compasses in A, and extend the other to F, and describe the obscure Arch F I; a right Line drawn from F to I, shall divide the Circumference into five equal Parts.

Fifthly, Any of the *Semidiametres*, as E A, E G, E F, *&c.* will divide the Circumference into six equal Parts.

Sixthly, Half the Line B D, as H B, or H D, shall divide the Circumference into seven equal Parts.

Seventhly, Divide the Line A F (by the 1st Problem,) into two equal Parts at right Angles in M, and draw the Line E M, continuing it till it cut the Circumference in the point K, and draw the Line A K, which shall divide the Circumference into eight equal Parts.

Eightly, Divide the Arch D A B into three equal parts, one of them will be the Line D L, which will divide the Circumference into nine equal parts.

Ninthly, The Line E I will divide the Circumference into ten equal Parts. *Which was*, &c.

XXV. *Within a Triangle to inscribe a Circle.*

Let the Triangle given be A B C, in which a Circle is to be described, which shall touch all the three sides of the Triangle. Divide (by the 12th Problem) any two Angles of the Triangle into two equal Parts, as here I have divided the Angle A C B, by the obscure Line C *a*, and the Angle A B C, by the occult Line B *b*, which two Lines cross each other in the point D, which is the Centre, upon which you may describe a Circle which will touch all the sides of the Triangle, and will be the greatest Circle that can be inscribed within the same Triangle. *Which was*, &c. Figure XXV.

XXVI. *About a Triangle to describe a Circle.*

Let the given Triangle be E F G, about which it is required to describe a Circle which shall touch all the three Angles of the Triangle. Divide (by the 1st Problem) any two sides of the given Triangle, into two equal Parts at right Angles, (as here I have done the two sides E F and E G,) by the two occult Lines *c e* and *d f*, crossing each other in the point H: Upon which point, H as a Centre, if you open the Compasses to the distance of any of the Angles and describe a Circle, the Circumference of that Circle shall touch all the three Angles of the given Triangle E F G. *Which was*, &c. Figure XXVI.

XXVII. *Within a given Triangle to describe the greatest Geometrical Square that may be.*

Let the Triangle given be K L M, within which it is required to describe the largest Geometrical Square that may be. At the end of the Base of the Triangle, M, (by the 4th or 5th Problem,) erect the Perpendicular M *h*, making M *h* equal to the Base of the Triangle L M: Also (by the 6th Problem,) let fall the Perpendicular from the vertical Angle of the Triangle K, to cut the Base in *g*: Or, (by the 10th Problem,) through the vertical Angle, K, draw a Line Parallel to M *h*, as K *g*, cutting the Base in the former Point *g*: Then if you draw the obscure Line *g h*, it will cut the side of the Triangle K M in the Point N, through which Point N (by the 10th Problem) draw the Line N O, Parallel to the Base L M. This done, from the two Points O and N, draw the two Lines O P and N Q, Perpendicular to the Base L M, or Parallel to the Perpendicular K *g*, and you shall constitute the Geometrical Square N O P Q, which is the greatest that can be inscribed within the given Triangle K L M. *Which was*, &c. Figure XXVII.

XXVIII. *Within a Geometrical Square to inscribe an equilateral Triangle.*

Let the Geometrical Square given be R S T V, within which is to be inscribed the greatest equilateral Triangle that may be: Draw the two obscure Diagonal Lines R V and S T, crossing each other in the Centre X; then set one foot of your Compasses Figure XXVIII.

passes in V, and open the other to X, and with that distance describe the occult Arch *k k*, cutting the sides S V and T V of the Geometrical Square in the Points Z and Y. Lastly, draw the right Lines Z Y, R Z, and R Y; so shall you have constituted the equilateral Triangle R Z Y, which is the greatest equilateral Triangle that can be inscribed in the given Geometrical Square R S V T. *Which was*, &c.

XXIX. *Within a Pentagon to inscribe an equilateral Triangle.*

Figure XXIX. Let the Pentagon given, in which the greatest equilateral Triangle that may be is to be inscribed, be A B C D E. First divide any two sides of the Pentagon (by the 1st Problem) into two equal parts at right Angles, as is here done the two sides C D and A E; by the two obscure Lines A *o* and C *n*, crossing each other in H, the Centre of the Pentagon. Set one foot of the Compasses in H, and with the distance H A describe the occult Circle, which will touch all the Angles of the Pentagon: Also (with the distance A H) one foot being placed in A, with the other describe the occult Arch H *m*, which divide into two equal parts in the Point K; from A (through the Point K) draw the right Line A F, and with the distance A F upon the Point A, describe the occult Arch F G, and draw the Lines F G and A G; so shall you have constituted an equilateral Triangle A F G, which is the greatest that can be inscribed within the given Pentagon A B C D E. *Which was*, &c.

XXX. *About a Geometrical Square to Circumscribe a Triangle, whose Angles shall be equal to the Angles of a Triangle given.*

Figure XXX. Let L M N O be a Geometrical Square given, about which is to be Circumscribed a Triangle whose Angles are to be equal to the Angles of the given Triangle S T V. First, let one of the sides of the Square given (as N O) be continued on either side at liberty towards P and Q. Then, upon the opposite side of the Square L M, upon the angular Point M (by the same 11th Problem) make an Angle equal to the Angle S V T of the given Triangle. Also upon the Angular Point L (by the same 11th Problem) make an Angle equal to the Angle S T V, of the given Triangle, the containing sides of which two Angles, R M L and R L M being drawn, will meet in the Point R, and then will the two Triangles S T V and R L M be Equiangled, (by the Construction.) Lastly, extend the sides R M and R L of the Triangle R L M, till they cross the side of the Square N O, formerly extended in the Points P and Q; so shall the Triangle R P Q be circumscribed about the Geometrical Square L M N O, and the Angles of it shall be equal to the Angles of the given Triangle S T V. *Which was*, &c.

XXXI. *About an Equilateral Triangle to describe a Geometrical Square.*

Figure XXXI. Let A B C be an Equilateral Triangle given, about which a Geometrical Square is to be Circumscribed. Divide the side B C of the Equilateral Triangle, into two equal parts at right Angles (by the 1st Problem) in the Point H, and draw an obscure Line A H *m*. Then take in your Compasses the distance H C or H B, and set it from H to D. Then (by the 1st Problem) divide the Line A D into two equal parts in the point E, and at right Angles, by the occult Line *n* E *o*. Then take in your Compasses the distance A E, and set it from E to F, and from E to G. Lastly, draw the Lines A F, F D, D G, and G A, which will constitute the Geometrical Square A F G D, Circumscribed about the given Equilateral Triangle A B C. *Which was*, &c.

XXXII. *How to make a Geometrical Square which shall pass through any three Points given, which lie not in a right Line.*

Figure XXXII. Let the three given Points be K L M, by all which a Geometrical Square is to pass. Through two of the given Points (it matters not which) as K and L, draw a right Line and extend it at liberty: Then from the third Point M, (by the 6th Problem) let fall the Perpendicular M N, and extend M N, if need be. This done, take in your Compasses the distance from N to K, and set it upon the Perpendicular Line N M, (extended) from N to: Also, with the same distance N K or N O, set one foot in O, and with the other describe the occult Arch *c c*, and removing the Compasses to K, describe the obscure Arch *s s*, crossing the former in P. Lastly, draw the right Lines O P and K P, and they will constitute the Geometrical Square P O K N, which shall pass by the three given Points K L M. *Which was*, &c.

XXXIII. *A*

XXXIII. *A Triangle being given how to constitute another Triangle equal thereunto having the same Base and Altitude, but the Angle at the Base equal to an Angle given.*

Let Q R S be a Triangle given, to which it is required to make another equal, having the same Base and Altitude, but the Angle at the Base equal to the given Angle T. Through the Vertical Angle Q, of the given Triangle (by the 10th Problem) draw the obscure Line *m m*, Parallel to the Base of the Triangle R S. Then on the Angular point R, (by the 11th Problem) make the Angle V R S, equal to the given Angle T, and draw the containing side R V, till it cut the obscure Line *m m* in V. Lastly, draw the Line V S; so shall you have a new Triangle V R S, whose Base R S, and Altitude V *n*, are equal to those of the given Triangle Q R S, but the Angle at the Base equal to the given Angle T. *Which was*, &c. Figure XXXIII.

XXXIV. *Two Triangles of different kinds and Altitudes being given to reduce them both to one Altitude.*

Let the the two given Triangles be A B C, and C E D, the Altitude of A B C being A *n*, and the Altitude of C E D being E *r*; and it is required to reduce the Triangle C E D, to the Altitude of the Triangle A B C, the Triangle C E D still retaining the same quantity. Through the vertical Angle A of the Triangle A B C, draw the occult Line *o o* Parallel to the Base B C: Then extend the side C E of the Triangle C E D, till it cut the obscure Line *o o* in F, and from F to D, draw the occult Line F D: Then through the Point E, draw the obscure Line E G parallel to F D, (by the 10th Problem.) Lastly, draw the Line F G, and you shall have constituted a new Triangle C F G equal to C E D, and of equal Altitude with the Triangle A B C. *Which was*, &c. Figure XXXIV.

XXXV. *How to make a Triangle which shall be equal to a* Trapezia *given.*

Let H I K L be a *Trapezia* given, equal whereunto it is required to make a Triangle. Extend one side of the *Trapezia* at liberty, as the side K L. Then draw the Diagonal Line K I, and Parallel thereto (by the 10th Problem) through the Point H, draw the obscure Line H M, cutting the former side of the *Trapezia* K L, extended in M. Lastly, draw the Line I M, and you shall have a Triangle I M L, equal to the given *Trapezia* H I K L. *Which was*, &c. Figure XXXV.

XXXVI. *How to make a Triangle, which shall be equal to an irregular Figure of 5, 6, or 7 sides.*

Let N O P Q R, be an irregular Figure of five sides, and it is required to make a Triangle which shall be equal in quantity thereunto. Extend any one side of the irregular Figure given (as here P Q) on both sides at pleasure, to *c* and *d*. Then from the Angle N, (opposite to the extended side P Q) draw the two obscure Lines N P, and N Q: Also (by the 10th Problem) through the Angular Points O and R, draw the two obscure Lines O S and R T, parallel to N P and N Q, cutting the extended side P Q in the Points S and T. Lastly, draw the Lines N S and N T; so shall you have a Triangle N S T, equal in quantity to the irregular five sided Figure N O P Q R. *Which was*, &c. Figure XXXVI.

The same may be done for Figures of six, seven, or eight sides, or more, by often repeating of the same Work.

XXXVII. *Between two right Lines given, to find a mean Proportional Line.*

Let the two given Lines be A and B, between which it is required to find a third Line, which shall be a mean Proportional between the Line A and the Line B. Draw a right Line at Pleasure, as the Line C F. Then take the length of the given Line A in your Compasses, and set it from F to E: Also take the other given Line B in your Compasses, and set it from E to C; so is the given Line C F, equal to both the given Lines A and B, (the Point of joyning of the two Lines being the Point E.) This done, (by the 1st or 2d Problem) divide the Line C F, in two equal parts in D, and upon D, as a Centre, with the distance D C, or D F, describe the obscure Semicircle C G F. Lastly, from the Point of joyning of the two given Lines A and B, namely, from the Point E, (by the 3d Problem) erect the Perpendicular E G, cutting the obscure Semicircle Figure XXXVII.

micircle in G; so shall the Line E G be a mean Proportional between the two given Lines A and B. *Which was*, &c.

A Mean Proportional Line is such a Line whose length being multiplied in it self, the Product thereof shall be equal to the Product of the other two Lines, they being multiplied one by the other. So the given Line A being 4, and the given Line B being 9, the mean Proportional between E G, will be found to be 6. Now the Mean Proportional 6, being multiplied in it self, produceth 36. And the two given Lines, A 4 and B 9, being multiplied in each other, produce 36, also.

XXXVIII. *How to make a Geometrical Square which shall be equal to a Triangle given.*

Figure XXXVIII. Let the Triangle given unto which a Geometrical Square is to be made equal, be the Triangle H I K. Extend the Base of the Triangle I K at liberty towards P: Then from the vertical Angle H, (by the 6th Problem) let fall the Perpendicular H Q, which (by the 1st or 2d Problem) divide into two equal parts in R, between I K the Base, and Q R half the Perpendicular, find a mean Proportional (by the 37th Problem) by making K L equal to Q R, and dividing I L into two equal parts in M, and upon M as a Centre, at the distance M I or M L, describing the obscure Semicircle, I N L: Then from the Point of joyning K, erect a Perpendicular (by the 3d Problem) to I L as K N, cutting the obscure Semicircle I L N in N. So shall K N be the Mean Proportional, and shall be the side of the Geometrical Square sought. Wherefore, if (by the 16th Problem) you make a Geometrical Square of the Line K N, as N O K P, that Square shall be equal in quantity to the given Triangle H I K. *Which was*, &c.

XXXIX. *To make a Geometrical Square equal to a given Parallelogram or long Square.*

Figure XXXIX. Let V W S T be a Parallelogram given, unto which a Geometrical Square is to be made equal. The length of the Parallelogram is S T, take the breadth thereof S V, and set it from T to Z. Then (by the 1st or 2d Problem) divide the Line S Z, in two equal parts in X, and upon X at the distance X S or X Z, describe the obscure Semicircle S A B Z: Then from the Point of joyning of the two sides of the Parallelogram together, T, erect the Perpendicular T B, till it touch the Semicircle in B; so is T B a Mean Proportional Line, between the length and the breadth of the given Parallelogram, and is the side of a Geometrical Square, equal to the Parallelogram; so the Geometrical Square A B Y T, is equal to the given Parallegram. S T V W. *Which was*, &c.

XL. *To make a Geometrical Square, which shall be equal to the* Rhombus *given.*

Figure XL. Let C D E F be a *Rhombus* given, unto which a Geometrical Square is to be made equal. A Mean Proportional Line between the side of a *Rhombus*, and the Altitude or Perpendicular of a *Rhombus*, shall be the side of a Geometrical Square equal to that *Rhombus*. Continue one side of the given *Rhombus*, as E F, at liberty towards L, then take the length of the Perpendicular D H, and set it from F to M. Then divide E M into two equal parts in G, and upon G with the distance G E or G M, describe the Semicircle E K M, and from the point of joyning the side and Perpendicular of the *Rhombus* together, namely from F, erect the Perpendicular F K, to touch the Semicircle E K L in K; so is F K the Mean proportional between the side and perpendicular of the *Rhombus*. The Square made thereof K I L F, is equal to the given *Rhombus* C D E F. *Which was*, &c.

XLI. *To make a Geometrical Square equal to a* Rhomboyades *given.*

Figure XLI. Let the *Rhomboyades* unto which a Geometrical Square is to be made equal, be N O P Q, and a mean Proportional between the length of the *Rhomboyades* and the Perpendicular, shall be equal to the side of the Square, and may be thus found. Continue the side of the *Rhomboyades* P Q at Pleasure, then take the length of the Perpendicular O X, and set it from Q to R. Then divide the Line P R, (which is the length of the side, and the Perpendicular of the *Rhomboyades* added together, by the 1st or 2d Problem) in two equal parts in Z, and upon Z, as a Centre, at the distance Z P, or Z R, describe the obscure Semicircle P T R. Then if from the point of joyning

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ing Q, (by the third Problem) you erect a Perpendicular till it touch the obscure Circle, as Q T, the Line Q T shall be the side of the Geometrical Square S T V Q, which is equal to the given *Rhomboyades* N O P Q. *Which was*, &c.

XLII. *To make a Geometrical Square, equal to a* Trapezia *given.*

Let the *Trapezia* A B C D be given, and let a Geometrical Square be to be made equal thereunto: Now a Mean Proportional between the longest Diagonal of the *Trapezia*, and half of the two Perpendiculars being added together, shall be equal to the side of a Geometrical Square, equal to the *Trapezia*, which may be thus found. Through the *Trapezia* draw the longest Diagonal, as B D, and continue it at Pleasure towards E: Then from the Points A and C (by the 6th or 7th Problem) let fall the two Perpendiculars A F and C K, upon the Diagonal B D. This done, add the two Perpendiculars A F and C K together in one Line, and take the half thereof, which set from D to E; then divide B E into two equal parts in L, and upon L at the distance L B, or L E, describe the obscure Semicircle B H E: Also upon the Point of joyning the Diagonal and half the Perpendiculars together, namely, from the Point D, (by the 3d Problem) erect the Perpendicular, extending it till it cut the obscure Semicircle in H; so shall D H be the side of a Geometrical Square, I H K D, equal to the given *Trapezia*, A B C D. *Which was*, &c. Figure XLII.

XLIII. *To make a Geometrical Square which shall be equal to a regular* Polygon, *of any Number of sides given.*

Let the regular *Polygon* given, be the *Pentagon* M N O P Q, unto which a Geometrical Square is to be made equal. Now a Mean Proportional between half the Perimetre of the *Polygon*, and a Line draw from the Centre of the *Polygon*, Perpendicular to the middle of any of the sides, shall be equal to the side of a Geometrical Square, which shall be equal to the given *Polygon*. From the Centre of the *Polygon* R (by the 6th Problem) let fall the Perpendicular R S, upon the side O P, which side of the *Polygon* O P, continue at pleasure towards W. Now the length of two sides and half a side of this *Polygon* given, (it consisting of five equal sides,) is half the Perimetre; which set from S to V, and take the Perpendicular R S, and set it from V to W, dividing the Line S W into two equal parts in T; upon which Point T, at the distance T S, or T W, describe the obscure Semicircle S Z W; and from the Point of joyning, P, (by the 3d Problem) erect a Perpendicular, extending it from V, till it cut the obscure Semicircle in Z; so shall Z V be the side of the Geometrical Square V X Y Z, which Geometrical Square is equal to the given regular *Polygon*, (being the *Pentagon*,) M N O P Q. *Which was*, &c. Figure XLIII.

XLIV. *To make a Geometrical Square which shall be equal to two other Geometrical Squares given.*

Let the two given Squares be A and B, unto which a third is to be made equal to both the given Squares. Draw a Line at liberty, as D E, and upon the end E (by the 4th or 5th Problem) erect the Perpendicular E C; so shall the Angle C E D, be a right Angle. Then in your Compasses take the length of the side of the given Square A, and set it from E to F; also take the side of the Square B, and set it from E to G, and draw the Line F G; upon which Line describe the Geometrical Square F G H K, which shall be equal to the two given Squares A and B. *Which was*, &c. Figure XLIV.

Note, *That if from the right angular Point* E, *a Perpendicular be let fall upon the Line* F G *to* L, *and extended to* M; *the Line* M L, *shall divide the Geometrical Square* F G H K, *into two Parallelograms or long Squares*, H G M L, *and* M L K F, *of which* the Parallelogram F H M L, shall be equal to the given Square A, and the Parallelogram M L K G equal to the given Square B.

XLV. *To make a Geometrical Square which shall be equal to* 3, 4, *or* 5 *other Geometrical Squares.*

Let N O P Q, be the sides of four Geometrical Squares given, and it is required to make one Geometrical Square which shall be equal to all those four. First draw two Lines at liberty, as R S and S T, making a right Angle at S: Then take in your Compasses the side of the given Square N, and set it from S to V, and the side O, set from S to W; so shall S W be the side of the Square W D E V, equal to the two Figure XLV.

Squares of N and O. Secondly, take the ſide of this new Square V W, and ſet it from S to Y; alſo take the ſide of the Square P, and ſet it from S to X, and draw X Y the ſide of the Square Y F X G, which is equal to the three Squares of N O and P. Thirdly, take the Line X Y, the ſide of the laſt Square, and ſet it from S to A: Alſo take the ſide of the given Square Q, and ſet it from S to Z, and draw the Line Z A; ſo ſhall the Square made of Z A, namely, A B C Z, be equal to all the four Squares made of the four given ſides N O P Q. *Which was*, &c.

Note, *That which is ſaid in this and the laſt Problem concerning Geometrical Squares, the ſame is to be underſtood of Circles; for if the four Lines*, N, O, P, Q, *had been the Diametres of four Circles, and you had dealt with them as you did with them above as Squares, you would then have found the Line* A Z *the ſame, and it would be the Diametre of a Circle, equal to all the four Circles deſcribed on the Diametres* N O P Q.

XLVI. *A Geometrical Square being given, to augment the ſame to a Double, Quadruple, Octuple, &c. Proportion.*

Figure XLVI. Let K L M N be a Geometrical Square to be augmented to a 16th Proportion. Firſt, continue the ſides K L and K M, at pleaſure towards H and I, and alſo draw the obſcure Diagonal K N, extending it at pleaſure towards Z. Secondly, take in your Compaſſes the Diagonal K N, and ſet it from K to O, and from K to Q, and make the Square O P K Q, which will be double to the Square given. Thirdly, take the Diagonal K P, and ſet it from K to R, and from K to T, and make the Square R S T K, which will be four times as big as the given Square. Fourthly, take the Diagonal K S, and ſet it from K to V, and from K to X, and make the Square V W X K, which will be eight times as big as the given Square. Laſtly, take the Diagonal K W, and ſet it from K to Y, and from K to Æ, and deſcribe the Square Y Z Æ K, which will be ſixteen times as big as the given Square K L M N. *Which was*, &c.

Note, *here (as in the former Problem) that what is here ſaid concerning the ſide of a Geometrical Square, the ſame is to be underſtood of the Diametre of a Circle, as by the next Figure doth appear, where a Circle is augmented to ſixteen times its bigneſs.*

XLVII. *Two Geometrical Squares of unequal bigneſſes being given, to take the Leſſer of them out of the Greater, and to give the Remainder in a third Geometrical Square.*

Figure XLVII. The two Squares given are A B C D the greater, and E F G H the leſſer, and is to be taken out of the greater, and the remainder to be left in a third Square. Draw two Lines L K and K I, making a right Angle at K. Then take the ſide of the leſſer Square E F G H, and ſet it from K to N: Alſo take the ſide of the greater Square A B C D, and ſetting one foot in N, with the other make an obſcure mark, croſs the Line L K, as at M, and (if you pleaſe) draw the Line M N, the ſide of the greater given Square. Laſtly, take the diſtance K M, and ſet it from B to P, and from B to Q, and make the Square P B O Q, which ſhall be the third Square remaining. The *Gnomon* A O D, being equal to the leſſer given Square E F G H. *Which was*, &c.

XLVIII. *Several Triangles of ſeveral forms being given, how to make one Triangle which ſhall be equal to them all.*

Figure XLVIII. Let the three Triangles A B C, C D E, and E F G, be three Triangles of different kinds given, unto which one Triangle is to be made, which ſhall be equal to them all. Through the vertical Angle A, of the given Triangle A B C, (it being the Triangle that hath the greateſt Altitude of the three,) draw the obſcure Line in *m n*: Then (by the 34th Problem) reduce the other two Triangles into two Triangles of the ſame Altitude with the Triangle A B C: So ſhall the Triangle C D E be reduced into the Triangle K H E, and the Triangle E F G into the Triangle M L G, all three of the ſame Altitude. Now join the Baſe of all theſe three new Triangles together in one, as is done in the Scheme; ſo ſhall the Triangle A B C, be equal to it ſelf; the Triangle C H E, equal to the Triangle K H E; the Triangle E L G, equal to the Triangle M L G, all of the ſame Altitude, the common Baſe to them all being B G. Now two Lines being drawn from the two Points, B and G, (which are the two extreme ends of all the Baſes of the three new Triangles joined together,) unto any part of the obſcure Line *o p*, as to S, there ſhall be a Triangle made B S G, which ſhall be equal to all the other three, and ſo conſequently to the three Triangles given.

In

In like manner, If a Line had been drawn from A to G, the Triangle BAG would have been equal to all the three given Triangles: Alſo two Lines drawn from L to B and G, it would make a Triangle LBG, which would be alſo equal to the three Triangles given; and ſo infinite others might be made by Lines drawn from B and G, to any Point in the Line *op*. *Which was*, &c.

XLIX. *A Triangle being given, to make another Triangle equal thereunto, whoſe Baſe, or Perpendicular, ſhall be limited.*

Let the given Triangle be ABC, and it is required to make another Triangle equal thereunto, whoſe Perpendicular ſhall be equal to the Line F. Figure XLIX.

Take the Line F in your Compaſſes, and with that diſtance draw the Line EK, parallel to the Baſe AB, cutting the Line AC in E; then draw the Line EB, and extend the Baſe AB towards L: Alſo through C draw a Line parallel to the Line EB, as CD, cutting the Baſe AB, extended, in D: Laſtly, draw the Line ED, which ſhall conſtitute the Triangle AED equal to the Triangle ABC, whoſe Perpendicular EH is equal to the given Line F. *Which was*, &c.

L. *Upon a right Line given, to deſcribe a* Pentagon.

Let AB be the Line given. Firſt, take the Line AB in your Compaſſes, and ſetting one foot in A, with the other deſcribe the Arch BDF, and upon A erect the Perpendicular AC, cutting the Arch in C: Then divide the Arch BC into five equal parts at 1, 2, 3, 4, 5; and at the end of them, *viz.* at D, draw the Line AD: Then divide the Line BA into two equal parts, at right Angles in O, by the Line OE, cutting the Line AD in E: Then upon E, as a Centre, deſcribe the Circle ABFGH, and draw the Lines BH, HG, GF, and FA, conſtituting the Equilateral Equiangular *Pentagon*, upon the given Line AB. *Which was*, &c. Figure L.

LI. *To find out two right Lines, which ſhall have ſuch Proportion one to the other as two Squares have.*

Suppoſe the two Lines A and B to be the Sides of two Squares, and let it be required to find two right Lines, which ſhall have ſuch Proportion one the other as the Square made of the Line A hath to the Square made of the Line B. Figure LI.

Firſt, make the two Lines A and B to be the Baſe and Perpendicular of a right-angled Triangle, by making the Baſe DE equal to the Line A, and the Perpendicular DC equal to B; this done, draw the Line CE, and from the right Angle D let fall a Perpendicular to CE, as DF, dividing the Baſe CE into two parts, FC and FE; which two Lines FC and FE have ſuch Proportion one to another, as the Squares made of the given Lines A and B ſhall have one to another. *Which was*, &c.

LII. *To cut off a part from an* Iſoſceles *Triangle, with a Line drawn parallel to the Baſe thereof; ſo that the Line parallel to the Baſe, and the two Segments of the Legs, cut off by the parallel Line, ſhall be all three equal.*

Let ABC be an *Iſoſceles*, and let it be required to draw a Line DE, parallel to the Baſe BC, which parallel DE ſhall be equal to either of the Segments of the Legs cut off, DB and EC. Figure LII.

By the twelfth Problem divide the Angle ABC into two equal parts, by the Line BF, cutting the Leg (or Side) of the Triangle AC in E; alſo divide the Angle ACB into two equal parts by the Line CG, cutting the Side AB in D; then draw the Line DE which ſhall be parallel to BC, and alſo equal to DB and EC. *Which was*, &c.

LIII. *Between Two given Right Lines to find a Third in Proportion to them.*

Let A and B be two Lines given, and let a third Line be required, which ſhall be in Proportion to them. Figure LIII.

Draw two right Lines, as CD and DH, making any Angle, as the Angle CDH; then take the Line A, and ſet it from D to E; alſo, take the Line B, and ſet it from D to F, and from D to G, and draw the Line EF; then through the Point G draw a Line parallel to EF, as GK; ſo ſhall DK be the third proportional Line to the Lines A and B. *Which was*, &c. For,

As

As DE, equal to A 4,
Is to DF, equal to B 6,
So is DG, equal to B 6,
To DK, equal to 9.

LIV. *Between Three given right Lines, to find a Fourth in Proportion to them.*

Figure LIV. Let A, B, C, be three Lines given, and let a fourth (as D) be found in Proportion to them.

As in the foregoing Problem, so in this, make any Angle at pleasure, as the Angle EFG; then take in your Compasses the Line A, and set it from F to H; set also the Line B from F to I, and draw the Line HI; then take the Line C, and set it from F to K, (always upon the same Line that the first Line A was set upon;) then through the Point K, draw the Line KL, parallel to HI, cutting FE in L: So shall FL be the fourth Proportional. *Which was*, &c. For,

As FH, equal to A 12,
Is to FI, equal to B 14,
So is FK, equal to C 18,
To FL, equal to D 21.

These two last Problems doe the Work of the Golden Rule of Proportion in Lines.

LV. *To divide a Line given into two Parts, which shall be in Proportion one to the other, as two given Lines.*

Figure LV. Let AB be a Line given to be divided into two parts, which shall have Proportion one to the other as the Line C hath to the Line D.

From the end of the given Line A draw a Line AH at pleasure, making the Angle HAB; then take in your Compasses the Line C, and set it from A to F; also, take the Line D, and set it from F to E, and draw the Line EB; then through the Point F, draw the Line FG, parallel to EB, cutting the given Line AB in the Point G; so is the Line AB divided in the Point G into two parts, AG and GB, which are in Proportion one to the other as the Line C is to the Line D. *Which was*, &c. For,

As AF, equal to C 20,
Is to FE, equal to D 30,
So is AG 16,
To GB 24.

LVI. *Between two right Lines given, to find two mean proportional Lines.*

Figure LVI. Let the two Lines given be A and B, and it is required to find two mean proportional Lines betwen them.

First, Draw a right Line at pleasure, as CM, and at the end C erect the Perpendicular CL; then take the Line A, and set it from C to D; and the Line B, from C to E, and draw the Line ED, which divide into two equal parts in F; upon which Point (as a Centre) describe the Semicircle EGD; then take the lesser Line B, and setting one foot of the Compasses in D, with the other cross the Semicircle in G; to this Point G apply a Ruler, moving it up and down upon that Point, till the Ruler, being so moved, do cut the two Lines, CL and CM, in the Points H and K, at equal distances from F: So shall EH and DK be the two mean proportional Lines. *Which was*, &c.

LVII. *To divide a Line in Power according to any Proportion given.*

Figure LVII. Let CD be a Line given, to be divided in Power as the Line A is to the Line B.

By the 55th Problem divide the Line CD in E, as the Line A is to the Line B; then will ED (the lesser part) be to A, (the lesser Line,) as CE (the greater part) is to B, (the greater Line;) then upon the given Line CD describe the Semicircle CFD, and from the Point E erect a Perpendicular to cut the Semicircle in F; then draw the Lines CF and FD, which together in Power shall be equal to the Power of the given Line CD, and in Power one to another as the Line A is to the Line B. *Which was*, &c.

LVIII. *To enlarge a Line in Power according to any Proportion assigned.*

Let (in the former Figure) CE be a Line given to be inlarged in Power as the Line B is to the Line G.

By

By the 54th Problem enlarge the Line C E in Proportion as B to G; that is, as B is to G, so is C E to C D: Upon the Line C D describe the Semicircle C F D, and upon E erect a Perpendicular to cut the Semicircle in F; then draw the Line G F, which shall be in Power to C E, as the Line G to the Line B. *Which was*, &c.

LIX. *To diminish or enlarge a Plot, or Figure given, according to any Proportion assigned.*

Let A B C D E be a Plot or Figure given, to be diminished (or made less) in such Proportion as the Line L is to the Line K. Figure LIX.

By the 57th Problem divide one of the Sides of the Figure, (as the Side A B,) in Power as L to K, so that the Power of A F may be to the Power of A B, as L to K.

First, From the Angle A draw Lines to the other Angles C and D; then through the Point F draw a Parallel to B C, to cut A C in G; also through G, draw a Parallel to C D, as G H, cutting C D in H; lastly, through H draw a Parallel to D E, as H I, cutting A E in I: So shall the Plot A F G H I be like the Plot A B C D E, and in Proportion to it as the Line L is to the Line K. *Which was*, &c.

But if the Plot or Figure A F G H I had been given, and it had been required to enlarge the same in Proportion as K to L, then you must increase the Sides A F and A I, and draw the Lines A G and A H, increasing them also: Then (by Problem 58.) you must enlarge in Power the Line A F as K to L, which set from A to B, and through B draw a Line parallel to F G, till it cut the Line A G extended in C: Then through C draw a parallel Line to G H, cutting the Line A H extended in D; also through D draw a Line parallel to H I, cutting the Side A I extended in E: And so shall you have inclosed the Plot or Figure A B C D E like unto the Figure A F G H I, and augmented in Proportion thereunto as the Line K is to the Line L. *Which was*, &c.

LX. *Upon a Line given to describe a Figure like unto a Figure given.*

Let A B C D E be a Figure given, and let F G be a Line given; upon which Line it is required to describe a Figure like unto the Figure A B C D E, having all its Sides and Angles proportional. Figure LX.

Extend the Base Line A E of the Figure, on both Sides, as to H and I; then setting one foot of the Compasses in E, describe the Arch I K L M; also, set one foot in A, and with the other describe the Arch H N O P; which done, lay a Ruler on the Angle A, and every of the Points E, D, C, B; and mark upon the Arch the Points H, N, O P: Also, lay a Ruler to the Angle E, and every of the Points A, B, C, and D, and mark upon the Arch the Points I, K, L, M: Then with the distance E I, or A H, setting one foot of the Compasses in F, describe the Arch Q R S T; also, setting one foot in G, describe the Arch V W X Y: Then laying a Ruler to the Point F, and every of the Points R, S, T, draw the occult Lines F R, F S, F T: Also, lay a Ruler to the Point G, and every of the Points W, X, Y, draw other occult Lines, crossing the former in the Points 1, 2, 3: Lastly, draw the Lines F 1, 1.2, 2.3, 3 G; you shall include the Figure F 1, 1.2, 2.3, 3 G, like and proportional to the Figure A B C D E, as the given Line F G is to the Line A E of the greater Figure. *Which was*, &c.

LXI. *Within a Triangle, to inscribe a Parallelogram, whose Sides shall have such Proportion one to the other as two Lines given.*

Let A B C be a Triangle given, in which a Parallelogram is to be inscribed, whose Sides shall have such Proportion one to the other, as the Line D hath to the Line E. Figure LXI.

At the distance of the Line D draw a Parallel to the Base A C, as F G, cutting the Side A B in F: Then take the Line E, and set it upon the Parallel from F to G; and by G draw a Line parallel to the Side B C, as H I, cutting A B in H: Then (by the 54th Problem) the Proportion will be, As A H is to A F, so is A B to A K; from which Point K let fall a Perpendicular to the Base A C, as K L; also from K draw a Parallel to A C, as K M: Lastly, draw M N parallel to K L: So shall you include the Parallelogram L K M N within the Triangle A B C, whose Sides K L and M K have such Proportion one to the other as the Line D hath to the Line E. *Which was*, &c.

LXII. *To divide a Line by Extreme and Mean Proportion.*

Let A B be a Line given to be so divided. Extend the given Line A B towards C, and upon the Point A erect the Perpendicular A D, equal to A B; then take half A B, Figure LXII.

and

and set it from A towards C, and it will terminate in E; then setting one foot of the Compasses in E, extend the other to D, and describe the Arch D G, cutting the Line A B in G: So shall the Line A B be divided by extreme and mean Proportion in the Point G; A G being the greater, and B G the lesser Segment thereof. *Which was*, &c.

LXIII. *Two Points within a Circle being given, to find the Centre of the Arch of a great Circle which shall pass through those two Points, and also divide the Circumference of the Circle into two equal Parts.*

Figure LXIII. Let G B H D be a Circle given, whose Centre is C, and the two Points through which the Circle is to pass be X and Y.

First, Through one of the Points X, and the Centre C, draw a right Line X C, extending it towards F; upon this Line, from the Centre C, erect the Perpendicular C B, and draw B X, extending it till it cut the Circle in G; through which Point G and the Centre C, draw the Diametre G C H.

Secondly, Through the Points B and H draw the Line B H, extending it till it cut the Line X F in the Point R; so have you three Points R, Y, and X, through which (by the 23d Problem) draw the Arch of a Circle A X Y L, whose Centre will be at K; and this Arch will divide the given Circle in two equal parts in the Points A and L; for a right Line drawn from A to L will pass through the Centre C. *Which was*, &c.

LXIV. *From a Point assigned without a Circle, to draw a Line which shall cut off an Arch of the Circle, whose Chord shall be equal to any Line given, less than the Diametre of the Circle.*

Figure LXIV. Let A be the Circle, and let B be a Point without the same Circle, from which Point it is required to cut off an Arch of a Circle, whose Chord shall be equal to the Line C.

Take in your Compasses the length of the Line C, and set it any where upon the Circumference of the given Circle, as from D to E, and draw the Line D E at length: Then from B draw a Line to the Centre of the Circle, as B G; and with the distance B G, upon the Centre G describe the Arch B F, cutting the Line D E (extended) in F: Then take F D in your Compasses, and setting one foot in B, with the other cross the Circumference of the Circle in H; Lastly, draw the Line B K H, which cutteth off the Arch of the Circle K L H, whose Chord K H is equal to the given Line C. *Which was*, &c.

LXV. *To draw a right Line, which shall touch a Circle in a Point assigned.*

Figure LXV. Let A B C be the Circle given, and let A be that Point in the Circumference by which the touch Line is to be drawn.

From the Centre of the Circle, and through the assigned Point A, draw a Line D A F: Upon A erect a Perpendicular to D F, as A H, which prolong to I; so shall the Line H A I touch the Circle in the given Point A. *Which was*, &c.

LXVI. *Upon a right Line given to describe a Segment of a Circle, which shall be capable to receive an Angle equal to an Angle given.*

Figure LXVI. Let A B be a Line given, upon which to describe a portion of a Circle, capable to receive an Angle equal to the Angle C.

Upon the Point A make the Angle B A D, equal to the given Angle C, and also upon the Point A erect the Perpendicular A E; then divide the Line A B into two equal parts in H, and upon H erect the Perpendicular H F, cutting A E in F: Then upon F (as a Centre) and at the distance F A, or A B, describe the Segment A K S L B: So shall all the Angles that can be drawn upon the Line A B within that Segment be equal to the given Angle C, as the Angles A K B, A S B, A L B, and infinite others. *Which was*, &c.

LXVII. *From a Circle given to cut off a Segment, capable to contain an Angle equal to an Angle given.*

Figure LXVII. Let A C E S T be the Circle given, from which to cut off a Segment capable to receive an Angle equal to the Angle D.

Draw the Semidiametre B A, and to the Point A draw the touch Line A F, perpendicular to B A: Then make the Angle F A C equal to the Angle D, and draw the Line A C, cutting

cutting the Circle in C: Then ſhall all the Angles deſcribed upon the Line AC within the Segment AESTC, as AEC, ASC, ATC, *&c.* be all equal to the given Angle D. *Which was*, &c.

LXVIII. *Within a Circle to inſcribe a Triangle whoſe Angles ſhall be equal to a Triangle given.*

Let ABC be a Circle given, within which a Triangle is to be drawn whoſe Angles ſhall be equal to the Angles of the given Triangle DEF. Figure LXVIII.

Firſt draw a Touch-line GAH, touching the Circle in the Point A; upon which Point A deſcribe a Semicircle GKLH, and make the Angle GAK, equal to the Angle D; alſo make the Angle HAL, equal to the Angle E: Laſtly, draw the Lines AKB and ALC; ſo ſhall the Triangle ABC be like and equiangle to the given Triangle DEF. *Which was*, &c.

LXIX. *To divide a Triangle into two Parts according to any Proportion given, by a Line drawn from any Angle thereof.*

Let the Triangle given be NOP, and let it be required to divide the ſame into two parts by a Line drawn fron the Angle N, which ſhall be in proportion one to the other as the Line A is to the Line B. Figure LXIX.

By the 55th Problem, divide the Baſe of the Triangle OP into two Parts, in proportion one to the other, as the Line A is to the Line B, which will be in the Point R: A Line drawn from N to R ſhall divide the Triangle into two Triangles, NOR and NRP, which ſhall be in proportion one to the other as the Line A is to the Line B. *Which was*, &c.

Otherwiſe,

Let the Triangle given be ſuppoſed to be a piece of Land containing 7 Acres, and it is required to cut off from it 3 Acres, to lie towards the Angle O, and to leave 4 Acres lying next the Angle P, and let the Line, or Hedge, of ſeparation be drawn from the Angle N. From O, one end of the Baſe of the Triangle, draw a Line at liberty, as OQ, making any Angle with the Baſe of the Triangle OP: Then the Compaſſes being opened to any ſmall diſtance, run that diſtance ſeven times along the Line OQ, at the Points 1, 2, 3, *&c.* and from the ſeventh Point draw the obſcure Line 7P: Then (becauſe 3 Acres are to be cut off next to O) count 3 from O, at the Point 3, and through the Point 3 (by the 10th Problem) draw a Line Parallel to P7, as the Line 3R, cutting the Baſe of the Triangle in R: A right Line drawn from N to R ſhall divide the Triangle NOP into two Triangles, of which the Triangle NOR ſhall contain Three Acres, and the Triangle NRP Four Acres. *Which was*, &c.

LXX. *To divide a* Trapezia *into two Parts, according to any Proportion given, by a Line drawn from a Point given in any of the Sides thereof.*

Let ABCD be a *Trapezia* given to be divided into two parts, in Proportion one to the other, as the Line L is to the Line M, by a Line drawn from the Point LE, in the ſide AB. Figure LXX.

Firſt (by the 35th Problem) reduce the *Trapezia* ABCD, from the Point E, into the Triangle EFG. Then (by the 55th Problem) divide the Baſe of the Triangle into two parts in Proportion, as L to M, which Point of Diviſion will be in K; from E draw the Line EK; ſo ſhall the *Trapezia* ABCD, be divided into two *Trapezias*, AECK, and EBKD, which ſhall be in Proportion to one another, as the Line L, is to the Line M. *Which was*, &c.

Otherwiſe,

Suppoſe the *Trapezia* ABCD to be a piece of Land containing 9 Acres, and it is required to cut off 3 of them to lie next to the Hedge AC, and 6 towards the Hedge BD, by a ſeparating Line or Hedge drawn from E, in the ſide AB. Extend the ſide of the *Trapezia* CD, (which is oppoſite to the given Point E,) on both ſides at liberty towards F and G, and from the given Point E, to the two oppoſite Angles of the *Trapezia*, C and D, draw the two obſcure Lines EC and ED, and through the other two Angular parts of the *Trapezia* A and B, (by the 10th Problem) draw two other obſcure Lines parallel to EC and ED, as AF and BG, cutting the ſide of the *Trapezia* extended in the Points F and G; ſo drawing the two Lines EF, and EG, the *Trapezia* ABCD is reduced into the Triangle EFG. Then from F, the end of the Baſe of this new Triangle, draw the Line FH at liberty, making any Angle with the Baſe of the Triangle FG. Upon which Line (with any ſmall diſtance of the Compaſſes) run nine times at the Points 1, 2, 3, 4, *&c.* And from the Point 9, draw the obſcure Line 9G, and

through the Point 3 (becauſe 3 Acres are to be cut off) draw another obſcure Line parallel to G 9, as 3 K, cutting the Baſe of the *Trapezia* in K, a Line drawn from E to K, ſhall be the ſeparating Line, and ſhall divide the given *Trapezia* A B C D, into the the two *Trapezias* A E C K, containing three Acres, and the *Trapezia* E B K D, containing ſix Acres. *Which was*, &c.

LXXI. *To divide a Triangle into two parts, according to any Proportion given, by a Line drawn from a Point given in any of the ſides of the Triangle.*

Figure LXXI. Let A B C be a Triangle given, to be divided into two parts, in Proportion one to the other, as the Line K is to the Line L.

Firſt from the given point E, draw an occult Line to the oppoſite Angle at A, as A E. Then (by the 55th Problem) divide the Baſe of the Triangle B C, in the point D, in Proportion, as K to L; then draw the Line D F parallel to E A. Laſtly, from the point F draw a Line to the point E, which Line F E ſhall divide the given Triangle A B C into two parts, *viz.* into the Triangle F B E, and the *Trapezia* A F E C, in Proportion one to the other, as the Line K, is to the Line L. *Which was*, &c.

Otherwiſe,

If A B C were a piece of Land containing ſeven Acres, and it were required to divide it into two parts by a Line drawn from the point E, ſo that two Acres may lie towards the Angle B, and five Acres towards the ſide A C; the work is already done: for the Triangle F B C will contain two Acres, and the *Trapezia* A F E C, five Acres. *Which was*, &c.

LXXII. *To divide a Triangle into two parts, according to any Proportion by a Line drawn parallel to any of the ſides thereof.*

Figure LXXII. Let the Triangle given be A B C, and let the proportion into which it is to be divided by a Line parallel to the ſide A C, be as the Line K, to the Line L.

Firſt (by the 55th Problem) divide the Baſe of the Triangle B C in the point E, into two parts, in Proportion as K to L. Then (by the 37th Problem) find a mean Proportional Line between B C the whole Baſe, and B E the leſſer Segment of the Baſe, which will be B F; from which point F, draw the right Line F H, parallel to the ſide A C, cutting the ſide A B in H; ſo ſhall the Line F H divide the Triangle into two parts, namely the Triangle H B F, in ſuch proportion to the Line K, as the *Trapezia* A H F C is to the Line L. *Which was*, &c.

LXXIII. *To divide a* Trapezia *into two parts, which ſhall be in any Proportion, by a Line drawn parallel to one of the ſides.*

Figure LXXIII. Let A B C D be a *Trapezia* given to be divided into two parts, by a Line drawn parallel to the ſide B A, in ſuch ſort that the leſſer part ſhall have ſuch proportion to the greater part, as the Line E hath to the Line F, and to lay the leſſer part next to the ſide B A.

Becauſe the Line of ſeparation is to be parallel to the ſide B A, it muſt neceſſarily cut the two ſides B C and A D; wherefore, extend thoſe two ſides, till they concurr in the Point G. This done, (by the 35th Problem) reduce the *Trapezia* A B C D, into the Triangle A B H, which *Trapezia* thus reduced, the Baſe of the Triangle equal thereunto will be the Line A H. Then (by the 55th Problem) divide this Baſe A H, in the Point I, in ſuch proportion as the Line E is to the Line F. This done, (by the 37th Problem) find a mean Proportional Line between G A (the Point of Concurrence of the two ſides B C and A D, extended,) and G I, (the Point of the Proportional Diviſion,) which will be the Point K. Laſtly, through the Point K draw a Line parallel to A B, as K L; ſo ſhall you cut off the leſſer part A B L K, (next the ſide A B,) and the greater part K L C D, (next to the ſide C D,) in proportion one to the other, as the Line E to the Line F. *Which was to be done*, &c.

But if it had been required to divide the *Trapezia* by a Line drawn parallel to the ſide A D, then the Line of ſeparation would have paſſed through the ſides A B and C D, and thoſe two ſides ſhould have been extended till they met; but the reſt of the work would be ſtill the ſame.

LXXIV. *To divide an irregular Plot of many ſides, according to any Proportion aſſigned with a Line drawn from any Angle thereof.*

Let

Let A B C D E F be an irregular Plot, to be divided into two parts, in proportion one to the other as the Line G is to the Line H, with a Line drawn from the Angle C, and to lay the leſſer part towards E. Figure LXXIV.

Foraſmuch as the Line of Diviſion is to be drawn from the Angle C, it will neceſſarily fall upon the ſide A F, which extend on both ſides from A and F; and (by the latter part of the 36th Problem) reduce the irregular Figure A B C D E F, into the Triangle G K I, equal thereunto. Then (by the 55th Problem) divide the Baſe I K, into two parts in proportion, as G to H; ſo that as G is to H, ſo I L to L K; which Point of Diviſion will fall in L, becauſe the leſſar part is to be laid towards A. Laſtly, draw the Line C L, which ſhall divide the Plot into two parts, the leſſer part C D E F L, being in proportion to the leſſer Line G, as the greater part A B C L, is to the greater Line H. *Which was*, &c.

LXXV. *To divide a Plot according to any proportion aſſigned, by a Line drawn from a Point Limited in any of the ſides thereof.*

Let the Plot given be A B C D E F, and let the Point from which the Diviſion is to be made be G, in the ſide B C, and let the Proportion into which it is to be divided be ſuch as is between the Line H and the Line I, and let the leſſer part be laid towards the ſide A B. Figure LXXV.

The Point G, from whence the Diviſion is to be drawn, will neceſſarily fall upon the ſide A F; wherefore extende A F on both ſides, and (by the latter part of the 36th Problem) reduce the whole Plot into the Triangle G K L, (whoſe Perpendicular Altitude is G O.) This done, (by the 55th Problem) divide the Baſe of the Triangle G K L, in proportion as H to I, in the Point M, (the leſſer part being ſet from K.) Laſtly, from the aſſigned Point G draw the Line G M, which ſhall divide the Plot into two parts, in proportion one to the other, as the Line I to the Line H, and the leſſer part ſhall lie towards the ſide A B. *Which was*, &c.

LXXVI. *To divide a Triangle by a Line drawn parallel to a Line without the Triangle, according to any Proportion between two given Lines.*

Let A B C be a Triangle, and let D be a Line without the Triangle: It is required to divide the Triangle into two parts in Proportion one to the other, as the Line E is to the Line F. Figure LXXVI.

Firſt divide the Baſe of the Triangle (by the 55th Problem) in the Point G, as E is to F. Then through the Point B, draw a Line parallel to the Line D, cutting the Baſe of the Triangle A C in H. Then (by the 37th Problem) find a mean Proportional between A G and A H, which will be A I. Then through the Point I, draw a line parallel to B H or D, as the Line I K, which will cut off the Triangle A K I, in Proportion to the other part of the Triangle I K B C, as E is to F. *Which was*, &c.

LXXVII. *To cut off from a Triangle a part equal to a Figure given, by a Line drawn parallel to a Line without the Triangle.*

In the foregoing Figure, let A B C be a Triangle given, and D a Line drawn without the Triangle, and let it be required to draw a Line within the Triangle parallel to D, which ſhall cut off a part thereof equal to the Square L M N O. Figure LXXVII.

From the Angle B draw a Line parallel to D, as B H. Then increaſe the ſide of the Square O N, making N P equal to N O, and draw the Line L P; ſo ſhall the Triangle L O P, be equal to the Square L M N O. Then (by the 34th Problem) reduce the Triangle L O P, into the Triangle O Q R, of equal Altitude with the Triangle A B C. Then take the Baſe of this Triangle R O, and ſet it upon the Baſe of the Triangle A B C, from A to G, which done, (by the 37th Problem) find a mean Proportional between A G and A H, which will be A I. Laſtly, through the Point I draw a Line parallel to D, or B H, (which alſo is parallel to D,) as I K, which ſhall cut off the Triangle A I K, equal to the Square, L M N O. *Which was*, &c.

LXXVIII. *To divide a Triangle into two parts, according to any Proportion by a Line drawn from a Point without the Triangle.*

Let A B C be a Triangle, and D a Point without the Triangle, from which Point it is required to draw a right Line which ſhall divide the Triangle into two parts in Proportion one to the other, as the Line E is to the Line F. Figure LXXVIII.

First, increase the side of the Triangle A B, and through the given Point D, draw a Line parallel to the Base of the Triangle A C, till it cut the Base extended in G. This done, (by the 53d Problem) divide A B in H, as E is to F: Then, as E is to F, so A H to H B. And (by the 54th Problem) as G D is to A C, so A H to A I; which set from A to L. Then (by the 37th Problem) find a mean Proportional between G A and G I, as A K, which set from I to M. Lastly, draw the Line D M, which shall cut off the Triangle A M T, which shall be in proportion to the other part of the Triangle M B C T, as the Line E is to the Line F. *Which was*, &c.

LXXIX. *From a Point without a Triangle to cut off a part of the Triangle equal to a Figure given.*

Figure LXXIX. In the foregoing Figure, let A B C be a Triangle given, and D a Point without the same, from which it is required to draw a Line, to cut off from the Triangle A B C, a part equal to the *Trapezia* N O P Q.

First, increase the side of the Triangle A B, and through D draw a Line Parallel to A C, till it cuts the side A B extended in G. Then (by the 35th Problem) reduce the *Trapezia* N O P Q, into the Triangle N O R, and that again into the Triangle N S Q, whose Altitude is equal to the Altitude of the given Triangle A B C, (from C upon A B being increased.) This done, take the Base Q N, and set it upon the Base A B, from A to H. Then, (by the 54th Problem) as G D is to A C, so is A H to A I; which set from A to I. Then (by the 37th Problem) find a mean Proportional between A G and A I, as A K; which set from L to M. Lastly, a Line drawn from D to M, shall cut off the Triangle A M T, equal to the *Trapezia* N O P Q. *Which was*, &c.

LXXX. *To divide a Square into two equal parts, by a Line drawn from a Point without the Square.*

Figure LXXX. Let A B C D be a Square given, and E a Point without the same, from whence it is required to draw a Line which shall divide the Square into two equal parts.

First draw the Diametre A C, which divide into two equal parts in F. Then from the point E through the Point F, draw the Line E F G, which shall divide the Square into two equal parts, as into A B G H, and G C H D, and from a Point E without the Square. *Which was*, &c.

If the Point had been taken within the Square as at S, or in the side of the Square, as at H, the work had been still the same.

And the like may be done in a Parallelogram, *Rhombus*, or *Rhomboyades*, as in the Figures.

LXXXI. *To divide a* Trapezia *according to any Proportion, by a right Line drawn from a Point without, and to lay the greater or lesser part towards what side of the Figure shall be required.*

Figure LXXXI. Let A B C D be a *Trapezia* given, and let E be a Point without the same, from which a Line is required to be drawn to divide the same *Trapezia* into two parts which shall have such Proportion one to the other, as the Line F hath to the Line G, and to lay the lesser part towards the side A B.

The Line of Division to be drawn from E, will cut the sides B C and A D; wherefore extend them till they concurr in H, and then through the Point E, draw a Line parallel to A H, extending it till it cut the Line B H, extended in I. Then (by the 35th Problem) reduce the *Trapezia* into the Triangle A B K; so is A K the Base, and B S the Perpendicular. Then (by the 55th Problem) divide the Line A K in L, in Proportion as F to G, and (because the lesser part cut off is to lie next the side A B;) set the lesser Segment of the Base from A to L. Then (by the 54th Problem) as I E is to H L; so is H B to H M. Again, (by the 37th Problem) find a mean Proportional between I H and H M, as is H N, and (always) divide H M into two equal parts in O, and draw the occult Line O N, which set from O to P. Lastly, a right Line drawn from E to P, shall divide the *Trapezia* into two parts, A B P Q the lesser, lying next the side A B, and P C D Q the greater, lying next the side C D; and these two parts have such Proportion one to the other, as the Line F hath to the Line G. *Whichwas*, &c.

LXXXII. *To diuide a* Trapezia *into two Parts, (according to any possible Proportion) by a Line drawn through a Point within the* Trapezia.

Let

Let A B C D be a *Trapezia* given, to be divided into two parts, in Proportion one to the other, as the Line F is to the Line G, by a Line which ſhall paſs through the Point E, within the *Trapezia*. Figure LXXXII.

Continue the two ſides B C and A D, (through which the Line of ſeparation will paſs) till they concurr in H: Then reduce the *Trapezia* A B C D, (by the 35th Problem) into the Triangle A B I, whoſe Baſe is A I, and Perpendicular B S. Then divide the Baſe B I (by the 55th Problem) into two parts in K, as the Line F is to the Line G. Then through the given Point E, draw the Line M E L, parallel to A D, and let fall a Perpendicular from E to A D, as E T. Then to the two Perpendiculars B S, E T, and half the Line H K, find (by the 54th Problem) a fourth Proportional Line, that as the Perpendicular E T is to the Perpendicular B S, ſo ſhall half H K (*viz.* H V) be to H N, which ſet from H to N. Through the Point N, draw a Line parallel to B H, as N M, cutting the Line L E M in M. This done, (by the 47th Problem) take the Square of E L from the Square of E M, and the ſide of the remaining Square will be equal to the Line X, which ſet from N to P. Laſtly, a right Line drawn from P through the given Point E, as P E Q ſhall divide the *Trapezia* A B C D, into two parts, A B Q P, and P Q C D; which ſhall be in proportion one to the other, as the Line F is to the Line G. *Which was*, &c.

LXXXIII. *To divide a Plot according to any (poſſible) Proportion given; by a Line drawn through a Point aſſigned within the ſame Plot.*

Let the Plot given be A B C D E F, and it is required to divide the ſame into two parts, which ſhall be in proportion one to the other, as the Line H is to the Line I, by a right Line drawn through the Point G within the Plot. Figure LXXXIII.

It does evidently appear that the Line of ſeparation will cut the ſides B C and A F, of the Plot; wherefore extend them till they concurr in the Point K: Extend alſo the ſide A F (on the other ſide) towards M. Then (by the 36th Problem, the latter part) reduce the whole Plot into the Triangle C L M, whoſe Perpendicular is the ſhorteſt diſtance from C, unto the Baſe A F. This done, (by the 55th Problem) divide M L (the Baſe of the Triangle equal to the whole Plot) into two parts in Proportion, as H to I, which Point of Diviſion will fall in the Point N; ſo, that as H is to I, ſo L N may be to N M. Then from the Point within G, let fall a Perpendicular to the Baſe L M; (or take the neareſt diſtance thereto,) and (by the 54th Problem) ſay, as this perpendicular (or ſhorteſt diſtance) is to the Perpendicular from C, upon the ſame Baſe L M; ſo ſhall the half of K N, be unto K O, which diſtance ſet from K to O. Then through O, draw a Line parallel to K G, as O P; alſo through the given Point G, draw a Line parallel to the Baſe L M, as Q G P, till it cut the former parallel in P. Again, (by the 47th Problem) ſubtract the Square of Q G, from the Square of G P, and the ſide of the remaining Square will be a Line equal to S, which Line S take and ſet it from O to V. Laſtly, from this Point V, draw a right Line through the given Point G, as V G W, which ſhall divide the Plot into two parts, namely, into W B A V the leſſer, and W C D E F V the greater, in proportion one to the other, as the Line H to the Line I, by a Line drawn through the Point G. *Which was*, &c.

LXXXIV. *To divide a Plot according to any aſſigned Proportion, by a Line drawn from a Point aſſigned without the Plot.*

Let the Plot given be A B C D E F, it is required to divide the ſame into two parts, which ſhall have ſuch Proportion one to the other, as the Line G hath to the Line H, and from the Point X without the Plot. Figure LXXXIV.

It is apparent that the Line of ſeparation drawn from the Point X without, will cut the two ſides A F and C D; therefore extend them till they concurr in the Point K, and extend C D beyond K. Then (through the given Point X) draw a Line parallel to A K, extending it till it cut C K extended in L, as the Line X L. This done, (by the latter part of the 36th Problem) the Baſe of a Triangle equal to the whole Plot, as the Line L M, whoſe perpendicular is the ſhorteſt diſtance from C to the Baſe M N. Then (by the 55th Problem) divide M N in O, as G is to H, that is, as G is to H, ſo N O to O M. Then again (by the 54th Problem) ſay, as L X is to K O, ſo K C to K P. Now (by the 37th Problem) between L K and K P, find a mean Proportional, as K Q, which found, divide (always) K P into two equal parts in R, and draw the Line R Q, which ſet from R to S. Laſtly, from the Point X to S, draw the right Line X T S; ſo ſhall the Plot be divided into two parts S D E F T the leſſer, and S C A B T the greater, in proportion one to the other, as the Line G is to the Line H, by a Line drawn from the point X, without the Plot. *Which was*, &c.

OF

OF

GEOMETRY SOLID.

PART II.

SECT. I.

Geometrical DEFINITIONS.

I. *A Solid (or Body) is that which hath Length, Breadth, and Thickneſs, (or Depth.)*

THis Definition of a Solid is without any deſignation or form of Figure eaſily underſtood, only conceiving in the Mind, or beholding with the Eye, a piece of *Timber, Stone, Metall caſt in a Mold*, or any matter elſe.

II. *The Terms or Extremes of a Solid, is a Superficies.*

As the Terms of a Line are Points, the Terms of a Superficies are Lines, (as hath been already defined;) ſo the Terms of a Solid (or Body) are Superficies, whether the Dimenſions of the Superficies be equal or unequal: As for Example, let the length of a Solid be 5 Inches, the breadth 3 Inches, and the thickneſs (or depth,) 2 Inches: I ſay that the Terms of ſuch a Solid are 6 Superficies, *viz.* 2 whole lengths are 5 Inches, and breadth 3 Inches; two other whole lengths are 5 Inches, and breadth 2 Inches, (and theſe are the top, bottom, and the two ſides,) the other two Superficies are 3 Inches long, and 2 Inches broad, and thoſe are the two ends.

III. *A right Line is perpendicular to a Plain, when it maketh right Angles with all the right Lines, that are (or can be) drawn from that Point of the Plain upon which the right Line ſtandeth.*

Figure I. As, let A B C D be a Plain, and let O P be a Line thereupon erected: I ſay, if the Line P O, do make right Angles with all the Lines drawn upon the Plain from the Point O, as the Angles, P O E, P O F, P O G, (and as many as there can be right Lines drawn from the Point O,) all equal, (or right Angles,) then the Line P O is a Line erected perpendicular to the Plain, A B C D. Such a Line is ſometimes called a *Plumb-Line.*

IV. *A Plain is perpendicular to a Plain, when the right Lines drawn in one Plain, to the Line of common Section of the two Plains, and making right Angles therewith, do alſo make right Angles with the other Plain.*

Figure II. I ſay that the Plain A M B O, is perpendicular to the Plain G K L D, when the right Lines F G and H K, drawn upon the Plain A M O B, to the Line of common Section of the two Plains M B, do make right Angles therewith; the Plain A M O B doth alſo make right Angles with the Plain G L K D.

V. *The inclination of a right Line to a Plain, is when a Perpendicular is drawn from the higheſt Point of that Line to the Plain, and another Line drawn from the Point, which the Perpendicular makes in the Plain, to the end of the ſame Line which is in the ſame Plain; whereby the Angle is acute, which*

which is contained under the inſiſting Line, and the Line drawn in the Plain.

As to the Plain C D, I ſay the Line A B is an inclining Line thereto; for if a Perpendicular be let fall from the higheſt Point A of the Line A B, to the Plain C D, it will fall in E, and if a Line be drawn upon the Plain from B to E, the Angle A B E is acute, which is contained under the inſiſting Line A B, and the Line B E which is drawn upon the Plain.

VI. *The Inclination of a Plain to a Plain, is an acute Angle contained under theſe right Lines, which being drawn in either of the Plains, to the ſame Point of the common Section, do make right Angles with the common Section.*

The Plain A B inclineth to the Plain C D, by the acute Angle F H G, contained under the right Lines F H and G H, which being drawn in either of the Plains to the ſame Point H, of the common Section E B, do make right Angles with the common Section E B. Figure IV.

VII. *Plains are ſaid to be inclined to other Plains in the ſame manner, when the ſaid Angles of Inclination are equal one to another.*

VIII. *Parallel Plains are thoſe, which being prolonged never meet.*

IX. *Like Solid Figures are ſuch as are contained under like Plains, equal in Numbers.*

X. *Equal and like Solid Figures are ſuch as are contained under like Plains, equal both in Multitude and Magnitude.*

XI. *A Solid Angle is the Inclination of more than two right Lines which touch one another, and are not in the ſame Superficies.*

Otherwiſe,

A Solid Angle is that which is contained under more than two plain Angles, not being in the ſame Superficies, but conſiſting all at one Point.

XII. *A* Pyramide *is a Solid Figure, comprehended under many Plains ſet upon one Plain, (which is the Baſe of the Pyramide,) and gather together to one Point.*

When two, three, or more Superficies are raiſed up from one Superficies, which is the Baſe, and ever aſcending diminiſheth their breadths, till at length all their Angles concurr in one Point; there making a Solid Angle, and a Solid thus terminated, is called a Pyramide. Figure V.

Of Pyramides there are divers kinds according to the Variety of their *Baſes*. For,

If the *Baſes* of the Pyramide be a { Triangle, Quadrilateral, Pentagon, Hexagon, Heptagon, *&c.* } then it is called a { Triangular, Quadrangular, Pentagonal, Hexagonal, Heptagonal, } Pyramide.

Although theſe Pyramides cannot well be expreſſed in a plain Superficies, yet you may ſufficiently conceive of it by the Figures here ſet; if you imagine the Point A, together with the Lines A B, C B, and D B, to be elevated on high, till all the plain Angles of the ſeveral Superficies A C B, A D B, and C D B, do all meet and concurr in the Point B, there making the Solid Angle A C D B.

XIII. *A Priſme is a Solid Figure comprehended under Plains, whereof the two oppoſite are equal, like, and parallel, and the other are Parallelograms.*

By this Figure you may conceive what a Priſme is, if you imagine the Superficies, A B C D, to be the *Baſes*, and the two Superficies A E F B, and C E F D, to be erected upon the two ſides of the *Baſes* A B and C D, not perpendicularly, but bending the one to the other, till they join together in the Line E F; ſo is the Priſme contained under Figure VI.

der five Superficies, namely under the three Parallelograms A B C D, B A F E, F E C D, and under the two Triangles, F B D and E A C, which are equal, like, and parallel.

XIV. *A Sphere is a Solid Figure made when the Diametre of a Semicircle abideth unmoved, the Semicircle is turned round about, till it return to the same place from whence it began to be moved.*

Another Definition by *Theodosius*.

A Sphere is a Solid or Body contained under one Superficies, in the middle whereof there is a Point, from which all right Lines drawn to the Circumference are equal.

Figure VI. Upon the Line A B, being the Diametre, describe a Semicircle A C B, whose Centre let be D, the Diametre A B being fixed upon his ends or points, A and B, imagine the whole Superficies of the Semicircle to move round from some one point assigned, till it return to the same point again; so shall it produce a perfect Sphere or Globe, such as you see in the Figure.

XV. *The* Axis *of a Sphere, is that fixed right Line about which the Semicircle is moved.*

Figure VII. As the Line A D B, which is the Diametre of the Semicircle, and consequently of the Sphere or Globe.

XVI. *The Centre of a Sphere is the same Point with that which is the Centre of the Semicircle* D.

XVII. *The Diametre of a Sphere is a right Line drawn through the Centre, and terminated on either side in the Superficies of the Sphere,* as A B. *Fig.* VII.

XVIII. *A* Cone *is a Figure made when one side of a rectangled Triangle;* (viz. *one of those sides which contains the right Angle,*) *remaining fixed, the Triangle is turned round about, till it return to the place from whence it first moved; and if the fixed right Line be equal to the other which containeth the right Angle, then the* Cone *is a rectangled* Cone; *but if it be less, it is an obtuse Angled* Cone; *if greater, it is an accute Angled* Cone.

Figure VIII. As a Sphere was produced by the Motion of a Semicircle about its Diametre; so a Cone is produced by the turning about of a rectangled Triangle, upon one of the sides which subtendeth the right Angle from some one point, til it return to the same point again.

Let A B C be a rectangled Triangle, having the Angle A B C a right Angle, contained under A B and B C. Now suppose the side A B to be fastned, and suppose the Triangle A B C to be moved from some one point assigned, till it return to the same point again: so shall the Solid or Body thus described be a perfect Cone. Of Cones there be three kinds, *viz.* right, obtuse, and accute Angled.

First, if the right Line which abideth fixed be equal to that which is moved about, and containeth the right Angle, then is the Cone a rectangled Cone.

Secondly, But if it be less, then it is an obtuse Angled Cone. And,

Thirdly, If it be greater, then it is an accute Angled Cone.

XIX. *The* Axis *of a Cone is that fixed Line about which the Triangle moved. And the Base of the Cone is the Circle which is described by the right Line which is moved about.*

In this Cone the right Line A B is the *Axis*, and the Circle D E C is the *Basis* of the Cone A D E C.

XX. *A Cylinder is a Solid Figure, made by the moving round of a right Angled Parallelogram, one of the sides thereof abiding fixed, till the Parallelogram be turned about from some assigned Point, till it return to the same point again.*

Figure IX. Suppose A B C D to be a rectangled Parallelogram, having its side A B fixed, about which imagine the whole Parallelogram to be turned about, till it return to the Point,

Point where it began, then is that Solid or Body by this motion described, a perfect *Cylinder*.

XXI. *The* Axis *of a* Cylinder *is that right Line which abideth fixed, about which the Parallelogram is moved: And the* Basis *of the* Cylinder, *are the Circles described by the two opposite sides of the Parallelogram in their Motion.*

So in the Figure the right Line AB, about which the Parallelogram was moved, is the *Axis* of the *Cylinder*; and the two Circles DEFG, and CHKL, made by the motion of the two opposite sides of the Parallelogram, are the *Bases* of the *Cylinder*. Figure IX.

XXII. *Like Cones and* Cylinders *are they, both whose* Axes *and Diametres of their* Bases *are Proportional.*

XXIII. *A Cube is a Solid or Body contained under six equal Squares.*

It is a Solid Body in form of a common Dye, and in *Plano* cannot be expressed otherwise, than in the Figure which consists of six Parallelograms, which you must conceive to be (all of them) both equilateral and Equiangular: Now if you suppose one of them, as ABCD, to be the *Basis*, and EFGH, to be in the top over it; so that the Lines AE, CG, DH, and BF, are perpendiculars erected from the Points ABCD, then those Lines AE and CG will represent one of the upright Plains, namely, the upright Plain; AGCE and the Lines DH and BF will represent another of the upright Plains, namely, the Plain HFDB; and the Lines BF and AE another upright Plain, namely, EFAB; and the Lines GC and HD the other Plain, namely, GHCD. Figure X.

This Definition well conceived and understood, will render those that follow more intelligible.

XXIV. *A* Tetraedron *is a Solid Figure contained under four equal and equilateral Triangles.*

The form of this Solid is before sufficiently defined in the Definition of a Pyramide, and again in this Figure which is the same in effect, only in this the four Triangles are equilateral, and there one only was equilateral, and the other three *Isosceles*. Now in this Figure, if you conceive the Triangle ACD, to be the *Basis* of the *Tetraedron*, and the other three Triangles ACB, ADB, and CDB, issuing from the Sides of the *Basis*, AC, AD, and CD, and concurring in the Point B, you have the perfect representation of it, as a Solid Body. Figure XI.

XXV. *An* Octaedron *is a Solid Body, or Figure contained under eight equal and equilateral Triangles.*

For the representation of this Body in *Plano*, imagine first, that upon the upper face of the Superficies of the Parallelogram ABCD, be described a *Pyramis*, having his four Triangles AFB, AFC, CFD, and DFB, equilateral and equiangular, and concurring in the Point F. Also conceive that on the lower face of the same Parallelogram, be described another *Pyramis*, having his four Triangles AEB, AEC, CED, and DEB, equilateral and equiangular, and all of them concurring in the Point E. And this Description may be easily conceived. Figure XII.

XXVI. *A* Dodecaedron *is a Solid Figure or Body, contained under twelve equal, equiangular, and equilateral* Pentagons.

In the Figure imagine, first the Pentagon ABCDE to be the *Basis* of the Body; then imagine the *Pentagon* FGHKL to be a *Pentagon* opposite to the former, and between these two *Pentagons* there will be ten *Pentagons* pulled up, five from the five sides of the ground *Pentagon*, namely, Figure XIII.

From the side		The *Pentagon*	
	AB		ABONM.
	BC		BCQPO.
	CD		CDSRQ.
	DE		DEUTS.
	EA		EAMXU.

The other five *Pentagons* have each one of their ſides common with one of the ſides of the *Pentagon* F G H K L, which is oppoſite unto the *Baſis*, and thoſe are the other five *Pentagons*, namely,

The *Pentagon* { F G N M X. / G H P O N. / H K R Q P. / K L R S T. / L F X U T.

Which *Pentagons* if you imagine to be equal, equilateral, and Equiangular, and to be lifted up from the ſides of the *Baſis*, and the *Pentagon* oppoſite to the *Baſis* (both ways,) you will eaſily conceive the form of this Body called *Dodecaedron*.

XXVII. *An* Icoſaedron *is a ſolid Body, contained under Twenty equal and equilateral Triangles.*

Figure XIV. Foraſmuch as there are in this Solid Body Twenty Triangles, and every Triangle hath three Angles, the concourſe of the ſaid Triangles will be in Twelve Points, as at A, B, C, D, E, F, G, H, K, L, M, and N. Where Note that in this plain Figure the two Points M and N are but one; but you muſt imagine one of them to be erected upwards, the other oppoſite to it downwards. Now,

The five Triangles which concurr in the Point M, and are imagined to be erected upwards, are { B M D. / D M F. / F M H. / H M L. / L M B.

And

The five Triangles which concurr in the Point N, and are conceived to be erected downwards, are { A N C. / C N E. / E N G. / G N K. / K N A.

The other ten Triangles which include this Body, are theſe.

A B C, B C D, C D E, D E F, E F G, and F G H. G H K. H K L. K L A. L A B.

And thus is the Body compleated, and in the Figures of theſe five Bodies the repreſentation of them is expreſſed three ſeveral ways.

Firſt, As they are here deſcribed in *Plano*, upon a plain Superficies.

Secondly, As they repreſent themſelves in a Body, like Form, and

Thirdly, As they may be drawn upon *Paiſt-board*, or upon ſuch like matter as will be pliable and bow, and then in the Lines cut the *Paiſt-board* half way through, and bow, and fold in the ſeveral Plains of which the Body conſiſts, and they will cloſe together in ſuch ſort, that they will make the perfect Body deſigned.

XXVIII. *A* Parallelilipedon *is a Solid Figure contained under ſix Quadrilateral Figures; whereof thoſe which are oppoſite are parallel.*

XXIX. *A Solid Figure is ſaid to be inſcribed in a Solid Figure, when all the Angles of the Figure inſcribed, are comprehended, either within the Angles, or in the Sides, or in the Plains of the Figure wherein it is inſcribed.*

XXX. *A Solid Figure is ſaid to be circumſcribed about a Solid Figure, when either Angles, or Sides, or Plains of the circumſcribed Figure, touch all the Angles of the Figure about which it is circumſcribed.*

SECT.

SECT. II.

Geometrical THEOREMS.

I. *One part* A C *of a right Line, cannot be in a plain Superficies, and another part* C B *elevated upwards.*

PRoduce A C in the Plain directly to F: If you conceive C B to be drawn ſtreight from A C; then two right Lines A B and A F hath one common Segment. *Which is impoſſible.* Eucl. El. L. 11. Pr. 1. Figure I.

II. *If two right Lines* A B *and* C D *cut one another, they are in the ſame Plain; and every Triangle* D E B *is in one and the ſame Plain.*

Conſtruction. FOr imagine E F G, part of the Triangle D E B, to be in one Plain, and and the part F D G B to be in another, then E F, part of the right Line E D, is in a Plain, and the other part elevated upwards. *Which is abſurd.* Figure II.

Demonſtration. Therefore the Triangle E D B is in one and the ſame Plain, and ſo alſo are the right Lines, E D and E B; wherefore the whole Lines A B and D C are in one Plain. *Which was,* &c. Eucl. El. L. 11. Pr. 2.

III. *If two Plains* A B *and* C D *cut one the other, their common Section* E F *is a right Line.*

Demonſtration. IF E F the common Section be not a right Line, then in the Plain A B draw the right Line E G F, and in the Plain C D the right Line E H F; therefore two right Lines E G F and E H F include a Superficies. *Which is abſurd.* Eucl. El. L. 11. Pr. 3. Figure III.

IV. *If a right Line* E F, *be at right Angles erected upon two Lines* A B *and* C D, *cutting one the other at the common Section* E, *it ſhall alſo be at right Angles to the Plain* A C B D, *drawn by the ſaid Line.*

Conſtruction. TAke E A, E C, E B, and E D, equal one to the other, and join the right Lines A C, C B, B D, and D A. Draw any right Line G H through E, and join F A, F C, F D, F B, F G, and F H. Figure IV.

Demonſtration. Becauſe A E is equal to E B, and D E to E C, and the Angle A E D, equal to C E B; therefore A D is equal to C B; and likewiſe A C, is equal to D B; therefore A D is parallel to B C, and A C to D B: wherefore the Angle G A E, is equal to the Angle E B H, and the Angle A G E equal to the Angle E H B. But alſo A E is equal to E B; therefore G E is equal to E H, and A G to B H. Whence, by reaſon of the right Angles, (by the 4th *Hypotheſis,*) and ſo equal at E, the *Baſes* F A, F C, F B, and F D are equal; therefore the Triangles A D F and F B C, are equilateral one to another, and thence the Angle D A F, equal to B C F. Therefore in the Triangles A G F and F B H, the Sides F G and F H are equal; and ſo by conſequence the Triangles F E G and F E H are mutually equilateral: therefore the Angles F E G and F E H are equal, and ſo right Angles. In like manner F E makes right Angles with all the Lines drawn through E, in the Plain A D B C, and is therefore Perpendicular to the ſaid Plain. *Which was to be Demonſtrated.* Eucl. El. L. 11. P. 4.

V. *If two right Lines* A B *and* D C, *be erected perpendicular to one and the ſame Plain* E F; *thoſe right Lines* A B *and* D C *are parallel one to the other.*

Conſtruction. DRaw A D, whereunto let D G (equal to A B) be perpendicular in the Plain E F, and join B D, B G, and A G. Figure V.

Demonſtration. Seing in the Triangles B A D and A D G, the Angles D A B and A D G are right Angles, and A B equal to D G, and A D is common; therefore B D is equal to A G: whence in the Triangles A G B and B G D, equilateral one to the other, the Angle B A G is equal to B D G; of which ſeeing B A G is a right Angle, B D G ſhall be ſo alſo. But the Angle G D C is ſuppoſed right; therefore the right Line D G

is perpendicular to the three Lines DA, DB, CD, which are therefore in the same Plain wherein AB is. Wherefore, since AB and CD are in the same Plain, and the internal Angles BAD and CDA are right Angles, AB and CD shall be parallel. *Which was*, &c. Eucl. El. L. 11. Pr. 6.

VI. *If there be two parallel right Lines* AB *and* CD, *whereof one* AB *is Perpendicular to a Plain* EF, *then the other* CD *shall be perpendicular to the same Plain* EF.

Figure V. THE Construction and Demonstration of the last Theorem being transferred hither, the Angles GDA and GDB are right Angles; therefore GD is perpendicular to the Plain, wherein AD and DB are, (and also, in which AB and CD are.) Therefore GD is perpendicular to CD, but the Angle CDA, is also a right Angle; therefore CD is perpendicular to the Plain EF. *Which was to be Demonstrated.* Eucl. El. L. 11. Pr. 8.

VII. *If two right Lines* AB *and* AC *touching one another, be parallel to two other right Lines,* ED *and* DF, *touching one another, and not being in the same Plain, those right Lines contain equal Angles,* BAC *and* EDF.

Figure VI. *Construction.* LEt AB, AC, DE, and DF, be equal one to the other, and draw AD, BC, EF, BE, and CF. Then,

Demonstration. Seeing AB and DE are parallel and equal, also BE and AD are parallel and equal.

In like manner, CF and AD are parallel and equal; therefore also BE and FC are parallel and equal; therefore BC and EF are equal. Wherefore, since the Triangles BAC and EDF are of equal sides one to the other, the Angles BAC and EDF shall be equal. *Which was to be Demonstrated.* Eucl. El. L. 11. P. 10.

VIII. *At a Point given* C, *in a given Plain* AB, *two right Lines,* CD *and* CE, *cannot be erected perpendicular on the same side.*

Figure VII. FOr both the Lines, CD and CE, should then be perpendicular to the Plain AB, and consequently parallels. *Which is repugnant to the Definition of Parallels.* Eucl. El. L. 11. P. 13.

IX. *Plains,* CD *and* EF, *to which the same right Line* AB *is perpendicular, are parallel.*

Figure VIII. *Construction.* IF you deny this, then let the Plains CD and FE meet, so that their common Section be the right Line GH, in which take any Point, as I, and draw to it the right Lines IA and IB, in the said Plains.

Demonstration. Whereby in the Triangle IAB, two Angles, namely, IAB and IBA, are right Angles. *Which is absurd.* Eucl. El. L. 11. Pr. 14.

X. *If two right Lines* AB *and* AC, *touching one the other, be parallel to two other right Lines,* DE *and* DF, *touching one the other, and not being in the same Plain with them, the Plains* BAC *and* EDF, *drawn by those right Lines, are parallel one to the other.*

Figure IX. *Construction.* FRom A draw AG, perpendicular to the Plain EF, and let GH and GI be parallel to DE and DF; these also shall be parallel to AB and AC.

Demonstration. Therefore since the Angles IGA and HGA are right Angles, also CAG and BAG shall be right Angles; therefore GA is perpendicular to the Plain BC, but the same is perpendicular to the Plain EF; therefore the Plains BC and EF are parallel. *Which was*, &c. Eucl. El. L. 11. Pr. 15.

XI. *If two parallel Plains,* AB *and* CD, *be cut by some other Plain* HEIGF, *their common Sections* EH *and* GF *are parallel one to the other.*

Figure X. FOR if they be conceived to be otherwise, being in the same Plain that cuts them, they would meet somewhere, if produced; suppose in I: Wherefore, since the whole

whole Lines H E I and F G I are in the Plains A B and C D being produced, the Plains alſo ſhall meet. *Contrary to the Hypotheſis.* Eucl. El. L. 11. P. 6.

XII. *If two right Lines* A L B *and* C M D *be cut by parallel Plains,* E F, G H, *and* I K, *they ſhall be cut proportionally: For as* A L *ſhall be to* L B, *ſo ſhall* C M *be to* M D.

Conſtruction. LET the right Lines A C and B D be drawn in the Plains E F and I K, as alſo A D paſſing through the Plain G H in the Point N; and join N L and M L. Figure XI.

Demonſtration. The Plains of the Triangles A D C and A D B make the Sections B D and L N, and alſo A C and N M parallels: Therefore as A L is to L B, ſo is A N to N D; and as A N is to N D, ſo is C M to M D. *Which was,* &c. Eucl. El. L. 11. P. 17.

XIII. *If a right Line* A B *be perpendicular to ſome Plain* C D, *all the Plains extended by that right Line* A B (*as* E F, &c.) *ſhall be perpendicular to the ſame Plain* C D.

Conſtruction. LET there be ſome Plain E F drawn by A B, making the Section E G, with the Plain C D; from ſome Point whereof, as H, draw H I, parallel to A B, in the Plain E F. Figure XII.

Demonſtration. Then ſhall H I be perpendicular to the Plain C D, and ſo likewiſe any other Lines that are perpendicular to E G. Therefore the Plain E F is perpendicular to the Plain C D; and by the ſame reaſon, any other Plains drawn by A B ſhall be perpendicular to E F. *Which was,* &c. Eucl. El. L. 11. P. 18.

XIV. *If two Plains* A B *and* C D, *cutting one the other, be perpendicular to ſome Plain, as* G H, *their Line of common Section* E F *ſhall be perpendicular to the ſame Plain* G H.

Demonſtration. BEcauſe the Plains A B and C D are taken perpendicular to the Plain G H, it appears (by Definition 64.) that out of the Point F there may be drawn in both Plains, A B and C D, a Perpendicular to the Plain G H, which ſhall be but one, and therefore ſhall be the common Section of the ſaid Plains. *Which was,* &c. Eucl. El. L. 11. P. 19. Figure XIII.

XV. *If a Solid* A B *be contained under Parallel Plains, the oppoſite Plains thereof* (A G, D B, *&c.*) *are like and equal Parallelograms.*

Conſtruction. THe Plain A C, cutting the parallel Plains A G and D B, makes the Sections A H and D C parallels, and by the ſame reaſon A D and H C are Parallels. Figure XIV.

Demonſtration. Therefore A D C H is a Parallelogram, by the like Argument the other Plains are Parallelograms: Wherefore ſeeing A F is parallel to H G, and A D to H C, the Angle F A D ſhall be equal to C G H; therefore becauſe A F is equal to H G, and A D to H C; and ſo A F is to A D, as H G is to H C; the Triangles F A D and G H C are like and equal, and conſequently the Parallelograms A E and H B are like and equal: And the ſame might be ſhewn of the reſt of the oppoſite Plains. *Therefore,* &c. Wich was, *&c. Eucl. El. L.* 11. *P.* 25.

XVI. *If a Solid* Parallelipipedon A B C D *be cut by a Plain* E F, *parallel to the oppoſite Plains* A D *and* B C; *Then as the Baſe* A H *is to the Baſe* E H, *ſo ſhall the Solid* A H D, *be to the Solid* B H C.

Conſtruction. COnceive the *Parallelipipedon* to be extended on either Side, and take A I equal to A E, and B K equal to E B; and put the Plains I Q and K P parallel to the Plains A D and B C; then the Parallelograms I M and A H, D L and D G, I Q, A D, E F, *&c.* are like and equal. Figure XV.

Demonſtration. Wherefore the *Parallelipipedon* A Q is equal to A F, and by the ſame reaſon the *Parallelipipedon* B P is equal to B F; therefore the Solids I F and E P are as *Multiplex* of the Solids F E and E C, as the Baſes I H and K H, are of the Baſes A H and B H. And if the Baſe I H be greater, equal, or leſſer than K H, the Solid I E ſhall be greater, equal, or leſs than the Solid E P: And conſequently A H ſhall be to B H, as A F to E G. *Which was,* &c. Eucl. El. L. 11. P. 25.

The

The ſame may be accommodated to all ſorts of *Priſms:* Whence this *Corollary*; If any *Priſm* whatſoever be cut by a Plain parallel to the oppoſite Plains, the Section ſhall be a Figure equal and like to the oppoſite Plains.

XVII. *If a Solid* Parallelipipedon AB *be cut by a Plain* FGCD, *drawn by the diagonal Lines* DF *and* CG *of the oppoſite Plains* AE *and* HB; *that Solid* AB *ſhall be equally biſected by the Plain* FGCD.

Figure XVI. *Demonſtration.* FOR becauſe DC and FG are equal and Parallels, the Plain FGCD is a Parallelogram, and ſeeing the Parallelograms AE and HB, are equal and like; alſo the Triangles AFD, HGC, CGB, and DEF, are equal and like: But the Parallelograms AC and AG are equal and like to FB and FD; therefore all the Plains of the *Priſm* FGCDAH are equal and like to all the Plains of the *Priſm* FGCDEB; and conſequently this *Priſm* equal to that. *Which was*, &c. Eucl. El. L. 11. P. 28.

XVIII. *Solid* Parallelipipedons AGHEFBCD *and* AGHEMLKI, *being conſtituted upon the ſame Baſe* AGHE, *and in the ſame Highth*; [*i. e.* between the Parallel Plains AGHE and FLKD, and ſo underſtand it in the Demonſtration:] *whoſe inſiſting Lines* AF *and* AM *are placed in the ſame right Lines* AG *and* FL, *are equal one to the other.*

Figure XVII. *Demonſtration.* FOR, if from the equal Priſms AFMEDI and GBLHCK, the common Priſm NBMPCI be taken away, and the ſolid AGNEHR be added, the *Parallelipipedon* AGHEFBCD ſhall be equal to the *Parallelipipedon* AGHEMLKI. *Which was*, &c. Eucl. El. L. 11. P. 29.

XIX. *Solid* Parallelipipedons ADBCHEFG *and* ADCBIMLK, *being conſtituted upon the ſame Baſe* ADBC, *and in the ſame highth, whoſe inſiſting Lines* AH *and* AI, *are not placed in the ſame right Lines, are equal one to the other.*

Figure XVIII. *Conſtruction.* PRoduce the right Lines HEO, GFN, and LMO, KIP, and draw AP, DO, BQ, and CN.

Demonſtration. Then ſhall DC, AB, HG, EF, PQ, and ON, be as well equal and parallel one to the other, as AD, HE, GF, BC, KL, IM, QN, and PO: Wherefore the *Parallelipipedon* ADCBPONQ ſhall be equal to either of the *Parallelipipedons* ADCBHEFG, or ADCBIMLK: and conſequently theſe two are equal one to the other. *Which was*, &c. Eucl. El. L. 11. P. 30.

XX. *Solid* Parallelipipedons ABCD *and* EFGL *of the ſame highth, are one to the other as their* Baſes.

Figure XIX. *Conſtruction.* PRoduce EHI, and make the Parallelogram FI equal to AB, and compleat the *Parallelipipedon* FINM.

Demonſtration. It is clear, that the *Parallelipipedon* FINM (which is by conſtruction equal to the *Parallelipipedon* ABCD,) is in proportion to the *Parallelipipedon* EFGL as the *Baſis* FI (which is equal to AB) is to the *Baſis* EF. *Which was*, &c. Eucl. El. L. 1. P. 32.

XXI. *Like Solid* Parallelipipedons ABCD *and* EFGH *are in triple Proportion one to the other, of that in which their* Homologous *Sides* AI *and* EK *are.*

Figure XX. *Conſtruction.* PRoduce the right Lines AIL, DIO, and BIN; and make IL, IO, and IN, equal to EK, KH, and KF; and ſo the *Parllelipipedon* IXMT equal and like to the *Parallelipipedon* EFGH; and let the *Parallelipipedons* IXPB and DLYQ be finiſhed.

Demonſtration. Then ſhall AL be to IL, (equal to EK,) as DI to IO, (equal to KH.) And, as DI is to IO, ſo is BI to IN. That is, the Parallelogram AD is to the Parallelogram DL, as DL is to IX; an DL is to IX, as BO is to IT. That is, the *Parallelipipedon* ABCD is to the *Parallelipipedon* DLQY, as the *Parallelipipedon* DLQY is to the *Parallelipipedon* IXBP: And the *Parallelipipedon* DLQY is to the *Parallelipipedon*

rallelipipedon IXBP, as the *Parallelipipedon* IXBP is to the *Parallelipipedon* IXMT, (equal to the *Parallelipipedon* EFGH.) Therefore the proportion of ABCD to EFGH is triple of the Proportion of ABCD to DLQY; (or of AI to EK.) *Which was*, &c. Eucl. El. L. 11. P. 33.

Corollary.

Hence it appears, that if four right Lines be continually proportional; as the first is to the fourth, so is a *Parallelipipedon* described on the first, to a *Parallelipipedon* described on the second; they being like, and in like manner described.

XXII. *In equal solid* Parallelipipedons AD, CB, EH, *and* GF, *the Bases and Altitudes are reciprocal,* (*as* AD *to* EH, *so* EG *to* AC.) *And Solid* Parallelipipedons ADCB *and* EHGF, *whose Bases and Altitudes are reciprocal, are equal.*

Construction. First, Let the Sides CB and GE be perpendicular to the Bases; then if the Altitudes of the Solids be equal, the Bases also shall be equal, and the thing is clear. But if the Altitudes be unequal, from the greater take EG, take EI equal to AC, and at I draw the Plain IK, parallel to the Base EH. Then, Figure XXI.

Demonstration. 1. *Hypothesis:* AD is to DH, as the *Parallelipipedon* ADCB is to the *Parallelipipedon* EHIK; and the *Parallelipipedon* ABCD is to the *Parallelipipedon* EHIK, as the *Parallelipipedon* EHGF is to the *Parallelipipedon* EHIK: And as the *Parallelipipedon* EHGF is to the *Parallelipipedon* EHIK, so is GL to IL: And as GL is to IL, so is GE to IE, (equal to IF.) It is plain therefore, that as AD is to EH, so is GE to AC. *Which was*, &c.

2. *Hypothesis*, ADCB is to EHIK, as AD is to EH; and AD is to EH, as EG is to EI; and EG is to EI, as GL to IL: And GL is to IL, as the *Parallelipipedon* EHGF is to the *Parallelipipedon* EHIK. Wherefore the *Parallelipipedons* ADCB and EHGF are equal. *Which was*, &c. Eucl. El. L. 11. P. 34.

Moreover, Let the Sides be oblique to the Bases, and erect right *Parallelipipedons* upon the same Bases, in the same Altitude, the oblique *Parallelipipedons* shall be equal to them. Wherefore, since by the first part of the Demonstration, the Bases and Altitudes of those be reciprocal, the Bases and Altitudes of these also shall be reciprocal. *Which was*, &c.

Corollary.

All that hath been demonstrated of *Parallelipipedons* in the five last Theorems, do also agree to Triangular Prisms, which are half *Parallelipipedons*, as appears by the seventh foregoing Theorem.

1. Triangular Prisms are of equal highth with their Bases.
2. If they have the same, or equal Bases, and the same Altitude, they are equal.
3. If they be like, their proportion is treble to that of their Sides of like Proportion.
4. If they be equal, their Bases and Altitudes are reciprocal, and if their Bases and Altitudes be reciprocal, they are also equal.

XXIII. *If there two Prisms,* ABCFED *and* GHLMIK, *be of equal altitude; whereof one hath its Base* ABCF *a Parallelogram, and the other* GHM *a Triangle: And if the Parallelogram* ABCF *be double to the Triangle* GHM, *those Prisms* ABCFED *and* GHMLIK *are equal.*

Demonstration. FOR if the *Parallelipipedons* AN and GQ be compleated, they shall be equal, because of the equality of their Bases AC and GP, and of their Altitudes; therefore also the Prisms (the halves of the *Parallelipipedons*) shall be equal. *Which was*, &c. Eucl. El. L. 11. P. 40. Figure XXII.

Scholium.

From the preceding Demonstrations, the dimension of Triangular Prisms and Quadrangular *Parallelipipedons* is learn'd, *viz.* by multiplying their Altitude into the Bases.

XXIV. *Like* Poligon *Figures* ABCDE *and* FGHIK, *described in Circles,* ABD *and* FGI, *are in Proportion one to another, as the Squares described of the Diametres of the Circles* AL *and* FM.

Construction. DRaw AC and BL; also FH and GM. Figure XXIII.

Demon-

Demonstration. Because the Angle ABC is equal to the Angle FGH, and AB is to BC, as FG is to FH; therefore the Angle ACB is equal to the Angle FHG. But the Angles ABL and FGM are right Angles, and so equal; therefore the Triangles ABL and FGM are equiangular: Wherefore AB is to FG, as AL is to FM; therefore ABCDE is to FGHIK, as the Square of A is to the Square of FM. *Which was*, &c. Eucl. El. L. 12. P. 1.

Corollary.

Hence (because AB is to FG, as AL to FM; and AL is to FM, as BC to BH, *&c.*) the Contents of like *Polygon* Figures described in a Circle, are in proportion as the Diametres of the Circles.

XXV. *Pyramids* ABCDEF *and* GHIKLM, *consisting under the same Altitude, and having* Polygonous *Bases*, ABCDE *and* GHIKL, *are one to the other as their Bases* ABCDE *and* GHIKL.

Figure XXIV. *Construction.* DRaw the right Lines AC, AD, and GI, GK. *Fig.* 1.

Demonstration. Then is the Base ABC to the Base ACD, as the *Pyramis* ABCF is to the *Pyramis* ACDF: Therefore (by composition) ABCD is to ACD, as the *Pyramis* ABCDF is to the *Pyramis* ACDF. But also ACD is to ADE, as the *Pyramis* ACDF is to the *Pyramis* ADEF: Wherefore, of equality, ABCD is to ADE, as ABCDF is to ADEF; and thence (by composition) ABCDE is to ADE, as the *Pyramis* ABCDEF is to the *Pyramis* ADEF. Moreover ADE is to GKL, as the *Pyramis* ADEF is to the *Pyramis* GKLM: And inversely, as before, GKL is to GHIKL, as the *Pyramis* GKLM is to the *Pyramis* GHIKLM: Wherefore (again of equality) ABCDE is to GHIKL, as the *Pyramis* ABCDEF is to the *Pyramis* GHIKLM. *Which was*, &c. *Fig.* 2.

But if the Bases have not Sides of equal Multitude, the Demonstration will proceed thus: The Base ABC is to the Base GHI, as the *Pyramis* ABCF is to the *Pyramis* GHIK; and ACD is to GHI, as the *Pyramis* ACDF is to the *Pyramis* GHIK: Therefore the Base ABCD is to the Base GHI, as the *Pyramis* ABCDF is to the *Pyramis* GHIK. Moreover the Base ADE is to the Base GHI, as the *Pyramis* ADEF is to the *Pyramis* GHIK: Therefore the Base ABCDE is to the Base GHI, as the *Pyramis* ABCDEF is to the *Pyramis* GHIK. *Which was*, &c. Eucl. El. L. 12. P. 6.

XXVI. *In equal Pyramids*, ABCD *and* EFGH, *having Triangular Bases*, ABC *and* EFG, *the Bases and Altitudes are reciprocal: And Pyramids having Triangular Bases, whose Altitudes and Bases are reciprocal, are equal.*

Figure XXV. *Demonstration.* THE compleated *Parallelipipedons* ABICDMKL and EFNGHQOP, 1. *Hypothesis.* are Sextuple of the equal Pyramids ABCD and EFGH, (either of either,) and so equal one to the other: Therefore the Altitude H is to the Atitude D, as ABIC is to EFNG: and ABIC is to EFNG, as ABC is to EFG. *Which was*, &c.

2. *Hypothesis.* The Altitude H is to the Altitude D, as ABC is to EFG; and ABC is to EFG, as ABIC is to EFNG: Therefore the *Parallelipipedons* ABICDMKL and EFMGHQOP are equal: Consequently also the Pyramids ABCD and EFGH, they being subsextuple of the same, are equal. *Which was*, &c. Eucl. El. L. 12. P. 9.

The same is applicable to Polygonous Pyramids; for they be in like manner reduced to Triangulars.

Corollary.

Whatsoever is demonstrated of Pyramids does likewise agree to any sort of Prisms, being they are Triple of the Pyramids that have the same Base and Altitude with them.

Therefore,

1. The Proportion of Prisms of equal Altitude is the same with that of their Bases.
2. The Proportion of like Prisms, is Triple of that of the Sides of like Proportion.
3. Equal Prisms have their Bases and Altitudes reciprocal; and Prisms which are so reciprocal are equal.

Scholium.

From what is hitherto demonstrated, the Dimension of any Prisms and Pyramids may be collected.

XXVII. *If*

XXVII. *If a* Cylinder ABCD *be divided by a Plain* EF, *parallel to the opposite Plains* BC *and* AD; *then as one* Cylinder AEFD *is to the other* Cylinder EBCF, *ſo is the* Axis GI *to the* Axis IH.

Conſtruction. THE *Axis* being produced, take GK equal to GI, and HL equal to IH, equal to LM, and conceive Plains drawn at the Points K, L, and M, parallel to the Circles AD and BC. Figure XXVI.

Demonſtration. Therefore the Cylinder ED is equal to the Cylinder AN, and the Cylinder EC is equal to BO, equal to PO: Therefore the Cylinder EN is as multiplex to the Cylinder ED, as the *Axis* IK is of the *Axis* IG. And in like manner the Cylinder FP is as Multiplex to the Cylinder BF, as the *Axis* IM is of the *Axis* IH: But as IK is equal, greater, or leſs than IM, ſo is the Cylinder EN equal, greater, or leſs than the Cylinder FP. Therefore the Cylinder AEFD is to the Cylinder EBCF, as GE is to IH. *Which was*, &c. Eucl. El. L. 12. P. 13.

XXVIII. *Cones,* AEB *and* CFD, *and Cylinders,* AH *and* CK, *conſiſting upon equal Baſes,* AB *and* CD, *are to one another as their Altitudes* ME *and* NF *are.*

Conſtruction. THE Cylinder HA and the *Axis* ME being produced, take LM, equal to FN, and at the Point L draw a Plain parallel to the Baſe AB. Figure XXVII.

Demonſtration. Then ſhall the Cylinder AP be equal to the Cylinder CK: But the Cylinder AH is to the Cylinder AP, (equal to CK,) as ME is to ML, (equal to NF,) *Which was*, &c. Eucl. El. L. 12. P. 14.

The ſame may be affirmed of Cones, ſubtriple of Cylinders; as alſo of Priſms and Pyramids.

XXIX. *In equal Cones* BAC *and* EDF, *and Cylinders* BH *and* EK, *the Baſes and Altitudes are reciprocal,* (*as* BC *to* EF, *ſo* MD *to* LA.) *And Cones and Cylinders, whoſe Baſes and Altitudes are reciprocal, are equal one to the other.*

Demonſtration. IF the Altitudes be equal, then the Baſes are equal alſo, and the thing is evident; if unequal, then take away MO equal to LA. Figure XXVIII.

1. *Hypotheſis.* Then as MD is to MO, (equal to LA;) ſo is the Cylinder EK, (equal to BH,) to EQ: And as EK is to EQ, ſo is the Circle BC to the Circle EF. *Which was*, &c.

2. *Hypotheſis.* As BC is to EF, ſo is DM to OM, (equal to LA;) and as DM is to OM, ſo is the Cylinder EK to the Cylinder EQ; and as EK is to EQ, ſo is BC to EF; and as BC is to EF, ſo is BH to EQ: Therefore the Cylinder EK is equal to the Cylinder BH. *Which was*, &c. Eucl. El. L. 12. P. 15.

The ſame Argument may be uſed for Cones.

XXX. *Spheres* BAC *and* EDF *are in triple Proportion one to the other, of that in which their Diametres* BC *and* EF *are.*

Conſtruction. LET the Sphere BAC be to the Sphere G in triple proportion of that of the Diametre BC to the Diametre EF. Figure XXIX.

Demonſtration. I ſay G is equal to EDF: For (if it be poſſible) let G be leſſer than EDF, and conceive the Sphere G concentrical with EDF; in the Sphere EDF inſcribe a *Polyedron* not touching the Sphere G, and a like *Polyedron* in the Sphere BAC; theſe *Polyedrons* are in triple proportion of the Diametres BC and EF, that is, of the Sphere BAC and G: Conſequently the Sphere G is greater than the *Polyedron* inſcribed in the Sphere EDF, the part than the whole. *Which is abſurd.*

Again, If it be poſſible, let the Sphere G be greater than EDF; and as the Sphere EDF is to another Sphere H, ſo let G be to BAC; that is, in triple proportion of the Diametre EF to BC: Therefore ſince BAC is greater than E, we ſhall incurr the abſurdity of the firſt part; wherefore rather the Sphere G is equal to EDF. *Which was*, &c. Eucl. El. L. 12. P. 18.

Corollary.

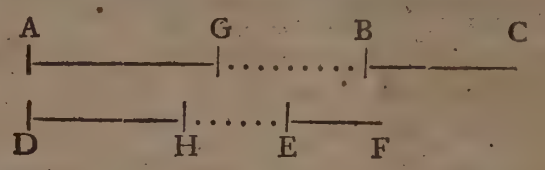
Hence as one Sphere is to another Sphere, ſo is a *Polyedron* deſcribed in that, to a like *Polyedron* deſcribed in this.

XXXI. *A Perpendicular Line,* DF, *drawn from* D, *the Centre of a Circle* ABC, *to* BC *the Side of a* Pentagon, *inscribed in the same Circle, is the half of these two Lines taken together,* (viz. *of the Side of the* Hexagon DE, *and the Side of the* Decagon EC,) *inscribed in the same Circle* ABC.

Figure XXX. *Construction.* MAke FG equal to FE, and draw GC.

Demonstration. Then CE is equal to CG; therefore the Angle CGE is equal to CEG, equal to ECD; therefore the Angle ECG is equal to EDC, equal to a quarter of ADC, equal to half CED, (or half BCD.) Consequently the Angle GCD is equal to ECG, equal to EDC: Wherefore DG is equal to GC, (or CE;) therefore DF is equal to EE (or DG) and EF, equal to half DE and CE together. *Which was,* &c. Eucl. El. L. 14. P. 1.

XXXII. *If two right Lines* AB *and* DE *be cut according to extreme and mean Proportion,* (*as* AB *to* AG, *so* AG *to* GB; *and as* DE *to* DH, *so* DH *to* HE;) *they shall be cut after the same manner, namely, into the same Proportion;* (*as* AG *to* GB, *so* DH *to* HE.)



Construction. MAke BC equal to BG, and EF equal to EH.

Demonstration. Then AB multiplied into BG is equal to five Squares of AG; wherefore the Square of AC is equal to four times AB multiplied in BG, together with the Square of AG, which is also equal to five Squares of AG. In like manner, the Square of DF shall be equal to five Squares of DH; therefore AC is to AG, as DF is to DH; whence (by Addition) AC and AG is to AG, as DF and DH is to DH; that is, twice AB is to AG, as twice DE is to DH: Consequently AB is to AG, as DE is to DH; whence (by Division) AG is to GB, as DH to HE. *Which was,* &c. Eucl. El. L. 14. P. 2.

XXXIII. *The same Circle* ABD *comprehends both* ABCDE, *the* Pentagon *of a* Dodecaedron, *and* LMN, *the Triangle of an* Icosaedron *in the same Sphere.*

Figure XXXI. *Construction.* DRaw the Diametre AG, and the right Lines AC and CG, and let IK be the Diametre of the Sphere, and the Square of IK equal to five Squares of OP; and make OP to PQ, as OQ to QP.

Demonstration. Because the Squares of AC and CG, which are equal to the Square of AG, equal to four Squares of FG, and the Squares of AB, FG, and CG, are equal; thence the Squares of AC and AB are equal to five Squares of FG. Moreover because CA is to AB, as AB is to CA, wanting AB; and OP to OQ, as OQ to QP; and so CA to OP, as AB to OQ: therefore three Squares of AC (or the Square of IK) is to five Squares of OP, (or the Square of IK,) as three Squares of AB, is to five Squares of OQ; therefore three Squares of AB is equal to five Squares of OQ. But because ML is the Side of a *Pentagon* inscribed in the Circle whose Ray is OP; thence fifteen Squares of RM, is equal to five Squares of ML, equal to five Squares of OP, and 5 of OQ, equal to three Squares of AC, and three of AB, equal to fifteen Squares of FG. Therefore RM is equal to FG, and consequently the Circle ABD is equal to the Circle LMN. *Which was,* &c. Eucl. El. L. 14. P. 3.

XXXIV. *If from* F *the Centre of a Circle, encompassing the* Pentagon *of a* Dodecaedron ABCDE, *a perpendicular Line* FG *be drawn to one Side of the* Pentagon CD, *the Rectangle contained under the said Side* CD,

CD *and the Perpendicular* FG, *being thirty times taken, is equal to the* Superficies *of the* Dodecaedron. *Alſo,*

If from the Centre L *of a Circle, inſcribing the Triangle of an* Icoſaedron HIK, *a perpendicular Line* LM *be drawn to one Side of the Triangle* HK, *the Rectangle contained under the ſaid Side* HK *and the Perpendicular* LM *being thirty times taken, ſhall be equal to the* Superficies *of the* Icoſaedron.

Conſtruction. DRaw FA, FB, FC, FD, FE; then ſhall the Triangles CFD, DFE, EFA, AFB, and BFC, be equal. Figure XXXII.

Demonſtration. But CD multiplied into FG, ſhall be equal to two Triangles CFD; therefore 30 times CD multiplied in GF, ſhall be equal to 60 times CFD, equal to 12 *Pentagons* ABCDE, equal to the *Superficies* of the *Dodecaedron*. *Which was*, &c.

Conſtruction. Draw LI, LH, LK.

Demonſtration. Then HK multiplied in LM, is equal to two Triangles LHK; therefore 30 times HK multiplied in LM, ſhall be equal to 60 Triangles HLK, equal to 20 times the Triangle IHK, equal to the *Superficies* of the *Icoſaedron*. *Which was*, &c. Eucl. El. L. 14. P. 4.

Corollary.

From hence it follows, That as CD multiplied into FG is in proportion to HK multiplied into LM, as the *Superficies* of the *Dodecaedron* is to the *Superficies* of the *Icoſaedron*.

XXXV. *The* Superficies *of a* Dodecaedron *hath to the* Superficies *of an* Icoſaedron *inſcribed in the ſame Sphere, the ſame Proportion that* H, *the Side of a Cube, hath to* AD, *the Side of an* Icoſaedron.

Conſtruction. LET the Circle ABCD encloſe both the *Pentagon* of a *Dodecaedron*, and the Triangle of an *Icoſaedron*, whoſe Sides are BD and DA; upon which, from the Centre E, let fall the Perpenciculars EF upon AD, and EGC upon DB; and draw CD. Figure XXXIII.

Demonſtration. Becauſe EC and CD is to EC, as EC is to CD; therefore EG (or half EC and CD) is to EF, (or half EC,) as EF is to FG, wanting EF, (or half CD.) But H is to BD, as BD is to H, wanting BD: Therefore H is to BD, as EG is to EF. Conſequently, H in EF is equal to BD in EG: Wherefore ſince H is to AD, as H in EF is to AD in EF; therefore H ſhall be to AD, as BD in EG ſhall be to AD in EF; and DB in EG ſhall be to AD in EF, as the *Superficies* of a *Dodecaedron* is to the *Superficies* of an *Icoſaedron*. *Which was*, &c. Eucl. El. L. 14. P. 5.

XXXVI. *If a right Line* AB *be cut in Extreme and Mean Proportion, then as the right Line* BF, *containing in power that which is made of the whole Line* AB, *and that which is made of the greater Segment* AC, *is to the right Line* E, *containing in power that which is made of the whole Line* AB, *and that which is made of the leſſer Segment* BC; *ſo is the Side of the Cube* BG, *to the Side of an* Icoſaedron BK, *inſcribed in the ſame Sphere with the Cube.*

Conſtruction. IN the Circle whoſe Semidiametre is AB, inſcribe BFGHI, the *Pentagon* of a *Dodecaedron*, and BKL the Triangle of an *Icoſaedron*; wherefore BG ſhall be the Side of a Cube inſcribed in the ſame Sphere. Figure XXXIV.

Demonſtration. Therefore the Square of BK is equal to three Squares of AB; and the Square of E is equal to three Squares of AC: Therefore the Square of BK is to the Square of E, as the Square of AB is to the Square of AC; and the Square of AB is to the Square of AC, as the Square of BG is to the Square of BF: Wherefore (by inverſion) the Square of BG is to the Square of IK, as the Square of BF is to the Square of E: Whence BG is to BK, as BF is to E. *Which was*, &c. Eucl. El. L. 14. P. 6.

XXXVII. *A* Dodecaedron *is to an* Icoſaedron *as the Side of a Cube is to the Side of an* Icoſaedron, *inſcribed in one and the ſame Sphere.*

Demonſtration. BEcauſe the ſame Circle comprehends both the *Pentagon* of a *Dodecaedron*, and the Triangle of an *Icoſaedron*, the Perpendiculars drawn from the Centre of the Sphere to the Plains of the *Pentagon* and Triangle, ſhall be equal

one to another. Therefore if the *Dodecaedron* and *Icosaedron* be conceived, divided into *Pyramids*, right Lines being drawn from the Centre of the Sphere to all the Angles, the Altitudes of all the *Pyramids* shall be equal one to the other. Wherefore since the *Pyramids* of equal highth are one to another, as their Bases; and the *Superficies* of the *Dodecaedron* is equal to 12 *Pentagons*, and the *Superficies* of the *Icosaedron* to 20 Triangles; the *Dodecaedron* shall be to the *Icosaedron*, as the *Superficies* of the *Dodecaedron* is to the *Superficies* of the *Icosaedron*; that is, as the Side of the Cube is to the Side of the *Icosaedron*. Eucl. El. L. 14. P. 7.

XXXVIII. *The same Circle* BCDE, *comprehends both the Square of the Cube* BCDE, *and the Triangle of the* Octoedron FGH, *inscribed in one and the same Sphere.*

Figure XXXV. *Demonstration.* LET A be the Diametre of the Sphere. Because the Square of A is equal to three Squares of BC, equal to six Squares of BI; and also the Square of A is equal to two Squares of GF, equal to six Squares of KF; thence shall BI be equal to KF: Therefore the Circle CBED is equal to the Circle GFH. *Which was*, &c. Eucl. El. L. 14. P. 8.

SECT. III.

Geometrical PROBLEMS.

I. *From a Point given on high,* (*as* A,) *to draw a right Line* (AI,) *perpendicular to a Plain below as* (BC.)

Figure I. *Construction.* IN the Plain BC draw any Line, as DE, to which from the Point A, let fall the Perpendicular AF; and likewise draw FH, in the Plain BC, cutting the said Line DE in F: Then let fall AI, perpendicular to FH, which AI shall be perpendicular to the Plain BC.

Demonstration. Through I let KIL be drawn parallel to DE, because DE is perpendicular to AF and FH; therefore DE shall be perpendicular to the Plain IFA, and so also KL is perpendicular to the same Plain; therefore the Angle KIA, is a right Angle: But the Angle AIF is also a right Angle; therefore AI is perpendicular to the Plain BC. *Which was to be done.* Eucl. El. L. 11. Prop. 11.

II. *In a Plain given* (BC,) *at a Point given therein* (A,) *to erect a Perpendicular Line* (AF.)

Figure II. *Construction.* FRom some Point without the Plain, as D, draw DE perpendicular to the Plain BC, then joining the Points A and E, by a Line AE, draw AF parallel to DE.

Demonstration. It is (by the Construction) apparent that AF is perpendicular to the Plain BC. *Which was to be done.* Eucl. El. L. 11. Pr. 12.

This and the preceding Problem, are practically performed by applying two Squares to the Point given.

III. *To make a Solid Angle* MHIK, *of three plain Angles*, A, B, *and* C, *whereof two* (*howsoever taken*) *are greater than the third. But it is necessary that those three Angles be less than four right Angles.*

Figure III. *Construction.* MAke AD, AE, BE, BF, CF, CG, equal one to the other, and of the subtended Lines DE, EF, and FG, (that is, of the Line HI, equal to the subtendent FG; of the Line HK, equal to the subtendent DE; and of the Line KI equal to the subtendent EF,) make the Triangle HIK, about which (by the 23d Problem) describe the Circle LHKI. But because AD is greater than HL, let the Square of AD, be equal to the Squares of HL and LM, and let LM be perpendicular to the Plain of the Circle HKI, and draw HM, KM, and LM. Wherefore since the Angle HLM is a right Angle; therefore the Square of MH, is equal to the Squares of HL and LM, equal to the Square of AD; and therefore MH is equal to AD. By the

the same reason, M K, M I, and A D, (that is, A E, E B, *&c.*) are equal. Therefore, since H M is equal to A D, and M I to A E, and E D to H I, the Angle A shall be equal to the Angle H M I, as likewise the Angle I M K equal to the Angle B, and the Angle H M K equal to the Angle C. Wherefore a Solid Angle is made at M, of the three given Plain Angles. *Which was to be done.*

Demonstration. A D is assumed to be greater than H I, but this is manifest; for if A D be equal or greater than H I, then the Angle A is equal or greater than the Angle H L I; in like manner B shall be equal or greater than H L K, and C equal or greater than K L I: Wherefore the three Angles A, B, C, shall either be equal to, or exceed four right Angles, (contrary to the *Hypothesis.*) Therefore rather let A D be greater than H L. *Which was to be demonstrated.* Eucl. El. L. 11. Pr. 23.

IV. *Upon a given right Line* (A B) *and at a Point given in it* (A,) *to make a Solid Angle* A H I K, *equal to a Solid Angle given,* C D E F.

Construction. FRom some Point (as F) of the given Angle, draw F G perpendicular to the Plain D C E, and draw the right Lines D F, F E, E G, G D, and C G; make A H equal to C D, and the Angle H A I equal to E D C, and A I equal to C E; and in the Plain H A I, make the Angle H A K equal to D C G, and A K equal to C G. Then erect K L perpendicular to the Plain H A I, and let K L be equal to G F, and draw A L. Then A H I L shall be a Solid Angle, equal to the Solid Angle given C D E F. *Which was to be done.* Eucl. El. L. 11. P. 26. Figure IV.

V. *Upon a right Line given* (A B) *to describe a* Parallelipipedon A K, *like, and in like manner situate with a Solid* Parallelipipedon *given,* (C D.)

Construction. OF the Plain Angles B A H, H A I, and B A I, which are equal to F C E, E C G, and F C G, make the Solid Angle A, equal to the Solid Angle C. Also make F C to C E, as B A to A H, and C E to C G, as A H to A I, (whence of equality F C is to C G, as A B to A I,) and so finish the *Parallelipipedon* A K, which shall be like to that which was given C D. Figure V.

Demonstration. For by the Construction, the Parallelogram B H is like to F E, and H I to E G, and B I to F G; and so the opposites of these, to the opposites of them. Therefore the six Plains of the Solid A K, are like to the six Plains of the Solid C D; and consequently A K and C D are like Solids. *Which was to be done.* Eucl. El. L. 11. P. 27.

VI. *Two unequal Circles* (A B C G, *and* D E F) *having the same Centre* (M,) *to inscribe in the greater Circle* (A B C G) *a Polygon Figure of equal and even sides, which shall not touch the lesser Circle* (D E F.)

Construction. THrough the Centre M draw the Line A C, cutting the Circle D E F in F; from F raise a Perpendicular F H. Divide the Semicircle A B C into two equal Parts in B, and the half thereof B C in I, and so continually, till the Arch I C become less than the Arch H C, from I let fall the Perpendicular I L. Figure VI.

Demonstration. It is manifest that the Arch I C measureth the whole Circle, and that the Number of Arches is even, and so that the subtended Line I C is the side of the *Polygon*, that may be inscribed without touching the lesser Circle D E F. For H G toucheth the Circle D E F, to which I K is parallel, and placed outwardly; wherefore I K does not touch the Circle D E F, much less do C I or C K, and the other sides of the *Polygon*, more remote from the Centre. *Which was to be done.* Eucl. El. L. 12. P. 16.

Corollary.

Observe that I K toucheth not the Circle D E F.

VII. *Two Spheres* A B C V, *and* E F G H, *consisting about the same Centre* D, *being given, to inscribe a Solid of many Sides* (*or* Polyedron) *in the greater Sphere* A B C V, *which shall not touch the Superficies of the lesser Sphere* E F G H.

Construction. LET both the Spheres be cut by a Plain passing by the Centre, making the Circles E F G H and A B C V, and the Diametres A C and B V drawn, cutting perpendicularly. In the Circle A B C V inscribe the equilateral *Polygon* Figure VII.

lygon V M N L C, *&c.* not touching the Circle E F G H. Then draw the Diametre N *a*, and erect D O perpendicular to the Plain A B C. By D O, and by the Diametres A C and N *a*, conceive Plains, D O C and D O N, erected, which ſhall be perpendicular to the Circle A B C V; and ſo in the Superficies of the Sphere, make the Quadrants D O C and D O N, in which let the right Lines C P, P Q, Q R, R O, and N S, S T, T *g*, *g* O, be fitted, equal, and of equal multitude with C N, N L, *&c.* make the ſame Conſtruction in the other Quadrants O L, O M, *&c.* and in the whole Sphere. Then I ſay the thing required is done.

Demonſtration. From the Points P and S, to the Plain A B C V, draw the Perpendiculars P X and S Y, which ſhall fall on the Sections A C and N *a*. Therefore, becauſe both the right Angles P X C, S Y N, and P C X, S N Y, inſiſting upon equal Circumferences, are equal, the Triangles alſo P C X and S N Y, are equiangular: Wherefore ſeing P C is equal to S N, alſo P X is equal to S Y, and X C equal to Y N; whence D X is equal to D Y; and therefore, as D X is to X C, ſo is D Y to Y N: therefore Y X and N C are parallel. But becauſe P X and S Y are equal, and ſince being Perpendicular to the ſame Plain A B C V, they are alſo parallels; therefore Y X and S P ſhall be equal and parallel. Whence S P and N C are parallel one to the other, and ſo the Quadrilateral N C P S, and by the ſame reaſon S P Q T, and T Q R G, as alſo the Triangle *g* R O, are ſo many Plains. In like manner the whole Sphere may be ſhewn full of ſuch Quadrilaterals and Triangles; wherefore the Figure inſcribed is a *Polyedron*.

From the Centre D draw D Z, perpendicular to the Plain N C P S, and join Z N, Z C, Z S, and Z P, becauſe D N is to N C, as D Y is to Y X, thence N C is greater than Y X (or S P) and likewiſe S P greater than T Q, and T Q greater than *g* R. And becauſe the Angles D Z C, D Z N, D Z S, and D Z P are right, and the Sides D C, D N, D S, and D P equal, and D Z common; thence Z C, Z N, Z B, and Z P, are equal one to the other; and conſequently, about the Quadrilateral N C P S, a Circle may be deſcribed, in which (becauſe N S, N C, and C P are equal; and N C greater than S P,) N C ſubtends more than the Quadrant; therefore the Angle N Z C, at the Centre, is obtuſe, and therefore the Square of N C is greater than twice the Square of Z C, for it is equal to the Squares of Z C and Z N. Let N I be drawn perpendicular to A C; therefore, ſince the Angle A D N (or the Angles D N C and D C N) is obtuſe, the half of it D C N, ſhall be greater than the half of a right Angle; and ſo that which remains of the right Angle C N I, ſhall be leſs than it; whence I N is greater than I C. Therefore the Square of N C (or the Squares of N I and I C) is leſs than twice the Square of I N; therefore I N is greater than Z C, and conſequently, D Z greater than D I; but the Point I, is without the Sphere E F G H, and ſo much more the Point Z; wherefore the Plain N C P S (whoſe next Point to the Centre is Z) does not touch the Sphere E F G H. And if a Perpendicular D ♪ be drawn to the Plain S P Q T, the Point ♪, and ſo all the Plain S P Q T, is yet farther removed from the Centre; which is alſo true in the other Plains of the *Polyedron*. Therefore the *Polyedron* O R Q P C N, *&c.* inſcribed in the great Sphere, does not touch the leſſer Sphere. *Which was to be done.* Eucl. El. 12. P. 7.

Corollary.

Hence it follows, *That if in any other Sphere a Solid* Polyedron, *like to the aboveſaid* Polyedron *be inſcribed, the proportion of the* Polyedron *in one Sphere, to the* Polyedron *in the other, is triple of that of the Diametres of the Spheres.*

For if right Lines be drawn from the Centres of the Spheres, to all the Angles of Baſes of the ſaid *Polyedrons*, then the *Polyedrons* will be divided into Pyramids, equal in Number, and like, whoſe *Homologous* Sides are Semidiametres of the Spheres, as appears, if the leſſer of theſe Spheres be conceived deſcribed within the greater about the ſame Centre.

For the right Lines drawn from the Centre of the Sphere to the Angles of the *Baſes*, will agree one to the other, by reaſon of the likeneſs of their *Baſes*, and ſo will like *Pyramids* be made. Wherefore, ſince every Pyramid in one Sphere, to every Pyramid like it in the other Sphere, hath proportion Triple to that of the *Homologous* Sides, that is, of the Semidiametres of the Spheres, and as one Pyramid is to one Pyramid, ſo all the Pyramids (that is, the Solid *Polydron* compoſed of theſe) are to all the Pyramids; that is, the Solid *Polyedron* compoſed of the others; therefore the *Polyedron* of one Sphere, ſhall have to the *Polyedron* of the other Sphere, proportion Triple of that of the Semidiametres, and ſo of the Diametres of the Spheres.

VIII. *To*

VIII. *To describe a Pyramid* EGFI, *and comprehend it in a Sphere given; and to demonstrate that the Diametre of the Sphere* AB *is in power* sesquialtera *of the Side* EF *of the Pyramid* EGFI.

Construction. ABout AB describe the Semicircle ADB; and let AC be equal to twice CB; from the Point C erect the Perpendicular Line CD, and join AD and DB; then at the interval of the Ray, HE equal to CD, describe the Circle HEFG, wherein inscribe the equalateral Triangle EFG; from H erect IH, equal to CA, perpendicular to the Plain EFG; produce IH to K, so that IK may be equal to AB, and join the right Lines IE, IF, and IG; so shall EFGI be the Pyramid required. Figure VIII.

Demonstration. For that the Angles ACD, IHE, IHF, and IHG, are right Angles; and CD, HE, HF, and HG, equal, and IH equal to AC: Therefore AD, IE, IF, and IG, shall be equal among themselves. But seeing AB (equal to twice CB) is to CB, as the Square of AC is to the Square of DC; thence shall the Square of AC be equal to twice the Square of CD; therefore the Square of AD is equal to the Squares of AC and CD; which is also equal to thrice the Square of CD, equal to thrice the Square of HE, equal to the Square of EF; and therefore AD, EF, IE, IF, and IG, are equal; and so the Pyramid EFGI is equilateral.

But if the Point C be placed upon H, and AC upon HI, the right Lines AB and IH shall agree, as being equal: Wherefore the Semicircle ADB being drawn about the *Axis* AB or IK, shall pass by the Points E, F, G; and so the Pyramid EFGI shall be inscribed in a Sphere. *Which was*, &c. Eucl. El. L. 13. P. 13.

Also it is manifest, That the Square of AB is to the Square of AD, as BA is to AC; and BA is to BC, as 3 is to 2.

Corollary.

1. The Square of AB is to the Square of HE, as 9 is to 2: For if the Square of AB be put 9, then the Square of AC (equal to the Square of EF) shall be 9; and consequently the Square of HE shall be 2.
2. If L be the Centre, then shall AB be to LC, as 6 to 1: For if AB be put 6, then AL shall be 3, and thence AC 4; wherefore LC shall be 1. Hence,
3. AB is to HI, as 6 is to 4; and 6 is to 4, as 3 is to 2. Whence,
4. The Square of AB is to the Square of HI, as 9 is to 4.

IX. *To describe an* Octaedron KEFGDL, *and comprehend it in the given Sphere wherein a Pyramid is; and to demonstrate that* AH, *the Diametre of the Sphere, is in power double of* AC, *the Side of the* Octaedron.

Construction. ABout AH describe the Semicircle ACH; and from the Centre B erect the Perpendicular BC, and draw HC and AC; then upon ED, equal to AC, make the Square EFGD, whose Diametres FD and EG, cut in the Centre I; From I draw IL, equal to AB, perpendicular to the Plain EFGD; produce IL to K, till IK be equal to IL, and join KE, KF, KG, KD, LE, LF, LG, and LD; then shall KEFGDL be the *Octaedron* required. Figure IX.

Demonstration. For AB, BH, FI, IE, *&c.* being Semidiametres of equal Squares, are equal one to the other; whence the *Bases* LF, LE, FE, *&c.* of the right angled Triangles LIE, LIF, FIE, *&c.* are equal; and consequently the eight Triangles LFE, LFG, LGD, LDE, KEF, KFG, KGD, and KDE, are equilateral; and make an *Octaedron* which may be inscribed in a Sphere, whose Centre is I, and IL, or AB the *Radius*, (because AB, IL, IF, IK, *&c.* are equal.) *Which was to be done.*

Moreover it is evident, that the Square of AH (or KL) is equal to twice the Square of AC (or LD.) *Which was to be demonstrated.* Eucl. El. L. 13. P. 14.

Corollary.

Hence it is manifest, That, 1. In the *Octaedron* the three Diametres BG, FD, and LK, do cut one the other perpendicularly in the Centre of the Sphere.

2. The three Plains EFGD, LEKG, and LFKD, are Squares cutting one another perpendicularly.
3. The *Octaedron* is divided into two like and equal Pyramids, EFGDL and EFGDK; whose common Base is the Square EFGD.
4. The opposite Bases of the *Octaedron*, are parallel one to the other.

X. *To*

X. *To describe a Cube* EFGHIKLM, *and comprehend it in the same Sphere wherein the former Figures were; and to demonstrate that* AB *the Diametre of the Sphere, is in power triple to* EF, *the Side of that Cube.*

Figure X. *Construction.* UPon AB describe the Semicircle ABC, and make AD one third part of AB; from D raise the Perpendicular DC, and join CB and CA: Then upon EF, being made equal to AC, make the Square EFGH; upon whose Plain let the right Lines EI, FK, HM, and GL, stand perpendicular, and all of them equal to EF; then connect them with the right Lines IK, KL, LM, and IM. The Solid EFGHIKLM is a Cube, as sufficiently appears from the Construction.

In the opposite Squares, EFKI and HGLM, draw the Diametres EK, FI, and HL, MG; by which let the Plains EKLH and FIMG be drawn, cutting one another in the Line NO, which shall divide the Diametres of the Cube EL, FM, GI, and HK, into two equal parts in the Point P, the Centre of the Cube.

Demonstration. Therefore P shall be the Centre of the Sphere passing by the angular Points of the Cube. Moreover, the Square of EL is equal to the Squares of EK and KL, or three times the Square of KL, equal to the Square of AC. But the Square of AB is to the Square of AC, as BA is to DA; and BA is to DA, as three is to 1. Therefore AB is equal to EL, wherefore we have made a Cube, *&c. Which was*, &c. Eucl. El. L. 13. P. 15.

Corollary.

1. Hence it is manifest, That all the Diametres of the Cube are equal one to the other, and do equally bisect one another in the Centre of the Sphere; and by the same means, the right Lines which conjoin the Centres of the opposite Squares, are equally bisected in the same Centre.
2. The Diametre of a Sphere containeth in power the Side of a *Tetraedron*, and of a Cube, *viz.* the Square of AB is equal to the Squares of BC and AC.

XI. *To describe an* Icosaedron ZGHIKFYVXRST, *and to incompass it in the Sphere wherein were contained the foresaid Solids; and to demonstrate that* FG, *the Side of the* Icosaedron, *is that irrational Line which is called a* Minor *Line.*

Figure XI. *Construction.* UPon AB, the Diametre of the Sphere, describe the Semicircle ADB, make BC to be one fifth part of BA; then from C erect the Perpendicular CD, and draw AD and BD.

At the distance of EF, equal to BD, describe the Circle EFKNG, wherein inscribe the equilateral *Pentagon* FKIHG: Divide equally into two parts the Arches FG, GH, *&c.* and join the right Lines FL, LG, *&c.* being the Sides of a *Decagon*: Then erect EQ, LR, MS, NT, OV, and PX, equal to PF, and perpendicular to the Plain FKNG, and connect RS, ST, TV, VX, XR; as also FX, FR, GR, GS, HS, HT, IT, IV, KV, and KX: Lastly, produce EQ, and take QY, equal to FL, and EZ equal to FL; and conceive the right Lines ZG, ZH, ZI, ZK, and ZF, to be drawn; as also YV, YX, YR, YS, and YT; so is the *Icosaedron* made.

Demonstration. For because EQ, IR, MS, NT, OV, and PX, are equal and parallel; also those Lines that join them, EL, QR, EM, QS, EN, QT, EO, QV, EP, and QX, are equal and parallel; and thence likewise LM, (or FG,) RS, MN, ST, *&c.* are equal one to the other; therefore the Plain drawn by EL, EM, *&c.* is distant equally from the Plain passing by QR, QS, *&c.* and the Circle QXRSTV, drawn from the Centre Q, is equal to the Circle EPLMNO, and RSTVX, is an equilateral *Pentagon.*

But EF, EG, EH, *&c.* and QX, QR, QS, *&c.* being conceived to be drawn; then because the Square of FR is equal to the Squares of FL and LR, or the Square of EF equal to the Square of FG; therefore FK and FG, and so also RS, FG, FR, RG, GS, GH, *&c.* shall be equal one to the other; and consequently the ten Triangles RFX, RFG, RGS, *&c.* are equilateral and equal. Moreover, because XQY is a right Angle, therefore the Square of XY is equal to the Squares of XQ and YQ, equal to the Square of VX or FG: Wherefore XY and VX, and likewise YV, YT, YS, YR, ZG, ZH, *&c.* are equal; therefore other ten Triangles are made equilateral and equal, both to one another, and to the ten former.

Moreover,

Moreover, Divide EQ equally in W, and draw the right Lines WF, WX, WV, and becauſe QX is equal to QV, and WQ the common Side; and FQX and EQV are right Angles; therefore WX ſhall be equal to WV: And by the ſame reaſon, all the Lines WX, WR, WS, WT, WV, WF, WG, WH, WI, and WK, are all equal. But becauſe ZQ is to QE, as QE is to ZE, therefore the Square of ZW is equal to five times the Square of EW, equal to the Squares of EQ (or EF) and EW, equal to the Square of WF: Therefore ZW is equal to WF. In like manner, WF is equal to YW; therefore the Sphere whoſe Centre is W, and WF the Ray, ſhall paſs by the twelve angular Points of the *Icoſaedron*.

Laſtly, Becauſe ZW is to EW, as ZY is to QE; and ſo the Square of ZW is to the Square of WE, as the Square of ZY is to the Square of QE: Therefore the Square of ZY is equal to five times the Square of EQ or BD. But the Square of AB is to the Square of BD, as AB is to BC; and AB is to BC as 5 to 1: Therefore ZY is equal to AB. *Which was to be done.*

Therefore if AB be put irrational, then EF, equal to the √ of the Square of AB, ſhall be alſo irrational; and conſequently FG, the Side of the *Pentagon*, and likewiſe of the *Icoſaedron*, is a *minor* Line. *Which was to be demonſtrated.* Eucl. El. L. 13. P. 16.

Corollary.

1. From hence is inferred, That the Diametre of the Sphere is in power quintuple of the Semidiametre of the Circle encompaſſing the five Sides of the *Icoſaedron*.
2. Alſo it is manifeſt, That the Diametre of the Sphere is compoſed of the Side of a *Hexagon*; that is, of the Semidiametre, and two Sides of the *Decagon* of a Circle encompaſſing the five Sides of the *Icoſaedron*.
3. It appears likewiſe, That the oppoſite Sides of an *Icoſaedron*, ſuch as RX and HI, are parallel: For RX is parallel to LP, which is parallel to HI.

XII. *To deſcribe a* Dodecaedron, *and comprehend it in the Sphere wherein the former Figures were comprehended; and to demonſtrate that the Side* RS *of the* Dodecaedron, *is an irrational Line of that ſort, which is called an* Apotome, *or Reſidual Line.*

Conſtruction. LET AB be a Cube inſcribed in the given Sphere, and let all the Sides thereof be divided equally in the Points E, H, F, G, K, L, *&c.* and join the right Lines KL, MH, HG, and EF; alſo make HI to be to IQ, as IQ is to QH; and take NO and NP, equal to IQ; then erect OR and PS, perpendicular to the Plain DB, and QT to the Plain AC; and let OR, PS, QT, be equal to IQ, NO, NP; whence DR, RS, SC, CT, and TD, being connected, DRSCT ſhall be a *Pentagon* of the *Dodecaedron* required: For draw NV parallel to OR, and having drawn NV out as far as the Centre of the Cube X, join the right Lines DS, DO, DP, CR, CP, HV, HT, and RX. Figure XII.

Demonſtration. Becauſe the Square of DO is equal to the Squares of DK (or KN) and KO, equal to three times the Square of ON, (or OK;) thence the Square of DR is equal to four times the Square of OR, equal to the Square of OP, or RS; therefore DR is equal to RS; and by the ſame reaſon DR, RS, SC, CT, and TP, are equal. But becauſe OR is equal and parallel to PS, therefore RS and OP, and conſequently RS and CD ſhall be alſo parallel; therefore theſe, with them that conjoin them, DK, CS, and VH, are in one and the ſame Plain. Moreover, becauſe HI is to IQ, as IQ is to QH; and IQ is to QH, as HN is to NV; and both TQ, HN, and QH, NV, are perpendicular to the ſame Plain; and ſo likewiſe Parallels THV ſhall be a right Line; and therefore the *Trapezium* DRSC, and the Triangle DTS, are in one Plain extended by the right Lines DC and TV; and DCTSR is a *Pentagon*, and that alſo equilateral. Farthermore, becauſe PK is to KN, as KN is to NP; and the Squate of DS is equal to the Squares of DP and PS, (or PN,) equal to the Squares of DK, PK and NP; thence the Square of DS is equal to the Square of DK and three Squares of KN, equal to four Squares of DK, (or DH,) equal to the Square of DC; therefore DS is equal to DC; whence the Triangles DRS and DCT are equilateral one to the other, and the Angle DRS equal to the Angle DCT, and likewiſe the Angle CSR equal to the Angle DCT; therefore the *Pentagon* DTCSR is alſo equiangular. Moreover, becauſe AX, DX, CX, *&c.* are Semidiametres of the Cube, thence XN is equal to IH, (or KN,) and ſo VX equal to KP; wherefore becauſe RVX is a right Angle, thence the Square of RX is equal to the Squares of XV and RV, (or NP,) equal to the Squares of KP and NP, equal

to three times the Square of KN, which is equal to the Square of AX, or DX, &c. Therefore RX, AX, and DX, and (by the ſame reaſon) XS, XT, and AX, are equal one to the other.

And if by the ſame method whereby the *Pentagon* DTCSR was made, twelve like *Pentagons* touching the twelve Sides of the Cube, be made, they ſhall compoſe a *Dodecaedron*; and a Sphere paſſing through their angular Points, whoſe Ray is AX, or RX,) ſhall comprehend that *Dodecaedron*. *Which was to be done.*

Laſtly, Becauſe KN is to NO, as NO is to OK; thence KL is to OP, as OP is to OK and PL: Therefore if AB, the Diametre of the Sphere, be ſuppoſed irrational, then ſhall KL, equal to one third part of the Root of AB, be alſo irrational; whence OP or RS, the Side of the *Dodecaedron* ſhall be a *reſidual* Line. *Which was to be demonſtrated.* Eucl. El. L. 13. P. 17.

Corollary.

From this Demonſtration it follows, that,

1. If the Side of a Cube be cut in extreme and mean Proportion, the greater Segment ſhall be the Side of the *Dodecaedron* deſcribed in the ſame Sphere.
2. If the leſſer Segment of a Line cut in extreme and mean Proportion, be the Side of the *Dodecaedron*, the greater Segment ſhall be the Side of the Cube inſcribed in the ſame Sphere.
3. It is alſo manifeſt, That the Side of the Cube is equal to the right Line which ſubtendeth the Angle of a *Pentagon* of the *Dodecaedron* inſcribed in the ſame Sphere.

XIII. *To find out the Sides of the five precedent Figures, and compare them together.*

Figure XIII. *Conſtruction.* LET AB be the Diametre of the Sphere given, and AEB the Semicircle; let AC be made equal to half AB, and AD to one third part of AB; then erect the Perpendiculars CE and DF, and alſo BG equal to AB; join AF, AE, BE, BF, and CG; and let fall the Perpendicular HI from H, the Point of interſection; and CK being taken equal to CI, from K erect the Perpendicular KL, and join AL: Laſtly, make AF in proportion to AO, as AO is to OF.

Demonſtration. Therefore as 3 is to 2, ſo is AB to DB; and as AB to DB, ſo is the Square of AB to the Square of BF, the Side of the *Tetraedron*.

And as 2 is to 1, ſo is AB to AC; and as AB to AC, ſo is the Square of AB to the the Square of the Side of the *Octaedron*.

Alſo, as 3 is to 1, ſo is AB to AD; and as AB to AD, ſo is the Square of AB to the Square of AF, the Side of an *Hexagon*.

Moreover, Becauſe as AF is to AO, ſo is AO to OF; thence ſhall AO be the Side of a *Dodecaedron*.

Laſtly, As BG (or twice BC) is to BC, ſo is HI to IC; therefore HI is equal to twice CI, equal to KI: therefore the Square of HI is equal to four times the Square of CI; conſequently the Square of CH is equal to five times the Square of CI; therefore the Square of AB is equal to five times the Square of KI; therefore KI, or HI, is a Ray of a Circle encloſing the *Pentagon* of an *Icoſaedron*; and AK, or IB, is the Side of a *Decagon* inſcribed in the ſame Circle; whence AL ſhall be the Side of a *Pentagon*, and alſo the Side of an *Icoſaedron*. Eucl. El. L. 13. P. 18.

Scholium.

Hence it is very apparent, That beſides the Five aforeſaid Figures, there cannot be deſcribed any other regular Solid Figure, (*viz.* ſuch as may be contained under ordinate and equal plain Figures;) for,

Three plain Angles (at leaſt) are required to the conſtituting of a Solid Angle, all which muſt be leſs than four right Angles: But ſix Angles of an equilateral Triangle, four of a Square, and ſix of a *Hexagon*, do ſeverally equal four right Angles; four of a *Pentagon*, three of a *Heptagon*, three of an *Octagon*, &c. do exceed four right Angles: Therefore only of three, four, or five equilateral Triangles, of three Squares, or three *Pentagons*, it is poſſible to make a Solid Angle: Wherefore beſides the five above mentioned Solid Bodies, there cannot any other Regular Body by formed.

The Proportions of the Sphere, and the Five Regular Figures inscribed in the same.

Let the Diametre of the Sphere be	2.00000
Then shall the Periphery, or Circumference, of the greater Circle be	6.28318
The *Superficies* of the greater Circle	3.14159
The *Superficies* of the Sphere	12.56637
The Solidity of the Sphere	4.18879
The Side of the *Tetraedron*	4.62299
The *Superficies* of the *Tetraedron*	4.6588
The Solidity of the *Tetraedron*	0.15132
The Side of the *Hexaedron*, or Cube,	1.1547
The *Superficies* of the *Hexaedron*	8.
The Solidity of the *Hexaedron*	1.5396
The Side of the *Octaedron*	1.11421
The *Superficies* of the *Octaedron*	6.9282
The Solidity of the *Octaedron*	1.33333
The Side of the *Dodecaedron*	0.71364
The *Superficies* of the *Dodecaedron*	10.51462
The Solidity of the *Dodecaedron*	2.78516
The Side of the *Icosaedron*	1.05146
The *Superficies* of the *Icosaedron*	9.57454
The Solidity of the *Icosaedron*	2.53615

XIV. *In a Cube given,* ABGHDCFE, *to describe a Pyramid,* AGEC.

Construction. FRom the Angle C draw the Diametres CA, CE, CG, and connect them with the Diametres AG, GE, EA, which are equal among themselves. Figure XIV.

Demonstration. For that all of them are the Diametres of equal Squares, therefore the Triangles CAG, CGE, CEA, and EAG, are equilateral and equal; and consequently, AGEC is a Pyramid, which insists upon the Angles of the Cube, and therefore is inscribed in the same. *Which was,* &c. Eucl. El. L. 15. P. 1.

XV. *In a Pyramid given,* ABDC, *to describe an* Octaedron, EGKIFH.

Construction. BIsect the Sides of the *Pyramid* in the Points E, I, F, K, G, H, which join with the right Lines EF, FG, GE, *&c.* Figure XV.

Demonstration. Forasmuch as these are all equal one to the other, consequently the eight Triangles EHI, IHK, *&c.* are equilateral and equal, and so make an *Octaedron* described in the given Pyramid. *Which was,* &c. Eucl. El. L. 15. P. 2.

XVI. *In a given Cube* CHGBDEFA, *to inscribe an Octaedron* NPQSOR.

Construction. COnnect the Centres of the Squares N, P, Q, S, O, R, with the twelve right Lines NP, PQ, QS, *&c.* Figure XVI.

Demonstration. All these being equal among themselves, do make eight equal and equilateral Triangles; wherefore the *Octaedron* NPQSOR is inscribed in the given Cube. *Which was,* &c. Eucl. El. L. 15. P. 3.

XVII. *In an* Octaedron *given,* ABCDEF, *to inscribe a Cube.*

Construction. LET the Sides of the Pyramid EABCD, whose Base is the Square ABCD, be equally bisected by the right Lines LM, MN, NO, and OL. Figure XVII.

Demonstration. All these are equal and parallel to the Sides of the Square ABCD, wherefore the Quadrilateral LMNO is a Square. In like manner, if the Sides of the Square LMNO be equally bisected in the Points G, H, K, I, and GH, HK, IK, IG, connected, GHIK shall be a Square. And if in the other five Pyramids of the *Octaedron*, the Centres of the Triangles be in the same sort connected with right Lines; then other Squares will be described like and equal to the Square GHKI: Wherefore six such Squares shall make a Cube, which shall be described within an *Octaedron*, for that its eight Angles touch the eight Bases of the *Octaedron* in their Centres. *Which was,* &c. Eucl. El. L. 15. P. 4.

XVIII. *In an* Icoſaedron *given to inſcribe a* Dodecaedron.

Figure XVIII.

Conſtruction. LET A B C D E F be a Pyramid of the *Icoſaedron*, whoſe Baſe is the *Pentagon*, A B C D E, and the Centre of the Triangles G, H, I, K, L, which connect with the right Lines G H, H I, I K, K L, and L G, then ſhall G H I K L, be a *Pentagon* of the *Dodecaedron* to be inſcribed.

Demonſtration. For the right Lines F N, F O, F P, F Q, paſſing by the Centres of the Triangles, do equally divide their Baſes into two Parts; therefore the right Lines M N, N O, O P, P Q, and Q M, are equal one to another, whence alſo the Angles M F N, N F O, O F P, P F Q, and Q F M, are equal; therefore the *Pentagon* G H I K L, is equiangular, and conſequently equilateral, being F G, F H, F I, F K, and F L, are equal. And if in the other eleven Pyramids of the *Icoſaedron*, the Centres of the Triangles be in like ſort connected by right Lines, then will *Pentagons* equal and like to the *Pentagon* G H I K L, be deſcribed. Wherefore twelve ſuch *Pentagons* ſhall conſtitute a *Dodecaedron*, which alſo ſhall be deſcribed in the *Icoſaedron*, ſeeing that the 20 Angles of the *Dodecaedron* conſiſt upon the Centres of 20 Triangles or *Baſes* of the *Icoſaedron*; whereby it appears, that a *Dodecaedron* is deſcribed in an *Icoſaedron*. *Which was to be done.* Eucl. El. 15. Pr. 5.

OF

OF THE

MENSURATION

OF

Geometrical Figures,

PLAINS and SOLIDS.

PART III.

Proœme.

THE Method which I shall observe in the Mensuration of Geometrical Figures, Plains, and Solids, shall be threefold; viz.

I. *By* Decimal Arithmetick, *of all the other ways the most exact and absolute, according to* Book I. Part II.

II. *By* Artificial *or* Logarithmical Arithmetick, *according to* Book I. Part III.

III. *By* Logarithmical Scales *and* Compasses, *as the most expeditious of any other, according to* Book I. Part IV.

SECT. I.

Of PLAINS.

I. *To find the* Area (*or Superficial content*) *of a Square.*

LEt A B C D be a Geometrical Square, whose four sides are all equal, each containing 24 Foot and a half, (or 24.5 Foot.) Figure I.

The RULE.

The Side of the Square multiplied in it self giveth the Area *or Superficial content thereof.*

1. *By Decimal Arithmetick.*

Thus 24.5 multiplied by 24.5, produceth 600.25, and so many Square Feet are contained in the Square A B C D, whose Side is 24.5.

2. *By Logarithms.*

The Logarithm of 24.5 is	1.3891661
The same again,	1.3891661
The Logarithm of 600.25	2.7783222

The Content of the Square, as before.

3. *By Scale and Compasses.*

Extend the Compasses from 1 to 24.5, the same extent will reach from 24.5, to 600.25, the Superficial Feet contained in the Square.

Note, that in what kind of Measure you do measure your Sides, of the same kind will the Content found be, as if the Sides be measured in Feet and parts of a Foot, the *Area* will be Feet also; if in Yards the *Area* will be Yards; if in Perches Perches; if in Miles Miles, *&c.*

II. *To*

II. *To find the* Area *of a Parallelogram or long Square.*

Figure II. LET EFGH be a Parallelogram whose Length E F, is 36.25 Poles or Perches, and its Breadth 12.5.

The RULE.

The Length being multiplied into the Breadth produceth the Area.

1. *Arithmetically.*

The Length 36.25 being multiplied by 12.5, the Product will be 453.125 Perches, for the *Area* or Content of the Parallelogram.

2. *By Logarithms.*

The Logarithm of 36.25, the Length is	1.5593080
The Logarithm of 12.5, the Breadth is	1.0969100
The Logarithm of 453.125 the *Area*	2.6562180

3. *Instrumentally.*

Extend the Compasses from 1, to 36.25, the same extent will reach from 12.50, to 435.125, the Content in Perches, as before.

III. *To find the* Area *of a* Rhombus.

Figure III. LET KLMN be a *Rhombus*, whose Sides is 7 yards, the *Area* or Content is required.

The RULE.

The Area *of a* Rhombus *is found by multiplying the Length of one of the Sides, into the Length of a Perpendicular, let fall from one of the obtuse Angles upon the opposite Side, as the Perpendicular* K O, *whose Length is* 5.25 *yards.*

1. *Arithmetically.*

The length of one of the Sides, 7 yards being multiplied into 5.25, the length of the Perpendicular K O, produceth 36.75 yards for the *Area* or Content of the *Rhombus.*

2. *By Logarithms.*

The Logarithm of 7, the Side is	0.8450980
The Logarithm of 5.25, the Perpendicular is	0.7212893
The Logarithm of 36.75 for the *Area*	1.5663873

3. *Instrumentally.*

Extend the Compasses from 1 to 7, the same extent will reach from 5.25, to 36.75, for the *Area*, as before.

IV. *To find the* Area *or Superficial Content of a Triangle.*

EVery Triangle is equal to half that Parallelogram, whose Length and Breadth are equal to the Base and Perpendicular of the Triangle.

To find the *Area* or Content of any Triangle, there are three ways, or Rules.

1. *The whole Perpendicular multiplied into the whole Base, produceth the double Content of the Triangle.*
2. *Half the Base multiplied into the whole Perpendicular, will give the* Area. *Or,*
3. *The whole Base multiplied into half the Perpendicular produceth the* Area.

Any side of a Triangle (in this Case) may be made the Base, but most commonly the longest Side is taken for the Base, and a Line let fall Perpendicularly from the Angle opposite to that Base, is called the Perpendicular, as in Examples following.

Figure IV.
1. *If the Triangle have one right Angle, then the longest Side which contains the right Angle is called the Base, and the shorter the Perpendicular, as in the (first Triangle* A B C,*) where* A B *is the Base, and* C A *the Perpendicular.*
2. *If the Triangle be equilateral, then any side may be made the Base, and a Perpendicular let fall from the Angle opposite to that Side shall be the Perpendicular, as in the (second Triangle* D E F*) where* E F *is made the Base, and* D G *the Perpendicular.*
3. *If the Triangle have two equal Sides, then it is best to make the unequal Side the Base, and a Line Perpendicular from the Angle opposite thereunto, shall be Perpendicular, as in the third Triangle* (H K L,) *where* K L *is the Base, and* H M *the Perpendicular.*
4. *If all the Sides be unequal and the Triangle have not a right Angle, then it is most convenient to make the longest Side the Base, and a Line let fall perpendicularly from the obtuse Angle opposite thereunto, the Perpendicular, as in the fourth Triangle,* N O P, *where* O P *is made the Base, and* N Q *the Perpendicular.*

1. *Arith-*

1. *Arithmetically.*

Theſe things premiſed, let A B C be a Triangle, whoſe Baſe A B let be 8 Paces, and the Perpendicular C A 4 Paces; 8 multiplied into 4, (the whole Baſe into the whole Perpendicular,) produceth 32, the half whereof 16 is the content of the Triangle in Square Paces, (by the Firſt Rule.)

Again, 4 (the whole Baſe) multiplied into 4, (the half Perpendicular,) produceth 16 for the *Area* or Content, as before, (by the Second Rule.)

Or, 8 the (whole Baſe) multiplied into 2 (the half Perpendicular) produceth 16 for the *Area*, (by the third Rule.)

2. *By Logarithms.*

The Logarithm of 8 (the Baſe) is	0.9030900
The Logarithm of 4 (the Perpendicular) is	0.6020600
The Logarithm of 32	1.5051500

Whoſe half 16, is the *Area* of the Triangle A B C (by the firſt Rule.)

Again,

The Logartihm of 4 (half the Baſe) is	0.6020600
The Logarithm of 4 (the whole Perpendicular) is	0.6020600
The Logarithm of 16 the *Area*, (by the Second Rule.)	1.2041200

Again,

The Logarithm of 8 (the whole Baſe) is	0.9030900
The Logarithm of 4 (the half Perpendicular) is	0.3010300
The Logarithm of 16 (the *Area*,) (by the third Rule.)	1.2041200

3. *Inſtrumentally.*

Extend the Compaſſes from 1 to 8, the ſame extent will reach from 4 to 32; the ſame extent will reach from 4 to 32; the half whereof is 16 for the *Area*, (by the firſt Rule.)

Or, Extend the Compaſſes from 1 to 4, the ſame extent will reach from 4 to 16, the *Area*, (by the Second Rule.)

Or, Extend the Compaſſes from 1 to 2, the ſame extent ſhall reach from 8 to 16, the *Area*, (by the third Rule.)

And by either of theſe three ways. the *Area* of

The Triangle	DEF HKL NOP	will be found to contain	38.25 31.25 39.875

V. *To find the* Area, *or Content, of any Triangle, by knowing the three Sides thereof, without the help of the Perpendicular.*

LET A B C be a Triangle, whoſe Sides are A B 20, A C 34, and B C 42; and let the Content thereof be required. The Figure V.

RULE:

Add all the three Sides together, and take half of that Number; then ſee what each Side ſeverally differs from that half Summ: which done, multiply that half Summ by any one of thoſe Differences, and the Product thereof by another Difference, and that Product by the third Difference; the Square Root of this Product is the Area *of the Triangle.*

1. *Arithmetically.*

The Side	AB is	20	Differences	28
	AC is	34		14
	BC is	42		06

The Summ — 96

The half Summ — 48

Being thus prepared, multiply 48 (the half Summ) by 28, (the firſt Difference) the Product will be 1344, this multiplied by 14, (the ſecond Difference) produceth 18816; and this multiplied again by 6, (the third Difference,) the Product is 112896, the Square Root whereof is 336, which is the *Area* of the Triangle A B C.

2. *By*

2. *By Logarithms.*

The RULE.

To the Logarithm of the half Summ of the Sides, add the Logarithms of the ſeveral Differences of each Side from the half Summ, half the Summ of theſe Logarithms, is the Logarithm of the Area *of the Triangle.*

Example.

The half Summ of the Sides	48	1.6812412
The firſt Difference	28	1.4471580
The ſecond Difference	14	1.1461280
The third Difference	6	0.7781512
Their Summ		5.0526784
The half Summ		2.5263392

Which is the Logarithm of 336, for the *Area*, or Content, of the Triangle, as before.

3. *Inſtrumentally.*

Having, as before, found the Summ, the half Summ, and the Differences of the reſpective Sides from the half Summ:

Firſt, Extend the Compaſſes from 1 to 48, (the half Summ,) that extent will reach the ſame way, from 28 (the firſt Difference) to 1344.

Secondly, From this Point take the diſtance to 1, and that extent will reach from 14 (the ſecond Difference) to 18816.

Thirdly, From this Point take the diſtance to 1, and that extent will reach from 6 (the third Difference) to 112896; the half diſtance between 1 and this Number, will fall upon 336, for the *Area* of the Triangle.

VI. *To find the* Area, *or Content ſuperficial, of a* Trapezia.

Figure VI. LET DEFG be a *Trapezia*, whoſe Content is required: From D to F, the longeſt Diametre (or Diagonal) of the Figure, draw the right Line DF, which ſhall reduce the *Trapezia* GDEF into two Triangles, EDF and GDF, the Line DF being the common Baſe to both Triangles: Then from the Angles E and G, let fall the Perpendiculars EH and GK.

To meaſure this *Trapezia*, there is the ſame varieties as there were in Triangles; and theſe are the *RULES.*

1. *Multiply the whole Baſe by the Summ of the Perpendiculars, the Product will be the double* Area.
2. *The Summ of the Perpendiculars multiplied in half the Baſe, produceth the juſt* Area.
3. *The whole Baſe multiplied by half the Summ of the Perpendiculars, the Product ſhall be the* Area.

1. *Arithmetically.*

Example: In this *Trapezia* DEFG, the common Baſe DF is 73 Pole or Perches, the Perpendicular GK is 28 Perches, and the Perpendicular EH 18 Perches; their Summ is 46, and their half Summ 23 Perches. Now,

1. If you multiply 73 (the whole Baſe) by 46 (the Summ of the two Perpendiculars) the Product will be 3358; the half whereof 1679 is the *Area* of the *Trapezia* in Square Perches, (by the Firſt Rule.)

2. If you multiply (the Summ of the Perpendiculars) 46, by 36.5 (the half of the Baſe) the Product will be 1679, for the *Area*, (by the Second Rule.) Or,

3. If you multiply 73 (the whole Baſe) by 23 (the half Summ of the Perpendiculars) the Product will be 1679, for the *Area* alſo, (by the third Rule.)

2. *By Logarithms.*

1. The Logarithm of 73 is	1.8633228
The Logarithm of 46 is	1.6627578
Their Summ is	3.5260806

The Logarithm of the double *Area*, whoſe half 1679 is the juſt *Area* (by the firſt Rule.)

2. The Logarithm of 46 is	1.6627578
The Logarithm of 36.5 is	1.5622929
Their Summ is	3.2250507

Which is the Logarithm of 1679, the *Area*, (by the Second Rule.) Or,

3. The

3. The Logarithm of 73 is ——— 1.8633228
The Logarithm of 23 is ——— 1.3617278
Their Summ is ——— 3.2250506
The Logarithm of 1679, the *Area*, (by the Third Rule.)

3. *Instrumentally.*

1. Extend the Compasses from 1 to 73, the same extent will reach from 46 to 3358, the half whereof is 1679, the *Area*, (by the First Rule.) Or,

2. Extend the Compasses from 1 to 46, the same extent will reach from 36.5 to 1679, the *Area*, (by the Second Rule.) Or,

3. Extend the Compasses from 1 to 23, the same extent will reach from 73 to 1679, the *Area*, (by the third Rule.)

VII. *To find the* Area *of an irregular Plot of many unequal Sides and Angles.*

SUch irregular Figures which consist of many Sides and Angles, (being none of the Forms before treated of,) cannot be measured at one operation, as all the former were, but will require many operations, according to the number of the Sides and Angles, which (in all irregular Plots or Figures, as well as in regular,) are always of an equal number; and before they can be measured, they must be reduced into some of the former regular Figures, as in the *Geometrical Problems* in this Book there are several ways shewn how that may be done. But to come to an Example:

Let A B C D E F G be an irregular Figure, as a Piece of Land, a Wood, *&c.* Now before this Piece can be measured, it must be reduced into Triangles, or *Trapezias*, or both, by drawing Lines from Angle to Angle within the Plot: Thus this irregular Plot A B C D E F G is divided into two *Trapezias* and one Triangle; for the Line drawn from F to C, cuts off one *Trapezia*, namely C D E F by the jagged Line C F; also another *Trapezia* A B C F is cut off by the other jagged Line A F, and then there is left of the irregular Figure, only the Triangle F A G. Figure VII.

Now by these Lines the whole Plot is reduced into two *Trapezias*, *viz.* C D E F, and A B C F, and the Triangle F A G. For this Triangle A F G it may be measured by the IV. Section, and the two *Trapezias* by the VI. Section hereof, the Bases and Perpendiculars being first measured, as in the Plot they are: And being thus prepared we will proceed to the casting of it up in Poles or Perches, supposing it to be the Plot of a Wood, or other Piece of Land.

1. *Arithmetically.*

First, We will begin with the Triangle FAG, whose Base AG is 34, and Perpendicular F O 10; now 34 the Base, multiplied by half the Perpendicular 5, produceth 170 Perches, for the content of the Triangle FAG.

Secondly, For the *Trapezia* A B C F, which is divided into two Triangles by the pricked Line AC, which is the common Base to the two Triangles A B C and A F C, which contains in length 35 Perches; also the Perpendicular F I contains 24 Perches, and the Perpendicular B H 8 Perches, the Summ of them both is 32, and the half is 16 Perches; which multiplied by 35 the whole Base, the Product is 560 Perches, for the content of the *Trapezia* A B C F.

Thirdly, For the *Trapezia* C D E F, which is divided into two Triangles by the pricked Line C E, which is a common Base to the two Triangles C D E and C F E, which contains in length 32 Perches; also the Perpendicular F L contains 17 Perches, and D K 13, the Summ of both being 30 Perches, and the half thereof 15 Perches, which multiplied by 32, the whole Base, the Product will be 480 Perches, for the Superficial content of the *Trapezia* C D E F.

The Result of the whole Plot.

		Whole Base.	½ *Perches.*	*Area.*
The	Triangle F A G	34	5	170
	Trapezia A B C F	35	16	560
	Trapezia C D E F	32	15	480
			The Summ	1210

Which Summ 1210, is the *Area* of the whole Plot A B C D E F G in Square (Poles, Rods, or) Perches; and that is 7 Acres, 2 Roods, and 10 Perches. But of this no more here, till we come particularly to treat of Surveying of Land.

2. *By Logarithms.*

1. The Logarithm of 34 is ——— 1.5314789
The Logarithm of 5 is ——— 0.6989700
Their Summ ——— 2.2304489 the Logarithm of 170 FAG

2. The Logarithm of 35 is ——— 1.5440680
The Logarithm of 16 is ——— 1.2041200
Their Summ ——— 2.7481880 the Logarithm of 560 ABCF

3. The Logarithm of 32 is ——— 1.5051500
The Logarithm of 15 is ——— 1.1760912
Their Summ ——— 2.6812412 the Logarithm of 480 CDEF
The whole *Area*, or Content of the Plot in Perches ——— 1210

3. *Instrumentally.*

1. Extend the Compasses from 1 to 34, the same extent will reach from 5 to 170; which set down for the *Area* of the Triangle FAG.

2. Extend the Compasses from 1 to 35, and that extent will reach from 16 to 560; which set down for the *Area* of the *Trapezia* ABCF.

3. The extent from 1 to 32, will reach from 15 to 480, for the *Area* of the *Trapezia* CDEF; all which being added together, their Summ will be 1210 Perches, for the *Area* or Content of the whole irregular Plot or Figure ABCDEFG in Square Perches.

VIII. *To find the* Area, *or Superficial Content of any Equilateral and Equiangular* Polygon.

REgular *Polygons* are such Figures, whose Sides and Angles are equal: and *Polygons* are denominated from the Number of their Sides and Angles; as,

If the Figure consist of { 5, 6, 7, 8, 9, 10, 12 } equal Sides and Angles, it is called a { *Pentagon.* *Hexagon.* *Heptagon.* *Octagon.* *Enneagon.* *Decagon.* *Dodecagon.* }

And every of these and such like Figures, (of how many equal Sides and Angles soever consisting,) are equal to that long Square (or Parallelogram,) whose length and breadth is equal to half the Perimetre of the *Polygon*, and a Perpendicular let fall from the Centre thereof to the middle of any of the Sides of the *Polygon:* Therefore for the measuring of them this is the *RULE:*

Multiply the whole Perimetre of the Polygon *by the half of the Perpendicular, (or half the Perimetre by the whole Perpendicular,) the Product shall give the* Area *of the* Polygon.

1. *Arithmetically.*

Figure VIII. *Example* 1. In the *Pentagonal Polygon* (of five Sides) ABCDE, where each Side is 25, and the Perpendicular HZ is 17.2. Each Side being 25, the whole Perimetre is 5 times 25, that is 125, which being multiplied by 8.6 half the Perpendicular, the Product will be 1075, for the *Area* or Superficial Content of the *Polygon*. Or, if you multiply 62.5 (half the Perimetre) by 17.2 (the whole Perpendicular) the Product will be 1075, for the *Area*, as before.

2. *By Logarithms.*

The Logarithm of 125 (the Periphery) is ——— 2.0969100
The Logarithm of 8.6 (half the Perpendicular) is ——— 0.9344984
The Logarithm of 1075, for the *Area*, ——— 3.0314084

Or,

The Logarithm of 62.5 (half the Perimetre) is ——— 1.7958800
The Logarithm of 17.2 (the whole Perpendicular) is ——— 1.2355284
The Logarithm of 1075, the *Area*, ——— 3.0314084

3. *Instrumentally.*

Extend the Compasses from 1 to 125, the same entent will reach from 8.6 to 1075, for the *Area* of the *Polygon* ABCDE. Or,

The extent from 1 to 62.5, will reach from 17.2 to 1075, as before.

Example

Example 2. In the *Octagonal Polygon* A B C D E F G H, each of whose Sides is 12, and so the whole Perimetre 96, this multiplied by 7.24 (half the Perpendicular) the Product 695.04 is the *Area* of the *Polygon*: Or half the Perimetre 48, by 14.48 (the whole Perpendicular O 12) will likewise produce 695.04, for the *Area*. Figure IX.

So likewise,

The Logarithm of 48 (half the Perimetre) is ——— 1.6812412
The Logarithm of 14.48 (the whole Perpendicular) is ——— 1.1607686
The Logarithm of 695.04, (the *Area*,) ——— 2.8420098

Also,

The extent of the Compasses from 1 to 48, will reach from 14.48 to 695.04, for the *Area*. And in this manner may any regular *Polygon* of how many Sides soever be measured.

IX. *Of the Mensuration of Circles and Parts of Circles.*

THE true Proportion of the Diametre of a Circle of to the *Peripheria* or Circumference thereof, was never as yet discovered, notwithstanding many learned Men have laboured very far therein; among which, that excellent *Van Culen* hath hitherto carried away the greatest commendation, he having set forth the same in *Decimal* terms to 36 Places, which are engraven upon his Toomb-stone in St. *Peter*'s Church in *Leyden*; which Numbers are these,

Diametre 1.000000.00000.00000.00000.00000.00000.00000
Circumference 3.141592.65358.97932.38462.64338.32795.02884

Of which large Number these six Places 3.14159 answering to the Diametre 1.00000 may be sufficient. *Archimedes* hath these small Numbers, as 7 to 22, so the Diametre of any Circle to the Circumference thereof; which two Numbers (though most in use) are not so exact, for as 7 is to 22, so is 1.00000 to 3.14285, being greater than those of *Van Culen* by $\frac{126}{1000000}$. But *Adrianus Metius* the elder, sometime Geometrician to the Estates of the United *Belgick* Provinces, hath in a Book of his, put out by *Simon à Quercu*, demonstrated the Proportion to be less than $\frac{377}{120}$, and greater than $\frac{333}{106}$, the intermediate Proportion of them is $\frac{355}{113}$; which is a little greater than what *Van Culen* found, whose difference is less than $\frac{1}{1000000}$; and therefore I shall make use of these excellent Numbers throughout this Discourse, viz. as 113 is to 355, so is the Diametre of any Circle to the *Periferia* or Circumference of the same. And now I proceed to Examples.

§ I. *The Diametre of a Circle being given, to find the Circumference.*

The *RULE*:

Multiply the Diametre of the Circle (always) by 355, *and that Product (always) divide by* 113, *the Quotient shall give you the Circumference.*

Example. Let A B C D be a Circle, whose Diametre B G let be 14.00, and let the Circumference thereof be required.

1. *Arithmetically.*

Multiply the Diametae 14 by 355, the Product will be 4970; which Product divide by 113, and the Quotiont will be 43.98, for the Circumference of the Circle A B C D.

2. *By Logarithms.*

The Logarithm of 113 ——— 2.0530784
The Logarithm of 355 ——— 2.5502283
The Logarithm of 14 (the Diametre) ——— 1.1461280
The Summ ——— 3.6963563
Subtract ——— 2.0530784
The Logarithm of 43.98, the Circumference ——— 1.6432779

3. *Instrumentally.*

Extend the Compasses from 113 to 355, the same extent will reach from 14 to 43.98, for the Circumference or *Periferia* of the Circle A B C D.

§ II. *The* Peripheria *or Circumference of a Circle being given, to find the Diametre.*

The RULE.

Multiply the Peripheria *or Circumference (always) by* 113, *that Product divided by* 355; *shall give the Diametre of the Circle.*

1. *Arithmetically.*

Example: Let the *Peripheria* of the Circle A B C D, be 43.982, and let the Diametre thereof B C be sought, multiply 43.982 (the Circumference) by 113, the Product will be 14; for the length of the Diametre B C.

2. *By Logarithms.*

The Logarithm of 355 is	2.5502283
The Logarithm of 113 is	2.0530784
The Logarithm of 43.982 the Circumference	1.6432779
Their Summ	3.6963563
From which subtract	2.5502283
The rest is the Logarithm of 14	1.1461280

Which is the length of the Diametre of the Circle B C.

3. *Instrumentally.*

The extent of the Compasses from 355 to 113, will reach (the same way) from 43.982 (the *Peripheria* of the Circle) to 14 the Diametre thereof.

§ III. *The Diametre of a Circle being given, to find the* Area.

The RULE.

Multiply the Diametre of the Circle in its self, and that Summ again multiply (always) by 22, *and that Product (always) divide by* 28, *the Quotient will be the Circles* Area, *or Superficial Content.*

1. *Arithmetically.*

Example. Let the Diametre of the Circle B C be 14, which multiplied in it self produceth 196, this Product multiplied by 22, makes 4312; which being divided by 28, the Quotient is 154 *ferè*, for the *Area* of the Circle, A B C D.

2. *By Logarithms.*

The Proportion is, as 28 is to 22, so is the Square of the Diametre, to the *Area.*

The Logarithm of 28	1.4471580
The Logarithm of 22	1.3424227
The Logarithm of 14 the Diametre	1.1461280
The same again,	1.1461280
Their Summ	3.6346787
From which subtract	1.4471580
The Logarithm of 154 *ferè*, the *Area.*	2.1875207

3. *Instrumentally.*

Extend the Compasses from 1 to 14 the Diametre, and set the same distance from 14 forwarder upon the Line, it will reach to 196, then take the distance from 28, downwards to 22; that distance will reach from 196, downwards to 154, the *Area*, fore. *ferè*

§ IV. *The Circumference or* Peripheria *of a Circle being given, to find the* Area.

The RULE.

Multiply the Peripheria *of the Circle in it self, and that Product multiply by* 7 (*always,*) *and this last Product divide by* 88 (*always,*) *the Quotient shall be the* Area *of the Circle.*

1. *Arithmetically.*

Example: Let the Circumference of the Circle A B C D be 43.982, which multiplied in it self produceth 1934.416, this Product multiplied by 7, produceth 13540.912, (or for brevity) 13541 which divided by 88, the Quotient will be 154 *ferè*; (or rather 153.938,) for the *Area* of the Circle A B C D.

By Logarithms.

The Proportion is, as 88 is to 7, so is the Square of the Circumference to the *Area.*

The

The Logarithm of 88 is ——— 1.9444827
The Logarithm of 7 is ——— 0.8450980
The Logarithm of 43.982, (the *Peripheria*) is ——— 1.6432749
The same again ——— 1.6432749
The Summ ——— 4.1316478
From which Subtract ——— 1.9444827
The Logarithm of 153.938 for the *Area* ——— 2.1871651

3. *Instrumentally.*

Extend the Compasses from 1 to 43.982, and set the same distance forward upon the Line, it will reach 1934.416; then take the distance from 88 downwards to 7, the same extent will reach from 1934.416 to 153.938.

§ V. *To find the* Area *of a Semicircle, the Diametre being given.*

The RULE.

Multiply half the Diametre in it self, and that Product multiply (always) by 22, *this last Product divided by* 14, *the Quotient shall be the* Area *of the Semicircle.*

1. *Arithmetically.*

Example: In the Semicircle A B C, whose Diametre B C is 14, and its half A O is 7, this 7 multiplied in it self produceth 49, multiply this 49 (always) by 22, the Product will be 1078; which divide by 14, and the Quotient will be 77, for the *Area* of the Semicircle.

2. *By Logarithms.*

The Proportion is, as 14 is to 22, so is the Square of the Semidiametre to the *Area.*
The Logarithm of 14 is ——— 1.1461280
The Logarithm of 22 is ——— 1.3424227
The Logarithm of 7 (the Semidiametre) is ——— 0.8450980
The same again ——— 0.8450980
Their Summ ——— 3.0326187
From which subtract ——— 1.1461280
The Logarithm of 77 for the *Area* ——— 2.8864907

3. *Instrumentally.*

Extend the Compasses from 1 to 7, the half Diametre A O, setting that distance forward from 7, it will reach to 49; this done, take (always) the distance between 14 and 22, and that extent will reach the same way, from 49 to 77, the *Area* of the Semicircle.

§ VI. *To find the* Area *of a Quadrant or Quarter of a Circle, whose Diametre is known.*

The RULE.

Multiply the Semidiametre of the Circle in it self, and that Product multiply (always) by 11, *and dividing this last Product (always) by* 14, *the Quotient shall be the* Area *of the Quadrant.*

1. *Arithmetically.*

Example: In the Quadrant A B O, whose Sides A O or B O are 7, multiply 7 in it self it produceth 49, which 49 multiplied (always) by 11, it produceth 539; which divide (always) by 14, the Quotient will be 38.5 for the *Area* of the Quadrant A O B.

2. *By Logarithms.*

The Proportion is, as 14 is to 11, so is the Square of the Semidiametre to the *Area* of the Quadrant.
The Logarithm of 14 is ——— 1.1461280
The Logarithm of 11 is ——— 1.0413927
The Logarithm of 7 (the Semidiametre A O) is ——— 0.8450980
The same again is ——— 0.8450980
Their Summ is ——— 2.7315887
From which subtract ——— 1.1461280
The Logarithm of 38.5 (the *Area* of the *Quadrant* ——— 1.5854607

§ VII. *Of*

§ VII. *Of the Menſuration and finding the* Area, *or ſuperficial Content of any Part, Portion, or Segment of a Circle.*

The Menſuration of Parts or Portions of Circles, of which there are ſeveral ſorts, Examples of all which do depend upon this following Theorem.

Every portion of a Circle which is contained under two Semidiametres, and an Arch Line (moſt properly called a Sector of a Circle) is equal to that long Square or Parallelogram, whoſe length and breadth are equal to the Semidiametre of the Circle, and of a Line equal to half the Arch Line of the Sector or Segment.

Therefore before Sectors or other Portions of Circles can be meaſured, a right Line muſt be found out, which ſhall be equal to the Arch Line contained between the two Semidiametres; and that is taught in the Geometrical Problems of this Book, however, I will here ſhew another way how to find a right Line which ſhall be equal to half any Arch Line, which will be more ſuitable to our preſent occaſion than the other.

Figure X. Now to find the length of a right Line, which ſhall be equal to half the Arch Line F G C, do thus: Firſt draw the right Line F C, which divide into four equal Parts in the Points H K L; then take one of thoſe Parts, as L C, and ſet it from C to G; then a Line drawn from G to H, ſhall be a right Line equal to half the Arch Line F G C, and will be found to contain 2.75 of ſuch parts as the Semidiametre of the Circle O F contains 7. Now,

To find the Area *of the Sector* F O C.

1. *Arithmetically.*

Multiply the Semidiametre 7 by 2.75, the half Arch Line G H, the Product will be 19.25, for the *Area* of the Sector of the Circle F O C.

2. *By Logarithms.*

The Logarithm of 7, the Diametre, is——————0.8450980
The Logarithm of 2.75, half the Arch Line is——————0.4393327
Their Summ——————1.2844307

Which is the Logarithm of 19.25, the *Area* or content of the Sector F O C.

3. *Inſtrumentally.*

The extent of the Compaſſes from 1 to 2.75, (the length of the half Arch Line) will reach, the ſame way from 7 (the Semidiametre) to 19.25 the *Area* of the Sector. Or, the extent of the Compaſſes from 1 to 7, will reach from 2.75 to 19.25, the *Area*, as before.

Note, But if only the Segment of a Circle were to be meaſured, as the Segment S T U, (Figure XI.) where there are no Semidiameters, you muſt then (by the Geometrical Problems) find the Centre of that Circle at X, of which S T U is an Arch, and then (by the former part of this Section,) find the *Area* of the whole Sector X, S T U, which done, (by the fourth Section hereof) find the *Area* of the Triangle X S U, which being ſubtracted from the *Area* of the whole Sector, X S T U, the remainder will be the *Area* of the Segment S T U.

Example: Let S T U be a Segment of a Circle to be meaſured, having found the Centre thereof at X, draw the two Semidiametres X S and X T; alſo find the half Arch Line (by the firſt part of this Section) T R: Then (by ſome Scale) meaſure the length of the Semidiametre X S or S U, which ſuppoſe to be 7; alſo by the ſame Scale, meaſure the half Arch Line R T, which let be 8.25, which multiplied by 7 the Semidiametre, the Product will be 57.75, for the *Area* of the whole Sector X S T U. This done, by the ſame Scale meaſure the Chord Line S U, which will be found to be 12.93. And now you have the three Sides of the *Iſoſcheles* Triangle X S U, by which (by the fifth Section hereof) you may find the *Area* of the Triangle to be 17.33, which being ſubtracted from 57.75, (the *Area* of the whole Sector) there will remain 40.42, for the *Area* of the Segment S T U.

X. *Of the Menſuration of Mixt, or Compound Plain Figures.*

Mixt or Compounded Figures are ſuch as conſiſt partly of right, and partly of Circular Lines; ſuch as are the Figures 12, 13, 14, and infinite others, all which muſt firſt be reduced into ſome of the former Figures, and then meaſured by the Rules before delivered.

Examples

Example of Figure XII.

This Figure consisteth of a Semicircle B C D, of two Sectors, B O B, D O E, and of a Triangle O A E: The Semicircle must be measured by the fifth Section of the 9th hereof, the two Sectors by the seventh of the 9th hereof, and the Triangle by the 4th or 5th hereof.

Example of Figure XIII.

This Figure consisteth of two Segments of different Circles, F G H, L I K, and of a *Trapezia* F H L K. The two Segments must be measured by the 8th Section of the 9th hereof, and the *Trapezia* by the 6th hereof.

Example of Figure XIV.

This Figure consisteth of two parts of Segments of Circles and a *Trapezia*; the whole Sectors M N S, and P R Q must first be found by the 7th of the 9th hereof, and the two Triangles M T S and U Q R, must be deducted from them; then must you measure (by the 6th hereof) the *Trapezia* N P T V, whose *Area* added to the former two parts of Sectors, shall give the whole *Area* or Superficial Content of the whole Mixt or Compounded Figure M N P Q.

And thus have you the true and most accurate way of measuring all sorts of Plain Geometrical Figures, consisting of right Lines, circular Lines, or Mixt or Compounded of both; by which *Board*, *Glass*, *Tapestry*, *Tiling*, *Floring*, *Painting*, *Plaistring*, *Paving*, and Land may be measured; particular Examples of all which shall be shewed in their due Places: But before I proceed to the Mensuration of Solids, (which is the next thing in order to be handled,) I shall lay before you these delectable and profitable Proportional Conclusions in the Circle, and that to a *Millimillinary* solution of the Truth, from my loving Loving Friend (now deceased) *John Wybard*, M. D.

Figure XV.

1. The Proportion of the Diametre of a Circle to the	Circumference,		3.141593 *ferè*.
	Side of the Square equal to the Circle, (which is the proper Squaring of the Circle.)	is as 1.000000 to	.886227 *ferè*.
	Side of the inscribed Square,		.707107, *ferè* $\sqrt{q\ \frac{1}{2}}$.

Contrarily,

2. The Proportion of the Circumference of a Circle to the	Diametre		.318318 *ferè*.
	Side of the Square equal to the Circle, (which is the second proper Quadrature of a Circle,)	is as 1.000000 to	.282095
	Side of the inscribed Square,		.225079

Also,

3. The Proportion of the Quadrat	Diametral, or Circumscribing.	is to the Circle it self, as 1.000000 to	.785398
	Circumferential		.079577
4. The Proportion of a Circle to the Quadrat	Diametral or Circumscribing	is as 1.000000 to	1.273240 *ferè*.
	Circumferential,		12.566371 *ferè*.

Having thus (in the foregoing Sections) prescribed and exemplified the Method for measuring of all sorts of right Lined Figures, regular and irregular; as also of Circles and parts of Circles, and of Figures Compounded of both, and that by three several means, *viz.* *Arithmetically* by *Decimals*, *Artificially* by *Logarithms*, and *Instrumentally* by *Scales* and *Compasses*; and at the close thereof laid down several Proportions relating to the Square, and the Circles inscribed and Circumscribed about the same. I think it

it convenient now to present my Reader with some other usefull and delectable Proportions relating to the equilateral Triangle, and the *Pentagon*, and of their Sides, Dimetres, Diagonals, *&c.* and also the Circles inscribed within, and those that are circumscribed about the same Figures; the equilateral Triangle and the *Pentagon* being the Sides (or Plains) of the Five Regular *Pythagorean*, or *Platonick* Bodies; of which Bodies (after I have prescibed the method for measuring of Solids, as I have already done of Plains,) I shall largely insist; but to proceed according to this prescribed Method.

I. *Of the Equilateral Triangle.*

Figure XVI. Let AKE be an equilateral Triangle, AEBK the Circumscribing Circle, CVD the Inscribed Circle, O the Centre of them both, AV the Perpendicular, OA the Ray or Semidiametre of the Circumscribing, and OV of the Inscribed Circle.

First, *For the Linear Proportions, or the Proportions of the Lines in the Ordinate, or Regular Triangle* AEK, I say that,

1. The Side AE, or AK, is to the Ray of the Circle,	Circumscribing AO	as 1 to	.57735 $\sqrt{q}\,\frac{1}{3}$.
	Inscribed OV.		.288675
	And so to the Diametre or Perpendicular AV,		.86602 $\sqrt{q}\,\frac{3}{4}$ (.75)
	And *vice versa*, the Perpendicular AV, is to the Side AE or AK,		1.1547
2. The Ray of the Circumscribing Circle AO, is to the	Side AE, or AK,	as 1 to	1.73205 $\sqrt{q}$ 3
	Ray of the Inscribed Circle OV,		0.5 *Dupla*
	And consequently to the Diametre, or Perpendicular AV,		1.5 *Subsesquialtera*
3. The Ray of the Inscribed Circle OV, is to the	Side AE, or AK,	as 1 to	3.4641
	Ray of the Circumscribing Circle AO.		2. *Subdupla*
	And so to the Perpendicular, or Diametre, AV,		3. *Subtripla*

Whereby it appears that the Ray (or *Radius*) of the Circumscribing or containing Circle is $\frac{2}{3}$ of the Perpendicular of the Triangle; and the Ray of the inscribed Circle is $\frac{1}{3}$ of the same.

Secondly, *For Superficial Proportions in the Ordinate Triangle, or Proportions of Superficial Dimensions.*

1. The Square of the	Side, Diametre,	is to the Triangle it self, as 1 to	.4330127 .57735 $\sqrt{q}$. $\frac{3}{4}$ (.75)

Which latter is the same with that of the Side to the Ray of the Circumscribing Circle.

And contrarily,

2. The whole Triangle is to the Square of its	Side, Diametre,	as 1 to	2.309401 1.73205 $\sqrt{q}$. 3.

And so consequently,

3. The	Side, Diametre,	is to the Side of the Square equal to the Triangle, as 1 to	.658037 .759836 *ferè.*

Hitherto of the Linear and Superficial Content of the Ordinate or Equilateral Triangle. Next,

II. *For*

II. *For the like Proportions in the Ordinate or Regular* Pentagon.

Let ABCDE be an Ordinate (or Equilateral) *Pentagon*, whose Side is AB, or BC, *&c.* AH is the Perpendicular, AG is the Ray, *Radius*, or Semidiametre of the circumscribing or containing Circle; GH the Ray of the inscribed or contained Circle; AH the Perpendicular; EB the Diagonal or Subtense of the *Pentagonal Angle*; which cutteth the Diametre or Perpendicular AH, by extreme and mean Proportion in the Point F. Figure XVII.

First, *Of the Linear Dimensions.*

1. The Side of *Pentagon* AB, is to the	Ray of the Circle { Circumscribing AG	as 1 to	.85065
	Ray of the Circle { Inscribed GH		.68819
	And so to the Diametre or Perpendicular of the *Pentagon* AH,		1.53884
	Diagonal, or Subtense of the *Pentagonal* Angle BE.		1.618034 *fere.*
2. The Ray of the Circumscribing Circle AG, is to the	Side of the *Pentagon* AB,	as 1 to	1.7557
	Ray of the Inscribed Circle GH,		0.809017
	(And so the *Pentagonal* Diametre AH)		1.809017
	Pentagonal Diagonal BE,		1.902113
3. The Ray of the Inscribed Circle GH is to the	Side of the *Pentagon* AB	as 1 to	1.453085
	Ray of the Circumscribing Circle AG,		1.236068 *fere.*
	(And so the Diametre or Perpendicular AH)		2.236068
	Diagonal or *Hypotenuse* BE,		2.351141
4. The Diametre of the *Pentagon* AH is to the	Side AB,	as 1 to	.649839
	Ray of the Circle { Circumscribing AG,		.552786
	Ray of the Circle { Inscribed GH		.44721
	(Of which two Circular Rays it is composed.)		
	Pentagonal Diagonal BE,		1.051462
	Greater Segment or Portion FH,		0.618034
	Lesser Segment or Portion FA,		.3819661
5. The Diagonal or *Hypotenuse* BE, is to the	Side AB	as 1 to	.618034
	Ray of the Circle { Circumscribing AG,		.525731
	Ray of the Circle { Inscribed GH,		.425325
	Diametre AH,		.951056
	Greater Segment thereof FH,		.587785
	Lesser Segment thereof AF,		.363271

Ramus in his XIX. Book of *Geometry* giveth these Proportions for the *Pentagon*, *viz.*

1. The Side of the *Pentagon* is to the
   - Ray of the Circumscribing Circle, as 6 to 5 *sesquiquinta*, which in Decimals is — as 1 to .83333
   - Ray of the Inscribed Circle, as 3 to 2 *sesquialtera*, which in Decimals is — as 1 to .66666
   - Diametre, as 2 to 3 *subsesquialtera*, which is irrational, *viz.* — as 1 to 1.50000

2. The Ray of the Circumscribing Circle is to the
   - *Radius* of the Inscribed Circle, as 5 to 4 *sesquiquarta*, which in Decimals is irrational, *viz.* — as 1 to .80000
   - Diametre of the *Pentagon*, as 5 to 9 *subsuperquadriquinta*, which in Decimals is irrational, *viz.* — as 1 to 1.80000

And so the *Radius* of the Inscribed Circle is to the said Diametre, as 4 to 9, *subdupla-sesquiquarta*, decimally irrational, *viz.* as 1 to 2.25000.

Secondly, *Of the Superficial Dimensions.*

1. The Square of the { Side AB / Diametre AH / Diagonal BE } is to the *Pentagon* it self, as 1 to { 1.720477 / 0.726543 *fere.* / 0.657164 *fere.* }

And Contrariwise,

2. The *Pentagon* it self is to the { Side AB, / Diametre AH, / Diagonal BE, } as 1 to { 0.581234 / 1.37638 / 1.52169 }

And so,

3. The { Side AB / Diametre AH / Diagonal BE } is to the Square of the *Pentagons* Side, as 1 to { 1.31167 / 0.85237 / 0.810656 }

These three last Proportions are the most exact and proper terms for an Ordinate *Pentagon* that may be; but *Ramus* hath them thus:

1. The Square of the
   - Side of the *Pentagon* is to the *Pentagon* it self, as 1 to $1\frac{2}{3}$, *sub-super-bitertia*, decimally defective, *viz.* — as 1 to 1.6666
   - Diametre of the *Pentagon* is to the *Pentagon* it self, as 1 to $\frac{20}{27}$, *superseptupartiens-vigesim.* in Decimals excessive, *viz.* — as 1 to 0.740740

2. The *Pentagon* it self is to the
   - Square of the Side thereof, as 1 to $\frac{2}{3}$, *super-bitertia*, decimally excessive, *viz.* — as 1 to .600000
   - Square of the Diametre, as 1 to $\frac{7}{20}$, *subsuper-septupartiens-vigesimas*, decimally defective, *viz.* — as 1 to 1.35000

And so,

3. The *Pentagonal* { Side / Diametre } is to the Side of the *Pentagonal* Square, as 1 to { 1.29099 deficient. / 0.86066 redundant. }

Let

Let these Proportions suffice for the Lineal and Superficial Mensurations of the Ordinate, Regular, and Equilateral Triangle, and *Pentagon*; and so I shall conclude this Tractate of Linear and Superficial Measure with the following Theorem, which I long since received of my honoured Friend Mr. *S. Foster*, sometime Professor of *Astronomy* in *Gresham*-College in *London*.

THEOREM.

The Circumference of a Circle is less than the Perimetre of any Ordinate Polygon *that Circumscribes the Circle, but greater than the Diametre of any Ordinate* Polygon *inscribed within the Circle.*

Declaration. THE Ark CBE is less than DBF, greater than CGE; that is, less than the Tangent of the Ark, greater than the Subtense. Figure XVIII.

Demonstration. 1. Greater than the Subtense, because between the two Points C and E, the Subtense CGE is shorter than any other curved Line; and therefore, than the Ark CBE drawn through the same terms, *per Defin.* 4. *Eucl.* 1.

2. Less than the Tangent DF; for by that which *Archimedes* proves in his first Proposition of the *Quadrature of a Circle*, a Triangle whose Side is AC, whose Base is equal to the Ark CBE, will be equal to the Sector CBEA. But the Triangle, whose Side or Altitude is AB, equal to AC, and whose Base is the Tangent DBF, is greater than the forenamed Sector by the exterior Figure CBEFD, and yet the Altitude (or *Radius*) AB or AC is in both equal: Therefore it will follow, that DF is greater than the Ark CBE. *Which was to be demonstrated.*

SECT. II.

Of SOLIDS.

I. *To find the Crassitude or Solid Content of a Cube.*

LET ABCDEFG be a Cube, whose Side AB, *&c.* let be 5 Foot, (or 5 of any other measure,) to find the Solid Content thereof, this is the Figure I.

RULE.

Multiply the Side of the Cube in it self, and that Product multiply again by the Side of the Cube; this last Product shall be the Crassitude, or Solid Content of the Cube.

1. *Arithmetically.*

Thus 5 (the Side) multiplied in 5, produceth 25, this 25 multiplied again by 5, produceth 125, and so many Cubical, or Solid Feet, (if measured by Feet) are contained therein.

2. *By Logarithms.*

The Logarithm of 5 (the Side) is——————————0.6989700
Which multiplied by 3, (or set down 3 times and added) produceth—2.0969100
The Logarithm of 125, which is the Solid Content of the Cube.

3. *Instrumentally.*

Extend the Compasses from 1 to 5, the same extent will reach from 5 to 25, and the same extent will reach from 25 to 125; which is the Solid Content of the Cube as before.

In this manner if the side of a given Cube were 25.75 Inches, and the Solidity were required.

1. *Arithmetically.*

25.75 multiplied by 25.75, produceth 663.0625, and that multiplied again by 25.75, produceth 17073.85 Inches for the Solidity of the Cube.

2. *By Logarithms.*

The Logarithm of 25.75 (the Side) is——————————1.4107772
Which multiplied by 3, produceth——————————4.2323316
Which is the Logarithm of 17073.85; for the Characteristick of the Logarithm being 4, denotes the Integer Number to consist of 5 Places, and so the Solidity is 17073.85.

3. *Instrumentally.*

The extent from 1 to 25.75, will reach from 25.75, to 663.06, and from 663.06, to 17073.85, as before. But these great Numbers are not fit to be applyed to Instrumental Practices.

II. *To find the Solidity of a* Parallelipipedon, *or (long Cube.)*

Figure II. LET HKLMNOP be a *Parallelipipedon*, whose Side of the Square at the end HO, *&c.* is 9, and the length thereof HK, *&c.* is 15 and a half, (or 15.5) To find the Solidity whereof this is the

RULE.

Multiply the Side of the Square at the end into it self, and that Product by the length, the last Product shall be the Solidity.

1. *Arithmetically.*

Multiply 9 (the side) by 9, the Product will be 81, which multiplied by 15.5 (the length) produceth 1255.5, for the Solidity of the *Parallelipipedon.*

2. *By Logarithms.*

The Logarithm of 9 (the Side) is	0.9542425
The same again	0.9542425
The Logarithm of 15.5 (the length)	1.1903317
The Logarithm of 1255.5 (the Solidity)	3.0988167

3. *Instrumentally.*

The extent from 1 to 9, will reach from 9 to 81, and the extent from 81 to 1, shall reach from 1 forward to 1255.5 for the Solidity as before.

III. *To find the Solidity of an* Oblong Parallelipipedon.

Figure III. I call that an *Oblong Parallelipipedon*, whose end is a Parallelogram, whose length and breadth are not equal, as the *Oblong Parallelipipedon* PQRSTVX, whose sides at the end are VP 3, and VT 5, and the length RQ, 22.5. To find the Solidity of such a Piece, this is the

RULE.

Multiply the length of the side at the end by the breadth thereof, and that Product multiply again by the length, the last Product shall give the Solidity.

1. *Arithmetically.*

Example: Multiply VT 5 the length at the end, by VP 3 the breadth at the end, the Product will be 15, which multiplied by RQ 22.5, the length produceth 337.5, for the Solidity.

2. *By Logarithms.*

The Logarithm of 5 (the length) is	0.6989700
The Logarithm of 3 (the breadth) is	0.4771212
The Logarithm of 22.5 (the whole length) is	1.3521825
Their Summ	2.5282737

Which is the Logarithm of 337.5, the Solidity of the *Parallelipipedon*, as before.

3. *Instrumentally.*

The extent of the Compasses from 1 to 5, will reach the same way from 3 to 15, and the extent from 1 to 15, will reach from 22.5 to 337.5, the Solidity.

IV. *To find the Solidity of a squared Stone or piece of Timber, whose end is a regular* Polygon, *of 5, 6, 8, or 9 equal Sides.*

Figure IV. LET ABCDEFGHK, be a Squared piece of Stone or Timber of 7 Sides, each Side being 4.5, and the length thereof 41. To find the Solidity of such a piece, this is the

RULE.

Find the Area *or Superficial content of the* Polygon *at the end, (by the 7th beforegoing,) which had, multiply that* Area *by the length of the piece, and that Product shall be the Solidity.*

1. *Arithmetically.*

Example: In this piece of 7 Sides, each Side being 4.5; I find the Perpendicular SO to be 4.67, and the half Perimetre of the *Polygon* to be 15.75, which multiplied together produce 73.5525; and this Product again multiplied by the length, CG or DH 18, produceth 1325.94, for the Solidity.

2. *By*

2. *By Logarithms.*

The Logarithm of 4.67, (the Perpendicular) is ——— 0.6693169
The Logarithm of 15.75 (half the Perimetre) is ——— 1.1972805
The Logarithm of 18 (the Length) is ——— 1.2552725
Their Summ ——— 3.1218699

Which is the Logarithm of 1324 *ferè*, which is the Solidity.

3. *Instrumentally.*

Extend the Compasses from 1 to 4.67, (the Perpendicular) that extent will reach from 15.75 (the half of the Perimetre) to 73.55, and the extent from 1 to 73.55, will reach from 73.55 to 1324 the Solidity of the piece as before.

V. *To find the Solid Content of a Prisme.*

LET ABCEHO be a Prisme, to find the Solidity whereof this is the RULE. Figure V.

Multiply the Area *of the Base by the length, that Product shall be the Solidity of the Prisme.*

1. *Arithmetically.*

Example: In this Prisme, whose Sides of the Triangular Base BE, *&c.* is 18, first (by the 4th or 5th beforegoing) find the *Area* of the Triangle BCE, (which is equal to AOH, the other end of the Prisme) 140.3, which multiply by the length OC, equal to AB, the length 72, the Product will be 10101.6, the Solidity of the Prisme.

2. *By Logarithms.*

The Logarithm of 140.3, (the *Area* of BCE) is ——— 2.1470577
The *Area* of OC or AB 72, (the length) is ——— 1.8573325
Their Summ ——— 3.0043902

Which is the Logarithm of 10101.6, the Solidity of the Prisme ABCEHO, as before.

3. *Instrumentally.*

The extent of the Compasses from 1 to 140.3, (the *Area* of the Base) will reach (the same way) from 72 (the length OC) to 10101.6, the Solidity.

VI. *To find the Solidity of a* Pyramis.

LET ABCDE be a Quadrantal *Pyramis*, (for a *Pyramis* is denominated from the Number of the equal Sides of the Base thereof, as here four; and therefore Quadrantal or Quadrilateral,) whose Solidity is required, to find which Solidity this is the RULE. Figure VI.

The Area *of the Base* BCDE *(found by the first hereof) multiplied by one third part of the Altitude, giveth the Solid Content.*

1. *Arithmetically.*

Example: Let the sides of the Base of the *Pyramis*, BC, CD, *&c.* be 18, and the height thereof AE 540: First, multiply 18 (the Side of the Base) in it self, the Product will be 324, (for the Superficial Content of the Base,) which Product being again multiplied by 180, (one third part of 540, the height AE,) the Product will be 58320; for the Solid Content of the *Pyramis.*

2. *By Logarithms.*

The Logarithm of 334, (the *Area* of the Base BCDE) is ——— 2.5105450
The Logarithm of 180, (one third part of 540, the Altitude) is ——— 2.2552725
Their Summ ——— 4.7658175

Which is the Logarithm of 58320, the Solidity of the *Pyramis* ABCDE, as before.

3. *Instrumentally.*

Extend the Compasses from 1 to 324, (the *Area* of the Base) the same extent will reach from 180 (one third part of 540 the Altitude,) to 58320, the Solid content of the *Pyramis.*

VII. *To find the Solid Content of Cone.*

THE finding of the Solid Content of a Cone, differeth nothing from the finding of that of a *Pyramis*, and the Figures themselves differ only in this, that the Base of a *Cone* is (always) a Circle, and the Base of a *Pyramis* is (always) either an equilateral

teral Triangle, a Square, or a regular *Polygon* of any Number of equal Sides, as 5, 6, 7, 8, &c. and to find the Solidity of such a *Cone*, this is the

RULE.

Multiply the Area *or Content of the Circular Base, by one third part of the height of the* Cone; *the Product shall be the Solid Content of the* Cone.

1. *Arithmetically.*

Figure VII. *Example:* Let O X Y S be a *Cone*, whose Base X Y S (found by the first or fourth Sections of the 9th hereof) let be 346.5, and the height thereof O X or O S 540, multiply 346.5, the *Area* of the Base, by 180, (one third part of the Altitude O S,) the Product will be 62370, for the Solidity of the *Cone*.

2. *By Logarithms.*

The Logarithm of 346.5, (the *Area* of the Base) is——————2.5397032
The Logarithm of 180, (one third part of the Altitude) is——————2.2552725
Their Summ is——————4.7949757

Which is the Logarithm of 62370, the Solidity of the *Cone*.

3. *Instrumentally.*

The extent of the Compasses from 1, to 346.5, will reach from 180 to 62370.

VIII. *To find the Solid Content of a Segment, or* Frustum *of a* Pyramis *or* Cone.

THE Frustum of a *Pyramis* or *Cone*, is when the smaller end thereof next the Vertical Point, is cut off parallel to the Base, and in this form are most squared Timber Trees, or round Timber growing, and such is the *Frustum* of the *Pyramis* R S T V, B C D E, (Figure VI.) whose Solidity let be required, and to find the same this is

RULE.

Multiply the Areas *at the two ends one into the other, and find the Square Root of that Product; then add the two Squares and this Root together, and multiply their Summ by one third Part of the Altitude, the Product shall be the Solidity of the* Frustum.

1. *Arithmetically.*

Example. The Square of the Base B C D E is 324, and the Square of the lesser Base, R S T V, (the Side being 12,) is 144; these two Squares multiplied together produce 46656, the Square Root whereof is 216; add these three together (*viz.* 144, 216, and 324,) their Summ will be 684; which multiply by 60 (one third part of 120 the Altitude of the *Frustum* R S T V, B C D E,) the Product will be 410404, which is the Solid Content of the *Frustum* R S T V, B C D E.

2. *By Logarithms.*

The Logarithm of (the Square B C D E) 324 is——————2.5105450
The Logarithm of (the Square R S T V) 144 is——————2.1583625
The Summ——————4.6689075
The half Summ——————2.3344537

Which is the Logarithm of the Root 216.

The Logarithm of the Summ of the three Squares 684 is——————2.8350561
The Logarithm of 60 (one third part of the highth) is——————1.7781512
The Summ——————4.6132073

Which is the Logarithm of 41040, which is the Solid Content of the *Frustum*, as before.

3. *Instrumentally.*

Extend the Compasses from 1 to 324, (the greater Square B C D E,) the same extent will reach from 144 (the lesser Square R S T V) to 46656; the half distance between 1 and 46656 will fall upon the Line at 216; these three (324, 216, 144) being added together make 684; wherefore extend the Compasses from 1 to 684, (the Summ,) and that extent will reach from 60 (one third of the highth) to 41040, the Solidity of the *Frustum*.

IX. *To find the Solid Content of a Cylinder.*

Figure VIII. LET A B C D be a Cylinder whose Solid Content is required, to find which this is the

RULE:

The Area *of the Base multiplied by the highth (or length) produceth the Solid Content.*

1. *Arithmetically.*

Example: Let the Diametre of the Cylinder A B be 7.0, then will the *Area* of the Circle be found to be 38.5; and let the highth thereof A C or B D, equal to F G, be 12.5;

12.00; multiply 38.5 the *Area*, by 12.00 the highth, the Product will be 462, for the Solidity of the Cylinder ABCD.

2. *By Logarithms.*

The Logarithm of 38.5 the *Area* of the Base, is ——— 1.5854607
The Logarithm of 12, the Highth, is ——— 1.0791812
The Summ ——— 2.6646419
Which is the Logarithm of 462, the Solidity of the Cylinder.

3. *Instrumentally.*

The Extent from 1 to 38.5, will reach from 12 to 462 the Solidity.

Thus is the Solidity of a Cylinder found by having the Diametre of the Circle at the end, and the length given; but if the Circumference and the Length were given, the *Area* of the Circle at the end must be found.

Example: In Figure VIII. where the Circumference is 22 *ferè*, then will the *Area* of the Circle be found to be 38.5, and that multiplied by 12 the Length, produceth 462, for the Solidity as before.

X. *The* Axis (*or Diametre*) *of a Sphere* (*or Globe*) *being given, to find the Superficial Content thereof.*

LET ACBD be a Sphere whose Diametre is given, and the Superficial Content is required, which to find this is the Figure IX.

RULE:

Multiply the Square of the Diametre (always) by 355, *and divide that Product by* 113, *the Quotient shall be the Superficial Content of the Sphere.*

1. *Arithmetically.*

Example: In the Sphere (Fig. IX.) whose Diametre AB is 21. First multiply 21 the Diametre in it self, the Product will be 441; which multiply again by 355, and that second Product will be 156555; which divide (always) by 113, and the Quotient will be 1385.44, which is the Superficial Content of the Sphere or Globe ACBD.

2. *By Logarithms.*

The Logarithm of AB, the Diametre 21, is ——— 1.3222193
The same again, ——— 1.3222192
The Summ is the Logarithm of 441, the Square of the *Axis*, ——— 2.6444385
To which add the Logarithm of 355, *viz.* ——— 2.5502283
Their Summ ——— 5.1946668
From which subtract the Logarithm of 113 ——— 2.0530784
The Remainder is ——— 3.1415884
Which is the Logarithm of 1385.44, the *Superficies* of the Globe, or Sphere.

3. *Instrumentally.*

The extent of the Compasses from 1 to 21, will reach from 21 to 441; and again, the extent from 113 to 441, will reach from 355 to 1385.4, the Superficial Content of the Sphere ACBD.

XI. *The* Axis *of a Sphere being given, to find the Solid Content of the same Globe, or Sphere.*

IN the forementioned Globe, or Sphere AGBD, whose *Axis* AB is 21, let its Solidity be required; to attain which this is the

RULE:

Multiply the Axis *or Diametre in its self, and that Product multiply again by the Diametre,* (i.e. *Cube the Diametre,*) *and that Cube number multiply (always) by* 355; *this last Product (always) divide by* 678, *and the Quotient shall be the Solidity of the Sphere, or Globe.*

1. *Arithmetically.*

Example: In the Sphere ACBD, (Fig. IX.) the *Axis* is 21, which multiplied in it self produceth 441, (which is Square of the *Axis*,) and 441 multiplied again by 21, the Product will be 9261, (which is the Cube of the *Axis*;) and this Cube Number multiplied by 355, produceth 3287655; which Number divided by 678, the Quotient will be 4849.048, for the Solid Content of the Sphere, or Globe, ACBD, whose *Axis* AB is 21.

2. *By*

2. *By Logarithms.*

The Logarithm of 9261 (the Cube of 21, the *Axis* AB,) is	3.9666579
The Logarithm of 355 is	2.5502283
Their Summ is	6.5168862
From which Summ subtract the Logarithm of 678	2.8312297
The Remainder is	3.6856565

Which is the Logarithm of 4849.05 *ferè*, the Solid Content of the Shpere, or Globe, ACBD.

3. *Instrumentally.*

The extent of the Compasses from 1 to 355 (the Proportional Number for the Circumference of a Circle,) will reach from 9261 (the Cube of the *Axis* 21) to 4849.05, the Solid Content of the Sphere.

XII. *The Superficial Content of a Sphere being given, to find the* Axis.

IN the fore-mentioned Sphere ACBD, (Fig. IX.) whose *Area*, or Superficial Content is 1385.44, let the *Axis* be required; to find which this is the

RULE:

Multiply the Superficial Content by 113 *(always,) and divide that Product (always) by* 355, *and the Square Root of the Quotient shall be the length of the* Axis.

1. *Arithmetically.*

Example: In the Sphere ACBD, whose Superficial Content is 1385.44, this multiplied by 113, (the proportional Number for the *Axis*, or Diametre,) the Product will be 156550.2; which Number being divided by 355, (the Proportional Number for the Circumference of a Sphere,) the Quotient will be 440.986 *defective*, (*i. e.* 441,) whose Square Root is 21, for the length of the *Axis* of the Sphere AB.

2. *By Logarithms.*

The Logarithm of 113 is	2.0530784
The Logarithm of 1385.4 (the *Area*) is	3.1415752
Their Summ	5.1946536
From which subtract the Logarithm of 355	2.5502283
The Remainder,	2.6444253
The half whereof is	1.3222126

Which is the Logarithm of 21, the Length of the *Axis*, or Diametre of the Sphere.

3. *Instrumentally.*

The extent from 355 to 113 will reach the same way, from 1385.4, to 441; and half the distance between 1 and 441, will reach from 1 to 21, the Length of the *Axis* AB.

XIII. *The Solidity of a Globe or Sphere being given, to find the Diametre, or* Axis.

IN the Sphere ACBD, (Fig. IX.) whose Solid Content is 4849.05, and let the Length of the *Axis* thereof be required; to find which this is the

RULE:

Multiply the Solidity (always) by 678, *(the proportional Number for a Circle's Diametre,* viz. 113, *multiplied by* 6,) *and that Product divide (always) by* 355, *(the proportional Number for a Circle's (or Sphere's) Circumference,) the Cube Root of that Quotient shall give the Length of the* Axis.

1. *Arithmetically.*

Example: In the Sphere ACBD, whose Solidity is 4849.05 *ferè*, this Solidity multiplied by 678, produceth 328766 *ferè*; and this Product divided by 355, the Quotient will be 9261, whose Cube Root being extracted, will be found to be 21, for the *Axis* required.

2. *By Logarithms.*

The Logarithm of 678 is	2.8312297
The Logarithm of 4849.05 (the Solidity) is	3.6856969
Their Summ is	6.5169266
From which subtruct the Logarithm of 355,	2.5502283
The Remainder is	3.9666983
One third part whereof is	1.3222327

Which is the Logarithm of 21 (near,) for the Length of the *Axis* of the Sphere.

3. *Instru-*

3. *Instrumentally.*

Extend the Compasses from 355 to 678, the same extent will reach the same way from 4849.05 to 9261; and the distance from 1 to 9261, being divided into three equal parts upon the Line, one of them will reach from 1 to 21, for the length of the *Axis* AB.

From what hath hitherto been delivered concerning the Mensuration of Circles and Spheres, may be gathered (in the Sphere) these excellent proportional

COROLLARIES.

Periphery of the greatest Circle, *&* *contra*, is the same with that of the Diametre of a Circle to the Circumference, *&* *contra*.

I. The Proportion of the *Axis* or Diametre of a Sphere to the { Side of the Cube, equal to the Sphere, (which is properly the Cubing of the Globe, or Sphere,) / Side of the Quadrat, equal to the convex Spherical *Superficies*, (which is properly the Squaring of the Spherical,) } is as 1 to { .806040 / 1.772454 *ferè*.

II. The Proportion of the greatest Circumference is to the { Side of the Cube, equal to the Sphere, (which may also be termed the Cubing of the Sphere,) / Side of the Quadrat, equal to the convex *Superficies*, (which may be termed the Squaring of the Spherical. } as 1 to { .256556 / .564189

III. The Proportion of the Cube of the { *Axis* or Diametre, / Greatest Periphery, } to the Sphere it self, is as 1 to { .523599 / .016887 *ferè*

On the contrary,

IV. The Proportion of the whole Sphere to the Cube of the { *Axis*, or Diametre, / Greatest Periphery, } is as 1 to { 1.909859 / 59.217626

V. The Proportion of the Quadrat of the { Diametre / Periphery } to the whole *Superficies*, is as 1 to { 3.14159 / 0.31831 *ferè* } the same with that of the { Diam. to Circumference. / Circ. to Diam. }

VI. The Proportion of the whole *Superficies* of a Sphere, to the Quadrat of the { Diametre, / Periphery, } is the same with that of the { Peripherical Quadrat to the whole *Superficies* (or of the Circumference to the Diametre.) / Diametral Quadrat to the whole *Superficies*, (or of the Diametre to the Circumference.) }

XIV. *The Side of an erect Cone, and the Diametre of the Base being given, to find the* Area *or Superficial content thereof.*

LET ABC be an erect Cone, whose Side AB, or AC is 18, and the Diametre of its Base BC is 12, and the *Superficies* thereof is required; to find which this is the Figure X.

RULE:

Multiply the Side of the Cone by half the Diametre at the Base, and that Product multiply (always) by 355, *which second Product divide by* 113, *the Quotient shall be the Superficial Content.*

1. *Arithmetically.*

Example: In the Cone ABC. First, multiply the Side of the Cone AB 18, into half the Base BC 6, the Product will be 38340; which Product divide by 113, and the Quotient will be 339.29, for the Superficial Content of the Cone ABC.

2. *By Logarithms.*

The Logarithm of 108 is	2.0334237
The Logarithm of 355 is	2.5502283
Their Summ is	4.5836520
From which subtract the Logarithm of 113,	2.0530784
The Remainder is,	2.5305736

Which is the Logarithm of 339.29, which is the Superficial Content of the Cone ABC.

3. *Instrumentally.*

Extend the Compasses from 113 (the Proportional Number for a Circle's Diametre) to 355, (the Proportional Number for the Periphery of a Circle,) the same extent will reach from 108 (half the Diametre multiplied in the Side) to 339.29, for the Superficial Content of the Cone ABC.

XV. *The* Axis *or heighth of an erect Cone, and the Diametre at the Base thereof being given, to find the Solid Content of that Cone.*

Figure X. LET DEF be an erect Cone whose *Axis*, DG is 18, and its Diametre at the Base BC is 12, and let the Solid Content thereof be required. To find which this is the

RULE:

Multiply the Diametre in it self, and multiply that Product by one third part of the Altitude, (or Axis,*) then multiply this second Product (always) by* 355, *and divide that Product (always) by* 452, *and the Quotient shall be the Solid Content of the Cone.*

1. *Arithmetically.*

Example: In the Cone (Fig. X.) DEFG, whose Diametre EF is 12, this multiplied in it self, produceth 144; which multiplied by 6, (one third part of the *Axis* DG 18,) the Product will be 864; this second Product multiply again by 355, and the third Product will be 306720; which being divided by 452, the Quotient will be 678.58, for the Solidity of the Cone DEFG.

2. *By Logarithms.*

The Logarithm of 355 is	2.5502283
The Logarithm of 864 is	2.9365137
Their Summ is	5.4867420
From which Subtract the Logarithm of 452,	2.6551384
The Remainder	2.8316036

Is the Logarithm of 678,585, the Solidity of the Cone DEFG.

3. *Instrumentally.*

Extend the Compasses from 452 to 355, that extent will reach (the same way) from 864, (the Square of the Diametre 12, multiplied into 6, one third part of the Cone's *Axis*,) to 678.58, which is the Cone's Solidity.

XVI. *The Side of a right Cylinder, and the Diametre of the Base being given, to find the Solid Content thereof.*

Figure XI. LET KLM be a right Cylinder, whose Side, *Axis*, or Height, is 12, and whose Diametre at the Base is 7, and let the Solid Content thereof be required. To find which this is the

RULE:

Multiply the Diametre in it self, and that Product in the length of the Axis; *which second Product multiply (always) by* 355, *and that third Product divide by* 452 *(always,) so shall the Quotient give the Solidity of the Cylinder.*

1. *Arithmetically.*

Example: In the Cylinder KLM, whose Side is 12, and Diametre of the Base 7. First, multiply 7 in it self, and it produceth 49; which multiply by 12 (the Length of the Side or *Axis* KO,) the Product will be 588; multiply this Product 588 by 355, (the Proportional Number for the Circumference of a Circle,) and that Product will be 208740; this Product must be (always) divided by 452, which is, by 113, the Proportional Number for the Diametre of a Circle, multiplied by 4,) and the Quotient will give 461.814, for the Solidity of the Cylinder.

2. *By*

2. *By Logarithms.*

The Logarithm of 355 is	2.5502283
The Logarithm of 588 is	2.7693773
Their Summ	5.3196056
From which ſubtract the Logarithm of 452	2.6551384
The Remainder is	2.6644672

Which is the Logarithm of 461.814, and that is the Solidity of the Cylinder KLM.

3. *Inſtrumentally.*

Extend the Compaſſes from 452 to 355, that extent will reach the ſame way from 588 to 461.81, the Solidity of the Cylinder KLM.

XVII. *The* Axis *and Periphery of a right Cylinder given, to find the Solid Content.*

LET the Cylinder KLM (Fig. IX.) be given, whoſe Circumference LNMO is 21.991, *&c.* and whoſe *Axis* KO is 12, and the Solidity thereof is required; which to find this is the Figure IX.

RULE:

Multiply the Circumference in it ſelf, and the Product thereof multiply by the Axis, *and that Product again (always) by* 113; *which Product divide (always) by* 1420, *and that Quotient will give the Solidity.*

1. *Arithmetically.*

Example: The Circumference of the Cylinder OLNM is 21.99, *&c.* which multiplied in it ſelf produceth 483.56, *&c.* that Product being multiplied again by 12, (the *Axis* KO,) produceth 5802.72, *&c.* And this laſt Product being again multiplied by 113, (the Proportional Number for the Diametre of a Circle,) will produce 655707.36, *&c.* which laſt Product being divided by 1420, (that is, by 355, the Proportional Number for the Circumference of a Circle, multiplied by 4,) that Quotient will be 461.8, *&c.* for the Solidity of the Cylinder, as before.

2. *By Logarithms.*

The Logarithm of 113 is	2.0530784
The Logarithm of 5802.7 is	3.7636361
Their Summ is	5.8167085
From which ſubtract the Logarithm of 1420,	3.1522883
The Remainder is	2.6644202

Which is the Logarithm of 461.8, *&c.* the Solidity of the Cylinder, as before.

3. *Inſtrumentally.*

Extend the Compaſſes from 1420 to 113, the ſame extent ſhall reach (the ſame way) from 5802.72 to 1461.8, the Solidity of the Cylinder.

SECT. III.

Variety of Dimenſions, Linear, Superficial, *and* Solid, *of the Five, Plain, Ordinate, or Regular Bodies, commonly called the* Pythagorean *or* Platonick *Bodies, Arithmetically, by way of Proportion, to a Millimillenary Part of the Integer, and analytically contracted.*

I. *LINEAR.*

1. The Side of a *Tetraedron* is to its
   - *Axis*, as 1. to .816497 *ferè*. } $\sqrt{\frac{2}{3}}$
   - Circumſcribing Sphere's *Axis*, as 1. to 1.224745 *ferè*. } alike.

2. The *Axis* of a *Tetraedron* is to its
   - Side, as 1. to 1.224745 *ferè*. } alike.
   - Circumſcribing Sphere's *Axis*, as 1. to 1.5 *ſub-ſeſquialtera*.

3. The *Axis* of a Sphere is to the inſcribed *Tetraedron*'s { Side, Axis, } as 1. to { .816497 *ferè*. .666666, *&c. ſeſquialtera*.

4. The Side of the { *Octaedron*, *Hexaedron*, *Icoſaedron*, *Dodecaedron* } is to its *Axis*, Diagonal, or Circumſcribing Sphere's Dimetient as 1. to { 1.414214 *ferè* $\sqrt{2}$. 1.732050 $\sqrt{3}$. 1.902113 2.802517

Contrarily,

5. The *Axis*, Diagonal, or angular Diametre, (or Circumſcribing Sphere's Diametre,) of the { *Octaedron*, *Hexaedron*, *Icoſaedron*, *Dodecaedron* } is to the Side, as 1. to { .707107 *ferè*. $\sqrt{\frac{1}{2}}$. .577350 $\sqrt{\frac{1}{3}}$. .525731 .356822

6. The Diametre of a Sphere is to the Side of the Circumſcribed { *Tetraedron*, *Octaedron*, *Hexaedron*, *Icoſaedron*, *Dodecaedron*, } as 1. to { 2.449490 *ferè*. 1.224745 *ſubdupla*. 1.000 *equalis*. 0.66158 0.449028 *ferè*.

7. The Side of the { *Tetraedron*, *Octaedron*, *Hexaedron*, *Icoſaedron*, *Dodecaedron* } is to the inſcribed Sphere's Diametre, as 1. to { .408248 .816497 *dupla*. 1.00, *&c. equalis*. 1.511522 2.227033 *ferè*.

The two laſt of theſe Proportions are conſequently of the Sides of thoſe two Bodies to their Altitudes.

8. The *Tetraedron*'s *Axis* is to its inſcribed Sphere's *Axis*, as 1. to .5, *dupla*.

And therefore contrarily,

9. The *Axis* of a Sphere is to the *Axis* of its Circumſcribing *Tetraedron*, as 1. to 2. *ſub-dupla*.

10. The *Axis* of a *Tetraedron*'s Ambient, or external Sphere, is to the *Axis* of its Inſcribed or internal Sphere, as 1. to ,3333, *&c.* infinitely, *viz.* 3 to 1, *tripla*.

And therefore converſely,

11. The *Axis* of a *Tetraedron*'s inſcribed Sphere is to the *Axis* of its Circumſcribing Sphere, as 1 to 3, *ſub-tripla*.

12. The

12. the *Axis* of the Circumscribing Sphere's Diametre of the { *Hexaedron* / *Octaedron* / *Dodecaedron* / *Icosaedron* } is to its Inscribing Sphere's Diametre, as 1. to { $\cdot 57735\sqrt{\frac{1}{3}}$ / $\cdot 794654$ }

And so consequently, the latter of these two Proportions is to be understood of the *Axis* of those two Bodies to their Altitudes.

13. The Inscribed Sphere's Diametre is to the Circumscribing Sphere's Diametre (or the *Axis* or Diagonal) of the { *Hexaedron*, *Octaedron*, } as 1. to 1.732051/3 ; { *Dodecaedron*, *Icosaedron*, } as 1. to 1.258401 *ferè*.

And so the latter of these two Proportions, is of the Altitude of these two Bodies to their *Axes*: And by the 10th and 11th beforegoing, it appears that these five Bodies, in respect of their Spherical Inscription and Circumscription, do require three distinct Spheres: For they being all severally inscribed within one Sphere, cannot then also be exactly circumscribed about one Sphere, but three several Spheres, whereof,

That which is inscribed in the
- *Tetraedron*, will be the least.
- *Hexaedron*, or Cube, and *Octaedron*, will be Equal, and the Mean.
- *Dodecaedron*, and *Icosaedron*, will be Equal, and the Greatest.

Contrarily,

These five Bodies being severally circumscribed about one Sphere, cannot then again be inscribed in one Sphere, but must have three several containing Spheres; of which,

That for the
- *Tetraedron*, will be the Greatest,
- *Hexaedron*, or Cube, and *Octaedron*, will be Equal, and the Mean.
- *Dodecaedron*, and *Icosaedron*, will be Equal, and the Least.

And the like may be observed in these five Bodies, as being Inscribed within, or Circumscribed about, one Sphere, in respect of the Circles circumscribing their Bases; the Diametres of these Circles, and of the Bodies circumscribing, and inscribed Sphere being compared together.

Wherefore,

14. The Diametre of the Sphere circumscribing the { *Tetraedron* / *Hexaedron* / *Octaedron* / *Dodecaedron* / *Icosaedron* } is to the Diametre of the Circle circumscribing its Base, as 1. to { $\frac{\cdot 942809}{\cdot 816497\ ferè}$ / $\cdot 607062$ }

So the second and third of these Proportions are also of the *Axes* or Diametres of these four Bodies, to the Diametres of their Bases ambient Circles; which in the *Hexaedron*, is of the Bodies Diagonal, to the Basial, or superficial Diagony.

15. The Diam. of the Sphere Inscribing the { *Tetraedron* / *Hexaedron* / *Octaedron* / *Dodecaedron* / *Icosaedron* } is to the Diametre of the Circle circumscribing its Base, as 1. to { $\frac{2.828427}{1.414214\ ferè}$ / 0.763932 }

And so the second of these Proportions is consequently of the *Hexaedron*'s (or Cube's) Side to its Basial Diagony.

16. The

16. The Diam. of the Circle circumscribing the Base of the { *Tetraedron* / *Hexaedron* / *Octaedron* / *Dodecaedron* / *Icosaedron* } is to the Diametre of its Circumscribing Sphere, as 1. to { 1.06066 / 1.224745 / 0.647278 }

And so the second and third of these Proportions are also of the Diametres of the Basial ambient Circles of these four Bodies, to their *Axes*, or Diagonies; which in the *Hexaedron* is of the Basial or Hedral Diagony to the whole Body's Diagony.

17. The Diametre of the Basial or Hedral ambient Circle of the { *Tetraedron* / *Hexaedron* / *Octaedron* / *Dodecaedron* / *Icasaedron* } is to its inscribed Sphere's Diam. as 1. to { 0.353553 / 0.707107 *ferè*. / 1.309016 }

And so the second of these Proportions is of the *Hexaedrons* Basial Diagony (or Diametre) to its Side.

And the last of these Proportions is of the Basial ambient Circle's Diametre to the Altitudes of those two Bodies.

From hence is seen by these four last Sections of Proportion, that these five Bodies being described either within or about one Sphere, have only three several Circumscribing or containing Circles for their Bases; whereof,

That which is for the { *Tetraedron*, is the greatest. / *Hexaedron*, and *Octaedron*, are both one, and the Mean. / *Dodecaedron*, and *Icosaedron*, are Equal, and the Least. }

Now let what is already delivered for the Linear Proportions of these Bodies (in respect of their several Lines of Dimension, considered absolutely in themselves, as also being inscribed and circumscribed, and that in relation to the Diametre of the Sphere circumscribing and inscribing) suffice. And from these I shall proceed to the like Proportions, in respect of the Circumference answering to the said Sphere's Diametre; and therefore,

1. The Circumference of a Sphere's greatest (or Central) Circle, is to the Side of the Inscribed { *Tetraedron*, / *Octaedron*, / *Hexaedron*, / *Icosaedron*, / *Dodecaedron*, } as 1. to { .259899 *ferè*. / .225079 / .183776 / .167345 / .113580 }

Contrarily,

2. The Side of the { *Tetraedron* / *Octaedron* / *Hexaedron* / *Icosaedron* / *Dodecaedron* } is to the Circumscribing Sphere's greatest Circumference, as 1. to { 3.847649 / 4.442883 / 5.441398 / 4.975664 / 8.804369 }

3. The Periphery of a Sphere's greatest Circle, is to its inscribed *Tetraedron*'s *Axis*, as 1. to .212206.

And so contrarily,

4. The *Axis* of a *Tetraedron* is to its ambient Sphere's greatest Periphery, as 1. to 4.712389.

5. The Periphery of a Sphere's greatest Circle, is to the Side of the ambient { *Tetraedron*, / *Octaedron*, } as 1. to { .779697 *ferè* / .389848 } *subdupla* / { *Hexaedron*, } The same as the Circumference to the Diametre, the Side of this Body being equal to the Sphere's Diametre. / { *Icosaedron*, / *Dodecaedron*, } as 1. to { .210589 *ferè*. / .142930 }

Contra-

Contrariwise,

6. The Side of the { *Tetraedron*, *Octaedron* } is to the Inscribed Sphere's greatest Periphery, as 1. to { 1.28255 } *dupla*
{ *Hexaedron* } { 2.56510 } The same as the Diametre
{ *Icosaedron*, *Dodecaedron* } { 4.748589 } to the Circumference;
{ 6.996430 } for the reason aforesaid.

7. The Periphery of a Sphere's largest Circle, is to the *Axis* of its ambient *Tetraedron*, as 1. to.63662

8. The *Axis* of a *Tetraedron* is to its inscribed Sphere's greatest Periphery, as 1. to 1.570796.

II. and III. *SUPERFICIAL* and *SOLID*.

Having thus laid down the Linear Proportions in these five Bodies, for the measuring of them, I shall now come to shew the Proportions, both Superficial and Solid, deduced from thence, and that (for brevity sake) conjunctly.

		Solidity.	*Superficies.*
1. The Cube and Square of the Side of the	*Tetraedron*	0.117851 13	1.732050 8
	Octaedron	0.471404 52	3.464102 *ferè.*
	Hexaedron	1.000000 *aequalis*	6.000 *sub-sextupla*
	Icosaedron	2.181695	8.660254
	Dodecaedron	7.663119	20.645729

Hence,

2. The Side of the	*Tetraedron*	0.49028	1.316074
	Octaedron	0.778346 *ferè.*	1.86121
	Hexaedron	1.	2.44949
	Icosaedron	1.29697	2.94283
	Dodecaedron	1.971523	4.543757

Hereby it appears, That these five Bodies having all of them the same Side, the *Octaedron* is quadruple to the *Tetraedron* in Solidity, and double in Superficiality: The *Hexaedron* (or Cube) is bigger than both those together, both in Solidity and Superficiality: The *Icosaedron* is greater than those three together in Solidity, but less in the Superficies; and is herein quintuple to the *Tetraedron*, and so double-*sesqui-alter* to the *Octaedron*; and the *Dodecaedron* is larger than all the other together, both in Solidity and Superficiality.

3. The { Cube, Square } of the *Tetraedron*'s *Axis*, is to its { Solidity, Superficies } as 1. to { 0.216506 35, 2.598076 21 }

Hence,

4. The *Axis* of the *Tetraedron* is to the Side of the { Cube, Square } equal to its { Solidity, Superficies } as 1. to { 0.600468, 1.611855 *ferè.* }

Then for the *Tetraedron* in reference to a Sphere, it followeth,

5. The { Cube, Square } of the Diametre of a Sphere is to the Inscribed *Tetraedron*'s { Solidity, Superficies, } as 1. to { 0.064150 0299, 1.154700 5 }

Hence,

6. The Diametre of a Sphere is to the Side of the { Cube, Square, } equal to the Inscribed *Tetraedron*'s { Solidity, Superficies, } as 1. to { 0.400312 3, 1.074569 9 }

Now for the other four Bodies, (1.) Simply in themselves: and (2.) in relation to their Ambient (or Circumscribing) Sphere.

			Solidity.	*Superficies.*
7. The Cube & Sq. of the *Axis* or Diam. of the	*Octaedron*'s	Circumscribing Sph. Diam. is to the Body's Solidity and Superficiality, as 1. to	.166667 *ferè.*	1.732050 8
	Hexaedron's		.192450 09	2.000 *sub-dupla.*
	Icosaedron's		.317018 9	2.393635
	Dodecaedron's		.348145	2.628656 *ferè.*

Hence

Hence,

			Solidity.	Superficies.
8. The *Axis* or Angular Diam. of the	*Octaedron*'s	Circumscribing Sphere's *Axis* or Diametre is to the Side of the Cube and Square, equal to the Body's Solidity & Superficiality, as 1. to	.550321	1.316074
	Hexaedron's		.57735	1.41421
	Icosaedron's		.68186	1.54714
	Dodecaedron's		.703483 *fere.*	1.621313

			Solidity.	Superficies.
9. The Cube and Square of a Sphere's Diametre is to the Solidity and Superficies of its Circumscribing	*Tetraedron*	as 1. to	1.7320508	10.3923048
	Hexaedron		The same as the Square and Cube of the Side.	
	Octaedron		0.8660254	5.1961524
	Dodecaedron		0.693786	4.162719
	Icosaedron		0.631757	3.79054
10. The Dimetient of a Sphere is to the Side of the Cube and Square, equal to the Solidity and Superficies of the ambient	*Tetraedron*	as 1. to	1.200937	3.223710
	Hexaedron		The same as in the Side of this Body, agreeing with the Inscribed Sphere's Dimetient.	
	Octaedron		0.953193	2.279507
	Dodecaedron		0.885269	2.040274
	Icosaedron		0.858058	1.946932

By this, and by the third and seventh Sections following, it appears that these five Bodies being all described about one Sphere, the *Tetraedron* is the biggest of all, both in Solidity and Superficiality, and is exactly double to the *Octaedron* in both dimensions, and very near as big as the *Hexaedron* and *Octaedron* both together, in respect of both dimensions; and the *Icosaedron* is the least of all in both Dimensions; which is almost contrary to the former couse observed in these Bodies, being described in one Sphere.

Now for the like Proportions in relation to the Circumference of a Sphere's greatest Circle; and First, of these Five Bodies, being inscribed in a Sphere.

			Solidity.	Superficies.
1. The Cube and Square of the greatest Circumference of a Sphere is to the Solid and Superficial Content of the Inscribed	*Tetraedron*	as 1. to	.00206893 69	.1169956
	Octaedron		.00537525 57	.1754934
	Hexaedron		.00620681 06	.20264236 7
	Icosaedron		.0102243 4	.2425259
	Dodecaedron		.0112282 2	.266338

Therefore,

2. The Circumference of a Sphere's greatest Circle is to the Side of the Cube & Squ. equal to the Solidity and Superficial Content of the Inscribed	*Tetraedron*,	as 1. to	0.127423	.342046
	Octaedron,		0.175173 *fere.*	.418919
	Hexaedron,		0.183776	.450158
	Icosaedron,		0.2170427	.492469
	Dodecaedron,		0.223925	.51608 *fere.*

Then for the Proportions Cubatory and Quadratary, in relation to a Sphere's greatest Diametral, or true Periphery, in respect of these Bodies Spheral Circumscriptibility; it followeth,

			Solidity.	Superficies.
3. The greatest peripherial Cube and Square of a Sphere is to the Solidity and Superficiality of its containing or including	*Tetraedron*,	as 1. to	.05586129 6	1.0529606
	Hexaedron,		.03225153 4	0.6079271
	Octaedron,		.02793064 8	0.5264803 4
	Dodecaedron,		.0223756 7	0.421771 5
	Icosaedron,		.0203751 4	0.384062 2

Whereupon,

4. The greatest diametral Periphery of a Sphere is to the Side of the Cube and Square, equal to the Solidity and Superficiality of the Circumscribing or Comprehending	*Tetraedron*,	as 1. to	.3822701	1.0261387
	Hexaedron,		.318309	.7796968
	Octaedron,		.3034079 8	.7255896
	Dodecaedron,		.2817898	.649439
	Icosaedron,		.2731284	.619727

Lastly,

Laſtly, for thoſe Bodies, and a Globe or Sphere compared wholly together, both Solidly and Superficially, and that according to their Inſcription and Circumſcription, the Proportions will be as followeth.

			Solidity.	*Superficies.*
5. A Globe or Sphere is to the Inſcribed	*Tetraedron*'s	Solidity and Superficiality together, as 1. to	.1225 1753	.3675526 *ferè.*
	Octaedron's		.3183 09886	.5513 28895
	Hexaedron's		.3675526 *ferè.*	.6366 1977
	Icoſaedron'		.6054614	.7619 1789
	Dodecaedron's		.6649087	.8367272

Contrariwiſe,

6. The	*Tetraedron*	is to the Circumſcribing Globe, or Sphere's Solidity and Superficiality together, as 1. to	8.162097	2.720699
	Octaedron		3.141 59265	1.8137 9936
	Hexaedron		2.720699	1.5707963
	Icoſaedron		1.651 6327 *ferè*	1.3124 77
	Dodecaedron		1.503966	1.195133

7. A Globe, or Sphere, is to its Ambient	*Tetraedron*, *Hexaedron*,	both Solidly and and Superficially together, (in one and the ſame reaſon,) as 1. to	3.307973 37
			1.9098593. The ſame as of a Sphere to the Cube of its *Axis*.
	Octaedron,		1.6539 8668. *Subduplum Tetraedri.*
	Dodecaedro,		1.325034
	Icoſaedron,		1.206567

Converſely,

8. The	*Tetraedron*	is to the Inſcribed Globe, or Sphere, both Solidly and Superficially together, as 1. to	.30229989
	Hexaedron		.5235 9877. The ſame as of the Cube of a Sphere's *Axis* is to the Sphere it ſelf.
	Octaedron		.6045 99788. Double to the Sphere inſcribed in the *Tetraedron.*
	Dodecaedron		.7546973
	Icoſaedron		.8287974

By theſe two laſt Sections of Solid and Superficial Proportions, between the Five Ordinate Bodies and a Globe, or Sphere, it appeareth that any of the ſaid Bodies being Circumſcribed about a Sphere, the Solid and Superficial Capacity thereof will be, numerically, one and the ſame. Alſo, Two diſtinct Spheres being Inſcribed in two ſeveral like Bodies, (whereof the Solid Meaſure of the one, and the Superficial Meaſure of the other, are numerically the ſame,) there will the Solidity of one of the Inſcribed Spheres, and the Superficiality of the other, be alſo numerically the ſame: A Note worth the obſerving.

SECT. IV.

Of certain Paſſions and Properties of the Five Regular Bodies, commonly called the Pythagorean, *or* Platonick, *Bodies.*

I. *Of the* TETRAEDRON.

1. A *Tetraedron* is divided into two equal Parts by three equal Squares, which cut one the other into two equal Parts at right Angles in the Centre of the *Tetraedron*, and whoſe Angles are ſet in the middle Sections of the Sides of the *Tetraedron*.

2. From a *Tetraedron* are taken away four Pyramids like unto the whole, which utterly take away the Sides of the *Tetraedron*; and that which is left is an *Octaedron* inſcribed in the *Tetraedron*, in which all the Solids inſcribed in the *Tetraedron* are contained.

3. A Perpendicular drawn from the Angle of the *Tetraedron* to the Baſe, is double to the Diametre of the Cube inſcribed in the *Tetraedron*. And,

4. A right Line coupling or joining the middle Sections of the oppoſite Sides of the *Tetraedron*, is triple to the Side of the ſame Cube.

5. The Side alſo of the *Tetraedron* is triple to the Diametre of the Baſe of the Cube. Wherefore,

6. The Side of the *Tetraedron* is in power double to the right Line which joineth the middle Sections of the oppoſite Sides. And,

7. The ſame right Line is in power *ſeſquialter* to the Perpendicular which is drawn from the Angle of the *Tetraedron* to the Baſe. Wherefore,

8. The Perpendicular is in power *ſeſquitertia* to the Line which coupleth the middle Sections of the oppoſite Sides.

9. A *Tetraedron* and an *Octaedron* inſcribed in it, and alſo an *Icoſaedron* inſcribed in the ſame *Octaedron*, do contain one and the ſame Sphere.

II. *Of the* OCTAEDRON.

1. FOur Perpendiculars of an *Octaedron* multiplied in four Baſes thereof, from the two oppoſite Angles of the ſaid *Octaedron*, and joined together by thoſe four Baſes, deſcribe a *Rhombus*; one of whoſe Diametres is in power double to the other, for it hath the ſame proportion that the Diametre of the *Octaedron* hath to the Side of the *Octaedron*; and an *Octaedron* and an *Icoſaedron* inſcribed in it, do contain one and the ſame Sphere.

2. The Diametre of the Solid of the *Octaedron* is in power *ſeſquialter* to the Diametre of the Circle which containeth the Baſe. And,

3. It is in power triple to the right Line which joineth the Centres of the oppoſite Baſes. And,

4. It is in power duple *ſuperbipartiens tertias* (*i. e.* as 8 to 3,) to the Perpendicular or Side of the foreſaid *Rhombus*. And moreover,

5. It is in length triple to the Line which joineth the Centres of the next Baſes.

6. The Angle of the Inclination of the Baſes of the *Octaedron*, doth with the Angle of Inclination of the Baſes of the *Pyramis*, make Angles equal to two right Angles.

III. *Of the* HEXAEDRON, *or CUBE.*

1. THE Diametre of a Cube is in power *ſeſquialter* to the Diametre of his Baſe. And,

2. Is in power triple to his Side. And,

3. Unto the Line which joineth the Centres of the next Baſes, it is in power ſextuple.

4. The Side of the Cube is to the Side of the *Icoſaedron* inſcribed in it, as the Whole is to the greater Segment.

5. Unto the Side of the *Dodecaedron*, it is as the whole is to the leſſer Segment.

6. Unto the Side of the *Octaedron*, it is in power double. And,

7. Unto

7. Unto the Side of the *Tetraedron* it is in power *ſubdupla*. Moreover,

8. The Cube is triple to the *Tetraedron*; but to the Cube the *Dodecaedron* is in a manner *dupla*. Wherefore,

9. The ſame *Dodecaedron* is in a manner ſextuple to the ſaid *Tetraedron*.

IV. *Of the* ICOSAEDRON.

1. FIve Triangles of an *Icoſaedron* do make a Solid Angle, the Baſes of which Triangles make a *Pentagon*; if therefore from the oppoſite Baſes of the *Icoſaedron* be taken away the other *Pentagon* by them deſcribed, theſe *Pentagons* ſhall in ſuch ſort cut the Diametre of the *Icoſaedron*, which coupleth the foreſaid oppoſite Angles; ſo that that part which is contained between the Plains of thoſe two *Pentagons*, ſhall be the greater Segment; and the reſidue, which is drawn from the Plain to the Angle, ſhall be the leſſer Segment.

2. If the oppoſite Angles of two Baſes joined together be joined by a right Line, the greater Segment of that right Line is the Side of the *Icoſaedron*.

3. A Line drawn from the Centre of the *Icoſaedron* to the Angles, is in power quintuple to half that Line which is taken between the *Pentagons*, or of the half of that Line which is drawn from the Centre of the Circle which containeth the foreſaid *Pentagon*, which two Lines are therefore equal.

4. The Side of the *Icoſaedron* containeth in power either of them, and alſo the leſſer Segment, namely, the Line which falleth from the Solid Angle to the *Pentagon*.

5. The Diametre of the *Icoſaedron* containeth in power the whole Line which joineth the oppoſite Angles of the Baſes joined together, and the greater Segment thereof; namely, the Side of the *Icoſaedron*.

6. The Diametre alſo is in power quintuple to the Line which was taken between the *Pentagons*, or to the Line which was drawn from the Centre to the Circumference of the Circle which containeth the *Pentagon*, compoſed of the Sides of the *Icoſaedron*.

7. The *Dimetient* containeth in power the right Line which joineth the Centres of the oppoſite Baſes of the *Icoſaedron*, and the Diametre of the Circle which containeth the Baſe. Moreover,

8. The ſaid *Dimetient* containeth in power the Diametre of the Circle which containeth the *Pentagon*, and alſo the Line which is drawn from the Centre of the ſame Circle to the Circumference: That is, it is quintuple to the Line drawn from the Centre to the Circumference.

9. The Line which joineth the Centres of the oppoſite Baſes contains in power the Line which joineth the Centres of the next Baſes, and alſo of the reſt of that Line of which the Side of the Cube inſcribed in the *Icoſaedron* is the greater Segment.

10. The Line which joineth the middle Sections of the oppoſite Sides is triple to the Side of the *Dodecaedron* inſcribed in it. Wherefore,

11. If the Side of the *Icoſaedron*, and the greater Segment thereof, be made one Line, the third part of the whole is the Side of the *Dodecaedron* inſcribed in the *Icoſaedron*.

V. *Of the* DODECAEDRON.

1. THE Diametre of a *Dodecaedron* containeth in power the Side of the *Dodecaedron*, and alſo that right Line unto which the Side of the *Dodecaedron* is the leſſer Segment, and the Side of the Cube inſcribed in it, the greater Segment; which Line is that which ſubtendeth the Angle of the Inclination of the Baſes, contained under two Perpendiculars of the Baſes of the *Dodecaedron*.

2. If there be taken two Baſes of the *Dodecaedron*, diſtant one from the other by the length of one of the Sides, a Line joining their Centres, being divided by extreme and mean Proportion, maketh the greater Segment of the right Line which joineth the Centres of the next Baſes.

3. If by the Centres of five Baſes ſet upon one Baſe, be drawn a plain *Superficies*, and by the Centres of the Baſes which are ſet upon the oppoſite Baſe, be drawn alſo a plain *Superficies*, and then be drawn a right Line joining the Centres of the oppoſite Baſes, that right Line is ſo cut that each of his parts ſet without the Plain *Superficies*, is the greater Segment of that Part which is contained between the Plains.

4. The Side of the *Dodecaedron* is the greater Segment of the Line which ſubtendeth the Angle of the *Pentagon*.

5. A Perpendicular Line drawn from the Centre of the *Dodecaedron* to one of the Baſes, is in power quintuple to half the Line which is between the Plains. And,

6. Therefore the whole Line which joineth the Centres of the oppoſite Baſes is in power quintuple to the whole Line which is between the ſaid Plains.

7. The Line which ſubtendeth the Angle of the Baſe of the *Dodecaedron*, together with the Side of the Baſe, are in power quintuple to the Line which is drawn from the Centre of the Circle which containeth the Baſe to the Circumference.

8. A Section of a Sphere, containing three Baſes of the *Dodecaedron*, cutteth off one third part of the Diametre of the ſaid Sphere.

9. The Side of the *Dodecaedron* and the Line which ſubtendeth the Angle of the *Pentagon*, are equal to the right Line which joineth the middle Sections of the oppoſite Sides of the *Dodecaedron*.

A SUP

A SUPPLEMENT TO THE Two foregoing BOOKS OF Arithmetick and Geometry, Treating of ALGEBRA.

Proœme.

THIS Tractate (as to the Subject Matter of it) should have come at the end of the First Book, (as a Fifth Part thereof;) but forasmuch as ALGEBRA *cannot be with profit attained unto without a competent knowledge in* Geometry, *as of such* Definitions, Theorems, *and* Problems *as are contained in the Second Book hereof, which treateth only of* GEOMETRY, *as well as of* Arithmetick *in Whole Numbers and Fractions, both Vulgar and Decimal, which are the Subjects of the First Book of* ARITHMETICK; *I therefore chose (as most beneficial) to place it here.*

Who was the first Inventor of this kind of Arithmetick (*commonly called* ALGEBRA;) *I do not find; but some do attribute the Invention if it to one* Gebar, *a learned Man, excellent in Numbers, Geometry,* &c. *who (in likeness to his Name) call it* Algebra. *But he cannot be the first Inventor; for a* Greek *Philosopher and Mathematician, named* Diaphontus, (*long before* Gebar's *time,*) *wrote Thirteen Books hereof, Six of which are yet extant; and the true Name of this Art is* ALGIEBAR, *and not* Algebra, *as by the* Arabian Avicen *may be proved, from what Dr.* John Dee, *in his learned Preface before* Euclid's *Elements, says concerning it,* viz. *That* Andreus Alpagus (*one most perfect in the* Arabick *Tongue*) *so translates* Scientia faciendi Algiebar & Almachabel, *i. e.* Scientia inveniendi Numerum ignotum, per Additionem Numeri, & Divisionem, & Æquationem: *That is to say, The Science of working* Algiebar *and* Almachabel, i. e. *The Science of finding an unknown Number, by Adding of a Number, and Division, and Æquation. Let this suffice concerning its Original and Name; and now I intend to give the Reader an account of what I shall doe in it in this Supplement.*

First then, for the Symbols here used, and the Method observed, it is that of Mr. Thomas Harriot, *in such Æquations as are proposed in Numbers; that of* Des Cartes *in such Æquations as are Solid and not in Numbers; and both of them as they were many years since intermixed by my much honoured Friend Mr.* Tho. Gibson, *whose Order and Method I here principally follow, with some small augmentation, but little or no alteration; and (at the close) I have added Sixteen Æquations arising from a Quantity divided into two unequal Parts; and the Second Book of* Euclid's *Elements demonstrated in* Species, *which I long since received from my worthy Friend Mr.* John Leek, *Reader of Mathematicks in* London, *in which the Characters and Symbols there used, are those of Mr.* Will. Oughtred, *that eminent and peerless Analist, and my worthy Friend.*

CHAP.

CHAP. I.

DEFINITIONS.

I. THE unknown Quantity of any Æquation is called (generally) *Poteſtas*, or a Power, Quantity, or Term.

II. A Rectangle is in Numbers the Product of two Numbers multiplying one another; in Geometry, it is the *Area*, Space, or Content of a Right-angled Quadrangular Figure, made alſo by the Multiplication of two Lines, which are called the Sides, of which one is the meaſure of the Breadth, the other of the Length.

III. A Rectangled *Parallelipipedon* is the Product of a Rectangle multiplied by a right Line, or Number; and if that Line, or Number, and the Length and Breadth of the Rectangle, be ſeverally equal, it is a Cube, or Die.

IV. A Priſm is a Solid contained within five *Superficies*, of which three are Quadrangular, and the other two, which are oppoſite, are Triangular.

Theſe Four may ſuffice in this place, becauſe they are treated of at large in the Firſt Sections of the Firſt and Second Parts of *Geometry*, in the Second Book hereof.

The Names of the Poteſtates, *or* Powers.

1. The Firſt Power is called a Side, or Root: The latter word *Root* is moſt uſed here, and it is ſignified thus *a*.

2. The Second Power is called a *Square*, and is written thus *aa*.

3. The Third is called a *Cube*, and it is thus written *aaa*, and ſometimes for brevity a^3.

4. The Fourth is a *Biquadrate*, or *Squared Square*, and is thus written *aaaa*, or ſometimes thus a^4.

5. The Fifth Power is called a *Surſolid*, written thus *aaaaa*, or thus a^5.

6. The Sixth a *Squared Cube*, written thus *aaaaaa*, or thus a^6.

7. The Seventh a *Second Surſolid*, and it is written thus *aaaaaaa*, or thus a^7.

8. The Eighth is called a *Squared Square Squared*, and it is written thus *aaaaaaaa*, or thus a^8, *&c.*

CONSECTARY.

I. It is hence manifeſt that theſe Powers uninterrupted are in continual Proportion, the Proportion of them being as *a* to Unity, or the Converſe.

II. It is alſo here plain, that every Power hath ſo many Dimenſions as the Letters wherewith it is written: For *aaaa*, or a^4 being written with four Letters, if one Letter ſtand for one Dimenſion, that is, Root, Side, Length, or Breadth, the other three ariſe by three ſeveral Multiplications, and every Multiplication adds a Dimenſion in this ſenſe.

A TABLE of the Poteſtates, *or* Powers, *of the Nine Digits.*

Roots, or Firſt Powers.	The Number of the Powers. I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.
I.	1	1	1	1	1	1	1	1	1
II.	4	8	16	32	64	128	256	512	
III.	9	27	81	243	729	2187	6561	19683	
IV.	16	64	256	1024	4096	16384	65536	262144	
V.	25	125	625	3125	15615	78125	390625	1953125	
VI.	36	216	1296	7776	47656	279936	1679616	10077696	
VII.	49	343	2401	16807	116649	823543	5764801	40353607	
VIII.	64	512	4096	32768	262144	2097152	16777216	134217728	
IX.	81	729	6561	59049	531441	4782969	43046721	387420489	

In this Table the Digits in Numeral Letters at the Head, I. II. III. *&c.* ſhew the Columns of the Second, Third, Fourth, *&c.* Powers.

The Digits on the Left Side in Numeral Letters ſhew the ſeveral Roots, or Firſt Powers, and their Proportion to Unity. All the reſt of the Table is evident, being compoſed by continual Multiplications into the Roots on the Left Side. Concerning the finding of the Roots to the higheſt Powers, ſee more in *Logarithmetical Arithmetick*, Book 1. Part 3. Chap. 5. Page 159, 160.

CHAP.

CHAP. II.

An Explanation of the Characters, *or* Symbols, *used in this Work.*

ONE ſingle Letter of the Alphabet is uſually put for any Quantity whatſoever, as well Line as Number, whether known or unknown.

But for the moſt part, where any Quantity is ſought, there *a*, or ſome other Vowel, is put for it, and the other Quantities known, are ſignified by Conſonants.

Theſe Letters are multiplied one into another by joining them together without any Prick, or Comma between; neither doth it at all import which is firſt or laſt written; for *bcd*, *bdc*, and *cbd*, are all one.

So *a* multiplied by $\left\{\begin{matrix} a \\ b \\ c \end{matrix}\right\}$ produceth $\left\{\begin{matrix} aa. \\ ab \\ ac \end{matrix}\right.$

The like of all others whatſoever, except Fractional Quantities:

As $\frac{ab+fg}{d}$ and $\frac{bc-fb+rc}{b+c}$

If the firſt of theſe were to be multiplied by *d*, it is done by taking away the *d* under the Line, and the Product is $ab+fg$: If the ſecond were to be multiplied by $b+c$, it is done by taking away the Denominator $b+c$, and the Product will be $bc-fb+rc$.

For all Fractions, as well in plain as Figurative Arithmetick, are nothing elſe but Quotients of one Number divided by another; and are multiplied again by taking away their Diviſor and Line of Separtion.

Diviſion is done in Figurative Arithmetick, moſt commonly, by applying ſome Line of Separation between the Dividend and the Diviſor.

So $\frac{a}{b}$ is *a* divided by *b*, and $\frac{abc}{f}$ ſignifies that *abc* is divided by *f*.

But yet if the Letter *f* had been found in the Dividend, the application of this Line had not been neceſſary; for it might have been better done by taking away that Letter *f* out of the Dividend.

So *afc* divided by *f*, the Quotient is *ac*.

And *ffcc* divided by $\left\{\begin{matrix} fc \\ ff \\ cc \\ ffc \\ fcc \\ f \\ c \end{matrix}\right\}$ the Quotient is $\left\{\begin{matrix} fc \\ cc \\ ff \\ c \\ f \\ fcc \\ ffc \end{matrix}\right.$

And the like is to be underſtood of all others.

Symbols of $\left\{\begin{matrix} \text{Majority,} & > \\ \text{Minority,} & < \\ \text{Æquality,} & = \\ \text{Addition,} & + \\ \text{Subtraction,} & - \\ \text{Root of any Quantity,} & \surd \end{matrix}\right.$

Proportionality $\left\{\begin{matrix} \text{Continued,} & :: \; ::: \; :::: \\ \text{Disjunct,} & : \; :: \; : \; :: \end{matrix}\right.$

So $\left\{\begin{matrix} b \geq c \\ b \leq c \\ b = c \\ b+c \\ b-c \end{matrix}\right\}$ ſignifies $\left\{\begin{matrix} b \text{ greater than } c. \\ b \text{ leſs than } c. \\ b \text{ equal to } c. \\ c \text{ added to } b. \\ c \text{ taken from } b. \end{matrix}\right.$

$\surd 72$, ſignifies the Square Root of 72, *&c.*

And $b' \; c'' \; d''' \; f''''$ ſignifies, that,
As *b* is to *c*, ſo is *c* to *d*, and ſo *d* to *f*.
Likewiſe $b' \; c'' \; f' \; g''$ ſignifies, that,
As *b* is to *c*, ſo is *f* to *g*.

Theſe things before expreſſed are almoſt generally received; and uſed, not only for brevity in writing, but perſpicuity in proving, as will be ſeen hereafter.

And here Note, That whereſoever — is not expreſſed, there + is underſtood, though it be not expreſſed.

If the ſign of Addition, namely +, ſtand before any Quantity, it ſhews that Quantity to be more than Nothing, that is, Something.

But if the ſign of Subtraction, to wit —, ſtand before any Quantity, it ſhews that Quantity to be leſs than Nothing, or a want of the ſaid Quantity.

So +4 ſignifies Four of any thing; but —4 ſignifies a want of Four, or Four leſs than Nothing.

In *ADDITION.*

The Addition of a Want of any thing, is all one with the Subtraction of the ſame thing.

So if to +12 you add —5, it makes +7.
And if to +12 you add —16, it makes —4.
But if to +12 you add +16, it makes +28.

In *SUBTRACTION.*

The Subtraction of —, is all one with the Addition of +.

So if from +12 you ſubtract —5, the Remainder is 17.

And if from +12 you ſubtract —16, the Remainder is +28.

The Addition of + to +, and Subtraction of — from —, is all one with common Addition and Subtraction: And generally for both,

In Addition, add the Quantities together with the ſame Sign.

In

In Subtraction, add them also; but all the Signs of that which is to be subtracted from the other must be changed.

Examples.

If to $+6-2+3$
Be added $+5+1-3$
The Summ is $+15-5=10$

But,

If from $+6-2+3$
Be subtracted $+5+1-3$
The Rem. will be $+6-2+3-5-1+3=4$

And this Rule is general, and generally known.

In *MULTIPLICATION.*

$\left.\begin{matrix}+\\+\\-\end{matrix}\right\}$ multiplied by $\left\{\begin{matrix}+\\-\\-\end{matrix}\right\}$ ever produceth $\left\{\begin{matrix}+\\-\\+\end{matrix}\right.$

And more Varieties than these there are not.

The Quantities that are accompanied with these Signs $+$ and $-$, (in both, Multipliers being placed one under another, as in common Multiplication,) must be multiplied every one below into every one above, and then the Work is done.

And thus, if $+bb+b-c$ were to be multiplied into $+f-g$, you must place them thus:

Multiply $+bb+b-c$.
Into $+f-g$

Saying		multiplied into		produceth
$+f$		$+bb$		$+bbf$
$+f$		$+b$		$+fb$
$+f$		$-c$		$-fc$
$-g$		$+bb$		$-bbg$
$-g$		$+b$		$-bg$
$-g$		$-c$		$+cg$

Their Sum $+bbf-bbg+fb-fc-bg+cg$

And this is the true Product.

In *DIVISION.*

The Number found out by Division is called the Quotient, [or *Parabola*,] that is, the Application of a Plain to a Longitude given, to find out a Latitude agreeing thereunto. And Note, That if a Number be applied to another Number, with a small separating Line drawn between them, it shews that the upper Number is to be divided by the under Number, unto which it is applied: As $\frac{12}{8}$ signifies, that 12 is to be divided by 8; and so $\frac{9}{27}$ signifies that 9 is to be divided by 27, *&c.*

But if this Line of Separation do not serve the turn, that is, if any desire (and it may be done) otherwise; it must then be by seeking what Quantity may be multiplied by the Divisor to produce the Dividend.

So if $bb+bc-bf-bg-cg+fg$, were to be divided by $b+c-f$.

Trial must be first made what Mixt Quantity multiplying $b+c-f$ will produce $bb+bc-bf-bg-cg+fg$.

In which there is this of *Compendium*, That seeing the Dividend consists of six Members, and the Divisor of three, the Quotient must be of two, that is, a Binomial only.

And because the Quantity g is found in the Dividend, and not in the Divisor, it must be in the Quotient.

The said Quotient therefore must be one of these three, $b+g$, $b-g$, or $g-b$.

It cannot be the first, for $+g$ into $-f$ would have produced $-fg$; but in the Dividend it is $+fg$, therefore it must be $-g$.

By the same reason, it cannot be the last; as also because $-b$ into $+b$ produceth $-bb$; but it is $+bb$ in the Dividend.

Therefore the Quotient sought must be $b-g$.

Some farther Rules for saving labour herein might be given, but these are sufficient.

Other Examples inserted for Practice.

I. In *ADDITION.*

To	$+5cd$	$5abcd$	$-15ggg$
Add	$-2cd$	$-abcd$	$11ggg$
Summ	$+3cd$	$4abcd$	$-4ggg$
To	$2b$	$3bc$	$8ab$
Add	aa	$2cd$	$-3bc$
Summ	$26+aa$	$3bc+2cd$	$+5ab-3bc$
Add	$16cde+4db+5p$	$3c^4+4ef-dg$	
	$-8cde-2db$	$-4c^4+2dg-ab$	
	$-3cde-db$	$2c^4+ef-dg$	
Summ	$5cde+db+5p$	$+c^4+5ef-ab$	

II. In *SUBTRACTION.*

From	$3cd$	$-5bc$
Subtract	cd	$-8bc$
Remainder	$3cd-cd$	$8bc-5bc$
Contracted	$2cd$	$3bc$

From $3aa-2cd+ab$
Subtr. $aa+3ab-3cd$
Rem. $3aa-2cd+ab-aa-3ab+3cd$

From $6ace+3cd-bc$
Subtr. $4ace-cd-5bc$
Rem. $6ace+3cd-bc-4ace+cd+5bc$
Contr. $2ace+4cd+4bc$

III. In *MULTIPLICATION.*

Multiplicand	$+ab$	$-ac$	$-aef$
Multiplier	$+ade$	$-ac$	$+aaf$
Product	$+aabde$	$+aacc$	$-aaaeff$

Multiply $ac+dg$
By $c+d$
$acc+cdg$
$acd+ddg$
Product $acc+cdg+acd+ddg$

Multi-

Multiply $da+bc$
By $da-ab$

$ddaa+dabc$
$-aadb-abbc$

Product $ddaa+dabc-aadb-abbc$

Multiply $a+b-c$
By $a+b-c$

$aa+ab-ac$
$+ab+bb-bc$
$-ca-cb-cc$

Product $aa+2ab-2ac+bb-2cb+cc$

Multiply $2a+2e-8$
By $2a-5$

$4aa+ae-16a$
$-10a-10e+40$

Product $4aa+4ac-26a-10e+40$

IV. In *DIVISION.*

Divide cd by fg, *facit* $\frac{cd}{fg}$ | $3dce$ by $15b$, $\frac{3dce}{15b}$

Let it be required to divide $ba+ca+be+ce$ by $b+c$: Place the Members as in common Diviſion, thus,

Diviſor.	*Dividend.*	*Quotient.*
$b+c$	$ba+ca+be+ce$	$a+e$
	$ba+ca$	
	$00\quad 00+be+ce$	
	$be+ce$	
	$00\quad 00$	

Then ſee how often b (the firſt Member in the Diviſor) may be had in ba (the firſt Member in the Dividend,) and the anſwer is a, which put in the Quotient, and thereby multiply the Diviſor $b+c$, and the Product is $ba+ca$, which place under the two firſt Quantities of the Dividend towards the Left Hand, and ſubtract it therefrom, ſo will the Remainder be 00: To this bring down the remaining part of the Dividend, namely $be+ce$, and divide be by e, and the Quotient will be e; by which multiply the whole Diviſor $b+c$, and the Product will be $be+ce$, which ſubtract from the latter part of the Dividend before brought down, the Remainder is 00.

And the Quotient of $\frac{ba+ca}{be+ce}$ is $a+e$, as in the Example above.

Concerning this matter have recourſe to the *Compendium* before.

CHAP. III.
Of ÆQUATIONS.

AN Æquation is when one or more ſpecial Quantities are equal to one or more other ſpecial Quantities, and written with the ſign of Equality between them, as $aa=ab$:

This is called a *Simple Square Æquation.*

And bb being a known Square, the Square Root thereof being extracted is equal to a, and that is the thing required.

But $aa+ba=cc$, and $aa-ba=dd$, and laſtly $-aa+ba=ff$, are (all of them) of that kind which are called *Mixed Æquations*; becauſe a (the thing required) is multiplied not only into it ſelf, but into another known Quantity, namely into b.

And Note, That this known Quantity in all Mixed Æquations is called the *Co-efficient.*

Note alſo, That the three ſorts of Mixed Æquations above expreſſed, are all that can happen in Quadraticks; and by ſome one of theſe, all Problems whatſoever, tranſcending plain Diviſion, (and not reaching Solids,) are to be reſolved by finding the Root a, according to theſe old Rules.

In the firſt, $aa+ba=cc$.

Unto the Quantity given, namely cc, add the Square of half the *Co-efficient*, it makes $+cc+\frac{bb}{4}$.

Which if it be in Lines, may be reduced into one Square; and from the Side of that Square take half the Co-efficient, and the Remainder ſhall be a, which was the thing deſired.

In the Second $aa-ba=dd$.

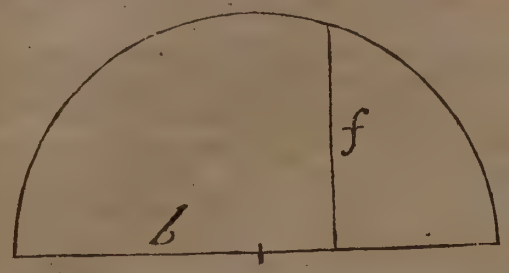
Unto dd add $\frac{bb}{4}$ (as in the former,) and the Summ thereof (being always in Numbers a Square, or in Lines to be reduced to a Square, as aforeſaid,) *Unto the Root, or Side, of that Square, add half the Co-efficient, the Summ thereof ſhall be* a, or the Root of the Æquation ſought for.

In the Third $-aa+ba=ff$.

From the Square of half the Co-efficient, which is $\frac{bb}{4}$, take the Quantity given, that is, ff, there will remain $\frac{bb}{4}-ff$; which being put into one Square, and the Side of that known, if that Side be either added to half the Co-efficient, or ſubtracted therefrom, either the Summ of that Addition, or the Remain of the Subtraction is equal to a.

For all Quadratick Æquations of this kind, (where aa, the greateſt unknown Power is wanting,) have two Roots, which being both together ever equal to the Co-efficient.

Now if upon the Co-efficient (as a Diametre) a Semicircle be deſcribed, and the Side of ff (the Quantity given) be applied therein, perpendicular to the Diametre b, the two Segments of b, are the two Roots ſought.

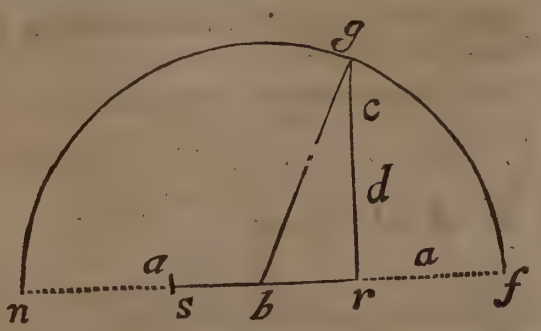


For in the Æquation $-aa+ba=ff$, it is (by the 14th of the ſixth of *Euclid*, or by Prob. 37, Lib. 2. Part 1. hereof.) as followeth:

$$b-a \quad f'' \quad f' \quad a''.$$

Wherefore either Segment may be a, and the other will be $b-a$, and f will be a mean between them.

Likewiſe, In the two former Æquations, the Work may be effected Geometrically, and proved alſo by this Scheme:



In which, as the Figure intimates, the Perpendicular repreſents the Side of cc in the Firſt Æquation, and the Side of dd in the Second.

Draw a Line bg, from the Centre b, to the top of the Perpendicular, the Centre b being firſt taken in the middle of the Line b, to wit, of the Co-efficient, for ſo it is uſually called.

And firſt, let the pricked Line be put for a, therefore (by the before-recited Propoſition,) it is,

$$a+b' \quad c'' \quad c' \quad a''$$

And if (as the Rule preſcribeth) to the Square of half b, you add the Square of c, the total ſhall be the Square of the Line bg; by the 47th of the firſt of *Euclid*, or by Theor. 12. Lib. 2. Part 1. hereof.

If therefore from the Line bg, or (which is all one) bf, you take the Line br, which is half the Co-efficient b, (for the whole Co-efficient b, is the ſame with sr,) the reſt, namely, the pricked Line rf, ſhall be equal to a; for,

$$rf=ns=a.$$

In like ſort concerning the ſecond Æquation, $aa-ba=dd$; if (according to the Rule) you add the Squares dd and $\frac{bb}{4}$ together, it gives the Square of the Line bg, to the Root of which, *viz.* bg, if you add half the Co-efficient, *viz.* br, or bs, the Summ ſhall be fs, or nr, either of them equal to a: and then, as nr, that is a, is to d, ſo is d to rf, or $a-b$, as it ought to be.

Having thus far proceeded in Æquations, I think it neceſſary to admoniſh the Reader, that in putting a (always) for the thing ſought, and working therewith as if it were known, quite through, as the Queſtion requires, he ſhall at laſt come to an Æquation, but it may be ſuch as wants reducing: of which as followeth.

CHAP. IV.

Of REDUCTION of ÆQUATIONS.

ÆQuations are reduced, by adding all that is neceſſary, or ſubtracting all that's unneceſſary, on both ſides the ſign of Equality $=$: Or by ſubtracting Contradictories, (if they happen on one and the ſame Side,) untill the Æquation (purged of all unneceſſary Members) remain with all that's abſolutely known on one Side, equal to (as little as may be) all that's unknown on the other Side.

One Example of this ſhall ſerve, which is this, as followeth.

In the Æquation,

$$aa-ba+dc+ba=gg+ba-dc.$$

To reduce this, you muſt remember, that the taking away a Want of any thing, is all one with the Addition of that thing.

Therefore ſeeing there is on the firſt ſide a want of ba, expreſſed by $-ba$, if you take away that $-ba$, you thereby add ba on that Side.

Wherefore (that it may ſtill be an Æquation) you muſt add ba on the other ſide alſo.

Then it will be,

$$aa+dc+ba=gg+2ba-dc.$$

Again, ſubtract ba on each ſide:

Then it is,

$$aa+dc=gg+ba-dc.$$

Once more, ſubtract ba on each ſide, that you may bring it to that ſide where aa ſtands:

Then it is,

$$aa-ba+dc=gg-dc.$$

Laſtly,

Lastly, (that the Consonants, or known things may come all on one Side) subtract dc on each side:

Then it will be,

$$aa - ba = gg - 2dc.$$

Take the Rectangle $2dc$ out of the Square gg, and let the rest be a Square ff:

Then it is reduced,

$$aa - ba = ff.$$

I have gone here a little about, only for the exercise of them that are wholly unacquainted herein; but now they shall see that this Reduction might have been quickly done another way: That is, seeing in the Æquation

$$aa - ba + dc + ba = gg + ba - dc.$$

there are in the first part Contradictories, *viz.* $-ba$ and $+ba$, they (destroying one another) might be taken away both at once:

And so it will be,

$$aa + dc = gg + ba - dc.$$

Then if you subtract dc and ba on both sides, it will be reduced to,

$$aa - ba = gg - 2dc$$

as it was before: And $gg - 2dc$, being put into on Square ff, the Æquation

$$aa - ba = ff$$

may be resolved as the Æquation $aa - ba = dd$ was, by the second Rule for Plain Æquations, a little before expressed.

And, as here, the Reduction was made by Addition and Subtraction only, so sometimes it is made by Multiplication, sometimes by Division; in both or either of which this a general

RULE:

That, *Whatsoever is done to any one Member, must be done to every Member quite through the Æquation.*

CHAP. V.

Of the Resolution of Æquations, *according to the general Method of Mr.* Thomas Harriot.

ALthough (having before given Rules for all sorts of mixed Squares) it may seem preposterously done hereafter to speak of Simple Squares; yet because the general Method of Mr. *Harriot* begins with Squares, I will do so, but only with one Example.

§ I. *Of Square Æquations.*

Let there be an Æquation of $aa = ff$: Or, Let it be exhibited in Numbers, $aa = 69169$.

First, Take notice that all Squares, whether Simple or Mixed, in Numbers are to be marked with Points, the first always over the place of Unity, (or Unites,) and so successively over every Binary or second Figure.

Cubes over every Ternary Figure.

Biquadraticks over every Quaternary Figure.

Surfolids over every Quinquenary Figure.

And so forwards.

This Square Number thus pointed is 69169.

In which, because there are three Points, there are three Figures in the Root.

So that a (being a single Letter) cannot fitly represent that Root, but some Trinomial, as if $b + c + d$ should be put equal to a, and the Square thereof should be equal to aa, or 69169.

But because it may as well be done by adding the *Gnomons*, that is, repetition of the second Working (as they are commonly called) so often as the Points are more that two, a Binomial will serve (with less trouble) to doe the same.

Let that Binomial be $b + c$

And put $b + c = a$

Their Squares shall be therefore equal.

That is, $bb + 2bc + cc = a$

That is, $bb + 2bc + cc = 69169$.

The Resolution.

The Homogeneal Number given,	69169
First single Root $b = 2$ and $bb =$	4.0000
Which 4.0000 being subtracted from the Number given 69169, then there remains of the Number given,	29169
Root Decuplate, $b = 20$	
Divisor, $2b$	40.00
The second single Root $c = 6$	
$2bc$	240.00
cc	36.00
	276.00
Subtract.	276.00
Remains of the Number given	1569
The Root increased $b = 26$	
Root increased and decuplated, $b = 260$	
Divisor is $2b = 520$	
The third single Root $c = 3$	
$2bc$	1560
cc	0009
Total	1569
Subtract	1569
Remains of the Number given,	0000

The Root increased 263, is therefore the true Root, as may be proved by Re-composi-

tion, or multiplying 263 by 263, for the Product will be 69169, which was the Number given.

The Cyphers which are put after in the Divisors and Subtracts, are only to fill up the number of Places, by which the Number given or rather, the remaining Points would else exceed.

For the like purpose is used the Decuplation of the Roots, as only to supply a place untill another Figure succeed in place of the Cypher.

And in nothing else doth this work differ from the ordinary Extraction of the Square Root as it is in Lib. 1. Part 1. Chap. 18. hereof.

The reason of it dependeth upon the fourth Theorem of the second Book of *Euclid*, where it is demonstrated, That,

If a Right Line be divided by chance into two Parts, the Square made of the Whole Line, is equal to the Squares of the Parts, and to the Rectangle made of the Parts twice. As in *Lib.* 2. Part 1. Theor. 4. Page 246. hereof.

So it his here as followeth:

The Square of the greater Part, that is, of 260,	$bb = 67600$
The Square of the lesser Part, that is, of 3,	$cc = 00009$
The Rectangle of the Parts, that is, 260 into 3 twice,	$2bc = 01560$
Equal to the whole Square,	69169

Nor do those Letters represent so naturally the things themselves in a Divided Superficies only, but as properly and clearly the parts of Solid Bodies; of which two or three Examples follow.

In which I admonish the Reader to be intent to the several Pointings of the Quantities according to their due order, as is before expressed; and also to the placings of the Divisors and Subtracts by Cyphers, as before also is intimated.

§ II. *Of Cubical Æquations.*

Let there be a Cube $aaa = fff$
Or proposed in Numbers $aaa = 41781923$.
Put as before, $b + c = a$.
Then their Cubes also shall be equal. That is,
$bbb + 3bbc + 3bcc + ccc = 41781923$.

The Resolution.

The Homogeneal Numb. given,	41781923
The first single Cubick Root $b=3$	
And $bbb = 27.000000$	
Subtract	27.000000
Remains of the Number given,	14781923
The first Root decuplated, $b=30$	
$3bb$	2700.000
$3b$	0090.000
Divisor	2790.000

Second single Root $c=4$	
$3bbc$	10800.000
$3bcc$	01440.000
ccc	00064.000
	12304.000
From the last Remain, *viz.*	14781923
Subtract	12304000
Remains of the Number given	02477923
The Root increased $b=34$	
The Root increased decuplate $b=340$	
$3bb$	346800
$3b$	001020
Divisor,	347820
The third single Root $c=7$	
$3bbc$	2427600
$3bcc$	0049980
ccc	0000343
	2477923
Subtract	2477923
Remains lastly of the Numb. given	0000000

The Root increased $b + c = 347$.

Which is the Root of the Cube 41781923; which may be proved by re-composition, that is, by Multiplying 347 by 347, and the Product again by 347; the last Product shall be equal to the Cube which was given to be resolved.

And, as above, in the Square the Canon of the Resolution was the Letters $bb + 2bc + cc$, being the true Square of $b + c$; and those Letters did answer exactly to the Parts of the Square divided alike in both dimensions; so here also the Canon of Resolution, or the Letters $bbb + 3bbc + 3bcc + ccc$, do exactly answer to the Parts or Members of a Cube, divided into two Parts, alike in all the three dimensions, as any one may prove upon a Cube made of some slender matter, and cut through all three ways; for he shall find the whole Cube (supposed equal to 41781923 as before,) justly made up of the two Cubes of the two Segments, that is, bbb and ccc, and three *Parallelipipedons*, whose Length and Breadth are equal to b, and their Thickness to c; those three are the $3bbc$; and lastly, three other *Parallelipipedons*, whose Length and Breadth are equal to c, and their Thickness to b; such are the $3bcc$.

See more concerning this matter before, in Book 1. Part 1. Chap. 18. Page 60: And also in this following Schematism.

The Cube of the greater Segment, which is 340, bbb,	39304000
The three great *Parallelipipedons* $3bbc$,	2427600
The three lesser *Parallelipipedons* $3bcc$,	49980
The Cube of the lesser Segment, which is 7, ccc,	343
The whole Cube given,	41781923

Note,

Note, *That the Greater Segment is the Aggregate of all the Single Roots except the last, being duly valued by a Cypher, as here it is,* 340: *But the lesser Segment is the last single Root only, as here* 7.

This is done here to let the Reader see, that he may be sure, let the Quantity to be resolved be Great or Little whatsoever, if he be carefull to make his Canon right, the Letters themselves will direct him how to frame his Divisors and Subtracts in order to the final Resolution, especially in these un-mixed Quantities; where the Points limit how far the Subtract shall advance at every operation; beginning first at the Point next the left hand, not farther; and to the Second Point only at the Second Work; and not otherwise in all that follow.

And in Mixed Æquations, if they be made up of Cube, with addition of certain Squares, or certain Roots, or both Squares and Roots; or by subtraction of the same; the Canon of the Resolution must ever be made by multiplying the assumed Root $b+c$ in the place of the Quesititious Root a, quite through the Æquation in all the Degrees thereof; for so shall arise all the several parcels of which the several Subtracts are orderly to be made.

In a Cubick Æquation, if all the Quantities be present, there is no need to point any but the Cubicks and Roots; yet I have here distinguished the Places of the Squares also, with little Astricks; which labour (when the Workman is intent about his business) may well enough be spared.

§ III. *Of the Resolution of Mixed Cubicks.*

Let the Æquation $aaa+daa-ffa=ggg$ be proposed in Numbers; as let it be

$aaa+32aa-75a=29282970$:

Therefore $d=32$, and $ff=75$,

and $ggg=29282970$.

Put $b+c=a$.

And make the Canon of Resolution by substituting $b+c$ in the place of a, quite through the several Quantities $aaa+daa-ffa$.

The Canon rightly made will be:

$+bbb$
$+3bbc+dbb-ffb$
$+3bcc+2dbc-ffc$
$+ccc+dcc$

These several parcels of the Canon being rightly subtracted from the *homogeneal Number* 29282970, the Number shall be thereby resolved, and the Root a found.

☞ *Note First*, That all the Parcels in the Canon which have not the Secondary Root in them, as $+bbb+dbb$, and $-ffb$, are to be subtracted at the first operation; the other remaining Parcels to be all subtracted as often as there shall be Points left above.

The Resolution.

The Homogeneal Numb. given,	29282970
The first single Root $b=2$	
$+bbb$	8.000000
$+dbb$	128.0000
	+928.0000
$-ffb$	.. 150.00
	+ 92650.00
Subtract	92650.00
Remains of the Number given	20017970
The first Root decuplate $b=20$	
$3bb$	1200.000
$3b$	.. 60.000
$2db$	.. 1280.00
d	... 32.00
	13912.00
$-ff$	 750
Divisor	1390450
The second single Root $c=9$.	
$3bbc$	10800.000
$3bcc$	. 4860.000
ccc	.. 729.000
$2dbc$	. 11520.00
dcc	.. 2592.00
	178092.00
$-ffc$	.... 675.0
	17793450
Subtract	17793450
Remains of the Number given	02224520
The Root increased $b=29$	
The Root increased decuplate $b=290$	
$3bb$	252300
$3b$	... 870
$2db$	. 18560
$-ff$	 75
Divisor	271805
The third single Root $c=8$.	
$3bbc$	.2018400
$3bcc$	... 55680
ccc	 512
$2dbc$	.. 148480
dcc	 2048
	2225120
$-ffc$	 600
	2224520
Lastly Subtract	2224520
Remains of the Number given	0000000

Whereby it appears that the whole Root 298 is the true Root whereby this Æquation is explicable, as may be proved also by Re-composition.

bbb		24389300
$3bbc$		.2018400
$3bcc$		...55680
ccc	is = to	512
dbb		.2691200
$2dbc$		..148480
dcc		2048
	In all	29305320
From which subtr. $ffb+ffc=$		...22350
	Remains	29282970

Which was the whole Homogeneal Number given.

NOTE:

Whereas in composing the Divisor all the gradual Quantities are used; as in the former Example, $3b$ and d, as well as $3bb$ and $2db$: It is to be noted that in practice, those smaller Particles $3b$, *&c.* may be omitted, the other without them ministring light enough for chusing the secondary Roots.

And having now instanced in an Example where all the Powers were present; in these one or two that follow, (to make the Work shorter,) I shall leave out one or other of them.

In the Æquation $aaa+ffa=ggg$
Propounded in Nmmbers:
$aaa+320406a=8348132.$

It sometimes happens that the Co-efficient abounds with more Binary Figures than the Homogeneal doth with Ternaries: In such a case, that there may be room made to begin the Extraction, the Co-efficient must be devolved to the next Point farther to the right Hand, or to the Second, Third, Fourth, or Fifth, or farther if need require, and there the Work is to begin. The Co-efficient is always the known Quantity, which multiplies any of the unknown inferiour Quantities.

§ IV. *Example of Devolution.*

$aaa+320406a=8348132.$
Put $b+c=a$
The Canon will be,
$bbb+3bbc+3bbc+3bcc+ccc+ffb+ffc=8348132$

Resolution.

The Homogeneal Number given	8348123
The First single Root $b=2.$	
$+bbb$ 0008.000	
$+ffb$ 6408120	
$=$ 6416120	
Subtract	6416120
Remains of the Number given	1932012
The first Root decuplate $b=20$	
$3bb$ 00001200	
ff 00320406	
Divisor 321606	

The second single Root $c=6$

$3bbc$	7200
$3bcc$	2160
ccc	216
ffc	.1922436
	1932012
From the last Remainder	1932012
Subtract	1932012
Remains of the Number given	0000000

Wherefore the whole Root is equal to the Root increased, 26, as may be proved in manner as aforesaid.

It sometimes also happens in the Æquation, $aaa-ffa=ggg$, Put into Numbers, as $aaa-105000a=203125.$

That the Co-efficient abounds with more Binary Figures than the Homogeneal with Ternaries: wherefore that there may be place for the Resolution, put before the Homogeneal toward the Left Hand, so many Cyphers as will afford that to receive as many Cubical Points, as the Co-efficient doth Quadratical; and at the first empty Point, as it were by anticipation, begin the Resolution: In which there is this of *Compendium*, That the first Square Root extracted out of the Co-efficient, is either equal to the first single Root of the Homogeneal sought, or less than it by Unity.

But if the Æquation had but two Dimensions, as $aa-254a=65024$, then the first Figure of the Co-efficient, namely 2, is the first Root.

§ V. *Example of Anticipation.*

The Homogeneal Numb. given $+023125$
$b+c=a$
The Canon is,
$bbb+3bbc+3bcc+ccc-ffb-ffc.$

The Resolution.

The first single Root $b=3$	
$+bbb$ 27.000000	
$-ffb$ 3150.0000	
$-$ 4500000	
Subtract	4500000
Remains of the Number given	$+4703125$
The first Root decuplate $b=30$	
$+3bb$ 2700.000	
$-ff$ 105000.0	
Divisor. 1650000	
$3bbc$ 5400.000	
$3bcc$ .360.000	
ccc ...8.000	
5768.000	
$-ffc$ 2100000	
$+$ 3668000	
Subtract	$+3668000$
Rmains of the Number given	$+1035125$

The

The Root increaſed $b=32$
Root increaſed duplicate $b=320$
$3bb$ 307200
$+ff$ 105000
Diviſor 202200

The third ſingle Root $c=5$
$3bbc$ 1536000
$3bcc$ . . 24000
ccc 125
1560125
$-ffc$. 525000
1035125

From the laſt Remainder $+1035125$
Subtract 1035125
Remains of the Number given 0000000

Which ſhews that the Root increaſed $b+c=325$ is the true Root of the Æquation: And it may be proved by Re-compoſition, as before.

In the Æquation $-aaa+ffa=ggg$, which is explicable by two Roots, as ſhall be ſhewed in the next Chapter § 5, to find them both.

Put the Æquation into Numbers.
As $-aaa+52416a=1244160$.
Put $b+c=a$.

Therefore $\left\{\begin{array}{l}-bbb+ffb\\-3bbc\\-3bcc+ffc\\-ccc\end{array}\right\}=1244160$.

§ VI. *Extraction of the Greater Root.*

The Homogeneal Number given 1244160
The firſt ſingle Root, $b=2$
ffb 104832.00
$-bbb$ 8.000000
$+$ 24832.00
Subtract $+24832.00$

Remains of the Number given -1239040
The firſt Root decuplate $b=20$
ff 52416.0
$-3bb$ 1200.000
Diviſor -675840

The ſecond ſingle Root $c=1$
ffc . 52416.0
$-3bbc$ 1200.000
$-3bcc$ . . 60.000
$-ccc$ . . . 1.000
$-.736840$
Subtract $-.736840$

Remains of the Number given -502200
The Root increaſed and decupled $b=210$
ff 52416
$-3bb$ 132300
Diviſor -79884

The third ſingle Root $c=6$
$+ffc$ 314496
$-3bbc$ 793800
$-3bcc$. 22680
$-ccc$ 216
-502200

From the laſt Remainder -502200
Subtract -502200
Remains of the Number given 000000
Root increaſed $b+c=216$, which is the true Root ſought.

§ VII. *Eduction of the Leſſer Root by Devolution.*

The Homogeneal Number given 1244160
$b=2$
ffb 104832.0
$-bbb$ . . . 8.000
$+1040320$
Subtract $+1040320$

Remains of the Number given $+$ 203840
The Root increaſed and decupled $b=20$
ff . 524.16
$-3bb$ . . 1200
Diviſor 51216

The ſecond ſingle Root $c=4$
ffc 209664
$-3bbc$ . . 4800
$-3bcc$ . . . 960
$-ccc$ 64
$+$ 203840
Subtract $+203840$
Remains of the Number given 000000
The Root increaſed, $b+c=24$.

Wherefore 24 is the true Root ſought, which may be proved by Re-compoſition, as hath been ſhewed before.

So this Æquation is explicable by two Roots, that is, 216 and 24.

VIETA, Lib. de Recognitione Æquationum, Cap. 18. *Prop.* 2. ſaith, That in the Æquation $-aaa+ffa=ggg$, the Co-efficient ff is compoſed of three Proportional Squares, and the Homogeneal ggg is made by the Multiplication of the aggregate of the two firſt, or the two laſt, (for all is one,) into the Side of the other, and the Root a, may be the Side either of the firſt or third. And it is evident: For
Make $cc'+dd''+bb'''=ff$,
And put $c=a$:
Therefore,
$ccc+ddc+bbc-ccc=ddc+bbc$.
Or put $b=a$:
It is $bbb+ddb+ccb-bbb=ddb+ccb$.
Both which are manifeſt.

Compen-

Compendium I.

Hence it may be shewed, That either of the Quesititious Roots, as a, being found, and called c, the other Root e may be found by a Quadratick Æquation only.

For suppose $cc+ce=ff-ec$: Then it is $ee+ce+cc-ff$:

And cc^{I} ce^{II} ee^{III}. *Eucl. L.* 6. 23.

But by Construction, cc^{I} dd^{II} hh^{III}, and $cc+dd+hh=ff$: So then, $hh=ee$, and $h=e$.

But it was shewed before, that h might be a Root of this Æquation $-aaa+ffa=ggg$ and therefore e also is a Root of the same; and this *Compendium* is proved.

Example in Numbers.

In the last Æquation $aa=cc=46656$.

$ff=52416$

From which take $cc=46656$

05760

Remains $ff-cc=05760$

But $ee=576$

And $ce=5184$

5760

The Summ is $cc+ce=05760$

00000

Therefore $ee+ce=ff-ec$.

Which was to be, *&c.*

In the Æquation $-aaa+faa=ggg$.

The Co-efficient f is composed of three proportional Lines, and ggg is equal to a Solid made by a Square (whose Side is equal to the two first, or the two last) multiplied into the remaining Line: And the Aggregate of the first and second may be a, and the Aggregate of the second and third shall be e.

But 1^{I} 2^{II} 4^{III}.

And suppose $-aaa+7aa=36$.

Then a may be 3, and e is 6. *Vieta, de Recog. Cap.* 18. *Prop.* 6.

Compendium II.

And therefore the Root a found, the Root e may be found by a Plain Æquation: For suppose the middle Proportional y,

It is $f-y-c^{I}$ y^{II} c^{III}

And $fc-cy-cc=yy$, or $yy+cy=fc-cc$.

And making $fc-cc=xx$, it is $yy+cy=xx$.

And the Root y being found, by the first Rule of *Chap.* 3. it is lastly, (making c^{I} y^{II} d^{III},) $y+d=e$.

Here followeth Four Rules and Examples grounded upon Mr. *Harriot*'s 6 Sections.

CHAP. VI.

Of the Fabrick of Æquations, *their Roots, Increment, and Decrement,* Multiplication *and* Division *of them, and their Number.*

RULE I. *EVery Æquation being composed of some known, and some unknown Quantities, hath its original by Roots composed of a Quantity known, and of one other Quantity unknown; and those Roots multiplied together, produce certain particular Members, with + and — respectively signed (for in every Æquation both these Signs are present,) which orderly placed make up the Æquation.*

As the Æquation $a.a-ba-ca+bc=0$. And because it was at first $a-b=0$, therefore $a=b$, and the like of c. And from hence it follows, that where the first Term (or highest Power) in a Quadratick Æquation is signed —, there the Æquation hath two Roots, as here by subtracting on both parts $+aa-ba-ca$, the Æquation will be $bc=-aa+ba+ca$, and must have two Roots.

1. These Compound Quantities so multiplying I shall call *Binomials*, whether $a+b$, or $a-b$, not having any need (in this Place) to distinguish between *Binomials* and *Residuals*.

2. The Æquation $aa-ba+ca=bc$, if it be $b<c$, put $c-b=d$; then the Æquation will be $+aa+da=bc$, and is of the first kind mentioned in *Chap.* 3. But if it be $b>c$, put $b-c=f$, and the Æquation will be $+aa-fa=bc$; and is like the second sort in the same Chapter.

The Original of the Æquation

$aa-ba+ca-bc=0$,

here proposed, is $+a-b=0$, multiplied by $+a+c=0$, that is $a=b$, by $a=-c$. This Æquation hath but one true Root, which is b, and one false, which is c.

3. By this which hath been said, it is plain, That some Æquations have as many Roots as Dimensions; some not so many, but none can have more; for the Number of Dimensions being the same with the number of the Multipliers (if all diverse) can be but all Roots: Nor can the Æquation be divided by any other thing than one of these Binomials by whose Multiplication it was made.

But if the Multipliers (how many soever) be still the same, there can be but one Root. For let $+a-b=0$ be multiplied Biquadratically,

The Product is

$+aaaa-4baaa+6bbaa-4bbba+bbbb$.

Where

Where it is plain, there can be no other Root but b; I mean, none greater or leſs than it; becauſe in truth here are 4 Roots, but every one ſingularly equal to b.

For if there may, let it be d, and let d be greater or leſs than b, it imports not which: And ſeeing $d=a$, ſubtract d in the place of a, quite through the Æquation, it will be

$$dddd - 4bddd + 6bbdd - 4bbbd - bbbb = 0.$$

Which if $d>b$, or elſe $d<b$, is at firſt ſight impoſſible: For the difference between the $+$ and $-$ is always equal to the Power of the difference between b and d, which Power is here a *Biquadrate*, therefore $d=b$. And again, ſeeing this Æquation may be derived by putting b equal to d, for ſubtracting b in the place of d, quite through, it will be

$$+2bbbb + 6bbbb = 4bbbb + 4bbbb.$$

Which is manifeſt; therefore again $b=d$, which is contrary to the ſuppoſition; therefore b is the only Root of this Æquation: For indeed, the Æquation propoſed being made only of Multiplications of $a-b=0$, cannot be divided, that is reſolved, by any other Binomial than $a-b$, of which it was made.

4. Hence it is, that the laſt Term in every Æquation may be called the *Homogenial*, becauſe it is naturally made by Multiplication of the Roots of the Æquation, though the Co-efficients in ſome ordinary Æquations are diſguiſed with other Characters, which happens by Addition or Subtraction of them, to reduce the Canonical Æquation to fewer Members; whereby the redundancy of the Signs $+$ and $-$ is to be taken away; this is to be ſeen above in this *RULE*, where the Æqu. $+aa-ba+ca-bc=0$, is reduced to $+aa+da-bc=0$, by making $d=c-b$; and $+aa+ba+ca-bc=0$, reduced to $+aa-fa-bc=0$, by making $b-c=f$: Where the Co-efficient d or f, is not a Part of the Homogeneal bc, but a difference by which b is greater or leſs than c; by help of which difference, the Æquation which conſiſted canonically of Four Members, hath now but Three.

5. And this Reduction is uſefull, for as *Des Cartes* ſaith, and may be ſeen true by the way of Multiplication above ſhewed; *Every Æquation hath ſo many true Roots, as the Signs* $+$ *and* $-$ *therein are changed:* Which in the Canonical Æquation $+aa-ba+ca-bc=0$ are changed three times; whereas the Æquation hath not three true Roots, but one true, and one falſe, that is b and c, and the common Æquation reduced changeth the Signs but once; that is, from $+da$ to $-bc$ in the former, or from $+aa$ to $-fa$ in the latter: And from thence it may be known that the Æquation hath but one *true* Root. The like conſideration ought to be had in others.

And whereas the ſaid *Des Cartes* doth often mention *falſe* Roots, it is to be noted that ſuch are leſs than Nothing, as $+a+b=0$: Or $+a=-b$, and if any true Root, as $+a-c=0$, be multiplied by this $+a+b=0$, there will ariſe an Æquation $+aa+ba-ca-bc=0$, where the Sign $+$ follows twice, the Sign $-$ twice, and they are once changed, which ſhould intimate (according to *Des Cartes*,) two *falſe* Roots and one *true*; for he ſaith, *So many times as* $+$ *or* $-$ *come twice together, ſo many falſe Roots there are:* This Æquation therefore muſt be reduced, by making $b-c=d$, if $b>c$, or elſe, if $b<c$, then make $c-b=f$, ſo it will be either $+aa+da-bc=0$, or $+aa-fa-bc=0$, which confirms that which *Des Cartes* ſaith of twice $+$ or $-$, namely, That there are as many falſe Roots in the Æquation, as $+$ or $-$ come twice together, and ſo many *true* Roots as $+$ and $-$ are changed.

And where the Roots are all falſe the Æquation is impoſſible, as $a+b=0$ multiplied by $a+c=0$, produceth $aa+ba+ca+bc=0$ which cannot be. And therefore when there is an Æquation pretended like $aa+ba+ca=-bc$, preſent judgment may be made.

6. The ſame *Des Cartes* ſaith alſo, *That all the falſe Roots in any Æquation may be turned to true ones, and the true ones to falſe, by changing the Signs of the Second, Fourth, and every even Term.* And this is evident; for of the Æquation $a^4 - 2a^3 + 10aa - 30a - 87 = 0$, by ſuch changing is made

$$+a^4 + 2a^3 + 10aa + 30a - 87 = 0;$$

where the firſt had three *true* Roots, and but one *falſe*; the latter hath three *falſe*, and but one *true*. This Æquation was taken at all adventures, to ſerve for an Example only; whereas any other will doe the like.

RULE II. *Every unknown Root of an Æquation may be increaſed or decreaſed, by ſuppoſing another unknown Quantity* $+$ *or* $-$, *the Decrement or Increment, and of that Binomial compoſing the Æquation, as it was before of the firſt unknown Quantity: And if this Increment be put equal to ſuch a part of the Co-efficient of the ſecond Term, as Unity is of the Dimenſions of the firſt Term, (if the Sign of the firſt and ſecond Terms be both* $+$ *or both* $-$,) *or if the Decrement be made equal to ſuch a part of the ſaid Co-efficient as Unity is of the Dimenſions, as aforeſaid, (if the Signs of the firſt and ſecond Terms be one* $+$ *the other* $-$,) *then by ſuch Increaſe or Decreaſe of the Root, the ſecond Term of the Æquation ſhall be taken away and annulled.*

EXAMPLE.

In the Æquation $+aaa+baa-bbc=0$, The Root a may be increaſed by making $e-q=a$, and ſubſtituting $e-q$ in the place of a, quite through the Æquation, and thereby ſhall ariſe a new Æquation.

$$\left.\begin{matrix} +eee - 3qee + 3qqe - qqq \\ +bee - 2bqe + bqq \\ -bbc \end{matrix}\right\} = \left\{\begin{matrix} +aaa \\ +baa \\ -bbc \end{matrix}\right.$$

Which is equal to the former, as you see agreeing in the particulars; and the Root being found, a may be had by casting away q from e.

And because the Number of the Dimensions of the first Term aaa is 3, if, according to the latter part of the *Rule*, the Quantity q be proportioned, by making $3'\ 1''\ b'\ q''$, then $b=3q$, and $+bee$ will destroy $-3qee$; and so the second Term ee will be quite taken out of the Æquation, as is manifest; for the Æquation so purged $+eee-3qqe+2qqq-bbe=0$, and by subtracting on each part $+2qqq-bbc$, having first made $bbc-2qqq=ddd$, it will be then $+eee-3qqe=ddd$. The manner of such Reduction of Solids shall follow in the next Chapter.

In like sort, the Root a might have been decreased by any Quantity, as x, which if it be proportioned to b as aforesaid, would take away the second Term of an Æquation, where the Signs of the first and second Terms are not like, as in the Æquation $+aaa-baa-bbe=0$, by putting $3x=b$, and $ex=a$: The Problem will be fully performed by making $e+x$, the Root of the new Æquation, as before was $e-q$, observing the same order in composing the particulars, due respect being had to the Signs $+$ and $-$, where they ought to be altered: The former reduced Æquation $+e^3-3qqe=ddd$ might be farther reduced (if need require) to $+eee+bqe=ddd$.

Note, This Augmentation and Diminution of the Roots in such manner as to take away the second Term of any Æquation, is of excellent use in such Æquations as have three or four Dimensions, and cannot by any Division with any Binomial made of a $+$ or $-$ some other known Quantity, as b, c, or the like, be reduced to fewer Dimensions; whereby it is certain, that such an Æquation is Solid, and cannot by any artifice already (or likely to be) invented, be resolved by Ruler and Compasses, but by any of the Conick Sections it may: In this Case it is either necessary, or extremely facilitating, to take away the second Terms (if there be any) from the Æquation, as shall be seen hereafter.

RULE III. *THE unknown Root of any Æquation may be multiplied (or divided) by any known Quantity multiplying (or dividing) the second Term of the Æquation by the said Quantity; the third by the Square, the fourth by the Cube thereof; and so forward continually in this Order, as often as there are Terms in it; having first assumed another unknown Quantity, so multiplex to the said unknown Root, as is required.*

EXAMPLE.

In the Cubical Æquation $a^3+baa+cca-bcd=0$, Let it be required to multiply the Root a by 4,

Assume $e=4a$, and write

$$eee+4bee+16cce+64bcd=0:$$

Which is an Æquation, and the Root e is quadruple to a, as may be proved thus:

Put $a=4$: $b=3$: $c=2$ and $d=21\frac{1}{3}$.

Then $\left\{\begin{array}{l} aaa=64 \\ baa=48 \\ cca=16 \end{array}\right\}$ But $bcd=128$.

In all 128

Therefore $a^3+baa+cca-bcd=0$.

Again, put $e=16$, all else the same still,

Then $\left\{\begin{array}{l} eee=4096 \\ 4bee=3072 \\ 16cce=1024 \end{array}\right\}$ But $64bcd=8192$.

In all 8192

Therefore $e^3+4bee+16cce-64bcd=0$.

And $e=16=4a$. *Which was to be proved.*

The Utility of this *Rule* will appear in reducing Æquations affected with Fractions to Whole Numbers, by multiplying the Root by the Denominator, or Denominators, of the Fraction; for by such means, the Co-efficient of the second Term is multiplied by the same as before, multiplying a by 4; multiply also b by the same number 4. And many times by this *Rule* Æquations may be freed from Surd Numbers also, especially if such be found in the second Term, as is easie to be seen by try-all; for if there be an Æquation so affected.

As $aaa+\sqrt{8}aa+\frac{29}{24}a-4\sqrt{2}=0$.

Put $e-\sqrt{8}=a$

And write $+eee+8ee+9\frac{2}{3}e-128=0$.

So the Surds are vanished.

But if yet it be required to avoid the Fraction $9\frac{2}{3}e$, then make $y=3e$, and multiplying 8 by 3, $9\frac{2}{3}$ by 9, and 128 by 27, there will be a new third Æquation,

$$+yyy+24yy+87y-3456=0:$$

Which consists of entire Numbers, having one true Root which is 9; and the Root of the middle Æquation was 3, which is the third thereof; and the Root of the first Æquation was $3-\sqrt{8}$. And now I hope this Rule, and the Use of it, is plain enough.

Note I. It may be Noted, that if the Surds in the second and last Terms of the first Æquation, *viz.* $aaa+\sqrt{8}aa+\frac{29}{24}a=4\sqrt{2}$, had been utterly incommensurable, the the Reduction had not been so fecible; for although $4\sqrt{2}$ multiplied by the Cube of $\sqrt{8}$, that is, by $8\sqrt{8}$, produceth $32\sqrt{16}$, which is equal to the entire Number 128; yet if it had been $2\sqrt{3}$, or $2\sqrt{5}$, or any such Primes to be multiplied by $8\sqrt{8}$, the Product would have been $16\sqrt{24}$, or $16\sqrt{40}$; though this last may (by the *Note* after the Consectary in *Chap.* 8.) be reduced by multiplying it again by $\sqrt{40}$ unto the entire Number 640: Nevertheless this second Multiplication by a Surd, renders the Æquation inexplicable, at least by the precedent *RULE*.

Note

Note II. It may be farther Noted, that if inſtead of $e=4a$, one would put $e=fa$, Lines being not ſo liquid as Numbers, the Æquation would then be,

$eee-fbee+ffcce-ffbcd=0$,

increaſing the Dimenſions of the leſſer Terms: For remedy whereof, three Lines are to be found in proportion one to another, as are the Magnitudes fb, fc, $fffb$; of which, let the firſt Line be ſuppoſed to contain Unity, as often as the Superficies fb doth, (for which purpoſe, Unity muſt be a Line ſet, and agreed on before,) the names of theſe Lines, when found, may be called g, h, k, and the Æquation may be written,

$+eee-gee+hce-kbc=0$.

Note III. But it is again to be Noted, That where the Lines f, b, and c, are commenſurable in Length, the three Lines k, h, and g, may be very eaſily found; for then they may be ſignified by Numbers, and if f be put for Unity, then $e=a$, and the Work fruſtrate: But when the ſaid Lines are incommenſurable in Length, this Reduction is always hard, if not impoſſible: For thoſe incommenſurable Lines do moſt commonly repreſent ſuch Surd Numbers, as cannot by any Reduction be compared.

RULE IV. *THE Æquation* $aaa-3bba=2ccc$, *or any other like it, by putting* $\frac{ee+bb}{e}=a$, *may, if* $c>b$, *be brought to* $eee=ccc+ddd$, *or if* $c=b$, *to* $eee=ccc$, *or laſtly, if* $c<b$, *then to* $eee=ccc+\sqrt{-d^6}$ *Which laſt may be called an impoſſible Æquation.*

Put e' b'' $\frac{bb'''}{e}$: And becauſe a is equal to the Summ of the Extremes, which are $e+\frac{bb}{e}$,

Therefore, from thence it will be

$$\left.\begin{matrix}\frac{+e^6+3bbe^4+b^4ee+b^6}{eee}\\ \text{And } \frac{-3bbe^4-3bbbbee}{eee}\end{matrix}\right\}=\left\{\begin{matrix}+2ccc\\ -3bba\end{matrix}\right.$$

Therefore rejecting the Contradictories, and Multiplying all by eee, it is

$+e^6+b^6=2c^3e^3$:

Therefore $+e^6-2c^3c^3e^3=-bbbbbb$;

And $+e^6-2c^3e^3+c^6-b^6$;

Therefore (for $e^3-c^3=\sqrt{e^6-2e^3c^3+c^6}$) $+e^3=ccc+\sqrt{c^6-b^6}$.

If now (in the firſt Caſe) c be greater than b.

Then put $c^6-b^6=d^6$:

Then it will be $eee=ccc+\sqrt{dddddd}$

That is, $eee=ccc+ddd$: Which is the Æquation promiſed in the firſt Caſe.

Secondly, If b be equal to c, then $c^6-b^6=0$ and it will eaſily follow, ſeeing (as is ſhewed above) that $e^6-2c^3e^3+c^6=0$, therefore the Root of it $e^3-c^3=0$: that is $eee=ccc$, the ſecond Æquation preſcribed.

Laſtly, By the third Caſe, ſeeing c is leſs than b, put $cccccc-bbbbbb=-dddddd$, then it will be $eee=ccc+\sqrt{-dddddd}$: the Æquation preſcribed in the third Caſe, and (becauſe of the inexplicability of $\sqrt{-dddddd}$) impoſſible.

COMPENDIUM.

Whereas Mr. *Harriot* ſaith, *Propter* $\sqrt{-d^6}$ *inexplicabilitem, &c.* The ſaid Quantity $\sqrt{-d^6}$ is not explicable, becauſe $-d^6$ ariſeth by multiplying $+d^3$ by $-d^3$, between which two there is no mean; for no one thing can produce d^6 but d^3 only, and $-d^6$ is not produced by $+d^3$ or $-d^3$, becauſe by both: This therefore may ſerve for a *Compendium* to ſave labour, which might elſe be loſt in ſeeking that which is impoſſible to be found.

Note, I uſe b^6 for $bbbbbb$, and b^4 for $bbbb$, and b^3c^3 for $bbbccc$, and the like (as *Des Cartes* hath done) only for abridgment, as in the Definitions of the Powers is already ſhewed: And $ccc+\sqrt{c^6-b^6}$, with that Line over, to diſtinguiſh between $\sqrt{c^6-b^6}$ as one Quantity, and $\sqrt{c^6}$ taken by it ſelf, and $-b^6$ taken apart alſo; for by ſuch miſtakes many great Errors enſue.

I will add no more, but theſe Four *RULES* may be multiplied by any one that doth not find theſe ſufficient for his purpoſe at his own pleaſure.

CHAP. VII.

Of REDUCTION *of* SOLIDS.

HAving ſpoken in *Chap.* VI. *Rule* 2. of making $bbc-2qqq=ddd$, and in *Rule* 4. of $c^6-b^6=dddddd$, I think it not amiſs (in this place) to ſhew how ſuch Addition and Subtraction of Solids may be performed.

And it may be Noted, That ddd is, for brevity ſake, there uſurped for ggc, or ſome other Solinomial Rectangle *Parallelipipedon* equal to the Binomial Rectangle Solid $bbc-2qqq$; for if this Binomial could (by Plain Geometry) be given in a Cube, as is ddd, ſomething elſe might be done, which is here omitted.

Now therefore, ſeeing $b=3q$, as there it is, the Æquation may be written,

$$9qqc-2qqq=ddd,$$
Or rather, $9qqc-2qqq=ggc$.

Make $\frac{qq}{c}=f$, therefore $2qqq=2qfc$:

Secondly, Make $9qq-2qf=gg$, which was firſt to be done.

Secondly, To reduce c^6-b^6 into one entire Solid, though not into a Squared Cube, as a^6, as is uſurped by Mr. *Harriot*, for brevity in writing, or facility in reaſoning (Page 100 of his Book,) ſuppoſing that be done, which cannot be done by ſtreight Lines and Circles hitherto.

Now therefore ſeeing c^6-b^6 is produced by Multiplication of $ccc+bbb$ into $ccc-bbb$, make $\frac{cc}{b}=f$, and $\frac{bb}{c}=g$, and $f+g=q$, and $f-g=p$: Therefore $bcf=ccc$, and $bcg=bbb$, and $bcq=ccc+bbb$.

Secondly alſo, $bep=ccc-bbb$, and therefore $bbccpq=cccccc-bbbbbb$, which was ſecondly to be done.

Example in Numbers.

Put $b=2$, and $c=3$.

Then $cccc^3=729$, and $bbbbbb=64$, and then $cccccc-bbbbbb$, that is $729-64=665$, which is produced by multiplying $27+8$ by $27-8$; that is 35 by 19. Now make $f=\frac{9}{2}$, and $g=\frac{4}{3}$; then $f+g=5\frac{5}{6}=q$, and $f-g=3\frac{1}{6}=p$. And $bcq=35$, and $bcp=19$. And laſtly, $bbccpq=665$.

Moreover, If you make $pq=xx$, the Solid is farther reduced to $bbccxx$, which although it be not a Squared Cube, yet it hath a Square Root, namely bcx, which may be of good uſe in many caſes to reſolve Æquations into Analogiſms, of which kind of Denomination, by help of the 14th of the 6th of *Euclid*.

Note, The three Caſes of the Æquation $a^3-3bb=2c^3$, mentioned in the beginning of the Fourth Rule of the laſt Chapter, are called by Mr. *Harriot*, the firſt, *Hyperbolical*; the ſecond, *Parabolical*; the third, *Elliptical*; becauſe of ſome ſimilitude between them and thoſe Sections: Of which three Caſes, the firſt is reſolved by a Conick Section, the ſecond by a Circle, and the third not at all.

Multiplication and *Diviſion* of Solids, is altogether as eaſie as *Addition* or *Subtraction*; for if one would divide ccc by bb, make $\frac{cc}{b}=x$; and again, make $\frac{cx}{b}=z$; then z is the Quotient required.

Example in Numbers.

Put $b=2$, and $c=3$, then $\frac{ccc}{bb}=6\frac{3}{4}$: to find which, make $\frac{cc}{b}=x-\frac{9}{2}$; then $cx=\frac{27}{2}$, and $\frac{cx}{b}=\frac{27}{4}=\frac{ccc}{bb}=6\frac{3}{4}$; as it ſhould be.

Again, If c^3 ſhould be divided by b, it is now $\frac{ccc}{bb}=z$, and multiplying by b, it is $\frac{ccc}{b}=bz$: Again, Multiplying by cc, it is $\frac{c^5}{b}=bccz$; and $bccz$ is the Quotient required.

But if it be required to bring the Quotient to a Biquadrate, make $bz=bd$; then $ccdd=bccz$, and make $cd=ff$; then the Quotient will be $fffff$.

Multiplication is naturally ſo eaſie, that there needs no more to be ſaid of it than what hath been ſaid already in *Chap.* 2.

Now of Æquations conſiſting of three Terms in continual Proportion, as $a^4+bbaa=c^4$: Or ſecondly, $a^6-bbba^3=c^6$: Or laſtly, Let it be $-a^8+bbba^4=c^8$; let them firſt be propoſed in Numbers, as $a^4+2aa=24$: If by *Rule* 1. of *Chap.* 3. it be wrought, it will be found to be $\sqrt{25}-1=aa$, and $aa=4$, or $a=2$.

Otherwiſe, If the Square of half the Co-efficient be added on both Parts, then
$$a^4+bbaa+1=25.$$
And their Square Roots alſo are equal, that is, $aa+1=5$, and $aa=4$, or $a=5$, as before, and the latter may prove the former.

Secondly, In the ſecond let it be
$$a^6-10aaa=459:$$
Add 25 to each Part, then it is
$$aaaaaa-10aaa+25=484.$$
Now each part of the Æquation is a Square, and their Roots alſo are equal; that is,
$aaa-5=22$; that is, $aaa=27$, and $a=3$.

Thirdly, In the laſt, if $-a^8+700a^4=46875$ from the Square of $\frac{700}{2}$, that is, from 122500, take the Homogeneal 46875, there remains 75625, whoſe Square Root is 275. And either $350+275$, or $350-275$, that is, either 625 or 75 is equal to $aaaa$, and $a=5$, or $\sqrt{qq975}=a$: which Character $\sqrt{qq}$, ſignifies the Biquadratical Root.

Note, The firſt and laſt of theſe Three Æquations, may be done as well in Lines as Numbers (by the ſaid three *RULES* of *Chap.* 3.) and ſo any Æquation of 4, 8, 16, or 32 Dimenſions; but Æquations of 6, 12, or 24 Dimenſions, cannot be effected ſo, becauſe there is ever one or more Cubick Roots to be extracted, which without two Means cannot be done.

For if it may, then I ſay, That two Means between any two Lines may thereby be found; for in the ſecond Æquation $a^6-bbbaaa=c^6$, by *Rule* 2. *Chap.* 3. $c^6+\frac{1}{4}b^6$ is a Square; make $\frac{cc}{b}=d$, then $bd=cc$, and $bbdd=cccc$, and $bbccdd=cccccc$; then make $\frac{1}{4}\frac{bb}{c}=f$; therefore $fc=\frac{1}{4}bb$, and $fcb^4=\frac{1}{4}b^6$. Now becauſe $\frac{bb}{c}=4f$, make $b=4f$, then $fccchh=\frac{1}{4}b^6$.

Make

Make $fc = ll$, then $ccbbll = \frac{1}{4}b^6$: Again, make $bb + bb = mm$, and $dd + ll = aa$; and then it will be $ccmmnn = bbccdd + ccbbll$; that is, $c^6 + \frac{1}{4}b^6$: To the Square Root hereof, cmn, add $\frac{1}{2}bbb$; thus make $\frac{1}{2}\frac{bb}{m} = p$; then $mp = \frac{1}{2}bb$, and $bmp = \frac{1}{2}bbb$. Make $\frac{bp}{n} = q$; then $mnq = \frac{1}{2}bbb$. Lastly, Make $c + q = x$, then it is $cnm + \frac{1}{2}bbb = mnx = aaa$, by *Rule* 2. of *Chap.* 3. Now if m, n, and x, be proportional, then the middlemost is equal to a; but that is uncertain, and cannot be made otherwise: But by making $rr = mn$, it will be $rrx = aaa$, and a will then be the leſſer of the two Means between r and x, if $r < x$, or the greater Mean if $r > x$. And ſo, if r and x had been given, and two Means between them required to be found; then by Retrogradation orderly one might come to the ſaid Æquation $a^6 - bbbaaa = c^6$, of which, if the Root a be found, two Means are alſo found between r and x, which was to be proved.

CHAP. VIII.

Of SURD NUMBERS.

RULE I.

THE Square Root of any Number being multiplied by that Number, produceth the Square Root of the Cube of the Number.

For $\sqrt{a}$ multiplied by a produceth $a\sqrt{a}$, but $a\sqrt{a} = \sqrt{aaa}$ for taking *equimultiplices* they will be equal; as if the firſt, namely, $a\sqrt{a}$, be multiplied ſtill by $\sqrt{a}$, the product is $a\sqrt{aa}$; that is, aa. And if $\sqrt{aaa}$ be multiplied by $\sqrt{a}$, it produceth $\sqrt{aaaaa}$; that is, aa alſo: wherefore $a\sqrt{a} = \sqrt{aaa}$. And therefore $3\sqrt{3} = \sqrt{27}$, either of which is the Cube of $\sqrt{3}$, and the like of all others.

RULE II.

Surd Numbers are multiplied and divided like Whole Numbers, the Product retaining ſtill the Character of the Root.

That is $\sqrt{2}$ multiplied by $\sqrt{3}$, produceth $\sqrt{6}$, and ſo of all others.

Note, Where I ſhall have occaſion (if any be) to ſpeak of a Cubick Root, I ſhall ſign it thus $\sqrt{c}$, and the Biquadratick Root thus $\sqrt{qq}$.

RULE III.

To multiply, Divide, Add, or Subtract the Roots of Surd Numbers, and firſt of,

MULTIPLICATION.

Beſides that which hath been ſaid in the laſt *Rule* above, theſe Roots of Surds may be multiplied and divided, and known by other names, ſo as ſometimes the Products, or Quotients ſhall be rational. Firſt therefore any Square Root doubled is the Square Root of the Quadruple. As,

$2\sqrt{5} = \sqrt{20}$, and $2\sqrt{20} = \sqrt{80}$.

$3\sqrt{5} = \sqrt{45}$, $4\sqrt{5} = 80$, $5\sqrt{5} = \sqrt{125}$.

$2\sqrt{10} = \sqrt{40}$, $3\sqrt{10} = \sqrt{90}$.

$4\sqrt{10} = \sqrt{160}$, and $5\sqrt{10} = \sqrt{250}$, *&c.*

Infinitely ſtill multiplying the Numerator 2, 3, 4, 5, *&c.* into it ſelf, and the Product into the Surd Number, as if $3\sqrt{10} = \sqrt{90}$, it ariſeth from 3 times 3 into the Surd Number $\sqrt{10}$, and the like of all others whatſoever.

For put $\sqrt{a} = \sqrt{10}$ to be multiplied by another Number, as by $a = 10$, the Product is $a\sqrt{a} = 10\sqrt{10}$; which by the firſt *Rule* is $\sqrt{aaa} = \sqrt{1000}$; that is, the Numerator 10 into it ſelf, making 100, which multiplied again by the Surd $\sqrt{10}$, gives 1000.

And if it had been at firſt $\sqrt{a} = \sqrt{10}$ multiplied by any other Number, as $e = 3$, the Product muſt by the ſame method be $e\sqrt{a} = e\sqrt{10}$; that is, (by the ſame reaſon as the former,) $\sqrt{eea} = \sqrt{ee10} = \sqrt{90}$.

And it is plain, that if any Root be multiplied by

2	the Product ſhall be the Root of the	*Quadruple,*
3		*Noncuple,*
4		*Sedecuple,*
5		*Vigintiquintuple,*
6		*Trigintiſextuple.*

And ſo forward infinitely, according to the Proportion of the Squares of the Multipliers.

Alſo by Decuplation, as if $5\sqrt{5} = \sqrt{125}$, then $5\sqrt{50} = \sqrt{1250}$. Or if $4\sqrt{4} = \sqrt{64}$, then $4\sqrt{40} = \sqrt{640}$; and (as before) if $4\sqrt{10} = \sqrt{160}$, then $4\sqrt{100} = \sqrt{1000}$. Alſo by Sub-decuplation, if $2\sqrt{10} = \sqrt{40}$, then $2\sqrt{1} = \sqrt{4}$.

Or if $5\sqrt{20} = \sqrt{500}$, then $5\sqrt{2} = \sqrt{50}$; and (according to that aforeſaid) $3\sqrt{37} = \sqrt{333}$, and $3\sqrt{36} = \sqrt{324}$, that is the Square Root of three times three times 36.

And this may often be of uſe, not only in Numbers but *Species*, and is therefore to be had in memory by him that would be ready in Multiplication of Surd Numbers, or Surd Quantities.

Farthermore, it may be uſefull to remember that in *Reciprocal Surds*, as $4\sqrt{5}$ and $5\sqrt{4}$, theſe two have that Proportion one to another, as 4 hath to a mean betwixt 4 and 5.

As for Example $4\sqrt{9}$, hath that Proportion to $9\sqrt{4}$, as hath 4 to 6, which is a mean betwixt 4 and 9; for $4\sqrt{9} = 12$, and $9\sqrt{4} = 18$; but 4^{I} 6^{II} 12^{I} 18^{II}, or more generally,

$a\sqrt{e^{I}}$

$a\sqrt{e}\ e\sqrt{a}$ ll $a\sqrt{ae}$ ll; for multiply the Means, it is $ae\sqrt{a}$; and multiply the Extremes, it is $a\sqrt{aee}$, and divide each of them by a, the first is $e\sqrt{a}$, the other is $\sqrt{aee}$; but by the former part of this *Rule* $e\sqrt{a}=\sqrt{aee}$; wherefore this is proved.

Consectary.

Hence it is evident, That Roots of themselves inexplicable may be so multiplied as the Product may be rational. For if $\sqrt{20}$ be multiplied by $4\sqrt{5}$, the Product will be $4\sqrt{100}=40$: For $2\sqrt{5}=\sqrt{20}$, and $2\sqrt{20}=\sqrt{80}$, therefore $4\sqrt{5}=\sqrt{80}$; but $\sqrt{80}$ multiplied by $\sqrt{20}$ gives $\sqrt{1600}=40$.

I need say nothing of Division, for that is no more but by the same steps to go back again, as $\sqrt{1600}$ divided by $\sqrt{80}$, the Quotient is $\sqrt{20}$; and so of the rest which hath been said in Multiplication.

NOTE,

These things being so, it will not be hard to find some Numbers to compare with any surd Number, so as to make that Work rational and exprimible which seemed not so; for there is not any Surd Number can be given, which may not by some Multiplication be made a rational Number.

For let it be $\sqrt{5}$, $\sqrt{7}$, $\sqrt{8}$, or any of these, as $\sqrt{7}$; multiply it first by $\sqrt{7}$, that produceth 7; but multiply $\sqrt{7}$ by any Square Number whatsoever, as by 4, omitting the Sign $\sqrt{}$, it gives 28, then again multiply $\sqrt{7}$ by $\sqrt{28}$, it produceth $\sqrt{196}=14$.

For this is all one as to multiply one Square Number by another, which must needs produce a Square Number. So here the Square Number 4 was multiplied by 7, and after by 7, that is, by 49, which Multipliers cannot produce any other than a Square Number, to wit, 196. *Eucl.* 9. 1.

And whatsoever hath hitherto been said of *Quadraticks*, may serve for *Cubicks* also, due respect always had to the degree of the quantity and Root; for any $\sqrt{c}$ multiplied by 2 gives $8\sqrt{c}$, by 3 it gives $27\sqrt{c}$, by 4 it gives $64\sqrt{c}$; that is, $2\sqrt{c}8=\sqrt{c}64$, and $3\sqrt{c}8=\sqrt{c}216$, and $3\sqrt{c}27=\sqrt{c}729$, the Proportion still increasing as the Cubes of their Multipliers.

And the like consideration had, this may be applicable to Biquadraticks, or any higher order.

And still whatsoever hath been said of Multiplication serves in a retrograde way for Division also.

RULE IV.

For *ADDITION.*

Surd Roots are usually added and subtracted by the Signs + and −, as the Square Root of 2 added to the Square Root of 8, the Summ is $\sqrt{2}+\sqrt{8}$, or subtracted, the Remainder is $\sqrt{8}-\sqrt{2}$.

But these may be added into one Summ, for seeing 8 is quadruple to 2, therefore $2\sqrt{2}=\sqrt{8}$, and the Summ is $3\sqrt{2}$, and the Remainder is $\sqrt{2}$. Likewise the reciprocal Surds $8\sqrt{2}=2\sqrt{8}$, are capable of Addition, Subtraction, Multiplication, or Division; for they are, being added, $3\sqrt{32}$, that is, $\sqrt{288}$; subtracted, $\sqrt{32}$; multiplied, $\sqrt{4096}$; divided $\sqrt{4}$; but such as are neither commensurable nor reciprocal cannot be amassed into one Summ.

And the Summ of the former Addition of $\sqrt{8}+\sqrt{2}$ being already reduced to $3\sqrt{2}$, may be yet farther reduced to $\sqrt{18}$; for $3\sqrt{2}$ is equal to the Square Root of three times three times two, as hath been more than once shewed.

And generally when the Surds given are denominated by Numbers in quadruple Proportion, as $\sqrt{2}$ to $\sqrt{8}$, and $\sqrt{3}$ to $\sqrt{12}$, *&c.* the lesser and the greater being twice added together, as 2 to 16, or 3 to 24, the Square Root of the Summ of the two Square Roots given to be added, that is, $\sqrt{2}+\sqrt{8}=\sqrt{18}$, and $\sqrt{3}+\sqrt{12}=\sqrt{27}$.

The reason is, $\sqrt{1}+\sqrt{4}=\sqrt{9}$, which 9 is composed of the lesser one, and the greater twice; that is as often as the $\sqrt{1}$ is contained in the $\sqrt{4}$.

But if the Numbers be prime one to another, they must be added or subtracted by the Signs + and −, for these *Rules* reach not to Primes.

And having said this little to acquaint such as are wont to be affraid of Operations where Surds are present, which this will render something easie, which perhaps seemed hard; and others which were hard, less difficult, I will now leave this ragged Subject, and recreate a little with a few Problems, the performing of which may serve to recall into Use and Practice that which hath been spoken of Solids in the former Chapter.

CHAP.

CHAP. IX.
PROBLEMS.

PROB. I.

ANY Right Line being given, to divide it into two parts, so as the Rectangle of the Whole and one of the Parts, may be to the Square of the other Part, in such Proportion as is betwixt any two Right Lines given.

Let the right Line given be b.
The Segment to be squared a,
Then the other Segment is $b-a$;
And let the two Lines given be r and s.
Then $bb-ba^{I}-aa^{II}$ r^{I} s^{II}.
And $raa=sbb-sba$. *per* 16. 6. *Euclid.*
That is, $raa+sba=sbb$.

Make $\frac{sb}{r}=d$, and divide all by r.

Then it is, $aa+da=db$; make $db=ff$.
Then lastly, it is, $aa+da=ff$. And a is easily found by *Rule* 1. of *Chap.* 2.

And if it had been required to have had the Rectangle $+$ or $-$ some other Plain to have had any limited Proportion to the Square aa, the Work had been almost the same with some small Addition.

PROB. II.

To make a Scalenon *Triangle, of which the Base, Perpendicular, and Proportion of the other Sides shall be given.* (I account that the Base which subtends the divided Angle.)

Let the Base given be b, the Perpendicular c, and the Proportion of the other Sides as r to s: of which let r be the lesser, and for the lesser Segment of the Base put a.

Therefore by supposition,

r^{I} s^{I} $\sqrt{cc+aa^{I}}$ $\sqrt{cc+bb-2ba+aa^{II}}$.

So that the Squares of them are also proportional: that is,

rr^{I} ss^{II}, $cc+aa^{I}$ $cc+bb-2ba+aa^{II}$.

And by Multiplying the Means and Extremes, it is $ssaa+sscc=rrcc+rrbb-2rrba+rraa$.

That is,

$ssaa-rraa+2rrba=rrcc+rrbb-sscc$.

Make $\frac{ss}{r}=x$.

And divide all the Æquation by r, then it is,

$xaa-raa+2rba=rcc+rbb-xcc$.

Secondly, Make $x-r=f$, and $gg=bb+cc$.
Then it is, $faa+2rba=rgg-xcc$.

Again, Make $\frac{rg}{f}=h$, and $\frac{xc}{f}=k$, and divide $faa+2rba=rgg-xcc$ by f.

Then it will be,

$$aa+\frac{2rb}{f}a=hg-ck.$$

Lastly, Make $\frac{2rb}{f}=q$, and $hg-kc=mm$.

The Æquation finally reduced will be then $aa+qa=mm$, and a may be found by the first *Rule* for Square Æquations, *Chap.* 2.

PROB. III.

Any Number being given, to find two other Numbers, so as all the three may constitute a Rectangle Triangle.

Unto the Square of the Number given add Unity, the half of the Summ shall be the *Hypotenuse*: Or from the said Square take Unity, the half of the Remainder shall be the middle Side.

For let the Number given be a, the Square is aa, to which adding Unity, the Summ is $aa+1$, the half whereof is $\frac{1}{2}aa+\frac{1}{2}$, for the *Hypotenuse*.

Secondly, From aa take Unity, the rest is $aa-1$, the half whereof is, $\frac{1}{2}aa-\frac{1}{2}$, for the middle Side.

But the lesser Side (by supposition) is a.

The Square of the lesser Side is aa.
The Square of the Middlemost is $\frac{1}{4}aaaa-\frac{1}{2}aa+\frac{1}{4}$
Both these Squares are $\frac{1}{4}aaaa+\frac{1}{2}aa+\frac{1}{4}$

But the Square of the *Hypotenuse*, *viz.* of $\frac{1}{2}aa+\frac{1}{2}$ is equal to these, that is, $\frac{1}{4}a^{4}+\frac{1}{2}aa+\frac{1}{4}$: Therefore by the 48th of the first of *Euclid*, the Proposition is proved.

Corollary.

Hence it is plain, that the two greater Sides of any Rectangle Triangle differ by Unity, for if two Squares differ by two, their halves differ by one.

Note, If it be required to have all the three Sides in Whole Numbers, then the lesser Side must be an odd Number.

PROB. IV.

The Difference of the Sides of a Rectangle, *with the* Area *and* Diagonal *in one Summ being given in Numbers, to find out the Sides.*

Let the difference of the Sides be 7.
And the *Area* and Diagonal together 73.
And put the lesser Side equal to a.
Then the greater is $a+7$.

These two multiplied produce $aa+7a$, which is equal to the *Area*.

And therefore $73-aa-7a$ is the Diagonal.

The

The Square of which is,
$+5329-146aa$
$+aaaa+14aa+$
$+49aa-1022a$

Which reduced, and rightly ordered is,
$+aaaa+14aaa-97aa-1022a+5329.$

Which by the 47th of the first of *Euclid*, is equal to the two Squares of the other Sides a, and $a+7$, whose Squares are aa, and $aa+14a+49$.

That is, $+aaaa+14aaa-97aa-1022a+5329=2aa+14a+49.$

That is, $+aaaa+14aaa-99aa-1036a+5280=0.$

That is, $-aaaa-14aaa+99aa+1036a=5280.$

In which Æquation, becauſe $aaaa$ hath four dimentions, and the Homogeneal 5280, but four places, the Root a cannot conſiſt of more than one Place, or Figure, which muſt be found out by trying every one of the nine Digits, if need be, and will be found at laſt to be 5: therefore the other Side is $5+7=12$, the *Area* 60, and Diagonal 13.

But if a had been more or leſs than 5, yet (except ſomething elſe lead a readier way) it is good to try 5 at firſt, if it be too little, then 7; if that alſo be too little, then 9; ſo there will be no need to try the even Numbers, 6, 8, *&c.* for if 5 bee too little, and 7 too great, it muſt be 6; the like reaſon will ſerve for 8, 4, 2; ſo that he which gueſſeth moſt unfortunately need not try above four or five digits, which is no great matter, the like happening ſometimes in ſeeking the Quotient in plain Diviſion, for no Man is ſure to gueſs right at firſt.

But that we may exemplifie this in bigger Numbers, where a may conſiſt of more places.

Let the difference of Sides be 71.

The *Area* and *Diagonal* together 1177.

Working as in the former Example, there will ariſe an Æquation, which being reduced and ordered as before, will be,

$-a^4-142aaa-2685aa+167276a=1380288.$

And putting $b+c=a.$

Then the Canon of the Reſolution will be,

$-bbbb-4bbbc-6bbcc-4bccc-cccc$
$-142bbb-426bbc-426bcc-142ccc$
$-2685bb-5370bc-2685cc$
$+167276b+167276c.$

To be orderly ſubtracted from the Homogeneal Number given 1380288, as followeth.

The Number given	$+1380288$
The firſt ſingle Root $b=1$	
$-bbbb$	1.0000
$-142bbb$	142.000
$-2685bb$	2685.80
In all	4205.00
$+167276b$	167276.0
Subtr. (the diff. of + and −)	$+125226\overline{0}$
Remains of the Number given	$+012802\overline{8}$

The firſt Root decuplate $b=10$

$-4bbb$	4000
$-6bb$	0600
$-4b$	0040
$-426bb$	42600
$-426b$	4260
$-5370b$	53700
Then	-105200 is all the −
And	$+167276$ is all the +

Diviſor $+62076$ is their difference.

The ſecond ſingle Root $c=3$

Remains of the Number given $+012802\overline{8}$

$-4bbbc$	12000
$-6bbcc$	.5400
$-4bccc$	.1081
$-ccccc$	..81
$-426bbc$	127800
$-426bcc$	.38340
$-142ccc$	..3834
$-5370bc$	161100
$-2685cc$	.24165
In all	-373800

$+167276c=501828$
all the leſs being 373800
The difference is $+128028$

Which being taken from the Remains of the Number given $+128028$, there remains finally nothing; ſo that the given Æquation is juſtly reſolved by the Root $b+c=13$.

The leſſer Side a is therefore 13, to which if the difference given, namely 71, be added, the middle Side 84 is thereby compoſed.

Again, if to that middle Side 84, be added Unity, the *Hypotenuſe* of a Right-angled Triangle is compoſed, whoſe three Sides are 13, 84, 85.

The *Superficies* of this Triangle is half the Parallelogram, or Rectangle required.

For 84 multiplied by 13, gives 1092, for the *Area* of the Rectangle, to which adding 85 the Diagonal, compoſeth the Number 1177, as was required in the Propoſition.

COMPENDIUM.

Seeing the two greater Sides of any Rectangle Triangle exceed one another by Unity, (as by the former *Corollary*,) the difference betwixt the two leſſer Sides being given, the difference betwixt every two Sides is alſo given.

So that putting a for the leſſer Side of the Rectangle, the greater Side is $a+71$, and the Diagonal $a+72$, whoſe Square is $+aa+144a+5184$, to which the two Squares of the Sides being $aa+aa+142a+5041$, are equal. That is,

$2aa+142a+5041=aa+144a+5184.$

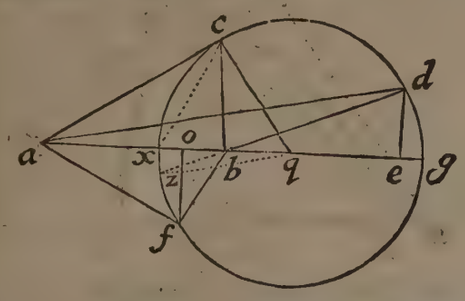
And ſubtract. from each part $aa+144a+5041$, there will remain $+aa-2a=143$. And a will be found 13, by the ſecond *Rule* of *Chap.* 2.

RESUMPT.

RESUMPT.

In the ſecond Problem of this Chapter it hath been ſhewed, How upon a Baſe and Perpendicular, and Proportion of the remaining Sides given, to deſcribe a Triangle.

It is there to be underſtood of an Acute-angled Triangle, in which the Perpendicular falls within the Triangle. Now therefore let it be otherwiſe.



As in this Figure of the Triangle *adb*, let there be given the Baſe *ab=b*, the Perpendicular *de=c*, and the Proportion of the other Sides, namely *bd=r*, and *da=s*; whereas in the ſecond Problem, the leſſer Segment of the Baſe was called *a*, and the greater *b—a*: Now here the Line *be* may be called *a*, and *ae* ſhall be *b+a*, and the reſt of the Work will be like that before in *Prob.* 2.

NOTE:

And it may be Noted, That if from *a* and *b* (the ends of the Line *ab*) be drawn other binary Lines, how many ſoever, ſo as they hold the ſame Proportion as *s* to *r*, and concurr in other Points, as *c*, *f*, *g*, *&c.* thoſe Points are all in the Circumference of a Circle whoſe Centre is in the Line *ab*, produced towards *g*.

For upon *ab* deſcribe the Rectangle Triangle *abc*, whoſe two Sides *ac*, *cb*, may be as *s* to *r*, and divide the Angle *acb* into two equal Parts by the right Line *cx*, and draw *cq* perpendicular to *ac* in *c*; then the Angle *xcq=90—xca*; likewiſe the Angle *cxq=90—xcb*, but *xcb=xca*, therefore *xcq=cxq*, and *cq=xq*.

And becauſe of the ſimilitude of the Triangles *acq* and *cbq*, it is *aq' cq'' bq' ''*; that is, *aq' xq'' bq' ''*. Now by ſuppoſition *ag' bg'' ac' bc''* And it hath been proved *aq' xq'' bq' ''*. Therefore by Compoſition alſo it will be,

aq+xq' xq+bq'' xq' bq''.

But *ac' bc'' aq' xq''*. And *xq' bq'' aq' xq''*. Therefore *ag' bg'' aq+xq' xq+bq''*. And therefore *ag=aq+xq*. And *xq=gq*. But it hath been proved that *xq=cq*, therefore *xq=cq=gq*.

Again making the Centre *q*, and the ſpace *xq*, *cq*, or *gq*; deſcribe the Circle *xcg*, and prolong *db* to the Circumference in *z*, and draw *qz*: And becauſe it hath been proved above, that *ag' bg'' ac' bc''*.

That is *ag' bg'' xq' bq'*, therefore alſo by Diviſion *ag—bg' bg'' xq—bq' bq''*.

That is *ab' bg'' xb' bq''*. Therefore the Rectangles *abq=xbg*. *Eucl.* 6. 16.

And becauſe by Suppoſition *ac' bc'' ad' bd''*. And *ac' bc'' qc'' qb''*. Therefore *ad' bd'' qc' qb''*, but *qc=qz*, therefore *ad' bd'' qz' qb''*, and the Angles *abd*, *zbq*, being equal, the Triangles *abd*, *zbq*, are equiangular. *Eucl.* 6. 6.

And therefore *ab' db'' bz' bq*. *Eucl.* 6. 4. and the Rectangles *dbz=abq*. *Eucl.* 6. 16. But it was now proved, that *abq=xbg*, therefore *xbg=dbz*.

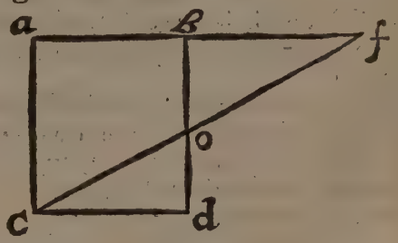
So that the Points *xzg*, being in the Circumference of the Circle *xzg*, the Point *d* muſt be in the ſame Circumference, *Eucl.* 3. 35.

The like Proof may ſerve to ſhew that the Point *f* is in the ſame Circumference; which is all that was to be proved.

This Circumference however deſired by the Ancients, effected by modern Mathematicians, ſeems yet to have little uſe, more than to help the conſtruction of the Triangle, which (but now I ſhewed) may be done without it.

PROB. V.

In any Rectangle [b d c a] *given, from an Angle in it,* [c,] *to draw a Right Line,* [c f] *cutting one oppoſite Side* [b d *in* o,] *and concurring with the other,* [b a,] *produced in* [f,] *ſo as the intercepted Line* [f o] *may be equal to* [z] *any other Right Line given.*



Put *bd=b* *ed=d*
do=a *bf=e*.

And becauſe the Triangles *bof*, *dce*, are like, Therefore it is *b—a' a'' e' a''*,

And *db—da=ae*, that is, *ae+da=db*.

So we are quickly come to the Æquation required, which Æquation having as many unknown Quantities (as *a*, *e*,) as known (to wit, *b* and *d*,) is hitherto uſeleſs.

That the Problem therefore may be ſolved, we muſt work another way, and bring it to a Solid Æquation, by making (for more convenience) *cd=b*, *fo=c*, and *bd=d*, and *bo=a*.

Then $d—a'\ a''\ b'\ \frac{ba''}{d—a}$ and $\frac{ba}{d—a}=bf$,

And the Square thereof $\frac{bbaa}{dd—2da+aa}+aa$ is equal to *cc*, by the 47th of the 1. of *Euclid*.

Multiply all by the Denominator *dd—2da+aa*

It makes

$ddaa-2daaa+aaaa+bbaa=ddcc-2dcca+ccaa,$

That is,

$aaaa-2daaa+bbaa-ccaa+ddaa+2dcca=ddcc.$

Make $bb+dd-cc=ff$, then ff shall be signed $+$ because here by supposition it shall be $bb+dd>cc$. And the Æquation will be $aaaa-2daaa+ffaa+2dcca=ddcc$.

Expunge the second term which is $-2daaa$, by the second *Rule* of the 6th *Chap.* And because the Rule is not fully exemplified there in the operation thereof, I will here work it at large. Because *aaaa* hath four Dimensions.

Therefore make $4'$ $1''$ $2d'$ $\frac{1}{2}d''$.

Again, because the first and second term have different signs, therefore put $e+\frac{1}{4}d=a$. *Chap.* 6. *Rule* 2.

The new Æquation arising thereof will be,

$$\left.\begin{array}{r} +e^4+2de^3+\frac{6}{4}ddee+\frac{4}{8}ddde+\frac{1}{16}dddd \\ -2de^3-3ddee-\frac{6}{4}ddde+\frac{2}{8}dddd \\ +ffee+dffe+\frac{1}{4}ddff \\ +2dcce+ddcc \\ -ddcc \end{array}\right\}=0$$

The Homogeneal $-ddcc$, is here put on the same side with the rest, because (for the present) it seems better to stand so, that it may be the last term, in relation to that which is gone before.

In this last Æquation, it is manifest that the second term $2deee$, is (through contraction of $+$ and $-$) abolished, as was required.

And now because the Quesititious Root e must be found by help of a *Parabola*, as before in the like case was used, it is necessary to reduce the Æquation to some such form as hath been shewed before.

First therefore to reduce the third term, because $d>f$, and $+\frac{6}{4}dd$, taken from $-3dd$, rests $-\frac{6}{4}dd>ff$; make $\frac{6}{4}dd-ff=gg$, so all of the third term shall be $-ggee$.

Likewise for the fourth term, if

$$+\tfrac{4}{8}ddd-\tfrac{6}{4}ddd+dff,$$

be summed up together the aggregate will be $-ddd+ffd$; make $dd-ff+2cc=hh$, then all the fourth term will be $+dhhe$.

Now for the last term, $-\frac{1}{4}d^4-\frac{1}{16}dddd=\frac{5}{16}dddd$, and therefore making $\frac{1}{16}dd-\frac{1}{4}ff=ll$, the aggregate of the least Term is thereby $-ddll$, for $ddcc$ is through contraction of the signs annulled.

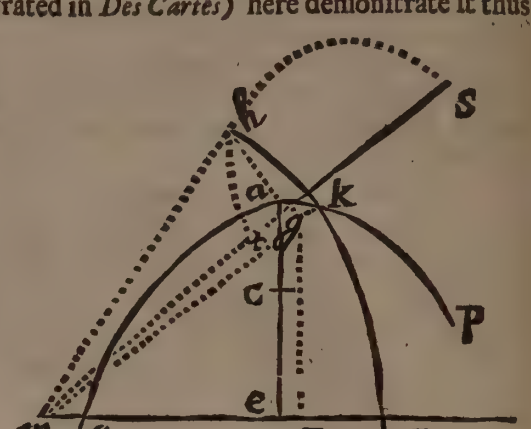
And now the Æquation is,

$$eeee-ggee+dhhc-ddll=0,$$

Make $\frac{gg}{d}=m$, and $\frac{hh}{d}=n$, and $\frac{ll}{d}=p$; then the Æquation will be $eeee-dmee+ddne-dddp=0$, and making $d=1$, then the Æquation fully reduced, and rightly prepared is $+eeee-mee+ne-p=0$.

Or $eeee=mee-ne+p$, which is altogether the same with that in the former Chapter, and the working of it is there shewed.

Except only because there the quantity f is signed $-$, and here the like quantity p is signed $+$, I shall (although this case only is demonstrated in *Des Cartes*) here demonstrate it thus:



Describe the *Parabola* fap, according to the Parametre d, (that is as,) and let ae be the *Axis*, and make $ac=\frac{1}{2}d$, $ce=\frac{1}{2}m$, and at right Angles at e make $me=\frac{1}{2}n$, and draw the Line mas, making $as=d$, and $ax=p$, and upon xs, as a Diametre, describe the Semicircle xhs, and from a to h raise the Perpendicular ah, cutting the Circle in h, and with the *Radius* mh describe the arch hky, cutting the Section in k, and from k let fall a Perpendicular to me produced in q, and draw the Lines mk and mh:

DEMONSTRATION.

To prove $gk=e$.

Suppose it done, and because $kg=qe=e$, and $me=\frac{1}{2}n$, therefore $mq=\frac{1}{2}n+e$, and the square of it is $\frac{1}{4}nn+ne+ee$.

And because $ae=\frac{1}{2}d+\frac{1}{2}m$, and $gk=e$, and because of the *Parabola* $ga=ee$, therefore also eg or $qk=\frac{1}{2}m+\frac{1}{2}d-e$ e, and the Square thereof is $+\frac{1}{4}mm+\frac{1}{2}md+\frac{1}{4}dd-mee-dee+eeee$. That is (because d is equal to Unity)

$$+\tfrac{1}{4}mm+\tfrac{1}{2}m+\tfrac{1}{4}-mee-ee+eeee,$$

to which add the former Square of $\frac{1}{2}n+e$,

And then the whole is

$$+\tfrac{1}{4}mm+\tfrac{1}{2}m+\tfrac{1}{4}-mee-ee+e^4+\tfrac{1}{4}nn+ne+ee,$$

That is,

$$\tfrac{1}{4}mm+\tfrac{1}{2}m+\tfrac{1}{4}-mee+e^4+\tfrac{1}{4}nn+ne,$$

equal to the Square of the *Radius* mk.

Again, because $me=\frac{1}{2}n$, the Square of it is $\frac{1}{4}nn$, and because $ae=\frac{1}{2}m+\frac{1}{2}$, the Square of that is $\frac{1}{4}mm+\frac{1}{2}m+\frac{1}{4}$, which added together make $\frac{1}{4}nn+\frac{1}{4}mm+\frac{1}{2}m+\frac{1}{4}$, for the Square of ma, to which add the Square of ah, that is p, (for $ax=p$, and $as=1$, and consequently by *Eucl.* 6. 13. the Square of ah is equal to p,) the aggregate is $\frac{1}{4}nn+\frac{1}{4}mm+\frac{1}{2}m+\frac{1}{4}+p$, equal to the Square of the *Radius* mh, but $mh=mk$. And therefore this aggregate is equal to the former aggregate, and one may see that these Quantities $+\frac{1}{4}mm+\frac{1}{4}nn+\frac{1}{4}+\frac{1}{2}m$, are common to both; and therefore the residuals are equal.

Namely,

Namely, $+eeee-mee+ne=+p$,
That is in *Des Cartes* his form.
$eeee=+mee-ne+p$,
Which was the Æquation to be resolved, and therefore $gk=e$, which was to be proved. And therefore if gk be added to $\frac{1}{2}d$, the Summ is equal to a the Quesitious Root of the first Æquation, and is equal to bo. So that the Point o which was first sought, is hereby found; and the Problem satisfied; which was to be done.

ADVERTISEMENT.

Now that the different ways of writing Æquations may cause no confusion, let it be supposed to be written ever thus, eeee, mee, ne, p=o, *the Signs* + *and* — *to be supplied as the occasion requires.* Then,

1. If it be $-m$, the centre of the desired Circle is within the Section, or at least below the *Vertex*.

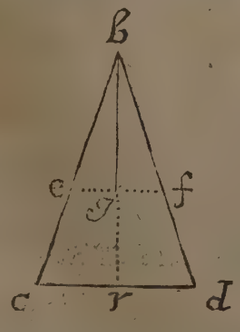
2. If it be $+m$, the said centre is above the *vertex*, and $\frac{1}{2}m$ is applied upon the *Axis* produced, from the Point c, which is always in the *Axis* within the Section, distant from the *Vertex* a by half the Parametre; and therefore in this case the Line m cannot be less than the Parametre d, otherwise the Point d would still fall within the Section.

3. If it be $+n$, those Perpendiculars (let fall from the several intersections of the *Parabola* and Circle to the *Axis*,) which are on the same side with the Centre, are the false Roots, and the other the true Roots, but if it be $-n$, then just contrary.

4. Lastly, if it be $-p$, then one auxiliary Circle will serve, as here it doth; but if it be $+p$, then there must be another also.

PROB. VI.

Upon a Line given as a Base to describe an Isosceles *Triangle, so that an inward parallel Base may cut off two Segments of the Sides betwixt the Bases, so that either Segment may be equal to the inward Base, the Perpendicular from the* Vertex *to the said inward Base being also by supposition given.*



Let there be given the Line cd, and the Line bg.

And let it be required upon cd, as a Base, to describe the *Isosceles bcd*, so as the Line bg, falling at right Angles with fe, equidistant to cd, the Lines df, ef, and ec, may be equal each to other.

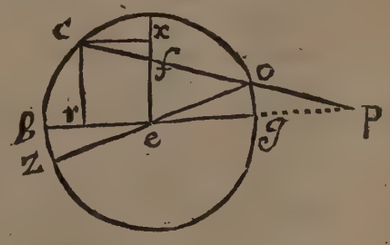
Put $gr=e$, and $fe=a$, and $cd=b$, and $bg=d$, then d', $d+e''$ a' b''.

That is $da+ea=db$. *Eucl.* 6. 16. which is the same Æquation as was first in the former Problem; and therefore if there be in the room of $da+ea=db$ substituted another Æquation like that in the former, such is the Æquation $a^4-2daaaa+ffaa+2dcca-ddcc=0$. And that purged from the second term, as before, there ariseth a second Æquation,
$+eeee-mee+ne-p=0$.
And lastly, If d be a Parametre, according to which a *Parabola* is described, the Root e, and consequently the Root a, may be found as in the former.

And thus having shew the method general for all Æquations which attain but 3 or 4 Dimensions, and exemplified it by Problems which lead to such Æquations; I now say that was the end of my present business. And if any still desire a longer reach, I referr him to *Des Cartes*, who hath proceeded to Æquations of 5 and 6 Dimensions; by which four Mean Proportionals, and quinquisection of Angles, and other sursolid Problems may be found and effected.

NOTE I.

The fifth Problem of this Chapter, as it is more composed than trisection, so it comprehends it, as may be seen by (not only *Pappus* and others who applied it to that end, but) the following Example. In which let there be an Arch of a Circle, namely bc, and let it be required to divide the Arch bc into three equal parts, or (which is as good) to find the third part of the Arch bc.



Find out the Centre e, and describe the Circle, and draw the Diametre bg, and produce it to p, or farther, as is needfull, and make cr the right Sine of the Arch bc, and from the Centre e draw ex, parallel to cr, and complete the Rectangle $cxer$, and by the fifth Problem, draw cp to cut ex, in f, so as fp may be equal to bg, then I say that go is equal to a third part of bc.

From o through e, draw oez.

Now because eo is *Radius*, fp the Diametre, and the Angle fep a right Angle, therefore the Lines fo, po, eo, are severally equal.

And the Angles $z\,e\,b = g\,e\,o$, and $p\,e\,o = e\,p\,o$. And alſo $f\,o\,e = 2\,p\,e\,o$. *Eucl.* 1. 32. Therefore alſo $f\,o\,e = 2\,z\,e\,b$.

But $f\,o\,e$, that is $c\,o\,z$, being in the Periphery, is meaſured by half the Arch $c\,z$, *Euclid* 3. 20. wherefore $b\,z$, which is the meaſure of half the Angle $c\,o\,z$, is a fourth part of the whole Arch $c\,z$, and conſequently a third part of $b\,c$, and therefore $g\,o$, which is equal to $b\,z$, is alſo equal to a third part of $b\,c$, which was to be proved.

NOTE II. *Mechanically.*

Seeing in this Scheme the Line $f\,o$ is ever equal to the Semidiametre $e\,g$, if in common practice there is $b\,c$, or any other Arch (not greater than a Quadrant) to be triſected, laying a thin Ruler to touch the Point c, and cut the diametre $b\,g$, produced in p, the Point required, the Compaſſes being open to the juſt Length of the *Radius* $e\,g$, ſetting one foot in $e\,x$, and ſhifting the Ruler till the other foot fall in the Periphery at o, the Point o ſhall always be diſtant from g, by a third of $b\,c$, the doing of which (although it muſt not be called Mathematical, yet) is very near as eaſie, and as free from erring as from a Point given to a Point given to draw a ſtreight Line, or upon a Centre given with a diſtance given to deſcribe a Circle; and from a given Point in it to ſet off an Arch equal to an Arch given: And therefore I recommend this as a moſt ſimple and ſhort and ſafe way for Mechanick uſe.

NOTE III.

If the Arch to be triſected be greater than a Quadrant, then triſect the Complement thereof to a Semicircle; and the third of this Complement taken from 60 Degrees (which is always a given Arch) leaves the third of the Arch required.

Inſcription of Chords in a Circle, and making equi-crural Triangles, whoſe Angle at the Baſe ſhall be to the Angle at the *Vertex* in any given Proportion, are the ſame thing; for to inſcribe a figure of 3, 4, 5, 6, 7, 8, 9, 10, 11, ſides *&c.* finds ſuch Triangles whoſe ſaid Angles ſhall be as $\frac{2}{2}, \frac{3}{2}, \frac{4}{2}, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}, \frac{8}{2}, \frac{9}{2}, \frac{10}{2}$, *&c.* as is eaſie to be ſeen by the operation.

Quadrature of the Circle is that in which (as yet) only *Archimedes* hath laboured with any ſucceſs, he having demonſtrated that the Circumference is to the Diametre leſſer than as 22 to 7, and greater than $21\frac{70}{71}$ to 7, within which ſtrict limits, a French Man many years ſince found it to be in whole Numbers thus:

As the whole Circumference is to the Perimetre of the inſcribed Square, ſo is 10 to 9, that is, Quadrant' Subtenſe", 10' 9", which is eaſie for practice; and may be proved thus:

Put the Diametre $= 7$, the half $= 3\frac{1}{2}$, of which the Square is $= \frac{49}{4}$, and doubled is $\frac{49}{2}$, whoſe Square Root is the ſide of the inſcribed Square. The whole Perimetre therefore is $4\sqrt{\frac{49}{2}}$, and the whole Circumference is found by this Analogiſm:

$$9' \quad 10'' \quad 4\sqrt{\tfrac{49}{2}}' \quad \tfrac{40}{9}\sqrt{\tfrac{49}{2}}''$$

It reſts to be proved that $\frac{40}{9}\sqrt{\frac{49}{2}}$ is greater than $21\frac{70}{71}$, and leſſer than 22.

Now 4.949 is ſomething leſs than $\sqrt{\frac{49}{2}}$, which multiplied by 40, and divided by 9, the Quotient is 21.995, which yet is greater than $21\frac{70}{71}$, for $\frac{995}{1000} > \frac{70}{71}$.

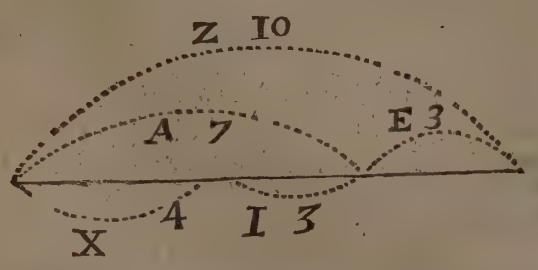
Again 4.950 is ſomething greater than $\sqrt{\frac{49}{2}}$, which multiplied by 40, and divided by 9, as before, the Quotient is 22; ſo that $\frac{40}{9}\sqrt{\frac{49}{2}} < 22$, and $\frac{40}{9}\sqrt{\frac{49}{2}} > 21\frac{70}{71}$. Which was to be proved.

ÆQUATIONS

Ariſing from a Quantity divided into Two unequal Parts:

AND

The Second BOOK of *EUCLIDE*'s ELEMENTS demonſtrated by *Species*.



If Z 10 be divided into A 7 and E 3.

1. Then $\left\{\begin{matrix} Z=A+E \\ 10=7+3 \end{matrix}\right\}$ Becauſe the Whole is equal to all its parts.

2. And $\left\{\begin{matrix} Z-E=A \\ 10-3=7 \end{matrix}\right\}$ By Tranſpoſition.

3. And $\left\{\begin{matrix} Z-A=E \\ 10-7=7 \end{matrix}\right\}$ By Tranſpoſition.

Becauſe A is ſuppoſed the greater Part, from it take I = E, the leſſer Part, and let the Remainder, which is the difference of the Parts, be noted with X.

But I + P = A, becauſe all the Parts are equal to the Whole, but E = I by ſuppoſition.

4. Therefore $\left\{\begin{matrix} E+X=A \\ 3+4=7 \end{matrix}\right\}$ By Interpretation.

5. And

5. And $\left\{\begin{matrix} E=A-X \\ 3=7-4 \end{matrix}\right\}$ By Tranſpoſition.
6. And $\left\{\begin{matrix} X=A-F \\ 4=7-3 \end{matrix}\right\}$ By Tranſpoſition.
But A—X=E by the fifth Æquation.
And Z—A=E by the third Æquation.
7. Therefore $\left\{\begin{matrix} A-X=Z-A \\ 7-4=10-7 \end{matrix}\right\}$ By Interpretat.
8. And $\left\{\begin{matrix} 2A-X=Z \\ 14-4=10 \end{matrix}\right\}$ By Tranſpoſition.
9. And $\left\{\begin{matrix} 2A=Z+X \\ 14=10+4 \end{matrix}\right\}$ By Tranſpoſition.
10. And $\left\{\begin{matrix} A=\frac{1}{2}Z+\frac{1}{2}X \\ 7=5+2 \end{matrix}\right\}$ By Diviſion.
11. And $\left\{\begin{matrix} X=2A-Z \\ 4=14-10 \end{matrix}\right\}$ By Tranſpoſition.
But E+X=A, by the fourth Æquation.
And Z—E=A, by the ſecond Æquation.
12. Therefore $\left\{\begin{matrix} E+X=Z-E \\ 3+4=10-3 \end{matrix}\right\}$ By Interpret.
13. And $\left\{\begin{matrix} 2E+X=Z \\ 6+4=10 \end{matrix}\right\}$ By Tranſpoſition.
14. And $\left\{\begin{matrix} 2E=Z-X \\ 6=10-4 \end{matrix}\right\}$ By Tranſpoſition.
15. And $\left\{\begin{matrix} E=\frac{1}{2}Z-\frac{1}{2}X \\ 3=5-2 \end{matrix}\right\}$ By Diviſion.
16. And $\left\{\begin{matrix} X=Z-2E \\ 4=10-6 \end{matrix}\right\}$ By Tranſpoſition of the 13th Æquation.

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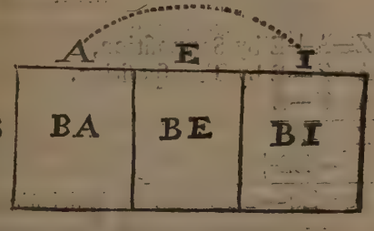
Propoſition I.

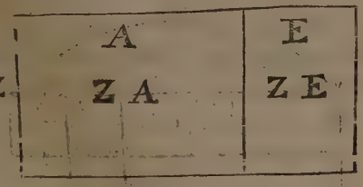
If there be two Right Lines, and the one of them be divided into any Parts, the Rectangle comprehended by the two Right Lines is equal to the Rectangles comprehended by the undivided Line, and each Segment of the divided Line.



If Z=A+E+I by Suppoſition,
Multiplied by B.
Then ZB=BA+BE+BI, by Multiplication.

Propoſition II.

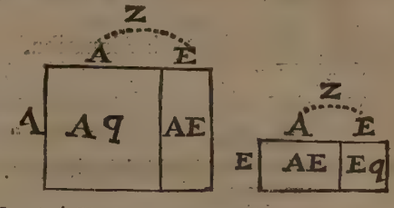
If a Right Line be divided at pleaſure, the Rectangles comprehended by the Whole and each Segment, is equal to the Square made of the Whole.



If Z=A+E, by Suppoſition.
Z
Then Zq=ZA+RE, by Multiplication.

Propoſition III.

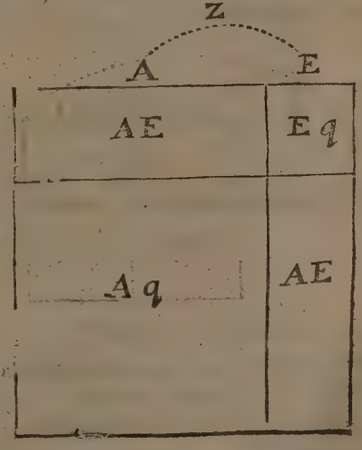
If a Right Line be divided at pleaſure, the Rectangle comprehended by the Whole and one Segment, is equal to the Rectangle of the Segments, and the Square of the ſaid Segments.



If Z=A+E, by Suppoſition,
A
Then ZA=Aq+AE
And ZE=AE+Eq } by Multiplication.

Propoſition IV.

If a Right Line be divided at pleaſure, the Square which is deſcribed on the Whole Line is equal to the Squares of the Segments, and to twice the Rectangle of the Segments.

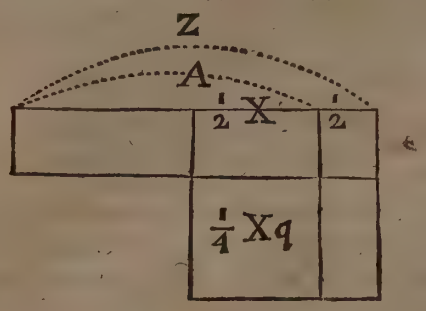


If Z=A+E
Z=A+E } by Suppoſition,
Aq+AE
AE+Eq
Zq=Aq+2AE+Eq by Multiplication.

Propo-

Propoſition V.

If a Right Line be divided into equal and unequal Parts, the Rectangle comprehended under the unequal Segments of the Whole, with the Square of the intermediate Part, is equal to the Square of the half Line.



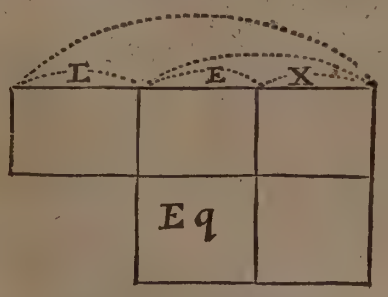
For, If Z=A+E by Suppoſition,
Then A=$\frac{Z+X}{2}$ by the tenth Æquation.
And E=$\frac{Z-X}{2}$ by the 15th Æquation.

$\frac{Zq+ZX}{ZX-Xq}$

Then AE=$\frac{Zq-Xq}{4}$ by Multiplication.
And AE+$\frac{Xq}{4}$=$\frac{Zq}{4}$ by Tranſpoſition.

Propoſition VI.

If a Right Line be divided into two equal Parts, and unto it another Right Line be added, the Rectangle comprehended by the Whole, with the Part added, together with the Square of the Half, is equal to the Square which is made of the Half and Part added, as of one Line.



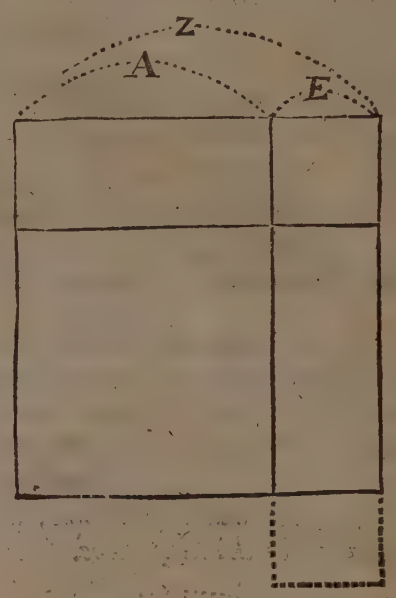
If 2E+X=Z by Suppoſition,
And E+X=A by the fourth Æquation,
Then Z=A+E by Interpretation,
And X=A—E by the ſixth Æquation.

$\frac{Aq+AE}{-AE-Eq}$

Then ZX=Aq—Eq by Multiplication.
And ZX+Eq=Aq by Tranſpoſition.

Propoſition VII.

If a Right Line be divided at pleaſure, the Squares of the Whole Line and one of the Segments together, are equal to twice the Rectangle of the Whole, and the ſaid Segment, and the Square of the other Segment.



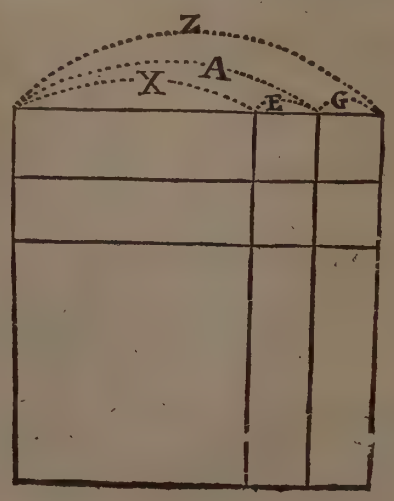
If Z=A+E by Suppoſition,
ThenA=Z—E by Tranſpoſition.

$\frac{A=Z-E}{Zq-ZE}$
$\frac{-ZE+Eq}{}$

And Aq=Zq—2ZE+Eq by Multiplication,
And 2ZE+Aq=Zq+Eq by Tranſpoſition.

Propoſition VIII.

If a Right Line be divided at pleaſure, the four Rectangles comprehended by the Whole and one of the Segments, with the Square of the remaining Segment, is equal to the Square which is made of the Whole Line, and the Segment as one Line.



If

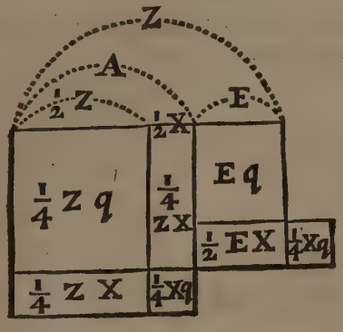
If $Z=A+E$ by Supposition,
Then $2A=Z+X$ by the Ninth Æquation.
And $2E=Z-X$ by the Fourteenth Æquation.

$Zq+ZX$
$-ZX-Xq$

$4AE=Zq-Xq$ by Multiplication,
And $4AE+Xq=Zq$ by Transposition.

Proposition IX.

If a Right Line be divided into two equal Parts, and into two unequal Parts, the Squares which are made of the unequal Segments of the Whole Line, are double to the Squares of the Half Line and intermediate Segments.



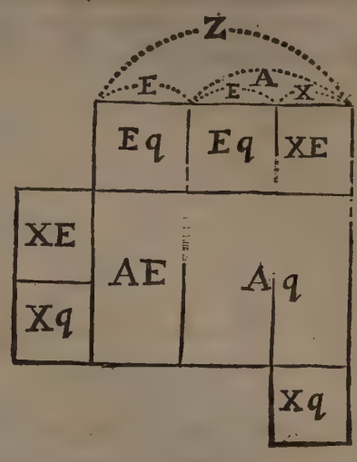
If $Z=A+E$ by Supposition,
Then $\left\{\begin{matrix}X=A-E\\X=A-E\end{matrix}\right\}$ by the Sixth Æquation.

$AA-AE$
$-AE+EE$

Then $XX=AA-2AE+EE$ by Multiplication.
And $ZZ=AA+2AE+EE$ by 4. Prop. *Eucl.* 2.
Therefore $ZZ+XX=2AE+2EE$ by Addition,
And $\frac{ZZ+XX}{2}=AA+EE$ by Division.

Proposition X.

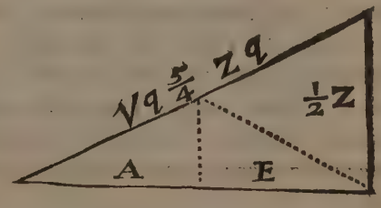
If a Right Line be divided into two equal Parts, and unto it be added another Right Line, the Squares which are made of the Whole Line and the Parts added both together, shall be double to the Squares which are made of the Half, and of the Half and Part added.



If $Z=A+E$ by Supposition,
And $X=A-E$ by the Sixth Æquation.
Then $Zq+Xq=2Aq+2Eq$, as before.

Proposition XI.

To divide a Line by Extreme and Mean Proportion, that is, so as the Rectangle comprehended by the Whole and one of the Parts, may be equal to the Square of the other Part.



If $Z=A+E$
And $\sqrt{q\,\frac{5}{4}Zq}-\frac{1}{2}Z=A$ by Construction,
Then $ZE=Aq$
For $\sqrt{q\,\frac{5}{4}Zq}=\frac{1}{2}Z+A$ by Transposition.

$\sqrt{q\,\frac{5}{4}Zq}=\frac{1}{2}Z+A$

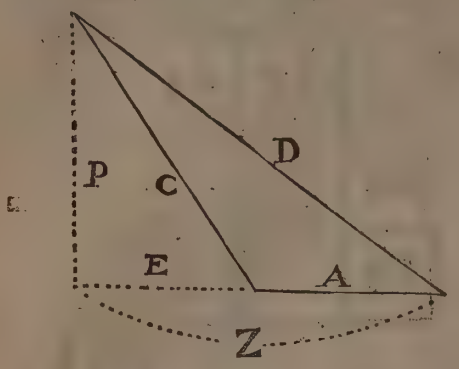
$\frac{5}{4}Zq+\frac{1}{2}ZA$
$+\frac{1}{2}ZA+Aq$

Then $\left\{\begin{matrix}\frac{5}{4}Zq=\frac{1}{4}Zq+Aq\text{ by Multiplication,}\\Zq=ZA+Aq\text{ by subtracting }\frac{1}{4}Zq.\end{matrix}\right.$
But $Zq=ZA+ZE$ by the second Proposition.
Therefore $ZA+Aq=ZA+ZE$ by Interpret.
And $Aq=ZE$ by Subduction.
Which was to be proved.

Proposition XII.

In obtuse Angled Triangles, the Square which is made of the Side subtending the obtuse Angle is greater than the Squares which are made of the Sides comprehending the obtuse Angle, by twice the Rectangle comprehended by one of the Sides which are about the obtuse Angle, (on which, being produced, a Perpendicular falleth,) and the

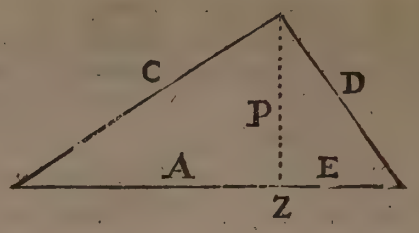
the Part which is between the Perpendicular and the obtuſe Angle.



In the Obtuſe Angled Triaugle C D A.
Zq=Aq+2AE+Eq by the 4th Prop.
And Zq+Pq=Aq+2AE+Eq+Pq by Additit:
But Dq=Zq+Pq by the 47th of the firſt *Eucl.*
Ergo Dq=Aq+2AEq+Eq+Pq by Interpret.
But Cq=Eq+Pq by the 47th of the firſt *Eucl.*
Ergo Dq=Aq+Cq+2AE by Interpretation.
Which was to be demonſtrated.

Propoſition XIII.

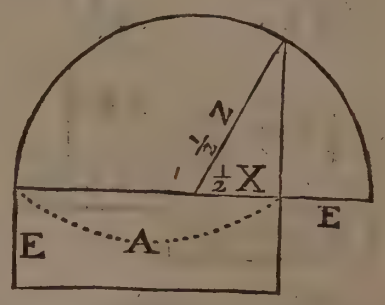
In Acute Angled Triangles the Square which is made of the Side ſubtending the Acute Angle is leſs than the Squares which are made of the Sides comprehending the Acute Angle, by twice the Rectangle comprehended by one of the Sides which are about the Acute Angle, (on which a Perpendicular falleth,) and the Part which is between the Perpendicular, and the Acute Angle.



In the Acute Angled Triangle C D Z.
Zq+Eq=2ZE+Aq by the ſeventh Prop.
And Zq+Eq+Pq=2ZE+Aq+Pq by Addit.
But Eq+Pq=Dq by the 47th of the firſt *Eucl.*
Ergo Zq+Dq=2ZE+Aq+Pq by Interpret.
But Cq=Aq+Pq by the 47th of the firſt *Eucl.*
Ergo Zq+Dq=Cq+2ZE by Interpretation.
Which was to be proved.

Propoſition XIV.

To make a Square equal to a Right Lined Figure given.



Æ+¼Xq=¼Zq by the fifth Propoſition,
Æ=¼Zq−¼Xq by Tranſpoſition.
But ¼Zq−¼Xq=Pq by the 47th of the firſt *Eucl.*
Ergo Æ=Pq by Interpretation.
Which was to be proved.

CURSUS

CURSUS MATHEMATICUS.

BOOK III.

Comprehending

PRACTICAL ASTRONOMY,

Or the DOCTRINE of

PRIMUM MOBILE.

In Two PARTS.

CONTAINING

I. The *Doctrine* of the SPHERE; and a Deſcription of all the *Circles*, *Lines*, *Points*, *&c.* thereupon deſcribed; and to what *Uſe* each of them ſerveth.

With the Uſe of the

Celeſtial Globe in Aſtronomy.

II. The *Doctrine* of SPHERICAL PROJECTION, Teaching, *Steriographically* to Project the SPHERE in *Plano*, upon any CIRCLE thereof, *Direct* or *Oblique*: Alſo *Orthographically* (by *Ellipſes*) commonly called the ANALEMMA.

AND

The *Uſes* of the forementioned PROJECTIONS exemplified in the ſolution of ſeveral *Problems Aſtronomical*, relating to the Motion of the *Sun* and *Stars*.

By *WILLIAM LEYBOURN*, Philomat.

LONDON:
Printed *Anno Dom.* M DC XC.

Cursus Mathematicus.

Book III.

PRACTICAL ASTRONOMY:

Or, The Doctrine of *PRIMUM MOBILE.*

Part I.

Containing the Doctrine of the SPHERE, and the Use of the Celestial GLOBE.

PROOEM.

IT may be expected I should say something here concerning the Name, Original, *and* Progress *of* ASTRONOMY; *but that Part of that Science, in this place to be treated of, being only* Practical *and* Introductory *to the other Parts thereof which follow: I shall reserve the forementioned* Definition, &c. *till I come to treat of the* Theorical *Part of Astronomy, as more pertinent thereunto.*

CHAP. I.

Of a Material SPHERE *or* GLOBE.

WHO was the first Inventer of the Artificial *Sphere* or *Globe* is not evident; some think (with *Pliny*) that it was found out by *Atlas*, and carried into *Greece* by *Hercules*; others have ascribed it to *Anaximander Milesius*: some to *Musæus*, as *Diogenes Laertius*; others to other Authors, amongst whom *Architas Tarentinus* is not forgotten: But all these were out-stript by *Archimedes* the *Syracusan* Mathematician, who flourished *Anno Mundi* 3739. *ante Nat. Christ.* 209. who is said to have composed a *Sphere* of *Transparent Glass*, representing unto the life the whole frame of the Heavens; wherein the *Sun*, *Moon*, and *Stars*, with their true *Motions*, *Periods*, and *Limits* were shewed to the sight, in such sort as if it were natural: Of which *Claudian* (an emi-

nent Poet in the time of *Theodosius* the *Emperor*, who was born at *Alexandria*, and flourished *Anno Christi* 390. elegantly wrote in his *Epigrams*, which in English is thus rendered:

In a small Glass, *when* Jove *beheld the* Skies,
He smil'd, and thus unto the gods replies;
Could Man thus far extend his studious care,
To mock my Labors in a brittle Sphere?
Heavens Laws, Mans ways, *and* Natures sovereign Right
This Sage *of* Syracuse *translates to sight.*
A Soul *within, on various Stars attends,*
And moves the quick-work unto certain ends.
A feigning Zodiack *runs his proper year,*
And a false Cynthia *makes new* Months appear.
And now bold Art, *takes on her to command*
And rule the heavenly Stars *with* humane hand.
Who can admire Salmonean *harmless* Thunder,
When a slight hand *stirs* Nature *up to* Wonder?

This is one Version of *Claudius*, but excellently thus translated by *T. Randolph.*

Jove *saw the Heavens, framed in a little Glass;*
And laughing, to the gods, these words did pass:
Comes then the Power of mortal cures so far?
In little Orbs my Labors acted are.
The Statutes of the Poles, *the Faith of* Things,
The Laws of gods, this Syracusian brings
Hither by Art: Spirits inclos'd attend
Their several Spheres, and with such motions bend
The living work; each Year the feigned Sun,
Each Month returns the counterfeited Moon.
And viewing now her World, bold industry
Grows proud to know the Heavens his Subjects be.
Believe, Salmonius *hath false Thunders thrown,*
For a poor hand is Natures rival grown.

But to pass by this piece of Art, and also that Sphere lately composed by *Cornelius Trebelius* and presented to King *James* the First of *England*, I shall come and give you an account of an Artificial Sphere or Globe, with all the *Circles*, *Lines*, and *Points* thereon *described* (and imagined so to be) in the *Heavens*; and to shew the *Use* or *Office* to which each of them is appropriate.

Now, *A* Material Sphere *or* Globe, *is an artificial representation of the* Celestial *and* Terrestrial *parts of the* World, *under the name of two* Globes, *viz.* Celestial *and* Terrestrial.

Upon the *Celestial Globe* is depincted the several *Constellations* of the fixed *Stars*, and also those *informis*, the *Via Lactea*, *&c.* in their true positions and situations in respect of *Longitude*, *Latitude*, *Right Ascension* and *Declination*, as they are posited in the *Heavens*.

Upon the *Terrestrial Globe* is described all the *Countries*, or *Parts* of the *known World*, with all the *Capes*, *Headlands*, *Coasts*, *Deserts*, *Islands*, *Peninsulas*, *Bays*, *Seas*, *Creeks*, *Rivers*, and whatsoever else hath been by the industry of heroick *Navigators* discovered in any part of the *World*, and these are also set upon the *Terrestrial Globe* (as the *Stars* were upon the *Celestial*) according to their true situation in respect of *Longitude* and *Latitude*.

CHAP.

CHAP. II.

Of the CIRCLES *of the Sphere or Globe.*

BESIDES the *Conſtellations* upon the *Celeſtial*, and the *Countries* and *Seas* upon the *Terreſtrial Globes*, there are upon either *Globe* deſcribed ſeveral *Circles* (the which Circles muſt be imagined to be in the Heavens really) and the principal of them are in number *Ten*; of which ſome are great *Circles* and ſome ſmall.

The number of the *great Circles* are *Six*, and the number of the *ſmall Circles* are *Four*.

A *great Circle* is ſuch a *Circle* as doth divide the body of the *Globe* or *Sphere* into two equal parts.

A *ſmall Circle* is that which divideth the *Body* of the *Sphere* or *Globe* into two unequal parts.

And beſides theſe *Six* principal *great Circles*, and the *Four ſmaller Circles*, there are divers other *Circles* both *great* and *ſmall* which are leſs material than the other *Ten*; as alſo, ſeveral *Lines* and *Points* upon (or imagined ſo to be) the *Globe*, as the *Rumbs*, the *Axis*, the *Poles*, the *Zenith*, the *Nadir*, *&c.* all which (in their proper places) as well as the *Ten* beforementioned, ſhall be deſcribed, and the Uſes to which each is appropriate, plainly declared.

The Six great Circles are,

1 The Horizon.
2 The Meridian.
3 The Equinoctial.
4 The Zodiack.
5, 6 The two Colures.

The Four leſſer Circles are,

1 The Tropick of Cancer, or the Northern Tropick.
2 The Tropick of Capricorn, or the Southern Tropick.
3 The Circle Artick, encompaſſing the North Pole.
4 The Antartick Circle encompaſſing the South Pole.

CHAP. III.

Of theſe ſeveral Circles, Lines, *and* Points *deſcribed upon the Globe, and to what* Uſe *or* Uſes *each of them ſerve.*

OF the Six *great Circles* of the *Sphere*, *Four* of them are deſcribed upon the *Superficies* of the *Globe*, but *two* of them are appendant to the *Globe*, and thoſe are

1 The *Horizon.* 2 The *Meridian.*

Circles, indeed, they are not (as the learned *Gregory* expreſſeth) to ſpeak properly, becauſe in a ſtrict ſenſe, no Line is ſuppoſed to have any breadth, whereas both theſe, *viz.* the *Meridian* and *Horizon* have breadth allotted them, ſo that ſuch things might be written or engraven upon them as might render them more uſeful, the *Globes* within them being in any Poſition; and therefore, they being of a Circular form (notwithſtanding the impropriety of the Speech) Uſe will have it ſo; and we muſt call them the *Meridian* and *Horizontal Circles.*

§. I. *Of the* HORIZON,

The *Horizon* is a great Circle appendant to the *Globe* in which the *Meridian* is placed, and conſequently the whole body of the *Globe* is moved, or may be ſet to any poſition: This is a great Circle dividing the *viſible* part of the Heavens from the *not viſible*; that is, the lower *Hemiſphere* from the higher. As, ſuppoſe you were at *Sea*, or upon a high Hill upon the *Land*, and looking round about you, you imagin that the

the Sky and Water, or the Sky and Earth did meet and touch each other: Now this Line of ſeparation of the *Sky* and *Water*, or *Sky* and *Earth* is the *viſible Horizon*, and ſuch a Circle is imagined to be in the Heavens.

Fig. I. and II.

This Circle is repreſented in the Figures by the broad Frame or Circle noted with H A O, which divideth the *Globe* into two *Hemiſpheres*, namely the upper or viſible *Hemiſphere* H Z O, which we ſee, from the under or inviſible *Hemiſphere* H N O, which we ſee not.

To what Uſes doth this Circle ſerve?

1. When the *Sun*, *Moon*, or any other *fixed Star* or *Planet* cometh to touch this Circle on the *Eaſt ſide* thereof, the *Sun*, *Moon*, or *Star*, is then ſaid to *riſe*, or begin to appear. And on the contrary, when the *Sun*, *Moon*, or any *Star* cometh to touch this *Circle* on the *Weſt* part thereof, it is then ſaid to *ſet*.

2. By this *Circle* the increaſe and decreaſe of the *Days* and *Nights* are limited. For if the *Sun*, *Moon*, or a *Star* doth riſe between the Points H and A, it will continue above the Horizon leſs than 12 hours: If it riſe juſt at the Point A, it will continue juſt 12 hours above the *Horizon*: And if it riſe between the Points A and O, it will continue more than 12 hours above the Horizon.

3. By this Circle is diſcovered at what time the *Sun*, *Moon*, or any *fixed Star* or *Planet* doth *riſe* or *ſet*: For that hour Circle upon the Globe which toucheth the Horizon in that Point where the *Sun* or *Star riſeth*, is the hour at which the *Sun* or *Star riſeth*.

4. The *Amplitude*, or Point of the Compaſs, upon which the *Sun*, *Moon*, or *Star riſeth* and *ſetteth* is diſcovered; for if they riſe between H and A, they riſe between the *Eaſt* and the *South*, and ſet between the *Weſt* and the *South*: But if it *riſe* between A and O, it *riſeth* between the *Eaſt* and the *North*, and ſetteth between the *Weſt* and the *North*. But if it riſe juſt in A, it *riſeth* due *Eaſt* and *ſetteth* due *Weſt*.

What Lines are deſcribed upon this horizontal Frame or Circle.

1. On the outer Verge thereof, between two Circles, it is divided into 32 equal parts, in which are written the Names of the 32 *Rumbs* or *Points* of the *Mariners Compaſs*, and placed in their reſpective places, anſwerable to their places in the Heavens.

2. Within this Circle is a *Kalendar* or *Almanack*, ſhewing the *day* of the *Month*, the day of the *Week*, and the *fixed Feaſts* for ever.

3. The innermoſt Verge is divided into 12 equal parts, and in them is placed the 12 *Signs* of the *Zodiack*, anſwerable to the time of the year in which the *Sun* cometh to be in that *Sign* and *Degree*.

§. II. *Of the MERIDIAN.*

This is the ſecond *great Circle* which is appendant to the Globe, and it is made of Braſs, and in it the body of the Globe hangeth: This alſo is a great *Circle*, and paſſeth through Z the *Zenith*, and N the *Nadir* Points, and through P the *North Pole*, and S the *South Poles* of the World: Alſo it divideth the *Horizon* into two equal parts in the H South, and O the North Point thereof.

This Circle is repreſented in the Figures by Z P O æ N S H Æ.

Unto what Uſes doth this Circle ſerve?

1. By it the Globe is ſet in the *Horizon* to any *Longitude*, or place of the World.

2. To this Circle, when the Sun, Moon, or any Star cometh, it is ſaid to be upon the Meridian, or at the higheſt it can be that Day or Night; ſo that when the *Sun riſeth* towards the *Eaſt* in the Morning, it aſcendeth higher and higher, till it cometh to this *Circle*, and then it is *Noontide* or *Noon*, after which it deſcendeth lower and lower, till it ſet in the *Weſt* part of the Horizon.

3. The Sun, or a known Star being upon this *Circle*, and the *Altitude* thereof then obſerved, is one of the beſt and moſt uſual ways for a *Seaman* to find in what *Latitude* he is, as ſhall be ſhewed in its due place.

4. The

4. The Point Z in this *Circle* is the *Zenith* Point in the Heavens, and is that Point in the Heavens, which is directly over your head, in what part of the World soever you are. And N is the *Nadir*, or Point directly under your feet, and is opposite to Z the *Zenith*.

5. In the two Points P and S are placed the ends of two Wires, which pass through the whole body of the *Globe*, which *Wire* is called the *Axis* of the *World*, and the two Points P and S, through which the Wire passeth, are the two *Poles* of the *World*, P the *North Pole*, and S the *South Pole*.

Of the Graduations or Divisions that are upon this Brazen Meridian.

It is divided into 4 equal parts in the Points Æ, P, æ, S, each of which parts or quarters is divided into 90 equal parts or degrees, and numbred from Æ and æ towards P the North Pole, by 10, 20, 30, *&c.* to 90; and also from Æ and æ towards S the South Pole, by 10, 20, 30, *&c.* to 90 at S.

§. III. *Of the* EQUINOCTIAL.

The *Equinoctial* is a great Circle described upon the body of the *Globe*, dividing it into two equal parts, and passeth through A, the *East* and *West* Points of the *Horizon*; and also it passeth through the *Meridian* in the Points Æ and æ, which Points are each of them equidistant from P and S, the two Poles of the World, and so the line or *Axis* of the *World* P S, which passeth through the body of the *Globe*, cutteth this Circle at Right Angles in the Point A.

This Circle in the Figures is represented by the Line Æ A æ.

To what Uses doth this Circle serve?

1. Unto this *Circle* when the Sun cometh (which is twice every year, namely, about the 10th. of *March*, and the 12th. of *September*) it maketh the *Days* and *Nights* of *equal length* through the whole World; for at those times, the *Sun rises* due *East*, and *setteth* due *West*, which at no other times of the year it doth, but *rises* and *sets Northward* of the *East* and *West* Points, from the 10th. of *March* to the 12th. of *September*; and *Southward* of the *East* and *West*, from the 12th. of *September* to the 10th. of *March*.

2. All such *Stars* as have no declination, but are situate under this Circle, do always rise and set due East and West, and are always 12 hours above the Horizon, and as many under the same.

3. All the Inhabitants of the Earth that live under this Circle (as those of the *Island* of S. *Thome*, *&c.*) have their days and nights always equal 12 hours, and the Sun to them, when in this Circle, *rises* and *sets* due *East* and *West*, and at Noon the Sun is in their *Zenith*, or directly over their *heads*; so that at that time there is no shadow of any upright thing to be seen.

Of the Graduations or Divisions of this Circle.

1. It is divided into 24 equal Parts representing the 24 hours of a *Natural day*, through which Points of division, and through either of the *Poles* P and S, there are drawn 12 *Meridians* or *hour-Circles*, and every of these is divided into 15 equal degrees, so that the whole Circle is divided into 360 degrees (as all *great Circles* of the *Sphere* are) beginning at A, and so on to 90 at æ, and then to 180 at A, and then on to 270 at Æ, and then to 360 at A again.

§. IV. *Of the* ZODIACK *and* ECLIPTICK *Circle.*

The *Zodiack* is a *great Circle* dividing the *Globe* into two equal parts in the *East* and *West* Points of the *Horizon*, and cutteth both the *Horizon* and *Equinoctial* at *oblique Angles*, for it cuts the *Horizon* at an Angle equal to the Suns greatest *Meridian Altitude* in any *Latitude*: And it cuts the *Equinoctial* always at an *Angle* equal to the Suns greatest *Declination*. Through the middle of this *Circle* (for this *Circle* is supposed to have breadth) there passeth a Line, which is called the *Ecliptick*, or *Via Solis*, for that the Sun in his motion never deviateth from this *Line*, though the other Planets do some more and some less, which is the reason that there is breadth attributed to it.

This

This Ecliptick Line or Circle is noted in the Figures with ♋, and the Characters of the others *Signs* upon the ſame in their proper places.

To what Uſes this Circle ſerves.

1. By it is found out the *Suns Place.*

2. It determins the four *Quarters* or *Seaſons* of the *Yeqr*; for while the *Sun* is between ♋ and A, it maketh *Spring* and *Summer*, and while it is between A and ♎, it maketh *Winter* and *Autumn.*

3. It determins the *Suns Declination* from the *Equinoctial:* For while the *Sun* is moving from A, towards ♋, it hath *North Declination* increaſing: While it is paſſing from ♋ to A, it hath *North Declination* decreaſing: While it is moving from A towards ♎, it hath *South Declination* increaſing: And while paſſing from ♎ to A, it hath *South Declination* decreaſing.

4. From this Circle the *Latitudes* of the *Planets* are counted, *&c.*

Of the Graduations or Diviſions of this Circle.

It is divided into 4 equal parts, by the interſections of it with the *Equinoctial* and *Meridians*, and each of theſe Quarters is divided into three *Signs* of the *Zodiack*, and each of thoſe *Signs* into 30 degrees, in all 360 degrees, as the *Equinoctial* was: It hath the Six Northern Signs ♈ ♉ ♊ ♋ ♌ ♍ character'd upon the upper half towards P the North Pole, and the Six Southern Signs ♎ ♏ ♐ ♑ ♒ ♓ on the lower half towards S the South Pole.

This *Ecliptick* Circle hath two *Poles*, in which, as all the *Meridians* or *Hour-Circles* deſcribed upon the *Globe*, do meet in the *Poles* of the *World*; ſo all the *Circles* of *Longitude*, drawn through the 12 Signs of the *Zodiack*, do meet in the *Poles* of the *Ecliptick*, each *Pole* of the *Ecliptick* being diſtant from his correſpondent *Pole* of the *World* 23 deg. 30. min. and one is call'd the *North*, the other the *South Pole*, according to their poſition in reſpect of the *Poles* of the *World.*

In the *Figures* theſe two Poles are noted with C the *North*, and G the *South Pole.* Alſo in the *Figures* I have deſcribed only one *Circle* of *Longitude*, which cutteth, or paſſeth through the *Ecliptick* in the Points ♑ and ♋, as is the Circle C ♑ G.

§. V. *and* VI. *Of the Two* COLURES.

Theſe are two *great Circles* of the *Sphere*, and do paſs through rhe *Poles* of the *World*, and there do cut each other at *right Angles.* The one of theſe *Colures* cutteth the *Horizon* in the Point A, and the *Equinoctial Circle* alſo in the Points ♈ and ♎, and therefore is called the *Equinoctial Colure.* The other cutteth the *Horizon* in the Points H and O, and the *Ecliptick* Circle in the Points ♋ and ♑; and is therefore called the *Solſticial Colure*, the Sun then being at its greateſt diſtance Northward or Southward.

In the Figures the *Equinoctial Colure* is noted with P A S, and is the ſame with the *Axis* of the World. The *Solſticial Colure* is noted with H P O S and is the ſame with the *Meridian* Circle.

Unto what Uſes theſe Circles ſerve.

1. When the Sun is in that *Sign* or Point of the *Ecliptick* which meeteth with the *Equinoctial Colure*, which is in the firſt Scruple of ♈ or ♎, then are the *Days* and *Nights equal* all the World over: But when the Sun cometh to thoſe Points of the *Ecliptick* which are cut off the *Solſticial Colure* (which are in the beginning of ♋ and ♑) then the Sun being ♋, it maketh the *longeſt Day* to all the *Northern*, and the *ſhorteſt Day* to all *Southern* Inhabitants of the World: And being in ♑, it maketh the *ſhorteſt Day* to the *Northern*, and the *longeſt* to the *Southern* Inhabitants.

2. Upon theſe *Circles* the Suns Declination is counted.

3. Theſe two *Colures* repreſent the 4 *Cardinal Points* of *Heaven*, *viz.* The *Solſticial Colure* repreſenteth the *Meridian*, or *North* and *South*: And the *Equinoctial Colure* or *Prime Vertical Circle*, the Points of *Eaſt* and *Weſt.*

Of

Of the Four leſſer Circles *of the Sphere.*

§. VII. *Of the Tropick of* CANCER.

The *Tropick* of *Cancer* is a leſſer Circle, drawn upon the Superficies of the *Globe*, parallel to, or equidiſtant in all places from the *Equinoctial*, on the *North* ſide thereof, at 23 deg. 31 min. diſtant therefrom, which is equal to the *Suns* greateſt *declination*, and alſo to the obliquity of the *Ecliptick Line*. This *Tropick* is called the *Northern Tropick*, for that it lyeth on the *North* ſide of the *Equinoctial* towards the *North Pole*. It is noted in the Figures with ♋ 6 ♋.

§. VIII. *Of the Tropick of* CAPRICORN.

The *Tropick* of *Capricorn* is a leſſer Circle, drawn on the Superficies of the *Globe*, parallel to the *Equinoctial* on the *South* ſide thereof, towards the *South Pole*, at the ſame diſtance of 23 deg. 31 min. as the *Tropick* of *Cancer* was on the *North* ſide. This Tropick is called the *Southern Tropick*, becauſe it lieth on the *South* ſide of the *Equinoctial* towards S, the *South Pole*. It is noted in the Figures with ♑ 6 ♑.

To what Uſes do theſe two Circles ſerve?

1. They bound and limit the *Suns Courſe*; for the Sun in his paſſing through the *Signs* of the *Zodiack*, never exceedeth theſe bounds or Limits; for the *Sun* being in *Aries*, he aſcends upwards to *Taurus* and *Gemini*, and ſo to *Cancer*, making the *Spring Quarter*, and being then at his higheſt, he deſcends downwards from *Cancer* to *Leo* and *Virgo*, and ſo to *Libra*, making the *Summer Quarter*. And having thus made his progreſs through the Six *Northern Signs*, he begins to take his journey Southwards; So being in *Libra*, he deſcends downwards to *Scorpio* and *Sagittarius*, and ſo to *Capricorn*, where he is at his greateſt remotion from the *Equinoctial Southward*, in which progreſs he maketh the *Autumnal Quarter*; then returning back from *Capricorn* to *Aquarius* and *Piſces*, he returns to *Aries* again, where he began; and in this laſt progreſs from *Capricorn* to *Aries*, he createth the *Hyemnal* or *Winter Quarter*.

Thus doth bright Sol, *by certain Circuiting,*
Make Winter, Autumn, Summer, *and the* Spring.

§. IX. *and* X. *Of the* ARTICK *and* ANTARTICK *Circles.*

Theſe are two ſmall Circles drawn about the *Poles* of the World, parallel or equidiſtant to the *Equinoctial* and *Tropicks*; and the *Circle Artick* is ſo far diſtant from the *North Pole* P, as the *Tropick* of *Cancer* is from the *Equinoctial*, and the Antartick is at the ſame diſtance from the South Pole. The Artick Circle is noted in the Figure with B 6 C, and the Antartick with G 6 E.

To what Uſes do theſe Circles ſerve?

1. In theſe two Circles are the two *Poles* of the *Ecliptick*, from whence, towards either of theſe *Poles*, the *Latitude* of the *Planets* and *Fixed Stars* are counted.

2. They do limit and ſeparate the *Frigid Zone* from the *Temperate Zone*, for all that ſpace between

The { Artick Circle / Antartick Circle } and the { North / South } Pole is the { North / South } *Frigid Zone.*

All the ſpace between the two *Tropicks* comprehends the two *Torrid Zones*, and all the ſpace between

The { Tropick of ♋ / Tropick of ♑ } and the { Artick / Antartick } Circle is the { North / South } *Temperate Zone.*

CHAP. IV.

Of ſome other Circles of the Sphere neceſſary to be known.

THESE forementioned are the Ten Principal *Circles* of which a *Material Sphere* or *Globe* is compoſed: There are other *Circles*, which are not deſcribed actually upon the *Globe*, but are ſupplied by other means, with which *Circles* the *Reader* ought to be acquainted; and although they are not deſcribed upon the *Material Sphere*, yet they are in the *Projection* of the *Sphere* in *Plano*; and they are principally theſe Four, of which Four, Two are *great Circles*, and Two are *ſmall*.

I. *Meridians*, or *Hour Circles* } Great.
II. *Azimuths*, or *Vertical Circles* }
III. *Almicanthars*, or *Circles or Parallels of Altitude* } Leſſer.
IV. *Circles*, or *Parallels* of the *Suns or Stars Declination.* }

Of theſe ſeverally.

§. I. *Of* Meridians, *or HOUR CIRCLES.*

Hour-Circles are great Circles of the *Sphere*, meeting together in the *Poles* of the *World*, and croſſing the *Equinoctial* at *Right Angles*, dividing it at every 15th. degree, and then every one of theſe Diviſions, is *one Hour*; for 24 times 15, is 360, the quantity of *Degrees* in every *Circle*. But if theſe *Hour-Circles* paſs through other parts of the *Equinoctial*, dividing it unequally, then do thoſe *Hour-Circles* repreſent *unequal ſpaces of time*, according as they are diſtant from the *Firſt* (or General) *Meridian*. Of theſe *Meridians* or *Hour-Circles*, there are two deſcribed in the Projection, Figure II. The one cutting the *Equinoctial* at 3 hours diſtance from the *General Meridian* (which therefore repreſents the *Hour-Circle* of 9 or 3 of the Clock), and it is noted with P 3 S. The other *Hour-Circle* is drawn through the Point where the *Tropick of Cancer* cuts the *Horizon*, at which place the *Sun riſeth* when he is in *Cancer*, and this *Meridian* or *Hour-Circle* is noted in the Figure with P O S.

§. II. *Of AZIMUTHS, or VERTICAL Circles.*

Azimuths, or **Vertical-Circles**, are great Circles of the Sphere, meeting together in the *Zenith* and *Nadir Points*, as the *Hour-Circles* did in the *Poles* of the World, and theſe *Azimuths* do cut the *Horizon* at Right Angles, as the *Meridians* or *Hour-Circles* did the *Equinoctial*. In the Projection Figure II. there are two of theſe drawn, the one croſſing the *Equinoctial* where the *Hour-Circle* of 3 did cut it, and falleth upon the *Horizon* at R, and is noted with Z R N. The other cutteth the *Horizon* in D, and is noted with Z D N.

§. III. *Of ALMICANTHARS, or Circles of ALTITUDE.*

Almicanthars, alſo called **Circles**, or **Parellels** of **Altitude**, are leſſer Circles of the Sphere, and are drawn parallel to the *Horizon*. Theſe *Parallels* are drawn from the *Horizon* (and parallel thereto) through every degree of the *Meridian*, up to the *Zenith* Point. By theſe *Almicanthars* are meaſured the *Altitude* or *Height* of the Sun, Moon, or Stars; in the Figure there is only one of them drawn, and that is at 18 deg. of the *Meridian* above the *Horizon*, and it paſſes through the Point of interſection of the *Azimuth Circle* Z 3 N, and the *Hour-Circle* P 3 S, in the Point 3, it is repreſented in the Figure by the Circles C 3 E, it is drawn through the 18th. deg. of the *Meridian* counted from the *Horizon*; And when the Sun, Moon, a Star, or Planet, comes to be upon any part of this Circle, it is ſaid to be 18 deg. high, or 18 deg. *above the* **Horizon**.

§. IV. *Of Circles or Parallels of DECLINATION.*

Theſe **Parallels** of **Declination**, (for ſo they are moſt commonly called) are leſſer Circles of the *Sphere*, and are drawn parallel to the *Equi-*

noctial

noctial on both sides thereof through every degree of the *Meridian*, as the *Parallels* of *Altitude* were from the *Horizon*. Those *Parallels* that lie on the *North* side of the *Equinoctial*, towards P the *North Pole*, are called *Parallels of North Declination*, and they do cross the *Ecliptick* through the six *Northern Signs*. Those *Parallels* that are drawn on the *South side* of the *Equinoctial*, and towards S the *South Pole*, these cross the other half of the *Ecliptick* (under the *Horizon*) in the six *Southern Signs*; and these are called *Parallels of South Declination*. These *Parallels* (so far as they relate to the Course or Motion of the *Sun*) need be drawn but only between the two *Tropicks* (because the Sun in his Course never exceedeth those Bounds); but if they relate to the Stars, they may then be extended farther, even to P and S, the *North* and *South Poles*. In the Figure I have drawn only two of these; one on the *North* side, at 11 deg. 30 min. from the *Equinoctial*; and the other on the *South* side thereof, at 20 degrees distant therefrom. The *Northern Parallel* is represented by the Circle F G; and the *Southern Parallel* by C M; and when the Sun is in that degree of the *Ecliptick*, which the *Parallel of Declination* crosseth; as at Q, the Sun is then said to have 11 deg. 30 m. of *North Declination*: And when the Sun is in that point of the *Ecliptick* which the *Southern Parallel of Declination* cutteth (which is at F,) the Sun (or a Star there) is said to have 20 deg. of *South Declination*.

Some other Terms here necessary to be explained.

The **Latitude of a Place** is an Arch of the *Meridian*, intercepted between the intersection of the *Meridian* with the *Horizon*, and the *Poles* of the World; so in the Figure, the *Meridian* and *Horizon* intersect each other in the *North Point* of the *Horizon* at O, and P the *North Pole* is distant from O 51 deg. 30 min. and that is the *Latitude* to which the *Globe* or *Sphere* is here elevated, which is to the *Latitude* of *London*.

The **Latitude of a Star**, is its distance *from the Ecliptick* in any *Parallel* therefrom, either *Northward* or *Southward*.

The **Longitude of a Place or Country**, is the distance (counted upon the *Equinoctial* Circle) of the *Meridian* which passeth over the *Country* whose *Longitude* is required, and the *First* or *General Meridian*, which the Ancients counted from the Island of *Azores*: but, indeed, the *Longitude* may begin at any place, and so it is best to account by the *difference of Longitude*, of two *Places*; as in the Figure. Let one *Place* or *Country* be at F, and let another *Place* or *Country* be at T; The *General Meridian* P Z F Æ S, passeth over the Place at F, and cuts the *Equinoctial* at Æ. The *Meridian* P ♈ S passeth over the other *Place* at T, and cuts the *Equinoctial* at ♈; so that Æ ♈ is the *difference of Longitude* between the two *Places* F and T, which is 45 deg. or *Three hours* of *Time*.

The **Longitude of a Star**, is the Stars distance, or place in the *Ecliptick*, counted from the beginning of *Aries*; as if a *Star* be in the beginning of *Taurus*, that is one *Sign*, or 30 deg. from *Aries*, and that is the *Stars Longitude*.

The **Amplitude** of the Sun or a Star, it is that *Arch* or *Portion* of the *Horizon*, which is contained between the *East Point* of the *Horizon*, and that *Point* of the *Horizon* where the *Sun* or *Star rises*; as in the Figure: The distance between ☉, the place where the Sun is *rising* in the *Horizon* and ♈, is the *Amplitude* or *distance of the Suns rising from the true East Point* ♈, towards the *North*. Also if a Star were rising at R his *Amplitude* is R ♈ southward of the East.

The **Right Ascension** of the Sun or of a Star, is that part of the *Equinoctial* that *riseth* or *setteth* with that Star in a *Right Sphere*: But in an *Oblique Sphere* (that is, in any *Latitude* where the *Axis* of the *World* cutteth the *Horizon* at *Oblique Angles*) **Right Ascension** is then that Portion (or Arch) of the *Equinoctial* contained between the beginning of *Aries*, and that part of the *Equinoctial* which cometh to the *Meridian* with that Star.

The **Oblique Ascension** is that part of the *Equinoctial*, contained between the beginning of *Aries*, and that part of the *Equinoctial* that *rises* with the *Star*.

The **Ascensional Difference** is the Difference between the *Right* and *Oblique Ascension*.

The **Zenith** is the *Pole* of the *Horizon*, and is an imaginary *Point* in the Heavens just over our heads, in what part of the World soever we are, and it is 90 degrees distant from the *Horizon* round about.

The **Nadir** is an imaginary Point just under our feet, as the *Zenith* is over our heads; so that if a right Line were drawn (or extended) from the *Zenith* to the *Nadir*, it would pass directly through the Centre of the World (or Globe).

§. V. *Of some other things appendant unto a* Material Globe, *besides the* Meridian *and* Horizon.

Besides the forementioned *Meridian* and *Horizon* there are other parts which must be applyed to, and used with the *Globes*, before any *Problems* can be wrought or performed by them; as principally these, 1. The *Hour Circle* and *Index*. 2. The *Quadrant* of *Altitude*:

I. Of the *Hour Circle* and its *Index*.

Upon the *Brass Meridian* of every *Globe* is fitted a small Brass Circle, whose Centre is the Pole of the Globe, which is divided into 24 equal parts, representing the hours of the Day and Night, which in the revolution or turning of the *Globe* about, is pointed to by an Index, which is fixed to the *Axis* of the Globe.

II. Of the *Quadrant* of *Altitude*.

There is another appendant relating to the Brass Meridian, called the *Quadrant* of *Altitude*, which is a thin Plate of Brass, divided into 90 equal parts or *Degrees*, which is fitted with a *Nut* and *Screw*, to be moved to any degree upon the Brass *Meridian*, and there to be fixed.

To some Globes, when desired, there is added a *Circle of Position*, and a *Box* and *Needle* fixed to the foot of the *Horizon*, but these are not usual.

CHAP. V.

Of the several Positions *that a* Globe *or* Sphere *may be posited in.*

THERE are but Three Positions in which a *Sphere* or *Globe* can be seated in its *Horizon*, *viz.* *Direct*, *Parallel*, or *Oblique*; of which, the two first, are *Particular*, and the third more *General*.

§. I. *Of a Direct* SPHERE.

Fig. III.

The Globe may be so placed in the Frame of the *Horizon*, that both the *Poles* thereof may rest upon (or lye directly in) the North and South Points of the *Horizontal Circle*, neither *Pole* having any Elevation, the *Zenith-Point* being in the *Equinoctial Circle*, and the *Axis* of the World directly in the plain of the *Horizon*; and so the People living under that Circle have no *Latitude*, the Pole having no Elevation.

The *Globe* or Sphere being in this position, is said to be in a *Direct* position, the *Zenith* being directly in the *Equinoctial Circle*, and the *Poles* of the World directly in the *North* and *South* Points of the *Horizon*: and this *direct* position of the *Sphere* is particular to those who live under the *Equinoctial*, who have one benefit and priviledge above all the Inhabitants of the World besides, for that they can see both the *Poles*, and behold all the *Stars* (in both *Hemispheres*) to *rise*, *culminate*, and *set*.

§. II. *Of a* Parallel SPHERE.

Fig. IV.

The *Globe* may be so placed in the Frame of the *Horizon*, that one of the Poles shall be in the *Zenith*, and the other in the *Nadir*-Points; that is, either Pole shall be 90 deg. (or one quarter of a Circle) distant from the *Horizon* on either side thereof; and in this position will the *Equinoctial Circle* be in the *Horizon*, and the *Axis* of the World will cut both the *Equinoctial* and the *Horizon* at Right-Angles: One *Pole* having 90 deg. of Elevation, and the other as much of Depression.

The *Globe* or *Sphere* being in this position, is said to be *Parallel*; because the *Equinoctial*, and all the Circles of *Declination*, (which now are Circles of *Altitude* also) and the Axis of the World it self, do lie all of them Parallel to the *Horizon*. And those People (if any be) that inhabit these parts of the World, see only those Stars that

that are between the *Equinoctial* and the Elevated Pole; that is, if the *South*-Pole be elevated, they see the *Southern*; and if the *North*-Pole be elevated, they see the *Northern Constellation* only. And those People have but one day and one night in the whole year, and those most miserable cold, not to be imagined: For the Sun, at his highest, never extendeth to 24 degrees of Altitude; which is no more than it is with us in *London* at Noon upon the 14 or 15 of *February*. And this position of the Sphere is particular to these Inhabitants only.

§. III. *Of an* Oblique *SPHERE.*

Fig. V.

Fig. V.

The third (and most usual) position of the *Sphere* or *Globe* is more general; for it hath relation to all People living between the *Equinoctial* and either of the *Poles*: and according as the *Poles* of the *Globe* are elevated or depressed, accordingly are the People said to be situate: Thus, if the *Globe* be placed in the Frame of the *Horizon*, so that the Pole be elevated 10 deg. above the *Horizon*, then is the Globe elevated or fitted to resolve such Questions or *Problems Astronomical* as relate to those People who (have the Pole elevated, or) do live in the *Latitude* of 10. degrees.

This position of the *Globe* or *Sphere* is called *Oblique*, because the *Axis* of the *World*, the *Equinoctial*, and all the rest of the *Parallels* of *Declination*, are cut by the *Horizon* at *Oblique Angles*; whereas, in the two former positions they cut one the other at *Right Angles*. Now the People which live in these middle *Latitudes*, (I mean between the *Artick* and *Antartick* Circles) which (yet to our knowledg) is the most habitable part of the World, do see Stars both of the Northern and Southern *Hemispheres*; but yet they see not all the Stars in either Hemisphere. For in any Oblique Latitude, the Inhabitants see, and may observe all such Stars that are of the contrary Hemisphere to them, whose declinations are lesser than the complement of their *Latitude*; and all those Stars of the contrary Hemisphere, whose declination is greater than the complement of their *Latitude* they never see; for they never ascend above their *Horizon*: and on the contrary, those Stars of their own Hemisphere whose declinations exceed the complement of their Latitude, do never set, but are continually above their Horizon.

CHAP. VI.

How to rectifie the GLOBE, *fitting it for Use in any* Latitude *or Place of the World.*

BEING provided of a pair of *Globes*, the *Meridian*, *Horizon*, and *Hour-Circle* truly turned and divided; also the *Ball* truly hung, and the Meridian and Horizon (in all positions) cutting each other at Right Angles, the Papers truly joyned in their pasting, *&c.* all which are to be performed by the Workman, (though the Buyer ought also to have inspection thereinto) you may proceed to rectifie them in this manner.

1. Put the Brass-Meridian into two Notches that are in the North and South parts of the Horizon; the graduated or divided part thereof towards the East Point, and the blank or plain side of the Meridian towards the West Point of the Horizon; and let the Meridian rest in the Notch which is in the foot or bottom of the Horizon.

2. Place the Hour-Wheel about the Pole, so that the Hour-Lines of 12 and 12 do lye directly over the East or Graduated side of the Meridian, and that the point of the Axis do pass directly through the Centre of the Hour-Circle; so shall the two Twelves, one of them represent 12 at Noon, and the other 12 at Midnight; and the two Sixes, the one 6 in the Morning, and the other 6 at Night. Then put the little Index or Pointer upon the Axis, so that it may move as you turn the Globe about; and so is your Hour-wheel rectified.

3. Elevate the Pole of your Globe (whether the North or South-Pole) according to the Latitude of that part of the World you are in: as for Example, for *London*, whose Latitude is 51 deg. 30 min. North; the Meridian being in the Notches of the Horizon, and also in that in the foot of the Frame, as is before directed, move the

Meridian

Meridian up or down in the Notches, till you find 51 deg. 30 min. of the Meridian juſtly to touch the upper part of the North-part of the Horizon; for then is your Globe ſet exactly to the Latitude of 51 deg. 30 m.

4. For the rectifying of the Quadrant of Altitude, this alſo muſt have reſpect to the Latitude: Wherefore, the Latitude being 51 deg. 30 min. count 51 deg. 30 min. upon the South-part of the Meridian, from the Equinoctial-Circle towards the North (or elevated) Pole; and put on the Nut which is at the end of the Quadrant, ſo that the edge of the diviſions of the Quadrant may lie directly under the degrees of the Latitude, *viz.* under 51 deg. 30 min. and then ſcrew the Nut faſt; and ſo is the Quadrant of Altitude rectified alſo. And theſe are the four principal things that are to be rectified, for the reſolving of moſt Problems in Aſtronomy, *&c.* of which theſe following are the chief.

CHAP. VII.

ASTRONOMICAL PROBLEMS.

PROB. I.

The day of the Month known, either according to the Julian *or* Gregorian *Account, to find the* Sun's *place in the* Ecliptick.

SEEK the day of the Month (in either Account) according as you find them placed in the Kalendar, and right againſt it, in the innermoſt Circle next to it, you ſhall have the degree and minute in which the *Sun* ſhall be that day at noon.

EXAMPLE.

Let the day propoſed be the 18. of *October* (in the *Julian*, or the 28. of *October* in the *Gregorian* Account) which is S. *Luke*'s day: Find this Month and Day in the Kalendar, and right againſt it (in the innermoſt Circle) you ſhall find 5 deg. 32 min. of *Scorpio*, in which Sign and Degree the *Sun* will be upon that 18 or 28 of *October*.

In like manner, upon the 24. of *April* (*Julian*) or the 4. of *May* (*Gregorian*) (which is all one day) the *Sun* will be found to be in 14 deg. 32 min. of *Taurus*. And ſo of any other day, as in the following Table, which ſheweth, that upon

Jul.		Greg.			d.	m.		
January	7		17		28	9	*Capricorn*	♑
February	12		22		4	38	*Piſces*	♓
March	23	*April*	3	The *Sun*'s Place	13	23	*Aries*	♈
May	16		26	at Noon will be	5	39	*Gemini*	♊
July	27	*Aug.*	6	in	14	16	*Leo*	♌
September	3		13		21	0	*Virgo*	♍
November	5		15		23	41	*Scorpio*	♏

PROB. II.

By knowing the Place of the Sun *in the* Ecliptick, *the day of the Month in either of the Accounts may be obtained: As followeth.*

SEEK the Sign, Degree, and Minute in which the *Sun* is, in the innermoſt Circle of the Horizon; and right againſt it, you ſhall have the day of the Month in both Accounts.

EXAMPLE.

Let the *Sun* be in 24 deg. of *Gemini*; look in the innermoſt Circle for the ſign *Gemini*, and againſt it, in the Kalendar you ſhall find the 4. of *June* (*Julian*) or the 14. of *June* (*Gregorian* Account) which is the day of the Month.

In like manner, when the *Sun* is in 14 deg. 53 min. of *Capricorn*, the day of the Month will be found the 25. of *December* (*Julian*) or the 4. of *January* (*Gregorian* Account).

The

The Latitude (51 *deg.* 30 *min.*) *and the* Sun's *place in the Ecliptick* (viz. *in* 29 *of* Taurus) *being given, to find*

PROB. III.

1. *The* Sun's *Declination.*

Definition.] THE Sun's *Declination is an Arch of the Meridian, comprehended between the Equinoctial Circle, and that point of Ecliptick in which the* Sun *is.*

Practice.] Look for the 29 deg. of *Taurus* in the Ecliptick-Line of the Celestial Globe, and bring that Point to the Brass-Meridian; then shall the degrees of the Meridian contained between the Equinoctial and this Point, be 20. So that the Declination of the *Sun* is in a Northern Sign.

In like manner,

	deg.	*min.*			*deg.*	*min.*	
When the *Sun* is in	16	00	♈	The *Sun*'s declination will be found to be	15	46	North.
	25	00	♍		19	05	South.
	29	00	♒		11	52	South.
	13	00	♌		5	9	North.

PROB. IV.

2. *The* Sun's *Amplitude.*

Definition.] THE Amplitude, *is an Arch of the Horizon comprehended between the East or West-points thereof, and that point upon which the* Sun *doth rise or set.*

Practice.] Bring 29 deg. of *Taurus* to the Horizon, and there you shall find 33 deg. 20 min. to be contained between that, and the East or West-point of the Horizon, towards the North, because the *Sun* is in a Northern sign, that is very near the North-East by East point of the Compass, as appears by the points of the Compass upon the Horizon: And that is the Amplitude for that day.

And in like manner,

	deg.	*min.*			*deg.*	*min.*	
When ☉ is in	16	00	♌	The *Sun*'s Amplitude will be from the East or West found	26	28	Northward.
	25	00	♍		31	41	Southward.
	29	00	♒		19	17	Southward.
	13	00	♈		7	34	Northward.

PROB. V.

3. *The* Sun's *Right Ascension.*

Definition.] THE *Right Ascension of the* Sun *or of a Star, is that Arch of the Equinoctial which is contained between the beginning of* Aries, *and that point of the Equinoctial which comes to the Meridian, with that point of the Ecliptick in which the* Sun *or Star is.*

Practice.] Bring the 29 deg. of *Taurus*, or 20 deg. of North-Declination to the Meridian; so shall you find (upon the Equinoctial) 56 deg. 46 min. to be contained between the beginning of *Aries* and the Meridian: And such is the *Sun*'s Right Ascension when he is in 29 deg. of *Taurus.*

In like manner.

	deg.	*min.*			*deg.*	*min.*
When the ☉ is in	16	00	♌	The *Sun*'s Right Ascension will be	138	26
	25	00	♍		232	38
	29	03	♒		330	51
	13	00	♈		11	57

PROB.

PROB. VI.

4. *The* Oblique Aſcenſion.

Definition] *THE Oblique Aſcenſion of the* Sun, *or of a Star, is that Arch of the Equinoctial which is comprehended between the beginning of* Aries *and that point of the Equinoctial which comes to the Eaſt-point of the Horizon, with that point of the Ecliptick in which the* Sun *or Star is.*

Practice.] Bring 29 deg. of *Taurus* to the Eaſt-ſide or ſemicircle of the Horizon, then ſhall you find 20 deg. 30 min. of the Equinoctial to be contained between the beginning of *Aries* and the Eaſt-point of the Horizon; and that is the Oblique Aſcenſion of the *Sun* or Star, it being in the 29 deg. of *Taurus*.

In the ſame manner,

	deg.	*min.*			*deg.*	*min.*
The *Sun* being in	16	00	♌	The Oblique Aſcenſion will be found to be	117	10
	25	00	♍		258	27
	29	00	♎		346	30
	13	00	♈		005	26

PROB. VII.

5. *To find the* Aſcenſional Difference.

Definition] *THE difference of Aſcenſion is no other than the difference of degrees between the Right and Oblique Aſcenſion.* Wherefore ſubſtract the leſſer from the greater, and the remainder will be the Aſcenſional Difference, which will be found to be 27 deg. 14 min. Or, *the Aſcenſional difference is that ſpace of time contained between 6 of the clock either in the morning or the evening, and the time of the* Sun's *riſing or ſetting.* Wherefore,

PROB. VIII.

6. *To find the time of the* Sun's Riſing *and* Setting.

Practice. BRing 29 deg. of *Taurus* to the Meridian, and ſet the Index of the Hour-wheel to 12 of the clock Southward: then turn the Body of the Globe Eaſtward, till 29 deg. of *Taurus* touch the Eaſt-ſide of the Horizon; and then will the Index of the Hour-wheel point out 11 min. after 4 in the morning, at which time the *Sun* riſeth. And if you turn the Body of the Globe about Weſtward, till the 29 deg. of *Taurus* doth touch the Weſt-ſide of the Horizon, then ſhall the Index of the Hour-circle point at 49 min. after 7 at night, at which time the *Sun* ſetteth.

PROB. IX.

7. *To find the Length of the Day and Night.*

Practice.] TUrn the Globe about till 29 deg. of *Taurus* touch the Eaſt-ſide of the Horizon; and then ſet the Index of the Hour-circle to the North (or undermoſt) 12. Then turn the Globe Weſtward, till 29 deg. of *Taurus* touch the Horizon on the Weſt-ſide, and then ſhall the Index of the Hour-wheel point at 3 hours 38 min. more than 12 hours. So that the day is then 15 hours and 38 min. long. And if you count the hours between the North 12 and the Hour-Index, you ſhall find them to be 8 hours and 22 min. which is the length of the night, the *Sun* being in 29 deg. of *Taurus*.

And

And ſo,

	deg.	min.			*Aſcenſ. diff.* deg.	m.	*Sun's riſe.* h.	m.	*Sun's ſet.* h.	m.	*length of day.* h.	m.	*length of night.* h.	m.
	16	00	♌		21	12	4	35	7	25	14	50	9	12
The *Sun*	25	00	♍	you'll	25	52	7	43	4	17	8	34	15	28
being in	29	00	♎	find	15	00	7	00	5	00	10	00	14	00
	13	00	♈		6	30	5	34	6	26	12	52	11	8

PROB. X.

8. *The* Sun's *Meridian-Altitude, and his depreſſion at Midnight.*

THis may be effected, by adding or ſubſtracting of the *Sun*'s Declination to or from the Complement of the Latitude: For,

Definit.] *It is an Arch of the Meridian, comprehended between the interſection of the Meridian with the Horizon, and that part of the Meridian upon which the* Sun *is at Noon or Midnight.*

Practice.] Turn the Globe about, till the 29 deg. of *Taurus* be juſt under the Meridian; then ſhall you find the number of degrees of the Meridian which are comprehended between that Point and the Horizon, to be 58 deg. 30 min. which is the Meridian-Altitude. And if you bring the 29 deg. of *Scorpio*, (which is the oppoſite point of the Ecliptick in which the *Sun* is) to the Meridian, the number of the degrees of the Meridian between that point and the Horizon, will be found to be 18 deg. 30 min. which is the *Sun*'s depreſſion at midnight.

In like manner,

	deg.	*min.*			*deg.*	*min.*		*deg.*	*min.*
	16	00	♌		54	36		22	24
When the	25	00	♍	You ſhall find	19	25	and the	57	25
Sun is in	29	00	♎	the *Meridian*	26	38	*depreſ.*	50	22
	13	00	♈	*Altitude*	43	39		33	21

PROB. XI.

9. *When the* Twilight *begins and ends.*

Definition.] THE *Twilight beginneth when the* Sun *is* 18 *deg. below the Horizon before its riſing, and it endeth when the Sun comes to be* 18 *deg. below the Horizon after its ſetting.*

Practice.] The Globe rectified, and the *Sun* in 29 d. of *Taurus*, find the oppoſite point thereunto, which is the 29 deg. of *Scorpio*, and bring that point, as alſo the Quadrant of Altitude, both of them on the Weſt-ſide of the Meridian, and then move both the body of the Globe, and the Quadrant of Altitude alſo, till the 29 deg. of *Scorpio* lie directly under 18 deg. of the Quadrant of Altitude: Which done, keep them both together, and then ſee how many hours the Index is removed from 12, which you ſhall find to be 1 hour and 8 minutes. So that *Twilight* begins at 8 min. after 1 in the morning. And this being taken from 4 ho. 11 min. the time of the *Sun*'s Riſing that day, there will remain 3 ho. 3 min. which is the length or continuance of the *Twilight*. Alſo if you double the time of the beginning of *Twilight* 1 ho. 8 min. you ſhall have the length of dark night, which will be but 2 ho. 16 min.

In like manner, if you would know when the *Twilight* endeth after *Sun*-ſetting, you muſt bring the 29 deg. of *Scorpio* (the point oppoſite to the *Sun*) on the Eaſt-ſide of the Meridian, making it, and 18 deg. of the Quadrant of Altitude to meet, then the Index will ſhew 10 hours 52 min. and till that time of night doth *Twilight* continue. And ſo,

	deg.			*ho.*	*m.*			*ho.*	*m.*	
	29	♋		0	10			11	50	
The *Sun*	8	♌	Day-break	5	52	aft. mid.	Twilight	6	8	at night.
in	2	♍	will be at	4	16	night.	laſts till	7	54	
	0	♎		2	41			9	19	

And if you go about to find the time of the beginning and end of *Twilight*; all the time that the *Sun* is passing from 2 deg. of *Gemini* to 30 deg. of *Cancer*, which is from about the 12 of *May* to the 12 of *July*, you shall find that there will be no *Twilight* at all, but all that time continual day: For all that space of time the *Sun* never descendeth so much as 18 deg. under the Horizon, in the Latitude of 51 deg. 30 min.

PROB. XII.

10. *What* Altitude *the* Sun *shall have at* 6 *of the Clock in the Morning or Evening.*

Definition] THE *Altitude of the* Sun, *or of a* Star, &c. *is an Arch of an* Azimuth *or Vertical Circle, passing through the* Zenith *and* Nadir *Points, and the body of the* Sun *or* Star, *counted from the Horizon to the* Sun *or* Star: *And all such* Azimuths *or Vertical Circles are represented by the* Quadrant *of* Altitude.

¶ *Note*, That this Problem is only in use when the *Sun* is in the six Northern signs: For the *Sun* is never above the Horizon at six, when he is in Southern signs.

Practice.] Bring the 29 deg. of *Taurus* to the Meridian, and set the Index of the Hour-circle to 12; then turn the Globe about Eastward, till the Index of the Hour-circle come just to 6 of the Clock; then holding the Globe there, lay the Quadrant of Altitude just over the 29 deg. of *Taurus*, and there you shall find it to cut 15 deg. 30 m. of the Quadrant. And such *Altitude* shall the *Sun* have at 6 of the clock in the morning, and the same at 6 at night.

And so,

	deg.	min.			deg.	min.
When the *Sun* is in	16	00	♌	The *Sun*'s Altitude at 6 will be found to be	12	32
	13	00	♈		4	2

PROB. XIII.

11. *What* Azimuth *the* Sun *shall have at* 6 *of the Clock.*

Definition] THE Azimuth *is an Arch of the Horizon comprehended between the East, West, North, or South-points thereof, and the intersection of a Vertical-circle passing through the* Sun *or Star whose* Azimuth *you seek.*

¶ *Note*, This Problem is of Use only when the *Sun* is in Northern Signs.

Practice.] Bring the 29 of *Taurus* to the Meridian, and set the Index of the Hour-wheel to 12; then move the Globe till the Index lie upon 6; and holding the Globe there, lay the Quadrant of Altitude just over 29 deg. *Taurus*: Then shall you find, that there are 77 deg. 14 min. of the Horizon contained between the intersection of the North-part of the Meridian, and the Quadrant of Altitude, which is the Azimuth from the North; or 12 deg. 46 min. from the East, which is the Azimuth from the East: or 102 deg. 46 min. from the South, which is its Azimuth the South.

In like manner,

	deg.	min.				deg.	min.		deg.	min.		deg.	min.
The *Sun* in	16	0	♌	The *Sun*'s Azimuth at 6 will be found to be from the	*North*	79	49	*Ea. or W.*	10	11	*South*	100	11
	13	0	♈			86	47		3	13		93	13

PROB. XIV.

12. *At what hour the* Sun *shall be upon the East or West* Azimuths.

¶ This Problem is only of use when the *Sun* is in Northern Signs.

Practice.] BRing the 29 deg. of *Taurus* to the Meridian, and the Index to 12 of the Clock. Also bring the beginning of the degrees of the Quadrant of Altitude to the East-point of the Horizon, and turn the Globe about till the 29 deg.

of

of *Taurus* do touch the degrees of the Quadrant of Altitude; then shall the Index point at 7 min. past. 7, at which time in the morning will the *Sun* be exactly upon the East Azimuth, or point of the Compass. And if you carry the Quadrant of Altitude to the West-point of the Horizon, and turn the Globe about till 29 deg. of *Taurus* touch the edge of degrees thereof, the Hour-Index will point at 4 of the Clock and 53 min. at which time in the afternoon will the *Sun* be upon the West-Azimuth, or point of the Compass.

In the same manner.

	deg.	min.			deg.	min.		deg.	min.
When the *Sun* is in	16	00	♌	It will be due *East* at	6	57	*West* at	5	03
	13	00	♈		6	17		5	43

PROB. XV.

13. *What Altitude the* Sun *shall have when he is upon the* East, *or* West-Azimuths.

¶ This Problem is in use only when the *Sun* is in Northern signs.

Practice.] BRing 29 deg. of *Taurus* to the Meridian, and the Quadrant of Altitude to the East or West-points of the Horizon: Then turn the Globe about, till the 29 deg. of *Taurus* touch the Quadrant of Altitude, and you shall find it to touch at 25 deg. 55 min. of the Quadrant: and such Altitude hath the *Sun*, when he is upon the East or West-Azimuth.

In like manner,

	deg.	min.			deg.	min.
When the *Sun* is in	16	00	♌	Its Altitude when East or West will be found	20	19
	13	00	♈		6	36

PROB. XVI.

14. *What Altitude the* Sun *shall have at any time of the day.*

Practice.] BRing 29 deg. of *Taurus* to the Meridian, and set the Hour-Index to 12 of the Clock. Then turn about the Globe till the Hour-Index point to the given hour (suppose 9 in the morning, or 3 in the afternoon:) there keep the Globe; and laying the Quadrant of Altitude over the 29 deg. of *Taurus*, you shall find 43 deg. cut thereby; and such Altitude shall the *Sun* have at 9 in the morning, or 3 in the afternoon. And by this Problem the *Sun*'s Altitudes in any sign or degree of the Ecliptick at all hours may be found; as in this following *Synopsis* or Table.

So the *Sun* being in the beginning of		*Cancer*		*Gemini* or *Leo*		*Taurus* or *Virgo*		*Aries* or *Libra*		*Scorp.* or *Pisces*		*Aquar.* or *Sagitt.*		*Capric.*	
At the ho.		d.	m.	d.	m.	d.	m.	d.	m.	d.	m.	d.	m.	d.	m.
XII.	His Altitude will be	62	00	58	42	50	00	38	30	27	1	18	18	15	00
XI. I.		59	43	56	34	48	12	36	58	25	40	17	6	13	52
X. II.		53	45	50	55	43	12	32	37	21	51	13	38	10	30
IX. III.		45	42	43	0	36	0	26	7	15	58	8	12	5	26
VIII. IV.		36	41	34	13	27	31	18	8	8	33	1	15		
VII. V.		27	17	24	56	18	18	9	17	0	6				
VI.		18	11	15	40	9	0								
V. VII.		9	32	6	50										
IV. VIII.		1	32												

This Table will be of good use to such as have occasion to make *Cylinders* or *Quadrants*, to find the hour of the day; or *Rings* and other *Instrumental Dyals*; and to insert the *Tropicks* and other Parallels of the *Sun*'s course in fixed *Sun-Dyals*, &c.

PROB. XVII.

15. *What Altitude the* Sun *shall have, he being upon any* Azimuth.

Practice.] SEt the Quadrant of *Altitude* to the *Azimuth* you intend to find the *Altitude* upon; (suppose the 30 deg. of *Azimuth* from the South towards the East:) Bring the Quadrant of *Altitude* thither; and keeping of it there, turn the Globe about till 29 deg. of *Taurus* touch the degrees of the Quadrant of Altitude, and you shall find them to concur at 55 deg. 34 min. of the Quadrant: And such is the *Sun*'s *Altitude*, when he is 30 deg. from the South-part of the Meridian, either Eastward or Westward. And by this means you may find the *Azimuth* at all times. An Example of the *Sun*'s *Altitude* upon every Tenth *Azimuth* from the South, in the beginning of each Sign here followeth,

The *Sun* in the beginning of		*Cancer*		*Leo* or *Gemini*		*Virgo* or *Taurus*		*Libra* or *Aries*		*Scorpio* or *Pisces*		*Sagitt.* or *Aquar.*		*Capric.*	
Its Altitude on these degrees of Azimuth from the South, *viz.*	d. *Az.*	d.	m.	d.	m.	d.	m.	d.	m.	d.	m.	d.	m.	d.	m.
	0	62	0	58	42	50	0	38	30	27	0	28	18	15	0
	10	61	43	58	24	49	38	38	4	26	30	17	45	14	25
	20	60	51	57	28	48	33	36	36	25	10	16	5	12	41
	30	59	52	55	52	46	40	34	34	22	27	13	15	9	45
	40	57	20	53	29	43	51	31	21	18	48	9	14	5	34
	50	54	3	50	12	40	11	27	5	13	58	3	57	0	6
	60	49	56	45	53	35	23	21	41	8	0				
	70	44	40	40	25	29	27	15	13	1	0				
	80	28	11	33	46	21	29	7	52						
	90	30	38	26	10	14	25								
	100	22	27	18	2	6	45								
	110	14	14	9	58										
	120	6	34	2	30										

(found to be / will be found)

This Table hath the like use for the making of *Cylinders*, *Quadrants*, and other Instruments that give the *Azimuth* by the height of the *Sun*, as the preceding Table hath for the *Hour*, and may be so applied.

The Latitude (51 *deg.* 30. *min.*) *the* Sun's *place in the Ecliptick*, (29 *deg.* 0 *min. of* Taurus) *and his Altitude* (12 *deg.*) *being given: To find*

PROB. XVIII.

1. *The* Sun's Azimuth *at any time.*

Practice.] THE Globe being rectified, *&c.* and the *Quadrant of Altitude* fixed, and brought to the *Horizon*; turn 29 deg. of *Taurus* toward the East, if in the morning; or towards the West, if in the Evening, till it come to lie just under 50 deg. 47 min. of the *Quadrant of Altitude*; and then note at what degree in the *Horizon* the *Quadrant of Altitude* resteth, which will be at 48 deg. from the East, if in the morning; or 48 deg. from the West, if in the afternoon Northward, which is the *Azimuth* from the East or West towards the South. And this *Azimuth*, if reckoned by the Points of the Compass upon the *Horizon*, will be North-East, if in the morning; North-West, if in the evening, when the *Sun* is in 29 deg. of *Taurus*, and

and hath 50 deg. 47 min. of *Altitude*. Now if you count the degrees of the *Horizon* between the *Quadrant* of *Altitude* and the North-part of the *Meridian*, you shall find them to be 138 deg. which is the *Azimuth* from the North: And if you count them from the South-part of the *Meridian*, you shall find them to be 42 deg. which is the *Azimuth* from the South.

In like manner,

	deg.	*m.*	
The Latitude being	51	30	
The *Sun*'s place	1	00	*Aquarius*.
The *Sun*'s Altitude	12	00	

Then will the *Azimuth* be found to be 56 deg. from the East or West towards the South; which (by the Points of the Compass upon the *Horizon*) will appear to be S. E. by S. if in the morning, or S. W. by S. if in the evening.

PROB. XIX.

2. *The Hour of the day.*

Practice.] BRing 29 deg. of *Taurus* to the Meridian, and set the Index to 12 of the Clock: Then, if it be in the forenoon, set the *Quadrant* of *Altitude* on the East-side of the *Meridian*; but on the West-side, if it be in the afternoon. And turn the Globe about, till the 29 deg. of *Taurus* meet with 12 deg. of the *Quadrant* of *Altitude*; and then shall the Index of the Hour-circle point at 5 of the clock and 36 min. if it be in the morning; or at 24 min. after 6 of the clock, if it be at night. And that is the true hour of the day.

In like manner,

	deg.	*min.*	
The Latitude being	51	30	
The *Sun*'s place	1	00	*Taurus*.
The Altitude	36	00	

Then will the hour of the day be found to be either 9 in the morning, or 3 in the afternoon. And which of these hours it is, may best be known by a second observation of the *Altitude*: For if the *Altitude* do increase, it is the forenoon; but if it decrease, it is the afternoon.

Again,

	deg.	*min.*
The Latitude being	52	30
The *Sun*'s place the beginning of *Taurus*.		
The *Sun*'s Altitude	25	56

The hour of the day would be found to be either 8 min. past 4 in the afternoon. Or if it were in the forenoon, 52 min. after 7 in the morning.

PROB. XX.

3. *To find the Difference of* Latitude, *or, to know how many degrees the* Pole *must be Elevated or Depressed, to make the* Longest day *in any* Latitude *an Hour longer, or an Hour shorter than it is in your* Latitude.

LET it be required to find in what Latitude the *Longest day* shall be an hour longer than it is at *London*.

Practice.] Rectifie the Globe to the Meridian of *London*, the Index of the Hour-circle to 12, and the Solstitial Colure to the Meridian; so shall the number of hours contained between the Brass-Meridian and the Horizon, upon the Tropick of *Cancer*, be half the length of the *Longest day* at *London*, namely, 123 deg. which in time is 8 hours and 13 min. for the *Semidiurnal-Arch* of the *Artificial day* at *London*. At this intersection of the Tropick of *Cancer* with the Horizon, make a small mark upon the Tropick, and move the Globe Westward, till the Equinoctial Circle hath passed 7 deg.

30 min. or one half hour of time from the Meridian; and then obſerve where the Tropick of *Cancer* interſects with the Horizon again, and there upon the Tropick make another ſmall mark. Then bring the Solſtitial Colure back again to the Meridian, and there fix the Ball of the Globe; and move the Braſs Meridian in the Horizon upwards, till your ſecond prick which you made upon the Tropick do juſt touch the Horizon; then will the Globe ſtand at that Latitude in which the *Longeſt day* will be an hour longer than the *Longeſt day* at *London* is; and that Latitude will be found to be 56 deg. 27 min.

But if you would know in what Latitude the *Longeſt day* ſhall be one hour ſhorter than at *London*, then having made a mark upon the Tropick at the termination of the *Longeſt day* there, move the Globe Eaſtward, till 7 deg. 30 min. have paſſed the Meridian; and where the Tropick and Horizon interſect, make there a mark upon the Tropick, and bring the Solſtitial Colure to the Meridian, and fix the body of the Globe there; then depreſs the Meridian in the Horizon, till your ſecond mark do touch the Horizon; then will the Globe reſt at that Latitude wherein the *Longeſt day* will be one hour ſhorter than the *Longeſt day* at *London* is; which Latitude will be found to be 35 deg. 58 min.

PROB. XXI.

4. *How much muſt the* Sun's *Declination increaſe or decreaſe, to make the Day Artificial one Hour longer or ſhorter than it was at the time propoſed.*

SUppoſe in Latitude 51 deg. 30 min. the *Sun* to have 10 degrees of North-Declination, and I would know how much the Declination muſt increaſe Northward, to make the day one hour longer than it is.

Practice.] Elevate the Globe to the Latitude of *London*, 51 deg. 30 min. and bring the Equinoctial Colure to the Meridian; alſo, bring the *Sun*'s place, or the parallel of 10 deg. his Declination, to the Horizon; and upon the Horizon make a ſmall mark, to which bring the Equinoctial Colure, and at this interſection make a mark upon the Colure. Then (if the days lengthen, move the Globe Eaſtward, or Weſtward if they ſhorten) till 7 deg. 30 min. of the Equinoctial have paſſed the Meridian; and then, again, where the former Colure interſects with the Horizon, make another mark, upon the Colure. This done, bring the Colure to the Meridian again, and ſee what number of degrees of the Meridian (or degrees of the Colure it ſelf) are contained between the two pricks; which you ſhall find to be 4 deg. and 40 min. And ſo much muſt the Declination increaſe Northward, to make the day lengthen one hour, at that time of the year.

PROB. XXII.

5. *To find what number of Days are contained between the Days of Lengthning or Shortning one Hour, at any time of the Year.*

LET the time be as in the laſt Example, where the *Sun* hath 10 deg. of North-declination; which will be about the 5 of *April*.

Practice.] Having made two pricks in the Colure, as in the laſt Example, bring the firſt prick to the Horizon, where you ſhall find it to ſtand againſt the 5 of *April*. Then move the Globe till the other prick touch the Horizon; which it will do about the 19 of *April*, at which time the day will be one hour longer than it was upon the 5. of *April*; between which two days, there are contained 13 complete days, and parts of both the two other days. So that you may conclude, that in 14 days time (at that ſeaſon of the year) the days do lengthen one hour, and ſhorten as much when the *Sun* is in oppoſite ſigns.

PROB.

PROB. XXIII.

6. *To find the length of the Longest or Shortest day in any Latitude.*

Definition] THE *Longest Artificial day in any Latitude, is made by the* Sun's *passing through the Tropick of* Cancer *in all places that have North-Latitude, or through the Tropick of* Capricorn *to all that have South-Latitude.*

Practice.] To find the length of the Longest day at *London*, in Latitude 51 deg. 30 m. elevate the Globe thereto, and bring the Solstitial Colure to the Meridian, and the Hour-Index to 12, then count the number of hours upon the Tropick of *Cancer*, that are contained thereupon between the intersection thereof with the Horizon, on the East-side, and its intersection on the West-side, which you will find to be 16 hours, and almost half an hour, *viz.* 26 min. for the length of the Longest day. And the number of hours contained between the East and West intersections of the Tropick of *Capricorn* and the Horizon, is the length of the Shortest day, which at *London* will be found to be 7 hours, and somewhat above half an hour, *viz.* 34 min.

Half the length of the Day is the time of the *Sun's Setting*, and half the length of the Night is the time of the *Sun's Rising*.

PROB. XXIV.

7. *Of the Reason of the Inequality of Days Natural, and Days Civil.*

Definition] A *Natural day is that space of time in which the* Sun *moveth from the Meridian of some one place or Country, to the same Meridian again. These days are not always of an equal length, but are longer at some times of the year than at other times, but at all times they are longer than the Civil day is. A Civil day being that space of time containing just 24 hours, reckoned from* 12 *of the Clock at Noon one day, unto* 12 *of the Clock at Noon the next day; in which time the Equinoctial maketh one entire Revolution about the* Axis *of the World. The difference between these two sorts of Days is but small; and there is a double cause for this small inequality.*

1. *Because the* Sun's *apparent motion differs from his true motion, as being slower when he is in his* Apogæum, *than in his* Perigæum; *he moving scarce* 58 *min. in a day when he is in the one, and above* 61 *min. when he is in the other, and so increaseth in Right Ascension.*

2. *The difference of Right Ascensions in several equal parts of the Ecliptick: For when the* Sun *is near either of the Tropicks, the Ascensional Differences are greater than when the* Sun *is about the Equinoctial: For about* Aries *or* Libra, *the Right Ascension of* 10 *deg. is but* 9 *deg.* 10 *min. whereas the Right Ascension of* 10 *deg. of* Cancer *or* Capricorn *is* 10 *deg. and* 53 *min.*

Now this difference being so small, in one days time it is not perceptible by the Globe; wherefore if you would find this difference, it would be requisite to take some number of days, as 10, 20, *or* 30, *and in them it will be apparent. And to effect it by the Globe, do thus.*

Practice.] First, find the place of the *Sun*, both at the beginning and ending of those days you would compute the difference of. 2. Find the *Right Ascensions* answerable to each place in the Ecliptick, as also the difference of *Right Ascensions* answerable to the motion of the *Sun* in each respective number of days. 3. Compare the difference of the *Right Ascensions* together, by substracting the lesser from the greater; and the difference converted into time, shall be the number of minutes that the one number of days exceedeth the other.

EXAMPLE.

EXAMPLE.

Let it be required to find what difference there is in the length of the first 20 days of *December*, and the first 20 days of *March*.

			d.	m.		d.	m.	d.	m.	d.	m.
Sun's place	*Decemb.*	1 ♐	20	07	Its *Right Ascension*	259	01	22	24		
		21 ♑	10	33		281	25				
	March	1 ♓	21	24	is	Differ. of *Ascens.*				4	02
		21 ♈	11	11		352	06	18	22		
						10	28				

Here by this Synopsis you may perceive that the difference of these 20 days taken at several times of the year is 4 deg. 2 min. which converted into time is 16 minutes, that is, a quarter of an hour and one minute; and so much longer are the first 20 days in *December*, than the first 20 days of *March*.

And by this means you shall find that the Month of *January* is longer than the Month of *June* (both Months consisting of 31 days) by 1 deg. 20 min. which is only $4\frac{1}{3}$ min. in time.

			d.	m.		d.	m.	d.	m.	d.	m.
Sun's place	*January*	1 ♒	21	47	Its *Right Ascension*	293	29	31	24		
		31 ♒	22	17		324	53				
	June	1 ♊	20	40	is	Differ. of *Ascens.*				01	20
		31 ♋	18	16		79	43	30	04		
						109	47				

Thus the Month of *January* is longer than the Month of *June* by 4 minutes and $\frac{1}{3}$ of a minute of time.

¶ All the fore-going Problems may be performed upon either Globe; the *Horizon*, *Meridian*, *Quadrant* of *Altitude*, *Hour-Circle*, and most of the Circles upon the Globes themselves, being in both the same. But it is most proper to use the Terrestrial Globe for Geographical and Nautical Problems; and the Celestial Globe for such as concern Astronomy; and these following are chiefly such, and therefore best to be wrought by the Celestial Globe.

PROB. XXV.

To find the Longitude *and* Latitude *of any Star.*

Defini-tion.] THE Longitude *of a Star is an Arch of the Ecliptick, contained between the beginning of* Aries, *and the intersection of an Arch of a great Circle, which passeth through both the Poles of the Ecliptick, and also through the body of that Star.*

The Latitude *of a Star is that part of an Arch of a great Circle, which passeth through both the Poles of the Ecliptick, and through the body of the Star, and is contained between the Ecliptick-line and that Star.*

The Poles of the Ecliptick are distant from the Poles of the world $23\frac{1}{2}$ deg. on either side.

Practice.] For the *Longitude*, skrew the *Quadrant* of *Altitude* over that Pole of the Ecliptick which is neareſt to the Star whose *Longitude* you seek. Then laying the Quadrant just over the centre of the Star, look what degrees of the Ecliptick, (counting them from the beginning of *Aries*) and those degrees are the degrees of the Star's *Longitude*. So the *Quadrant* of *Altitude* skrewed over the North-pole of the Ecliptick, and laid upon the bright Star *Capella*, the Quadrant shall cut 77 deg. 16 min. of the Ecliptick Circle, counted from the beginning of *Aries*; and that is that Star's *Longitude*.

For the *Latitude*, the *Quadrant* fitted as before, and laid over the centre of *Capella*, the Star shall cut 22 deg. 50 min. of the *Quadrant* of *Altitude*; and such is the *Latitude* of that Star, North, for that it lies on the North-side of the Ecliptick-Line.

PROB.

PROB. XXVI.

To find the Right Ascension *and* Declination *of a Star.*

Definition.] THE Right Ascension *of a Star is that Arch of the Equinoctial which is contained between the beginning of* Aries *and that point which comes to the Meridian with that Star.*

The Declination *of a Star is an Arch of the Meridian contained between the Equinoctial and any Star.*

Practice.] For the *Right Ascension,* (the Globe being rectified) bring *Capella* to the Meridian, and then shall you find 73 deg. 7 min. of the Equinoctial contained between the beginning of *Aries* and the Meridian; and that is the *Right Ascension* of *Capella.*

For the *Declination,* bring *Capella* to the Meridian, so shall you find 45 deg. 37 min. of the Meridian contained between the Equinoctial and *Capella*; and that is the *Declination* of that Star. And in this manner you may find the *Longitude, Latitude, Right Ascension,* and *Declination* of any other Star upon the Celestial Globe: As in this following Table of the principal Fixed Stars of the first Magnitude, you shall find.

Stars Names.	*Longitude* deg.	m.	*Latitude* deg.	m		*Right As.* deg.	m	*Declin.* deg.	m.	
Arcturus	119	39	31	2	B	210	13	20	58	B
Lucida Lyra	280	43	61	47	B	276	27	38	30	B
Algol	51	37	22	22	B	41	46	39	39	B
Capella	77	16	22	50	B	73	7	45	37	B
Aldebaran	65	12	5	31	A	64	17	15	48	B
Regulus	145	17	0	26	B	147	43	13	33	B
Caudæ Leonis	167	3	12	18	B	173	4	16	25	B
Spica Virgin.	199	16	1	59	A	196	56	9	31	A
Antares	245	13	4	27	A	242	23	25	37	A
Fomahant	329	11	21	00	A	339	46	31	17	A
Regel	72	17	31	11	A	74	44	8	37	A
Syrius	99	35	39	30	A	97	42	16	14	A
Procyon	111	18	15	57	A	110	34	6	3	B

PROB. XXVII.

To find the distance of two Stars.

Practice.] 1. IF *the two Stars be both of them under the same Meridian,* Bring them under the General (or Brass) Meridian, and see what degrees of the Meridian are contained between them, for that is their distance.

2. *If they lie not under the same Meridian, but have the same declination, or lie in the same Parallel,* Bring one of them to the Meridian, and see what degrees of the Equinoctial are cut thereby: then bring the other Star to the Meridian, and count what degrees of the Equinoctial are contained between the Meridian and the degrees before found, for that is the distance of those two Stars.

3. *If the two Stars do neither lie under the same Meridian, nor in the same Parallel,* Then lay the Quadrant of Altitude (it being loose) to both the Stars, and the degrees of the Quadrant contained between the two Stars is their distance. And if the Quadrant be too short, you may use the Circle of Position, or take their distance with a pair of Calope-Compasses, and measure their distance upon the Equinoctial, or any other great Circle.

Thus, The Right Shoulder of *Auriga,* and the Right Shoulder of *Orion,* being under the same Meridian, their distance will be found to be 37 deg. 38. min.

Also, *Arcturus* and the Lions Neck, being near in the same Parallel, their distance will be found to be 57 deg.

Likewise, *Lyra* the Harp, and *Marchad* in the Wing of *Pegasus,* will be found to be distant 63 degrees.

PROB. XXVIII.

To know what Stars will be upon the Meridian at any hour of the Night.

Practice. THE *Sun* being in 29 deg. of *Taurus*, what *Stars* will be upon the Meridian at 10 of the Clock and 12 min. at Night, bring 29 deg. of *Scorpio* (which is the oppoſite Sign to *Taurus*) to the Meridian, and ſet the Index of the Hour-Circle to 12, then turn the Globe about weſtward, till the Index point at 12 min. after 10 of the Clock, and there hold the Globe; and all thoſe *Stars* which lie under the Braſs Meridian are then upon the Meridian, of which *Arcturus* is the chief.

PROB. XXIX.

To know what day in the year any Star ſhall be upon the Meridian at 12 of the Clock at Night.

Practice. BRing the *Star* to the Meridian, and mark what degree of the Ecliptick is juſt under the Meridian at the ſame time: Then find that degree of the Ecliptick in the *Horizon*, and note what day of the year ſtandeth againſt it, for that day of the year will that *Star* be upon the South-part of the Meridian at 12 at night: and when the Sun is in the oppoſite Point of the Ecliptick, the ſame *Star* will be upon the North-part of the Meridian at 12 at noon.

PROB. XXX.

The Sun's *Place, and the Altitude of a known Star given, to find the hour of the Night.*

THE *Sun* being in 21 deg. of *Capricorn*, the Altitude of the *Great Dog* 14 deg. I demand the hour of the Night.

Practice. The Globe Rectified, *&c.* being 21 deg. of *Capricorn* to the Meridian, and the Index to 12 of the Clock. Then move the Globe and Quadrant of Altitude ſo together, that the *Great Dog* meet with 14 deg. of the Quadrant; and then ſhall the Index point at 8 of the Clock and 22 minutes; which is the true hour of the Night.

And thus,

	deg.				deg.		deg.	min.
When the	20	♐	and the	*The Bulls Eye*	39	The hour	7	12
Sun is in	20	♏	Altitude	*The Bulls Eye*	30	will be	9	2
	5	♊	of	*Arcturus*	50		11	3

PROB. XXXI.

The Altitude of Aldebaran *(or any other Star) being given in a known Latitude; To find the Star's* Azimuth.

Practice. THE Quadrant of Altitude being fixed in the *Zenith*, move it and the Globe, till the degrees of Altitude given do meet with the centre of the Star; then ſhall the end of the Quadrant of Altitude ſhew you upon the Horizon the *Azimuth* in which the Star then is. And thus, if you bring the Quadrant of Altitude on the Eaſt-ſide of the Globe, moving it and the Globe both, till the centre of *Aldebaran* do meet juſt with 42 deg. of the Quadrant, you ſhall then find the Quadrant of Altitude to reſt at 33 deg. of the Horizon, counted from the Eaſt; or at 57 deg. if you count them from the South: and that is the *Azimuth* of *Aldebaran* when he hath 42 deg. of Altitude; and that is near the S. E. by E. Point of the Compaſs.

The

The Latitude of the Place (51 deg. 30 min.) and the Declination of a Star (suppose the *Bulls eye*, *Aldebaran*) given: To find

PROB. XXXII.

1. *Its* Right Ascension.

Practice. THE Globe Rectified to the Latitude, *&c.* bring *Aldebaran* to the Meridian: then count how many degrees of the Equinoctial are contained between the Meridian and the beginning of *Aries*; which will be 64 deg. 17 min. and that is the *Right Ascension* of that Star; which in time (by allowing 15 deg. for an hour, and 1 deg. for 4 min. of time) is 4 hours 16 min. its *Right Ascension* in time.

And in the same manner may you find

			d.	m.		h.	m.
The Right Ascension of	*Arcturus*	to be	210	13	in time	14	1
	Syrius		97	42		6	30
	Algol		39	39		2	38

PROB. XXXIII.

2. *Its* Ascensional Difference.

Practice. BRing the Star to the Meridian, and the Hour-Index to 12. Then bring the Star either to the East or West-side of the Horizon, and there you shall find 1 hour and 27 min. contained between the Index and 6 of the Clock: and such is the *Ascensional Difference* of *Aldebaran*.

In like manner, you may find

			d.	m.		h.	m.
The Ascensional Difference of	*Arcturus*	to be	28	40	or in time	1	55
	Syrius		21	28		1	26
	Algol		———			———	

Algol his Declination being more than the Complement of the Latitude, never rises nor sets, but is always above the Horizon.

PROB. XXXIV.

3. *Its* Amplitude.

Practice. BRing *Aldebaran* to the Horizon on either side of the Globe, and you shall find it to touch the Horizon at 25 deg. 56 min. from the East or West Northward; which is the *Amplitude* of the *Bulls eye*'s rising or setting. And according to the points of the Compass it riseth E. N. E. 2 deg. 26 min. Northerly; and sets W. N. W. 2 deg. 26 min. Northerly.

And thus may you find

				d.	m.
That	*Arcturus*	Riseth from	Northward	35	6
	Syrius	the East or	Southward	26	41
	Algol	West	Never rises or sets		

PROB. XXXV.

4. *The* Semidiurnal Arch, *and the time that* Aldebaran (*or any other Star*) *continues above the* Horizon.

Practice.] BRing *Aldebaran* to the Meridian, and set the Hour-circle to 12. Then turn the Globe Westward, till *Aldebaran* touch the Horizon: then shall the Hour-Index point at 7 ho. 27 min. And so long time is *Aldebaran* above the Horizon, before he comes to the Meridian; and continues so many hours and minutes above the Horizon, after he hath past the Meridian, and sets in the West. And those 7 ho. and 27 min. is the *Semidiurnal Arch* of that Star; which doubled, is 14 ho. 54 min. And so long doth that Star continue above the Horizon after the time of his rising.

And in this manner you may find

			d.	m.		h.	m.
The Semi-	*Arcturus*	☌	7	55	And his conti-	15	50
diurnal	*Syrius*	☍	4	34	nuance above	9	8
Arch of	*Algol*	☍	12	00	the Horizon	24	00

PROB. XXXVI.

5. *At what hour* (*any time of the year*) Aldebaran *comes to the Meridian.*

Practice.] LET the time be the first of *January*, at which time the *Sun* is in 22 deg. of *Capricorn*. Bring 22 deg. of *Capricorn* to the Meridian, and set the Hour-Index to 12. Then turn the Globe about till *Aldebaran* be under the Meridian, and then you shall find the Index to point at 42 min. after 8 of the Clock, at which time *Aldebaran* will be upon the Meridian that night.

In like manner you may find, that

				h.	m.
Upon	*October* 28	*Arcturus*	will be up-	11	10
	January 21	*Syrius*	on the Me-	9	33
	January 1	*Algol*	ridian at	7	12

PROB. XXXVII.

6. *At what hour* (*at any time of the year*) Aldebaran (*or any other Star*) *riseth or setteth.*

Practice.] LET the time be *January* 1. By the last before-going, you found that *Aldebaran* came to the Meridian at 8 ho. 42 min. And by the last but one, you found his Semidiurnal Arch to be 7 ho. 27 min. This being taken from 8 ho. 42 min. the time of his being South, leaveth 1 hour 15 minutes, the time of its Rising: so that upon the first of *January Aldebaran* did rise at 15 min. after 1 in the afternoon. Again, if you add his Semidiurnal Arch 7 hours 27 min. to the time of its being South 8 hours 42 min. the sum will be 16 hours 9 min. from which take 12 hours, and the remainer will be 4 hours 9 min. So that *Aldebaran* will set at 9 min. after 4 of the Clock the next morning. And in like manner you may find, that

				h.	m.		h.	m.
Upon	*October* 28	*Arcturus*	Rises	3	8	Sets	7	12
	January 21	*Syrius*	at	5	3	at	2	3
	January 1	*Algol*						

Algol never Rises nor Sets.

Prob.

PROB. XXXVIII.

7. *At what Horary distance from the Meridian* Aldebaran *will be due East or West: And what* Altitude *he shall then have.*

Practice. BRing *Aldebaran* to the *Meridian*, and the Hour-Index to 12. and the Quadrant of *Altitude* to the West-point of the *Horizon*: then turn the Globe Eastward, till the Centre of the *Star* be just under the edge of the Quadrant; then shall the Index point at 5 hours and 40 min. So that when *Aldebaran* is due East or West, he will be 5 hours 40 min. of time short of, or gone beyond the *Meridian*.

And when the Centre of *Aldebaran* is just under the Edge of the Quadrant of *Altitude*, you shall find it to touch 20 deg. 21 min. And such is the *Altitude* of *Aldebaran* when he is upon the East or West *Azimuth*. In like manner may you find, that

		h.	m.		d.	m.
Arcturus	will be upon the East or	4	49	and his	27	13
Syrius	West *Azimuth*, when he is	5	04	*Altitude*	20	56
Algol	distant from the Meridian	3	15	will be	54	37

PROB. XXXIX.

8. *What* Altitude *and* Azimuth Aldebaran (*or any other Star*) *shall have when six hours distant from the* Meridian.

Practice. BRing *Aldebaran* to the *Meridian*, and the Index to 12; Then turn the Globe about till the Index point at 6: then lay the Quadrant of *Altitude* over the Centre of the Star, and you shall find it to lie under 12 deg. 18 min. of the Quadrant: and such is the *Altitude* of *Aldebaran*. At the same time look what degrees of the *Horizon* are cut by the *Quadrant of Altitude*, and you shall find 8 degrees between it and the East or West Points Northwards. And such is the *Azimuth* of *Aldebaran*. And according to this Rule you shall find, that when

		Alt.		*Azim.*		
		d.	m.	d.	m.	
Arcturus	is 6 hours distant	16	15	76	36	from the
Algol	from the *Merid.* his	12	38	79	43	North.

Syrius is never 6 hours distant from the Meridian, nor any other Star that hath South-declination.

PROB. XL.

9. *To find what* Altitude *and* Azimuth *any Star hath when he is at any horary distance from the* Meridian.

Practice. THis is no other than the last. For having brought the Star to the *Meridian*, and the Index to 12, move the Globe till it come to the designed hour. Then the *Quadrant of Altitude* being laid over the Star, shall at the same time shew you both the *Altitude* and *Azimuth* thereof as before. This needeth no Example.

Prob.

PROB. XLI.

10. *Having the* Azimuth *of a Star, to find at what horary distance that Star is from the* Meridian, *and what* Altitude *that Star then hath.*

Practice. BRing the *Star* to the *Meridian*, the *Index* to 12, and the *Quadrant of Altitude* to the *Given Azimuth*; then turn the Globe about till the Centre of the *Star* lie just under the *Quadrant of Altitude*; the *Index* at that time shall give the *horary distance*, and Quadrant the *Altitude* of the *Star*.

Example: *Aldebaran* being seen upon 80 degrees of *Azimuth* from the North-Westward, that is, near upon the W. by N. Point of the Compass; the *Star* brought to the Meridian, and the *Quadrant of Altitude* to 80 degrees, and the *Hour-Index* to 12. If you bring the *Star* to the *Quadrant of Altitude*, you shall find the *Index* to point at 6 hours, which is the *Stars horary distance* from the *Meridian*. And the *Quadrant of Altitude* will shew 12 deg. 18 min. the *Altitude of Aldebaran* at that time.

PROB. XLII.

Concerning the Poetical Rising and Setting of the fixed Stars, viz.

Of The { Cosmical / Acronical / Heliacal } *Rising and Setting.*

And how to find either of them by the Globes.

1. *Of the* Cosmical *Rising and Setting.*

Definition.] A Star is said to *Rise Cosmically*, when it Riseth with the *Sun*, or with that degree of the Ecliptick in which the *Sun* then is. And the *Cosmical Setting* is, when a Star setteth in the Morning, or goeth down under the West-*Horizon* in the Morning at such time as the *Sun* is Rising in the East.

Practice.] Upon the 27 of *May*, the *Sun* then being in 17 degrees of *Gemini*, I would know what Stars do then Rise and Set *Cosmically*. Rectifie the Globe to your *Latitude*, and bring the 11 degree of *Aquarius* to the East-part of the *Horizon*: then look what Stars are about the edge of the Eastern Semi-circle of the *Horizon*: for all those Stars do that day Rise *Cosmically*. And those Stars which touch or are near the Rim of the West Semi-circle of the *Horizon* do Set at that time *Cosmically*. So shall you find

May 27 { *Aldebaran* or the *Bulls eye*, with divers other small-ler Stars, } Rising and { The right leg of *Serpentarius* and several other smaller Stars } Setting *Cosmically*.

2. *Of the* Acronical *Rising and Setting of the Stars.*

Definition.] A Star is said to Rise *Acronically* when it Riseth in the East-*Horizon*, at such time as the *Sun* goes down or Sets in the West-*Horizon*; and the *Acronical* Setting is when a Star goeth down under the *Horizon* with the *Sun*.

Pra-

Practice.] Upon the 18 of *October*, in the *Latitude* of 51 deg. 30 min. the *Sun* then being in 5 deg. of *Scorpio*, I would know what Stars do on that day Rise and Set *Acronically*. Rectifie the Globe to the *Latitude*, bringing the place of the *Sun* 5 deg. of *Scorpio* to the West-part of the *Horizon*, then shall all those Stars which you see on the Verge of the East-side of the *Horizon*, be Rising *Acronically*. And all those that are about the Verge of the Western part of the *Horizon* are then Setting *Acronically*. And so upon the forementioned day, you shall find

A Star in the *Whales Tail*, and several other smaller Stars } Rising, and { The *Tail* of the *Lyon*, the South Balance, and several other smaller Stars } Setting *Acronically*.

3. *Of the* Heliacal *Rising and Setting of the Stars.*

Definition.] A Star is said to *Rise Heliacally*, when having been some time combust, (or hid under the *Sun-Beams*) begins now to appear, it being at greater distance from the *Sun*. And a Star is said to *Set Heliacally*, which hath some small time before been seen, but now, by the near approach of the *Sun*, becomes *cumbust* and hidden under his *Beams*. Now to know when a Star begins to become *cumbust*, and when to be freed from his *cumbustment* of the *Sun-beams*, no certain Rule can be given: for the *Magnitude of the Star*, the *difference of the Climate*, the *cloudiness* or *serenity* of the *Air* may much alter. But the Opinion of the Ancient Astronomers was, that

A Star of the { 1 2 3 4 5 6 } *Magnitude* may be seen when the Sun is but { deg. 12 13 14 15 16 17 } below the Horizon.

And those which are only Nebulous, cannot be seen till the *Sun* be 18 deg. under the *Horizon*.

Practice.] Rectifie the Globe to the *Latitude*, and the *Quadrant of Altitude* in the *Zenith*; then bring the given Star (suppose *Regulus*, or the *Lions Heart*) to the East-side of the Horizon, and the *Quadrant of Altitude* to the West-side; then *Regulus* being a Star of the First Magnitude (by the former Rule of the Ancients) may be seen when the *Sun* is but 12 deg. below the Horizon: wherefore see what degree of the Ecliptick doth cut the *Quadrant of Altitude* in 12 degrees, which you shall find to be 9 deg. of *Pisces*, the opposite degree to which is 9 deg. of *Virgo*; to which Sign and degree when the *Sun* cometh, (which will be about the 23 of *August*,) then will *Regulus*; the Lions heart, *Rise Heliacally*: Then for the *Heliacal Setting*, bring the *Star* to the West-side of the Horizon, and turn the *Quadrant of Altitude* to the East-side, and see what degree of the Ecliptick is elevated upon the *Quadrant*, as the Magnitude of the *Star* you deal with doth require. For when the *Sun* comes to the opposite degree of the Ecliptick, that *Star* shall Set *Heliacally*. So

The *Pleiades*	shall Rise *Heliacally* upon	*June* 4	and Set *Heliacally* upon	*April* 20
Aldebaran		*June* 26		*April* 22
Arcturus		*Sept.* 26		*Novem.* 19

Concerning the *Cosmical* and *Acronical* Rising of the Stars, see the following Table.

A Ta-

A TABLE *shewing the time of the Year when* 50 *eminent Stars do Riſe, both* Coſmically *and* Acronically.

Calculated for the Year 1690. and may ſerve for many Years paſt, and to come, without any conſiderable Error.

Names of the STARS.	Their *Coſmical* Riſing.		Their *Acronical* Riſing.	
Marchab. Pegaſi	*January*	1	*March*	9
Right Shoulder of *Aquarius*		6	*February*	14
Extreme Star in the Wing of *Pegaſus*		28		19
Following Tail of the *Goat*	*February*	5	*January*	26
Bright Star in the *Rams-Head*	*March*	2	*April*	20
The following Horn of the *Ram*		5		15
The former Horn of the *Ram*	*April*	10		13
North-Tail of the *Whale*		12	*March*	19
The brighteſt of the *Pleiades*		22	*May*	10
The Star in the Knot in the Net of ♑	*May*	1	*April*	29
North-Horn of the *Bull*		15	*June*	7
North-Eye of the *Bull*		20	*May*	12
Belly of the *Whale*		23	*March*	16
Bulls Eye, *Aldebaran*		27	*May*	11
South-Horn of the *Bull*	*June*	5	*June*	17
Lower Head of *Gemini, Hercules*		21	*July*	20
Bright Foot of *Gemini*		25	*June*	5
Middle Star in *Orions Girdle*	*July*	2	*May*	4
North-*Aſellus*		12	*July*	25
Preſepe		14		19
South-*Aſellus*		17		17
Leſſer Dog, *Procyon*	*July*	18	*June*	6
Great Dog, *Palilicium*		30	*May*	3
Lions Heart	*Auguſt*	8	*Auguſt*	9
Lions Back		10	*October*	19
Hydra's Heart		20	*June*	16
Lions Tail		22	*October*	16
Hares Thigh	*Septemb.*	9	*April*	14
Vindemiatrix		13	*November*	8
Arcturus		15	*Decemb.*	13
Virgins Girdle		19	*October*	18
Bright Star of the *Crown*		28	*January*	7
Virgins Spike	*October*	3	*Septemb.*	24
Right Shoulder of *Hercules*		6	*January*	9
Left Shoulder of *Hercules*		10		21
Head of *Hercules*		21		8
North Balance		21	*Novemb.*	17
South Balance		23	*October*	24
Swans Bill		31	*February*	16
Right Shoulder of *Ophiucus*	*Novemb.*	5	*January*	4
Left Knee of *Ophiucus*		6	*December*	6
Lower Wing of the *Swan*		4	*March*	11
Vultures Tail		11	*January*	27
Right Knee of *Ophiucus*		16	*December*	6
Scorpions Heart		22	*Novemb.*	4
The *Eagle*		25	*Decemb.*	31
Pegaſus Scheat	*Decemb.*	11	*March*	24
Andromeda's Girdle		22	*April*	29
Andromeda's Head		25		6
Upper Horn of the *Goat*		25	*January*	13

PRO-

PROBLEMS Relating to ASTROLOGY.

INTRODUCTION.

ASTROLOGY, consisteth of two Parts, the one *Mathematical* (or *Astronomical*) the other *Astrological*, or *Judiciary*.

The *Astronomical* Part teacheth, how in a *Scheme* (or *Figure*, as they call it) to represent the *Face* of the *Heavens* upon a *Plain*, which they divide into XII. *Parts*, calling them *Houses*: And such a *Figure* do they erect for any Hour of the *Day* or *Night*, at any time of the *Year*, and in any *part* of the *World*.

The *Astrological* Part is, from the sight of the *Figure* so erected, to give a determinate *Judgment* of what shall be required of them at that time, for which the *Figure* was erected, whether of an *Annual Revolution*, *Election*, the *Nativity* of a Person, *Goods stoln*, &c.

The principal Authors that have given their Opinions concerning the dividing of the *Heavens* into XII. *Mansions* or *Houses*, are 1. *Ptolomy*, 2. *Alcabitius*, 3. *Campanus*, and 4. *Regiomontanus*; which last is now generally received among the *Astrologers* of these times, and by them deemed (for what reason I cannot tell) the *Rational way* of *Regiomontanus*.

Now, because (as I said before) that the Erection of a *Figure* of the Heavens is the *Mathematical* or *Astronomical part* of *Astrology*, I shall therefore shew how by the *Globes* to erect a *Figure* of the Heavens according to the various ways of the Four formentioned *Authors*, and that for the same *Latitude* and *Time*, that the difference between them may appear.

PROB. I.

How to Erect a Figure *of the Heavens in the* Latitude *of* London 51 *deg.* 30 *min.* N. *for the* 10*th. day of* March, *at* 49 *min. after* 9 *in the* Forenoon; *at which time the* Sun *entered into the first scruple of* Aries *in the Year* 1675.

I. *According to the* (*esteemed*) Rational *way of* Regiomontanus.

Definition 1.] THE Heavens are divided into XII *Houses* or *Mansions*, by 12 *Semicircles of Position*; for which purpose, to some *Globes*, there is made one of *Brass*, which is fixed in the Intersections of the *Meridian* and *Horizon*, by the *Elevation* or *Depression* whereof the Heavens may be divided into Parts or Houses through each degree of the *Ecliptick*.

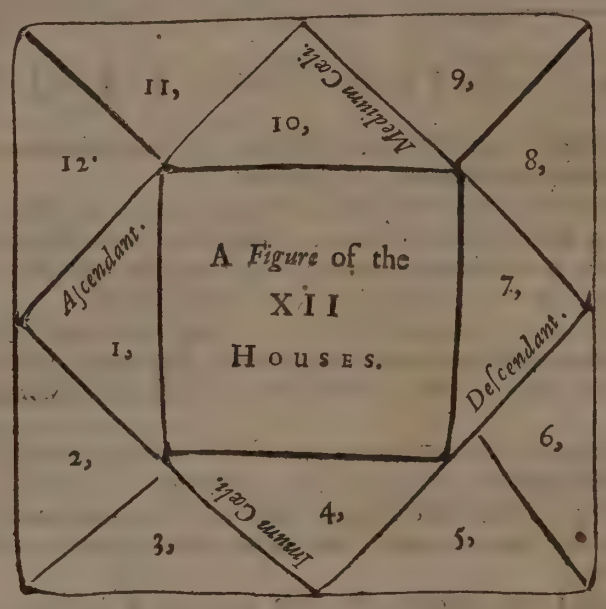
Definition 2.] Of these XII *Houses* or *Mansions* of Heaven, *Four* are called *Cardinal*, as (1.) the *Horoscope*, or *Ascendent*, or *Cuspis* of the *First House*. (2.) The *Medium Cœli*, or Angle of the *South*, or *Cuspis* of the *Tenth House*. (3.) The *Descendent*, or *Angle* of the *West*, or the *Cuspis* of the *Seventh House*. (4.) The *Imum Cœli*, or the *Angle* of the *North*, or the *Cuspis* of the *Fourth House*.

Definition 3.] *Regiomontanus* divides the Heavens into XII *Houses* according to his way, by the *Circle* of *Positions* passing through every 30th. degree of the *Equinoctial*, and cutting the *Ecliptick* at several Points, which are the *Cuspises* of the several *Houses*: —— So that when the *Globe* is set to the *Latitude*, and the Hour-wheel *Rectified* and brought to the Given hour, you have the *Cuspises* of the Four *Cardinal Houses* given: For,

The degree of the *Ecliptick* cut by the { East-side of the *Horizon*, South-part of the *Meridian*, West-side of the *Horizon*, North-part of the *Meridian*, } Gives the *Cuspis* of the { First, Tenth, Seventh, Fourth } House.

The *Cuspises* of the other 8 Houses are found by the motion of the *Circle* of *Position*, as shall be shewed by and by.

Definition 4.] The *Houses* are *denominated* by 1, 2, 3, 4, *&c.* to 12, from the *Ascendant* downwards to the *Imum Cœli*, up again to the *Descendant*, and again by *Medium Cœli* down to the *Ascendant*. As in the following *Scheme:*



Let this suffice for *Definition*, and now we will come to the *Practice* by the *Globes.*

Practice.] FIrst, to the day proposed, the 10th. of *March*, find (by the first *Astronomical Problem*) the Suns place in the *Ecliptick* at Noon, which you shall find to be in 0 deg. 5 min. of *Aries*.

Secondly, Set the *Globe* to the *Latitude* 51 deg. 30 min.

Thirdly, Bring the *Suns place* at *Noon* (0 deg. 5 min. of *Aries*) to the *Meridian*.

Fourthly, Turn the *Globe* about till the *Hour-Index* point to the *Hour given*, viz. to 49 min. after 9 in the *Morning*.

Lastly, The *Globe* being in this *Position*, fix it.

The *Globe* being fixed in this *Position*, you shall find that the *East-semicircle* of the *Horizon* doth cut the *Ecliptick* in 0 deg. 29 min. of *Cancer*, which is the *Sign* then *Ascending*, and must be placed upon the *Cuspis* of the *first House*.

Then cast your eye upon the Intersection of the *South*-part of the *Meridian* and the *Ecliptick*, and there you shall find the *Ecliptick* cut by the *Meridian* in 25 deg. of *Aquarius*, and that Point of the *Ecliptick* is then in the *Medium Cœli*, and must be set upon the *Cuspis* of the *Tenth House*.

Also you shall find that the *West-semicircle* of the *Horizon* cuts the *Ecliptick* in 00 deg. 29 min. of *Capricorn*; which point is then *Descending*, and must be placed upon the *Cuspis* of the *Seventh House*.

Lastly, You shall find that the *North*-part of the *Meridian* doth cut the *Ecliptick* in the 25th. deg. of *Leo*, which point is then upon the *Imum Cœli*, and must be placed upon the *Cuspis* of the *Fourth House*.

Thus have you found the Points of the *Ecliptick* which do occupy the *Cuspises* of the *Four Cardinal Houses*: Now for the other *Eight Houses*.

Let the *Globe* still rest in its former *Position*, and then,

First, Bring the *Circle of Position* to its place on the *East-side* of the *Horizon*; and being there fixed, raise it upwards towards the *Meridian*, till 30 deg. of the *Equinoctial* be intercepted between the *Horizon* and the *Circle of Position*; and then you shall find that the *Circle of Position* will intersect the *Ecliptick* in 20 deg. of *Taurus*; which degrees must be set upon the *Cuspis* of the *Twelfth House*. Secondly,

Secondly, Move the *Circle of Position* yet higher towards the *Meridian*, till 30 deg. more of the *Equinoctial* be intercepted between it and the *Horizon*, (in all 60 deg.) and when it ſo doth, you ſhall find the *Circle of Poſition* will cut the *Ecliptick* in 36 deg. of *Piſces*; which point muſt be ſet upon the *Cuſpis* of the *Eleventh Houſe*.

The *Meridian* gives the *Cuſpis* of the *Tenth Houſe* in 25 deg. of *Aquarius*, as before.

Thirdly, Move the *Semicircle of Poſition* from the *Eaſt-ſide* of the *Horizon* to the *Weſt-ſide*, and move it downwards from the *Meridian*, till 30 deg. of the *Equinoctial* be intercepted between the *Meridian* and *Circle of Poſition*, and then you ſhall find that the *Circle of Poſition* will interſect the *Ecliptick* in 7 deg. of *Aquarius*; which point muſt be ſet upon the *Cuſpis* of the *Ninth Houſe*.

Fourthly, Move the *Circle of Poſition* yet lower by 30 deg. *i. e.* 60 deg. from the *Meridian* downwards, and then you ſhall find the *Poſition-Circle* to cut the *Ecliptick* in 21 deg. of *Capricorn*; which point muſt be ſet upon the *Cuſpis* of the *Eighth Houſe*.

The *Deſcendant* or *Cuſpis* of the *Seventh Houſe* is the Interſection of the *Weſt-ſide* of the *Horizon* and *Ecliptick*, which is in 00 deg. 29 min. of *Capricorn*, as before.

And thus have you found the *Cuſpiſes* of the *Four Houſes* above the *Horizon*, beſide the *Aſcendant* and the *Medium Cœli*; viz. of the 12, 11, 9 and 8 houſes. Now the *Cuſpiſes* of the Four other Houſes under the *Earth* have the ſame degrees of the oppoſite Signs upon them: For,

20 deg. of *Taurus* 26 deg. of *Piſces* 7 deg. of *Aquarius* 21 deg. of *Capricorn*	being upon the *Cuſpis* of the	12 11 9 8	houſe.

20 deg. of *Scorpio* 26 deg. of *Virgo* 7 deg. of *Leo* 21 deg. of *Leo*	will be on the *Cuſpis* of the	6 5 3 2	houſe.

For the Six Signs
Aries, Taurus, Gemini, Cancer, Leo, Virgo,
are oppoſite to
Libra, Scorpio, Sagitt. Capric. Aquar. Piſces.

And this is the manner how (by the *Globe*) to erect a *Figure* according to the (reputed) *Rational way* of *Regiomontanus*.

Now if you would inſert the places of the *Planets* into your *Figure*, (for it is them that the *Aſtrologer* principally giveth *Judgment* by) your beſt way will be to have recourſe to ſome good *Ephemeris* (if you cannot *Calculate* them from *Aſtronomical Tables*) and ſo may you find the places of the ſeveral *Planets* at the time for which this *Figure* was *Erected* (viz. *March* 10. 1675. 49 min. after 9 in the forenoon) to be as followeth: *Viz.*

		deg.	
♄ *Saturn*	is in	28	*Aries.*
♃ *Jupiter*		19	*Sagittarius.*
♂ *Mars*		17	*Gemini.*
☉ *Sol*		00	*Aries.*
♀ *Venus*		18	*Aquarius.*
☿ *Mercury*		15	*Aries.*
☽ *Luna*		24	*Capricorn.*
☋ *Dragons Tail*		17	*Capricorn.*
☊ *Dragons Head*		17	*Cancer.*

Having thus obtained the places of the *Planets* either by *Calculation*, by *Ephemerides*, or *Inſtrument*, you may place them in their Correſpondent places in your *Figure*, as is here done, and ſo is your *Figure* ready to give your Judgment upon.

Lat. N. 51 d. 30 m.
A Figure of Heaven at the Suns entrance to ♈, March 10. 49 min. after 9. in the Forenoon 1675, according to REGIOMONTANUS.

II. *To Erect a* Figure *of the Heavens for the forementioned time,* Anno 1675, March 10 *day at* 49 *min. paſt* 9 *in the Morning.*

According to Campanus.

Definition.] THE *Cuſpiſes* of the Four *Cardinal Houſes* according to *Campanus*, viz. the *Aſcendent*, *Mid-heaven*, *Deſcendent*, and *Imum Cœli*, are the ſame as they were according to *Regiomontanus*: But, as *Regiomontanus* divided the *Houſes* by the *Circle of Poſitions* paſſing through each 30th degree of the *Equinoctial*, and interſecting the *Ecliptick* in the *Cuſpiſes* of the ſeveral *Houſes*; *Campanus* divides the 12 *Houſes* of Heaven by the *Poſition-Circles* paſſing through each 30th degree of the *prime Vertical Circle*, (or *Azimuth* of *Eaſt* and *Weſt*) and ſo interſecting the *Ecliptick* in the *Cuſpiſes* of the ſeveral *Houſes*. So that to Erect a *Figure* according to *Campanus* way, you muſt do as followeth. *viz.*

Practice.] You muſt firſt ſet the *Globe* to the *Latitude*, bring the *Suns place at noon* for the day given to the *Meridian*, and the *Hour-index* to 12, and the *Circle of Poſition* to the *Eaſt-ſide* of the *Horizon*; then turn the *Globe* about till the *Index* point at the *given hour*, and then fix the *Globe*. Thus far as in the former way of *Regiomontanus*; and the *Cuſpiſes* of the *Firſt*, *Fourth*, *Seventh*, and *Tenth Houſes* will be the ſame as in his way. But now to proceed according to *Campanus*.

The *Globe* ſeated and fixed in this *Poſition*, bring the *Quadrant of Altitude* to the *Zenith*, and there fix it; and bring the lower part of the *Quadrant of Altitude* to the *Eaſt*-point of the *Horizon*, and there keeping of it fixed:

1. Elevate the *Circle of Poſition*, till it cut 30 deg. of the *Quadrant of Altitude*, and then ſee what degrees of the *Ecliptick* are cut by the *Poſition-Circle*, which you ſhall find to be 22 deg. of *Aries*; which point muſt be ſet upon the *Cuſpis* of the *Twelfth Houſe*.

2. Move the *Circle of Poſition* yet 30 deg. higher upon the *Quadrant of Altitude*, namely, to 60 deg. from the *Horizon*, and then ſee what degree of the *Ecliptick* is cut thereby, and you ſhall find that the *Poſition circle* cuts the *Ecliptick* in 12 deg. of *Piſces*, which muſt be ſet upon the *Cuſpis* of the *Eleventh Houſe*.

The *Meridian* cuts the *Ecliptick* in 25 deg. of *Aquarius*, which is the *Cuſpis* of the *Tenth Houſe*, or *Mid-heaven*, as before.

3. Re-

3. Remove the *Circle of Position*, and also the *Quadrant of Altitude*, from the *East* to the *West-side* of the *Horizon*, and there move him from the *Meridian downwards*, till 30 deg. of the *Quadrant of Altitude* be intercepted between the *Zenith* and the *Circle of Position*, and then see where the *Position-circle* cuts the *Ecliptick*, which will be in 12 deg. of *Aquarius*; and those degrees must be set upon the *Cuspis* of the *Ninth House*.

4. Let down the *Circle of Position* 30 deg. more upon the *Quadrant of Altitude*, that is, 60 deg. distant from the *Zenith*, and then look what degrees of the *Ecliptick* are cut by the *Position Circle*, which you will find to be 28 deg. of *Capricorn*; which degrees must be set upon the *Cuspis* of the *Eighth House*.

The *Cuspis* of the *Seventh House* is 00 deg. 29 min. of *Capricorn*, being the Intersection of the *West-side* of the *Horizon* with the *Ecliptick*, as before.

And by this Artifice you have obtained the degrees belonging to the 1, 12, 11, 10, 9, and 8 houses above the *Horizon*, according to the way prescribed by *Campanus*: Now the other *Six* Houses under the *Horizon*, viz. the 2, 3, 4, 5, 6 and 7, are easily found, they being the opposite Points of the *Ecliptick* to the *Six* above the *Horizon*. So,

				d.	*m.*
The *Cuspis* of the	1	house	being *Cancer*	0	29
	12		*Aries*	22	0
	11		*Pisces*	12	0
	10		*Aquarius*	25	0
	9		*Aquarius*	12	0
	8		*Capricorn*	28	0
The *Cuspis* of the	7	house	will be *Capricorn*	0	29
	6		*Libra*	20	0
	5		*Virgo*	12	0
	4		*Leo*	25	0
	3		*Leo*	12	0
	2		*Cancer*	28	0

And thus have you the *Cuspises* of the several Houses according to the way prescribed by *Campanus*; which *Figure* having the *Planets* placed therein, will stand as in the following *Scheme*.

Lat. N. 51 d. 30 min.
A Figure of Heaven at the Suns entrance into Aries, March the 10th 1675.
at 49 min. after 9 in the Forenoon, according to CAMPANUS.

III. *To*

III. *To Erect a* Figure *of Heaven for the forementioned time*, viz. March 10. 1675, *at* 49 *min. after* 9 *in the Forenoon.*

According to Alcabitius.

Definition.] ALcabitius would have the XII *Houſes* of Heaven to be divided by *Domifying Circles*, or *Circles of Poſition* drawn from the *Poles* of the *World* through every 30th deg. of the *Equator*, beginning at the point of the *Ecliptick Aſcending*; and ſo counting 30 deg. upon the *Equinoctial* from thence, ſhall be the *Cuſpiſes* of the ſeveral Houſes. —— Wherefore, to Erect a *Figure* according to this mode, do as followeth.

Practice.] You muſt *Rectifie the Globe* to the *Latitude*, bring the *Suns place* at noon to the *Meridian*, the *Index* of the *Hour-wheel* to 12 at noon, and turn the *Globe* about to the hour given, and then fix it; ſo ſhall the *Aſcendent* be the ſame as in the two former ways, *viz.* 00 deg. 29 min. of *Cancer*.

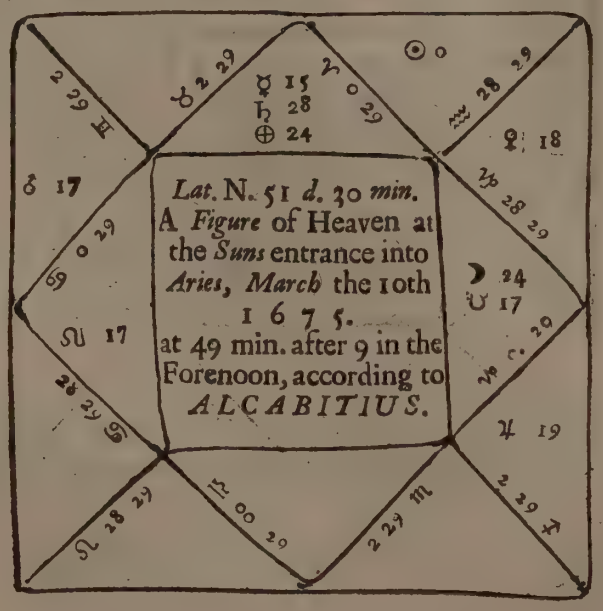
Then look what *Meridian* paſſeth through the *Aſcendent*, and count 30 deg. more upon the *Equinoctial*; and that *Meridian* where it paſſeth through the *Ecliptick* ſhall be the *Cuſpis* of the *Twelfth Houſe*; and 30 deg. forwarder that *Meridian* ſhall cut the *Ecliptick* in the *Cuſpis* of the *Eleventh Houſe*, and ſo onward till you come to the *Cuſpis* of the *Deſcendent*. And thus,

If from that *Meridian* which paſſeth through 00 deg. 29 min. of the *Ecliptick*, you count 30 deg. more upon the *Equinoctial*, you ſhall find that

			d.	m.	
The	First	*Meridian* of 30	0	29	*Cancer*,
	Second	deg. diſtant	2	29	*Gemini*,
	Third	from the *Aſ-*	2	29	*Taurus*,
	Fourth	*cendent*, will	0	29	*Aries*,
	Fifth	cut the *Eclip-*	28	29	*Aquarius*,
	Sixth	*tick* in	28	29	*Capricorn*,

Which are the *Cuſps* of the { 1, 12, 11, 10, 9, 8 } houſes.

And the oppoſite *Signs* and *Degrees* of the *Ecliptick* ſhall give the *Cuſpiſes* of the *Six* other under the Earth; namely, of the 7, 6, 5, 4, 3, and 2. And thus your *Figure* being erected, and the *Planets* placed therein, you will find it to be as in the following *Scheme*.


Lat. N. 51 d. 30 min.
A Figure of Heaven at the Suns entrance into Aries, March the 10th
1 6 7 5.
at 49 min. after 9 in the Forenoon, according to
ALCABITIUS.

IV. *To*

IV. *To Erect a* Figure *for the fore-mentioned time*, viz. March 10. 1675. *at* 49 *min. after* 9 *in the Forenoon, the* Sun *then entering* Aries *in the Meridian of* London.

According to the way preſcribed by Ptolomy.

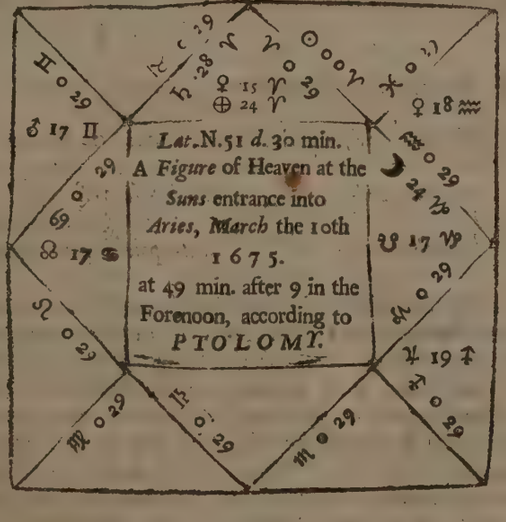
Definition.] *Ptolomy*, adviſeth that the *Heavens* ſhould be divided into 12 *Houſes* or *Manſions*, by *Domifying Circles* drawn through the *Poles* of the *Ecliptick*, and through every 30th deg. thereof from the *Aſcendant* downwards, round about. —— Wherefore to Erect a *Figure* of the Heavens according to this way of *Ptolomy*, do thus.

Practice.] Rectifie the *Globe* and *Hour-circle* as before, and you ſhall have the ſame *Aſcendant*, viz. 00 deg. 29 min. of *Cancer* to *Aſcend*, which is the *Cuſpis* of the *Firſt Houſe*; then 30 deg. forwarder, downwards, will be 00 deg. 29 min. of *Leo*, for the *Cuſpis* of the *Second Houſe*, &c.

			d.	m.	
So the *Cuſpis* of the	1	houſe	0	29	*Cancer.*
	2		0	29	*Leo.*
	3		0	29	*Virgo.*
	4		0	29	*Libra.*
	5		0	29	*Scorpio.*
	6		0	29	*Sagittary.*
And	7		0	29	*Capricorn.*
	8		0	29	*Aquarius.*
the *Cuſpis* of the	9	houſe	0	29	*Piſces.*
	10		0	29	*Aries.*
	11		0	29	*Taurus.*
	12		0	29	*Gemini.*

(will be)

And ſo a *Figure* Erected for the forementioned time, according to this preſcription of *Ptolomy*, and the Planets placed therein, will appear as in the following *Scheme* or *Figure*.



And

And thus have I shewed you the manner of Erecting of a *Figure* of the Heavens according to the Prescriptions of the four forementioned Authors, and have placed the Planets, Dragons-head and Tail, and the place of Fortune in each of them; by which you may see, that according to these four Varieties of Erections, the Planets keep not in the same Houses, as by the following *Synopsis* appears. For,

	accor. to *Regiomon.*	accor. to *Campan.*	accor. to *Alcabit.*	accor. to *Ptolomy.*
Saturn	XI	XII	X	X
Jupiter	VI	VI	VI	VI
Mars	XII	XII	XII	XII
Sol	XI	XI	IX	IX
Venus	IX	IX	VIII	VIII
Mercury	XI	XI	X	X
Luna	VIII	VII	VII	VII
Dragons-tail	VII	VII	VII	VII
Dragons-head	I	I	I	I
Part of Fort.	XI	XII	X	X

is in ... House, ... House, ... House, ... House.

PROB. V.

Having the Longitude *and* Latitude, *or the* Right Ascension *and* Declination *of a* Star, Planet, *or* Comet, *how to find the* Place *thereof upon the* Globe, *and to insert it therein if need require.*

WHat the *Longitude, Latitude, Right Ascension* and *Declination* of a *Star* or *Planet* is, you have heretofore defined; and in the last *Problem* you are taught how to *Erect* a *Figure* of the *Heavens Four* several ways. Now if you would see (upon the *Globe* it self) in what *House* any of the *Planets* are, without Erecting a *Figure*, you were best to make *Marks*, or set *Characters* of those *Planets*, *Comets*, or the like, upon the *Globe*, before you delineate your *Figure* upon Paper; which to effect, do thus.

I. *By the* Longitude *and* Latitude *given.*

If the *Latitude* of the *Star*, *Planet*, or *Comet*

be { *Northward* / *Southward* } Elevate the { *North-pole* / *South-pole* } of the *Globe* to 66 *d.* 30 *m.*

then will the *Pole* of the *Ecliptick* be in the *Zenith*, and the *Ecliptick-circle* will lie in the very Plain of the *Horizon*; in which Position of the *Globe*, screw the *Quadrant of Altitude* in the *Zenith*, over the *Pole* of the *Ecliptick*. —— This done, bring the point of *Longitude* of the *Planet in the Ecliptick* to the *Quadrant of Altitude*, and count the *Latitude* of the *Planet* upon the *Quadrant*, and under the degree of *Latitude* shall be the *point* upon the *Globe* in which the *Planet* or *Star* in the Heavens is.

And so the *Longitude* of the *Seven Planets* being as it is expressed in the following *Table*, such *Longitude* and *Latitude* they had at the time of the *Erection* of the former *Figure*, *March* 10. 1675, at 49 min. after 9 in the Forenoon. Now, if you find their respective Points upon the *Globe* (as I have now shewed you how to do) you may then, not only see in what *House* each of them shall be, but also what *Fixed Stars* are there, and what *Fixed Stars* are in the other *Houses* also.

The

The *Longitude* of	♄ *Saturn*	27	49	♈	And the *Latitude*	2	17	M.	A.
	♃ *Jupiter*	18	55	♐		0	38	S.	D.
	♂ *Mars*	17	24	♊		1	48	S.	A.
	☉ *Sol*	00	00	♈		0	00		
	♀ *Venus*	17	56	♒		3	44	S.	D.
	☿ *Mercury*	15	15	♈		1	07	S.	A.
	☽ *Luna*	24	47	♑		0	35	S.	D.

II. *By the* Right Aſcenſion *and* Declination *given.*

Suppoſe a *Star, Planet,* or *Comet* to have 147 deg. 43 min. of *Right Aſcenſion,* and 33 deg. 33 min. of *North Declination,* as the Star *Regulus,* or the *Lion's Heart* hath, and you would find its place upon the *Globe*;

Count 147 deg. 43 min. the Star's *Right Aſcenſion* upon the *Equinoctial* from the beginning of *Aries,* and bring that point of the *Equinoctial* to the *General Meridian*; and keeping the *Globe* there, count 33 deg. 33 min. the Star's *Declination* upon the *Meridian* upwards, (becauſe the Star's *Declination* is *Northward,*) and that point ſhall be the place of *Regulus* upon the *Globe.* And thus may any part, or point, in the *Heavens* be found upon the *Globe,* if either the *Longitude* and *Latitude,* or the *Right Aſcenſion* and *Declination* of that point be firſt known.

PROB. III.

To know in what Houſe, *or under what* Circle *of* Poſition, *any* Star, Planet, *or Point of the* Ecliptick *is.*

REctifie the *Globe* to the *Latitude,* the *Quadrant of Altitude* to the *Zenith,* the place of the *Sun* in the *Ecliptick* to the *Meridian,* and the *Hour-circle* to 12; then turn the *Globe* about to the hour given, and bring the *Quadrant of Altitude* to the *Eaſt* or *Weſt* points of the *Horizon,* and there fix the *Globe*: Then move the *Circle of Poſition* upwards, till it touch the *Star, Planet,* or other Points of the *Ecliptick,* which you deſire to know the *Circle of Poſition* of, then ſhall the *Poſition-circle* croſs the *Quadrant of Altitude* in the number of that *Circle of Poſition* in which that *Star, Planet,* or other Point of the *Ecliptick* is.

Thus. If at the time of the Erection of the former *Figure* it were required to know in what *Circle of Poſition* (or houſe) the *Pleiades* or *Seven Stars* were; Rectifie the *Globe* as before, and bring the *Poſition-circle* to the *Pleiades*; then ſhall the *Circle of Poſition* cut the *Quadrant of Altitude* in 42 deg. and under that *Circle of Poſition* is the *Pleiades* or *Seven Stars* at that time, and ſo conſequently in the XIIth Houſe.

I might here ſhew how to find the place of the *Thing* which *Aſtrologers* call the *Part of Fortune.* Alſo how (as they call it) to *Direct a Figure,* and to find out *Revolutions,* &c. But foraſmuch as theſe things are not ſo *Mathematical* as to require a *Globe,* or other *Mathematical Inſtruments,* either to *Demonſtrate* them by, or to eaſe the *Operation*; I ſhall refer them that have occaſion for ſuch *things,* to the *Arithmetical working* of them by the Pen, as moſt convenient, and reſerve the *Globes* for other purpoſes.

PRACTICAL ASTRONOMY:

PART II.

Of the DOCTRINE of Spherical Projection.

CHAP. I.

Of Spherical Projection *in General.*

A SPHERE or GLOBE, which is the beſt *Inſtrument* to inform the Phanſie (may appear by what hath been already declared in the *Deſcription* and *Uſes* thereof in the firſt part) yet it hath its Inconveniencies in the reſolving of *Problems* or *Queſtions* thereby; and therefore, for ſuch Purpoſes, ſeveral other *Inſtruments* (leſs cumberſom) have been by divers Perſons invented for the ſolution of *Aſtronomical, Geographical,* and *Nautical Queſtions,* or *Problems,* which are of daily uſe with *Mathematicians* and *Mariners:* As the *Planiſphere* of *Gemma Frigius*; afterwards improved by *John Blagrave* of *Reading*; the *Aſtrolabe* of *John Stophler*; the *Analema*; and of *Cylinders, Rings, Quadrants, &c.* an infinite number: All which *Inſtruments,* are either real *Projections* of the *Sphere*; or Parts of *Projections* thereof. But to paſs by theſe (and ſuch like) I ſhall here declare the manner how to *project* the *Sphere* (*in Plano*) upon the Plain of any *Great Circle* thereof: By which, not only *Queſtions* concerning *Aſtronomy, &c.* may be reſolved; but alſo how *Spherical Triangles* may be laid down or projected, and the *Sides* and *Angles* of the ſame meaſured, anſwerable to any *Problem* propoſed; and alſo declare, how many ſeveral *Queſtions* will ariſe from any *Spherical Triangle* (Right or Oblique angled) ſo projected.

Now to *project* the *Sphere in Plano,* is to deſcribe upon a Plain the *Circles* of the *Sphere* or *Globe* (it being ſituate in any Poſition) in ſuch ſort, that the *Lines* and *Circles* ſo deſcribed, ſhall perform whatſoever the *Globe* it ſelf can, it being in that Poſition: And the *Varieties* of Poſitions that a *Globe* or *Sphere* may be looked upon, are but Three, *viz.* either 1. *Perpendicular*; 2. *Direct*; or 3. *Obliquely* to the Eye.

1. *Perpendicular:* A *Circle* is ſaid to be *perpendicular* to the ſight, when the Superficies of it is *parallel* to it: As ſuppoſe the *Eye* to be placed in any part of the edge of a *round Table,* and thence to behold the whole Superficies of that *Circular Plain*; then is the *Superficies* of that *Circular Plain parallel,* and the *Circle* it ſelf *Perpendicular* to the Eye.

2. *Direct:*

2. *Direct*: A Circle is ſaid to have a *Direct* Aſpect to the *Eye*, when the *Superficies* of it is directly oppoſite to it; interſecting the ſuppoſed Line from the *Eye* to the *Centre* of the *Circle*, at *Right Angles*.

3. *Oblique*: A Circle is ſaid to have an *Oblique Aſpect* to the *Eye*, when the ſuperficies of it is neither *Parallel* with, nor *Oppoſite* to the *Viſual Line*, but interſecteth it at *Oblique Angles*.

Now, how a *Circle* ſeen by the *Eye* in any of theſe *Three Poſitions*, may be *projected* in *Plano*, ſhall be explained in the *Three* following *Problems*.

PROB. I.

Upon a Plain *to* Project *a* Circle, *which is* Perpendicular *to the* Sight; *and to meaſure the parts thereof, it being ſo Projected.*

Fig. I. *Fig.* I.

A *Circle* in this Poſition being *Projected* will become a *Streight Line*, and may (upon either hand) be extended infinitely: As,

Let A B C D be ſuch a *Circle*, in the edge whereof let the *Eye* be placed in A, the *Diameter* B D being extended infinitely on both ſides to F and G: Or farther, this *Right Line* ſhall be the *Projection* of the *Circle* ABCD, and may be diſtinguiſhed into Parts in this manner; Divide the Circumference of the *Circle* into any number of Parts, ſuppoſe Eight, at the Points H, B, I, C, K, D, and L, Lines drawn from A (the place of the *Eye*) through theſe Points, ſhall cut the *Projected Line* FG, in the Points F, B, O, E, P, D, G, and the ſtreight Lines FE, BE, and OE, ſhall be the Projection of the Arches HC, BC and IC, and theſe Arches are the double Meaſures of thoſe Lines; being the double Meaſures of the Angles at A, and EF being a Tangent Line anſwerable to the *Radius* of the Circle AE, which is the *Radius* of the Circle by which theſe Angles are meaſured, the Lines EF, EB, and EO, are the Tangents of thoſe Angles FAE, BAE, and OAE. Now becauſe the Tangent of 90 degrees cannot be limited, therefore a *Circle* thus *projected* will become an *Infinite Line*. And what is here ſaid of the

Lines { E F, E B, E O } on the one ſide, the ſame is to be underſtood of the *Lines* { E G, E D, E P } on the other ſide.

Angles { F A E, B A E, O A E } on the one ſide, the ſame is to be underſtood of the *Angles* { G A E, D A E, P A E } on the other ſide.

PROB. II.

Upon a Plain *to* Project *a* Circle, *which hath a* Direct Aſpect *to the* Eye; *and to meaſure the Parts thereof, it being ſo* Projected.

A *Circle* in this Poſition being *projected*, will be a perfect *Circle*: Thus in a *Globe*, if the *Pole* of the World were elevated to the *Zenith* (and ſo making it a *Parallel Sphere*) and the *Eye* poſited in that *Point*, all the *Circles* between the *Pole* and the *Equinoctial* are in this Poſition perfect *Circles*, and alſo *Parallel* one to the other; and all their Centres meeting in one *Axis* or *Radius*, they being *projected* will all meet in one Point, namely in E.

Fig. I. *Fig.* I.

Let ABCD be a *Fundamental* (or *Primitive*) *Circle* (for ſo, hereafter, I ſhall call the outermoſt and perfect *Circle* comprehending the *Projection*). If you open your Compaſſes to any extent leſs than the *Radius* of the *Primitive Circle* EC; as ſuppoſe to EM, a *Circle* drawn upon the Point E, at the diſtance EM, ſhall be the *Projection* of that *Circle* ſo poſited, the Parts whereof are correſpondent to the Parts of the *Primitive Circle*; that is, the *Arch* NM in the *leſſer Circle*, is equal to the *Arch* IC in the *greater*; for it contains the ſame number of Degrees.

PROB. III.

Upon a Plain *to Project a* Circle, *which hath an* Oblique Aſpect *to the* Eye.

A *Circle* in this Poſition being *projected*, will be the *Arch* of a *Great Circle* of that *Sphere* upon which it is *projected*; in the *Deſcription* of ſuch a *Circle*, the greateſt difficulty will be to find the *Centre* of it; for which, ſometimes there are but *Two*, ſometimes *Three Points* given: When there are only *Two* Points given, a *Third* muſt be found; which how to find, is taught in the 63 Problem of the Second Book of *Plain Geometry*: And when *Three* Points are given, that is alſo taught in the 23 Geometrical Problem concerning the *Deſcribing of the Oblique Circles*; as alſo, *How to Meaſure* them being deſcribed; I muſt refer to its due place, for that ſeveral other *Præcognita* are not yet taught.

CHAP. II.

Of ſuch Circular Scales *as are to be inſcribed upon a* Ruler *(or Sector rather) for the more eaſie* Projecting *of the* Sphere in Plano, *and for divers other Purpoſes, with the Conſtruction or Making of the ſame* Scales.

I Call them *Circular Scales*, becauſe (though they be Projected upon a Ruler into *Streight Lines*) yet they are all of them deduced from a *Circle*; as by the Fabrick or making of them will appear.

Now the *Scales* that I would adviſe to be ſet upon a *Ruler* (or rather a *Sector*) as moſt commodious, not only for *Projecting* of the *Sphere in Plano*, but for infinite other good Uſes (as ſhall hereafter be taught) are theſe following, *viz.*

A Scale of {
1. *Chords* to 90 degrees, of ſeveral *Radiuſes*,
2. *Right Sines* to 90 deg.
3. *Secants* extending to 75 deg. at leaſt,
4. *Tangents* to 75 deg.
5. *Half Tangents* to 90 deg. equal to 45 of the Whole Tangent,
6. *Verſed Sines* to 180 deg.
7. *Rumbs* to Eight Points,
8. *Equal Parts* into 20, 60, 100, *&c.*
} all to the ſame *Radius*.

To theſe may be added,

A *Meridian Line.*
A *Line* of *Longitudes.*
Chords to ſeveral *Radiuſes.*

The Fabrick or Geometrical Conſtruction of theſe Scales, *as they are deduced from a* Circle.

Fig. II. *Fig.* II.

1. UPon a Sheet of fine Paſtboard, or ſuch like Matter, deſcribe a Semicircle of any *Radius* or Semidiameter whatſoever, as OR; [this in the Figure is made to two Inches and a half *Radius*, as OR, that is five Inches the whole Diameter VS: And to that *Radius* are all the following Projections drawn.]

Having

Having drawn a Right Line VS, upon the middle thereof at O, and at the distance OR; describe the Semicircle VRS; then is OR the *Radius* or Semidiameter, and VS the Diameter.

2. Divide the two Quadrants OVR and ROS, each of them into 90 equal Parts or Degrees, beginning at V and S, and ascending upwards towards R, numbering them by 10, 20, 30, *&c.* to 90 at R, as in the Figure.

3. If you draw Right Lines from 10 in the one Quadrant, to 10 in the other, and from 20 in one, to 20 in the other, and so up to 90 at R, they will be Lines parallel to VS, and will cut or divide the Line OR into 90 unequal Parts, and so create OR a Line of Right *Sines*.

4. Upon the Point S, erect a Perpendicular to VS, as ST, then, if from the Centre O, you draw Right Lines through every tenth degree of the *Quadrant*, you shall divide the Perpendicular Line ST into unequal Parts, at the Points 10, 20, 30, *&c.* up to T, and farther, if the length of the Ruler will permit. So shall the unequally divided Line ST be a *Tangent Line*, 45 degrees whereof is equal to the *Radius*, or to the whole Line of *Sines* OR.

5. If from the Centre O, and through every tenth degree of the Quadrant, you draw right Lines to the *Tangent* Line, those Lines shall be *Secants*, as the Line OEL (drawn through 60 deg. of the Quadrant SR) is the *Secant* of 60 deg. And the Line drawn from O, through 50 in the Quadrant, to 50 in the *Tangent* Line, is the *Secant* of 50 deg. and so of the rest, which Scales of *Tangents* and *Secants* let be extended to 75 deg. at least, or farther, if the *Ruler* will permit. And these *Secants* may be transferred from the *Tangent* Line ST, to the *Secant* Line OE, or RE rather, because the *Secants* always *begin* where the *Sines* do *end*.

6. The Line VS, which is equal to twice OR, the Line of *Sines*, is the Line of *Versed Sines*, and is no other than the Line of *Right Sines* doubled, as you may plainly see, but it is numbered from V by 10, 20, 30, 40, *&c.* towards S, ending at S with 180. deg.

7. Draw the Line SR, and with one Foot of your Compasses placed in S, open the other to 80 in the Quadrant SR, and describe the little Arch 80, 80, cutting the Line SR in 80; do the like for 70, 60, 50, *&c.* so shall the Line SR be divided into 90 unequal Parts, and become a Line of *Chords*, 60 of whose degrees are equal to the whole Line of *Sines*; and so from S to 60 deg. is called the *Radius* of the Line of *Chords*, it being equal to the Semidiameter of the Circle OR.

8. The Line of *half Tangents*, is no other than the Tangent of 45 deg. (which is equal to the whole Line of *Sines*) doubly numbered, that is Five deg. are called 10, Ten 20, Twenty 40, Forty 80, *&c.*

9. There is mention made of a *Scale* of *Rumbs*, which is nothing else but a Line of *Chords*, divided into *Eight* unequal parts as the *Scale* of *Chords* is into 90. and those subdivided into *Halves* and *Quarters* answerable to the Eight *Points*, half *Points*, and Quarter *Points*, of one Quarter, of the *Mariners Compass*. This *Scale* is in the Figure noted with V R, and is made by dividing the Quadrant OUR into *Eight* equal parts in the Points 1, 2, 3, 4, 5, 6, 7. Then setting one foot of the Compasses in V, extend the other to 7, and draw the Arch 7, 7. do the like from V to 6, 6, *&c.* so shall the Line V R be a Scale of *Rumbs*.

10. There is also mention made of a *Scale* of *Longitudes*, which in the Scheme is noted with R M, and is thus deduced from the *Circle*. From the Point R draw a right Line R M, parallel to V O; then setting one foot of the Compasses in R, extend the other to V, and describe the obscure Arch V M, so shall the *Scale* of *Longitudes* begin at M; then keeping one foot of the Compasses still in R, extend the other to 10, 20, 30, 40, 50, 60 degrees of the Quadrant, and draw the obscure Arches 10, 10 —— 20, 20 —— 30, 30, *&c.* to 60 degrees; so shall the Line M R be a *Scale* of *Longitudes*.

11. For the *Meridian Line*, that will not be divided from the Circle, wherefore the construction thereof must be referred to another place, where we shall have occasion to make use of it: For in the *Projecting* of the *Sphere in Plano* (which is the matter now in hand) the Scales of *Chords*, *Sines*, *Secants*, *Tangents*, and *Half Tangents* are sufficient: The other *Scales* properly belong to *Navigation*; of which hereafter.

12. The Line of *Equal Parts*, whose division is into 100, (equal to the *Radius* of the *Sines* and *Tangents*, were best to be drawn the whole length of the *Ruler*, or be made a *Diagonal Scale* on the other side of the *Ruler*: but these things are at the liberty

liberty of the Maker or Owner; and therefore let this ſuffice for the Conſtruction of the *Scales* upon the *Ruler*, only in ſome ſpare place thereof, let there be two Lines of Equal Parts, each equal to the *Radius* O R; the one divided into 20, and noted with L for Leagues; the other into 60, noted with M for Miles.

And here note by the way, that the Five Scales firſt mentioned, *viz. Chords*, *Sines*, *Secants*, *Tangents*, and *Half Tangents*, are all that are needful in Spherical Projection; and therefore I would adviſe to have thoſe Five put upon one Side of the *Ruler*, with a Scale of Equal parts added: As for the other Scales of Verſed *Sines*, *Rumbs*, Equal parts, *Meridian* Line, and Line of *Longitude*, they are Scales uſeful in Navigation, and ſo may be either omitted or put on the other Side of the *Ruler*.

Theſe *Scales* being put upon a *Ruler*, are fitted for all the Uſes following; but whereas I have often ſaid [*or rather upon a Sector*] the Reaſon is this: For if you uſe them as they are upon a *Ruler*, you are conſtrained to draw your *Projection*, or other *Schemes* whatſoever, of the ſame bigneſs of that to which the *Radius* of your *Scale* is made: Whereas (if they be put upon a *Sector*) you may *Project* your *Sphere* to any *Radius*, *bigger* or *leſs* at pleaſure: This I thought good to intimate, before I come to ſhew the Uſes of the *Scales*.

CHAP. III.

The joint uſe of the Scales of Sines, Tangents, half Tangents, Secants and Chords, in Projecting of the Sphere in Plano, *both General and Particular.*

THE *Sphere* may be Projected *in Plano* upon any *Circle Great* or *Small*, *Direct* or *Oblique*, I ſhall inſtance in ſome few, and thoſe ſhall be both *General* or *Particular*; I call thoſe *General Projections* which ſerve generally for all *Latitudes*, as the *Analemma*, the *General Aſtrolabe* of *Gemmifriſius*, afterwards augmented and amplified by *John Blagrave* of *Reading* Gentleman, and by him called *The Mathematical Jewel*; I call thoſe *Projections Particular*, as are deſcribed upon the *Plain* of ſome particular *Horizon*, or ſuch as have a particular *Horizon Projected* upon them.

By this kind of *Projecting* of the Sphere *in Plano*, many (if not all) *Spherical Concluſions* may be performed, and true *Schemes* of the *Sphere* drawn ſuitable to any *Queſtion*, which will be a great help in *Trigonometrical Calculations*.

CHAP. IV.

How to Project the Sphere in Plano *upon the Plain of the General* Meridian, *by Circular Lines, which will be a* General Projection: *Alſo how to fit an* Horizon *thereunto, ſuitable to any aſſigned* Latitude.

Fig. I.

Fig. I.

FIrſt, draw a Circle B C D E, upon the Centre A, where it is to be croſſed with two diameters at Right Angles; let the four Quadrants be divided each of them into 90 degrees, beginning at C, and ending with 90 at B and D in the upper Semicircle, and beginning at B and D, and ending with 90 at E in the lower Semicircle.

circle. These Circles thus drawn, let the outermost B C D E represents the *General Meridian*, C the *North-Pole*, E the *South-Pole*, and B A D the *Equinoctial*; C A E the *Equinoctial Colure* and Prime Vertical Circle, or *hour of Six*.

Secondly, From your Scale of half Tangents take 15 deg. (one hour's distance) and set them upon the Equinoctial from the Centre A, to 15 on either side towards B and D. Also take 30 deg. (which is two hours distance) and set them from A on both sides to 30 and 30 towards B and D, do so with the Tangents of 45, 60, and 75 deg. which are whole hour distances: But instead of taking 15, 30, 45 d. *&c.* out of the Scale of half Tangents, you may if you will (and that is better) take the half Tangents of 5, 10, 15, 20, *&c.*

Thirdly, Those *hour Circles* must all of them pass through E and C the two *Poles* of the World, and must cut the *Equinoctial* B D at right Angles in the Points 15, 30, 45, *&c.* and for the drawing of them, you must out of your *Scale* of *Secants* take the *Secant* of 15 deg. (which is the Secant complement of 75 deg.) and set it from 75 to 30 upon the *Equinoctial Circle* on both sides, so shall 30 and 30, be the two Centres for the first hour distance from the *Meridian* on both sides, namely of E 75 C ——— Also for the second hour's distance, take the *Secant* of 30 deg. (which is the Complement of 60 deg.) and set it from 60 to 60 on the *Equinoctial Line*. So shall 60 and 60 be the two Centres for the two second hours from the *Meridian*, namely of C 60 E, and E 60 C ——— Likewise for the third hour distance, take the *Secant* of 45 deg. (which is also the Complement of 45 deg.) and set it from 45 to D and B, so shall D and B be the Centres of the two hour Circles E 45 C, and C 45 E. ——— Again, for the fourth hour, take the *Secant* of 60 deg. (which is the Complement of 30 deg.) and set it from 30 and 30 on both sides upon the *Equinoctial*, extended to F and G; so shall F and G be two Centres, whereby to describe the two *Meridians* (or hour Circles) E 30 C, and C 30 E. —— Do the like for the *Secant* of 75. *&c.* And if you should put the *Meridians* on by 10, 20, 30 deg. *&c.* the *Secant* Complement is always the Centre.

Fourthly, The *Meridians* (or hour Circles) being drawn, the next thing to be done is to draw the *Parallels of Declination*, and they are all small Circles parallel to the *Equinoctial* B D. And to find the Points through which they must pass, take out of your Scale of half Tangents the half Tangents of 10, 20, 30, *&c.* deg. and set them upon the *Equinoctial Colure* from A, both ways, to 10, 20, 30, *&c.* towards E and C. ——— These Points being found upon the *Colure* C E, Out of your Scale of Tangents take 10 deg. (the Complement of 80 d.) and set it from 80 to *c*, (upon the Line E C extended;) so shall *c* be the Centre of the Parallel of 80: also take 20 (the Complement of 70) out of the Scale of Tangents, and set it from 70 to d, so shall *d* be the Centre of the Parallel of 70 deg. And so of all the rest. ——— And in this nature may the two Tropicks be inserted; for the half Tangent of 23 d. 30 m. being set from A to *a* and *b*, shall give the points a and *b*, through which the Tropicks must pass, and the Tangents of 66 deg. 30 m. (the complement of 23 d. 30 m.) set from *a* and *b* upon the Colure E C extended, will be the Centres whereby to describe the two Tropicks ♋ *a* ♋ and ♑ *b* ♑.

Fifthly. A Right line ♋ A ♋ being drawn between the two Tropicks shall be the *Ecliptick*, which must be divided and numbered according as the Scale of half Tangents, but into Sines of the *Zodiac*, as you see in the figure.

Sixthly, To this *Projection* there ought to be a moveable *Horizon*, which will make the Projection *Universal*, which *Horizon* must be the full length of the Diameter B D, and must be divided and numbered as the Scale of half Tangents is divided, and as you see in the Figure: And this *Horizon* is to turn about upon the Centre A. and therefore I call it moveable.

Some Uses of this Projection.

I. *How to fit the Projection for Use in any Latitude.*

If you would fit this Projection for Use in the Latitude of 40 degrees North, count 40 deg. upon the *Meridian* (or outermost Circle) from C towards D, and thither bring the *moveable Horizon*, which being there fixed with a bit of Wax, a Screw, or the like, the Projection will be fitted to perform many Spherical Conclusions in that Latitude, a few whereof shall be here delivered.

II. *What*

II. *What Declination ſhall the Sun have when he is in* 29 *deg. of* Taurus?

Find the Point of the 29th deg. of *Taurus*, in the Ecliptick Line, and the Parallel of Declination which paſſeth by that Point is the 20th Parallel from the Equinoctial, towards ☋ the *North-Pole*; ſo that when the Sun is in 29 deg. of *Taurus*, it hath 20 deg. of North Declination.

Examples of this Projection in the Latitude of 40 *deg.*

The Sun having 20 *deg. of North Declination, to find,*

I. The Sun's *Meridian Altitude.*

The Sun's Declination being 20 deg. North, count how many degrees are contained between that Parallels Interſection with the General *Meridian*, and the *Horizon*, which you will find to be 70, and that is the Sun's *Meridian Altitude.*

II. To find the *Aſcenſional Difference.*

See how many degrees of the *Horizon* are contained between the Centre and the Point where the Parallel of the Sun's Declination cutteth the *Horizon*, which you will find to be 17 d. 47 m. and that is the *Aſcenſional Difference*; , which in Time is 1 hour and 11 min.

III. The *Semidiurnal* and *Seminoctiurnal* Arks.

Count upon the Parallel of Declination, from the *Meridian*, how many hours and parts of hours are contained between the South part of the *Meridian* and the *Horizon*, which you will find to be 7 h. 26 m. and that is the *Semidiurnal Ark.*—— Alſo count how many hours and minutes upon the Parallel of Declination are contained between the North part of the *Meridian* and the *Horizon*, which you will find to be 4 h. 34 m. and that is the *Seminoctiurnal Ark.*

Note. The *Semidiurnal Ark* doubled is the *Length of the day.* The *Seminoctiurnal doubled* is the *Length of the Night.* —— The *Aſcenſional Difference* is ſo much as the Sun *Sets* or *Riſes* before or after *Six* a Clock.

IV. To find the Sun's *Amplitude.*

The number of degrees upon the *Horizon*, between the Centre and the Parallel of Declination, are 26 d. 31 m. And that is the Sun's *Amplitude* from the Eaſt or Weſt.

V. *To find what Altitude the Sun ſhall have at Six a Clock.*

Bring the moveable *Horizon* to the *Zenith* of the Place, and note through what degree thereof the Parallel of the Sun's Declination doth paſs, for that is the complement of the Sun's Altitude at Six a Clock: So the Sun having 20 de. of North Declination, that Parallel will croſs the Horizon in 12 de. 42 m. And ſuch Altitude will the Sun have at Six a Clock, when he hath 20 deg. of North Declination in the Latitude of 40 degrees.

Chap.

CHAP. V.

How to Project the Sphere in Plano, *suitable to any assigned Horizon; suppose* London, *whose Latitude is* 51 *deg.* 30 *m.*

Fig. II. *Fig.* II.

SUppose a Globe to be elevated to the Latitude of 51 deg. 30 m. and to be pressed flat down into the Plain of the *Horizon*, then will the outer *Circle* or Limb thereof, represent the *Horizon* of *London*, and all the *Circles* contained in the Upper Hemisphere of the Globe, may be artificially represented within this *Primitive Circle* W N E S; as the *Meridians*, *Equinoctial*, *Ecliptick*, *Circles* of *Altitude*, *Parallels* of *Declination*, *&c.* by Circular Lines, amongst which the *Azimuths* only, in this *Projection*, become *Streight Lines*, all meeting in the Center Z.

With the *Radius* (or 60 deg.) of your Scale of *Chords*, upon Z as a Centre, describe the *Circle* N E S W, representing the *Horizon* of *London*, which cross with the two Diameters W E and N S at right angles in Z: So shall the Centre Z be the *Zenith* thereof, N S the *Meridian*, and W E the *Prime Vertical Circle*, or *Azimuth* of *East* and *West*.

Let the four Quadrants of this *Circle* be each of them divided into 90 equal parts, or degrees, and numbred from N and S, towards E and W, by 10, 20, 30, *&c.* to 90 at W and E.

Thus much for the *Limb*, or outermost *Primitive Circle*, now for the work within.

First, Extend the *Meridian* Line N S out at length, on both sides, to B and O: And (because the Latitude is 51 d. 30 m.) take 33 deg. 30 m. the Complement of the Latitude, out of the Scale of *Half Tangents*, and set it from Z the *Zenith* to P; so shall P be the *North Pole* of the World, distant from the *Zenith* Z 38 d. 30 m. and elevated above the North part of the *Horizon* N P, 51 d. 30 m. equal to the *Latitude*. The point P being found, out of the Scale of *Secants* take the *Secant* of the Latitude 51 d. 30 m. and set it upon the *Meridian Line* (extended) from P to B; so shall B be a Centre, and B P *Radius*, upon which to describe the *Circle* W P E, which is the *Hour Circle* of Six, cutting the *Horizon* in E and W, the East and West Points thereof, and passing through P the Pole of the World; and this *Circle* is also the *Equinoctial Colure*.

Secondly, Out of the Scale of *Half Tangents* take the Latitude 51 d. 30 m. and set it from Z to Æ; so shall Æ be the Point in the *Meridian*, through which the *Equinoctial* must pass. Also take 38 d. 30 m. the Complement of the Latitude, out of the Scale of *Secants*, and set it upon the *Meridian* from Æ to C; so shall C be a Centre, and C Æ a *Radius*, upon which to describe the *Equinoctial Circle* W Æ E, cutting the *Horizon* in the East and West Points, and the *Equinoctial Colure* in the same Points also, and the Ecliptick in the Points ♈ and ♎.

Thirdly, Forasmuch as the *Ecliptick* cutteth the *Equinoctial* on either side thereof, at an Angle of 23 deg. 30 min. add 23 deg. 30 min. to 51 d. 30 m. and the Summ is 75 deg. also subtract 23 deg. 30 min. from 51 deg. 30 min. and the remainder will be 28 deg. Wherefore take 75 deg. out of your Scale of *half Tangents*, and set them from Z to ♈, and take the *half Tangent* of 28 deg. and set it from Z to ♋; so shall the two Points ♈ and ♋ be the two Points in the *Meridian*, through which the two halves, *viz.* the Northern and Southern Semicircles of the *Ecliptick*, must pass. And now, if to 38 d. 30 m. the Comp. of the Latitude, you add 23 d. 30 m. the Obliquity of the *Ecliptick*, the Summ is 62 d. the *Meridian Altitude* of the Sun when he is in *Cancer*; and if you subtract 23 d. 30 m. from 38 d. 30 m. the remainder will be 15 d. for the Sun's *Meridian Altitude* when he is in *Capricorn*: Wherefore take 62 deg. the *Meridian Altitude* in *Cancer*, out of your Scale of *Secants*, and set it from ♋ to O; so shall O be the Centre of the Northern Semicircle of the

Ecliptick, noted with ♈, ♉, ♊, ♋, ♌, ♍. And the *Secant* of 15 deg. the Sun's *Meridian Altitude* when he is in ♍, being set from ♍ to D, shall give D for the Centre of the Southern Semicircle of the *Ecliptick*, charactred with ♎, ♏, ♐, ♑, ♒, ♓. Both which Semicircles being drawn, will cut the *Horizon* in the Points of *East* and *West*, and the *Meridian* in the Points ♋ and ♑.

Fourthly, For the Dividing of the two Semicircles of the *Ecliptick*; out of your Scale of *half Tangents* take 62 deg. and set it from Z to F; so shall F be the *Pole* of the Northern Semicircle of the *Ecliptick* ♈, ♋, ♎; and the *half Tangent* of 15 d. set from Z to G, shall give the Point G, for the *Pole* of the Southern half of the *Ecliptick* ♈, ♑, ♎, from which two *Poles*, F and G the *Ecliptick* must be divided in this manner: A Ruler laid to F, and every 30 deg. of the *Primitive Circle* W S E, shall cut the Northern Semicircle of the *Ecliptick* in the Points ♉, ♊, ♋, ♌, ♍; and being laid to G, and every 30th deg. of the Line, it shall cut the Southern Semicircle in the Points ♏, ♐, ♑, ♒, ♓, for the Points of the Sun's Entrance into any of the *Signs*; and so may every fifth, tenth, or every single degree of the *Ecliptick* be divided.

Fifthly, For the two *Tropicks*, which are Parallels to the *Equinoctial*, and do bound the Course of the Sun: The Intersections of the *Tropicks* with the *Meridian* depend upon the *Equator*; for from Æ 23 d. 30 m. farther shall be ♑, the Intersection of the *Southern Tropick* with the *Meridian*, and from Æ 23 deg. 30 m. nearer shall be ♋, the Intersection of the *Meridian* with the *Northern Tropick*. Now to find the Points ♋ and ♑ upon the *Meridian*, do thus: From Z to Æ, being the *half Tangent* of 51 deg. 30 m. add thereto 23 deg. 30 m. the Sun shall be 75 deg. wherefore, the *half Tangent* of 75 deg. being set from Z, shall reach to ♑, the Intersection of the *Southern Tropick* with the *Meridian*: The Intersections of the other intermediate Parallels of Declination with the *Meridian* shall be found in like sort, by their degrees of distance from the *Equinoctial*; and they will be such as in this Table.

A Table of the Intersections of the Parallels of Declination with the Meridian.

The Half Tangent of	De.	M.	Being set from Z upon the Meridian ZS, shall give the Point of Intersection of the Parallel of	De.	Degrees of South Declination with the Merdian,	And	The Half Tangent of	De.	M.	D.	Degrees of North Declination with the Meridian.
	75	00		♑				28	00	♋	
	74	30		23				28	30	23	
	73	30		22				29	30	22	
	72	30		21				30	30	21	
	71	30		20				31	30	20	
	70	30		19				32	30	19	
	69	30		18				33	30	18	
	68	30		17				34	30	17	
	67	30		16				35	30	16	
	66	30		15				36	30	15	
	65	30		14				37	30	14	
	64	30		13				38	30	13	
	63	30		12				39	30	12	
	62	30		11				40	30	11	
	61	30		10				41	30	10	
	60	30		9				42	30	9	
	59	30		8				43	30	8	
	58	30		7				44	30	7	
	57	30		6				45	30	6	
	56	30		5				46	30	5	
	55	30		4				47	30	4	
	54	30		3				48	30	3	
	43	30		2				49	30	2	
	52	30		1				50	30	1	
	51	30		Equi.				51	30	Equi.	

Sixthly, Then for the *Centres* of the *Tropicks*, and the other intermediate *Parallels* of *Declination*, they depend upon their Intersections on the *Southern* part of the *Meridian*,

dian, it being extended, and their distances from Z shall be such *Tangents* as are exhibited in this *Table*.

A Table for the Centres *of the* Tropicks *and* Parallels *of* Declination *from the* Zenith.

Degrees of South Declination.	De.	Mi.	Tangents from Z.	Degrees of North Declination.	De.	Mi.
♑	58	20		♋	27	47
23	57	49		23	27	57
22	56	45		22	28	16
21	55	44		21	28	36
20	54	42		20	28	50
19	53	43		19	29	19
18	52	41		18	29	42
17	51	46		17	30	5
16	50	50		16	30	28
15	49	55		15	30	52
14	49	1		14	31	17
13	48	9		13	31	42
12	47	17		12	32	8
11	46	28		11	32	36
10	45	38		10	33	3
9	44	50		9	33	32
8	44	3		8	34	2
7	43	18		7	34	33
6	42	34		6	35	3
5	41	50		5	35	36
4	41	8		4	36	8
3	40	27		3	36	43
2	39	47		2	37	17
1	39	8		1	33	53
Equi.	38	30		Equi.	38	30

There is yet another way whereby Geometrically to find the *Centres* of the *Tropicks*, and other *Parallels* of *Declination*; for

A Table of Amplitudes.

If from the Scale of Chords you take	De.	M.	And set them both ways from W and E towards ♋ and ♑, you shall have the intersection of the Parallel of	Deg.	From the Equinoctial, upon the Horizon, or Primitive Circle.
	1	36		1	
	3	13		2	
	4	44		3	
	6	26		4	
	8	3		5	
	9	40		6	
	11	17		7	
	12	55		8	
	14	33		9	
	16	12		10	
	17	51		11	
	19	31		12	
	21	11		13	
	22	52		14	
	24	34		15	
	26	7		16	
	28	1		17	
	29	46		18	
	31	32		19	
	33	20		20	
	35	9		21	
	37	0		22	
	38	53		23	
	39	50		♋ ♑	

And thus have you three Points whereby to draw every *Parallel* of *Declination*, *viz.* One upon the *Meridian* NS, and Two upon the *Horizon* or *Primitive Circle*; so that their *Centres* may be found by the 23d. *Problem* of the Second Book of *Plain Geometry*: All which *Centres* will fall in the *Meridian* N S extended.

Seventhly, For the *Hour Circles:* The Point B being found to be the *Centre* for the *Hour Circle* of 6, through the Point B draw a Line at length perpendicular to the *Meridian* PB, extending it on both sides thereof, and also Parallel to WE, as the Line KBL; for in this Line will all the *Centres*, by which the *Hour Circles* are to be described, fall, which Line ought to be very carefully drawn. Now to find the *Centres*, do thus,

Eighthly, Upon P (as a *Centre*) at the distance P B, describe the *Semicircle* ABG, which divide into 12 equal parts in the Points * * * *, *&c.* Then lay a Ruler to P, and every of the Points * * *, *&c.* Lines drawn from those Points will cut the *Tangent* Line KBL in the Points 3, 2, 1, and 11, 10, 9, *&c.* which are the *Centres* of the first, second, and third *Hours* from the *Meridian*, on both sides thereof: For one foot of the *Compasses* being set in 1, or 11 in the Line BL, and the other extended to P, the *Pole* of the *World*, (where all the *Hour Circles* must meet,) it will describe the *Arch* or *Hour Circles* of 7, and 5, *&c.* And in the same manner may the rest of the *Hours* be drawn, and also the *halves* and *quarters* of *Hours*, or every 5th. or 10th. deg. of the *Equinoctial*, or more if you please, be inscribed, by first dividing the *Semicircle* ABG into such parts, and transferring them to the *Tangent* Line KBL.

These *Hour Circles*, if they were drawn out, would all of them pass through P the *Pole* of the *World*, but in this place, and for our present purpose, it will be sufficient to draw them only between the *Tropicks*, because the *Sun* never exceedeth those Bounds.

Lastly, To this *Projection* there belongeth an *Index*, as MHK, to move upon the *Centre* Z, which *Index* is to be divided and numbred as the *Scale* of *half Tangents* is upon the *Scale*, and as you see done in the *Figure*.

A Synopsis *of the making of this Projection.*

Z P = ½ t	38	30	The *Pole*.
P B = Se.	51	30	*Cent. Equ. Colure.*
Z Æ = ½ t	51	30	The *Equinoctial*.
Æ C = Se.	38	30	*Cent. Equin.*
♈ D = Se.	15	00	*Cent.* of *Eclip.*
♋ O = Se.	62	00	*Cent.* of *Eclip.*
Z F = ½ t	62	00	*Pol. Eclip.*
Z G = ½ t	15	00	*Pol. Eclip.*
Z ♑ = ½ t	75	00	*Trop. Capric.*
Z ♋ = ½ t	28	00	*Trop. Canc.*

Some USES of this *Projection.*

I. *To find the Sun's Declination, his place in the Ecliptick being given.*

Seek the *Sun*'s place in the *Ecliptick Circle*, and then observe what Parallel of *Declination* doth cross the *Ecliptick* in that Point in which the *Sun* is; for that is the *Sun*'s *Declination* for that day.

So the *Sun* being in the 29 deg. of *Taurus*, the *Parallel* of *Declination* which passeth through that Point is the 20th. *Parallel* from the *Equinoctial* towards the *North Pole*; and so the *Declination* at that time is 20 deg. *North*, because the *Sun* is in a *Northern Sign*.

II. *By knowing of the Sun's Declination, to find the Ascensional Difference, and Time of the Sun's Rising and Setting.*

Find the *Parallel* of *Declination* in which the *Sun* is, and trace it along with your Eye (or with a Needle's point) till it touch the *Horizon*, and there see upon or between what *Hours* it falleth, for that is the time of the *Sun*'s *Rising* or *Setting*; and so many *Hours* and *Minutes* as that is before or after *Six*, the same is the *Ascensional Difference*.

So the *Sun* having 20 deg. of *North Declination*, that *Parallel* being followed to the *Horizon*, will fall at 11 *Minutes* after 4 of the Clock in the Morning, at which time the *Sun Rises*, or at 49 min. after 5 in the Evening, at which time the *Sun Setteth*; and the distance contained between the *Parallels* intersection with the *Horizon*; and the *Hour Line* of S, is 1 hour and 49 min. and that is the *Ascensional Difference*.

III. *The*

III. *The Amplitude of the Sun's Rising or Setting.*

Find the Point where the *Parallel* of *Declination* doth cut the *Horizon*, the Number of Degrees contained between that Point and the Point of *East* or *West*, are the Degrees of the *Sun's Amplitude*.

So the *Sun* having 20 deg. of *North Declination*, the Degrees of the *Horizon* contained between this *Parallel* and the Point of *East* or *West*, are 33 deg. 20 min. and that is the *Amplitude*; and so many degrees doth the *Sun Rise* or *Set* distant from the true *East* or *West* Points of the *Horizon* towards the *North*, because the *Sun* hath *North Declination*.

IV. *At what hour of the Day the Sun will be due East or West.*

Look for the Point where the *Parallel* of *Declination*, in which the *Sun* is, doth cut the Line of *East* and *West*, or the *Prime Vertical Circle*, *W. E.* and what *Hour* it cutteth, that is the *Hour* at which the *Sun* will be due *East* or *West*.

So you shall find that the *Parallel* of 20 deg. of *North Declination* will cut the *Prime Vertical Circle* (on the *East* side) at 7 min. after 7 in the *Morning*, at which time the *Sun* will be due *East*. —— And it doth intersect it (on the *West* side) at 53 min. after 4 in the *Afternoon*, at which time the *Sun* will be due *West*.

V. *What Altitude the Sun shall have when he is [illegible] the East or West Azimuth.*

Lay the *Index* upon the *East* or *West* Line of the *Projection*, and look what Degrees of the *Index* are cut by the *Parallel* of *Declination*, for those are the Degrees of *Altitude* which the *Sun* shall have when he will be upon the *East* or *West Azimuth*.

So the *Declination* being 20 deg. *North*, and the *Index* laid upon the Line of *East* or *West*, the *Parallel* of 20 deg. of *North Declination* will pass under 25 deg. 55 min. And such *Altitude* shall the *Sun* have when he is upon the *East* or *West Azimuths*.

VI. *What Altitude the Sun shall have at Six of the Clock.*

Look where the *Parallel* of *Declination* crosseth the *Hour Circle* of Six, and to that Point bring the *Index*, the Degrees of the *Index* which lie over that Point, are the Degrees of *Altitude* that the *Sun* shall have at that *Hour*.

So the *Declination* being 20 deg. *North*, the *Index* being laid over the Intersection of this *Parallel*, with the *Hour Circle* of Six, will have cut by this Intersection 15 deg. 30 min. And such *Altitude* shall the *Sun* have at Six of the Clock.

And in this manner may you find what *Altitude* the *Sun* shall have any hour of the day, in any *Parallel* of *Declination*.

VII. *What Azimuth the Sun shall be upon at Six of the Clock.*

Lay the *Index* over the Intersection of the *Parallel* of *Declination* and the *Hour Circle* of Six, then the Degrees of the *Limb* of the *Circle*, cut by the *Index*, are the Degrees of *Azimuth*.

So the *Index* laid upon the Intersection of the *Parallel* of 20 deg. of *North Declination*, with the *Hour Circle* of Six, it will cut in the *Limb* of the *Horizon* 77 deg. 14 m. if you count them from the *North*; or 12 deg. 46 min. if you count them from the *East*; or 102 deg. 0 min. if you count them from the *South*: And any of these is the *Azimuth* at 6 of the Clock.

The Sun's *Declination* and *Altitude* being given: To find,

I. *The Sun's Azimuth.*

Count the Degrees of the *Sun's Altitude* upon the *Index*, and move the *Index* about, till those Degrees do lie just over the *Parallel* of *Declination*, then shall the Degrees cut by the *Index* in the *Limb* of the *Horizon*, give the *Azimuth*.

So

So the *Sun*'s *Declination* being 20 deg. *South*, and his *Altitude* 12 deg. If you bring 12 deg. of the *Index* to touch the *Parallel* of 20 deg. of *South Declination*, you ſhall find the *Index* to cut in the *Limb* 34 deg. from the *South*, and that is the *Sun*'s *Azimuth* at 12 deg. of *Altitude*.—But if you count the *Azimuth* from the *North*, it is 146 deg. and from the *Eaſt* or *Weſt* 56 deg.

II. *The Hour of the day.*

Count the Degrees of the *Sun*'s *Altiude* upon the *Index*, and move it about till thoſe Degrees do cut the *Parallel* of *Declination*, and the *Hour* that paſſeth by that Interſection, is the *Hour* of the *day*.

So the *Sun*'s *Declination* being 20 deg. *North*, and the *Altitude* 12 deg. bring 12 deg. of the *Index* to meet with the *Parallel* of 20, and the *Hour Circle* under this Interſection will be found to be 6 hours 30 min. for the *Hour* of the *day*.

Note, That in all the foregoing *Problems*, whatſoever is ſaid concerning the *Sun*, the ſame is to be underſtood alſo of any *fixed Star*, whoſe *Declination* is known.

The Tropicall *Fig.* III. *Projection*

CHAP. VI.

To Project the Sphere upon one of the Leſſer Circles; *viz. The* Tropick *of* Capricorn.

1. WITH 60 deg. of the Scale of *Chords*, upon the *Centre* P, deſcribe the *Circle* ♎ Æ ♈ for the *Equinoctial*; croſs it with two *Diameters* at *right Angles* in the *Centre* P with ♎ P ♈, the *Equinoctial*, and Æ P æ, the *Solſtitial Colure*, which is alſo the *Meridian* of the Place.——2. Out of your *Scale* of *half Tangents*, take 113 deg. 30 m. that is, the diſtance of the *Tropick* of *Capricorn*, from P the *North Pole* of the *World*, and ſet them upon the *Meridian* from P, Southward, to ♑.——3. Take the *half Tangent* of 66 deg. 30 m. the Complement of the *Sun's* greateſt *Declination* (or his diſtance from the *North Pole*) and ſet it upon the *Meridian*, Northward, from P to ♋, and at that diſtance upon P deſcribe the *Circle* for the *Tropick* of *Cancer*.——4. Divide the ſpace between ♋ and ♑ into two equal parts in the Point E; ſo ſhall E be the *Centre* whereon to deſcribe the *Ecliptick Circle* ♈ ♋ ♎ ♑.——5. Take the *half Tangent* of 51 d. 30 m. the *Latitude*, out of the Scale of *half Tangents*, and ſet it upon the *Meridian* Northward, from P to R; ſo ſhall R be the Interſection of the *Horizon* with the North part of the *Meridian*.——6. Take the *Tangent* of 38 d. 30 m. the Compl. of the *Latitude*, and ſet it upon the *Meridian*, Southward, from P to H; ſo ſhall H be the *Centre* of the *Horizon*, and H R the *Semidiameter* thereof, whereupon the *Horizon* A R B is to be deſcribed, cutting the *Ecliptick* in the Points ♈ and ♎, if there be no former Error committed in the Work.—— And thus have you projected the *Meridian*, *Equinoctial*, *Ecliptick*, *Colures*, *Horizon*, and *Tropicks*; the next thing to be done, is, to divide the *Ecliptick*, which may be done from a Table of *Right Aſcenſions*, but *Geometrically* thus.——7. Divide the *Equinoctial* into 12 equal parts, then laying a *Ruler* to E (the *Centre* of the *Ecliptick*, and every of thoſe 12 Diviſions of the *Equinoctial*) it will cut the *Ecliptick Circle* in the Points ♈ ♉ ♊ ♋ ♌ ♍ ♎ ♏ ♐ ♑ ♒ ♓; and according as you divide the *Equinoctial* into every 10th, 5th, or every ſingle Degree, ſo may you divide the *Ecliptick* alſo. 8. For the *Azimuths*, or *Vertical Circles*, to the *Radius* of the *Equinoctial Circle*, take the *half Tangent* of 38 deg. 30 min. the Complement of the *Latitude*, and ſet them from P, downwards, (upon the South part of the *Meridian*) to Z; ſo ſhall Z be the *Zenith* Point, where all the *Azimuth Circles* muſt concur; and how to project them is in this manner:——9. Take 51 deg. 30 min. the *Latitude*, out of your Scale of *Tangents*, and prick them down on the North part of the *Meridian* from P to X, and through X draw a *Tangent-Line* parallel to ♈ P ♎, extending it infinitely on both ſides of the *Meridian*.——10. Make Z X a new *Radius*, equal to a *Tangent* of 45 deg. and on that *Tangent-Line*, ſet the *Tangents* of 10, 20, 30, 40, *&c.* which Points, with their reſpective diſtances from Z, the *Zenith*, ſhall be the *Centres* whereon the *Azimuths* muſt be deſcribed.——11. For the *Circles* of *Poſition* draw a right *Line*, drawn through H, the *Centre* of the *Horizon*, parallel to ♈ P ♎ (the *Equinoctial Colure*) then divide the South *Semicircle* of the *Equinoctial* into ſix equal parts; a *Ruler* laid from P the *Pole*, to each of thoſe ſix diviſions, will cut the former *Parallel-Line* in the Points *a b c d*, and thoſe ſhall be the *Centres* whereby to deſcribe the *Circles* of *Poſition*. And thus is this *Projection* finiſhed: But an *Index* muſt be made to move upon the *Centre*, ſuch as is deſcribed in the *Figure*. Fig. III.

A brief Synopſis *of this Projection.*

P the *Pole* of the *World*, as alſo of the *Equinoctial*, and the two *Tropicks*.
P ♑ = *half Tangent* of 113 deg. 30 m.
P ♋ = *half Tangent* of 66 deg. 30 min.
P R = *half Tangent* of the *Latitude*.
P H = *Tangent* of 38 deg. 30 min.
H the *Centre* of the *Horizon*.
R the *Centre* through which the *Circles* of *Poſition* are to paſs.
P E = *Tangent* of 23 deg. 30 min.
E the *Centre* of the *Ecliptick*.

If into this *Projection* the *Parallels* of *Latitude* to the *Ecliptick* be drawn, it will be an abſolute Inſtrument to find the *Riſing* and *Setting* of the *Moon* and other *Planets* that have *Latitude*.

CHAP.

CHAP. VII.

How to Project the Sphere upon any Oblique Circle; *and to find in what* Latitude *and difference of* Longitude (*from the place where the* Oblique Circle *deviates from the* Meridian *and* Zenith *of a known* Latitude) *shall be an* Horizontal Plain.

Fig. IV. AS *Great Circles* of the *Sphere* are *Horizons* in some place of the World or other; so all *Plains* on which *Sun-Dials* are made, are *Horizontal-Dials* in some one part of the World: And such a *Plain* (or *great Circle*) which I mean here, will best be represented by the Roof of a House. Now suppose in the *Latitude* of *London*, 51 deg. 30 m. that the side of a House should decline from the South, Westward, 24 deg. 20 m. and the Roof thereof incline towards the North part of the *Horizon* 36 deg. I would know in what *Latitude*, and how much differing in *Longitude* that part of the World is, in which this *Oblique Plain* to the *Meridian* and *Zenith* (or *Horizon*) of *London*, will be an *Horizon*.

Fig. V. Having drawn a *Primitive Circle* HXOD, to represent this *Oblique Plain* (or *Circle*).

1. Draw a *Diameter* HO, crossing it with the *Diameter* CF at *right Angles* in Q.

2. Take the *half Tangent* of the *Plain's Inclination*, 36 deg. and set them from Q to Z; so shall Z be the *Zenith* of *London*, and Q of the place enquired.

3. Take the *half Tangent* of 54 deg. the Complement of the *Plain's Inclination*, and set it from Q to B; so shall B be one Point through which the *Horizon* of *London* must pass.

4. Take the *Tangent* of 36 deg. the *Plain's Inclination*, and set it from Q to C, or the *Secant* of 36 deg. and set it from B to C; either of which distances shall give the Point C, for the *Centre* of the *Horizon* of *London* HBO.

5. Take the *Tangent* of 54 deg. the Complement of the *Inclination*, and set it from Q to F.

6. Take 24 deg. 20 min. the *Declination* of the *Plain*, from the *Scale* of *Chords*, and set them from H to *c*, from D to *d*, and from O to *e*.

7. A *Ruler* laid from Z to *c*, *d*, and *e*, will give the Points W, S, and E, for the *West*, *South*, and *East* Points of the *Horizon* of *London*.

8. Draw FG perpendicular to QF, (or parallel to HO,) and extend it as far as you shall see requisite.

9. Draw a Line through the Points E and W, (which, if you have committed no former Error, will pass also through the *Centre* Q) and extend it till it cut the Line FG last drawn, so shall the Point of Intersection be the *Centre* of the *Meridian* PZS, which is the *Meridian* of the place where the *Declining Inclining Plain* in *London* will be an *Horizontal Plain*.

10. A *Ruler* laid from W to Z, will cut the *Primitive Circle* in *a*, from which Point, set 38 deg. 30 min. the Complement of the *Latitude* of *London*, to *b*; and a *Ruler* laid from W to *b*, will cut the *Meridian* in the Point P, for the *North Pole* of the World.

11. Set 90 deg. of your *Chords* from *b* to *f*, and from *f* to *g*; a *Ruler* laid from W to *f*, will cut the *Meridian* in Æ, for the Point where the *Equinoctial* must cut the *Meridian*; and laid from W to *g*, will cut the *Meridian* (extended) in M, to the *South-Pole*; and a right Line drawn through P Q and M, shall be the *Axis* of the World.

12. The *Equinoctial Circle* must be drawn through the Points W Æ E; and to find the *Centre* thereof,

13. Divide the right Line WE into two equal parts in R, and upon R raise the *Perpendicular* R T, drawing it forth till it meet with the *Axis* of the World PQM (extended) in the Point T, which is the *Centre* of the *Equinoctial Circle*: Or, if from R, through C, (the *Centre* of the *Horizon*) a right Line be drawn, and extended till it concur with the *Axis* of the World PQM extended, it would meet in T, the *Centre* of the *Equinoctial* also.

14. Divide

14. Divide M P into two equal parts at *Right Angles* in D, and draw D G, extending it infinitely.

15. Upon P, at the distance P D (or any other) describe the *Semicircle* L D N, and laying a *Ruler* from P to G, the *Centre* of the *Meridian* P Z S, it will cut the *Semicircle* L D N in K; at which Point K, begin to divide the *Semicircle* into 12 equal parts, at the Points ⊙⊙⊙⊙, *&c.* then a *Ruler* laid from P, to each of those 12 equal parts ⊙⊙⊙⊙, *&c.* shall give the Points of the *Tangents* of 15, 30, 45, 60, *&c.* Degrees upon the *Tangent-Line* G D, in which shall be the *Centres* of the several *Meridians*; G being the *Centre* of the first *Meridian* from 12 of the Clock; or the *Meridian* of the Place.

Or having a *Sector*, (as formerly intimated) make P D equal to the *Tangent* of 45 d. and so the *Tangents* of 15, 30, 45, 60, *&c.* being set upon the *Tangent-Line* G D, both ways from D, shall give the *Centres* of the other *Meridians*.

To find in what Latitude and Difference of Longitude from London, *this Projection will be an Horizontal Projection.*

The *Projection* thus finished, if you take the distance Q P in your *Compasses*, and measure it upon your *Scale* of *Half Tangents*, you shall find it to contain 72 deg. 34 m. whose Complement is P X, 17 deg. 26 m. and in that *Latitude* will this *Declining Inclining Plain* be an *Horizontal Plain*.

And if you lay a *Ruler* from P to Æ, it will cut the *Primitive Circle* in *h*; and the distance N *h*, measured upon the *Scale* of *Chords*, shall be 14 deg. 41 m. which is the *Plain's Difference of Longitude*.

So that a *Plain* declining 24 deg. 20 m. and inclining to the *Horizon* 36 deg. in the *Latitude* of *London*, will be an *Horizontal Plain* in such part of the World, which lies in the *Latitude* of 17 deg. 16 min. and differs in *Longitude* from *London* 14 deg. 41 m.

Moreover, from this *Projection* may be deduced the several *Requisites* belonging to such a *Declining Inclining Plain* in the *Latitude* of *London*: For,

1. The *Elevation of the Meridian* above the *Horizon* of *London*, is represented by the *Arch* H A, which measured upon the *Scale* of *Chords*, will be found to be 69 d. 54 m.

2. The *difference of Meridians* (which is the same with the *Plain's difference of Longitude*) is represented by the *Angle* Q P Æ, whose Measure, upon the *Equinoctial*, is the *Arch* V Æ; or upon the *Primitive Circle* N *h*, which will be found to contain 14 deg. 41 min.

3. The *Altitude of the Stile* of the *Dial* above the *Sub-stile* thereof, represented by the *Arch* of the *Meridian* P A, whose Measure is the Complement of the *Arch* Q P, which measured upon the *Scale* of *Half Tangents*, will be found to be 72 deg. 34 m. whose Complement P A 17 deg. 26 min. is the *Altitude* of the *Stile* of the *Dial* above the *Sub-stile*, equal to the *new Latitude*.

4. The *Distance of the Sub-stile* from the *Meridian*, represented by the *Arch* of the *Primitive Circle* A X, which measured upon the *Scale* of *Chords*, will be found 4 deg. 30 m. and so far must the *Sub-stile* be removed from the *Meridian* upon the *Dial-Plain*.

5. And for the *Hour-Distances* upon the *Plain*, the *Projection* will give you them also: For, a *Ruler* laid from Q to the Intersections of the several *Meridians* upon the *Primitive Circle*; Lines drawn thereby will be true *Hour-Lines* of such a *Declining Inclining Plain*.

Thus in this *Projection*, you have (in some measure) the whole *Art of Dialling demonstrated*; as I have at large shewn in a particular Treatise of *Geometrical Dialling*.

A Synopsis *of this Oblique Projection.*

H X O D the *Primitive Circle*: Or, the *Declining Inclining Plain*.
Q Z = the *half Tangent* of 36 deg. *Plains Inclination*.
Q B = the *half Tangent* of 54 deg. = *Co-Plains Inclination*.
Q C = *Tang.* 36 deg. Or B C = *Secant* 36 = *Plain's Inclination*, and is the *Centre* of the *Horizon*.
Q F = *Tang.* 54 deg. = *Co-Plains Inclination*.
H *c* = D *d* = O *e* = 24 deg. 20 min. = the *Plain's Declination*. A *Ruler* laid from Z to *c*, *d*, and *e*, will give W, S, and E, in the *Horizon*.
G F ⊥ to Q F, or || H O.
E W extended, gives G for the *Centre* of the *Meridian*. A *Ruler* laid from W to Z,

gives *a b*=the *Chord* of 38 deg. 30 min.=the *Co-Latitude*. W *b* gives P for the *Pole* of the World: *b f*=*b g*=*Chord* of 90 deg. A *Ruler* laid from W *f* gives Æ the *Equinoctial*; W *g* gives M the *South Pole*; and P Q M is the *Axis* of the World.

Through W Æ E draw the *Equinoctial Circle*.

W R=R E=half W E. Extend the *Axis* Q P, and draw R C till it concur with Q P in T; so shall T be the *Centre* of the *Equinoctial*.

Divide M P into two equal parts in D, and draw D O infinitely on both sides P D M. Upon P, at the distance P D, describe the *Semicircle* L D N. A *Ruler* laid from P to G, will cut the *Semicircle* in K; at K begin to divide L D N into 12 equal parts: A *Ruler* laid from P to those equal parts, will give the *Tangents* of 15, 30, 45, *&c.* upon G D O, and be the *Centres* of the *Meridians*; G being the *Centre* of 12.

Lines drawn from Q, through the Intersections of the *Meridians*, with the *Primitive Circle*, (representing the *Declining Inclining Plain*) shall be the true *Hour-Lines* upon such a *Declining Plain*.——And this *Declining Inclining Plain* will be an *Horizontal Plain* in the *Latitude* of 17 deg. 26 min. and differing in *Longitude* from *London*, Eastward 14 deg. 41 min. which will be about the Kingdom of *Cassena*.

CHAP. VIII.

How to Project the Sphere upon the Plain *of the* Equinoctial: *And to make an* Horizon *thereto suitable to any* Latitude.

THis is an universal *Projection*, and therefore will require a particular *Horizon* for every several *Latitude*: For the projecting and making whereof, mind the following *Rules*.

1. Let the *Primitive Circle* noted with A ♎ B ♈ represent the *Equinoctial*, divide it into 24 equal parts, and mark them with their proper *Hours*; the *Centre* whereof, noted with ⊙, representing the *Pole* of the World.

2. The *Diameter* A ⊙ B, having XII. at each end thereof, is the general *Meridian*, and the *Solstitial Colure*.

3. The *Diameter* ♈ ⊙ ♎ having VI at each end thereof, and drawn at *Right Angles* to the former, is the *Hour Circle* of VI. of the Clock, and also the *Equinoctial Colure*.

4. The *Circle* O P F S, is a *Polar Circle*, and is distance from ⊙ the *Pole* of the World, 23 deg. 30 min. equal to the obliquity of the *Ecliptick*, which *Circle* is thus to be described——Take 23 deg. 30 min. out of your *Scale* of *half Tangents*, and with that distance upon the *Pole* ⊙, describe the Circle O P F S, which shall be the *Polar Circle*.

5. The *Circle* E ♋ W ♑, representeth either of the *Tropicks*, and is distant from the *Equinoctial* A ♈ B ♎, 23 deg. 30 min. equal to the obliquity of the *Ecliptick*; or it is 66 deg. 30 min. distance from ⊙ the *Pole* of the World.——To project it, Out of your *Scale* of *half Tangents* take 66 deg. 30 min. and with that distance on the *Pole*, or *Centre* ⊙, describe the *Circle* E ♑ W ♋, representing either *Tropick*.

And here note, That all *Circles* described upon the *Pole* ⊙, are all *Circles* or *Parallels* of the *Sun* or *Stars Declinations*, and may be inserted by help of the *Scale* of *half Tangents*, as the *Polar* and *Tropical Circles* were.

6. The *Circle* ♈ ♋ ♎ is the *Ecliptick*, whose greatest obliquity from the *Equinoctial* is 23 deg. 30 min. and so falls just in the Point ♋——It is thus to be described: The *half Tangent* of 66 deg. 30 min. will reach from ⊙ to ♋——For its *Centre*, take out of your *Scale* of *Secants*, the *Secant* of 23 deg. 30 min. and that distance shall reach from ♋ upon the *Meridian* to G, which Point G is the *Centre* upon which the *Ecliptick* ♈ ♋ ♎ must be described——For its Pole, it is the Point P, the Intersection of the *Polar Circle* with the *Meridian*, through which Point all the *Circles* of the *Stars Longitude* are to pass.

7. The *Circle* ♈ H ♎, represents the *Horizon* of *London*, whose *Latitude* is 51 deg. 30 min.——To describe which, take the *half Tangent* of 51 deg. 30 min, and that shall reach from ⊙ to H, the Intersection of the *Meridian* and *Horizon*.——For its *Centre*, take the *Tangent* of 51 deg. 30 min. and that shall reach from H to I, which

Point

Point I is the *Centre* of the *Horizon*, upon which it must be described. — For its *Pole*, the *half Tangent* of 38 deg. 30 min. (its Complement) being set from ⊙ to Z, giveth the Point Z for the *Zenith* of the place, which is also the *Pole* of the *Horizon*, through which Point must all the *Azimuth Circles* pass.

These forementioned, are all the *Circles* which are described upon this *Projection*: It remains now to shew how to inscribe other *Spherical Circles* upon them, which will render them far more useful than before.

8. The *Circle* K P L is a *Circle* of *Longitude*, passing through 65 deg. 12 min. which is the *Longitude* of *Aldebaron*, or the *Bull's Eye*. — To describe the *Circles* of *Longitude*: First, they must all pass through the Point P, and through every degree of the *Ecliptick*; so that the *Ecliptick* being divided from the Point P, the *Pole* thereof; as I shewed how to divide the *Horizon* from Z, the *Pole* thereof; you have three Points given, through which they may be described: And to describe them, the *Tangent* of 66 deg. 30 min. will reach from P, upon the *Meridian*, extended upon it; and where that extent endeth, will be the *Centre* of the 00 deg. and 180 deg. of *Longitude*, *viz.* of the *Circle* ♈ P ♎; the *Centres* of the rest will be in the same right Line, drawn at right *Angles* to the *Meridian*, through the former *Centre*, and may be found by their natural *Tangents*, as was before taught.

9. The *Circle* * *a* *, is a *Parallel* of the *Bull's Eye's* Declination, 15 deg. 48 min. and is found by taking the *half Tangent* of 74 deg. 12 min. its Complement, and with that distance describe the *Circle* * *a* * upon the *Centre* ☋.

10. The *Circle* ♈ Z ♎ is the *Prime Vertical Circle*, or *Azimuth* of East or West. To describe it, it is to pass through Z the *Zenith*, as all *Azimuths* must do; and seeing the *Zenith* is 38 deg. 30 min. distance from the *Pole*, the *Secant* Complement thereof 15 deg. 30 min. taken from the *Scale* of *Secants*, and set from Z upon the *Meridian* (extended) shall give the Point Y for the *Centre* of the said *Prime Vertical Circle*, — And in a Line drawn at right *Angles* to that *Meridian*, through this Point Y, as the Line Y D, shall the *Centres* of all *Azimuth Circles* fall — And to find them, you must make Z Y a *Radius*, and the natural *Tangents* of 10, 20, 30, *&c.* set from Y upon that Line both ways, shall be the *Centres* for the *Azimuths* of 10, 20, 30, *&c.* degrees from the East or West, as the Point D is the *Centre* of the *Azimuth* R * Z V, an *Azimuth* of 39 deg. 42 min. from the East, or of 50 deg. 18 min. from the South; and the Point *d*, on the other side, is the *Centre* of the *Azimuth* M * Z, passing through the *Bull's Eye* at 6 of the Clock, being 80 deg. from the *Meridian*, or 10 from the East.

11. The *Circle* *m e* * *n*, is a *Circle* of *Altitude* of 20 deg. 20 min. passing through the *Bull's Eye* when he is upon the West *Azimuth* — To describe this, or any other *Parallel* of *Altitude* — Lay a *Ruler* from ♈ to H, and it will cut the *Equinoctial Circle* in ♃ — Then out of your Line or Scale of *Chords* take 20 deg. 20 min. and set it from ♃ to ☊; then a *Ruler* laid from ♈ to ☊, shall cut the *Meridian* in the Point *e*, through which the *Parallel* must pass. — Again, lay a *Ruler* to ♈, and the opposite Intersection of the *Meridian* and *Horizon*, so shall the *Ruler* cut the *Equinoctial* in ♂; then from ♂ to ☿ set 20 d. 20 m. and a *Ruler* laid from ♈ to ☿, shall cut the *Meridian* extended in *g*; divide the Space between *e* and *g* into two equal parts in *f*, so shall *f* be the *Centre* of the *Parallel* of *Altitude* of 20 d. 20 m. *m e* * *n*. And the like is to be done for any other.

And thus have I briefly shew'd how to *Project* several *Circles* of the *Sphere* upon this *Projection*, which will render it far more useful, as by some *Problems* following will appear.

A Synopsis *of this Projection.*

♈ A ♎ B the *Equinoctial Circle.*
E ♑ W ♋ the two *Tropicks.*
⊙ ♋ or ⊙ ♑ each of them equal to the *half Tangent* of 66 deg. 30 min.
⊙ S or ⊙ P each of them equal to the *half Tangent* of 23 deg. 30 min.
♋ G each of them equal to the *Secant* of 23 deg. 30 min.
G being the *Centre* of the *Ecliptick.*
⊙ H equal to the *half Tangent* of *Latitude* 51 deg. 30 min.
H I equal to the *Tangent* of 51 deg. 30 min.
I the *Centre* of the *Horizon.*
⊙ Z equal to the *half Tangent* of 38 deg. 30 min.
⊙ *Centre* of the *Tropicks.*
⊙ *Centre* of the two *Polar-Circles.*
P O S F the *Polar Circles.*

♈ ☉ ♎ the *Equinoctial Colure.*
A ☋ B the *Solstitial Colure.*
♈ ♋ ♎ the *Ecliptick.*
♈ H ♎ the *Horizon.*
Z the *Zenith.*
P the *Pole* of the *Ecliptick.*
K P L a *Circle* of *Longitude* passing through the *Bull's Eye.*
☉ the *Pole* of the *World.*
M * Z, or Z * N an *Azimuth* passing through the *Bull's Eye* at 6 of the Clock.
♈ Z ♎ the *Prime Vertical Circle,* or *Azimuth* of East and West.
* *a* * a *Parallel* of the *Bull's Eye's* Declination 15 deg. 48 min.
m e * *n* a *Parallel* of *Altitude* passing through the *Bull's Eye* when he is upon the East or West *Azimuth, viz.* 20 deg. 20 min.
f the *Centre* of the *Parallel* of *Altitude m e n* 20 deg. 20 min.
R * Z V an *Azimuth* of 50 deg. 18 min. from South.
* *r v* a *Parallel* of *Altitude* of 45 deg.
♈ P ♎ the *Circle* of *Longitude* belonging to 00 deg. and 180 deg.

PROB. I. *To find the* Latitude *of a* Star.

THE *Latitude* of a *Star,* is its distance from the *Ecliptick:* Wherefore through the *Star* draw a *Circle* of *Longitude,* as the *Circle* KPL is a *Circle* of *Longitude,* passing through *Aldebaron* or the *Bull's Eye,* the *Pole* whereof is at D; then lay a *Ruler* from D the *Pole* of the *Circle,* to the *Star,* and note where it cuts the *Primitive Circle*; also lay the *Ruler* from D, to the Intersection of the *Circle* of *Longitude* with the *Ecliptick,* and note also where the *Ruler* cuts the *Primitive Circle:* The distance between these two Marks, measured upon the Line of *Chords,* is the *Latitude* of that *Star*; and for *Aldebaron* will be found to be 5 deg. 31 min.

PROB. II. *To know at what* Horary *distance from the next Meridian, any* Star *will be upon the East or West* Azimuth.

FIrst draw a *Parallel* of the *Star's Declination*; then draw an *Azimuth Circle* of East and West, and note where they intersect; then from the *Centre,* to the Point of their Intersection, lay a *Ruler,* or extend a *Thred,* which will fall upon the *Star's Horary* distance from the *Meridian.*——So the *Parallel* of the *Bull's Eye's Declination,* being the Circle * *a* *, intersecting the *Prime Vertical Circle,* or *Azimuth* of East or West ♈ Z ♎ (on the West side thereof) in the Point *, the Line ☉ * being drawn through that Point of Intersection, will cut the *Hour-Circle* in 5 hou. and 8 m. from the next *Meridian:* And at such distance will the *Bull's Eye* be, when he is upon the East or West *Azimuth.*

PROB. III. *To find what* Altitude *any* Star *shall have when he is due East or West.*

UPon the Intersection of the *Meridian* and *Horizon,* lay a *Ruler* to the Point where the *Parallel* of the *Stars Declination* cuts the *Azimuth Circle* of East or West, and mark where it cuts the *Primitive Circle*; the distance of that Point from ♈ to ♎ being measured upon the Line of *Chords,* shall there shew the *Altitude* of the *Star.*—So the *Bull's Eye* being upon the East or West *Azimuth,* will be found to have 20 d. 20 m. of *Altitude.*

PROB. IV. *To know what* Altitude *any* Star *shall have when he is six Hours distance from the* Meridian.

DRaw a *Parallel* of the *Star's Declination,* and note where it cuts the *Hour-Line* of Six, and through that Point of Intersection describe an *Azimuth Circle,* and find the *Pole* thereof; from which *Pole* lay a *Ruler* to the *Star,* and note where it cuts the *Primitive Circle*; also lay a *Ruler* from the *Azimuth-Pole,* to the Point where the *Azimuth* which passeth through the *Star* cuts the *Horizon,* and mark where it cuts the *Primitive Circle*; the distance between this and the former Mark, being measured upon the *Scale* of *Chords,* shall shew the *Altitude* of the *Star.*——And by thus doing, the *Altitude* of *Aldebaron* at 6 hours distance from the *Meridian,* will be found to be 12 deg. 20 min.

Prob.

PROB. V. *To know what* Azimuth *any* Star *shall have, when he is Six Hours distance from the Meridian.*

THrough the Point of Interſection of the *Stars Parallel* of *Declination* with the Hour of Six, draw an *Azimuth Circle* as before; then from Z the *Zenith*, to the Interſection of the *Azimuth*, paſſing through the *Star* and the *Horizon*, lay a *Ruler*, and mark where it cuts the *Primitive Circle*; the diſtance from the *Meridian* to this Point ſhall give the *Star*'s *Azimuth* from the South.—And thus the *Bull*'s *Eye*, when he is ſix Hours diſtance from the *Meridian*, will be found to have 80 deg. of *Azimuth* from the South.

PROB. VI. *The* Declination *and* Altitude *of a* Star *being given, to find the* Star's *Hour.*

FIrſt deſcribe a *Parallel* of the *Star*'s *Declination*, and another of the *Star*'s *Altitude*, and through their Point of Interſection draw a right *Line* from ⊙ the *Centre* of the *Projection*, which *Line* ſhall ſhew the *Star*'s Horary diſtance from the *Meridian*.—Thus the *Declination* of the *Bull*'s *Eye* being 15 deg. 48 min. whoſe *Parallel* is * *a* *, and the *Altitude* of the *Bull*'s *Eye* obſerved to be 45 deg. which is the *Parallel* of *Altitude* * *v r*, interſecting each other in the two Points * and *r*: If you lay a *Ruler* from ℗, the *Pole* or *Centre*, to * or *r*, it will in either caſe cut the *Equinoctial Circle* in 2 hours 17 min. from the *Meridian*, which is the *Star*'s hour, or his Horary diſtance from the *Meridian*.

PROB. VII. *The* Declination *and* Altitude *of a* Star *being given, to find the* Azimuth *which that* Star *is upon.*

FIrſt you muſt deſcribe the *Star*'s *Parallel* of *Altitude*, and *Parallel* of *Declination*, as in the laſt *Problem*, and through their Point of Interſection, and the *Zenith* Point Z, deſcribe an *Azimuth*; and to find what *Azimuth* that is, do thus:

Look where the *Azimuth-Circle* cuts the *Horizon*, and lay a *Ruler* from the *Zenith* Z, to this Point of Interſection, and mark where the *Ruler* cuts the *Primitive Circle*; the diſtance between the *Meridian*, and the place where the *Ruler* cuts the *Primitive Circle*, meaſured on the *Scale* of *Chords*, will be the *Azimuth* which the *Star* was then upon.—And thus the *Bull*'s *Eye* having 45 degrees of *Altitude*, his *Azimuth* from the South part of the *Meridian* will be found to be 50 deg. 18 min.

How to inſert into this Projection an Horizon *ſuitable to any aſſigned* Latitude; *ſuppoſe* 40 *Degrees.*

From your *Scale* of *half Tangents* take 50 deg. and ſet them upon the *Meridian* from ⊙ to T, for the *Zenith* Point; and the *half Tangent* of 40 deg. (the *Latitude*) and ſet it from ⊙ to Q: So the *Secant* of 50 deg. (its Complement) ſet from Q, ſhall be the *Centre* of the *Circle* ♈ Q ♎, and be an *Horizon* for 40 deg. of *Latitude*. And ſo for any other; but it muſt be divided from its *Pole* (or *Zenith*) T.

How within two Hemiſpheres *of the foregoing Projection, to inſert the ſeveral Conſtellations of the* Fixed Stars, *and by them to perform the ſeveral Uſes of the* Cœleſtial Globe.

Each of the *Hemiſpheres* conſiſteth of three parts, one whereof is *Fixed*, and the other two are *Moveable*.

I. *Of the* Fixed *Part.*

The *Fixed Circle* is the *Hour Circle*, which is divided into 24 equal Parts or Hours, and numbered from XII. on either ſide, towards the Eaſt and Weſt, by I. II. III, *&c.* to XII. on the contrary ſide. And again, each of theſe Hours is ſubdivided into 12 equal Parts, each Part containing 5 minutes of Time.

Without this *Circle* of Hours, is another *Circle*, divided into 32 equal parts, and denominated by the Names of the 32 Points of the *Mariner*'s *Compaſs*; diſcovering the Poſition of the *Stars* in the Heavens at any time. And of theſe two *Circles* do the *Fixed* parts of the *Hemiſpheres* conſiſt.

II. *Of the* Moveable *Parts.*

1. Within theſe *Fixed Circles* move the two *Hemiſpheres*, which are limited within two other *Circles* called *Equinoctials*, which move (with them) upon the *Poles* of the World, as their *Centre*. The

The innermoſt of which two *Circles* is divided into 360 equal parts, or degrees, and in a narrow Margin ſo numbered, from the Eaſt, by the South, Weſt, and North, to the Eaſt again, by 10, 20, 30, *&c.* to 360.—And this *Circle* or *Equinoctial* is again ſubdivided into 12 equal parts, each part containing 30 deg. or one Sign of the *Zodiack*, and named by the Names, and charactered by the Characters of the Twelve Signs, thus *ARIES* 10, 20, *Aries* 30, *&c.*

The outermoſt of theſe two *Circles* is a *Circle* of *Months*, and is divided into 12 unequal Parts or *Months*, each *Month* having its proper Name, and its due number of Days, and are thus named and numbered, *January* 10, 20, 31, *&c.*

Between the *Centre* of the *Hemiſpheres* (which are the two *Poles* of the World) and the *Equinoctials*, are inſerted the ſeveral *Stars* according to their *right Aſcenſions*, *Declinations*, and *Magnitudes*, the Northern *Stars* upon the Northern, and the Southern upon the Southern *Hemiſphere*: All which *Stars* are drawn into ſuch *Conſtellations* as (by the *Poets*) have been feigned or attributed to them; and ſuch *Stars* as are *In formis*, are there placed, according to their *right Aſcenſions*, *Declinations*, and *Magnitudes* alſo.

From the *Centre* (or *Pole*) of each *Hemiſphere*, to the beginning of the 360 degrees, which is at the beginning (or firſt deg.) of *Aries* is drawn a *Semidiameter*, which contains one Quarter of the *Equinoctial Colure* (and therefore I ſhall ſo call it) which is divided into 90 unequal degrees, and is numbred from the beginning of *Aries* upwards towards the *Pole*, by 10, 20, 30, 40, *&c.* to 90, at the Pole it ſelf.

Again, from the firſt degree (or beginning) of *Aries*, to the beginning of *Libra*, in both *Hemiſpheres*, there is drawn an *oblique Arch* of a *Circle*, which is divided into ſix unequal Parts or Signs, and each Sign is ſubdivided into 30 unequal Parts, of which the Northern *Hemiſphere* contains the ſix Northern, and the Southern the ſix Southern Signs, each Sign being numbered and charactered thus, *Aries* 10, 20, 30, *Taurus* 10, 20, 30, *&c.*

This *Circle* is the *Ecliptick*, and cutteth the *Solſtitial Colure* in the Points *Cancer* and *Capricorn*, which are 23 deg. 30 min. diſtance from the *Equinoctial* on each *Hemiſphere*.

There is alſo upon the *Pole*, or *Centre* of each *Hemiſphere*, two *Circles* deſcribed, the one paſſing through 23 deg. 30 min. and the other through 66 deg. 30 min.; that which paſſeth through 23 deg. 30 m. in the [Northern / Southern] *Hemiſphere*, is the *Tropick* of [*Cancer* / *Capricorn*]; and that which paſſeth through 23 deg. 30 m. of the [Northern / Southern] *Hemiſphere*, is the [*Arctick* / *Antarctick*] *Circle*——None of theſe *Circles* have any diviſions at all upon them, neither is there any need; but by them you may diſcover what *Stars* are bounded and limited by them.

All theſe Parts of the *Hemiſpheres* hitherto deſcribed, are general, and are of the ſame uſe in all *Latitudes*: But to make the uſes of theſe *Hemiſpheres* the more ample, there is,

2. Another *Moveable Part* added, which is an *Horizon*, which *Horizon* muſt be peculiar to ſome certain *Horizon* or part of the World, (and although a General or Univerſal *Horizon* might eaſily be contrived, yet the Conveniencies thereof would not be equivolent to the inconveniencies.) And how to make ſuch an *Horizon* for any aſſigned *Place* or *Latitude*, and how to divide the ſame; as alſo to divide the *Equinoctial Colure* and the *Ecliptick*, is already ſhewn. And here take notice, that one of the *Semidiameters* of this particular *Horizon* (made for the *Latitude* of *London*, 51 deg. 30 m.) is divided and numbered in all reſpects, and with the ſame diviſions as is the *Equinoctial Colure*. But the Arch of the *Horizon* is divided, firſt into two equal parts, and each of thoſe numbered by 10, 20, 30, *&c.* to 90, from the Eaſt and Weſt Points thereof, to the North and South, and is again divided into 16 unequal parts, and named backwards and forwards, from Eaſt and Weſt, by the Denominations of the Points of the Compaſs — This *Horizon* is to be applyed to, and to move upon the *Pole*, or the *Centre* of either *Hemiſphere*; and ſo renders the Uſes of the *Hemiſpheres* general for that *Latitude*, — Note here, that I call the ſtreight part of the *Horizon* the *Diameter*, and the circular part, the Arch of the *Horizon*.

And thus have you an account of the ſeveral parts of theſe *Hemiſpheres*; it reſteth now, that we ſay ſomething of the Uſes of them.

The USES of the *Hemiſpheres*.

PROB. I. *The Day of the Month being given: To find the* Sun's Place.

SEek the Day of the Month in the *Moveable Circle* of Months, and thereto bring the Diameter of the *Horizon*; ſo ſhall it cut in the *Equinoctial*, the Sign, Degree, and Minute, that the *Sun* is in at Noon, that day.

Example,

Example, Let the day be the 20th. of *April*, and let the *Sun's Place* be required—Lay the Diameter of the *Horizon* upon the 20th. of *April*, and it will cut the 10th. deg. of *Taurus*, which is the *Sun's Place* that day. And ſo upon the 8th. of *Auguſt* the *Sun's Place* will be found to be in 25 deg. and an half of *Leo*.

PROB. II. *The* Sun's *Place in the* Zodiack *being given, to find the* Day *of the* Month.

BRing the Diameter of the *Horizon* to the *Sun's Place*, and it will cut the *day of the Month*.

So the Diameter being laid to the 10th. deg. of *Taurus*, it will cut the 20th. day of *April*: And laid to 29 deg. and an half of *Leo*, it will cut the 8th. day of *Auguſt*.

PROB. III. *The Place of the* Sun *in the* Ecliptick *being known, to find his* Declination.

FInd the degree of the *Sun's Place* in the *Ecliptick*, in either of the *Hemiſpheres*, and bring the divided parts of the Diameter of the *Horizon* thereto; ſo ſhall the degrees of the Diameter of the *Horizon*, which are intercepted by the *Sun's Place* in the *Ecliptick*, be the degrees of the *Sun's Declination*.

So if the *Sun* be in 17 deg. of *Taurus*, the divided part of the Diameter of the *Horizon* being brought thereto, there will be cut upon the Diameter 17 deg. and that is the *Sun's Declination*.——Alſo, if the *Sun* be in 10 deg. of *Sagittarius*, the *Declination* will be found to be 22 deg.

And here note, if the *Sun's Place* be found in the Northern *Hemiſphere*; the *Sun's Declination* is Northward: But if in the Southern *Hemiſphere*, the *Declination* is South alſo.

PROB. IV. *The* Sun's Declin. *being given, to find his* Place *in the* Ecliptick.

IF the *Sun* hath North or South *Declination*, make uſe of the correſpondent *Hemiſphere*—Then upon the Diameter of the *Horizon* count the degrees of the *Sun's Declination*, and turn the Diameter about till the deg. of the *Sun's Declination* do croſs the *Ecliptick*, and where it ſo croſſeth, there is the *Place* of the *Sun*.—So the *Sun's Declination* being 10 deg: North, then 10 d. of the Diameter of the *Horizon* will cut 25 d. 55 m. of *Aries*, or 4 d. 5 m. of *Virgo* in the Northern *Hemiſphere*——And if the *Sun* have 10 d. of South *Declination*, it will then cut the *Ecliptick* in 25 d. 55 m. of *Libra*, or in 4 deg. 5 min. of *Piſces* in the Southern *Hemiſphere*. For the *Sun* makes four Gradations of *Declination* in a Year, in the ſeveral Quarters of the *Ecliptick* contained between *Colure* and *Colure*.

PROB. V. *The Place of the* Sun *being known, to find his* Right Aſcenſion.

BRing the Diameter of the *Horizon* to the *Sun's* Place in the *Eclptick*, then will the Diameter of the *Horizon* cut alſo the *Circle* which is divided into 360 deg. in the degrees of the *Sun's Right Aſcenſion*.——So the *Sun* being in 29 deg. of *Taurus*, if you bring the Diameter of the *Horizon* to that deg. counted in the *Ecliptick*, it will cut the *Circle* which is divided into 360 deg. in 56 deg. 50 min. And ſuch is the *Sun's Right Aſcenſion* at that time.——But if the *Sun's* Place had been in 29 deg. of *Scorpio*; then his *Right Aſcenſion* (in the Southern *Hemiſphere*) would be found to be 236 d. 50 m.

PROB. VI. *The* Sun's Declination *being given*: *To find his* Amplitude.

IN reſolving many *Problems*, it will be requiſite to have a Thred fixed in the *Centre* of the *Hemiſpheres*; with a ſmall Bead to move upon it, as in this, and ſome other *Problems* which follow.——First, lay the Diameter of the *Horizon* upon the two Sixes; and ſet the Bead which is upon the Thred to the *Sun's Declination*, counted upon the Diameter of the *Horizon*: Then move the Thred along till the Bead do juſt touch the Arch of the *Horizon*, and what degrees of the Arch of the *Horizon* the Bead chanceth to fall upon, thoſe degrees (or that Point of the Compaſs) counted from the Eaſt and Weſt Points of the *Horizon*, are the degrees of the *Sun's Amplitude* or Diſtance of his *Riſing* or *Setting*, from the true Eaſt or Weſt Point.——So the *Sun* having

having 23 deg. 30 min. of North *Declination*, ſet the Bead thereto; and (the Diameter of the *Horizon* lying juſt upon VI. and VI.) move the Thred till the Bead upon it do juſt touch the Arch of the *Horizon*; which it will do at 40 d. (or within 6 d. and a quarter of the North-Eaſt, or North-Weſt Points of the Compaſs) which is the *Sun's Amplitude* — But if the *Sun's Declination* had been 20 d. the Bead ſet thereto, and moved to the Arch of the *Horizon*, it would then cut 33 d. 20 m. (or 25 m. ſhort of the North-Eaſt by Eaſt, or South-Weſt by South Points of the Compaſs) for the Amplitude.

PROB. VII. *The* Sun's Declination *given, to find the Time of the* Sun's Riſing *and* Setting.

REctifie the Bead to the *Declination*, and lay the *Horizon* to the two Sixes, as before; then move the Thred till the Bead touch the Arch of the *Horizon*; and then ſhall the Thred ſhew upon the *Hour-Circle*, the hour and minute of the *Sun's Riſing* or *Setting*. — So the *Declination* being 23 deg. 30 min. the Bead rectified and brought to the Arch of the *Horizon*, the Thred will then cut the *Hour-Circle* at 47 m. after III. in the Morning, at which time the *Sun Riſeth*; and if you carry the String yet on, till the Bead touch the Arch of the *Horizon* in the other ſide of XII. the Thred will then fall upon 13 min. after VIII. at Night; at which time the *Sun Setteth* — But if the *Sun* have 20 d. of North *Declination*, you ſhall find that he will *Riſe* 11 m. after IV. and *Set* at 49 min. after VII.

PROB. VIII. *How to find the* Right Aſcenſion *of any* Star.

MOve the Diameter of the *Horizon* about, till it lie upon the *Centre* of the *Star*; and upon the *Circle* of 360 d. the Diameter of the *Horizon* ſhall there ſhew the degrees of *Right Aſcenſion*. — So the Diameter being laid upon *Cor Leonis*, ſhall cut 147 d. 17 m. — And laid upon the *Bull's Eye*, it ſhall cut 64 d. 17 m. which are the *Right Aſcenſions* of thoſe *Stars*.

PROB. IX. *To find the* Declination *of a* Star.

MOve the graduated part of the Diameter of the *Horizon* about, till it cut the *Star* deſired, and count what degrees of the Diameter are cut by the *Star*, for thoſe are the degrees of the *Star's Declination*. — So if the Diameter of the *Horizon* be brought about till it touch *Cor Leonis*, that *Star* ſhall be cut by 13 deg. 33 min. which is that *Star's Declination*. — And the Diameter being turned about to the *Bull's Eye*, that *Star* ſhall be cut by 15 deg. 46 min. and that is his *Declination*.

PROB. X. *To find what* Stars *do never* Set, *but are always above the* Horizon *in any* Latitude.

COunt your *Latitude* upon the graduated Diameter of the *Horizon*, from the *Pole* or *Centre* downwards, (calling 80 deg. 10 deg. and 70 deg. 20 deg. *&c.*) ſo keeping your Finger, or ſetting a Pin, in that Point; turn the *Horizon* round about, and all thoſe *Stars*, which are between your Finger and the *Pole*, do never *Riſe* nor *Set*, but are always above that *Horizon*. — For all ſuch *Stars*, whoſe *Declination* are greater than the Complement of any *Latitude*, do never *Riſe* or *Set* in that *Latitude* — And all ſuch *Stars* in the contrary *Hemiſphere*; whoſe *Declination* do exceed the *Latitude*, thoſe *Stars* do never Riſe, or appear in that *Latitude*. — And thus, if you lay the Diameter to *Lucida Lyra*, you ſhall find that *Star* to lie juſt under 51 deg. 30 min. (if you count the degrees from the *Pole* downwards) ſo that in the *Latitude* of 51 deg. 30 min. when that *Star* is to the Weſt-ward of the South part of the *Meridian*, it continually deſcends lower and lower, till being arrived to the North part of the *Meridian*; and then it immediately begins to aſcend, till it arrives to the Southern part of the *Meridian* again.

PROB. XI. *To know what* Stars *ſhall be upon the* Meridian *at any Hour of the Night throughout the Year.*

LAy the Diameter of the *Horizon* upon XII. and XII. and move the *Hemiſphere* about, till you bring the Day of the Month juſt againſt the propoſed Hour, ſo ſhall all thoſe *Stars* which lie under the Diameter of the *Horizon*, be upon the *Meridian* at that Hour of the Night. Thus,

Thus, if it were required to know what *Stars* ſhall be upon the *Meridian* upon the 14th. of *January*, at Eight of the Clock.

Lay the Diameter of the *Horizon* upon XII. and XII. then turn the *Hemiſphere* about, till you bring the 14. day of *January* to lie juſt againſt VIII. in the *Hour-Circle*; then will all thoſe *Stars* which lie juſt under the Diameter of the *Horizon*, be upon the *Meridian* at VIII. of the Clock that Night, of which you ſhall find *Aldebaron*, or the *Bull's Eye*, to be the principal.——And at 4 minutes after XI. that Night will the *Little Dog* be upon the *Meridian*.

PROB. XII. *To know what* Stars *will be upon the* Meridian *at Midnight, any Night in the Year.*

LAy the Diameter of the *Horizon* upon the Day of the Month, ſo ſhall all the *Stars* which lie under the Diameter be upon the *Meridian* at XII. of the Clock. —— This needeth no Example, but you may find, that

upon { *February* 6. / *February* 16. / *March* 3. / *March* 27. } the { *Lion's Heart* / *Lion's Back* / *Lion's Tail* / *Virgin's Spike* } will be upon the *Meridian* at XII. at Night, *&c.*

PROB. XIII. *To know at what Hour of the Night any* Star *will be upon the* Meridian.

LAY the Diameter of the *Horizon* upon XII. and move the *Hemiſphere* about, till the *Bull's Eye* lies juſt under the *Diameter*, then juſt againſt the firſt day of *January* in the *Circle* of Months you ſhall find 55 min. paſt VIII. in the *Hour-Circle*, which ſhews that at that time the *Bull's Eye* will be upon the *Meridian*.

PROB. XIV. *To find the difference of* Longitude, *or the* Horary *Diſtance of any two* Stars.

BRing one of the *Stars* againſt XII. in the *Hour-Circle*, then lay the Diameter of the *Horizon* upon the other *Star*; and what Hour that cuts (counted from XII.) upon the *Hour-Circle*, is the diſtance of *Longitude*, or *Horary* Diſtance of theſe two *Stars*.——Thus, Let the two *Stars* be the *Bull's Eye*, and the *Lion's Tail*.

Bring the *Lion's Tail* to XII. and then if you lay the Diameter of the *Horizon* upon the *Bull's Eye*, you ſhall find it to cut 7 hours and 15 min. which is 109 degrees; and ſuch is the difference of *Longitude* of theſe two *Stars*, or their *Horary Diſtance*.

PROB. XV. *To find the* Hour *of the* Night *by the* Stars.

SEE what *Star* you find to be either upon the North, or South part of the *Meridian*; and accordingly bring the ſame *Star* to the like *Meridian* in the *Hemiſphere*: Then look the Day of the Month in the *Circle* of Months, right againſt which you have the true *Hour* of the *Night*.——Thus, if upon the 20th. of *January* you ſhould find *Aldebaron*, or the *Bull's Eye* to be upon the South part of the *Meridian*; bring *Aldebaron* juſt againſt XII. in the *Hemiſphere*, then againſt the 20th. of *January* you ſhall find 22 min. before 8, or 38 min. after 7, which is the true *Hour* of the *Night* on the 20th. of *January*, when *Aldebaron* will be upon the *Meridian*.

PROB. XVI. *To find the* Semidiurnal *or* Seminocturnal *Arch of any* Star.

LAy the Diameter of the *Horizon* upon VI. and bring the *Star* juſtly to touch the Arch of the *Horizon*: Then lay the Diameter of the *Horizon* upon the *Star*, and count how many Hours are contained between either of the XII. for the one is the *Semidiurnal*, and the other the *Seminocturnal* Arch of that *Star*.———Thus, if you lay the Diameter of the *Horizon* upon VI. and bring *Aldebaron* to touch the Arch of the *Horizon*; and then lay the Diameter of the *Horizon* upon *Aldebaron*, you ſhall find it to cut upon 36 min. after IV. if you count the Hours from XII. ſo that the *Seminocturnal* Arch of that *Star* is 4 hours 36 minutes: And if you count the Hours from the other XII. you ſhall find them to be 7 hou. and 24 min. which is the *Semidiurnal* Arch of that *Star*, it being a *Star* having North *Declination*.

PROB. XVII. *To find the Amplitude, or the Point of the Compass upon which any* Star *Riseth or Setteth.*

LAY the Diameter of the *Horizon* upon VI. and bring the *Star* to touch the Arch of the *Horizon*, so shall the degrees of the Arch of the *Horizon*, contained between the East or West Points of the *Horizon*, shew the *Amplitude* of that *Star's Rising* and *Setting*, from the *East* or *West*, towards the *North* or *South*. Thus, if you lay the Diameter of the *Horizon* upon VI. and bring the *Bull's-Eye* to the Arch thereof, you shall find it to lie under 26 degrees of the *Horizon*; and so many degrees doth that *Star Rise* distance from the true *East* Point towards the *North*, and *Sets* so many degrees distance from the *West North-ward* also, (because the *Star* hath *North Declination*) and so his *Amplitude* of *Rising* is *North East* by *East*, 8 degrees *Easterly*. And his *Amplitude* of *Setting* is *North-West* by *West*, 8 degrees *Westerly*.

PROB. XVIII. *To find at what Hour any* Star *Riseth or Setteth.*

LAY the Diameter of the *Horizon* upon VI. and bring the *Star* to lie just under the Arch of the *Horizon* on the *East-side*. Then seek the day of the Month in the *Circle* of Months, and the Hour which standeth against it, is the Hour of the *Stars Rising*: And if you bring the *Star* to lie under the Arch of the *Horizon* on the *West-side* against the day of the Month, you shall have the time of the *Star's Setting*. —— Thus, upon the 25th. of *December*, if you would know at what Hour the *Bull's-Eye Riseth* and *Sets*, lay the Diameter of the *Horizon* upon VI. and bring the *Bull's-Eye* just under the Arch of the *Horizon* on the *East-side*, and against the day of the Month you shall find 2 Hours from the *Meridian*, which is 2 of the Clock in the Afternoon for the time of that *Star's Rising*. And the *Star* being brought under the *West* part of the *Horizon*, the day of the Month shall be against IIII Hours 46 minutes, which is 46 minutes after 4 in the Morning, at which Hour *Aldebaron*, or the *Bull's-Eye*, Setteth.

PROB. XIX. *To know what time of the Year any* Star *in the* Hemisphere *will be upon the* Meridian *at* 12 *a Clock at Night.*

BRing the *Star* to XII on the *South* part of the *Meridian*, then upon that day of the Month which standeth against the North XII, shall that *Star* be upon the *Meridian* at Midnight. —— So, if I would know upon what Night in the Year the *Bull's-Eye* will be upon the *Meridian* at 12 at Night: Bring the *Bull's-Eye* against the *South* XII, and against the *North* XII you shall find the 16th. of *November*, upon which Night at 12 of the Clock *Aldebaron* (or the *Bull's-Eye*) will be upon the *Meridian*.

CHAP. IX.

How to Project the Sphere *upon the Plain of the* Meridian *by Streight* Lines *and* Ellipses.

Fig. VI. THis is a General *Projection*, and for the Projecting thereof; with 90 degrees of your *Scale* of *Sines*; upon C, as a *Centre*, describe the *Circle* P Æ S æ; crossing it with two *Diameters* P S and Æ æ, dividing it into four Quadrants, each of which divide into 90 equal Parts or Degrees, beginning at P and S, and ending at Æ and æ.

The *Circle* thus divided and numbered, represents the General *Meridian* —— Æ æ the *Equinoctial* —— P S the *Prime Vertical Circle*, or *Azimuth* of *East* or *West* —— P the *North*, S the *South Pole* —— ♋ ♋ the *Tropick* of *Cancer* —— ♑ ♑ the *Tropick* of *Capricorn* —— A B the *Arctick Circle* —— C D the *Antarctick Circle* —— ♈ ♈ the

♈ ♑ the *Ecliptick* —— The other Parallels between Æ *æ* and P and S, are Parallels of the *Sun's* or a *Star's Declination.*

For the Projecting of which Lines, *&c.* 1. Take 23 deg. 30 m. from the *Scale* of *Chords*, and set them upon the *Meridian* from Æ upwards to ♋ and downwards to ♑, and also from *æ* to ♋ and to ♑, and draw the two Lines ♋ ♋, and ♑ ♑, parallel to the *Equinoctial*, for the two *Tropicks* of *Cancer* and *Capricorn*.

2. Draw the Right Line ♋ C ♑ for the *Ecliptick Circle*. And thus have you all the Great *Circles* of the *Sphere* projected in *Streight lines*, except the *General Meridian*, which is a *Circle*, and for the other *Meridians* (or *hour Circles*) between the *Meridian* and the *Colure* P C S they are all *Ellipses*, the manner how to draw them shall now follow.

The Line C Æ being equal to the *Scale of Sines*, take from thence 75 deg. set it upon the *Equinoctial* from C to 1, and from C to 11, for the hour Points of One and Eleven. Also take 60 deg. from the *Scale* of *Sines*, and set them from C to 2, and from C to 10, for the hour Points of Two and Ten. Again, take out of the *Scale* of *Sines*, the *Sines* of 45, 30, and 15 deg. and set them upon the *Equinoctial* both ways from C, to the Points 3, 4, and 5, and 9, 8, and 7; and those shall be the Points through which those *hour Circles* must pass upon the *Equinoctial*.

But to find other correspondent Points upon the *Tropicks*, and other Parallels of *Declination*, you must by help of a Sector, divide the *Tropicks* 6, ♋ and 6 ♑; as also the intermediate Parallels of *Declination*, as the *Scale* of *Sines* upon your Ruler is divided; so shall you have Points, through which Lines being drawn, making no Angles, shall be the *hour Circles*, and will be all of them *Elliptical*, for the describing of which there are several sorts of Instruments, one of the best whereof you have by the Figure of the Projection described.

Lastly, The *Ecliptick* must be also divided as the *Scale* of *Sines* is divided, and must have the Characters of the Signs put to them as in the figure.

How the Annalemma *may be made Universal for all Latitudes, both North and South, from the Equinoctial to the Poles.*

1. Let the *Ellipsis* be drawn through every degree of the *Equinoctial*, and that quite up to the *Poles*, or at least so many of them as the largeness of the Instrument will admit.
2. Let all the Parallels of *Declination* be also drawn up to the *Poles*.
3. In the *Planisphere* let there be some eminent *Fixed Stars* placed according to their *Right Ascension* and *Declination*.
4. Let there be the Months and Days of each Month put in, answerable to the *Sun's* Place or *Declination*, but without the *Circle* between the *Tropicks* on either side.
5. Let there be a Moveable *Horizon*, divided as a Line of Versed *Sines* to move upon the *Centre* of the *Planisphere*, yet so that it hinder not the Cursor to pass by the *Centre*.
6. Under the divisions of the *Horizon*, at a competent distance must be cut a hollow Grove, if the *Horizon* be made of Wood, but a Doughtale, if made of *Brass*, upon which a Cursor must move.
7. The Cursor is a piece of Brass about an Inch in breadth, and in length somewhat more than the *Radius* of the Instrument, it must be made to slide along the *Horizon* without interruption, and to stand fixed when occasion is, the two sides or edges are to be divided as one half of the *Horizon* is, *viz.* into a Line of *Sines* equal to the *Radius* of the Instrument.

All these Appurtenances and Lines, are perspicuous in the Figure thereof.

This Instrument thus fitted and being about 15 or 16 Inches Diameter, will be the most absolute Instrument that an Artist can make use of, both for

Portability in Carriage.
Ease in Use.
Exactness in Performance.
Universality in all Parts of the World.

As shall be made appear by these few Uses (amongst many others that might be added) following.

Some USES of THE ANALEMMA IN ASTRONOMY, &c.

THE *Analemma* being a General Inſtrument, may be fitted to any particular *Horizon* or *Latitude*: But ſome Problems or Uſes thereof, being general, may be performed thereby, without having reſpect to any *Latitude*, and ſuch are theſe which follow.

SECT. I.

The Day of the Month being given, to find,

I. *The Sun's Declination.*

SEek the day of the Month in the *Circle* of Months, and that Parallel of *Declination*, which falleth (or ſhooteth) upon the day of the Moneth is the degree of the *Sun's Declination* for that day.

Example. Upon the 17*th.* of *Auguſt.*]

Seek the 17*th.* of *Auguſt* in the *Circle* of Months, and you ſhall find the Parallel of 10 deg. of *Declination* to ſhoot upon it, and ſuch is the *Sun's Declination* upon the 17*th.* of *Auguſt.*

¶ If the day of the Month be found between the *Equinoctial Circle* and the *North Pole*, (where all the *Meridians* meet in the *Centre*,) then the *Sun* hath *North Declination*, but if on the other ſide, it hath *South Declination.*

II. *What two days in the year are of equal Length.*

Example. Upon the 17*th.* of *Auguſt.*]

By the foregoing you found the *Declination* to be 10 deg. Wherefore caſt your eye along the Parallel of 10 deg. and you ſhall find it to ſhoot upon the 5*th* of *April*, (upon the other ſide of the Inſtrument,) which day is of the ſame Length with the 17*th.* of *Auguſt.*

III. *The Sun's Place in the* Ecliptick.

All the Parallels of the *Sun's Declination* croſs the *Ecliptick Line*, in that degree in which the *Sun* is.

Example. When the *Sun* hath 10 deg. of *North Declination.*]

You found (by the firſt) that upon the 17*th.* of *Auguſt* the *Sun* had 10 deg. of *North Declination.* —— Wherefore trace the Parallel of 10 deg. till you find it to croſs the *Ecliptick*, and you ſhall find it to cut the *Ecliptick* in 4 deg. and 26 min. of *Virgo*, and in that *Sign* and Degree is the *Sun* upon the 17*th.* of *Auguſt.*

On the contrary (upon the 5*th.* of *April*) the *Sun* having the ſame *Declination*, the Parallel of *Declination* cutteth the *Ecliptick* in the ſame place, (but in a contrary Sign, *viz.*) in 25 deg. and 34 min. of *Aries*, and in that Sign and Degree is the *Sun* upon the 5*th.* of *April.*

IV. *The*

IV. *The Sun's* Right Aſcenſion.

Example. When the *Sun* is in 4 deg. 26 min. of *Virgo.*]

Seek out upon the *Ecliptick Line* for 4 deg. 26 min. of *Virgo,* and ſee which of the *Meridians* (or *hour Circles*) doth paſs through that point; and with your eye follow that *Meridian,* till you come to the *Equinoctial:* —— Then count how many degrees of the *Equinoctial,* are contained between the firſt Point of *Aries* (which is the *Centre* of the Inſtrument) and that *Meridian*; ſo ſhall you find that the *Meridian,* which paſſeth through 4 d. 26 m. of *Virgo,* will cut the *Equinoctial* in 56 deg. and 50 min. from the *Centre,* and ſuch is the *Right Aſcenſion,* when the *Sun* is in 4 deg. 26 m. of *Virgo.*

Theſe Four Problems, and the converſe of them, have no relation to any Latitude; *thoſe which follow have either the* Latitude *of the Place given, or elſe by ſome other thing or things given, the* Latitude *is to be found; and therefore it will be neceſſary, firſt, to ſhew,*

How to fit the Analemma to any Latitude.

Count the degrees of the *Latitude* of the Place, from downwards to if it be *North,* or from to if it be *South,* and to thoſe degrees bring the end of the moveable *Horizon*; and there fix it by help of a Screw or ſuch like; and then the Curſor being upon it, the Inſtrument is fitted to perform many Aſtronomical Problems, ſuch as here follow.

SECT. II.

The Latitude of the Place, 51 d. 30 m. and Declination of the Sun (10 deg. North) given, to find,

I. *The Aſcenſional Difference.*

Obſerve where the Parallel of *Declination* croſſeth the *Prime Vertical Circle,* (or *hour Circle* of Six,) and from that Point count upon that Parallel, how many degrees or *hours,* are contained between it and the *Horizon,* for thoſe degrees or *hours* are the quantity of the *Right Aſcenſion,* or of the *time* that the *Sun Riſes* or *Sets* before or after Six a Clock.

So the *Latitude* being 51 d. 30 m. and the *Declination* 10 deg. *North,* if upon the Parallel of 10 deg. of *North Declination* you account the number of degrees contained between the *hour Circle* of Six and the *Horizon,* you ſhall find them to be 12 d. 45 m. or in time 51 min. from Six, and ſuch is the *Aſcenſional Difference* when the *Sun* hath 10 deg. of *North Declination.*

II. *The time of the* Sun's *Riſing and Setting.*

Whereſoever the Parallel of *Declination* cuts the *Horizon,* that *Meridian* that cutteth the *Horizon* in the ſame Point, is the time of the *Sun's Riſing.*

Example. The *Sun* having 10 deg. of *North Declination.*]

The *Horizon* being ſet to the *Latitude,* you ſhall find it to touch the *Horizon* at 9 m. after 5 in the Morning, at which time the *Sun* will *Riſe,* when he hath 10 deg. of *North Declination.* —— And at the ſame time you may ſee alſo, that he Setteth at 51 min. after 6 at night.

III. *The length of the Day and Night.*

The Hours and Minutes of the *Sun's Riſing* being doubled, give the Length of the Night, —— And the Hours and Minutes of the *Sun's Setting* give the Length of the Day.

But by the Inſtrument Thus:

Count every hour from the *South* part of the *Meridian* to the *Horizon* for two hours, (upon the Parallel of *Declination,*) and every degree for eight minutes of time;

ſo

ſo ſhall you find the Length of the Day to be 13 hours 42 min. And counting the Hours from the *North* part of the *Meridian*, the Length of the Night will be found to be 10 hours and 18 min.

IV. *The Sun's Amplitude, (or the diſtance that the Sun Riſes or Sets from the true Eaſt or Weſt Points of the* Horizon.)

Example. The *Sun* having 10 deg. of *North Declination.*]

The number of degrees upon the *Horizon* (counted from the *Centre* of the Inſtrument to the interſection of the Parallel of *Declination* therewith, are the degrees of the *Sun*'s Amplitude.

So if you follow with your eye the Parallel of 10 deg. of *North Declination*, till it meet with the *Horizon*, you ſhall find the Parallel to cut the *Horizon* in 16 deg. 15 m. And ſo far doth the *Sun* [Riſe / Set] diſtant from the true [Eaſt / Weſt] Points of the *Horizon*, towards the *North*, (becauſe the *Sun* hath *North Declination*,) and that is the *Amplitude.*

V. *The* Suns' Meridian Altitude.

The *Sun's Meridian Altitude* is its height at Noon.

Example. At 10 deg. of *North Declination.*]

Bring the Curſor to the Interſection of the Parallel of 10 degrees of *North Declination* with the South part of the *Meridian*, and the degrees upon the Curſor are the degrees of the *Sun's Meridian Altitude.* ——

So the Curſor being brought to the Interſection of the Parallel of 10 deg. of *Declination* with the *South* part of the *Meridian*, you ſhall find at that Interſection, 48 d. 30 m. cut upon the Curſor, and ſo many degrees of *Altitude* will the *Sun* have at Noon in that Parallel.

VI. *The* Sun's *Depreſſion at Midnight.*

The *Sun*'s Depreſſion at Midnight is ſo many degrees as the *Sun* is under the *Horizon* at 12 in the Night.

So, the ſame time, the *Sun* having 10 deg. of *North Declination*, move the Curſor to the Interſection of the Parallel of 10 deg. of *South Declination* with the *Meridian*, and there you ſhall find out (upon the Curſor) 28 deg. 30 m. for the *Sun*'s Depreſſion (under the *Horizon*) at Midnight.

VII. *When the Twilight begins and ends.*

Twilight is ſaid to begin when the *Sun* is 18 deg. under the *Horizon*, before its *Riſing*, —— And it ends when the *Sun* is fallen 18 deg. below the *Horizon* after its *Setting*:

Example. The *Declination* being 10 deg. *North.*]

Move the Curſor along the *Horizon*, till you make 18 deg. thereof to meet with the Parallel of 10 deg. of *South Declination*, and that you ſhall find it to do at 2 hours 56 min. at which time Twilight begins in the Morning: —— And alſo at the ſame time it cuts 9 hours and 4 min. at which time in the Evening Twilight ends.

And if you ſubſtract the time of the *Sun's Setting*, from the time of the Twilight's ending, the remainder is 2 hours 13 min: and ſo long doth the Twilight continue both in the Morning before *Sun Riſing*, and in the Evening after *Sun Setting.*

Alſo if you double the length of the Twilight it makes 4 hours 26 min. which taken from 18 hours, 18 m. (the length of the Night before found) there will remain 5 hou. 2 min. for the length of the dark night.

VIII. *What* Altitude *the* Sun *ſhall have at 6 a Clock.*

Example. The *San* having 10 deg. of *North Declination.*]

Move the Curſor along, till it meet with the Interſection of the hour of 6, and the Parallel of 10 d. of *N. Declination*, and then ſhall you find 7 deg. 55 min. cut upon the Curſor; and ſuch *Altitude* will the *Sun* have at 6 of the Clock, Morning or Evening, when he hath 10 deg. of *North Declination.*

IX. *What*

IX. *What* Altitude *the* Sun *ſhall have, at any hour of the Day.*

The *Sun* having 10 deg. of *N. Declination*, and let the Hour be half an hour after 8 in the Morning.]

Find the Point upon the Inſtrument, where the Parallel of 10 deg. of *North Declination* croſſeth the Hour Line of half an Hour paſt VIII. and bring the Curſor to that Point, and there you ſhall ſee the Curſor will touch that Point at 30 deg. and 40 min. and ſuch *Altitude* will the *Sun* have at half an Hour after VIII. in the Morning, or at half an hour after III. in the Afternoon.

And working in this manner, in the *Latitude* of 51 d. 30 m. when the *Sun* hath 20 deg. of *North Declination*, you ſhall find his *Altitude* at all hours of the Day to be as in this Table.

Hours		deg.	m.
	XII	58	42
XI	I	56	34
X	II	50	55
IX	III	43	06
VIII	IIII	34	13
VII	V	24	56
	VI	15	40
V	VII	6	50

X. *What* Azimuth *the* Sun *ſhall have at 6 a Clock.*

The *Sun's Azimuth* at ſix a Clock is his diſtance from the true *Eaſt* or *Weſt* Points of the *Horizon* at that time.

Example. The *Sun* having 10 deg. of *North Declination*]

Bring the Curſor to the Interſection of the Parallel of 10 deg. of *Declination*, with the (vertical *Circle*, or) hour Line of 6. Then ſhall you find the edge of the Curſor to cut 6 deg. and 8 min. upon the *Horizon*, and that was the *Sun's Azimuth* from the *Eaſt* or *Weſt*, towards the *North* at that time.

XI. *What* Altitude *the* Sun *ſhall have, when he is juſt upon the* Azimuth *of* Eaſt *or* Weſt.

Example. The *Sun* having 10 d. of *North Declination*,]

Bring the Foot (or the bottom of one of the Edges) of the Curſor, to the *Centre* of the Inſtrument, and there hold it; then ſee what degrees of the Curſor are cut by the Parallel of 10 deg. of *North Declination*, which you will find to be 13 deg. and 20 min. And ſuch *Altitude* ſhall the *Sun* have, when he is upon the *Azimuth* of *Eaſt* or *Weſt*, the *Sun* then having 10 d. of *North Declination*.

XII. *At what hour the* Sun *ſhall be upon the* Eaſt *or* Weſt Azimuth.

Example. The *Sun* having 10 deg. of *North Declination*.]

Bring the Curſor to the *Centre* of the Inſtrument (as before) then ſee what hour *Circle* (or *Meridian*) paſſeth through the Interſection of the Parallel of *Declination* and the Curſor, which you will find to be 26 min. after 5 in the Afternoon, or 34 min. after 6 in the Morning; and at thoſe hours will the *Sun* be upon the *Eaſt* or *Weſt Azimuths*.

SECT. III.

Thirdly, The Latitude *of the Place,* (51 *d.* 30 *m.*) *the* Declination *of the* Sun (10 *d.* North) *and the* Altitude *of the* Sun (30 *d.* 30 *m.*) *to find,*

I. *The Hour of the Day.*

MOve the Curſor upon the *Horizon*, till 30 deg. 30 m. thereof doth juſt touch the Parallel of 10 deg. of *North Declination*; then obſerve what *Meridian* (or hour *Circle*) paſſeth by that Interſection, which you ſhall find to be 30 min. after 8, if in the

the Morning; or 30 min. after 3, if in the Afternoon. And that is the Hour of the Day.

II. *To find the* Sun's Azimuth: *Let the* Latitude, Declination, *and* Altitude *be as in the laſt.*

Move the Curſor till 30 degrees thereof cuts the Parallel of 10 degrees of *North Declination*; then bring the *Horizon* with the Curſor upon it to the *Equinoctial*; ſo ſhall that hour *Circle* which lyeth under 30 deg. of the Curſor, be the *Azimuth*, which counted from the *South* part of the *Meridian* will be found to be 66 deg. 8 m. from the *North* 113 deg. 52 m. And from the *Eaſt* or *Weſt* 23 d. 52 m.

SECT. IV.

Several ways to find the Latitude.

I. *By the* Sun's Declination, *and his* Meridian Altitude.

TAke in your Compaſſes the Number of degrees of the *Meridian Altitude*, then ſetting one foot thereof upon the *Meridian* at the Point where the Parallel of *Declination* toucheth it, and put the other foot downwards upon the *Meridian*, and to that point bring the moveable *Horizon*; ſo ſhall the degrees of the *Meridian*, comprehended between the Elevated *Pole* and the *Horizon*, be the *Latitude*.

So the *Meridian Altitude* being 67 deg. and the *Declination* 10 deg. *North*, the *Latitude* will be found to be 33 degrees.

II. *By the* Sun's Declination *and* Amplitude, *from* Eaſt *or* Weſt.

Turn the *Horizon* about, till the degrees of *Amplitude* counted thereupon from the *Centre*, do cut the Parallel of *Declination*; then will the *Horizon* give you the *Latitude* upon the outermoſt *Circle*.

So the *Amplitude* being 22 deg. 46 m. and the *Declination* 10 deg. *North*, the *Latitude* will be found to be 63 deg. 20 m.

III. *By the* Sun's Declination *and* Altitude *when he is upon the* Eaſt *or* Weſt Azimuth.

Set the Curſor to the *Centre*, and count upon it the *Altitude*, then turn the Index about till the degrees of *Altitude* on the Curſor do croſs the Parallel of *Declination*; then will the *Horizon* give you the *Latitude*.

So the *Sun's Altitude* being 10 *deg.* 39 min. when the *Sun* is due *Eaſt* or *Weſt*, the *Latitude* will be found to be 70 deg.

IV. *By the* Sun's Declination, *his* Altitude, *and* Azimuth *from the* South, *to find the* Latitude.

Set the Curſor to the *Azimuth* from the *South*, upon the *Horizon*, then Count the *Sun's Altitude* upon the Curſor, turning the *Horizon* about till the *Altitude* counted upon the Curſor do croſs the Parallel of *Declination*; then ſhall the *Horizon* reſt at the *Latitude* ſought.

So if the *Sun's Azimuth* from the *South* ſhould be 72 deg. 28 m. the *Altitude* 56 d. and the *Declination* 10 d. *North*, the *Latitude* will be found to be 30 deg.

The End of the Third Book.

CURSUS MATHEMATICUS.

BOOK IV.

OF COSMOGRAPHY:

In Two PARTS.

COELESTIAL } OR { *ASTRONOMICAL.*
TERRESTRIAL } OR { *GEOGRAPHICAL.*

CONTAINING,

I. The *Cosmical* or *Mundane Systems* of the World, *Ancient* and *Modern*: Of the *Fixed Stars* and *Planets*: Their Number, Names, Motions, Distances, Magnitudes, Nature, Substance, Figure, Light, *&c.* And of the SATELLITES about some of them.

II. The *Terriaquean System*, consisting of Land and Water, it's Species and Magnitude: The Division of it by *Zones*, *Climes* and *Parallels*; And into *Regions*, *Kingdoms*, *Provinces*, &c. With their Situations in respect of *Longitude* and *Latitude*.

WITH

The Use of the GLOBES in GEOGRAPHY and NAVIGATION.

AND

The Description, Projection, and Use Of MAPS and CHARTS; both *General* and *Particular*.

By *WILLIAM LEYBOURN*, Philomat.

LONDON,
Printed *Anno Dom.* M DC XC.

OF COSMOGRAPHY.

PART I.

Cœleſtial, or Aſtronomical.

PROOEM.

Of the Name, Original and Progreſs of

ASTRONOMY.

ASTRONOMY is a *Science* which teacheth the *Laws* and *Rules*, whereby the *Motions* of the *Stars* are Regulated and Determined: It was anciently called alſo *Aſtrology*; for, that which *Plato* called *Aſtronomy*, *Ariſtotle* and others termed *Aſtrology*. Thus *Thales* is the firſt among the *Greeks*, that is ſaid to *Aſtrologiſe*, who never treated of that Judiciary Art: But in after times this Synonomy ceaſed; for this Cœleſtial Science, known anciently to the *Greeks*, by that part of it only which was called *Meteorologick*, as conſidering the *Motions* of the *Stars*, with the *Reaſons* thereof, came, in after Ages, to receive the Addition of another part, called *Apoteleſmatick*; which teacheth to *Divine* and *Prognoſticate* of future events from the Sight and Aſpects of the *Stars* and *Planets*: But afterwards in *Greece*, there was given to this latter a peculiar and diſtinct appellation, namely *Aſtrology*, there being reſerved to the former only that of ASTRONOMY; and ſo deſcribed to be *That Science which contemplates the Motion, Diſtance, Colour, Light, Order, Place, Magnitude, and the like Adjuncts of the Fixed Stars and Planets*; without any reſpect to the Judiciary part.

As to its ORIGINAL, we cannot reaſonably refer it to a better Patron than Admiration. So ſays *Gaſſendus*, *Originem ipſi fecit Admiratio*: For our Forefathers admiring the *Splendor*, *Variety*, *Multitude* and *Magnitude* of the *Stars*; together with their Conſtant and Regular *Motion*, transferred their Admiration into Obſervation; and that (in proceſs of Time) into *Tables*, or *Parapegmata*, for the information of Poſterity. And upon this ground we may (with *Ricciolus*) affirm ASTRONOMY to be as ancient as the STARS themſelves; and that it was at firſt (with other divinely infuſed Arts) reduced to Experiment and Practice by *Adam* himſelf, and by his Poſterity perpetuated, as we find by *Joſephus*, who (in his third and eighth Chap. of the eleventh Book of *Jew. Antiq.*) writes, that *Seth*, having been inſtructed therein by *Adam*; and underſtanding the World was twice to periſh, once by *Deluge*, afterwards by a general *Conflagration*, reduced this Art to an Epitomy, and inſcribed it on two *Pillars*; the one of Brick, againſt the violence of Fire; the other of *Stone*, againſt the Inundation of Waters: One of which (that is the Pillar of Stone) he affirms to have been extant in his Time, in a Place called *Syrias*, or

Seirath, conceived (by Mr. *Is. Voſſius*, *Lib. de Ætat. Mundi*) to be the Land bordering upon Mount *Ephraim*, not far from *Jerico*.

ASTRONOMY being thus brought into the World, was improved by the following Patriarchs, who, by reaſon of their long Lives, had the opportunity of *obſerving* and *noting* many *Aſtral Revolutions*: Amongſt whom, in this Science, the moſt celebrated is *Enoch*, whoſe Books upon this Subject, are ſaid to be extant at this day; from whence *Tertullian* and *Origen* produce ſeveral Citations. But to what perfection this Science was brought before the Flood, is uncertain: But it appears by the Teſtimony of *Origen*, from the forecited Books of *Enoch*, that the *Stars* were then reduced into *Aſteriſms*, under peculiar and diſtinct Denominations.

After the *Flood*, and the diſperſion of Mankind over the face of the Earth, the Study of *Aſtronomy* began to be improved by ſeveral Nations, who, doubtleſs, had derived the knowledg thereof from *Noah* and his Poſterity: So that it may ſeem no wonder, if at one and the ſame time, divers Perſons, in divers Regions, applied themſelves to the Obſervation and Study of this *Aſtral Science*. And from hence ariſe the great Conteſts among ſeveral Nations, for the Glory and Honor of its *Invention*. But it is clear, that Mankind diſperſed themſelves out of *Aſia*, into *Africa*, *Europe*, and other parts of the World; the Glory therefore ought, in the firſt place, to be attributed to the *Aſiaticks*, and among them, chiefly to the *Babylonians*, *Chaldeans*, and *Bactrians*; among whom *Evahdnes*, *Belus*, *Zoroaſter* and *Otanes*, are principally to be celebrated; as alſo, *Naburian*, *Sudinus*, and *Seluchus* the *Chaldean*; and before them the Patriarch *Abraham* and his Father *Thare*.

From the *Aſſyrians* and *Chaldeans* it came next to the *Egyptians*, brought thither by *Abraham*, as *Euſebius* proves from the Authority of *Joſephus*, *Melo*, and others; but *Eupolemus* ſeems to infer, that *Abraham* firſt taught the ſame to the *Phenicians*, before his deſcent into *Egypt*.

There are ſome that averr, that the *Egyptians* were ſo far from receiving of the knowledg of *Aſtronomy* from the *Chaldeans*, that they affirm the *Chaldeans* to be inſtructed herein by the *Egyptians*; but it ſeems ſtrange, if this Science were known to the *Egyptians* before the *Babylonians* and *Chaldeans*, that yet the *Egyptians* Obſervations ſhould be ſo much later than thoſe of the *Babylonians*: For we find few of the *Egyptians* to precede the time of the death of *Alexander* the Great, than which, even thoſe of the *Greeks* are earlier; whereas the Obſervations of the *Babylonians* appear to have been made almoſt 2000 Years before that time.

There are others (as *Lucan*) who would rob both the *Chaldeans* and *Egyptians* of this Honor, and aſſign the Invention thereof to the *Ethiopians*; but this ſwims againſt the ſtream of Tradition. The *Mauritanians* alſo come in for a ſhare in this Invention, affirming they were inſtructed therein by *Atlas* (the Son of *Libya*) their King.

From the ſeveral Nations before mentioned, *Aſtronomy* ſeems to have been divided into Three Sects, the *Aſſyrian*, the *Egyptian*, and the *Atlantick*; of which laſt, yet the *Greeks* and *Romans* make no reckoning; for among them were only enumerated theſe Three Sects, *Chaldaick*, *Egyptian*, and *Grecian*; yet it doth not appear that the *Greeks* had made any conſiderable progreſs in *Aſtronomy* before the death of *Alexander* the Great, only ſome Obſervations of Eclipſes made by *Thales* and *Anaxagoras*: Yet ſuch was their ſelf-Conceit, as confidently to affirm, that *Aſtronomy* ow'd its Invention to them, and particularly to the *Rhodians*. Others of them aſcribe its Original to their Poet *Orpheus*. But theſe being fabulous, perſuade us rather to conclude with the Opinion of thoſe who maintain *Aſtronomy* to have been firſt brought into *Greece* by *Thales* the *Mileſian*, who derived it from the *Egyptians*.

From him it was improved by *Anaximander*, *Anaximenes*, *Anaxagoras*, *Democritus*, *Empedocles*, *Euctemon*, *Meton*, *Endoxus*, and others of the *Athenian* School, till the time of *Alexander* the Great, his founding the City of *Alexandria* in *Egypt*: After which, the *Ptolomies*, his Succeſſors, erected an *Academy* there, for all manner of Studies; the *Grecian Aſtronomy* made its retreat thither, and under thoſe Princes flouriſh'd in equal Glory with the *Egyptian*. From thence we hear of the famous Names of *Antolychus*, *Calippus*, *Timochares*, *Ariſtyllus*, *Conon*, *Hipparcus*, *Soſigines*, *Theon*, *Ptolemy*, *Paulus Alexandrinus*, *Theon Alexandrinus*, and his Daughter *Hypatia*, "Who by barba-
"rous Chriſtians (or Monſters rather) of *Alexandria*, out of mere Envy (for her admi-
"rable Skill in *Aſtronomy*, and other *Mathematical Learning*, which ſhe publickly pro-
"feſs'd and taught) was moſt inhumanly murthered, and torn in pieces in the very
"Cathedral Church.

Among

Among the *Romans* it was long ere this Science gain'd acquaintance, Arms being there more in fashion than Arts: The first that History makes mention of is, *Cajus Sulpitius Gallus*, he was Legat to *Æmilius Paulus* in the War against *Perses*, King of *Macedon*; who first amongst them published a Discourse of *Eclipses*. After him we hear of *Lucius Taruntius*, *Nigidius Figulus*, *Varro*, and *Cicero*, Students in *Astronomy*.

But the *Latins*, for Promoters of this Science, are indebted to none more than to their great Dictator C. *Julius Cæsar*, who

——— Media inter Prælia semper
Stellarum, Cœlique Plagis, Superisque vocabat.

He always liesure found, amidst his Wars,
To mark the Course of Heaven, and learn the Stars.

He (with the assistance of *Sosigines*, the *Egyptian*) reduced the *Roman* Year to the course of the *Sun*, which we yet retain: He wrote (in Greek) of the *Stars*. From him *Mathematical Arts*, particularly *Astronomy*, began to flourish among them. His Nephew and Successor *Augustus* (by his Example) encouraging the same.

Thus have you a clear and concise Account of the *History* of *Astronomy* from its *Original* to the time of *Augustus Cesar*: If the inquisitive Reader would have a continuation of this *History*, let him peruse Esquire *Sherburn*'s Translation, and learned Comment upon *Marcus Manilius* Poem; in an *Appendix* unto which he will find a *Catalogue* of the most eminent *Astronomers*, as well *Ancient* as *Modern*, their *Works* and *Writings*, according to succession of *Time*, from the first *Birth* of *Astronomy*, to the Year of our Lord 1675. from whose *Labours* I have extracted this short Account.

OF THE

System of the WORLD.

IT may be called the *Cosmical* or *Mundane System*, which is no other than the Co-ordination of the *Parts* of the *World*; whose MATTER is the *Number* of the *Elements* and *Heavenly Orbs*: Its FORM, the *Order* and *Disposition* of them among themselves: and *Relatively*, as to the *Centre* of the UNIVERSE.

To explain and illustrate this *Subject* concerning MATTER and FORM, it will be requisite (and to the Reader's great satisfaction) to represent unto him the several *Opinions*, as well of the *Ancients* as *Moderns*, touching the same.

And forasmuch as the MATTER of this SYSTEM, which is the *Number* of Heavens, is variously, and with much Confusion discoursed of by the Learned; some *encreasing*, and others *diminishing* the Number of them: I shall therefore exhibit this following *Synopsis* (from *Ricciolus*, in his *Almagestum Novum. Lib.* 9. *Sect.* 3.) of the *Number* and *Distribution* of the *Heavens*; with the several Authors or Assertors of the same.

A SYNOPSIS *of the Number of the* Heavens *according to several Authors.*

I. *Heaven*,
- That is, One *World*, according to *Aristotle*.
- And that sydereal and fluid, according to S. *Chrysostom*, *Tertullian*, S. *Bonaventure*, *Tycho Brahe*, *Longomontanus*, *Kepler*, *Bullialdus*, and *Tellez*.
- And that sydereal, but solid, according to *Ægidius*, *Hurtadus*, *Cisalpinus*, and *Aversa*.

II. *Heavens*,
- That is, the *Empyræum* created the first day, and the *Firmament* created the second Day, according to *Clemens*, *Acasius*, *Theodoret*, *Anastasius Synaita*, *Procopius*, *Suidas*, S. *Bruno*, and *Claudianus Mamertus*.
- That is, the Sydereal and Aereal, according to *Justin Martyr*.
- That is, one of the fixed Stars, the other of the Planets, according to S. *Gregory Nissen*.
- That is, the *Primum Mobile*, and the Sydereal, according to *Mastrius* and *Bellutus*.

III. *Heavens*,

III. *Heavens,*
- To wit, the *Emperæum, Sydereal,* and *Aereal,* according to S. *Basil,* S. *Ambrose, Damascene, Cassiodorus, Genebrardus, Suarez, Tannerus, Hurtadus, Oviedus, Tellez,* and *Borrus.*
- That is, one of the Fixed Stars; one of the Planets, and the other Aereal, according to *Gregory Nyssene*
- The *Emperæum, Watery,* and *Sydereal,* according to *Thomas Aquinas.*
- That is, the *Watery, Sydereal,* and *Aereal,* according to *Cajetan.*

IV. *Heavens,*
- That is, the *Emperæum,* that of the *fixed Stars,* that of the *Planets,* and the *Aereal*; according to S. *Athanasius.*

V. *Heavens,*
- And all *Sydereal* and *Solid, viz.* The *Fixed Stars* of *Saturn, Jupiter,* and the *Sun,* (with *Mars, Venus,* and *Mercury* included,) and of the *Moon*; according to *Oviedus.*
- That is, the *Emperæum,* the *Water,* that of the *Fixed Stars,* that of the *Planets,* and the *Aereal*; according to *Ricciolus.*

VII. *Heavens,*
- But confusedly reckoned; by *Philastrius.*
- That is, the *Air*; the *Æther, Olympus,* the *Fiery Heaven*; the *Firmament,* the *Angelical Heaven,* and the Heaven of the *Trinity*; are reckon'd by *Beda.*
- That is, the lower Region of the *Air,* the upper Region thereof, the lower Region of the *Fire,* the upper Region thereof, the *Sydereal* Heaven, the *Chrystalline* Heaven, and *Empyræum*; according to *Rabanus.*

VIII. *Heavens,*
- And all *Sydereal,* that is, the Heaven, or Sphere of the *Fixed Stars,* and the *Seven Spheres* of the *Seven Planets*; according to the *Babylonians, Egyptians, Eudoxus, Plato, Calippus, Aristotle, Cicero, Philo,* S. *Basil,* S. *Ambrose, Damascene, Bonaventure, Remigius, Thomas Aquinas, Aben-Ezra, Carthusianus, Lyranus, Tostatus, Brugensis Riccius, Orontius, Cremoninus, Philalthæus Anticus, Ruvius.*

IX. *Heavens,*
- That is, the *Emperæum,* and eight *Sydereal solid* Heavens; according to *Arriaga.*
- That is, the *Primum Mobile,* the eight *Sydereal* Heavens; according to *Macrobius, Haly, Alpetragius, Rabbi Josue, Rabbi Moyses, Scotus, Abraham Zagutus, Sacroboscus, Claromontius,* and, as some (but falsly) conjecture, *Hipparcus,* and *Ptolemy.*

X. *Heavens,*
- That is, the *Primum Mobile,* the Sphere of the Motion of the *Fixed Stars,* the eighth Sphere, for the Motion of Trepidation in Longitude, and that of the seven *Planets*; according to *Alphonsus* and his Followers, *Fernelius, Purbachius, Regiomontanus, Amicus, Appianus, Maurolicus,* and *Longius.*
- That is, the *Primum Mobile,* the Sphere of the Motion of Trepidation in Longitude, that of the *Fixed Stars,* and the Sphere of the seven *Planets*; according to *Arzahel, Thebit,* and *Isaac Israelita.*
- That is, the *Emperæum,* the *Primum Mobile,* and eight *Sydereal* Heavens; according to *Guilielmus Parisiensis,* and *Johannes Antonius Delphinus.*

XI. *Heavens,*
- That is, the *Emperæum,* and the ten moveable Spheres of *Alphonsus* and his Followers, before-mentioned; according to *Petrus Alliacensis,* the College of *Conimbra, Martinengus,* and (sometime) *Clavius.*
- To wit, the *Primum Mobile,* the Sphere of the first Libration in Latitude, The Spere of the second Libration in Longitude, the Sphere of the *Fixed Stars,* and those of the seven *Planets*; according to *Johannes Vernerus, Leopoldus de Austria, Johannes Antonius Maginus,* and (in his latter days) *Clavius.*

XII. *Heavens,*
- If to these foregoing you add the *Emperæum,* there will be XII. Heavens.

XIV. *Heavens,*
- That is, seven Spheres or Orbs, counting from the *Primum Mobile* to the *Aplané* or *Sphere* of the *Fixed Stars*; and seven of the *Planets*; according to *Johannes Baptista Turrianus,* and *Fracastorius.*

Having

Having by the foregoing *Synopsis* sufficiently explained the *Number* of the *Heavens*, which is the *Matter* of the *Mundane System*; we are in the next place to give an account of their *Order* and *Disposition*, which is the *Form* thereof.

According to its *Form*, the *Mundane System* is consider'd under a double *Hypothesis*, either as having the *Earth* seated in the *Centre* of the *Universe*, *immoveable*, and the *Sun* and *Heavens* moving about it; or having the *Sun* in the *Centre*, exempt from any *Motion* of *Lation*, and the *Earth* moving about it in the Solar Orb.

And it is worthy Observation, (as *Ricciolus* notes,) that the two Sects of *Philosophers* and *Mathematicians*, the one maintaining the *Earth*'s *Mobility*, the other its *Immobility*, have had in the Schools, through several Ages, like *Castor* and *Pollux*, alternately their *Rise* and their *Fall*. For first, *Pythagoras*, with some others of his Sect, placing the *Earth* in the *Centre* of the World, making use of *Eccentricks* and *Epicycles*, seated the *Sun* in the midst of the *Planets*. But others of the *Pythagoreans*, retaining *Epicycles* and *Eccentricks*, placed the *Sun* in the *Centre* of the World, and the *Earth* in the midst of the seven *Planets*.

Afterwards *Plato*, placing the *Earth* in the *Centre* of the World, seated the *Sun* in the midst of the *Planets*, yet so, as for the most part, he supposed *Venus* and *Mercury* to be carried above him, whom *Eudoxus*, *Calippus*, and *Aristotle* followed, endeavouring to explain the Motion of the *Planets* by *Concentrick Circles*: So that they conceived neither *Venus* nor *Mercury* moved in *Epicycles*, above and beneath the *Sun*; but either always *above* or always *beneath* the *Sun*.

After this, others again, reducing *Eccentricks* and *Epicycles*, placed the *Earth* in the midst of the World, as *Archimedes*, *Hipparchus*, *Sosigines*, *Cicero*, *Vitruvius*, *Pliny*, *Macrobius*, and *Capella*; but differed among themselves, as to the various Order and Disposition of the *Planets*. Others again, as *Aristarchus*, *Philolaus*, and some *Pythagoreans*, removed the *Earth* out of its central Position, and made it move about the *Sun*, by them placed in the midst of the World.

In process of time, *Ptolemy*, reviving the first and ancient *Pythagorean System*, re-established *Eccentricks* and *Epicycles*, placing the *Earth* in the midst of the World, and the *Sun* in the midst of the *Planets*. Which Opinion continued, and was embraced by the greatest part of Astronomers, until the XIV. Century after Christ; though the Order of the *Planets* were changed by *Alpetragius* and *Geber*. On the other side, *Turrianus* and *Fracastorius*, retaining the *Ptolemaick* Order, brought in again *Concentrick Circles*, and those more in number, by many, than either *Aristotle* or *Eudoxus* had introduced. About which time *Copernicus*, raising as it were from the Grave, the *Hypothesis* of *Philolaus* and *Aristarchus*, touching the *Earth*'s *Annual Motion* about the *Sun*, in the *Centre* of the Universe, set forth and illustrated the same with such fair and probable Arguments, that it hath prevailed with most *Astronomers* at this day. He made *Venus* and *Mercury*, with *Plato*, *Vitruvius*, *Macrobius*, *Capella*, and *Bede*, to move, now above, now beneath the *Sun*. Hence *Tycho Brahe*, and with him *Longomontanus*, took occasion of introducing another *System*; wherein not only *Venus* and *Mercury*, but also *Mars*, *Jupiter*, and *Saturn*, are supposed to move about the *Sun*; and the *Sun* and *Moon*, with the *Fixed Stars* about the *Earth*, immovable in the *Centre* of the Universe; retaining the *Eccentricks*, but rejecting the *Epicycles* of *Ptolemy*. At length *Argolus*, altering the *Tychonick System*, makes the three superior *Planets* to move about the *Earth*; *Venus* and *Mercury* about the *Sun*; seating the *Earth* in the midst of the World. From whom yet *Ricciolus* dissents, making *Saturn*, *Jupiter*, the *Sun*, *Moon*, and *Fixed Stars*, to move about the *Earth*; and *Venus*, *Mercury*, and *Mars* about the *Sun*. These various Sentiments of the Learned, touching the Order and Disposition of the several parts of the *Mundane System* (as they have been in divers Ages successively asserted), being thus briefly enumerated, we shall next give somewhat a more ample Description and Illustration of the several *Hypotheses*, and exhibit withal to the Reader's view, the Schemes of each particular *System*.

OF

Of the Several SYSTEMS, BOTH ANCIENT & MODERN.

I. *Of that of* Pythagoras.

Fig. I. IN this *Syſtem* the *Terraqueous Globe* is ſeated in the midſt or Centre, about it the *Elementary Region*; next above that the *Moon*, then *Mercury*; next above him *Venus*, the *Sun* as Moderator of all, being placed, as in a Throne in the midſt of the Planets, environed not only by the three foregoing, called the *Inferior*, but by *Mars* likewiſe, *Jupiter* and *Saturn*, called the *Superior Planets*; above *Saturn* is the Sphere of the *Fixed Stars*, called by ſome the *Firmament*.

This was the firſt *Pythagorean Syſtem*, embraced by *Archimedes*, the *Caldeans*, *Ariſtotle*, *Cicero*, *Livy*, *Ptolemy*, *Alphonſus*, *Purbachius*, and the greateſt part of *Aſtronomers*, until the time of *Maginus* and *Clavius*.

II. *Of the* Platonick *and* Porphyrian Syſtems.

Fig. II. CONcerning the Syſtems of *Plato* and *Porphyrius*, there is a wonderful Concordancy, they both agreeing, that the *Earth* with the *Elementary Region* were placed in the Centre; above that the *Moon*; and immediatly above that the *Sun*; above him *Mercury*; then *Venus*; above her *Mars*; then *Jupiter*; laſtly, *Saturn*. In the ſecond place is that of *Porphyrius*, and ſome other *Platoniſts* beforementioned, agreeing in all but this, That whereas *Plato* immediatly above the *Sun* placed *Mercury*, and then *Venus*, *Porphyrius* immediatly above the *Sun* placed *Venus*, and then *Mercury* above her. Of which laſt *Macrobius*, Lib. 1. in *Somn. Scip.* C. 3. gives this accompt. *The late Platoniſts* (ſays he) *repudiated the dimenſions of* Archimedes, *as not obſerving double and triple Intervals: For they held that what was the Diſtance, or Interval from the Earth to the Moon, the ſame was double from the Earth to the Sun; and the Diſtance from the Earth to* Venus, *was triple from that from the Earth to the Sun; and from the Earth to* Mercury, *four times that from the Earth to* Venus; *and how far the Diſtance was from the Earth to* Mercury, *Nine times that was the Diſtance from the Earth to* Mars; *and the Diſtance from the Earth to* Jupiter, *Eight times that from the Earth to* Mars; *and the Diſtance from the Earth to* Saturn, *ſeven and twenty times that from the Earth to* Jupiter. The ſame order is likewiſe aſſerted by the Author of the Book *De Mundo*. Theſe three foregoing Syſtems are compoſed all of *Concentrick Orbs*.

III. *Of the* Ægyptian Syſtem.

Fig. III. THis Syſtem, followed by *Vitruvius*, *Martianus Capella*, *Macrobius*, *Beda*, and *Argol*, is compoſed partly of *Concentrick Orbs*, partly of *Eccentrick*, as may appear by the Scheme; wherein we may perceive the *Earth* ſeated in the midſt of the World, with the *Elements*, and immediatly above them the *Moon*, then the *Sun*; about whom, as their Centre, firſt *Mercury*, then *Venus* are ſuppoſed to move in *Epicycles*; above theſe *Mars*, next *Jupiter*, and then *Saturn*; of this *Syſtem* thus *Macrobius*, in *Somn. Scipion.* Lib. 1. C. 9. *The* Egyptian *ſubtilty is not without reaſon, which is this; the Circle wherein the Sun runs his courſe is ſurrounded with the Circle of* Mercury, *as Inferior, and by that of* Venus, *as Exterior. And hence it is that theſe two Stars, when they are in the*

the upper part of their Orbs or Circles, the Sun is held to be above them. Thoſe therefore who affirm the Spheres of theſe Planets to be under the Sun, are perſuaded to believe it, from this appearance of their Courſe, when they run in the Inferior part of their Orbits, which is more remarkable and more clearly diſcernable, for when they are in the upper part of their Circles they are more concealed; and therefore this perſuaſion of theirs hath prevailed, and accordingly the order of thoſe Planets hath been received almoſt by all Perſons; But more perſpicacious Obſervation diſcovered this better order, and juſtly does he ſtyle it a preſpicacious Obſervation, which ſo many Ages ſince, by the only ſagacity of Wit, ſo clearly diſcovered what we find now really detected by the help of the *Teleſcope*; and therefore *Ricciolus* ſays of this Syſtem, that it is *Pulcherrimum, ac, pro hac quidem parte, Veriſſimum Syſtema.*

IV. *Of the* Copernican Syſtem.

WE come now to the moſt Celebrious, and at this day, moſt generally receiv- Fig. IV.
ed *Mundane Syſtem*, from its reviver, called the *Copernican*, but owing its original to the *Samian* and *Italick School*, as being propoſed and aſſerted in the one by *Philolaus* of *Crotona*, in the other by *Ariſtarchus Samius*, both *Pythagoreans*; whence it is called the ſecond *Pythagorick Syſtem*, as differing from the former before deſcribed; that fixing the *Earth* immovable in the midſt of the World; this, on the contrary, giving to the *Earth*, not only a *diurnal* Motion about its *Axis*, but alſo an *Annual* about the *Sun*, as the Centre of the *Univerſe*: An *Hypotheſis* not unknown to the *Romans*; and therefore *Seneca* in his *Natural Queſt.* Lib. 7. propoſes it as a thing neceſſary to be diſcuſſed; *To know whether the Earth ſtanding ſtill, the Heavens be moved about it; or the Heavens ſtanding ſtill, the Earth be carried round.* And again, *it is a thing worthy Contemplation to be aſſured, ſays he, in what condition we are; whether in a ſeat of all others moſt ſlow, or the moſt ſwift: Whether God turns all things about us, or we our ſelves are turned about.* This *Syſtem*, about two Ages ſince, was reſuſcitated from Oblivion and the Grave, by *Cardinal Caſanus*, but imperfectly, until *Copernicus* came and gave it a perfect conſummation, followed by the greateſt Wits of this and the foregoing Age; to wit, *Georgius Joachimus Rheticus, Mæſtlinus, Rothmanus, Stermius, Kepler, Galilæo, Schickardus, Jordanus Brunus, Cælius Calcagninus, Didacus Aſtunica, Foſcarinus*, and *Gaſſendus*; *Lausbergius* and *Bullialdus*, *Antonius Laurentius Politianus, Wendelinus, Henrigonus, Rinatus Des Cartes*, only differing in this from *Copernicus*, that they allow to the *Fixed Stars* a proper Motion, which *Copernicus* denies. In this *Syſtem* we may perceive the *Sun* placed in the Centre of the *World*; next above him *Mercury*, finiſhing his Courſe in the ſpace of Eighty days, or thereabouts; then *Venus*, making her Revolution in *nine Months* time; above her the *Earth*, with the *Elementary Sphere*, in the *Annual Orb*, which it runs through in 365 days and a half, by a Motion from *Weſt* to *Eaſt*; that is, in the ſame Circle wherein the *Egyptians* and *Ptolemaick Syſtem* place the Sun; beſides which Annual Motion, *Copernicus* aſſigns to the *Earth* a *Diurnal* Revolution, in which it turns about its own Centre and *Axis*, inclined in the plane of the *Ecliptick*, in the ſpace of 24 hours, from *Weſt* to *Eaſt*: The Moon, by a Menſtrual Revolution, being carried about the *Earth*, as in an *Epicycle*; *Mars* running about the *Sun*, as the *Centre* of the, Univerſe in two years time; *Jupiter* above him in *twelve*; and *Saturn* in thirty; the Sphere of the *Fixed Stars* being diſtant by ſo vaſt an Interval from the Sphere of *Saturn*; that the *Annual Orb*, in which the *Earth* moves, appears in reſpect to it, no other than a point.

V. *Of the* Tychonean Syſtem.

THE foregoing *Syſtem*, tho in appearance, of all others, the moſt rationally Fig. V.
grounded, could not yet give ſo general a ſatisfaction to the curious, but that ſome of them conceive it might admit of Alteration, or Emendation. And thence the illuſtrious *Tycho* took occaſion to introduce the following *Syſtem* of his, which is no other than a certain Transfiguration of the *Copernican*, after this manner. Firſt, the *Earth* is ſuppoſed *fixed* without Motion in the Centre of the Univerſe, Concentrick to which is, firſt, the Sphere or Circle of the *Moon's Motion*; next that of the *Sun's Annual Courſe*; then the Sphere of the Fixed Stars. The Sun being placed as

the Centre of the other Planets; in whoſe Orb (as it were two *Epicycles*) are drawn the Circles of *Mercury* and *Venus*. At a greater diſtance is that of *Mars*, interſecting, when in oppoſition to the *Sun*, part of the ſolar Sphere, and therefore being *Achronical*, is nearer to the *Earth* than the *Sun*; above *Mars* is the Sphere of *Jupiter*; above *Jupiter* that of *Saturn*; above that the *Fixed Stars*; ſo that in this *Syſtem* the *Sun* is the *Centre* of *five Planets*; that is, not only of *Mercury* and *Venus*, according to the *Egyptian Syſtem*, but alſo of *Mars*, *Jupiter*, and *Saturn*, according to the *Copernican*; which likewiſe it reſembles, as ſuppoſing fewer Converſions, and rejecting the ſolidity of the *Ptolemaick* or *Purbachian* Orbs. But the *Copernican*, even in the Opinion of *Ricciolus*, ſeems to carry with it a greater Simplicity and Concinnity.

Among the ſeveral *Syſtems* which are grounded upon the *Earth's Immobility*, there are two (beſides the *Tychonick*) which ſeems to *Ricciolus* the moſt probable; the one *Semi-Ptolemaick*; the other *Semi-Tytonick*.

The *form* of the *Firſt* admits the *Centre* of the *Earth*, as the *Term* from whence the *Eccentricity* of the ſeveral *Orbs* are Meaſured; ſuppoſing *Venus*, *&c. Mercury* to be carryed about the *Sun* in *Epicycles*; and the *Eccentricities* of the other *Five Planets*, as alſo their *Epicycles*, not to be always of the ſame quantity; which may be apprehended by the *Figure* of the *Egyptian Syſtem* already deſcribed (Fig. III.) ſuppoſing only, that *Mars* in oppoſition to the *Sun*, be made to approach nearer to the *Earth*, than the *Sun* does.

The *form* of the *Second* is repreſented in the Scheme (Fig. VI.) Wherein the *Sun* is ſuppoſed to be the *Centre* of the *Sphere* of *Mercury*, *Venus*, and *Mars*; and the *Earth* the *Centre* of the Circle of the *Moon's Motion*, and of the *Fixed Stars*; as alſo of thoſe of *Saturn* and *Jupiter*: This *Hypotheſis* owes its Invention to *Ricciolus*, the Reaſons impelling him to embrace it, being theſe following:

Firſt, He obſerved that *Saturn* and *Jupiter* had *Secondary Planets* moving about them; but *Mars*, *Venus*, and *Mercury* none: Whence he conceived it probable, that *Saturn* and *Jupiter*, exerciſed as it were their proper Monarchies in the Heavens; and were no *Attendants* or *Satellites* of the *Sun*; but moving about the *Earth* as their *Centre*. That *Mars* was one of the *Sun's* greateſt and remoteſt *Satellites*, including within its *Sphere* that of the *Earth*; but *Venus* and *Mercury* his nearer *Satellites* or Guards.

Secondly, Becauſe in the variation of the *Eccentricities* of the *Planets*, he obſerved a greater Connexion of *Mars*, *Venus*, and *Mercury* with the *Sun*, than of *Saturn* and *Jupiter*.

Thirdly, Becauſe he conceived *Saturn* and *Jupiter*, being more *ſlow* and *Ponderous Planets*, to have a greater affinity to the *ſlow Sphere* of the *Fixed Stars*, and to reſpect (as do the *Fixed Stars*) the *Earth*, rather than the *Sun*, as the *Centre* of their Motions.

Fourthly, Seeing that *Mars*, *Venus*, and *Mercury*, in their Courſes, enter into the *Solar Heaven*, he conceived it more probable, that thoſe *Planets* ſhould have the *Sun* for the *Centre* of their *Motions*, and not to have any diſtinct *Æthereal Regions* aſſigned them, but to move all in one common *Region*; but *Saturn* and *Jupiter* to have their *Regions* allotted them.

Fifthly, For that it ſeems very probable, that among the *Planets* there ſhould be one *Mean*, or *Middle*, between the *Superior* and the *Inferior*; which, in *his Motion*, ſhould have ſome things *Common* with *Saturn* and *Jupiter*; and ſome with *Venus* and *Mercury*, viz. *Mars*.

VI. *Of the* Hypotheſis *of* Ricciolus.

Fig. VI. THis *Syſtem*, being *Semi-Ptolemaick*, and *Semi-Tychonick*, in it is to be noted, That the intervals of the ſeveral *Orbs* are ſo ordered, that the loweſt of the Concave Heaven of *Saturn* does not touch the upper part of the *Convex* of *Jupiter*; or the loweſt of *Jupiter* the uppermoſt of *Mars*, as in the *Ptolemaick*; but there is a void interval between the *Fixed Stars* and the uppermoſt *Convex* of *Saturn's* Orb, of 9824 Semidiameters of the Earth; and between the Concave of *Saturn* and the *Convex* of *Jupiter* an interval of 10045 of the ſame Semidiameters; and between the Concave of *Jupiter* and *Convex* of *Mars* an Interval of 5310 ſuch Semidiameters; and between the Concave of *Venus* and *Convex* of the *Moon's Orb* 1850 Semidiameters.

Beſides,

Beſides, the thickneſs of the whole Heaven of *Saturn*, computed together with its *Satellites*, 21361 Semidiameters. That of the *Sun*, *Mars*, *Venus*, and *Mercury*, being but as one *Heaven*; is ſuppoſed to contain 19091 of the Earth's Semidiameters.

These three laſt *Syſtems* ſuppoſe the *Heavens*, or the *Æthereal Region*, to be previous, fluid, and of a Thin, Liquid, and Tranſparent Subſtance, like the *Air*, but more Pure, and not conſiſting of *Solid Orbs*, as the *Peripateticks*, and thoſe of the *Ptolemaick* School do affirm.

OF THE FIXED STARS.

THEIR

I. *Number.* II. *Conſtellations.* III. *Figure.* IV. *Magnitudes.* V. *Diſtances.* VI. *Motions.* VII. *Of new Stars.*

HAving largely treated of the ſeveral *Syſtems* of the *World*, Ancient and Modern, I come now to treat of the *Fixed Stars.*

They are ſaid to be *Fixed*, becauſe they always keep (at leaſt ſeemingly) the ſame *invariable diſtance* from one another, and from the *Ecliptick*; as if they were ſo many *Studs* of *Gold* fixed in the *Chryſtal Firmament*, as was the Opinion of *Empedocles* and *Aximenes*. Hence the *Sphere* wherein they are conceived to be ranged, is called *inerrantes*, in regard of the unalterable *Order* obſerved in their *intervals* or *diſtances*. And for this reaſon *Ricciolus* conceives the Multitude of the *Fixed Stars* (as it were an *Army* drawn up in Battel array) might be called the *Militia of Heaven.* Having thus given the Definition of them, I proceed.

I. *Of their* Number.

If we look to thoſe only which are moſt Notable and Viſible, as reduced to the Six vulgar degrees of *Magnitude*, we ſhall find them, according to *Ptolemy*'s computation, to amount to but 1022; yet *Pliny* reckons them to be 1600. But if we reflect upon the abſolute Number of all the Stars in the Firmament, we may conclude them (tho not with *Jordanus Brunus* to be infinite, yet) to be innumerable, at leaſt by Humane Calculation; either as looked on by the bare Eye only, or by the help of a *Teleſcope*; by means of which laſt, *Galilæo* (in his Book intituled *Nuncio Sidereo*) reports, that he diſcovered in the Cloudy *Star* of *Orion*, no leſs then 21 other: In the *Nebulous Star* in *Præſepe* (or Moringer) 36: In the *Aſteriſm* of the *Pleiades* above 40: In the Space between the *Girdle* and *Sword of Orion*, no fewer than 80: And within little more than one degree's ſpace in the *Conſtellation* of *Orion*, above 500 *Stars.* *Reitha* likewiſe (in his *Radio Sydereomyſtic.* Pag. 197.) affirms, that he obſerved within the confines of the whole *Conſtellation of Orion* above 2000 *Stars.*

Some of the *Jewiſh* Doctors reckon not above 12000 *Stars* in all, but thoſe of the *Cabala* no leſs than 29000 *Myriads*; which Number *Schickardus* conceives to be too tranſcendent, and believes, that the whole *Area* of the Heavens would not receive above 26712 *Myriads*, tho they were placed contiguous one to another. But as to this particular, I ſhall conclude with this ſaying of *Scottus*: *That this* Globe *of the* Earth *which we Inhabit, being but as one* Point *in reſpect of the large* Expanſion *of the* Heavens, *is not to us, the Inhabitants thereof, fully diſcoverable: It is therefore intolerable arrogance, to imagin that our* Sight (*tho never ſo ſtrengthened by help of* Teleſcopes) *can diſcover all the* Stars *in the* Cœleſtial Expanſion, *and extream folly to go about to range them within the limits of any definite* Number; *that being the Work of God alone*, who numbers the Multitude of the Stars, and calleth them by their Names: *To whom be all Praiſe, Honour and Glory.* Amen.

II. *Of the* Conſtellations.

THE Ancient *Aſtronomers* being deſirous to bring the *Stars* into ſuch *Rule* and *Order* as might eaſily be remembred; they have *Reduced* them into certain *Conſtellations*, and attributed to them *Names*, whereby they might be known.

A *Conſtellation* (otherwiſe called an *Aſteriſm*) is a certain *Number of Stars*, gathered together into one *Form*, repreſenting ſome Living Creature, or other thing, by which they are particularly known: Of theſe *Conſtellations*, the Number of them drawn now upon our Lateſt and Beſt *Celeſtial Globes* and *Hemiſpheres*, are 64, of which the *Zodiack* contains 12. The *Northern Conſtellations* 23. The *Southern Conſtellations* 29, which make 64. To which may be added a *Star* of the *Second Magnitude*, taken notice of by my Honoured Friend Sir *Charles Scarborough*; who, with ſome other of his *Mathematical acquaintance*, being together, (upon that Evening on which preceding day, the *Serene Majeſty* of King *Charles* II. arrived at *London*, after his long abſence,) obſerved a *Star* to appear in very great Splendor, which *Star* lying *informis* between *Urſa Major*, and *Coma Berenices*, he ſaid to the reſt of his company, that that *Star* deſerved a *Royal Epithet*, which he termed *Cor Caroli*, upon that day it appearing ſo tranſcendently bright, which *Star* included within a *Hart*, having an *Imperial Crown* over it; is ſo engraven upon our Late and Beſt *Globes* and *Hemiſpheres* and named *Cor Caroli*.

Having thus given you an account of the *Number* of the *Stars*, and of the ſeveral *Conſtellations* into which they are reduced, I ſhall here lay before you,

A Table of the Number *of the* Stars *contained in the ſeveral* Signs *and* Conſtellations; *obſerved by the bare Eye, without the help of a* Teleſcope; *according to*

	The Order of the *Signs* and *Conſtellations.*	*Ptolomy* *Alphonſus* *Copernicus* *Clavius.*	*Grembergerus*, from *Clavius*, *Tycho* and *Pifferus.*	*Bayerus* from divers others.	*Keplerus* and *Bulialdus.*
The Six Northern Signs.	*Aries*	18	22	29	23
	Taurus	44	52	48	52
	Gemini	25	30	31	30
	Cancer	13	16	35	17
	Leo	35	40	43	40
	Virgo	32	41	42	43
The Six Southern Signs	*Libra*	17	20	15	20
	Scorpio	24	24	29	27
	Sagittarius	31	31	31	31
	Capricorn	28	28	29	28
	Aquarius	45	45	41	45
	Piſces	34	40	37	42
The Northern Conſtellations.	*Urſa Minor*	7	21	8	20
	Urſa Major	35	57	32	56
	Draco	31	30	33	32
	Cepheus	13	13	17	12
	Bootes	23	29	34	29
	Corona	8	8	20	8
	Hercules	28	31	48	31
	Lyra	10	11	13	11
	Cygnus	19	20	35	28
	Caſſiopea	13	26	25	45
	Perſeus	29	35	38	44
	Auriga	14	27	32	27
	Ophiuchus	29	24	30	56
	Serpens	18	13	37	26
	Sagitta	5	8	8	8

Aquila

The Northern Constellations	*Aquila*	15	12	32	12
	Antinous	—	7	—	7
	Delphin	10	10	10	10
	Equiculus	4	4	4	4
	Pegasus	20	23	20	24
	Andromeda	23	26	26	26
	Triangulum	4	4	5	4
	Coma Berenices	0	14		15
The Southern Constellations	*Cetus*	22	25	27	25
	Orion	38	62	49	62
	Eridanus	34	39	42	39
	Lepus	12	13	13	13
	Canis Major	29	29	19	29
	Canis Minor	2	5	8	5
	Argo Navis	45	50	63	53
	Centaurus	37	37	40	37
	Lupus	19	19	20	19
	Hydra	27	34	29	33
	Crater	7	8	11	8
	Corvus	7	7	7	7
	Ara	7	7	8	7
	Corona Austr.	13	13	13	13
	Piscis Notius	18	17	12	17
	Grus	—	—	13	13
	Phœnix	—	—	14	15
	Indus	—	—	12	12
	Pavo	—	—	16	23
	Apus	—	—	12	11
	Apis	—	—	4	4
	Camæleon	—	—	8	10
	Triangulum Au.	—	—	5	5
	Pisces Volens	—	—	7	7
	Dorado	—	—	7	7
	Toucan	—	—	8	8
	Hydrus	—	—	15	21

The Total Number of the Fixed Stars.

		Ptolomy	*Griemberg*	*Bayer*	*Kepler*
Magnitude	I	15	15	17	15
	II	45	55	63	58
	III	208	201	196	218
	IV	474	456	415	494
	V	217	304	348	354
	VI	49	186	341	340
Obscure and Nebulous		14	8	3	13
Sporides, or Inform	In the *Zodiack*			45	
	In the *Northern*			200	
	In the *Southern*			81	
In all		1022	1225	1709	1392

III. *Of their Figures.*

AS to their *Figure*, it is apparently *Round* or *Spherical*, and ſo maintained to be by the *Stoicks*, and with them *Manilius*: Yet *Plato* gives us the different Opinions of ſome of the *Ancients*. As,

Cleanthes held them to be *Pyramidal*, or *Pointed*.

Anaximenes, conceived them to be like *Studs* or *Nails* fixed in the *Chryſtalline Firmament*.

Others imagined them to be *Fiery*, or *Lucid Plates*, or flat *Pictures*, not of any thickneſs or profundity.

Scheriedrus, and *Antonius Maria de Reitha*, will have them to be of divers *Figures* or *Faces*, of a *Polyangular* ſhape; and ſuch the larger ſort of *Teleſcopes* do repreſent them; or, as *Kepler* (in his *Epit. Copernic.* pag. 498.) deſcribes them, like ſo many *Lucid Points* or *Sparks*, caſting forth their *Rays of Light* every way.

So that we are to apprehend their *Figure* to be only *Phyſically Spherical*, and not *Mathematically* ſuch; for in the firſt acceptation, they may be ſaid to be *Round Bodies*, however according to the later, their *Superficies* may be found to be *uneven*, and to conſiſt of many *Sides* and *Angles*.

IV. *Of their Magnitudes.*

IT will not be amiſs in the firſt place, before we undertake to ſay any thing as to the *Magnitudes* of the *Stars*, ingeniouſly to confeſs with *Schickardus*, That *Veras illorum Magnitudines verè ignoramus*.

But to ſatisfie the *Reader*, I ſhall in the following Five *Tables* give him ſome Account thereof, according to divers eminent *Aſtronomers*. Of which Tables, the

I. Shews how many Minutes and Seconds their Apparent Diameters contain.

II. How many Diameters of the Earth their true Diameters contain.

III. What the Solidities of their Bodies are compared with that of the Earth.

IV. From the Obſervations of *Ricciolus*, ſhewing as well their Apparent as their True Magnitudes.

V. Of the true Magnitude of one of the greateſt, and one of the leaſt *Fixed Stars*, according to the diſtance in the *Copernicon Hypotheſis*, *&c.*

In which Tables you may obſerve a great Diſparity; which ariſes, partly from the various diſtances aſſigned to them by ſeveral Authors from the Earth the ſuppoſed *Mundane Centre*: Partly from the divers Eſtimates of their Apparent Diameters made by the bare Eye, by *Tycho*, and others more Ancient; and by *Teleſcopes* by our *Modern Aſtronomers*.

I. *A* TABLE *of the apparent* Diameters *of the* Fixed Stars *in Minutes and Seconds.*

Their Magnitudes.	I. Magnit.		II. Magnit.		III. Magnit.		IV. Magnit.		V. Magnit.		VI. Magnit.	
According to	*Min.*	*Se.*	*Min.*	*Se.*	*Min.*	*Se.*	*Min.*	*Se.*	*Min.*	*Se.*	*Min.*	*Se.*
Maginus	10	00	5	30	4	00	3	00	2	00	1	00
Tycho, Lausbergius, Blanc.	2	00	1	30	1	05	0	45	0	30	0	20
Lausbergius	1	00	0	40	0	30	0	20	0	10	0	05
Hortenſius	0	08	0	06	0	05	0	04	0	03	0	02
Kepler	3	00	2	06								

II. *A* TABLE *of the true* Diameters *of the* Fixed Stars, *and how many* Diameters *of the Earth each contains: In* Diameters, *and* 1000 *parts of a* Diameter.

Their Magnitudes, According to	I.	II.	III.	IV.	V.	VI.
Maurolicus and *Clavius*	4 . 750	4 . 222	4 . 166	3 . 800	3 . 289	2 . 625
Fernelius	4 . 500	4 . 500	4 . 177	3 . 750	3 . 250	2 . 583
Tycho	4 . 1.500	3 . 055	2 . 200	1 . 528	1 . 204	0 . 400
Lausbergius	4 . 071	2 . 713	2 . 036	1 . 358	6 . 776	3 . 388

III. *A*

III. *A* TABLE *of the* Solidity *of the* Fixed Stars *to the Globe of the Earth.*

Their Magnitudes, According to	I.	II.	III.	IV.	V.	VI.
Alfraganus	100 1	90 1	72 1	54 1	36 1	18 1
Fernel. Maurolicus, Clavius	107 1	90 1	72 1	54 1	36 1	18 1
Tycho, Boyer, Blanc.	68 1	28.5 1	11 1	4.5 1	1.05 1	0.33 1

IV. *A* TABLE, *from the Observations of* Ricciolus, *shewing the true and apparent* Magnitudes *of* Stars *of several Magnitudes, deduced from their forementioned supposed distances from the Earth.*

Degrees of Magnitude.	The Names of the Stars of several degrees of Magnit.	The apparent Diameter.	The greatest Distance accord. to *Ricciolus* of 210000 Semidiam. of the Earth.		The least Distance accord. to *Ricciolus* of 100000 Semidiameters of the Earth.		The greatest Distance according to the *Ptolemaic* 40000 Semidiameters of the Earth.		The greatest Distance according to *Tycho* 14000 Semidiameters.	
			The true Distance contains the Earth's Diameter.	*The Body conta. the Eart. Body.*	*The true Diamet. contains the Earth's Diameter.*	*The Body contains the Eart. Body.*	*The true Diamet. contains the Ear. Diameter.*	*The Body contains the Earth's Body.*	*The true Diameter contains the Earth's Diameter.*	*The Body contains the Earth's Body.*
		Se. Thi.	*Di. Par.*		*Di. Par.*		*Di. Par.*		*Di. Par.*	
1	*Sirius*	18 00	17 . 45	5355	8 . 70	815	3 . 50	42 . 00	0 . 61	0 . 200
1	*Arcturus*	16 42	16 . 66	3932	8 . 00	512	3 . 20	32 . 00	0 . 46	0 . 143
1	*Aldebaran*	15 24	14 . 20	2810	7 . 60	402	3 . 00	27 . 00	0 . 57	0 . 125
1	*Spica Vir.*	15 05	13 . 11	2660	7 . 33	374	2 . 70	18 . 00	0 . 48	0 . 100
1	*Regulus*	14 05	13 . 60	2202	6 . 80	249	2 . 60	16 . 00	0 . 43	0 . 833
1	*Rigil*	13 40	13 . 00	2197	6 . 40	220	2 . 50	15 . 50	0 . 41	0 . 625
2	*Procyon*	12 20	12 . 00	1728	6 . 00	216	2 . 30	12 . 00	0 . 39	0 . 588
2	*Aquila*	11 00	10 . 60	1120	5 . 28	137	2 . 10	8 . 40	0 . 37	0 . 200
2	*Polaris*	7 53	7 . 60	402	3 . 90	62	1 . 55	3 . 25	0 . 27	0 . 300
3	*Algol*	7 03	6 . 60	260	3 . 40	34	1 . 35	2 . 20	0 . 24	0 . 014
4	*Propus*	6 10	6 . 00	216	2 . 90	26	1 . 15	1 . 50	0 . 20	0 . 008
5	*Pleias*	4 01	4 . 80	92	2 . 70	18	1 . 08	1 . 25	0 . 18	0 . 006
6	*Alcor*	4 00	4 . 00	64	2 . 15	9	0 . 80	0 . 70	0 . 15	0 . 003

But the *Astronomers* of the *Copernican Opinion* maintain the *Magnitudes* of the *Fixed Stars* to be far *greater* than the foregoing *Tables* shew them to be: I shall therefore annex this following *Table.*

V. *A*

V. *A* TABLE *shewing the true* Magnitude *of the* Fixed Stars; *that is of* SYRIUS, *one of the* Greateſt; *and* ALCOR *one of the* Leaſt: *Supposing the apparent Diameter of* SIRIUS *to be* 18 *Seconds, and* ALCOR 4 *Seconds, according to the Diſtance of the* Copernican Hypotheſis, *maintaining the* Parallax *of the* Fixed Stars, *made by the* Earth's Motion, *not to exceed* 10 *Seconds, and imagining the* Diameter *of the Annual* Orb *to be ſuch as upon thoſe Principles it is ſtated to be.*

According to	The Diſtances to be aſſerted, in Semidiamet. of the EARTH.	The true Magnitude of SIRIUS. The Diamet. of Sirius, contains Diameters of the Earth.	The true Magnitude of SIRIUS. *The Body of* Sirius, *contains the* EARTH's *Body.*	The true Magnitude of ALCOR. Diameter of Alcor cont. Diameters of the EARTH.	The true Magnitude of ALCOR. *The Body of* ALCOR *contains the Earth's Body.*
Copernicus	47,439,800	4170	71,677,713,000	1992	4,378,454,048
Herigonius	49,502,400	4350	82,312,875,000	2068	8,844,058,432
Galilæus	49,832,416	4380	82,427,672,000	2092	9,155,362,688
Bulialdus	60,227,920	5300	,148,877,000,000	2530	15,941,277,000
Lausbergius	61,616,122	5424	,159,371,956,024	2588	17,333,761,472
Keplerus	142,746,428	12550	1,967,656,371,000	6000	216,000,000,000
Vendelinus	604,589,312	53200	15,056,882,800,000	25380	1,767,384,872,000

These Magnitudes may ſeem exorbitant to ſome Perſons, but in the Judgment of the intelligent *Schikardus*, *Eorum Speculationes qui Cœlum ſtellatum longiſſimè à nobis removent, & conſequenter Aſtra plurimum amplificant, veritati ſunt propriores quoniam minora neutiquam admittit conceſſa Orbis annui Parallaxis.* And *Ricciolus* treating upon this Subject, thought fit, in the Front of his Diſcourſe, to prefix this *Theorem*, as a moſt certain truth, that, *Parallaxis & Diſtantia Fixarum non poteſt certâ & evidenti Obſervatione humanitùs comprehendi, i.e.* To make any true and evident Obſervations of the true Parallaxes and Diſtances of the *Fixed Stars*, is a Work beyond humane comprehenſion.

V. *Of the Diſtance of the* Fixed Stars *from the Earth (or rather, the Sun.*

THis is a Queſtion of that difficulty, that *Pliny* pronounced the Inveſtigation thereof to no leſs than a piece of Madneſs: And therefore *Ricciolus* treating of this Subject (in his *Almagiſt. Nov.* l. 6. c. 7.) thought fit in the beginning of his Diſcourſe to prefix this *Theorem*, as a moſt certain Truth, *viz.* That *Parallaxis & Diſtantia Fixarum non poteſt certâ & evidenti Obſervatione humanitùs comprehendi.*

For indeed we know not whether the *Stars* are all in the ſame *Spherical Superficies* equally diſtant from the *Centre* of the *World*, or ſome higher, ſome lower; and this was the Opinion of the Ancient Stoick Philoſophers; and alſo of *Manilius*, who, ſpeaking of ſome *Stars* in the Head of *Orion* appearing more obſcure than the reſt, gives the reaſon of that *Phænomenon* to be

Non quod clara minus, ſed quod magis alta recedunt.
In his advanced Head three others ſhine
Deep in the Skies immerſt; nor yet leſs bright,
Though ſuch they ſeem, 'cauſe more remov'd from ſight.

And this *Hypotheſis* is ſo ſeemingly rational, that *Tycho*, *Galileus*, and *Kepler*, have embraced the ſame; and therefore we may rationally ſuppoſe, that their *Diſtances* are as divers as thoſe of the *Planets.*

Ricciolus (in his *Almagiſt. Nov.* l. 6. c. 7.) reckons up five manner of ways to attain (in ſome probability) to the knowledg of their incomprehenſible *Diſtances.*

Firſt, From the Suppoſition of their leaſt ſenſible *Parallax.*

Secondly, From the Proportion of their *Periodical Motion*, and Diſtance of ſome of the more certainly known *Cœleſtial Lights.*

Thirdly, From the Difference of *Refractions* of the *Sun*, *Moon*, and other of the *Planets*, and that of the *Fixed Stars.*

Fourthly,

Fourthly, From the computation of the bare *Diſtance* of *Saturn* from the *Earth*, without any regard to his *Shadow*.

Fifthly, From the *Diſtance* of *Saturn* joyned with the length of his Shadow; which the forecited *Ricciolus* ſuppoſes the moſt probable, which he thus exemplifies.

The *Shadow* of *Saturn* is ſuppoſed to extend to 118680 *Semidiameters* of the *Earth*, which joyned with the greateſt diſtance of *Saturn*, being accounted 90155 *Semidiameters* of the *Earth*: The diſtance of the *Fixed Stars* can be computed no leſs than 208835 of thoſe *Semidiameters*; that they may be clearly exempt from the reach of *Saturn* and his *Satellites* Shadow, will be found to be 210000 *Semidiameters* of the *Earth*. But for farther ſatisfaction in this particular, view the following *Tables*.

A TABLE *of the Diſtance of the* Fixed Stars *from the Earth, (or rather the* Sun.)

According to the following Authors.	Semidiam. of the Earth. Leaſt Diſtance.	Greateſt Diſtance.	Horizontal Parallax of the Fixed Stars. Sec.	Thir.
Albategnus, Junctinus	19000	Uncertain	10	58
Alfraganus, Barocius	20220	40440	10	14
Maurolycus	20077	20086	10	16
Fernelius, Clavius	22612	45225	9	08
Maginus,	20110	40220	10	15
Tycho	13000	14000	15	06
Marius Bettinus	2290		90	00
Ant. Mar. Rheita.	20000000		00	1
Ricciolus, according to his Fourth Method	100000	Uncertain		
To his Fifth Method	210900		2	0
			1 *fere*	

The *Parallexes* placed in the laſt Column anſwer to the *Leaſt Diſtances*; for according to their *Greateſt Diſtances* aſſigned by *Alfraganus, Maginus, Fernelius*, and *Clavius*, 5 or 6 *Seconds* would be ſufficient.

A TABLE *of the Diſtance of the* Fixed Stars *aſſerted by the Followers of* Copernicus: *in*

AUTHORS.	Semidiam. of the EARTH.	The greateſt Parallax of the *Fixed Stars* made from the Annual Motion of the Earth. The half from the Semidiamiter. Min.	Sec.	The whole from the Diameter of the Annual Orb. Min.	Sec.
Copernicus	Indefinite	*	*	*	*
Galilæus	13.046.400	0	20	0	40
Kepler formerly	34977.067	0	09	0	17
Kepler afterward	60.000.000	0	12	0	24
Lausbergius formerly	10.312.227	0	30	1	00
Lausbergius in latter days	41.958.000	0	07	0	15
Hortenſius	10.312.227	0	30	1	00
Herigon	1.440.000	3	0	6	00

A TABLE *of the* Diſtance *of the* Fixed Stars, *ſuppoſing the* Earth's Annual Motion *about the Sun, and the* Copernican *Diſtance of the* Earth *from the* Sun.

According to the Calculation of	Semidiameters of the Earth.	Total Parallax of the Fixed Stars. Min.	Sec.
Tycho and *Maginus*	7 . 850 . 000	1	00
Longomontanus	7 . 906 . 818	1	00
Scheinerus	13 . 133 . 376	0	40
	10 : 320 . 000	0	48

A TABLE *of the* Distance *of the* Fixed Stars *from the* Earth, *supposing the* Earth's Motion; *and the* greatest Parallax *of the* Fixed Stars, *to be Ten Seconds*; *and the* Distance *of the* Sun *and* Earth, *as this Table exhibits.*

According to these AUTHORS.	Distance of the Sun and Earth in Semidiameters of the Earth.	Distance of the Fixed Stars in Semidiameters of the Earth.
Copernicus	1150	47 . 439 . 800
Mæstlinus and *Galilæus*	1208	49 . 832 . 416
Kepler	3469	142 . 746 . 428
Lansbergius	1498.5	61 . 616 . 122
Bullialdus	1460	60 . 227 . 920
Herigonius	1200	49 . 502 . 400
Vendelinus	14656	604 . 589 . 312

VI. *Of the proper* Motion *of the* Fixed Stars.

THE *Motion* of the *Fixed Stars* is two-fold: The First is their *Circumrotation* about their own *Centres*, termed *Motus Virtiginis*; in which they are carried about with extraordinary celerity. Their Second is their *Motion* of *Revolution* from *West* to *East*, in which they are observed to move but very slowly: And concerning this, I shall here insert the Three following *Conclusions* of *Ricciolus* in his *Astronom. Reformat.* pag. 259.

First, That the *Motion* of the *Fixed Stars* is *Equal* and *Uniform.*

Secondly, That their *Annual Motion* is not less than 49 Seconds, nor greater than 51 Seconds.

Thirdly, That it seems most probable, that their *Annual Motion* is 50 Seconds 40 Thirds.

From this supposition of their *Annual Motion* of 50 Seconds 40 Thirds; it follows, that they complete not one degree in the *Ecliptick* sooner than in 71 years, 19 days, and 12 hours, very near: So that they run not the whole *Circle* of 360 degrees, in less time than 25579 years; which is the *Annus Magnus Platonicus*, (though by the Ancients computed to extend to no less than 36000 years;) which 25579 *Sydereal years* are equal to 25580 *Equinoctial years*; which will clearly appear by the following *Table.*

A TABLE

A TABLE of the Motion of the Fixed Stars in Longitude.

Years	Min.	Sec.	Th.
1	0	50	40
2	1	41	20
3	2	22	00
4	3	22	40
5	4	13	20
6	5	04	00
7	5	54	40
8	6	45	20
9	7	36	00
10	8	26	40
11	9	17	20
12	10	8	00
13	10	58	40
14	11	49	20
15	12	40	00
16	13	30	40
17	14	21	20
18	15	12	00
19	16	2	40
20	16	53	20
21	17	44	00
22	18	34	40
23	19	25	20
24	20	16	00
25	21	6	40
26	21	57	20
26	22	48	00
28	23	38	40
29	24	29	20
30	25	20	00
31	26	10	40
32	27	1	20
33	27	52	00
34	28	42	40
35	29	33	20
36	30	24	00
37	31	14	40
38	32	05	20
39	32	56	00
40	33	46	40
41	34	37	20
42	35	28	00
43	36	18	40
44	37	9	20
45	38	0	00
46	38	50	40
47	39	41	20
48	40	32	00
49	41	22	40
50	42	13	20
51	43	4	00
52	43	54	40
53	44	45	20
54	45	36	00
55	46	26	40
56	47	17	20
57	48	8	00
58	48	58	40
59	49	49	20
60	50	40	00

Years.	Deg.	Min.	Sec.	Th.
61	0	51	30	40
62	0	52	21	20
63	0	53	12	00
64	0	54	2	40
65	0	54	53	20
66	0	55	44	00
67	0	56	34	40
68	0	57	25	20
69	0	58	16	00
70	0	59	6	40
71	0	59	57	20
72	1	0	48	00
73	1	1	38	40
74	1	2	29	20
75	1	3	20	00
76	1	4	10	40
77	1	5	1	20
78	1	5	52	00
79	1	6	42	40
80	1	7	33	20
81	1	8	24	00
82	1	9	14	40
83	1	10	5	20
84	1	10	56	00
85	1	11	46	40
86	1	12	37	20
87	1	13	28	00
88	1	14	18	40
89	1	15	9	20
90	1	16	0	00
91	1	16	50	40
92	1	17	41	20
93	1	18	32	00
94	1	19	22	40
95	1	20	13	20
96	1	21	4	00
97	1	21	54	40
98	1	22	45	20
99	1	23	36	00
100	1	24	26	40
200	2	48	53	20
300	4	13	20	00
400	5	37	46	40
500	7	2	13	20
600	8	26	40	00
700	9	51	6	40
800	11	15	33	20
900	12	40	00	00
1000	14	4	26	40
2000	28	8	53	20
3000	42	13	20	00
4000	56	17	46	40
5000	70	22	13	20
6000	84	26	40	00
7000	98	31	6	40
8000	112	35	33	20
9000	126	46	00	00
10000	140	44	26	40
20000	281	28	13	20
25579	360	00	00	00

A TABLE *of the* Right Aſcenſion *and* Declination *of* 100 *Eminent* Fixed Stars, *calculated for the Year of* Chriſt 1700 *compleated, according to* Ricciolus, *in his* Aſtron. Reſtorat.

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diffe. of Aſcenſ. in 100 y.		Declin. in 100 years.	
	De.	Mi.	Se.	De.	Mi.	Se.		Deg.	M.	M.	
Head of *Andromeda.*	358	14	08	27	27	06	B	1	17	34	A
Girdle of *Andromeda.*	13	11	20	34	2	49	B	1	23	33	A
South Foot of *Andromeda.*	26	21	51	40	52	30	B	1	29	30	A
Fomalhant of ♒	340	11	0	31	8	10	A	1	25	31	S
Right Shoulder of ♒	327	56	55	1	43	44	A	1	20	29	S
Left Shoulder of ♒	318	55	54	6	48	46	A	1	21	26	S
Left Hand of ♒	307	45	54	10	33	44	A	1	26	19	S
Bright * in the *Eagle.*	294	2	47	8	6	32	B	1	27	13	A
Firſt in ♈ Horn.	24	17	2	17	48	24	B	1	23	31	A
Second in ♈ Horn.	24	30	3	19	19	24	B	1	22	31	A
Bright one in *Aries.*	27	35	58	22	1	30	B	1	25	30	A
Goat of *Auriga.*	73	35	56	45	40	0	B	1	49	10	A
Right Shoulder of *Auriga.*	84	29	42	44	51	30	B	1	58	$4\frac{5}{6}$	A
Arcturus in *Bootes.*	210	33	2	20	48	2	B	1	11	$29\frac{1}{2}$	S
Left Shoulder of *Bootes.*	215	2	33	39	35	12	B	1	2	27	S
Præſepe in ♋.	125	46	2	20	43	4	B	1	28	19	S
Northern *Aſinego* in ♋.	126	26	0	22	3	0	B	1	30	20	S
Southern *Aſinego* in ♋.	126	54	3	19	15	0	B	1	27	20	S
Great Dog *Sirius.*	97	57	6	16	18	6	A	1	7	4	A
Little Dog *Procyon.*	110	54	33	5	59	12	B	1	20	12	S
Upper Horn of ♍.	300	24	34	13	22	6	A	1	25	16	S
Lower Horn of ♍.	301	07	29	15	38	2	A	1	27	17	S
Firſt in the Tail of ♍.	320	56	29	17	54	21	A	1	26	26	S
Second in the Tail of ♍.	322	43	30	17	22	22	A	1	25	27	S
Bright * in *Caſſiopea's* Chair.	354	14	33	57	32	16	B	1	15	34	A
Sheder or *Caſſiopea's* Breaſt.	5	56	00	54	55	16	B	1	22	34	A
In the flexure of *Caſſiopea.*	9	45	58	59	7	36	B	1	27	34	A
In *Caſſiopea's* Knee.	16	36	00	58	40	22	B	1	35	33	A
Cepheus his Girdle.	321	6	20	69	17	10	B	1	22	26	A
Bright * in the *Whale's* Jaw.	41	38	7	2	53	50	B	1	15	25	A
Northern in *Whale's* Belly.	24	12	0	11	44	50	A	1	15	31	S
Southern in *Whale's* Tail.	7	5	8	19	35	40	A	1	18	34	S
Northern in *Whale's* Tail.	1	4	12	10	24	54	A	1	17	34	S
Bright * in North Crown.	230	39	0	27	45	20	B	1	5	21	S
In the Beak of the Swan.	289	39	48	27	22	40	B	1	1	11	A
In the Swan's Breaſt.	302	55	52	39	20	5	B	0	$53\frac{1}{2}$	18	A
In the Swan's Tail.	307	47	17	44	14	52	B	0	$51\frac{1}{2}$	$20\frac{1}{2}$	A
Upper Wing of the Swan.	293	56	2	44	26	21	B	0	48	14	A
Lower Wing of the Swan.	308	29	10	32	51	24	B	0	0	21	A
Bright * in the Dragon.	267	25	20	51	35	2	B	0	35	2	S
In the Head of *Caſtor.*	108	50	46	32	30	26	B	1	44	11	S
In the Head of *Pollux.*	111	43	36	28	43	2	B	1	34	12	S
Bright * in Foot of *Gemini.*	95	3	32	16	37	32	B	1	28	2	S

Hercules

Names of the STARS.	Right Ascension for the Year 1700.			Declination for the Year 1700.				Diffe. of Ascens. in 100y.		Declin. in 100 year.	
	De.	Mi.	Se.	De.	Mi.	Se.		Deg.	M.	M.	
Hercules Head.	255	21	37	14	46	48	B	1	8	8	S
His Right Shoulder.	244	19	35	22	11	40	B	1	5	15	S
His Left Shoulder.	255	31	33	25	15	48	B	0	52	8	S
Hydræ's Heart.	138	12	22	7	21	30	A	1	15	25	A
Lion's Heart *Regulus*.	148	4	15	13	25	16	B	1	22½	28½	S
Lyon's Tail.	175	25	34	16	14	4	B	1	19	34	S
Bright * in *Juba Leonis*.	150	48	47	21	21	0	B	1	25½	29	S
Bright * in *Lumbis Leonis*.	164	32	20	22	7	44	B	1	27	34	S
Uppermost in the Neck.	149	58	52	24	53	54	B	1	28	29	S
Lowest in the Neck.	147	47	52	18	13	33	B	1	28	28	S
Thigh of the Hare.	78	51	30	20	59	00	A	1	5	7	S
Northern Scale of ♎.	225	15	26	8	14	46	A	1	21½	24	A
Southern Scale of ♎.	218	38	12	14	45	18	A	1	25	27	A
Bright Star in ♎.	276	39	32	38	32	16	B	0	50	4	A
Ophiucus Head.	260	15	38	12	49	22	B	1	11	7	S
Ophiucus Left Hand.	239	47	37	2	52	2	A	1	23	18	A
Ophiucus Right Knee.	252	39	40	9	53	30		0	50	10	A
Ophiucus Left Knee.	245	11	37	15	16	30		1	23	15	A
Ophiucus Right Shoulder.	162	8	38	4	44	40	B	1	13	5	S
Uppermost in *Orion*'s Head.	79	41	10	9	34	38	B	1	22	7	A
Orion's Right Shoulder.	84	43	4	7	18	20		1	22	4	A
Orion's Left Shoulder.	97	16	40	6	3	2		1	19	8	A
Orion's Foot, Rigel.	75	2	50	8	33	42	A	1	15½	9½	S
In *Orion*'s Belt, the 1	79	9	48	0	32	50		1	17	7	S
In *Orion*'s Belt, the 2	80	12	54	1	25	46		1	17	6	S
In *Orion*'s Belt, the 3	81	18	25	2	8	20		1	16	5	S
Pegasus Mouth.	322	27	36	8	32	14	B	1	18	26	A
Sa'd Alpharas in the Leg.	342	20	30	26	28	38	B	1	12	32	A
Mareab in the joyning of Wing.	342	28	10	13	35	58	B	1	15	32	A
Tip of *Pegasus* Wing.	359	27	25	13	32	56		1	16	34	A
Pegasus Side.	45	32	18	48	44	54		1	28	12	A
Bas Al Gol of *Perseus*.	42	12	42	39	46	30		1	37	25	A
Hindermost in the Head of the Southern Fish.	345	24	5	1	40	2		1	17	33	A
In the Knot in the Line of ♓	26	38	5	1	19	0		1	18	30	A
Bright * in the Head of ♐.	283	1	5	21	22	48	A	1	31	8	S
Antares, *Scorpion*'s Heart.	242	47	28	25	39	54		1	32	16	A
Northern Front of *Scorpius*.	236	58	15	18	53	36		1	28	19	A
Middlemost.	236	14	34	21	41	40		1	30	20	A
Southern Front of *Scorpius*.	235	18	0	25	9	54		1	37½	21	A
Bright * in the Serpents Neck.	232	24	0	7	24	36	B	1	15	21	S
Aldebaran, the *Bulls-Eye*.	64	41	35	15	52	10	B	1	26½	15	A
In the North Horn of ♉.	76	51	18	28	19	2		1	37	8	A
In the South Horn of ♉.	79	55	20	20	55	58		1	31	7	A
North Eye of ♉.	62	43	36	18	30	28		1	24	17	A
Lowest of the *Hyades*.	60	39	35	14	55	38		1	25	17	A
Bright * of the *Pleiades*.	52	27	35	23	9	24		1	29	21	A
Spica Virginis.	197	22	55	9	33	30		1	19½	32½	A
Girdle of *Virgo*.	190	10	22	5	2	54	B	1	18	34	S
Vindemiatrix in ♍.	191	52	20	12	34	58		1	17	43	S

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diffe. of Aſcenſ. in 100y.		Declin. in 100 year.	
	De.	Min.	Se.	D.	Mi.	Se.		De.	M.	Min.	
Bright Star in the Shoulder of the Great Bear.	161	17	5	63	22	2	B	1	41	32	S
Bright * in the Bear's Side.	160	52	20	57	59	2		1	37	32	S
The Bright * in the hindermoſt Thigh.	174	23	34	55	23	42		1	23	34	S
On the Back near the Tail.	180	8	2	58	41	42		1	20	34	S
The First in the Tail.	190	7	56	57	36	58		1	9	33	S
The Second in the Tail.	197	55	2	56	30	52		1	3	32	S
The Laſt in the Tail.	203	53	50	50	50	56	B	1	2	31	S
Laſt * in the Tail of the Leſſer Bear, Pole Star.	9	52	10	87	42	51	B	3	10	34⅘	A
The Bright * in the Shoulder, *Cynoſura.*	222	39	20	75	37	30		1	15	2½	A
The Right Aſcenſion of this * ſtill decreaſes.											

VII. *Of New* Stars.

THey are called *New Stars*, in regard they are now and then Viſible, and at other times diſappear, as that in *Caſſiopea*, diſcovered by *Tycho*, and three others in the ſame *Conſtellation*, lately obſerved by *Caſſini*, one in *Serpentarius*; two in *Eridianus*, one in the *Whale's Neck*; one in the Northern part of *Andromeda's Girdle*; one in the *Breaſt* of the *Swan*, and another in its *Beak*. The *Nature*, *Subſtance* and *Generation* of which *Stars*, being variouſly diſcourſed of by *Philoſophers* and *Aſtronomers*, I ſhall paſs by, only I ſhall here give you the Opinion of *Ricciolus* (in *Alm. Nov. Tom.* 2. l. 8. c. 17.) where he ſaith, *That the Globes of theſe kind of* Stars *have but one ſide of them ſhine, the other being altogether obſcure, and that they move, and have a Circumrotation about their own proper* Axis, *now turning their bright ſide towards us, at other times their Dark.* The Periods of their ſeveral *Motions* are not yet determined; but by Obſervations hitherto made, it appears, that that in the *Whale's Neck* makes its Revolution in *Eleven Months*, That in the *Swan's Breaſt* is ſuppoſed not to finiſh her Revolution ſooner than in 14 *years*. That in the *Beak of the Swan*, to finiſh hers in about *Ten Months*. The Periods of the others Revolutions being not as yet diſcovered.

A Table *of the* Right Aſcenſion *and* Declination *of* 100 *Eminent* Fixed Stars, *calculated for the Year of Chriſt* 1700 *compleated*; *according to* Ricciolus, *in* Aſtronom. Reſtorat. *And of their difference of* Right Aſcenſion *and* Declination *in* 100 *Years.*

Names of the STARS.	Right Aſcenſion for the Year 1700.			Declination for the Year 1700.				Diffe. of Aſcenſ. in 100y.		Declin. in 100 year.	
	De.	Min.	Se.	D.	Mi.	Se.		De.	m.	Min.	
Head of *Andromeda.*	358	14	8	27	27	06	B	1	17	34	A
Girdle of *Andromeda.*	13	11	20	34	2	49	B	1	23	33	A
Southern Foot of *Andromeda.*	26	21	51	40	52	30	B	1	29	30	A
Fomalhant of ♒.	340	11	0	31	8	10	A	1	25	31	S
Right Shoulder of ♒.	327	56	55	1	44	44	A	1	20	29	S
Left Shoulder of ♒.	318	55	54	6	48	46	A	1	21	26	S
Left Hand of ♒.	307	45	54	10	33	44	A	1	26	19	S
Bright * in the Eagle.	294	2	47	8	0	32	B	1	27	13	A

First

Names of the STARS.	Right Aſcenſion for the year 1700.			Declination for the year 1700.				Diff. of Aſſenſ. in 100.y.		Declin. in 100. years.	
	De.	Mi.	Se.	De.	Mi.	Se.		Deg.	M.	M.	
Firſt in Horn of ♈.	24	17	2	17	48	24	B	1	23	31	A
Second in ♈ Horn.	24	30	3	16	19	24	B	1	22	31	A
Bright one in *Aries*.	27	35	58	22	1	30	B	1	25	30	A
Goat of *Auriga*.	73	35	56	45	40	00	B	1	49	10	A
Right Shoulder of *Auriga*.	84	29	42	44	51	30	B	1	58	4½	A
Arcturus in *Bootes*.	210	33	2	20	48	2	B	1	11	26½	S
Left Shoulder of *Bootes*.	215	2	33	39	35	12	A	1	2	27	S
Præſepe in *Cancer*.	125	46	2	20	43	4	B	1	28	19	S
Northern *Aſinego* in ♋.	126	26	0	22	3	0	B	1	30	20	S
Southern *Aſinego* in ♋.	126	54	3	19	15	0	B	1	27	20	S
Great Dog *Cyrius*.	97	57	6	16	18	6	A	1	7	4	A
Little Dog *Procyon*.	110	54	33	5	59	12	B	1	20	12	S
Upper Horn of ♍.	300	24	34	13	22	6	A	1	25	16	S
Lower Horn of ♍.	301	7	29	15	38	2	A	1	27	17	S
Firſt in the Tail of ♍.	320	56	29	17	54	21	A	1	26	26	S
Second in the Tail of ♍.	322	43	40	17	22	22	A	1	25	27	S
Bright one in *Caſſiopea*'s Chair.	358	14	33	57	32	16	B	1	15	34	A
Scheder. or the Breaſt of *Caſſiopea*.	5	56	0	54	55	16	B	1	22	34	A
In the flexure of *Caſſiopea*.	9	45	58	59	7	36	B	1	27	34	A
In *Caſſiopea*'s Knee.	16	36	0	58	40	22	B	1	35	33	A
Cephus his Girdle.	321	6	20	69	17	10	B	1	22	26	A
Bright one in the *Whales* Jaw.	41	38	7	2	50	50	B	1	15	25	A
Northern in the *Whale's* Belly.	24	12	0	11	44	50	A	1	15	31	S
Southern in the *Whale*'s Belly.	7	5	8	19	35	40	A	1	18	34	S
Northern in the *Whale*'s Tail.	1	4	12	10	24	54	A	1	17	34	S
Bright one in the North. Crown.	230	39	0	27	45	20	B	1	5	21	S
In the Beak of the Swan.	289	39	48	27	22	40	B	1	1	11	A
In the Swan's Breaſt.	302	55	52	39	20	5	B	0	53½	18	A
In the Swan's Tail.	307	47	17	44	14	51	B	0	51½	20½	A
Upper Wing of the Swan.	293	56	2	44	26	21	B	0	48	14	A
Lower Wing of the Swan.	308	29	10	32	51	24	B	0	0	21	A
Bright one of the *Dragon*.	267	25	20	51	35	2	B	0	35	2	S
In the Head of *Caſtor*.	108	50	46	32	30	26	B	1	44	11	S
In the Head of *Pollux*.	111	43	36	28	43	2	B	1	34	12	S
Bright one in the Twins Foot.	95	3	32	16	37	32	B	1	28	2	S
Hercules Head.	255	21	37	14	46	48	B	1	8	8	S
Hercules Right Shoulder.	244	19	35	22	11	40	B	1	5	15	S
Hercules Left Shoulder.	255	31	33	25	15	48	B	0	52	8	S
Heart of *Hydra*.	138	12	22	7	21	30	A	1	15	25	A
Lion's Heart *Regulus*.	148	4	15	13	25	16	B	1	22½	28½	S
Lion's Tail.	175	25	34	16	14	4	B	1	19	24	S
Bright one in *Juba Leonis*.	150	48	47	21	21	0	B	1	25½	29	S
Bright one in *Lumbis Leonis*.	164	32	20	22	7	44	B	1	27	34	S
Uppermoſt in the Neck.	149	58	52	24	53	54	B	1	28	29	S
Loweſt in the Neck.	147	47	52	18	13	33	B	1	28	28	S
Thigh of the Hare.	78	51	30	20	59	0	A	1	5	7	S
Northern Scale of *Libra*.	225	15	26	8	14	46	A	1	21½	24	A
Southern Scale of *Libra*.	218	38	12	14	45	18	A	1	25	27	A
Bright one in *Lyra*.	276	39	32	38	32	16	B	0	50	4	A

Names of the STARS.	Right Ascension for the Year 1700. De.	Mi.	Se.	Declination for the Year 1700. De.	Mi.	Se.		Diff. of Ascens. in 100 y. Deg.	M.	Declin. in 100. years. M.	
Head of *Ophiucus.*	260	15	38	12	49	22	B	1	11	7	S
Left Hand of *Ophiucus.*	239	47	37	2	52	2	A	1	23	18	A
Right Knee of *Ophiucus.*	252	39	40	9	53	30	A	0	50	10	A
Left Knee of *Ophiucus.*	245	11	37	15	16	30	A	1	23	15	A
Right Shoulder of *Ophiucus.*	162	8	38	4	44	40	B	1	13	5	S
Uppermoſt in the Head of *Orion.*	79	41	10	9	34	38	B	1	22	7	A
Right Shoulder of *Orion.*	84	43	4	7	18	20	B	1	22	4	A
Left Shoulder of *Orion.*	97	16	40	6	3	2	B	1	19	8	A
Foot of *Orion Rigel.*	75	2	50	8	33	42	A	1	15½	9½	S
1 In the Belt of *Orion.*	79	9	48	0	32	50	A	1	17½	7	S
2 In the Belt of *Orion.*	80	12	54	1	25	46	A	1	17	6	S
3 In the Belt of *Orion.*	81	18	25	2	8	20	A	1	16	5	S
Mouth of *Pegaſus.*	322	27	36	8	32	14	B	1	18	26	A
Sa'd Alpharas in the Leg.	342	20	36	26	28	38	B	1	12	32	A
Marcab in the joining of the Wing.	342	28	10	13	35	58	B	1	15	32	A
End of *Pegaſus* Wing.	359	27	25	13	32	56	B	1	16	34	A
Bright one in the Side of *Perſeus.*	45	32	18	48	44	54	B	1	28	12	A
Ras Al Gol of *Perſeus.*	42	12	42	39	46	30	B	1	37	25	A
The hindermoſt in the Head of the Southern Fiſh.	345	24	5	1	40	2	B	1	17	33	A
In the Knot in the Line of ♓.	26	38	5	1	19	0	B	1	18	30	A
Bright one in the Head of ♐.	283	1	5	21	22	48	A	1	31	8	S
Antares, Heart of *Scorpius.*	242	47	28	25	39	54	A	1	32	16	A
Northern Front of *Scorpius.*	236	58	15	18	53	36	A	1	28	16	A
Middlemoſt *Scorpius.*	236	14	34	21	41	40	A	1	30	20	A
Southern Front of *Scorpius.*	235	18	0	25	9	54	A	1	37½	21	A
Brighteſt in the Neck of *Scorpius.*	232	24	0	7	24	36	B	1	15	21	S
Aldebaran, or Southern Eye of ♉.	64	41	35	15	52	10	B	1	26½	15	A
In the Northern Horn of ♉.	76	51	18	28	19	2	B	1	37	8	A
Southern Horn of ♉.	79	55	20	20	55	58	B	1	31	7	A
Northern Eye of ♉.	62	43	36	18	30	28	B	1	24	17	A
Loweſt of the *Hyades* ♉.	60	39	35	14	55	38	B	1	25	17	A
Bright one of the *Pleyades* ♉.	52	27	35	23	9	24	B	1	29	21	A
Spica Virginis.	197	22	55	9	33	30	B	1	19½	32½	A
Girdle of *Virgo.*	190	10	22	5	2	54	B	1	18	34	S
Vindemiatrix in *Virgo.*	191	52	20	12	34	58	B	1	17	33	S
The Bright one in the Shoulder of the Greater Bear.	161	17	5	63	22	2	B	1	41	32	S
The Bright one in its Side.	160	52	20	57	59	2	B	1	37	32	S
The Bright one in the hindermoſt Thigh.	174	23	34	55	23	42	B	1	23	34	S
On the Back near the Tail.	180	8	2	58	41	42	B	1	20	34	S
The First in the Tail.	190	7	56	57	36	58	B	1	9	33	S
The Second in the Tail.	197	55	2	56	30	52	B	1	3	32	S
The Laſt in the Tail.	203	53	50	50	50	56	B	1	2	31	S
The laſt in the Tail of the leſſer Bear, now the Pole Star.	9	52	10	87	42	51	B	3	10	34½	A
The Bright one in the Shoulder, heretofore called *Cynoſura.*	222	39	20	75	37	30	B	1	15	2½	A

A Table

A TABLE *ſhewing the Right Aſcenſion, and Diſtance from the South Pole of* 37 *eminent Fixed Stars in the Southern Hemiſphere, with the Difference of their Right Aſcenſion, and of their Diſtance from the Pole in* 100 *years.*

The Names of the STARS.	Right Aſcenſion		Diſtance from the South Pole			Differ. of R. Aſcen. 100 year.			Diffe. of Di. à Pole		Mag.
	de.	min.	de.	min.		de.	min.		min.		
The *Star* next the leſſer Cloud.	1	30	10	56		1	10		20		3
The Head of the *Phœnix.*	2	28	45	55	S	1	15		33		2
Achernar.	21	15	31	15		0	56		31		1
The Head of *Hydra.*	26	58	26	48	S	0	46	S	29	S	3
The *Northern* bright *Star* of the *Dove.*	82	4	S 55	47		0	54		4	S	2/3
The *Southern* of them.	84	58	S 54	8	S	0	52	S	2	S	2/3
Canobus.	94	13	37	34		0	33	1/4	2	S	1
That between the Great Dog's Thighs.	101	32	S 61	32		0	58	S	7		2/3
That in the Belly of the Dog.	103	52	64	11	S	1	1		8	S	2/3
In the Tail of the Dog.	107	52	61	23		0	59		10	S	2/3
The bright one in the Maſt of the Ship.	118	3	50	58		0	52	S	16		2
The bright one in the Deck of the Ship.	119	51	S 43	41		0	46	S	16	S	2
The bright one under the Keel.	123	52	S 31	36	S	0	31	S	19		2
The *Southern* of the two that follow the Section of the Keel.	128	50	S 36	32		0	41	S	21		2
The *Southern* in the Section of the Keel.	137	0	32	8		0	40		24	S	2
The Root of the Royal Oak.	137	5	21	40	S	0	19		24	S	2
The *Northern* in the Section of the Keel.	137	54	S 36	25	S	0	46		25		2/3
The preceding Arm of the Croſs.	179	39	S 33	6		1	16	S	33		3
In the Foot of the Croſs.	182	20	28	45	S	1	19	S	33	S	2
The Head of the Croſs.	183	27	S 34	45	S	1	20		33		2
The bright one in the Loins of the Centaur.	186	1	S 42	52	S	1	20	S	33		2/3
The following Arm of the Croſs.	187	24	32	10		1	24		33		2
The preceding Shoulder of the Centaur.	195	40	S 55	3		1	23		32		3
In the left Knee of the Centaur.	205	25	S 31	16	S	1	39	S	30		2
That near the *Pole* in the Tail of the Bird of Paradiſe.	205	21	8	19		3	2		29	S	4
The following Shoulder of the Centaur.	206	58	S 55	18	S	1	27		29	S	3
The Right Foot of the Centaur.	214	32	30	35	S	1	49	S	27		1
In the Point of the *Southern* Triangle.	243	42	21	43	S	2	32	S	14		2/3
The Sting of the *Scorpion*'s Tail.	257	52	53	13		1	41		6	S	2/3
In the fifth Joint of the *Scorpion*'s Tail.	258	26	47	16		1	46	S	6		2/3
In the *South* end of *Sagittary*'s Bow.	270	39	S 55	33	S	1	39		1		2
In the left Shoulder of *Sagittary.*	278	47	S 63	20		1	33		5	S	3
In the Eye of the *Peacock.*	299	56	32	22		2	1		17		2
The *Crain*'s Head.	323	33	S 51	13		1	32		27		3
The *Crain*'s Wing.	326	51	41	33		1	36	S	28		2
In the *Crain*'s Rump.	335	40	41	27	S	1	31	S	30	S	2
Fomalhaut.	339	54	S 58	41	S	1	23		31	S	1

If the *Right Aſcenſion* be leſs than 90 deg. or greater than 270 deg. the Diſtances of theſe *Stars* from the *South Pole,* are to be encreaſed for years to come, and diminiſhed for years paſt: The contrary in the remaining Semicircle of *Right Aſcenſion.*

Of

OF THE PLANETS, And of the SATELITES About some of them.

I. *Of the* SUN.

THE most *Glorious Luminary* the *Sun*, presents his self to our view in the first place, wherein we will consider,

1. *Its Nature and Substance.*

With *Anaxagoras*, *Democratus*, &c. of old; and with *Kepler*, *Scheinerus*, *Bullialdus*, *Kircher*, *Ricciolus*, and others of late: It cannot with fairer probability, be conjectured to be other than a formal *Fiery Body*, consisting of true and proper *Elementary Fire*, partly *Liquid*, partly *Solid*. The *Liquid* being an *Ocean of Light*, and moving with *Fiery Billows*, and flaming *Ebullitions*; as hath been by the abovesaid Authors observed by help of large *Telescopes*. The *Solid* parts being like the *Land* in this our *Terraqueous Globe*; divided into *Continents*, *Islands*, *Mountains*, *Rocks*, *&c.* that thereby the vehement Motion of the *Solar Ocean* might be restrained, repelled, dissipated and broken; to the end it might more effectually communicate its *Pansspermatick Virtue* to all those *Bodies*, to which it is to afford *Light* and *Influence*. It is also probable, that the *Solid* parts of the *Sun*, within which the *Fluid Fire* is contained, as *Fire* in a *Fornace*; are thoroughly ignified in the same manner as the *Bricks* in the *Roof* and *Sides* of a *Fornace* are, made red hot, and look of the same *Colour* with the Fiery Mass of melted Matter within them: But this (with some other the like probable Conjectures) will be more clearly represented in the annexed Scheme, taken from the joint *Observations* of *Scheinerus* and *Kircherus*.

In which Figure the Letters F G represent the *Axis* of the Globe of the *Sun*.
D E the *Sun*'s Æquator.
B F C The North Part of the *Sun*.
H G I The South Part of the *Sun*.
B C H I The Torrid Part of the *Sun*.
A The Seat of Light.
L M N O, *&c.* Evaporations from the Spots.

By these *Observations* lately made by the *Telescope*, it is manifest, that the *Sun* is not (as *Aristotle* and his followers fansie) of an unalterable Substance, but subject to divers vicissitudes and alterations, as not only the Generation and Production, but the Dissolution and Corruption of several strange *Phænomena* in the Body thereof (unknown to the Ancients) clearly demonstrate: Of which the most eminent are the *Maculæ*, *& Faculæ Solares*.

2. *Of the Spots*, &c.

The *Maculæ* or Spots are certain Cloudy Obscurities, appearing upon the Disque of the *Sun*, supposed to be a fuliginous obscure Matter or Vapour, sometimes closely compacted into one, sometimes dissipated into several places, issuing from its fervent fiery Body, by force of its extream Heat. Whether these *Maculæ* be in the *Sun* it self, or distant from it, is not certainly determined: Yet if you consider their first *Appearance* or *Birth*, their *Disappearance* or *Decay*, their *Shape* or *Figure*, *Increase*, *Diminution*, *Separation*, *Conjunction*, *Motion*, and other like accidents; it is credible, and (by what Observation makes out) most probable, that they are in the very *Body* of the *Sun*, or not far from the Superficies thereof; for that, when they are observed near the Bor-

der

der or Edge of the *Sun*'s Diſque, going off from its viſible *Hemiſphere*; and turning about to the other, they cannot be perceived to be raiſed above the Superficies of the *Sun*.

3. *Of their Shapes or Figures.*

Their *Shapes* and *Figures* are of unequal form and greatneſs, and moſt irregular; which argues them not to be *Stars* or *Planets* moving about the *Body* of the *Sun*, as ſome of the Ancients have ſuppoſed. Some of theſe are more durable and laſting than others; and thoſe are conceived to be the *Solid* parts of the *Sun*, appearing variouſly figured, and of different *Magnitudes*, by reaſon of the virtiginous Motion of the *Sun* about its own *Axis*, repreſenting them to our ſight, diverſly ſituated; which diverſity of their appearance, may alſo ariſe from the manner of the *Sun*'s *Circumgyration*, which is ſuch, that its *Axis* does not always keep the ſame Inclination to the Plain of the *Ecliptick*.

4. *Of the* Faculæ, *or Lights in the* Sun.

They are conceived to be *Fiery Globes*, burſting out of the *Solar Mountains*, and ſhining amidſt the Cloudy Vapors: Which by reaſon of the innate fervor of the *Sun*'s Globe, boil up like *Mountains of Light*, ſcattering and diſperſing the *darker Spots*, and diſcovering, as it were, a *Fiery Ocean* of unuſual *Splendor*; *Scheinerus* defines them to be ſmall *Plats* or *Spaces* in the *Sun* brighter than the reſt of his Body. And *Galilæo*, thus, *In the Face of the* Sun (ſaith he) *there appear certain Marks brighter than the reſt, in which is obſerved the ſame Motion as in the Spots, which that they are inherent in the very Body of the* Sun, *cannot be doubted, in regard it is not credible, there can be any other Subſtance beſides brighter than that of the* Sun.

5. *Of the* Sun's Magnitude *and* Diſtance *from the Earth.*

The true *Magnitude* of the *Sun* (ſays *Ricciolus*) is to be had from its true *Semidiameter*; for that being doubled gives its true *Diameter*; from whence his other kinds of *Magnitudes* are derived: Of which take the following Table.

A TABLE *of the true* Magnitude *of the* Sun, *compared with the* Earth.

The Sun's	True Diame. Contains	Its Circumference Contains	The *Area* of its greateſt Circle Contains	Its Convex Superficies Contains	Its Solidity Contains
According to the following Authors.	Simple Diame. of the Earth.	Simple Diameters of the Earth	Square Diameters of the Earth	Square Diameters of the Earth	The Solidity of the Earth.
Ptolomy, *Morolycus*, *Clavius* & *Barocius*	5 . 50	17 . 28	24 . 00	134 . 00	166 . 37
Ariſtarchus more than	6 . 33	20 . 14	30 . 66	127 . 00	254 . 00
Ariſtarchus leſs than	7 . 16	22 . 43	38 . 00	155 . 00	368 . 11
Albategnius	3 . 70	18 . 71	26 . 00	108 . 00	186 . 00
Copernicus	5 . 45	16 . 50	22 . 00	91 . 00	161 . 87
Tycho and *Blancanus*	5 . 19	16 . 30	22 . 00	85 . 00	140 . 00
Longomontanus	5 . 81	18 . 66	26 . 00	95 . 00	196 . 00
Keplerus	15 . 00	47 . 10	176 . 00	706 . 00	3375 . 00
Lausbergius	7 . 56	24 . 00	46 . 00	176 . 00	434 . 00
Bullialdus	7 . 00	22 . 30	39 . 00	156 . 00	343 . 00
Wendelinus	64 . 00	200 . 96	3216 . 00	12864 . 00	262144 . 00
Kircherus	5 . 20	16 . 00	21 . 00	83 . 00	140 . 00
Rheita.	10 . 00	31 . 40	78 . 00	314 . 00	1000 . 00
Ricciolus	33 . 83	106 . 15	885 . 00	30056 . 00	3860 . 00

Ricciolus ſaith, that the attaining to the knowledg of the *Sun*'s true diſtance from the *Earth*, is a ſublimity that hath hitherto exceeded the ſubtilty of all Aſtronomers: Yet he propoſeth Three ſeveral ways of finding it out,

First,

First, By means of the *Horizontal* (or any other *Parallax*) of the *Sun*.
Secondly, By the *Eclipses* of the *Sun* and *Moon*.
Thirdly, By the *Moon*'s *Distance* from the *Earth*.

But however this may be found out, this is to be considered, That the Sun is sometimes in *Apogeo*, or its *fartheſt* diſtance from the *Earth*: Sometimes in *Perigeo*, or its *neareſt* diſtance to the *Earth*. By the half of the difference of which two *Diſtances*, the *mean diſtance* is to be determined. But concerning this matter, ſee the following *Table*, ſhewing the ſeveral Opinions, as well of the *Ancient* as *Modern Aſtronomers* concerning the ſame.

A TABLE *of the* Sun's Diſtance *from the Centre of the Earth, in Semidiameters of the ſame.*

According to theſe AUTHORS.	Greateſt Diſtance of the *Sun* in *Apog.*	Mean Diſtance	Leaſt Diſtance in *Perigeo.*
Hipparcus his *Data*	1586.00	1472.00	1357.00
Or	1429.00	1379.00	1231.00
Poſidonius	13141.00	——	——
Ptolemæus and *Maurolicus*	1210.00	1168.00	1126.00
Clavius and *Barocius*	1216.00	1168.00	1126.00
Albategnius and *Alphonſus*	1146.00	1107.00	1068.00
From the *Data* of *Albitegnius*, as *Lansb.*	7936.00	——	——
Alfraganus	1220.00	1215.00	1210.00
Fernelius	1309.00	1256.00	1204.00
Copernicus and *Maginus*	1179.00	1142.00	1105.00
From the *Data* of *Copernicus*, *Lansberg.*	942.00	——	——
Michael Neander	1197.00	1160.00	1122.00
Meſtlinus in { *Max. Excentr.*	1208.50	1160.00	1111.75
Meſtlinus in { *Min. Excentr.*	1197.50	1160.00	1122.75
Jo. Offucius	——	1152.00	——
Tycho and *Blancanus*	1182.00	1150.00	1117.00
Longomontanus	1334.00	1288.00	1242.00
Kepler { in *Ephem.*	1800.00	1768.00	1736.00
Kepler { in *Comment. Martis*	——	——	——
Kepler { in *Stella Nova*	——	1432.00	——
Kepler { in *Epit. Aſtron.*	3469.00	——	——
Kepler { *ex Paralax* in *Tab. Ru.*	3438.00	3381.00	3327.00
Lansberg. in *Min. Excentr.*	1550.86	1498.51	1446.26
Iſmael Bullialdus	1485.88	1460.00	1433.86
Athanaſ. Kircherus	1940.75	190675	1872.66
Anton. Maria de Rheita	2073.00	2000.00	1927.00
Godefridus Vendelinus	14905.00	14656.00	14407.00
Galilæus	——	1208.00	——
Marius Bettinus	——	1145.00	——
Langrenus	——	3420.00	——
Ricciolus	7580.00	7327.00	7074.00
Or	7600.00	7300.00	7000.00

II. *Of the* MOON.

THE *Moon* (being the neareſt *Planet* to our *Terreſtrial Globe*) I come therefore next to treat of; and

1. *Of her* Opacity, *or* Shadineſs.

That ſhe is altogether deprived of any proper or innate Light of her own, is evident by her *Total Eclipſes*, wherein ſhe is wholly deprived of her Luſtre, which, (if ſhe had any of her own) would rather in that greateſt Obſcurity become the more conſpicuous. From hence is inferred, that the *Light* ſhe hath is from the *Sun*: And alſo as ſhe is an *Opacous*, ſo alſo ſhe is a *Denſe Body*, apt both to *receive*, and alſo to *reflect* the *Light* ſhe receiveth from the *Sun*.

2. *Of*

2. *Of her* Aſperity, *or* Unevenneſs.

That the *Moon* is full of *Eminencies* and *Depreſſions*; like the *Hills* and *Valleys* of our *Terreſtrial Globe* hath been the conjecturable Opinion of the *Ancients*; but by us, in this Age, it is manifeſt to our *Sight*, by help of the *Teleſcope*; and aſcertain'd to reaſon, by thoſe leſſer Spots (called new ones) which vary their Situations and Magnitudes, according to the acceſs and receſs of the *Sun*, to which their Site is always oppoſite, evidencing them to be the *Shadows* of the more eminent parts of the *Moon*'s *Globe*, emulating our *Alps*, *Hills*, and *Mountains*; and in many places ſurpaſſing them in height.

3. *Of her* Heterogeneity, *or* Parts.

The *Heterogeneity* of the Parts of the *Moon*, was long ſince believed by the Ancients, but by us, at this day, certainly demonſtrated by the various Reflections of the Light; from whence we may conclude, the *Brighter Parts* of the *Moon* to be the more *Denſe*, *Solid*, and *Opacous Parts*, as our *Earth*; and the *Obſcure Parts* (commonly called the *Spots*) to be as our *Ocean*, *Seas*, *Lakes*, and *Rivers*.

From what hath hitherto been ſaid, we may probably infer, That the *Moon* is compoſed of *ſolid* and *liquid Parts*, as is our *Terraqueous Globe*, which we inhabit. And although moſt *Aſtronomers*, as well Ancient as Modern, conceive the *Moon* to be, as it were, another *Earth*; yet we are not to imagine, that it is compoſed of the ſame *Sand*, *Clay*, *Stones*, *&c.* as this *Terreſtrial Globe* is; or that the *Lunary Seas*, *Lakes*, *Pools*, *&c.* are of the ſame *Water* with our *Seas*, *Lakes*, or *Pools*, but happily of a quite different Matter, and to us incomprehenſible: And therefore, as to its *Nature* and *Subſtance*, we ſhall be ſilent.—— But, ſince mention hath been made of the *Maculæ*, or *Spots* in the Face of the *Moon*; as well ſuch as are obvious to the bare Eye, as thoſe diſcernable by the help of the *Teleſcope*; it will be neceſſary to ſay ſomething concerning *them*.

4. *Of the* Maculæ, *or* Spots *in the* MOON.

The *Maculæ*, or *Spots* in the *Moon* are diſtinguiſhed into two; *viz.* the *Ancient* or *Modern*.

1. *Of the* Ancient Maculæ, *or Spots*.

The *Ancient Maculæ*, are thoſe great and large *Spots*, at all times, diſcernable by the bare Eye, without the help of a *Teleſcope*; reſembling large *Seas*, *Lakes*, *&c.* heretofore known, and taken notice of by the Ancients; of which *Plutarch* hath written a particular Treatiſe, under the Title, *De Facie in Orbe Lunæ*; tranſlated, and Commented upon by *Kepler*.

2. *Of the* New Maculæ, *or Spots*.

The *New Maculæ*, are thoſe *leſſer Spots*, which are not diſcernable by the bare Eye, but require the help of a *Teleſcope*; and theſe are various; differing in *Magnitude*, *Figure*, *Situation*, *Colour*, *&c.* obſerved, and exactly deſcribed by ſundry eminent Modern Aſtronomers: Among whom *Langrenus*, in his *Selenographia*, hath noted them by the Names of ſeveral Perſons eminent for their Dignity or Honor, or for their Skill in the Mathematicks.

Hevelius (as if the *Moon* were another *Earth*) hath deſcribed them by *Geographical Denominations*, transferring to them the Names proper to our Terreſtrial *Continents*, *Promontories*, *Mountains*, *Iſlands*, *Seas*, *Lakes*, *&c.* of which he hath in his Works given us a *Map*, and large *Selenographical Table* of theſe Names, alphabetically diſpoſed, too large for this place. He hath alſo another *Scheme* of the *Full*, and ſeveral *Faces* of the *Moon*, with the *Maculæ*, or *Spots*, as they appear to the bare Eye, but more eminently by the *Teleſcope*. The *Scheme* whereof I have here inſerted.

Grinaldus hath ſignalized theſe *Maculæ* or *Spots*, by the Names or Perſons peculiarly addicted to, or eminent in the Study of *Aſtronomy*, with a *Scheme* thereof; and a large *Nomenclature* of the *Names*; both which I have for brevity omitted, contenting my ſelf with that of *Hevelius*.

The *Schemes* and *Tables* here mentioned, are of great uſe and advantage to all Students in *Aſtronomy*, and neceſſary for making their own *Obſervations*, and for the better underſtanding of thoſe made by others.

5. *Of the* Diſtance, Apparent Diameter, *and* Magnitude *of the* MOON.

Concerning the Motion of the *Moon* (in this place) I ſhall ſay nothing, it being beſides the preſent purpoſe; but of the *Diſtance*, *Apparent Diameter*, and *Magnitude* of the *Moon*, take theſe following *Tables*, according to ſeveral Authors.

A TABLE *of the* Moon's Diſtance *from the Centre of the Earth, in Semidiameters of the ſame; and its Horizontal Parallax.*

According to the following AUTHORS.	*In* Oppoſition, *or* Conjunction.					
	Diſt. from Cent. of the Ear.			*Horizontal Parallax.*		
	Apog. *Sem. M.*	Med. *Sem. M.*	Perig. *Sem. M.*	Apog. *Min. Se.*	Med. *Min. Se.*	Perig. *Min. Se.*
Ptolemæus	64 10	59 0	53 50	53 34	58 16	63 51
Copernicus	65 30	60 19	55 8	52 56	57 00	62 54
Tycho	58 8	56 30	54 52	59 9	60 51	62 39
Longomontanus	57 38	56 00	54 23	59 37	61 26	63 14
Lausbergius	64 10	59 5	54 00	53 34	58 8	63 39
Bulialdus	61 40	59 5	56 30	55 56	58 16	60 52
Ricciolus	64 15	59 00	53 45	53 30	58 16	63 55
Kepler in *Ephimerides*	62 52	59 00	55 8	54 41	58 16	62 20
Kepler in *Epit. Aſtron. &c.* / *Tab. Rudolph.*	59 0	56 28	54 00	58 22	60 53	63 41
Hyparcus, for ſome time	83 0	77 00	71 00	41 30	45 00	48 30
Afterwards	72 30	67 30	62 38	47 30	51 30	55 30
Alphonſus	61 8	57 45	54 23	53 19	59 21	63 17
Argol	60 4	56 00	51 57	56 45	61 17	65 36
Vandelinus	63 56	60 00	56 4	53 46	57 18	61 18
Kircherus	67 50	62 37	57 24			
	In its *Quadratures.*					
Ptolomæus	43 53	38 43	33 33	60 19	89 00	103 00
Copernicus	68 21	60 19	52 17	50 19	57 00	65 48
Tycho	60 36	56 30	52 24	56 44	60 51	65 36
Longomontanus	60 4	56 00	51 57	57 15	61 26	66 9
Lausbergius	66 58	56 5	51 12	51 20	58 8	67 6
Bulialdus	64 15	59 5	53 55	53 30	58 8	63 43
Ricciolus	66 42	59 00	51 20	51 32	58 16	66 56
Kepler in *Ephemeris*	61 36	59 00	56 24	55 26	58 16	60 58
Kepler in *Epit. Aſtron. &c.* / *Tab. Radulph.*	59 00	56 28	54 00	58 22	60 53	63 41

As to the Apparent Diameter of the *Moon*, the following *Table* will ſhew the Opinions of ſeveral *Aſtronomers*, Ancient and Modern, concerning the ſame.

A Syno pſis *of the Apparent Diameters of the* MOON.

AUTHORS.	In Conjunct. Oppoſit. and ſquare Aſpec.	In *Apogæo.* *Min.*	*Sec.*	In *Perigæo.* *Min.*	*Sec.*
Ptolemæus, in	☌ and ☍	31	20	35	20
	□	42	8	55	00
Albategnius, in	☌ and ☍	29	30	35	20
Alphonſus and *Purbaccus*, in	☌ and ☍	29	00	36	8

Copernicus

AUTHORS.	In Conjunct. Oppoſit. and ſquare Aſp.	In *Apogæo.*		In *Perigæo.*	
		Min.	*Sec.*	*Min.*	*Sec.*
Copernicus, *Reinholdus*, and Sometime *Maginus*, in	☌ and ☍	30	00	35	38
	□	28	45	36	44
Maginus, in Supplimen. to *Ephem.* in	☌ and ☍	30	30	34	40
Clavius and *Herigonius*, in	☌ and ☍	30	00	36	8
Tycho, *Origanus*, and *Mulerius*, in	☌	25	36	28	48
	☍	32	00	36	00
	□	32	32	36	00
Longomontanus, in	☌ and ☍	32	0	34	0
Keplerus in *Ephem.* in	☌ and ☍	30	0	34	10
In Epit. & Tab. Rudolph. in	☌ and ☍	30	0	32	44
Lausbergius, in	☌ and ☍	30	0	35	38
Pet. Gaſſendus, in	☌ and ☍	26	36	31	6
Iſma. Bulialdus, in	☌ and ☍	31	10	34	0
	□	29	56	36	4
Got. Vendelinus in	☌ and ☍	28	40	32	42
Sed. in Vertice, in	☌ and ☍	29	10	33	18
Kircherus, in	☌ and ☍	29	22	34	40
Ricciolus, in	☌ and ☍	28	00	33	30
	□	27	00	35	6

From the Diſtances and Apparent Diameters of the *Moon* already given, the *Magnitude* thereof may be deduced; which (without Calculating) is exhibited in the following *Table*; according to the Opinions of ſeveral Authors.

A TABLE *of the* Magnitude *of the* Moon, *compared with that of the* Earth *and* Sun.

According to	The true Diam. of the ☽ and ☉, unto that of the Earth; the Ear. ſuppos'd to be 100.			The Solidity of the *Moon* to the			
				Earth.		*Sun.*	
	Moon.	*Ea.*	*Sun.*	As	To	As	To
Ariſtarchus more than	31.50	100	633	1	16.00	1	5832
Ariſtarchus leſs than	39.80	100	716	1	32.00	1	8000
Ptolemæus	29.33	100	550	1	40.00	1	6648
Copernicus	28.52	100	545	1	42.87	1	6999.92
Mæſtlinus	28.33	100	546	1	44.00	1	7300
Tycho	29.16	100	520	1	41.91	1	5848
Longomontanus	26.84	100	580	1	51. 2	1	10000
Kepler	25.70	100	1500	1	59.00	1	204671
Lausbergius	28.00	100	757	1	45.52	1	19770
Bulialdus	27.91	100	700	1	45.98	1	15765.33
Vendelinus	26.62	100	6400	1	53.00	1	1383362
Kircherus	29.43	100	520	1	40.00	1	5600
Schyrlæus	28.42	100	1000	1	42.96	1	42875
Ricciolus, in *Almag.*	26.20						
Ricciolus, in *Aſtr. Re.*	27.50	100	3383	1	55.00	1	2123000

III. *Of* MERCURY.

1. *Of its* Place.

THis *Planet* is carried in an *Epicycle* about the *Sun*, as the *Centre* of its Motion, running ſometimes above, and ſometimes beneath the *Sun*; and ſometimes higher, and ſometimes lower than *Venus*: For, when *Venus* and *Mercury* are in their *Apogea*, and above the *Sun*, then *Mercury* is below *Venus*; but when in their *Perigea*, and beneath the *Sun*, then *Mercury* is above *Venus*: Alſo, when *Venus* is in her *Apogeum*, and *Mercury* in his *Perigeum*, then is he beneath *Venus*: When *Venus* is in her *Perigeum*, and *Mercury* in his *Apogeum*, then is he above *Venus*; as by the *Tytonick Syſtem* doth plainly appear.

2. *Of its* Figure.

It is (not *Mathematically*, but) *Phyſical Orbicular*, or *Round*, riſing here and there with extracting *Hills* and *Mountains*, as does this our *Earthly Globe*, being alſo like it in *Opacity*, and receiving its illumination from the *Sun*; whence at ſeveral times it is ſeen under ſeveral *Phaſes* and *Appearances*: For, in its greateſt digreſſion from the *Sun*, it appears *Dichotomous*, or divided into two parts in the middle; but when *Retrograde*, and approaching to an Oppoſition with the *Sun*, he appears in a Figure like that of the *New-Moon*. There are likewiſe (by help of good *Teleſcopes*) obſerved in him ſeveral *Spots* ſucceſſively following one another; ſome *Light*, ſome *Duskiſh*; which *Light Spots*, are (by *Kircherus*) conceived to be the ſaid *Terreſtrial* and *Montanous* parts of the *Mercurial Globe*, illuminated by the *Sun*; the *Dusky* ones to be the *Liquid*, or *Watery* parts thereof, which by their ſucceſſive motion likewiſe evince, that he hath a vertiginous *Rotation* about its own *Axis*; which (as *Rheita* affirms) is determined within the ſpace of about ſix hours.

3. *Of his* Diſtance *from the* Earth.

As to his *Diſtance* from the *Earth*, according to *Tycho* he is diſtant

in his	*Greateſt Diſtance*	1660	Semidiameters of the Earth.
	Mean Diſtance	1150	
	Leaſt Diſtance	630	

But according to *Ricciolus*, he is diſtant

in his	*Greateſt Diſtance*	10868	Semidiameters of the Earth.
	Mean Diſtance	8057	
	Leaſt Diſtance	5246	

4. *Of his* Magnitude.

Its *Apparent Diameter*, in its mean diſtance, (according to *Tycho*) is computed to be 2 min. 10 ſeconds; but by *Ricciolus*,

in its	*Greateſt Diſtance* only	9	Seconds.
	Leaſt Diſtance	25	

Its true *Magnitude*, according to *Ricciolus* (ſuppoſing its mean diſtance to be as before expreſſed, and its Diameter to contain one fourth part of the Earth's Diameter, is concluded to be leſs than the Earth's Body by one 256 part, which being reduced to the Meaſures of *Kepler*, its Diameter may be reckoned to contain 377 *Italian Miles*; the *Solidity* of its *Body* about 21253933 cubical *Italian Miles*.

IV. *Of* VENUS.

THE principal Remarks of this *illuſtrious Planet*, made by the Ancients, were theſe following:

Firſt, they obſerved her ſometimes to precede the *Sun*, riſing in the *Morning* before him; ſometimes to follow the *Sun*, ſetting in the *Evening* after him; now to be in *Conjunction* with the *Sun*, at other times receding from him, to the diſtance of 48 degrees.

Secondly, they obſerved her likewiſe to differ in *Magnitude*, appearing ſometimes *greater*, ſometimes *leſs*, and ſometimes in a *mean* or *middle Magnitude*; whence they concluded her to approach ſometimes *nearer* to the *Earth*, and to be in *Perigeo*; at other times to be *farther* from it, and to be in *Apogeo*.

Thirdly,

Thirdly, As to her *Riſing* and *Setting*, they obſerved that ſhe was never to be ſo remote from the *Sun*, as that ſhe ever ſet *Coſmically*, or roſe *Achronically*.

Beſides theſe *Obſervations* of the *Ancients*, the *Modern Aſtronomers*, by help of the *Teleſcope*, have noted ſeveral other ſignal *Phænomena*; as, that ſhe is liable to the ſame variety of *Changes* as the *Moon*, ſometimes almoſt *Full*, at other times *Gibbous*, and now and then *Horned*, as well when ſhe is *Veſpertine* as *Matutine*; manifeſted by the various Obſervations of *Galilæo*, *Schinerus*, *Kepler*, *Gaſſendus*, and *Hevelius*. She is ſometimes likewiſe ſeen by *Day*, at which times her true *Figure* is beſt to be diſcerned. But the rareſt and moſt ſignal *Phænomenon* of *Venus*, is when ſhe hath greateſt *Latitude* near her loweſt *Conjunction* with the *Sun*; for then being ſeen by a good *Teleſcope*, ſhe appears *Horned*, and as big as the *New Moon* to the bare Eye.

Of the *Structure*, *Nature*, and *Subſtance* of this *Planet* from the *Phænomena* already recited; it may be concluded that it is an *Opacous Body*, and hath its *Light* from the *Sun*; that it is *Spherical*, becauſe it is *Spherically illuminated*; that it is of a *ſolid* and *conſiſtent* ſubſtance, in regard it is not diſſipated or diſſolved, (though carried in a rapid Courſe through the liquid *Æther*.) It is of a *rough* and *uneven Superficies*, in regard it is repreſented to be ſuch by the *Teleſcope*, and otherwiſe could not in ſuch abundance reflect the *Sun*'s *Light* upon the *Earth*. That *Bodies* objected to its *Rays* are ſeen to caſt a Shadow. It is made up of *ſolid* and *liquid Matter*, as is our *Terraqueous Globe*; and it is found to have a *Vertiginous Motion* about its own *Axis* and *Centre*, which it compleats in the ſpace of Fourteen Hours.

As to its *Apparent* and *True Diameter*, its *Solidity*, and *Proportion* to the *Body* of the *Earth*, as alſo its *Diſtance* from thence, Authors do extremely vary; wherefore, according to *Ricciolus*,

The *Apparent Diameter* of *Venus* in its *greateſt* Diſtance from the *Earth* is 33 Seconds, 30 Thirds; in its *mean* Diſtance 1 Minute, 44 Seconds, 12 Thirds; and in its *leaſt* Diſtance 4 Minutes, and 8 Seconds.

Its *True Diameter* contains of the Earth's Diameter 1.015.——Its *Circumference* of the Earth's Diameter 3.6.——The *Area* of its greateſt Circle contains Square Diameters of the Earth 1.004.—— Its *Convex Superficies* contains Square Diameters 4.014.——The *Body* of *Venus* contains that of the Earth 1.5.—— Its *Diſtance* from the Earth when *greateſt* 12919.——In its *Mean Diſtance* 7580.——In its *Leaſt Diſtance* 2241 Semidiameters of the Earth.

V. *Of* MARS.

THis *Planet* is obſerved to move in a large *Orbit* about the *Sun*, as the Centre of its Motion; in which *Circle* or *Orbit* he includes the Earth with the *Moon*, *Mercury*, and *Venus*, and takes up a great part of Solar Region; and when in his *Perigeum*, becomes nearer to the *Earth* than the *Sun*; its *Parallax* at that time being *greater* than the *Sun*'s; and by the notable increaſe of his Light, he is ſometimes taken for a *Comet*.

As to his *Figure*, it is *Spherical*, and is obſerved by *Kepler*, when in Square with the *Sun*, to be *Dichotomous*, or biſected; at other times, between its Square and Oppoſition to the *Sun* to be *Gibbous*, and (as *Rheita* affirms) rarely to be ſeen perfectly *Round*. *Hevelius* denies that it can ever be ſeen *Horned*. *Fontanus* repreſents it in a *Round Figure*, with a *Black Spot* in the middle. Father *Sirſalis*, the Jeſuite, at *Naples* beheld him *almoſt Round*, with *two Spots* beneath the middle of his *Globe*. But *Ricciolus* is of opinion, that if he could be ſeen in Conjunction with the *Sun*, we might diſcern him to be *Horned*. From theſe ſeveral *Phaſes* of his *Dichotomy* and *Gibboſity*, Aſtronomers argue his Light to be derived from the *Sun*, as that of the *Moon*.

As to the *Structure* and *Nature* of this *Planet*, it is, as the reſt, *Solid* and *Liquid*; the *Solid* part is ſuppoſed, by *Kircherus*, to conſiſt of a fuliginous Subſtance, like that of Sulphur, Arſnick, or Orpiment, hard and incombuſtible, evaporating malignant and deſtructive Qualities, ſweating out in great plenty a bituminous Matter. Its *Liquid* Parts he imagins to conſiſt of a ſoft and clammy Subſtance, like melted Pitch mix'd with Sulphur. He hath a vertiginous Motion about his own *Centre*, which, according to *Rheita*, is compleated in the ſpace of 45 hours 6 minutes.

Its *Apparent Diameter* (according to *Ricciolus*) being in its *greateſt* Diſtance from the *Earth* 10 Seconds, 6 Thirds: In its *mean* Diſtance 22 Seconds: And in its *leaſt* Diſtance 1 Minute, 32 Seconds.

Its *True Diameter* contains of the *Earth*'s *Diameter* 0.052 parts: The *Solidity* of his Body, the *Earth*'s Body 0.014.

Its *Greateſt Diſtance* from the *Centre* of the *Earth*, is, by *Ricciolus*, computed to be 21005 Semidiamiters of the *Earth*: Its *mean* Diſtance 11000: And its *leaſt* Diſtance 2373 Semidiameters.

VI. *Of* JUPITER.

THis beneficent *Planet* is carried in an *Orbit* above *Mars*; as is manifeſt in the mutual Conjunctions of thoſe *Planets*, at which time he is obſcured and hidden from our ſight by the Interpoſition of *Mars*; as the *Sun* (in an Eclipſe) is by the *Moon*'s Body.

Rheita affirms, that he obſerved *Jupiter* to be inveſted round with a vapid *Atmoſphere*. And *Banditus* reports, that in *Anno* 1643. he obſerved the edges of *Jupiter*'s *Globe* to be very rough and uneven; riſing with Tumors like Hills and Mountains, and diſcovered in his Body two ſmall *Maculæ* or *Spots*, and two great ones like hollow *Caverns*, one *round*, the other *oval*, being in length about the ſeventh part of its *Diameter*. *Hevelius* likewiſe affirms, that the Face of *Jupiter* is variegated with *Maculæ* or *Spots*, in a manner, like that of the *Moon*, and rugged, with uneven Swellings and *Aſperities* as that is. But leaving theſe to farther diſcovery, I ſhall only take notice of the more ordinary *Phænomena* manifeſted by frequent *Obſervations*; and thoſe are,

Firſt, Certain *Faſciæ*, or *Belts*, girdling, as it were, the Diſque of *Jupiter*; whither eminent parts of his Body, like Ridges; or rather hollow Furrows or Trenches, is uncertain. Theſe are ſaid to be firſt diſcovered at *Naples*, by *Jo. Baptiſta Zuppus*, and *Daniel Bartolus*, Jeſuites there; and ſince by *Fontana*, *Grimaldi*, *Ricciolus*, and others. They appear ſometimes Three, ſometimes Two, at other times ſingle, and bordered ſometimes with the two other ſmaller Welts, and at other times but with one of them. They are now and then beheld in the middle of his Diſque, now above it, at other times below it, not always ſtreight, but ſometimes crooked and bending; their Convexities appearing ſometimes turned upwards, ſometimes downwards; which are evident Marks of the vertiginous Motion, or Rotation of that Planet about its *Centre*. Theſe ſometimes ceaſe to appear, and therefore being at ſuch times obſerved by *Gaſſendus*, he ſeems to ſuſpect thoſe *Phænomena*.

Secondly his *Satellites*, being Four Stars, ſo called, moving about the Body of *Jupiter*, as his *Guards*. Theſe were firſt obſerved in *Italy*, by *Galilæo* in the year 1610. In *Germany* by *Simon Marius*, by help of the *Teleſcope*; without which, by reaſon of *Jupiter*'s Splendor, and their ſmall diſtance from him (none of them being above 12 deg. from his Body) they are not to be diſcerned, and therefore altogether unknown to the Ancients. The number of theſe have been queſtioned by ſome Aſtronomers, *Rheita* affirming them to be no leſs than Nine. *Jo. Baptiſta Zuppus* affirms he obſerved in the Year 1644. no leſs than Twelve leſſer Stars moving about *Jupiter*. But the number of Four only is by *Gaſſendus*, *Hevelius*, and *Vincentius Reinerus*; who for Ten Years together moſt diligently obſerved them. Theſe Four *Stars* were by *Galilæus*, their firſt Diſcoverer, called *Sidera Medicæa*. The firſt next the Body of *Jupiter*, he called *Coſmus Minor*; the next (or *penintimus*) *Coſmus Major*; the third (or *penextimus*) *Maria Medicæa*; the fourth, or outermoſt, he called *Katharina Medicæa*. Authors ſince, have given other Names to theſe *Satellites*, which I here omit, reſting ſatisfied with the former, and ſhall exhibit unto you, according to the Obſervations of ſeveral Authors: 1. Their *Digreſſions* from the Body of *Jupiter*: 2. Their *periodical Revolutions*: 3. Their *diurnal* and *horary Motions*.

A TABLE *of the* Satellites *greateſt Digreſſion from the Body of* Jupiter, *in Semidiamiters of the ſame, acoording to ſeveral Authors.*

Satellites.	I.		II.		III.		IV.	
AUTHORS.	*Sem.*	*Mi.*	*Sem.*	*Mi.*	*Sem.*	*Mi.*	*Sem.*	*Mi.*
Galilæus	3	0	5	0	8	0	12	0
Sim. Marius	3	0	5	0	8	0	13	0
Rheita	3	0	4	0	6	0	10	0
Vendelinus	3	0	5	0	8	0	14	0
Hodierna	3	30	5	30	9	0	14	30

A TABLE

A TABLE *of the* Satellites Periodical Revolutions, *in their ſeveral Orbits.*

Of the	According to	Days	Hours	Minutes	Seconds.
I	*Galilæus*	1	18	30	*ferè*
	Marius	1	18	28	30
	Rheita	1	18	30	0
	Hevelius	1	18	28	0
	Hodierna	1	18	28	44
	Vendelinus	1	769 Th	ouſands of a	Day.
II	*Galilæus*	3	13	20	*circiter*
	Marius	3	13	18	0
	Rheita	3	13	20	0
	Hevelius	3	13	18	0
	Hodierna	3	13	18	15
	Vendelinus	3	554 Th	ouſands of a	Day.
III	*Galilæus*	7	4	0	*ferè*
	Marius	7	3	56	34
	Rheita	7	4	0	0
	Hevelius	7	3	57	0
	Hodierna	7	4	1	26
	Vendelinus	7	164 Th	ouſands of a	Day.
IV	*Galilæus*	16	18	0	*ferè*
	Marius	16	18	9	15
	Rheita	16	18	0	0
	Hevelius	16	18	9	0
	Hodierna	16	18	14	33
	Vendelinus	16	756 Th	ouſands of a	Day.

Their Diurnal and Horary Motion in their reſpective Orbits, each being divided into 360 *Degrees.*

Of The	According to	Diurnal De.	Diurnal Min.	Diurnal Se.	Horary. De.	Horary. Min.	Horary. Sec.
I	*Galilæus*				8	29	*Circiter*
	Marius	203	25	00	8	28	30
	Hodierna	203	23	44	8	28	$29\frac{1}{2}$
II	*Galilæus*				4	13	*ferè*
	Marius	101	17	22	4	13	00
	Hodierna	101	17	21	4	13	13
III	*Galilæus*				2	6	*circiter*
	Marius	50	14	57	2	6	30
	Hodierna	50	13	32	2	5	34
IV	*Galilæus*				0	54	30
	Marius	21	29	$3\frac{1}{2}$	0	53	30
	Hodierna	21	28	48	0	53	42

As to the *Greateſt Diſtance* of *Jupiter* from the *Earth*, the ſame is computed by *Riccioius*, in its Greateſt Diſtance to be 47552 *Semidiameters* of the *Earth*. —— In its Mean Diſtance 36500 *Semidiameters*; —— And in its Leaſt Diſtance 26441 *Terreſtrial Semidiameters.*

The *Apparent Diameter* of *Jupiter* when leaſt, in *Apogeo*, 38 Seconds, 18 Thirds. — When in its *Mean* diſtance from the *Earth* 49 Seconds, 46 Thirds. — When Greateſt, in its *Perigeum* one Minute, 8 Seconds, and 46 Thirds.

Its *True Diameter* contains of the *Earth's Diameter* 8.80 —— Its *Circumference* 27.6; of the ſaid *Semidiameters*. —— The *Area* of its *Greateſt Circle*, 64 Square *Diameters* of the *Earth* —— Its *Convex Superficies* 242 of the ſaid Square *Diameters*. —— The *Solidity of its Body*, contains that of the *Earth's* 685 times.

As to the *Structure* of its *Globe*, it may be concluded (with the reſt of the *Planets*) to be conpoſed of *Solid* and *Liquid* Parts; and that it is a *Body*, though Phyſically *Round*, yet full of uneven Aſperities, inveſted with a vapid *Atmoſphere*; carried about its own *Centre* by a *Vertiginous Motion*; finiſhed (according to *Rheita*) in 11 days, 20 hours, 1 Minute and 15 Seconds; and that its *Zones* or *Belts* are *Solid* parts, leſs capable of *Light* than the reſt of its *Solid* or *Liquid* parts are.

VII. *Of* SATURN.

Saturn, the higheſt, and of all the *Primary Planets* hitherto diſcovered, the moſt remote from the *Earth*, runs his courſe above all the reſt; but beneath the *Fixed Stars*; as is collected from his *Parallax* (of all the *Planets*) the *Leaſt*, and in a manner none at all. There are not any either of the *Fixed* or *Erratick Stars*, that afford to the obſerving Eye (by means of the *Teleſcope*) ſuch ſtrange and admirable *Phænomena*. The chief and moſt worthy of Note are theſe three following.

Firſt, That he appears *Girdled* about, with a certain *Paſcia* or *Zone*; or rather encompaſſed about with an *Armilla*, or *Ring* of Light.

Secondly, That he hath ſeveral *Lunulæ*, or leſſer *Planets*, like the *Jovial Satellites*, moving about his Body.

Thirdly, That his *Figure* appears variouſly, and incredibly diverſified, being ſometimes beheld Solitary, in a *Round* form: At other times repreſented with two *Rundles* adhering to each ſide; which again alter their *Figure*, and appear like certain *Anſæ* or *Handles*.

Touching the *Structure*, *Nature*, and *Subſtance* of this *Planet*, it may be probably concluded: Firſt, That it is compoſed *ex Solido & Liquido*, of a Plumbeous or Leaden Temper and Colour; That it is *Spherical*, but withal, full of uneven Aſperities. — Secondly, That it is an *Opacous Body*, and illuminated *ab extra*, and although the *Sun's Light* may approach it, yet is is not ſufficient to give a requiſite Luſtre, to ſo *Great* and ſo *Remote* a *Body*, and therefore muſt needs receive its Light from ſome other Fountain, —— Thirdly, That his *Satellites* have their proper Light, and thereby adminiſter ſupplies to that of their *Prince*. —— Fourthly, That he hath a *Gyration* about its own *Centre* and *Axis*; compleated (according to *Rheita*) in 29 days, 10 hours, 1 Minute and 16 Seconds.

His Diſtance from the *Earth* is by *Aſtronomers* variouſly computed; but (according to *Ricciolus*) in his *Greateſt diſtance*, he is found to be diſtant from the *Centre* of the *Earth* 90155 *Semidiameters* thereof: — In his *Mean Diſtance* 73000 —— And in his *Leaſt Diſtance* 57743 of the like *Semidiameters*.

His *Apparent Diameter*, according to *Ricciolus*; when *leaſt*, in his *Apogeo*, with his *Concom.* is 46 Seconds. —— When in his *Mean Diſtance* from the *Earth* 57 Seconds —— And when *Greateſt*, in his *Perigeo*, one Minute 12 Seconds.

His *True Diameter*, with his *Concometants*, contains of the *Earth's Diameter* 20.166 — His *Body* contains that of the *Earth's Body* 891 times.

VIII. *Of*

VIII. *Of* Comets *and* Fiery Meteors.

Comets and *Fiery Meteors* are generated either in the *Upper*, *Lower* or *Middle Region* of the *Air*, and thoſe that are generated in

The { Upper, Middle, Lower } *Region* are { *Fax*; *Ignis perpendicularis*, *Bolis*, *Capra ſaltans*, *Scintillæ Volantes Trabo*, &c. / *Stella cadens*; *Lanceæ ardens*; *Fulmen*, &c. / *Draco volans*, *Ignis fatuus*, *Ignis lambens*, *Sidus Helenæ*, *Caſtor & Pollux*.

All which ariſe from Vapours and Exhalations, which the *Earth* continually expires and diffuſes round about through its ambient *Atmoſphere*.

1. *Of* Comets.

Concerning the Matter, Place, and efficient Cauſe of *Comets*, *Aſtronomers* and *Philoſophers*, both Ancient and Modern, do much differ in their Opinions: Some will have them not to be any thing real or diſtinct from other pre-exiſtent Cœleſtial Bodies; but rather a mere appearance, made by the Reflection, or Refraction of the *Sun Beams*.

Others are of Opinion that they are *Fiery Meteors*, generated of copious exhalations from the *Earth* and *Sea*, and elevated to the Supreme Region of the *Air*, and hurried about by the ſwift Motion of the *Primum Mobile*, and take *Fire*, and laſt as long as the Sulphury, Unctious, fat Matter, of which they conſiſt, affords them Fuel to burn or ſhine.

Others again, are of Opinion that *Comets*, if not all, yet the moſt part, are created by God of nothing; or, at leaſt, formed of ſuch matter as beſt pleaſeth him; whether *Cœleſtial* or *Elementary*, and of ſuch *Shape* and *Figure*, as may ſerve to terrifie or admoniſh Mankind, and preſignifie Calamities to enſue.

2. *Of the* Tail, Train, *or* Buſh *of* Comets.

Ariſtotle, and ſuch as follow him, aſſert the *Buſh* or *Train* of a *Comet* to be an Exhalation ſet on Fire, and diverſified according to the diverſe diſpoſition of the Matter that feeds its flame.

Apion makes the *Buſh* of a *Comet* to be nothing elſe, but the *Rays* of the *Sun* tranſmitted through the *Semidiaphanous* Head thereof, as it were, through a Globe of Glaſs.

Tycho Brahe conceives it to be nothing elſe but the Beams of the *Sun*, penetrating the Head of the *Comet*, and terminated in ſome matter not altogether perſpicuous, and reflected towards us; for he ſuppoſes the *Æther* not to be thoroughly *Diaphanous*.

Kepler is of Opinion, that the *Tail* of a *Comet* is only enlightned by the *Sun*'s Beams paſſing through the Body of the *Comet*, which he imagins to be purely pellucid, yet denſe withal.

Gallileus is of Opinion, that the *Tail* of a *Comet* ought in its own nature to be ſtreight, as being produced by the *Sun*'s Beams, but appears to us to be crooked when near the *Horizon*, and inclined thereunto, by reaſon of the Refraction of the *Species*; or of the *Viſual Rays* made in the *Spherical Superficies* of the *Air*, which near the *Earth* is filled with groſs Vapours.

III. *Of the Various Forms and Shapes of* Comets *and their* Tails.

From the divers Figures and Appearances of theſe *Trains* or *Buſhes*, *Comets* are diſtinguiſhed into ſeveral Kinds or Species; which are reducible to theſe two *Heads* or *Claſſes*; *viz.*

1. *Criniti, ſeu Comati.* 2. *Barbati.*

To the First *Claſſis*, relate theſe: { 1 *Diſceus*, 2 *Pitheus*, 3 *Hippeus*, 4 *Argyrocomus*, 5 *Hircus* } To the Second, Theſe { 6 *Lampadias*, 7 *Ceratiæ*, 8 *Acontiæ*, 9 *Xiphias*, 10 *Lanchites*, 11 *Vern*, 12 *Tetragonius*.

1. *Discus, sive Disci-formis Cometa:* Is a *Comet* resembling in Shape or Form, a round *Dish* or *Platter*: Among these kind, the Chief of which is called *Rosa, sive Chryseus*, bright Shining, and of a Silver Colour, mixed with Gold or Amber Colour. As *Fig.* 1. Those of this sort, which are not absolutely Round, resemble the Figure of a *Shield*, and are accordingly called *Clypei-formes*. As *Fig.* 2, 3, 4.

2. *Pitheus, sive Doli-formis*, resembles the Form or Shape of a *Tun*; of which there are divers kinds; some of an *Oval* Figure; some like a *Tun* or *Barrel* set an end; some like one inclining, or cut short off; others have a *Train* or *Bush* annexed. As *Fig.* 5, 6, 7.

3. *Hippeus, seu Equinus*, resembles a *Horses Main*, not always of the same Figure; for sometimes it spreads its *Train* from the fore-part or Front; sometimes from the hinder-part; now of an *Oval Figure*; now like a *Rhomboides*, and therefore it is distinguished into *Equinus Barbatus*; *Equinus Angularis, sive Quadrangularis*; and *Equinus Ellipticus*. As *Fig.* 8, 9, 10.

4. *Argyrocomus, seu Argenti Comus:* differing little from that which is called *Rosa: Fig.* 1. But that it is of a *Whiter Colour*; and shines with such a dasling Silver-haired Light, as it can scarce be looked upon.

5. *Hircus*, or the *Goat*, it is invironed with a kind of *Main*, seemingly rough and hairy, by the slender *Fibræ* of its *Beams* or *Rays:* It is sometimes *Round* without any *Train* or *Bush*. As *Fig.* 14, 15, 16.

6. *Lampadias, sive Lampadi-formis*, it is a *Comet* resembling burning *Lamps* or *Torches*, and is of several Shapes; for, sometimes it hath its *Flame* or *Blaze* carried upwards like a *Sword*; sometimes double or triple pointed. As *Fig.* 11, 12, 13.

7. *Ceratiæ*, or the *Horned Comet*, it sometimes appears *Bearded*, sometimes with a *Tail* or *Train*; some have the *Figure* of a *New Moon*: Those that are Tailed, have sometimes a *Crooked Tail* bending upwards; sometimes downwards; others have the Tail of an unequal breadth and thickness every way; some have their *Bush Pointed*; others like a *Horn* or *Trumpet*. As *Fig.* 18, 19, 20, 21.

8. *Acontiæ*, Are *Comets* formed like a *Dart* or *Javelin*, with an oblong, or close compressed Head, and prolix extenuated *Tail* or *Train*. As *Fig.* 22, 23, 24.

9. *Xiphias, sive Ensi-formis*; a *Comet* resembling a *Sword*, the Head being fashioned like to a *Hilt*, the *Tail* being *Long*, *Streight*, and *Pointed*; yet sometimes bending like to a *Cimitar*; when it is of a Lesser, and more contracted form, it resembles a *Knife* or *Dagger*. As Fig. 25, 26, 27, 28.

10. *Lonchites, seu hastæ-formis*; is a *Comet* resembling a *Lance*, its Head being of an *Elliptical* form, its *Stream* of Light or *Tail* being very *Long*, *Thin* and *Pointed*. As Fig. 29, 30, 31.

11. *Vern, seu Pertica*, is a *Comet* almost of the same Species with the former; save that the *Head* is rounder, and its *Train* of Light *Longer*, and Sharper *Pointed*.

12. *Tetragonias, sive Quadratus*, is a *Comet* whose Head is for the most Part *Quadrangular*; it hath a *Long Train*, very *Thick* and *Uniform*, and is not unlike that *Meteor* called *Trabs*, or a *Fiery Beam* or *Columb*; but all these will better appear by the several *Schemes* or *Figures* of them hereunto annexed: And I shall forbear to add those distinctions which some have given them in reference to the *Planets*, they making them some *Solar*, others *Lunar*, *Mercurial*, *Venerial*, *Jovial* and *Saturnine*, nor of their *Magnitudes*, *Duration*, *Motion*, *Prognosticks*, or final *Causes:* But give the Reader an *Historical Abstract* of the *Times* of the several appearances, of such of **These splendid** Ænigma's, **Proposed by God, but never to be resolved by humane Wit,** (as *Ricciolus* ingenuously says of them,) as have appeared in the last *Century* of Years past, *viz.* from the Year 1590. to this time 1690. as followeth.

An Account of such Comets (*or splendid* Ænigma's,) *as have appeared to the World, since* Anno Christi 1590, *to this time.*

1590. On the 20th of *February*, a *Comet* appeared in the *Evening*, lasting till the 5th of *March*, between the Constellations of *Andromeda*, and the *Ram*, and the *Northern Fish*; its place being in the highest *Heavens*, and not nearer to the *Earth* than the *Sun:* It described by its *Motion*, an Arch of a *Great Circle*, and measured in its Course one fourth part of the *Hemisphere:* Its *Head* small, Shining with a Pale and Obscure Light, at the first biggest, but afterwards not exceeding 3 minutes in Diameter, and successively diminishing both in *Magnitude* and *Light:* It cast forth some small *Rays*, both thin and rare; and extended towards that part of the *Heavens* opposite to the *Sun*. As in Figure 42.

On

On the 10th of *July* before *Sun Riſing*, a *Comet* appeared, which laſted till the 21th. of *Auguſt* in the Solſtitial *Colure*, Sign *Cancer*: In its *Motion* it went from the *Tropick* of *Cancer* to the *Arctick Circle*, contrary to the Series of the Signs, *viz.* from *Cancer* to *Gemini* and *Taurus*, and in *Cepheus* it diſappeared. 1593.

A *Comet* appeared on the 9th of *July*, in the *Evening* in the *North*, among the *Stars* of *Urſa Major*; in its *Motion* going forward a little towards the hinder part of the *Great Bear*; ſo that it ran through *Cancer*, *Leo*, *Virgo*, and at laſt became Stationary as to *Longitude*, declining its Courſe to the fourth deg. of *Virgo*: It was of the Colour of *Saturn*, its Tail ſtretching to the part oppoſed to the *Sun*, towards the *Pole* of the *Ecliptick*, yet with ſome little deviation. 1596.

On the 26th of *September* in the *Evening*, about 7 of the Clock, and from thence all night, till the 5th of *November*, a *Comet* appeared under the *Great Bear*, a little higher than that *Star* which is towards the Square, in the 30th deg. of *Leo*, and in 36 deg. of *Northern Latitude*; its *Parallax* found not to exceed 3 min. and conſequently, its place in the higheſt *Heaven*, or *Æther*: It moved in direct *Motion*, from the foremoſt foot of the *Great Bear* under its Belly, paſſing by the midſt of *Bootes*, and ſtruck through the *Serpent*, coming under the Hand of *Orion*, arrived at his foremoſt foot, and ſtayed in his Leg. The Orbit in which it was carried, ſeemed to be a *Great Circle*, at laſt extremely bent towards the *Ecliptick*. On the 30th of *September* its diurnal *Motion* was 13 deg. but both before and after, its *Motion* was ſlower; at length *Retrograde* and *Stationary* as to *Longitude*. Its Head was not of an equal or even roundneſs, but here and there exuberated: Its apparent *Magnitude* greater than any of the *Fixed Stars*, or even that of *Jupiter*: Its Light Weak, Pale, and Wateriſh, like that of the *Moon* when near the Shadow of the *Earth*, towards its end diminiſhing more and more: Its Tail was ſomething Long and Thick, projected with ſome little deviation, againſt that part of the *Heavens* oppoſite to the *Sun*: *Vendelinus* ſaw it like a flaming Lance or Sword, 7 deg. in length. 1607.

The firſt *Comet* appeared upon the 25th of *Auguſt* at 3 of the Clock in the Morning, and laſted till the 25th of *September*; it appeared a little beneath the fore Foot of *Urſa Major*, inclining towards the Head of *Leo*, in the 10th deg. of that Sign, and in 22 deg. of *North Latitude*; on the 2d. of *September* it proceeded in *Motion Retrograde*, in antecedence of the Signs; afterwards one degree in its Courſe more remiſs; it appeared to be hairy; its Light not clear Shining, but Cloudy and duskiſh; its Tail ſhort and broad, ſpreading towards the *Weſt*. 1618.

The ſecond *Comet* appeared two hours before the *Sun's Riſing*, which laſted till the 23 of the ſame Month, between the *Autumnal Section*, and 18th deg. of *Libra*; declining from the *Ecliptick Southward* 15 deg. its *Motion Retrograde*. Its Head was not diſtinctly obſerved by the *Europeans*, by reaſon of its vicinity to the *Sun*, but in *Perſia* its Colour was found to be like the Vapour of Flower of Brimſtone ſet on Fire: Its Tail was like that of the *Eſtridge* bowed, in length 45 deg. In *Perſia* it was obſerved to be like a *Scymiter*, or rather like a Young *Palm Tree*, whoſe top Bows bend but a little. 1618.

On the 22 and 23 of *November*, a third *Comet* was ſeen in the Morning before *Sun Riſing*, which laſted till the 13th of *December*: It took its Riſe from the *Equinoctial Eaſtern* Point; its *Motion* was *Northward*, its Colour was like that of *Venus*, whoſe *Magnitude* it equalled, if not exceeded; it had a long Main or Treſſes. 1618.

On the 24th of *November*, a Fourth *Comet* was ſeen in the Morning before *Sun Riſing*, which laſted 60 days, *viz.* till the 24th of *January* next following: On the 29th of *November*, it was ſeen between the *Scales* of *Libra*, more *Eaſtward* than a Line drawn directly between the two *Scales*, and more approaching to the *Northern Scale*: It's true place being in the *Æther*, for at the firſt it was diſtant from the *Earth* 71 of its *Semidiameters*; at laſt it was higher than the *Sun* it ſelf; for its *Parallax* was found to be leſs than that of the *Sun*. Its *Motion* was to the *North*, with ſome inclination *Weſtward*, for it paſſed by the middle of *Libra*, and by *Bootes*, and when it had advanced ſo far as his Head, it ſhined all Night, then proceeded above his *Wriſt*, and over *Urſa Major*, its *Motion* becoming every day ſlower and ſlower. As to its progreſs in the *Ecliptick*, it run from the end of *Scorpio* to the middle of *Cancer*, above one third part of the *Heavens*; at firſt *Southwards*, at laſt *Northwards*: Its Colour was *Paliſh*, the lower part of its Head was perfectly Round, the upper part whence the Tail iſſued was uneven, and as it were indented; Its Light languid, whitiſh and cloudy, yet ſometimes a little Twinkling: In the midſt of the Head at the firſt, was a little Kernel or *Nucleus*; afterwards ſeparated into three or more, and at length parting into more, 1618.

leſſer and leſſer, was diſſolved; its apparent as well as true *Magnitude* being various and unconſtant: Its Tail towards the Head, was very narrow about the middle, and its extremity pretty large: It was extended with ſome inclination, now to the *South*, now to the *North*, but not preciſely towards the *Sun*: Through its middle, according to *Longitude*, ran a Clear *Line*, like the Pith of a Tree; nor ſeldom the whole Buſh or Train ſeemed to have a kind of *Scintillation* and *Fluctuation*; its Head likewiſe darting forth *Rays*, and ſuddenly withdrawing them: Its length was ſometimes extended to 45, 60, 75, nay to 104 degrees, its breadth to 3 degrees: Its Colour near the Head
Fig.XLV. *Ruddy* and *Firy*; towards the End, *Pale* and *Wateriſh*. Fig. 45.

On the 29th of *November*, a *Comet* appeared in the *Evening* about Eight of the Clock, which laſted but Two days in the *Conſtellation* of *Berenice's Hair*, not ſo far as 5 degrees from the left Leg of *Bootes*, about 10 deg. from *Arcturus*, in the 8th deg. of *Libra*, and in the 26 deg. of *North Latitude*; Its *Motion* was contrary to the ſucceſſion of the Signs *viz.* from the Head of *Bootes* towards the *Ecliptick* and *Spica Virginis*: It was ſomthing, leſs in appearance than *Arcturus*, but ſufficiently Light and Splendid: Its Tail was erected upwards towards the *Zenith*, in length 12 degrees like to a *Broom*, conſpicuous and ſplendid enough, where it iſſued from the Head, towards the Extremity more thin and dilute.

On the 20th of *December*, about 6 of the Clock in the *Evening*, a *Comet* appeared, which laſted till the 10th of *January*, in the 9th deg. of *Gemini*, and 31 deg. of *Southern Latitude*, not far from *Rigel*, in the *Left Foot of Orion*, ſo that by its Situation with the ſaid *Star*, and another above the *Foot of Orion* in *Eridanus*, it made in a manner an *Equilateral Triangle*. Its true place was in the *Æther*, or higheſt *Heaven*, for when it was neareſt the *Earth*, its diſtance then was 110 *Semidiameters* thereof; its *Parallax* at firſt being found to be 31 min. 15 ſec. but about the 12 of *January*, it was diſtant from the *Earth* 22509 of the *Earth's Semidiameters*, its *Parallax* not above 9 ſeconds, and conſequently, it was as high as the Orb of *Jupiter*. Its *Motion* was conſtantly *Retrograde* from *South* to *North*, by the *Hare*, *Foot of Orion*, *Taurus* to the *Pleiades*, and from thence as far as *Perſeus*; not directly, but obliquely from E. S. E. to N. N. W. in a Great *Circle* inclining to the *Ecliptick* and *Æquator*. At firſt, by its diurnal *Motion* compleating 11 deg. 16 min. at laſt 30 min. it ran through in the time of its duration 65 deg. 51 min. paſſing beyond and beſide the Courſe of the *Annual Orb*: At firſt diſtant from the *Earth* 94 *Semidiameters* thereof, at laſt 72. Its Head was Round, and little leſs than the *Moon* at Full; its Light Pale and Dull, like that of the *Moon* o'ercaſt by a thin Cloud; in it were obſerved ſeveral *Nuclei* or *Kernels*, every day varying their Situation; Its apparent *Magnitude* was not always the ſame, being found at firſt to be 30 min. afterwards but 3 min. 30 ſecon. Its Tail at the beginning extended *Eaſtward*, towards the part of *Heaven* oppoſed to the *Sun* (yet with ſome ſmall deviation) as far as to the Handle of *Orion's Sword*, like to a Sharp pointed *Cone*, in length 7 deg. of a Whitiſh, but ſomething obſcure Colour, caſting forth thin ſmall *Hairs* or *Rays*, which about the 7th of *January* it quite loſt: Its Tail ſtill increaſed in
F.XLVII. length, being at firſt only 3440, at laſt 165000 *Germane Miles*. Fig. 47.

On the third of *February*, a *Comet* was ſeen in the Morning 47 min. after 5 of the Clock, which laſted 53 days, it appeared *Eaſtward* beneath the *Dolphin*, between the *Eagles Head*, and that of the *Leſſer Horſe*, in 10 deg. of *Aquarius*, and 22 deg. of *North Latitude*, in the very *Æther* or higheſt Heaven, at firſt 2000, at laſt 9000 *Semidiameters* of the *Earth* diſtant from it, and conſequently (according to the Opinion of *Hevelius*) higher than the *Sun* it ſelf. Its Courſe was from the *Eaſt Weſtward*, by the *Head*, *Neck*, and *nether Wing* of *Aquila*, in a *Line* almoſt Parallel to the *Ecliptick* and *Æquator*, but *Retrograde*, and not in a Great *Circle*: Its Head was Round and of a Yellowiſh Colour, clear and conſpicuous, in the middle whereof, at firſt, was a ruddy *Nucleus* or *Kernel*, equal almoſt to *Jupiter*, encompaſſed with a certain thin and more dilute matter: Its Tail extended above 6 deg. in length towards the *Dolphin*, narrower where it joyned to the Head than in its extremity, and pointing to that part of the *Heaven* oppoſite to the *Sun*, but with ſome kind of deflection.

Upon the 14th of *December*, a *Comet* was ſeen which laſted almoſt 3 Months, at firſt in the Morning about 5 of the Clock, afterwards in the *Evening*, near the *Beak of the Crow*, in 8 deg. of *Libra*, and 22 deg. of *Southern Latitude*, in the higheſt *Æther*, its *Parallax* at the beginning 59 Seconds, in the middle 4 min. and in the end 16 min. ſo that at firſt it was 3500, then 1000, and laſtly 120000 *Semidiameters* of the *Earth*, diſtant from the *Terreſtrial Globe*, and higher than *Mars*: Its *Motion* was *Retrograde* from *Corvus*, by *Hydra*, *Argus*, the *Great Dog*, the *Hare* unto *Aries*; yet was

was not the *Line* of its Courſe carryed altogether under that of a *Great Circle*, it declining notably *Northward*: It ran through more than Five Signs of the *Zodiack*, *viz. Libra*, *Virgo*, *Leo*, *Cancer*, *Gemini*, *Taurus*, even unto *Aries*; and in reſpect of its Orbit, made a Progreſs of 154 degrees. Its Head was very conſpicuous, in Colour ſomewhat Yellow; in the midſt whereof was diſcerned a clear light, ſurrounded with another more obſcure, compoſed of ſundry *corpuſcula*, interſperſed with other more ſubtile matter, of a divers denſity, meeting at firſt with the *Nuclei*, after ſeparating and dividing. As to its apparent *Magnitude*, it exceeded ſix times the *Diameter* of the *Earth*: Its Tail extended 14 deg. in length *South-Weſt*, ſometimes longer, ſometimes ſhorter. On the 8th of *December*, it ſpread its *Rays* upward, in manner of a *Peacock*'s Tail; from which time after, it was directed toward the *Eaſt*. All the time of its duration, extending it as far as the Sign *Gemini*, toward that part of the *Heavens* oppoſed to the *Sun*, yet with ſome deviation, now *Northward*, now *Southward*: Towards its end it was ſometimes quite loſt, and then again recovered.

Upon the 6th of *April*, half an hour after One in the Morning, a *Comet* was ſeen, which laſted 14 days, in the *Breſt of Pegaſus*, in 15 deg. of *Piſces*, and 27 deg. of *Northern Latitude*: Its *Parallax* at the beginning 69 Seconds, which decreaſed ſucceſſively to 41 Seconds: Its diſtance from the *Earth*, at firſt, 3000 *Semidiameters* of the *Earth*; towards the End, 5000 of the ſame *Semidiameters* diſtant from it, and conſequently 60 times higher than the *Moon* at firſt, and at laſt equal to the heighth of the *Sun* it ſelf. Its Progreſs was by *Pegaſus*, under the Head of *Andromeda*, by the *Northern Fiſh*, as far as unto *Aries*; in motion continually direct; at the beginning, it ran through in one day 4 deg. 6 min. at laſt, 2 deg. 23 min. Its Head was Round, of a Yellowiſh Colour; its Light quick and bright, in the middle whereof was a ſingle *Nucleus* of a conſpicuous *Magnitude*, of a Gold Colour, encompaſſede qually about with another thinner kind of Matter; the apparent Diameter of the Head was 6 minutes: Its Tail at the beginning extended almoſt 17 deg. in length, projected between the *Mouth* and the *foremoſt Foot* of *Pegaſus Weſtward*: Where it iſſued from the Head, it was Thick and Lucid, and of the ſame denſe matter with that of the Head; but, towards the End, it was more thin and dilate: It ſtretched ſometimes in length to 25 degrees, ſtreaming towards the part oppoſed to the *Sun*, with ſome ſmall deviation *Southward*.

Upon the 5th and 10th of *March*, about the firſt hour of the Night, after the *Italian* way of account, a *Comet* was obſerved by *Signior Caſſini* at *Bologna*: The Head or Body was not ſeen, being hid under the *Horizon*; but the Tail was of a Stupendous length, being extended (as it appeared at *Lisbon* in *Portugal*) over almoſt the fourth Part of the viſible *Heaven*, from *Weſt* to *Eaſt*, from the *Whale*, through *Eridianus*, to the *Star* which precedes the *Ear of Lepus*, as it was obſerved at *Bologna* by *Caſſini*.

Upon the ſecond day of *March*, a *Comet* firſt appeared, which laſted till the end of *April*, and ſeen both *Morning* and *Evening*, as *Signior Caſſini* hath computed; at the beginning it was between the *Head of Meduſa*, and the *Pleiades*; afterwards, having continued its Courſe towards the *Root of the Southern Horn of Taurus*, and having paſſed the *Ecliptick*, went on above the top of *Orion's Head* to the *Via Lactea*. Its *Motion* falling into a *Line*, little differing from an arch of a *Great Circle*, cutting the *Ecliptick* in 10 deg. 45 min. of *Gemini*; its greateſt *Latitude* in 10 deg. 45 min. of *Piſces*; that is, between 39 and 40 deg. *Northward*; the ſame *Circle* cut the *Æquator* at 101 d. of the *Vernal Section Eaſtward*; and his greateſt *Declination Northward*, was 38 deg. 30 min. He made about 2 deg. 32 min. a day in the *Great Circle* of his apparent *Motion*. Its Head (as it was obſerved with a *Teleſcope* of 17 Foot long) appeared almoſt Round, but well diſtinguiſhed from the miſtineſs, which framed a kind of a *Chevelure* wherewith it was encompaſſed; and even the middle was a little confuſed, and ſeemed to have inequalities, as are ſeen in Clouds: The Tail was almoſt imperceptible, yet by the *Teleſcope* it was ſeen to turn oppoſite to the *Sun*; and appeared of the length of two *Diameters* of the Head.

Upon the 21 of *April*, there appeared a *Comet*, which roſe about 2 in the *Morning*, near *N. E.* by *N.* It had a ſhort Buſhy Tail, about 5 deg. in length, pointing towards the *Right Foot of Andromeda*. The Head of the *Comet* was of a Pale Colour, and as big as a *Star* of the *Firſt Magnitude*: Its *Longitude* was in 11 deg. of *Taurus*, and in 18 deg. of *North Latitude*. Upon the 23 of *April* it did Riſe after 2 in the *Morning*, the Tail ſtreaming towards the *Star* in the *Knee of Caſſiopea*, and being in *Longitude* in 15 deg. of *Taurus*, and in 17 deg. of *North Latitude*.

The End of the Firſt Part of Coſmography Cœleſtial.

OF COSMOGRAPHY.

PART II.

Terrestrial, or Geographical

PROOEME.

Of GEOGRAPHY.

THE whole Body *or* Globe *of the* Earth, *which we call the* Terrestrial Globe, *consisteth of these two Principal Parts*, viz. Earth *and* Water, *and as these two are depicted upon the* Globe, *or in* Maps, *the* Land *is bounded and distinguished from the* Water *by an irregular Line, which runs turning and winding into* Creeks *and* Angles *like as the* Shoar (*which it representeth*) *doth, and that part of the* Globe *which is inclosed by this Irregular or Winding* Line, *is the* Land, *and that without it is the* Water.

SECT. I.

That the Terrestrial Globe is Round and Spherical.

THat the whole body of the *Terrestrial Globe*, (consisting of *Earth* and *Water*) is of a *Spherical* Form, may be proved by divers sound Reasons urged by *Geographers*: As,

I. *From its* Latitude *from* North *to* South.

That the *Earth* is *Round* deduced from its *Latitude*, from *North* to *South* is thus proved: For, if a Man travel from *North* to *South*, or from *South* to *North*, he shall perceive new *Stars* in the *Heavens* to appear, and shew themselves, which before he could not see; which can be referred to no other Cause than the *Spherical* Convexity, or Swelling of the *Earth*. For otherwise, if the *Earth* were flat or plain, (as some have vainly imagined,) all the *Northern Stars* would appear to the Inhabitants of the *Southern Regions*, and the *Southern Constellations* to the *Northern* Inhabitants; which common sense and reason altogether contradicts.

II. *From its* Longitude *from* East *to* West: *That the Earth is Round, deduced from its* Longitude *from* East *to* West, *may thus be proved.*

1. From the *Rising*, *Culminating*, and *Setting* of the *Moon* and *Stars*; forasmuch as they do not *Rise* and *Set* with all *Nations* at the same time: For with the Inhabitants of the *East*, the *Sun*, *Moon*, and *Stars* do *Rise*, *Culminate*, and *Set*, sooner than they do with the *Western* Inhabitants; and that in such proportion, that every 15 degrees of the *Equinoctial*, measured out by the *Sun*'s diurnal Motion, differs *One whole hour*: This is proved by the experience and testimony of Ancient *Cosmographers*, that the *Sun Rises* in *Persia*, towards the *East*, four hours sooner than at *Spain*, in the *West*.

III. *From the* Difference *of* Time, *that Eccliples of the* Sun *and* Moon *do appear unto several Nations or Parts of the World.*

That the *Earth* is *Spherical*, deduced from *Eclipses*, doth thus appear; for we see plainly, that *Eclipses* of the *Moon* appear sooner to the *Eastern*, and later to the *Western* Inhabitants of the *World*, as *Ptolomy* reports: That in *Arbela*, a Town in *Assyria*, where *Alexander* overcame *Darius*, the last King of the *Persians*, there was observed an *Eclipse* at the fifth hour of the Night; which same *Eclipse* was seen in *Carthage* at the

the ſecond hour; —— Alſo an *Eclipſe* of the *Sun* at *Campania* (as *Pliny* reports) was ſeen between 8 and 9 in the Morning, which was not ſeen in *Armenia* till between 10 and 11 of the Clock.

1. *From the* Eclipſe *of the* Moon.

An *Eclipſe* of the *Moon*, which is cauſed by the Shadow of the Body of the *Earth*, being interpoſed between the *Sun* and the *Moon*; foraſmuch as this Shadow falling upon the *Moon*, doth to us appear *Circular*; it is manifeſt from *Optical* Principles, that the *Earth* (being the *Body* from whence that Shadow proceedeth) is alſo a *Spherical* or *Round* Body.

2. *From the* Eclipſe *of the* Sun.

Likewiſe, from the *Eclipſes* of the *Sun*, which are cauſed by the Interpoſition of the *Body* of the *Moon*, between the *Sun* and thoſe places where the *Sun* appears *Eclipſed*; I ſay, it could not be determined when, and in what place, (or places) ſuch an *Eclipſe* ſhould appear, and where not, if the form of the *Earth* were not *Orbicular* or *Round*.

IV. *From the* Riſing *and* Setting *of the* Sun, Moon, *and* Stars.

By the *Sun*, *Moon*, *Planets*, and *Fixed Stars*, *Riſing*, *Setting*, and coming to the *Meridian* in ſome places *ſooner*, and in others *later*, and that in a proportion anſwerable to the *Roundneſs* of the *Earth*. As the *Sun* (and conſequently the *Moon*, a *Star*, or *Planet*) is upon the *Meridian* at *London*, ſooner by almoſt four Hours than it is at *Virginia*, *New England*, and other *Eaſtern Regions*: And ſo in *Eaſt-India*, and other *Eaſtern Regions*, the *Sun*, *Stars*, *&c.* are ſooner upon their *Meridian*, than upon ours at *London*; from whence it is evident that the *Earth* is *Round*.

V. *From* Sailing *or* Travelling Northward, *or* Southward.

Firſt, if you travel from the *South* towards the *North*; you have the *Northern Star* (or *Pole*) more elevated; and the *Southern Pole* (or *Star*) more depreſſed; the *Elevation Northerly*, increaſing equally with the *Depreſſion Southerly*; and either of them proportional to the *Diſtance Sailed* or *Travelled*: The like happeneth in Travelling *Southwards*, you then *Elevate* the *South Pole* and *depreſs* the *Northern Pole* as much: —— But if again, you *Sail* or *Travel* from *Eaſt* to *Weſt*, or from *Weſt* to *Eaſt*, being under the *North Pole* (or in *North Latitude*) look what *Elevation* the *North Pole* hath at your Setting out, the ſame it ſhall have at your *Voyage* or *Journeys* end; as if you travel from *London* where the *North Pole* is elevated 51 deg. 32 m. to *Carleton Iſland*, which is directly *Weſt*, and near 3000 Miles diſtant therefrom; yet, the *North Pole* there, ſhall have the ſame elevation as it had at *London*; and conſequently, the ſame all the way of your *Journey* or *Voyage*, which could not poſſibly be, if the *Earth* were of any other form than *Round*.

VI. *By ſeeing of a Ship under Sail.*

Suppoſe ſtanding upon the *Sea Shore*, you ſee a *Ship* far off under *Sail*, making towards the *Land*; at firſt, you ſee only the *Top-Sails*, or higheſt parts; and withal, do manifeſtly behold the Convex Superficies of the *Sea*, as it were raiſed, and interpoſing her ſelf between our Sight, and the *Hull* (or lower parts) of the *Ship*, until *She* approaching nearer, and nearer (and that proportional to her diſtance run) you, at her arrival, ſee her whole *Body* (or *Bulk*) which doth evidently demonſtrate, the *Spherical Roundneſs* of the *Earth*.

SECT. II.

Of the abſurdities that would follow, ſhould any other ſhape than Roundneſs, *be attributed to the Figure of the Earth.*

1. The Elevation of the *Pole* would be the ſame in all parts of the *Earth*.
2. There would be the ſame *face* and appearance of the *Heavens* to all Perſons alike whereſoever inhabiting.
3. The *Sun*, *Moon*, and *Stars* would *Riſe* *Culminate*, and *Set* in all places alike, and at the ſame time.
4. All *Eclipſes* would appear to all People at the ſame hour.
5. The *Days* and *Nights* would be of the ſame length to all Inhabitants.
6. All *Shadows* would be every where alike, and one Region would not be *hotter* nor *colder* than another.

Sect.

SECT. III.

That Mountains *and* Valleys *upon the* Earth *do not oppose the* Roundness *thereof.*

It is conceived by the Ignorant, that the *Earth* cannot be said to be round, in respect of the vast *Mountains, Hills,* and *Valleys,* which are in the same: It is true indeed, that the *Globe* of the *Earth* is not Absolutely and Geometrically Round, as *Pliny* lib. 2. c. 21. saith, *that the Earth and Water make one Globe, not so absolutely Round as the Heavens, but much different.* Yet notwithstanding this, the three Reasons following will shew, that this Inequality (how great soever it seems to the sight) is altogether insensible, and bears no proportion to the huge vastness of the whole *Earth.*

I. *From the heighth of the highest Mountain.*

Let a *Mountain* be imagined to be in perpendicular heighth *Ten Miles*; but few *Mathematicians* will grant any to be so much (although a late *Mariner* hath published in Print, that he hath Sailed three times about the Peak of *Teneriffe,* and made diligent observations several ways and times, to attain to the perpendicular heighth thereof, which observations he hath at large Printed, and seems to demonstrate the Perpendicular heighth thereof to be 30 Miles; (*vix credo.*) But allowing a *Mountain* to be 10 Miles high, the *Semidiameter* of the *Earth* is (as hereafter will be shewed) 3600 Miles, so that this *Hill* compared to the whole *Diameter* of the *Earth,* which is 7200 Miles, is but as 10 to 7200 or as 5 to 3600, which indeed hath no sensible proportion.

II. *From the Eclipse of the Moon.*

The *Eclipse* of the *Moon,* being caused by the Shadow of the interposed *Earth,* is described by a *Spherical Figure,* without any unequal or rugged parts; which no doubt would appear, if those Parts challenge any due proportion, or sensible quantity in respect of the whole *Earth.*

III. *By the Shadow of a Spherical Body, having some irregular dents and swellings in it.*

As if a *Spherical* Body had in it here and there some irregular indents made in it, and in others some risings or swellings put or applyed to it, If such a *Spherical* Body should be held between the *Sun* Beams, so as to cast its Shadow upon some plain Superficies, it would give a *Round* or *Spherical* Shadow upon the Plain, in regard of the little quantity of those small irregularities in respect of the whole Body.

These are the general Reasons given by all *Geographers,* to prove the Rotundity of the *Terrestrial Globe,* and the absurdities which would follow if it were not so.

And now seeing we are treating of this matter, I cannot omit to insert in this place, what I lately met with in a Learned Discourse, written by Mr. *J. Childrey,* Entituled *Britannia Baconica*; or, the Natural Rarities of *England, Scotland* and *Wales*; according as they are to be found in every Shire: In which Book, when he comes to speak of *Carnarvanshire* in *Wales,* he hath this following Discourse; which I have here inserted.

The Air of this Shire (saith he) is sharp and piercing. *Here are extraordinary high Hills,* (*the highest in all* Wales) *on some of which, the Snow lies long*; *and on others it lies all the year long, hard crusted together: A thing not at all to be wondered at, since on the Alps, and many other Mountains much more Southerly than our Island is, it doth the like. The consideration of which hath bred an Opinion in me, that the Globe of the Earth and Sea, is of an Elliptick or Oval form, that is, like an Egg*; *and my Reason is this.*

I suppose, that every Year under both the Poles, *there falls a quantity of Snow* (*either little or much, in the time of the* Sun's *being in the contrary* Tropick, *and likely enough at other times of the year too*) *which the* Sun *when he hath greatest Power upon it, cannot melt all. And this is more than probable*; *because, not only in* Greenland, *but also here in this* Shire, *and* (*if we will believe* Munster) *on the top of the* Alps *too, there are Mountanous Crusts of Frozen Snow, that never were melted: So that now after so many years lapse, it cannot be, I think, but that the Diameter of the Earth from Pole to Pole*; *from the top of the Snow at one end of the Earth, to the top of it at the other end, is much longer than in any other part under the* Equator; *though, at the Creation, it were* (*as I believe*) *made Spherical: And so I suppose in longer process of time it will grow more oblong. And as it so in-*

creaseth

creaſeth in length, ſo I believe, the Sea will decreaſe in depth (though both very inſenſibly) becauſe Snow muſt conſiſt of ſomething; and that ſomething can be nothing but a very vapour condenſed and congealed, &c. And this Watry Vapour muſt be drawn out of the Sea, or out of that part of the Earth, which once (ſooner or later) received it from the Sea: And this Snow being thrown down at the Poles, and not melting, that ſo it may return from whence it came, and refill that which is imployed, muſt needs cauſe a decreaſe in the Sea.

Now that which tempts me to embrace this Paradox the more affectionately, is, for that it ſerves excellent well to ſolve a great doubt, which troubled Tycho *and* Kepler, *about* Central Eclipſes *of the* Moon *that happen near the* Æquator, *ſuch as that was which* Tycho *obſerved in the year* 1588, *and that which* Kepler *obſerved in the year* 1624, *concerning which, he ſpeaks to this purpoſe.* Notandum eſt hanc Lunæ Ellipſin (inſtar illius, quam Tycho anno 1588. obſervavit, totalem, & proximam Centrali) egregie calculum fefelliſſe: Nam non ſolum mora totius Lunæ in tenebris brevis fuit, ſed & duratio reliqua multò magis. Perinde quaſi Tellus Elliptica eſſet, dimetientem breviorem habens ſub Æquatore, longiorem à Polo uno ad alterum: *That* ☽, *we muſt note that this Eclipſe of the Moon* (viz. *that on the* 26. *of* September, Stylo Novo 1624.) *likewiſe that which* Tycho *obſerved in the year* 1588, *being a* Total, *and almoſt* Central *one, did notoriouſly deceive my Calculation; for not only the duration of the total obſcurity was ſhort, but alſo the reſt of the duration before and after the total obſcurity much ſhorter; as if (ſaith he) the Earth were Elliptical, having a ſhorter Diameter under the Equinoctial, than from one Pole to another. And yet I am not ſo devoted to my own fancies, but that one ſolid reaſon ſhall prevail with me to abandon the deareſt of them; though, for the preſent, I ſee abundance of reaſon for what I think.*

SECT. IV.

Of the Meaſure of the Earth.

THE Circumference of the *Terreſtrial Globe* of *Earth* and *Sea* (as of the *Circles* in the *Cœleſtial Spheres*) do contain 360 degrees; by which degrees, the Diſtances of Places upon the *Earth* and *Sea* are meaſured, as well as of the *Stars*, *Planets*, and *Comets* in the *Heavens*: So that the true knowledge of the quantity of ſuch a *Degree* in our known Meaſure, will be a Fundamental Principle to remove many doubts both in *Aſtronomy*, *Geography* and *Navigation*.

Now an *Engliſh Mile* (by Statute) contains in Length 8 *Furlongs*; and every *Furlong* 40 *Poles*, or *Perches*; every *Perch* 16 *Foot* and a half; So that by this reckoning, an *Engliſh Mile* containeth 5280 Foot in Length; although it be uſually taken, by *Mariners* in their keeping of their *Reckonings* of *Voyages* at *Sea*, that 60 ſuch *Miles* do make a *degree*: And this would be very ſtrange, that our *Engliſh Meaſure* being deduced from ſo ſlender and uncertain means, as 3 *Barley Corns* to an *Inch*, 12 *Inches* to a *Foot*, and 5280 of thoſe *Feet* to a *Mile*, ſhould ſo happily fall out to anſwer to one *Minute*: Yet by the *Obſervations* of the moſt diligent Inquirers into this neceſſary point, do find that about 66 of our *Engliſh Miles* do anſwer to one *degree*.

An account of ſome Experiments *made by* Ancient *and* Modern Mathematicians, *concerning this* Experiment.

	Feet &c.	Miles
Anno Chriſti 827. by the command of Prince *Almanon*, in *Meſopotamia*, it was found that one degree did contain (reduced to our *Engliſh Feet.*)	370222	$70\frac{311}{2640}$
Alhazen an *Arabian* declares the Compaſs of the *Earth* to be 24000000 *Paces*, and 333333 *Arabian* Feet in one degree, which reduced to our *Engliſh Feet*, are	367283	$69\frac{2263}{5280}$
Ptolomy of *Alexandria*, makes the quantity of one degree to contain 360000 *Rhynland Feet*, which reduced to our *Engliſh Feet*, are	371900	$70\frac{1315}{2640}$
Willebrordus Snellius, an *Hollander*, about *Anno* 1613, finds one degree to contain 342000 *Rhynland Feet*, which reduced to *Engliſh Feet*, are	353306,	67 ferè

Mr. *Ri-*

Mr. *Richard Norwood*, an able *Mathematician* and *Seaman*, (and my very good Friend) did in *Anno* 1635 make an *Experiment* hereof, between *York* and *London*, and finds one degree upon the *Earth* to contain 691 *English Miles*, which is Feet.

Feet &c.	Miles
367200	
691	

This experiment of his, he hath at large set down in his Book entituled the *Seamans Practice*, which (as himself confesses) was not done with that accurateness in all particulars as he could have done, had he had some other supplies, for what he did, was voluntary, and upon his own account: And the manner of his proceeding in it is undeniable: And therefore, in the mean time, till it shall please God to raise up some noble minded spirit to effect the same, we must be content with what we already have, for all the foregoing except the last, we have no account of the manner how they observed, but divers ingenious Men have attempted to find out the true *compass* of the *Earth*, but either the grounds they have gone upon, have been uncertain, or the distance of the two places of *Observation* have been too short, or the diligence of the *Observator* to be suspected. That way which hath been practised by some, which is by the height of an *Hill*, and a *Tangent Line* from thence to the Superficies of the *Sea*, is rather a fiction than a thing of actual performance; for, neither the perpendicular height of the Hill above the level of the *Water*, can with any certainty be obtained; nor such a *Tangent Line*, by reason of the *Refraction* of the *Vapours* continually rising out of the *Sea* can be estimated. And therefore, in this place I shall be bold to insert a way (not much differing from that Mr. *Norwood* practised in his perambulation from *York* to *London*) which hath been proposed by an ingenious Gentleman, and my very loving Friend Mr. *Im. Halton*, which take in his own words as followeth.

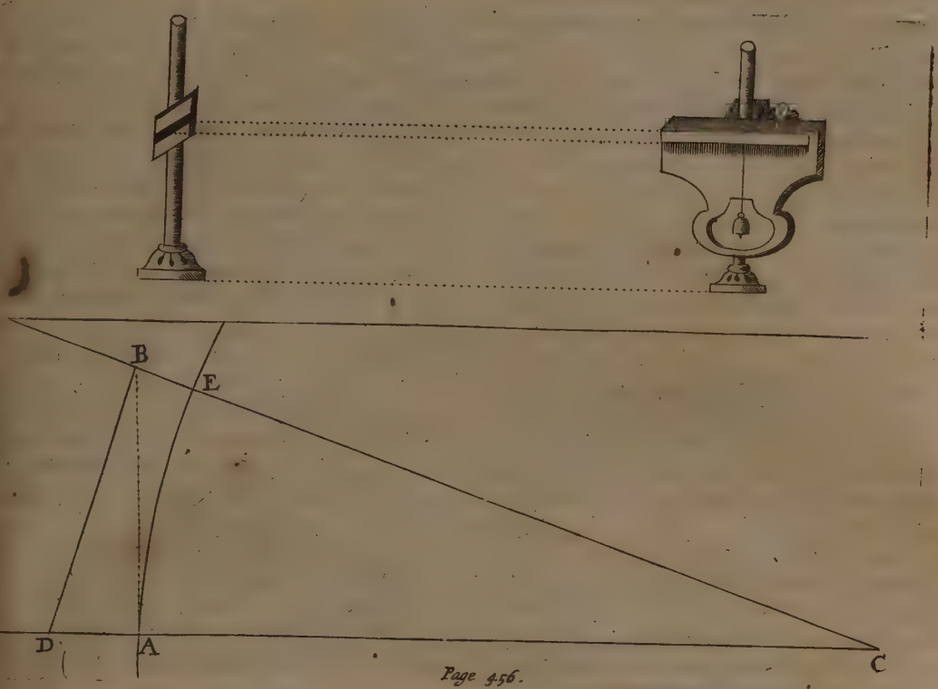
Let the height of the *Pole* at two Towns of this Land, distant *Northward* one from the other some *Scores*, or rather, *hundreds* of *Miles*, being with Instruments of sufficient Magnitude by some Learned Artists exactly observed; and that there were also employed certain skilful *Surveyors* (such as are indeed lovers of *Art* and *Truth*) to take the true distances and positions from *Place* to *Place* between the two *Towns*: Which survey I could wish were made with good *Theodolites*, or *Circumferentors*, and Plotted with a *Scale*, which should not be less than a *Foot* by Standard for *Ten Miles*, and the *Chains* fitted thereto; and that the measure be taken, not only along the *Highways*, but by side *Stations* also, where *Steeples*, and other *Places* eminent may be seen. If the two *Places* or *Towns* of the *Observations* were *London* and *Edenburgh* in *Scotland*, it would be preciseness sufficient; nay, if it were but *London* and *Cambridge*, it would yield a greater certainty than any that I know hath yet been used. This I say were an excellent work, and worthy the Heroical magnificence of some great Man; and yet not of any chargable performance neither; but it would bring a marvellous Light and furtherance to *Navigation* and *Astronomy*: And here I will make bold to propose a way, which any ingenious *Student*, whose *Sight* both of his *Eyes* and understanding is quick and perspicuous, may himself privately, and with much facility practice; the Reason whereof consisteth upon these three *Principles*.

I. That if with a *Levelling Instrument* set up in any place, parallel to the *Horizon*, a Man take a true *Level* unto another *Place*: the *visual Line* by which he levelled, shall be a *Tangent* to such an *Arch* of a *Great Circle* upon the *Earth*, as is contained between the *Station* and the *Mark*; because, that the *Visual Line*, together with the two *Lines* imagined out of the *Centre* of the *Earth*, do include a Right angled Triangle, having the *Right Angle* at the *Level*.

II. That if the same *Instrument* be set just even with the former *Mark*, and you level backward to the former *Station*; this last *Visual Line* shall over-shoot the former place of the *Instrument*: and shall include a new and greater *Right angled Triangle*, having the *Right Angle* at the second *Station*.

III. That the former of the two *Visual Lines* shall cut this latter, and greater *Right Angled Triangle* into two *Right Angled Triangles* like to it self, and one like to the other. By the 8*th Prop* of the 6*th* of *Euclid*.

As



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As in this *Scheme* The *Centre* of the *Earth* is C; the firſt *Place* or *Station* of the *Levelling Inſtrument* is A, and the *Viſual Line* thence is A B, to the *Mark* B, which is alſo the place of the *Inſtrument* in the *Second Station*, from whence the *Viſual Line* backward is B D, over-reaching the firſt place A. Here are three like *Right angled Triangles*; namely, the *Greateſt* C B D, cut into two other C A B, and B A D with the *Line* A B; Wherefore,

As A B, is to A D: ſo is A C, to A B. That is,

As the Diſtance between the two Stations A B, (for by reaſon of the vaſt greatneſs of the *Earth*, and the exceeding ſmall diſtance between the two *Stations* in compariſon thereof, the *Viſual Line* A B, ſhall be the ſame with the *Ground Line* A E;)

Is to the over-ſhooting of the Second Line of Level A D:

So is the Radius,

To the Tangent *of the Arch* A E (intercepted between the Two *Stations.*)

The quantity of which *Arch* being ſought out in the *Canon* of *Tangents.* Say again,

As the ſame Arch, (in *Sexageſme*, or *Decimal* parts of a *Degree*;)

Is unto a Degree in the like Parts;

So is the Diſtance between the two Stations in Feet;

To the Number of Feet anſwering to a degree upon the Earth.

Example.

Suppoſe the *Diſtance* between the two *Stations* to be 528 feet, (which is the *Tenth* part of a *Mile*) and that the ſecond *Line* of Level over-ſhooteth the former $\frac{138}{10000}$ of a Foot, or 0.0138 which you ſhall find will be near about the matter. Say,

As 528: is to 0.0138:

So is 10.00000: to 2.61; the *Tangent* of Min. 0.09 + Say again,

As 0.09 +: is to 60:

So is 528: to 351120, the number of Feet anſwering to a Degree upon the *Earth*, which is Miles $66\frac{1}{2}$.

Thus

Thus have I ſet down the *Rule*, and illuſtrated it with an *Example*. But in the practice (by reaſon of the *weakneſs* of our *ſight*, not able to diſcern a thing diſtinctly at any great diſtance, we are conſtrained to take but ſhort *Stations*; whereby the overſhooting of the ſecond *Line* of *Level* above the firſt, is but very ſmall) there is required great preciſeneſs: For the performance whereof it will not be amiſs, to ſet down ſome *directions* both concerning the *Inſtrument*, *Place*, and *Time*.

The *Levelling Inſtrument* to be uſed in this *Work*, I would not have to be either with a *Chanel* for *Water*, nor with *Sights*. For the *Water*, beſides that it doth continually exhale *Vapours*, hath a certain tenacity, whereby to avoid any drineſs near to it; it will rather collect it ſelf, and ſtand in a *heap*, than mix with its *Enemy*; and contrariwiſe, very gladly diffuſeth it ſelf in purſuit of any moiſture. And as for the *Sights*, If the *Sight-hole* be very *ſmall*, it hindereth our *ſeeing*; if any whit *large*, it admitteth too many viſive *Rays*, which dilating themſelves, cannot fix on the true and individual point of the *Object*. But I would have it only with a *Ledge* one inch thick, and 3 inches broad; and ſo broad alſo would I have a black *ſtroak* to be in a *ſquare white Board*, for the *Mark* to *Level* at; that having ſet the *Ledge* of the *Inſtrument* by the *Plumb Line* parallel to the *Horizon* in one *Station*, you ſtanding aloof off, and guiding your *Eye* along the two *Edges* of the *Ledge*, and your *Companion* at the other *Station*, *raiſing up*, or *letting down* the *Mark-board* as you ſhall direct him; you may ſee the upper *Line* of the *black Stroak* level with the *upper Edge*, and the *lower Line* with the *lower Edge*.

The *Place* for the trial of this *Experiment*, I would have to be a *Plain Field*, wherein you are to have for your uſe ready *meaſured* out by the Foot, directly *Eaſt* and *Weſt*, ſuch a *diſtance* as you can diſcern diſtinctly thereat, which to a good and perfect *Sight* may be 1000 Feet, or to an indifferent *Sight* 528 Feet, which is the *Tenth* part of a *Mile*: And at both ends of that *diſtance* (which are to be your *Stations*) the *Ground* to be handſomly *plained* and *beaten*, for the more exact ſetting up of your *Inſtruments* thereon.

The *Time* for making your *Obſervation*, I would have to be about *Midſummer*, in a ſeaſonable dry and calm *Weather*; when having ſet up your *Levelling Inſtrument* in the *Eaſtern Station*, you may take your firſt *Level* about 11 of the Clock in the *Forenoon*: which being done, you may remove your *Levelling Inſtrument* to the *Weſtern Station*; and about *One* of the Clock in the *Afternoon* (when the *Sun* is gone ſo far paſt the *Meridian*) take your *back Level*.

Theſe are the moſt *neceſſary* and *accurate cautions* that I can deviſe; and all little enough for ſo curious and ſubtle an inquiry: I have alſo ſet down the form of the *Levelling Inſtrument*, and of the *Mark Board* in *Figure*.

SECT. V.

Of the Diviſion *of the* Terreſtrial Globe.

THE whole *Superficies* of the *Globe* conſiſteth of *Land* and *Water*, and therefore, thoſe are the two Principal Parts into which it muſt be firſt divided: And thoſe again are ſub-divided: For,

1. The *Earthy* Part which we call *Land*, is divided into theſe Parts, *viz.*

1. *Continent*, and 2. *Iſland*.

And thoſe again are divided into theſe Parts, *viz.*

3 *Peninſula* } { 5 *Promontory.*
4 *Iſthmus* } { 6 *Cape.*

2. The Watery Part, which we call *Sea*, is alſo divided into theſe Parts, *viz.*

1 *Ocean.* } { 4 *Creek.*
2 *Sea.* } { 5 *Lake.*
3 *Straits.* } { 6 *River.*

Of

Of the *Earth* and its *Parts.*

1. Of a *Continent.*

A *Continent* is a great quantity of *Land* (not environed or separated by the *Sea,*) in which many Kingdoms and Countries are contained; as *Europe, Asia,* &c.

2. *Of an* Island.

An *Island* is a part of the *Earth* clasp'd in the embraces of the *Sea,* and hoopt as it were with a watery Girdle, as *Great Britain* and *Ireland.* These again are subdivided into *Peninsula*'s, *Isthmus*'s, *Promontories, Capes,* &c.

3. *Of a Pen.*

A *Peninsula* or *Pene-Insula* is a part of Land which being almost environed and incompassed round with *Water,* is joyned to the firm Land by some little *Isthmus,* as *Africa* is joyned to *Asia,* or *Morea* to *Greece.*

4. *Of an* Isle.

An *Isthmus* is a little narrow neck of *Land* betwixt two *Seas,* joyning a *Peninsula* to the *Continent*; as that of *Darien* in *America,* or *Corinth* in *Greece.*

5. *Of a* Promontory.

A *Promontory* is *Mons* in *Mari prominens,* a high Hill or Mountain lying out as an Elbow of land into the *Sea,* the utmost end of which is called a *Cape*; as the *Cape* of *Good Hope,* and *Cape Verde.*

The *Land* drawn upon the *Superficies* of the *Terrestrial Globe,* is divided in four principal Parts or Quarters, *viz. Europe, Asia, Africa,* and *America.*

Of EUROPE.

Europe of the Four Great Parts of the *World* is the leaſt, yet the moſt conſiderable in reſpect of the *Beauty* of her *Kingdoms* and *Commonwealths*; the *Politeneſs* of her *Inhabitants*, the *Government* of her *Cities*, the *Goodneſs* of her *Air*, her general *Fertility*, and her *Traffick* and *Commerce*.

Her *Situation* is all in the *North Temperate Zone*. The Length thereof from *Cape Finis Terræ*, on the *weſt* of *Spain*, to the *River Tanais* (now *Don*) is 2400 *Miles*, at 60 to a degree: The Breadth thereof from the *South* to the *North*, *i. e.* from *Cape Metapan* of *Morea*, to the moſt *Northern Promontory* of *Norway* about 2100 *Miles*.

On the *North*, *Europe* is bounded by the *Northern Ocean*, or the *Frozen Sea*; Towards the *weſt* by the *Atlantick Ocean*: By the *Mediterranean Sea* towards the *South*, and beyond that *Sea*, by part of *Affrica*: As for the *Eaſtern* bounds, from the *Mediterranean Sea* to the *North*, they are theſe: (1) By the *Archipelago*, or *White Sea*, otherwiſe the *Ægean Sea*. (2) By the Straight of *Gallipoli*, or the *Dardanels*, otherwiſe called the *Arm of St. George*, and formerly the *Helleſpont*. (3) By the *Marmora*, formerly the *Propontis*. (4) By the *Straight of Conſtantinople*, or the *Canal* of *Mar Maggiore*, formerly the *Thracian Boſphorus*. (5) By the *Black*, or *Mar Maggiore*, formerly *Pontus Euxinus*. (6) By the *Straight* of *Caffa* or *Veſpera*, otherwiſe the *South of St. Johns*; formerly the *Commerian Boſphorus*. (7) By a *Limen*, otherwiſe the *Sea* of *Zabaque* and *Tanais*, formerly *Palus Mœotis*. (8) By the River *Don* or *Tana*, formerly *Tanais*. (9) By the *Line* drawn from the moſt *Eaſterly* winding of *Donu*, to the *Northern Ocean* near *Obi*.

There are Four Principal *Languages* reckoned to be ſpoken in this Part of the *World*; viz. *High-Dutch*, *Latin*, *Greek* and *Slavoniſh*; of which the *High-Dutch* is of three ſorts. (1) *High-Dutch* in *Germany*; *Saxon* in *England* and *Scotland*; *Daniſh* in *Denmark*, *Sweden*, *Norway* and *Ireland*. (2) The *Latin* is corrupted into *Italian*, *French*, and *Spaniſh*. (3) The *Greek* had formerly four Dialects, the *Attick*, *Ionick*, *Dorick*, and *Æolick*. The *Slavonian* Language runs through all *Slavonia*, *Bohemia*, *Poland*, and *Muſcovy*. ——— There are Seven other Languages of leſs note, which are uſed in *Europe*, the *Albanies*, *Coſack*, *Hungarian*, *Finland*, *Iriſh*, *Britiſh*, and *Biſcagne*. The *Caſſack* hath ſome affinity with that of the Leſſer *Tartars*; *Finland* and *Lapland* ſpeak only *Finlandiſh*. The *Britiſh* is ſpoken in *Wales*, *Cornwall*, and *Britany* in *France*; *Biſcayn* only in *Biſcayni*.

The Kingdoms, &c. *of* EUROPE.

	Spain, with its Kingdoms or Principalities, *viz.*	Caſtile,	*Madrid.*
		Leon,	*Leon.*
		Navarr,	*Pampelona.*
		Biſca,	*Bilboa.*
		Aſturie,	*Oviedo.*
		Gallicia,	*St. Jago de Compoſtella.*
		Portugal,	*Lisbon.*
		Algarve,	*Pharo.*
		Andalouſia,	*Sevil.*
		Granada,	*Granada.*
		Murcia,	*Murcia.*
		Arragon,	*Caragoſa.*
		Valencia,	*Valencia.*
		Catalonia,	*Barcelona.*
		The Iſles of Baleares,	*Majorca.*
The three moſt Southern pts. are,	*Italy*, with its ſeveral Eſtates and Principalities, the chief of which are,	Piedmont,	*Turin.*
		Millain,	*Millain.*
		Genoua,	*Genoua.*
		Parma,	*Parma.*
		Mantua, & Modena,	*Mantua.*
		Venice,	*Venice.*
		Toſcany,	*Florence.*
		Eſtate of the Church,	*Rome.*
		Naples,	*Naples.*
		Iſle of Sicilly,	*Meſſina.*
		Iſle of Sardiny,	*Calari.*
		Iſle of Corſica,	*Baſtia.*
	Turkey (in *Europe*) with ſeveral Eſtates, the chief of which are thoſe of	Boſnie,	*Jajcza.*
		Servie,	*Belgrad.*
		Bulgarie,	*Sophia.*
		Romania,	*Conſtantinople.*
		Macedonia,	*Salonichi.*
		Theſſalonie,	*Armicho.*
		Epire,	*Perveza.*
		Achaia,	*Selines.*
		Peloponeſus,	*Petras.*
		Dalmacie,	*Rhaguſa.*
		Sclavonia,	*Poſtega.*
		Illyris,	*Zatha.*
		Croatia,	*Siſſeg.*

Europe

Europe with its Kingdoms, Isles, &c. may be considered in three times three parts, and then

- Together with the several Isles as they lie in the
  - *Ægean* or *Grecian* Seas, as — Negroponte, Crete, *the* Isles *of* Cyclades, &c.
  - *Ionian* Seas, as — Zant, Zeffalonia, Corfu, &c.
  - *Adriatick* Seas, as — Zara, Lesina, Curzola, Lissa, &c.
- The three innermost parts, and within the Continent are
  - *France*, with its twelve Governments, of General Estates, *viz.*
    - Picardy, — *Amiens.*
    - Normandy, — *Roen* or *Roven.*
    - *Isle of* France, — *Paris.*
    - Champagne, — *Troys.*
    - Bretagne, — *Nantes.*
    - Orlenois, &c. — *Orleans.*
    - Bourgogne, — *Dijon.*
    - Lyonnois, &c. — *Lyon.*
    - Guyenne & Gascogne, — *Bourdeaux.*
    - Languedoc, — *Toulouse.*
    - Provence, — *Marseille.*
    - Dauphin, — *Grenoble.*
  - The several Estates, which lie between
    - *France*,
      - *The Cathol. Low Country.* — *Anvers.*
      - Lorrain, — *Metz.*
      - *The* French *County*, — *Besansons.*
      - Savoy, — *Cambery.*
    - *Germany*,
      - *The Low Countries, or the United Provinces*, — *Amsterdam.* / *Rotterdam.*
      - *The* Swisses, — *Basle.*
      - *The* Grisons, — *Coire.*
  - *Germany*, with its several Estates and Principalities, the chief of which are
    - *On this side the* Rhine, — *Strasbourg.*
    - *Beyond the* Rhine, — *Cologne.*
    - Westphalia, — *Munster.*
    - Franconia, — *Noremberg.*
    - Sovabe, — *Ausbourg.*
    - Bavaria, — *Munchen.*
    - Austria, — *Vienna.*
    - Bohemia, — *Prague.*
    - *Higher* Saxony, — *Dresden.*
    - Brandenbourg, — *Berlin.*
    - Pomerania, — *Stetin.*
    - *Lower* Saxony, — *Hamburgh.*
  - *Poland*, with its several Estates, the chief of which are
    - Polonia, — *Cracow.*
    - Prussie, — *Dantzick.*
    - Mazovie, — *Warsaw.*
    - Lithuania, — *Wilna.*
    - Volhinia, — *Kyovia.*
    - Podolia, — *Kamieniec.*
    - Russia Nigra, — *Loewenberg.*
  - And some Estates, or Principalities towards the *Danube* and Black Sea, as
    - Hungaria, — *Buda.*
    - Transilvania, — *Hermenstat.*
    - Valaquie, — *Targoviske.*
    - Moldavia, — *Sotzowa.*
    - *Little* Tartaria, — *Nigropoli.*
  - *Scandinavia*, where are Kingdoms and Estates of
    - *Danemark*
      - Danemark, — *Capenhaguen.*
      - Norway, — *Trondhem.*
    - *Sweden*
      - Gothland, — *Calmar.*
      - Sweden, — *Stockholm.*
      - Finland, — *Abo.*
      - Livonie, — *Riga.*
- The three most Northern pts. Kingdoms and Isles, are
  - *Moscovia*, with its several Kingdoms, Dutchies, and Provinces, the chief of which are
    - Moscovy, — *Mosco.*
    - Wolomodire, — *Wolodomer.*
    - Dwine, — *St. Michael Archangel.*
    - Cazan (*Kingdom.*) — *Cazan.*
    - Astracan (*Kingdom.*) — *Astracan.*
  - The Isles of *Great Britain*, which are the Kingdoms of
    - England, — *London.*
    - Scotland, — *Edinburgh.*
    - Ireland, — *Dublin.*

Of ASIA.

ASIA is one of the Tripartite Divisions of our Continent, if we consider the advantages which the Author of Nature hat given it; or the manifold Actions which have passed in it. (1.) That the first *Monarchies* and *Religions* have had their rise here. (2.) That the chief *Mysteries* both of the *Old* and *New Law*, were here laid open. (3.) That from hence all Nations of the World, and all *Arts* and *Sciences* had here their first beginning; so that we may justly prefer it before the other *Parts* of the *World*.

It is seated in the *Oriental* part of our Continent, and most part in the *Temperate Zone*; what it hath under the *Torrid*, being either *Peninsula* or *Islands*, which the Waters refresh.

It extends from *Smirna* in the West, to the fartheſt part of *Tartaria* near *Jesso* in the East 4800 Miles; and from the lowermost part of *Molacca* in the South, to the Straights of *Weigats* in the North, it makes 4200 Miles: in which measures are not comprehended the *Islands* belonging to *Asia*, which are as *Great*, as *Rich*, and *Numerous*, as any in the other Parts.

From whom this Part of the World took its Name is uncertain. Some attributing it to *Asia* the Daughter of *Oceanus* and *Thetis*: others from *Asius*, the Son of *Atis* a King of *Lydia*: some from *Asius* the Philosopher: others from the *Venetian* Word *Asia*, signifying *Medium* or *Middle*.

The Religions in this Part followed are many; but the *Jews*, *Mahometans*, and *Idolaters*, far exceed the *Christians* in number. *Idolatry* began in the time of the *Assyrians*, *Judaism* among the *Hebrews*, *Christianism* in the *Holy-Land*; but was first so called at *Antioch*, and *Mahometism* in *Arabia*.

Asia, towards the West, is separated from *Africa* by the *East Sea*, and by the *Isthmus* of *Sues*: it is divided from *Europe* by several Seas and Straights, before mentioned in the Description of *Europe*. Towards the other Regions of the World, *Asia* is environed by the *Tartarian*, *Chinean*, *Indian*, *Persian* and *Arabian Seas*. *Caucasus* and *Taurus* are the higheſt Mountains, but several other Countries give them different Names.

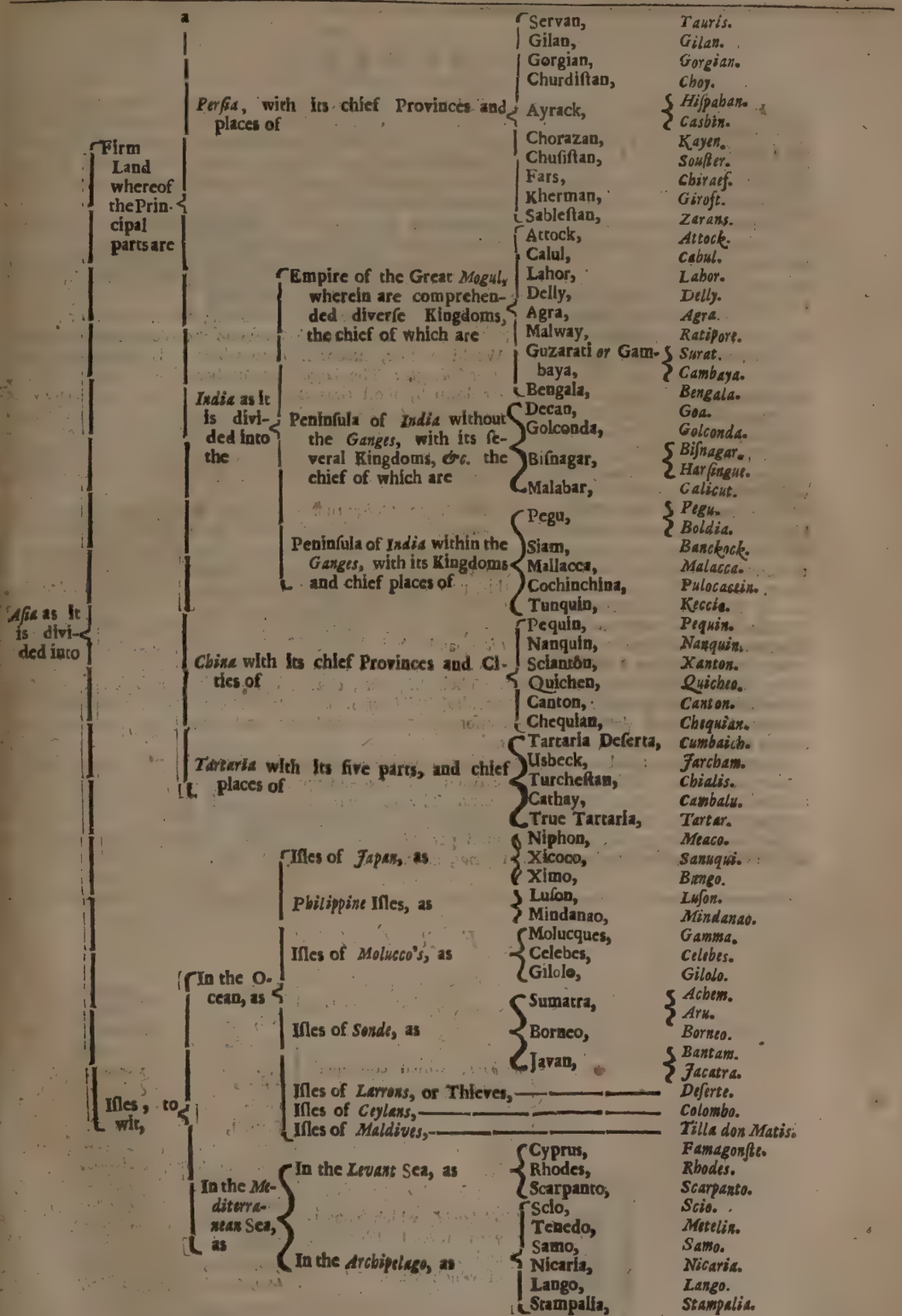
The Air of *Asia* is every where Temperate; and if we consider her *Gold*, *Silver*, *Precious Stones*, *Drugs*, *Spices*, and *Silks*; it is the *Richest*, as well as the most *Noble* Part of the World.

Four of the Seven Wonders of the World are in this Part, *viz.* (1.) The *Temple of Ephesus*. (2.) The *Mansoleum* in *Hallicarnassus*. (3.) The *Walls* of *Babylon*. (4.) The *Rhodian Colossus*. Besides, (5.) The *Olympian Jupiter* in *Europe*. (6.) The *Egyptian Labyrinth*. And (7.) The *Pyramids* in *Africa*.

Asia is in subjection to four Potent Monarchs, who are able to bring mighty *Armies* into the *Field*. (1.) The *Grand Seignior*, who resides in *Europe*. (2.) The *Sultan* of *Persia*. (3.) The *Cham* of *Tartary*, at this time King of *China*. (4.) The *Great Mogul*. Besides these, there are several Great Princes in *India*, and in many of the *Islands*.

ASIA.

- *Turkey* in *Asia*, which comprehendeth the parts and chief places of
  - Anatalia,
    - *Smyrna.*
    - *Epesus.*
    - *Lampsaco.*
    - *Bursa.*
    - *Scutari.*
    - *Tripoli.*
  - Sourie,
    - *Aleppo.*
    - *Tripoli.*
    - *Damascus.*
    - *Sayd.*
  - Diarbeck,
    - *Caraemit.*
    - *Achanchive.*
    - *Sacnosat.*
  - Turcomanie,
    - *Erzerum.*
    - *Cars.*
    - *Majaferequin.*
- *Georgie*, which comprehendeth the parts and chief places of
  - Mingrelie,
    - *Fazze.*
    - *Savatopoli.*
  - Gurgistan,
    - *Cori.*
  - Zuirie.
    - *Chipechet.*
    - *Strauu.*
  - Little Turcoman.
    - *Derbent.*
- *Arabia*, with its parts and chief places of
  - Arabia the Stony,
    - *Busseres.*
    - *Moab.*
  - Arabia the Desart,
    - *Anna.*
  - Arabia the Happy
    - *Medina.*
    - *Mecca.*
    - *Aden.*


Asia as it is divided into
Firm Land whereof the Principal parts are
Perſia, with its chief Provinces and places of
Servan, Tauris.
Gilan, Gilan.
Gorgian, Gorgian.
Churdiſtan, Choy.
Ayrack, Hiſpahan. Casbin.
Chorazan, Kayen.
Chuſiſtan, Souſter.
Fars, Chiraef.
Kherman, Giroft.
Sableſtan, Zarans.
India as it is divided into the
Empire of the Great Mogul, wherein are comprehended diverſe Kingdoms, the chief of which are
Attock, Attock.
Calul, Cabul.
Lahor, Labor.
Delly, Delly.
Agra, Agra.
Malway, Ratipore.
Guzarati or Gambaya, Surat. Cambaya.
Bengala, Bengala.
Peninſula of India without the Ganges, with its ſeveral Kingdoms, &c. the chief of which are
Decan, Goa.
Golconda, Golconda.
Biſnagar, Biſnagar. Harſingue.
Malabar, Calicut.
Peninſula of India within the Ganges, with its Kingdoms and chief places of
Pegu, Pegu. Boldia.
Siam, Banckock.
Mallacca, Malacca.
Cochinchina, Pulocastin.
Tunquin, Keccio.
China with its chief Provinces and Cities of
Pequin, Pequin.
Nanquin, Nanquin.
Sciantõu, Xanton.
Quichen, Quicheo.
Canton, Canton.
Chequian, Chequian.
Tartaria with its five parts, and chief places of
Tartaria Deſerta, Cumbaich.
Usbeck, Jarcham.
Turcheſtan, Chialis.
Cathay, Cambalu.
True Tartaria, Tartar.
Iſles, to wit,
In the Ocean, as
Iſles of Japan, as
Niphon, Meaco.
Xicoco, Sanuqui.
Ximo, Bungo.
Philippine Iſles, as
Luſon, Luſon.
Mindanao, Mindanao.
Iſles of Molucco's, as
Molucques, Gamma.
Celebes, Celebes.
Gilolo, Gilolo.
Iſles of Sonde, as
Sumatra, Achem. Aru.
Borneo, Borneo.
Javan, Bantam. Jacatra.
Iſles of Larrons, or Thieves, Deſerte.
Iſles of Ceylans, Colombo.
Iſles of Maldives, Tilla don Matis.
In the Mediterranean Sea, as
In the Levant Sea, as
Cyprus, Famagonſte.
Rhodes, Rhodes.
Scarpanto, Scarpanto.
In the Archipelago, as
Sclo, Scio.
Tenedo, Metelin.
Samo, Samo.
Nicaria, Nicaria.
Lango, Lango.
Stampalia, Stampalia.

Of *AFRICA*.

AFRICA Was by the Ancients called *Olympia*, *Hesperia*, *Oceania*, *Coryphe*, *Amonis*, *Ortygia*, and *Æthiopia*; by the *Greeks*, *Lybia*; by the *Ethiopians*, *Alkebu-lam*; by the *Arabians*, *Ifrichea*; by the *Indians*, *Befecath*; by the *Turks*, *Magribon*; but the generally received Appellation is *Africa*: derived either from *Apher*, an *Hebrew* Word, signifying *Duſt*, or from *Epher* or *Aphar*, one of the Nephews of *Abraham*; by the *Greek* Fablers, from *Afer*, a Companion of *Hercules*; by the *Arabians*, from *Faruch*, to *divide* or *ſeparate*; or from the ancient name of *Carthage*, called *Africa*.

It is ſituate (for the moſt part) in the *Torrid Zone*, the *Equinoctial* running through the very middle of it; ſo that the Ancients thought it un-inhabited, as parched by the *Sun's* exceſſive heat: but what to them ſeemed almoſt impoſſible, is now common; for the Secrets of her moſt remote *Shoars* are beaten up, and traced by continual *Voyages*: firſt by the *Portugals*, and afterwards by the *Engliſh* and *Dutch*; ſo that the Confines or Bounds of this part of the World are perfectly known. For,

It is bounded by four famous *Seas*, *viz.* The *Mediterranean* on the North; the *Red Sea*, or *Arabian Gulf*, on the Eaſt; the *Æthiopian* on the *South*; and the *Atlantick Oceans* on the Weſt; ſo that it is divided from all the other Parts of the World by *Seas*, except *Aſia*, to which it is joyned by a little *Iſtmus*. Its form is like a huge *Pyramid* or *Triangle*: The largeſt extent thereof, from Eaſt to Weſt, and from North to South, is variouſly ſet down by moſt *Geographers*, by reaſon of the irregularity of the Figure thereof; wherefore I ſhall here ſet down the *Meaſures* thereof, according to Mr. *Robert Morden* in his late *Geography*, Printed at *London* 1688. Thus, The degrees of *Longitude* from Eaſt to Weſt, contained between *Cape Verde*, and *Cape Gaardafuy*, is 72 degrees, which is 4320 Miles, 60 to a degree; and its breadth in *Latitude*, North and South, from *Cape Bon*, to the *Cape of Good Hope*, is 70 degrees, which makes 4200 Miles.

The greateſt *Rivers* in *Africa* are *Nilus* and *Niger*. *Nilus* runs above 3000 Miles, paſſes through the biggeſt *Lakes*, forms the biggeſt *Iſlands*, and *Waters* the moſt lovely *Valleys* in the World: the Heads of theſe *Rivers* are not well known, but moſt probable they are in *Ethiopia*; the *Cataracts* or *Falls* upon the Confines of *Ethiopia* and *Egypt*; and their *Mouths*, which throw themſelves into the Sea, below *Egypt*. —— The greateſt *Lakes* in this part are *Zair*, *Zambere* and *Zaflan*, all of them in *Ethiopia*. —— The chief *Mountains* are that of *Atlas*, and the *Mountains of the Moon*.

Africa is the *Barreneſt* and worſt *Peopled* part of our Continent. Her *Great Rivers* full of *Crocodiles*, Her *Mountains* and *Deſarts* filled with *Lions*, and other Ravenous and Cruel *Beaſts*; and the ſcarcity of Water produces many *Monſters*, which Creatures of ſeveral Species Couple and Engender at the Watering Places, where they often meet. Their *Elephants*, *Camels*, *Baboons*, and *Dromedaries*, are very ſerviceable to the *Africans*: They have alſo, *Wild Aſſes*, *Unicorns*, *Barbary Horſes*, *Cameleons*, *Little Monkies*, *Parots*, and *Auſtriches*, which afford them fine Feathers; and their *Civet Cats* they much eſteem for the excellency of their *Scents*.

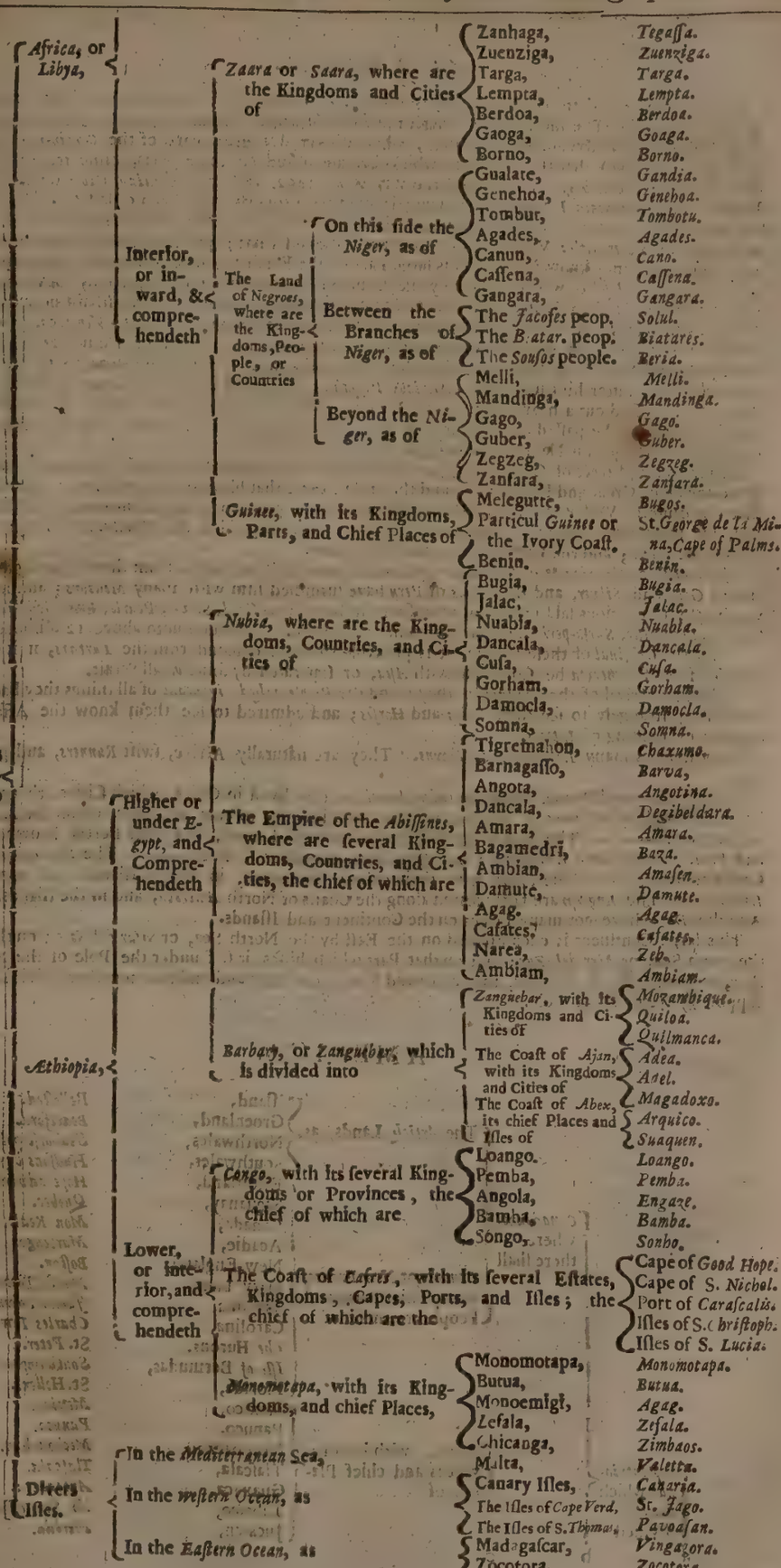
Among the great number of different *Tongues* which are ſpoken in *Africa*, the moſt general are the *Barbary* or *African*, which comes from the Ancient *Punick* and the *Arabick*; which two extend through all *Barbary*, *Bilidulgerid*, *Egypt*, and *Sarra*. The *Ethiopian* in the greateſt part of *Ethiopia*. The Language of the *Negroes* differs from both the other, and hath divers Idioms.

The *Religions* in *Africa*, are for the moſt part *Idolatrous*, as *Paganiſm*, and *Mahometaniſm*; yet there are mixed among them vaſt numbers of *Jews*, and *Chriſtians* of ſeveral ſorts.

AFRICA.

- *Exterior*, or outward, & Comprehendeth
  - *Barbary*, which containeth the Kingdoms of
    - Morocco, — *Morocco*.
    - Fez, — *Fez*. / *Sala*.
    - Telenſin, — *Telenſin*.
    - Algier, — *Algier*. / *Bugia*.
    - Tunis, — *Tunis*.
    - Tripoli, — *Tripolis*.
    - Barca, — *Barca*.
    - Deſart of *Barca*, — *Ammon*.
  - *Bilidulgerid*, which containeth ſeveral Kingdoms and Provinces, the chief of which are
    - Sus or Teſſet, — *Teſſet*.
    - Darha, — *Darha*.
    - Segellomeſſa, — *Segellomeſſa*.
    - Tafilet, — *Tafilet*.
    - Tegorarin, — *Tegorarin*.
    - Zeb, — *Neſta*.
    - Bilidulgerid, — *Fezzen*. / *Gademes*.
  - *Egypt*, which is divided into
    - Sayd *or* Bechia, — *Cairo*.
    - Errif, — *Alexandria*. / *Koſetta*.
    - Coaſt of the Red Sea, — *Sues*. / *Grodol*.

Africa


Africa, or Libya,
Interior, or inward, & comprehendeth
Zaara or Saara, where are the Kingdoms and Cities of
Zanhaga, Tegassa.
Zuenziga, Zuenziga.
Targa, Targa.
Lempta, Lempta.
Berdoa, Berdoa.
Gaoga, Goaga.
Borno, Borno.
The Land of Negroes, where are the Kingdoms, People, or Countries
On this side the Niger, as of
Gualate, Gandia.
Genehoa, Genehoa.
Tombut, Tombotu.
Agades, Agades.
Canun, Cano.
Cassena, Cassena.
Gangara, Gangara.
Between the Branches of Niger, as of
The Jacofes peop. Solul.
The Batar. peop. Biatares.
The Sousos people. Beria.
Beyond the Niger, as of
Melli, Melli.
Mandinga, Mandinga.
Gago, Gago.
Guber, Guber.
Zegzeg, Zegzeg.
Zanfara, Zanfara.
Guinee, with its Kingdoms, Parts, and Chief Places of
Melegutte, Bugos.
Particul Guinee or the Ivory Coast, St. George de la Mina, Cape of Palms.
Benin, Benin.
Africa as it is divided into
Higher or under Egypt, and Comprehendeth
Nubia, where are the Kingdoms, Countries, and Cities of
Bugia, Bugia.
Jalac, Jalac.
Nuabia, Nuabia.
Dancala, Dancala.
Cusa, Cusa.
Gorham, Gorham.
Damocla, Damocla.
Somna, Somna.
The Empire of the Abissines, where are several Kingdoms, Countries, and Cities, the chief of which are
Tigremahon, Chaxumo.
Barnagasso, Barua,
Angota, Angotina.
Dancala, Degibeldara.
Amara, Amara.
Bagamedri, Baza.
Ambian, Amasen.
Damute, Damute.
Agag. Agag.
Cafates, Cafates.
Narea, Zeb.
Ambiam, Ambiam.
Æthiopia,
Barbary, or Zanguebar, which is divided into
Zanguebar, with its Kingdoms and Cities of
Mozambique.
Quiloa.
Quilmanca.
The Coast of Ajan, with its Kingdoms and Cities of
Adea.
Adel.
Magadoxo.
The Coast of Abex, its chief Places and Isles of
Arquico.
Suaquen.
Congo, with its several Kingdoms or Provinces, the chief of which are
Loango. Loango.
Pemba, Pemba.
Angola, Engaze.
Bamba, Bamba.
Songo, Sonho.
Lower, or interior, and comprehendeth
The Coast of Cafres, with its several Estates, Kingdoms, Capes, Ports, and Isles; the chief of which are the
Cape of Good Hope.
Cape of S. Nichol.
Port of Carascalis.
Isles of S. Christoph.
Isles of S. Lucia.
Monomotapa, with its Kingdoms, and chief Places,
Monomotapa. Monomotapa.
Butua, Butua.
Monoemigi, Agag.
Zefala, Zefala.
Chicanga, Zimbaos.
Divers Isles.
In the Mediterranean Sea,
Malta, Valetta.
In the western Ocean, as
Canary Isles, Canaria.
The Isles of Cape Verd, St. Jago.
The Isles of S. Thomas, Pavoasan.
In the Eastern Ocean, as
Madagascar, Vingazora.
Zocotora, Zocotora.

Of AMERICA.

AMERICA, otherwise called the *New world:* (1) *New* for its late discovery; (2) *World*, for its Greatness: The most usual (though the most improper) Name is *America*, so called from *Americus Vespasius*, an adventurous *Florentine*, who discovered a great part of the *Continent* of it, in the year 1500, but since that the first light which he had to find out those Parts, came from the *directions* and *example* of *Columbus*, who first led the way *Anno* 1492. and that *Sebastian Cabot* touched at many Places, which *Americus Vespasius* never saw; it might as properly have been called *Columbiana*, *Sebastiana*, or *Cabotia*, as *America*; but oftentimes it is called the *West-Indies: West* in respect of its Situation, and *Indies* for its *Wealth:* What kind of Country it is, and what it produceth we do now know in part, and future Ages may know more. This large Part of the Earth was unknown to the *Europeans* till 1492. one *Christopher Columbus* was furnished by the Court of *Spain* with three Ships, only for discovery, with which at about 63 days Sailing upon the Ocean descried *Fire*, which was in an Island near the Coasts of *Florida*, called by the Natives *Gnnahani*, now *Salvador*. After whom *John Cabot*, a *Venetian*, *Anno* 1497, in behalf of *Henry* VII. of *England*, discovered all the *North-East* Coasts thereof, from *Cape Florida*, to the South, beyond *New-Found-Land* in the North; causing the *Royalets* to become *Homagers* to the Crown of *England*. Next after him succeeded *Americus Vespasius*, employed by *Emanuel* King of *Portugal*, in the year 1500. to find out a nearer way to the *Moluccos*, than by the *Cape of Good hope*; notwithstanding, in that his enterprise he passed no farther than the *Cape* of *Augustine* on the Coasts of *Brasil*, this vast Part of the *World* retains his Name, as aboveSaid.

As for this vast tract of Ground in general, it hath the advantage of being Temperate and Fruitful, by reason of its Great and Fair *Rivers*, and the fresh *Breezes* that blow in the *Torrid Zone*; whereby we find that the cause of Violent or Remiss Heats, do not always proceed from the nearness or distance of the *Sun*, but many times from the Situation of the *Place*; the disposal of the *Mountains* or *Lakes*; the quality of the *Soyle*; and the nature of the *Winds* that blow.

The *wealth* of *America* is so vast, that *Spain* hath, and still doth draw from thence prodigious quantities of *Gold* and *Silver*, and the *Mines* of *Peru* have furnished him with many *Millions*; and his Revenue yearly, (by impositions laid upon *Goods* transported thence; as *Gold*, *Silver*, *Pearls*, *Emraulds*, *Skins*, *Sugar*, *Tobacco*, *Cochenille*, *Sarsa-perilla*, *Ginger*, and other *Commodities*,) amounts unto above 12 Millions.

As for the *Original* of these People, it is probable they did descend from the *Tartars*, if it be so, that the *West* side of *America* be Continent with *Asia*, or separated by some small Strait.

At the first arrival of the *Spaniards*, they found the *People naked*, *ignorant* of all things they had not seen; wondering strangely to see their *Ships* and *Horses*; and admired to see them know the Affairs of one another by reading of *Letters*.

They have as many *Languages* as *Towns*. They are naturally Active, swift *Runners*, and good *Swimmers*.

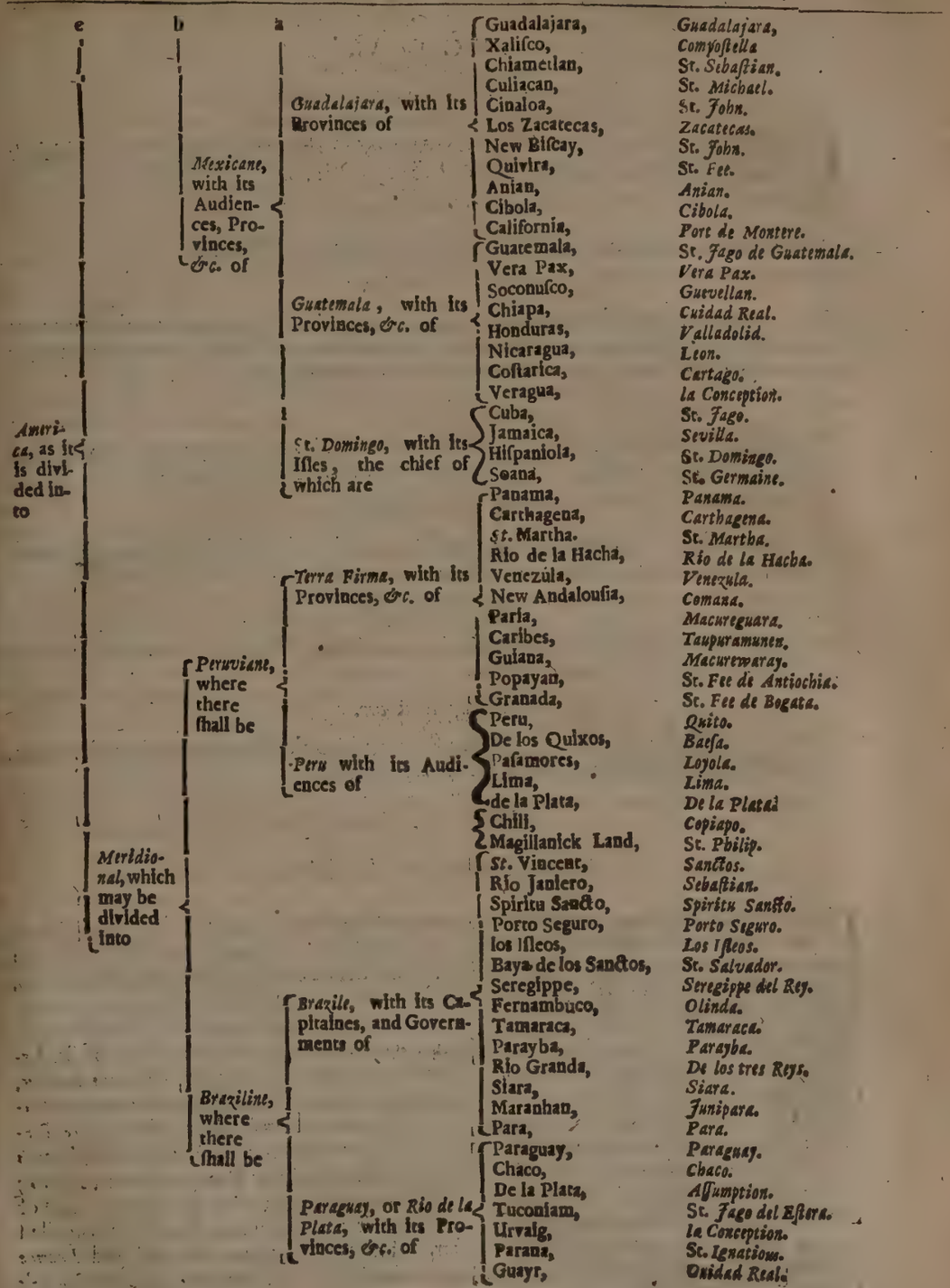
The *Mexicans* and *Peruans* were the only *Americans* that lived in Cities, which Cities, though Founded by those People which we call *Savages*, were no way inferior to ours in *Europe*, either for *Magnitude* or *Magnificence*. ——— The *Spaniard* possesses the Richest, Largest, and most Fertile Provinces, *Mexico* and *Peru*, formerly two Kingdoms; one *Hereditary*, the other *Elective*. ——— The *Portugueses* have the Coasts of *Brasil*. ——— The *French* have their *Colonies* in *Canada*, in several Islands, and upon the firm *Land*. ——— The *English* are fairly seated along the Coasts of North *America*, and in the Islands. ——— And the *Dutch* have got many Places on the Continent and Islands.

This large Continent is encompassed on the East by the North Sea, or *Mar del Nort*; on the West by the South Sea, or *Mar del Zur*; and on that Part which hides it self under the Pole of the *Erymanthean Bear*, to the Straits of *Magellan*, the North and South Seas meet and embrace together, dividing it (as is supposed) from either Pole.

AMERICA.

- **Septentrional**, which may be divided (a)
  - **Canadian**, where there shall be (b)
    - The *Artick* Lands, as
      - Island, — *Belleſted.*
      - Groenland, — *Beareford.*
      - Northwales, — *Seahorſe point.*
      - Southwales, — *Hudſons bay.*
    - *Canada*, or *New France*, whose chief Parts and People are those of (c)
      - Eſtoitland, — *Hope advanced.*
      - Saguanay, — *Quebec.*
      - Canada, — *Mon Real.*
      - Acadie, — *Martengo.*
      - New England, — *Boſton.*
      - Mary Land, — *Mary's Town.*
      - Virginia, — *James Town.*
      - Carolina, — *Charles Town.*
      - *The* Hurons, — St. *Peter.*
      - *Iſle of* Bermudas, — *Southampton.*
      - Florida, — St. *Hellens.*
  - *Mexico*, with it's Provinces and chief Places of (c)
    - Mexico, — *Mexico.*
    - Panuco, — *Panuco.*
    - Mechoacan, — *Mechoacan.*
    - Tlaſcala, — *Tlaſcala.*
    - Guaxaca, — *Antequera.*
    - Tabaſco, — *No. Sen. de la Victoria.*
    - Jucatan, — *Merida.*

Mexi-


e
b
a
America, as it is divided into
Meridional, which may be divided into
Mexicane, with its Audiences, Provinces, &c. of
Peruviane, where there shall be
Braziline, where there shall be
Guadalajara, with its Provinces of
Guatemala, with its Provinces, &c. of
St. Domingo, with its Isles, the chief of which are
Terra Firma, with its Provinces, &c. of
Peru with its Audiences of
Brazile, with its Capitaines, and Governments of
Paraguay, or Rio de la Plata, with its Provinces, &c. of
Guadalajara, Guadalajara,
Xalisco, Compostella
Chiametlan, St. Sebastian.
Culiacan, St. Michael.
Cinaloa, St. John.
Los Zacatecas, Zacatecas.
New Biscay, St. John.
Quivira, St. Fee.
Anian, Anian.
Cibola, Cibola.
California, Port de Montere.
Guatemala, St. Jago de Guatemala.
Vera Pax, Vera Pax.
Soconusco, Guevellan.
Chiapa, Cuidad Real.
Honduras, Valladolid.
Nicaragua, Leon.
Costarica, Cartago.
Veragua, la Conception.
Cuba, St. Jago.
Jamaica, Sevilla.
Hispaniola, St. Domingo.
Soana, St. Germaine.
Panama, Panama.
Carthagena, Carthagena.
St. Martha. St. Martha.
Rio de la Hacha, Rio de la Hacha.
Venezula, Venezula.
New Andalousia, Comana.
Paria, Macureguara.
Caribes, Taupuramuneu.
Guiana, Macureparay.
Popayan, St. Fee de Antiochia.
Granada, St. Fee de Bogata.
Peru, Quito.
De los Quixos, Baesa.
Pasamores, Loyola.
Lima, Lima.
de la Plata, De la Plata.
Chili, Copiapo.
Magillanick Land, St. Philip.
St. Vincent, Sanctos.
Rio Janiero, Sebastian.
Spiritu Sancto, Spiritu Sancto.
Porto Seguro, Porto Seguro.
los Isleos, Los Isleos.
Baya de los Sanctos, St. Salvador.
Seregippe, Seregippe del Rey.
Fernambuco, Olinda.
Tamaraca, Tamaraca.
Parayba, Parayba.
Rio Granda, De los tres Reys.
Siara, Siara.
Maranhan, Junipara.
Para, Para.
Paraguay, Paraguay.
Chaco, Chaco.
De la Plata, Assumption.
Tuconiam, St. Jago del Estra.
Urvaig, la Conception.
Parana, St. Ignatius.
Guayr, Cuidad Real.

Sect. VI.

SECT. VI.

Of the Water and its Parts.

THE Earth (as was said before) is encompassed about with the Water, which is either *Ocean*, *Seas*, *Straits*, *Creeks*, *Lakes*, or *Rivers*.

1. *Of the Ocean.*

The *Ocean* is a general Collection or Rendezvouz of all Waters.

2. *Of the Sea.*

The *Sea* is a part of the *Ocean*, and is either Exterior, lying open to the Shoar, as the *British* or *Arabian* Seas; or Interior, lying within the Land, to which you must pass through some *Strait*, as the *Mediterranean* or *Baltick* Seas.

3. *Of a Strait.*

A *Strait* is a narrow part or Arm of the *Ocean*, lying betwixt two Shoars, and opening a way into the *Sea*, as the *Straits* of *Gibralter*, the *Hellespont*, &c.

4. *Of a Creek.*

A *Creek* is a small narrow part of the *Sea* that goeth up but a little way into the Land, otherwise called a *Bay*, a *Station*, or *Road* for Ships.

5. *Of a Lake.*

A *Lake* is that which continually retains and keeps Water in it, as the *Lake Nicurgua* in *America*, and *Zaire* in *Africa*.

6. *Of a River.*

A *River* is a small Branch of the *Sea* flowing into the Land, courting the Bank, while their Arms display to embrace her silver Waves.

SECT. VII.

Of the Names of the Ocean.

FIrst of all, according to the four Quarters it had four Names; from the *East* it was called the *Eastern Ocean*, from the *West* the *Western*, from the *North* the *Northern*, and from the *South* the *Southern*. But besides these more general Names, it hath other particular appellations, according to the Regions or Countries it boundeth upon, and the Nature of the Sea: *viz.* as it lies extended toward the *East*, it is called the *Chinean* Sea, from the adjacent Country of *China*; so the *Archipelago* of St. *Lazarus*, from the multitude of Islands. Toward the *South*, it is called *Oceanus Indicus*, or the *Indian* Sea, because upon it lies the *Indians*. The Gulph of *Bengala*, from *Bengala* a City in the *Indies*. Where it touches the Coast of *Persia*, it is called *Mare Persicum*: so also *Mare Arabicum*, from *Arabia*; so towards the *West*, is the *Æthiopian* Sea. Then the *Atlantick* Ocean, from *Atlas* a Mountain or Promontory in *Africa*; by the *Spaniards* called *Mar del zur*, as also *Mare Pacificum*; and on the other side of *America* is called by them *Mar del Nort*. Where it touches upon *Spain*, it was called *Oceanus Cantabricus*, now the Bay of *Biscay*. The Sea between *England* and *France* is called the *Chanel*; between *England* and *Ireland*, the *Irish* Sea, by some St. *George's* Chanel: between *England* and *Holland*, it is called by some the *German* Ocean, by others the *British* Seas; *Vulgò*, the Narrow Seas: beyond *Scotland* it is called *Mare Caledonium*; higher towards the *North*, it is called the *Hyperborean* or *Frozen* Sea; more *Eastward* upon the Coast of *Tartaria*, the *Tartarian* Sea, *&c.* And this shall suffice concerning the *Ocean*, or Exterior *Seas*.

SECT.

SECT. VIII.

Of the Names of the Inland Seas.

THE *Baltick* Sea, of old *Sinus Codanus*, by some called the *East-Sea*, by the Inhabitants the *Belt*, lying between *Denmark* and *Sweden*, the entrance whereof is called the *Sound*.

Secondly, the *Euxine* Sea, or the *Black* Sea, by some *Mare Caucasium, Scythicum, Sarmaticum, Colchicum*; by the *Turks, Caradinizi*: to which joyns *Meotis Palus*, now *Mar de Zabacken*.

The third is the *Caspian* or *Hircanian* Sea; by the *Turks, Mar de Sala*; by the *Persians*, the Sea of *Backu*; by the *Moscovites, Chwalenskei Mare*. The length is from *North* to *South*, and the breadth from *East* to *West*, contrary to all the Antient *Geographers*; which is certainly discovered, not only by the exact observation of *Olearius*; but also by his curious enquiries of the true Situation of its Maritime places according to the Longitude and Latitude of the *Persians*, as also by the *Astronomical Calculations* of Mr. *Graves*, &c.

The fourth, is the *Arabian* Gulph, *Mare Erythæum*, or *Rubrum*, *Vulgò* the *Red* Sea; by others *Mare Rasso*, and *Mar de Mecca*.

The fifth, is the *Persian* Gulph, or the Gulph *de Elcatife*.

The sixth, is *Mare Mediterraneum*; by the *English*, the *Straits*; by the *Spaniards*, *Mar de Levant*: the beginning or entrance of it is called the *Straits* of *Gibralter*, rather *Gibraltarec, olim Gaditanus*. This Sea hath many names, as it toucheth upon several Countries; the particular account whereof I shall refer to *Geographical* Description of *Maps*, &c. The length of it is by our *New Globes* not 37 *deg.* of the *Equinoctial* from *Tangier* to *Scanderone*: by other *Globes* and *Maps* it is more than 42 *deg.* of the *Equinoctial*. And this shall suffice for a Description of the Water, and its Parts.

Now that all Places, Cities, Towns, Seas, Rivers, Lakes, *&c.* may be readily found out upon the *Globe*, all *Geographers* do or should place them according to their *Longitude* and *Latitude*: but several *Geographers* do so differ in their beginning of their *Longitude*, that it is very difficult to understand.

SECT. IX.

Of the Difference of Authors concerning Longitude.

TO *say the truth, by reason of the variety of* Meridians, *the* Longitudes *are grown to such an uncertainty and confused pass, that it is not every mans work to set them down. This indeed I have observed, that many* Geographers, *or rather describers of particular places, tell us that such a place is so many degrees of* Longitude; *but from what* Meridian, *others must guess. Some particularly profess to follow* Mercator: *but what are most men the wiser for this? for* Mercator's Meridian *was not always the same; sometimes through the* Canary *Islands, sometimes through the* Azores. *Others again will tell you their* Meridian *shall pass through the* Azores; *but whether from that of St.* Michael, *or that of* Corvo, *is not set down; and yet* 6 *deg. of difference. I shall therefore take this course: First, set down the several* Meridians *observed. Secondly, the difference of* Longitude *betwixt these* Meridians. *Lastly, which of these I have fixt upon.*

1. The *Great Meridian* by *Ptolomy* and most of the Ancient and Greek *Geographers* was made to pass through *Junonca*, one of the Fortunate, now thought to be the *Canary* Islands.

2. By the *Arabian* and *Nubian Geographers*, through the utmost point of the *Western* Shoar, near *Hercules*'s Pillars.

3. *Ortelius*, in his Sheet-Map of *Europe*, makes *London* to lie in 28 *deg.* of *Longitude*; but in the Sheet-Maps of *France* and *Belgia* it lies in but 21 *deg.* so that his first *Meridian* to me is yet unknown. The *Spaniards* since the Conquest of the *West-Indies*, contrary to all other, account their *Longitude* from *East* to *West*, beginning at *Toledo*.

4. Our Modern *Geographers*, as *Mercator*, *Cambden*, *Speed*, and others, removed it into the *Azores*; some placing it at St. *Michaels*, others at *Corvo*.

5. *Blaew*, the *Dutch Geographer*, begins his *Longitudes* from *Tenerif*, one of the *Canary* Islands; but upon his *Great Map*, the *Great Meridian* passeth through *Tercera*-Isle,

one

one of the *Azores*; which the reſt of their *Common Map*-makers, *De Wit*, *Viſher*, &c. as well as many of our *Engliſh*, are bound to follow through ignorance, tranſcribing as well his Errors as his Copies for the beſt.

6. *Sanſon*, the *French Geographer*, for ſome Reaſons (beſt known to himſelf) begins his *Longitudes* at *Ferro*, one of the *Canary* Iſlands; and therefore *Blome*, his Tranſlator, is bound to follow it, though poſſibly he cannot tell ſo much, and yet the Kingdoms great pretender to *Geography*.

7. A late *Engliſh Hydrographer* tells us, that with a great deal of Reaſon and Conſideration he placed his firſt *Meridian* at *Gracioſa*, one of the Iſlands of the *Azores*; but it is delineated upon his *Globes* and *Maps* through *Tercera*, almoſt 2 *deg.* more *Eaſtward*: a ſmall miſtake, that another muſt come after him to tell himſelf what *Meridian* he went by.

Secondly, The differences of theſe ſeveral *Meridians* I find are thus ſtated.

From *Ptolomy*'s *Meridian* to the *Arabian Meridian* was by *Abalfeda*, in his Introduction to his *Geography*, accounted to be 10 *deg.* of the *Æquator*; *Briet* ſaith but 8 *deg.*

			d.	*m.*	
From the *Pico* of *Tenerife* to	*Toledo*	is	15.	55	the *Spaniſh*
	Gracioſa		10.	25	the miſtaken *Engl.*
	Tercera		9.	0	the ſuppoſed *Dutch*
	Palma or *Ferro*		2.	5	the *French*
	Corvo		13.	25	*Mercator*, or o-
	St. *Michael*		8.	5	thers

Meridian.

And this laſt is the *Meridian* from which the *Longitudes* are reckoned in the new *Terreſtrial Globe*, and in ſeveral *Maps* that are lately ſet forth by Mr. *Robert Morden*, and Mr. *William Berry*, which I take to be the beſt that ever the World afforded.

SECT. X.

Of the Diviſion of the World by Zones, Climates *and* Parallels.

THE *Zones*, *Climates*, and *Parallels* are to be conſidered two ways, either in *themſelves*, or elſe in their *Adjuncts* or *Inhabitants* belonging to them; of both which we ſhall here give an account.

1. *Of* Zones.

A *Zone* ſignifieth as much as a *Girdle* or *Band*, becauſe by it theſe ſpaces in the Earth are with larger *Bands* compaſſed about. *A* Zone *therefore is a Space comprehended within two named and leſſer* Parallels, *or between one* Parallel *and the* Pole *it ſelf.*

The *Zones*, into which the *Terreſtrial Globe* is divided, are in number *Five*, which diviſion is generally received; and hath been familiar with the old Latin Poets, as appears by *Virgil*,

Quinque tenent Cœlum Zona, &c.

Five Zones *ingirt the Sky, whereof one Fries*
With fiery Sun-beams, and all ſcorched lies.
Both which the fartheſt off on either hand
The blew-ey'd Ice and brackiſh Shoars command.
'Twixt theſe two and the midſt the Gods do give
A wholſome place for wretched Man to live.

And *Ovid* to the ſame purpoſe,

Two Girdles on the Right Hand, on the Left
As many, cut the Skies; more hot's the Fifth.
So God dividing with an equal Hand,
Into ſo many parcels cuts the Land;
The midſt, through heat, affords no dwellers eaſe.
The deep Snow wraps up two: but betwixt theſe
And th' other Regions, are two places ſet,
Where Froſts *are mixt with* Fires, *and* Cold *with* Heat.

This

This distinction of the *Zones* being too general, I shall divide them according to the method of our times, in this manner.

The *Zones* are either *Temperate* or *Intemperate*, and the *Intemperate* are either *Cold* or *Hot*. 1. The *Intemperate Hot Zone* is that space of the Earth which is contained between the two Tropick Circles, of *Cancer* and *Capricorn*. This *Zone* circumscribes within it two Great Circles of the *Sphere*, *viz.* the *Equinoctial* running through the midst thereof at right Angles, and the *Ecliptick* Obliquely crossing it, and meeting in the two Tropicks twice in the Year.

The extent or breadth of this *Zone* is 47 degrees, which resolved into Miles, by allowing 60 degrees to one Mile, is Miles 2820: for between the *Equinoctial* and either *Tropick* are included 23 degrees and a half, which make that summ.

Within the compass of this *Zone* is included the greatest part of *Africa*. Also many Islands, as *Java*, *Samatria*, *Taprobana*: and a great part of the South *America*, called *Peruana*.

This *Zone* was imagined by the Ancients, as *Aristotle*, *Pliny*, *Ptolomy*, and many other *Philosophers*, *Divines*, and *Poets*, to be altogether inhabitable, by reason of extreme heat, and therefore they called it *Intemperate*: but later discoveries have proved the contrary.

2. *The Intemperate Cold Zones* are included between the *Polar Circles* and the *Poles* of the World, whereof one is *Northern*, and is contained between the *Arctick Circle* and the *North Pole*. The other *Southern*, contained between the *Antarctick Circle* and the *South Pole*.

The greatness and extent of these *Zones* are each of them 23 degrees and a half, which reduced into Miles make 1410 Miles.

The Northern Cold Zone, contains in it *Greenland*, *Fineland*, and divers other *Northern Regions*, some whereof are not yet fully discovered.

The Southern Cold Zone, hath the same *Magnitude* as the *Northern*; but for other matters they lie hid in the Gulf of obscurity, no Discovery having to us as yet been made. And whether these two *Zones* be uninhabited by reason of intemperate *Cold*, as the other hath been thought by reason of two much *Heat*, this is no place to discourse.

3. *The Temperate Zone*, is the Space contained between the *Tropicks* and the *Polar Circles*; whereof the one is *Northern*, and is contained between the *Tropick* of *Cancer*, and the *Arctick Circle*. The other *Southern*, contained between the *Tropick* of *Capricorn* and the *Antarctick Circle*.

These *Zones* are termed *Temperate* for these Reasons.

1. Because the Sun-Beams are cast obliquely, and so cannot cast so much heat as in those places where they are Perpendicularly darted downwards.

2. Because the *Zones* seem to be mixt of both Extremes, partaking in some measure of both the qualities of *Heat* and *Cold*: the one from the *Torrid*, the other from the *Frigid Zone*.

3. Because in these *Zones* the distances between *Summer* and *Winter* are very remarkable, having a middle difference of *Time* betwixt them, is compounded of both *Extreams*.

The extent or breadth of either of these *Zones* is 43 degrees, which resolved into Miles is 2820 Miles.

The Northern Temperate Zone contains in it the upper and higher part of *Africa*, stretching even to the *Mountain* of *Atlas*. In it is placed all *Europe*, even to the *Northern Islands* in the *Frigid Zone*, and a great part also of *Asia*.

The Southern Temperate Zone, is not so well known to us, it being far distant from our Habitation, and awaiting, as yet, the farther Industry of our Heroick and Industrious *English Navigators*.

2. *Of* Climates.

A *Climate* is a Space of Earth contained between two *Parallels*, distant from the *Equinoctial* towards either *Pole*.

That the *Zones* and *Climates* do agree in form, is evident by the definition of them, but they differ in *Greatness*, *Number* and *Office*.

1. In *Greatness*, because the *Zones* do occupy *Greater*, the *Climates Lesser* Spaces in the *Earth*.

2. In *Number*, the *Zones* being only *Five*, but there are many more *Climates*.

3. In *Office*, *Use* and *Effect*; because the *Zones* are to distinguish the mutation of the quality

quality of the *Air*, and *Shadows*, according to divers *Regions* of the *Earth*. But the *Climates* are uſed to ſhew the greateſt differences of hours in the day; the variation of the *Riſing* and *Setting* of the *Stars*: for places under the ſame *Climate* have the ſame quantity of *Days* and *Nights*, the ſame *Riſing* and *Setting* of the *Stars*: whereas places ſeated under divers *Climates* have a great variation in *Days* and *Nights*, and a divers *Riſing* and *Setting* of the *Stars*. For as often as the *Solſtitial* or *Longeſt day* of one place, differs from the *Longeſt day* of another, by the ſpace of half an hour, a new *Climate* is placed. Wherefore under the *Æquator* the Days are always *Equal*, namely twelve hours long; and therefore, beginning at the *Equinoctial*, if we approach towards either *Pole*, ſo far as that the longeſt *Artificial Day* amounts to twelve hours and a half, we may aſſure our ſelves that we are come to the firſt *Climate*. And proceeding nearer towards either *Pole*, till the *Longeſt day* be 13 hours, we are then arrived to the Second *Climate*, and ſo forward.

But although all *Climates* are placed according to the equal increaſe of half an hour in the length of the *Longeſt day*, yet the *Climates* themſelves compared one with the other are not all of the ſame greatneſs. For no one *Climate* is equal to another in the ſame *Hemiſphere*, but are ſtill leſſer one than the other, by how much they are removed from the *Equinoctial* towards either *Pole*: for the firſt *Climate* from the *Equinoctial* is reckoned to be in the *Latitude* of about 8 *deg.* 34 *min.* which is 514 Miles, the ſecond begins in 16 *deg.* 43 *min.* both which make 1003 Miles, which is ſomewhat leſs than the former, for 514 taken from 1003 there remains 489 for the Second *Climate*, and ſo of the reſt, decreaſing, whereas the laſt is not ſo many minutes as quarters of Miles.

Concerning the *Number* of *Climates* there have been great differences among *Geographers*; *But a* Climate *is now defined by later Writers to be a* Space *comprehended between three* Parallels *in the* Habitable Earth, *whence the length of the* Longeſt day *is increaſed by half an hour*. Hence it muſt needs follow, that from the *Equinoctial* (where the Day is always 12 hours long) to the *Polar Circles* (where the *Longeſt day* is 24 hours long) there ſhould be placed 24 *Climates*; and this is the *Certain* diſtinction of *Climates*: —— There is another diſtinction which they call *Uncertain*, which is between the *Polar Circle* and the *Pole* it ſelf: it is termed improper, becauſe in thoſe *Climates* the *Day* is not increaſed by *half Hours*, as in the former, but firſt by whole *Days*, then by *Weeks*, and laſt of all by whole *Months*; inſomuch that under the *Pole* it ſelf they have *ſix Months* perpetual *Day*, and ſo long again a perpetual *Night*. But *Climates* are Spaces *Parallel* to the *Equinoctial*, containing *three Parallels*, the *Middlemoſt* dividing the ſame into two *Parts*: The *Latitude* of each *Climate* from its *Southern* to its *Northern Limit* is of that extent, that the *Longeſt day* increaſes half an *Hour*.

The *Climates* are *Denominated* from the Chief *Cities* or *Places*, by, or through which they paſs; as the Firſt by *Meroe*; the Second by *Alexandria*, the Third by *Rhodus* and *Babylon*, the Fourth by *Rome*, *Corſica*, and the *Helleſpont*, the Fifth by *Venice*, the Sixth by *Podolia*, the Seventh by *Witeberg*, the Eighth by *Roſtoch*, the Ninth by *Ireland*, the Tenth by *Bohuſe* in *Norway*, the Eleventh by *Gothland*, the Twelfth by *Wyburgh* in *Finland*, the Thirteenth by *Arotia* in *Sweden*, the Fourteenth by the Mouth or Outlet of the River *Darecally* in *Sweden*; and the reſt by other Places of *Norway*, *Sweden*, *Alba*, *Ruſſia*, and the adjacent *Iſlands*.

Theſe are the *Northern Climates*; the *Southern* are diſtinguiſhed by the Title of *Anti*, as *Anti-Meroes*, *Anti-Alexandrias*, &c. in regard of their Oppoſite Poſition to the *Northern Climates*.

But for the better underſtanding of the *Climates*, and what appertains to them, ſee the following *Table*, which is divided into Five *Columns*: the Firſt ſhews the *Number* of the *Parallels*, the Second of the *Climates*, the Third the *Longeſt day* in each *Parallel*, the Fourth the *Latitude*, or Elevation of the *Pole* of the reſpective *Regions* through which they paſs, the Fifth the *Amplitude* of the *Climates*.

Number

Numb. of Parallels.	Climates.	Long. Da. in each Parallel.		Latitude of Places.		Amplit. of each Climate.	
		H.	*M.*	*De.*	*M.*	*De.*	*M*
1	I	12	15	4	18	0	0
2		12	30	8	34		
3	II	12	45	12	43	7	50
4		13	0	16	43		
5	III	13	15	20	33	7	03
6		13	30	23	11		
7	IV	13	45	27	36	6	09
8		14	0	30	47		
9	V	14	15	33	45	5	17
10		14	30	36	30		
11	VI	14	45	39	20	4	30
12		15	00	41	12		
13	VII	15	15	43	32	3	48
14		15	30	44	29		
15	VIII	15	45	37	20	3	13
16		16	0	49	1		
17	IX	16	15	50	53	2	44
18		16	30	51	58		
19	X	16	45	53	17	2	17
20		17	0	54	29		
21	XI	17	15	55	34	2	0
22		17	30	56	37		
23	XII	17	45	57	34	1	40
24		18	0	58	26		

Numb. of Parallels.	Climates.	Long. Da. in each Parallel.		Latitude of Places.		Amplit. of each Climate.	
		H.	*M.*	*De.*	*M.*	*De.*	*M.*
25	XIII	18	15	59	14	1	16
26		18	30	59	59		
27	XIV	18	45	60	40	1	13
28		19	0	61	18		
29	XV	19	15	61	53	1	01
30		19	30	62	25		
31	XVI	19	45	62	54	0	52
32		20	0	63	22		
33	XVII	20	15	63	46	0	44
34		20	30	64	6		
35	XVIII	20	45	64	30	0	36
36		21	0	64	49		
37	XIX	21	15	65	9	0	29
38		21	30	65	21		
39	XX	21	45	65	35	0	22
40		22	0	65	47		
41	XXI	22	15	65	57		
42		22	30	66	6	0	16
43		22	45	66	14		
44	XXII	23	0	66	20	0	11
45		23	15	66	25		
46		23	30	66	28		
47	XXIII	23	45	66	30	0	6
48		24	0	66	31		

3. *Of* Parallels.

A *Parallel* is a Space upon the *Terrestrial Globe*, wherein the increase of the *Longest Day* is a quarter of an *Hour*.

Concerning these *Parallels* little can be said more than hath been said concerning *Climates*; only we must consider that a *Parallel* may be taken in two senses, *viz.* 1. Either as a *Line* or *Circle*, as defined in the Description of the *Circles of the Sphere*, void of any breadth: Or, 2. As a *Space of the* Earth *intercepted between two* Circles Parallel *to the* Equinoctial Circle, and in this sense it is here to be understood.

Now these *Parallels* are denominated from the increase of the *Longest Day* by a quarter of an *Hour*, as the *Climate* was by the increase thereof by half an *Hour*, yet these *Parallels* do no where divide the *Climates* into two Equal parts. For in the *Climates* we are to consider two things, 1. Their *Latitude* from *North* to *South*; Or, 2. Their *Longitude* from *East* to *West*. In respect of the former we cannot call the *Parallel* half the *Climate*, because the three *Lines* of which the *Climate* consists (namely the *middle* and the two *extremes*) are not in any case of an equal distance: but if we consider the extent of the *Circumference*, as it stretcheth it self between *East* and *West*, we must needs acknowledg much more; as that of two *Parallels* dividing the same *Climate* between them: that that is manifestly the greatest which is next the *Equinoctial*, and that the least which is nearest the *Pole*; because the Circles which comprehend their *Parallel* Space continually decrease towards the *Pole*. So that if we imagin two Men to Travel round about the *Earth*, the one in a *Parallel* nearer the *Æquator*, and the other in a *Parallel* nearer the *Pole*, in the same Space of time, it must needs follow, that he should go far faster which is nearest the *Æquator*, than the other nearest the *Pole*.

SECT.

SECT. XI.

Of the diverſities of Days *and* Nights, *and of the various* Seaſons *and* Shadows *Incident to the ſeveral* Inhabitants *of the* Earth.

1. *Of them who are right under the* Equinoctial Circle.

IT is to be obſerved that they whoſe *Zenith* is in the *Equinoctial Circle*, have in one *Year* two *Summers*, which are extreme *Hot*, in regard the *Sun* twice paſſeth directly over their Heads, which is when he is in the beginning of *Aries* or *Libra*, and have likewiſe two *Winters*, which cometh to paſs when the *Sun* is in the firſt point of *Cancer* or *Capricorn*; for then the *Sun* is fartheſt diſtant from them, (notwithſtanding their *Winters* are very *Warm*, and not comparable in *Cold* to ours;) ſo that it appears that our two *Seaſons* of *Spring* and *Autumn*, are to them two *Summers*, and our two *Seaſons* of *Winter* and *Summer*, are to them two *Winters*. Moreover, they have *Four* ſundry *Shadows*, for when the *Sun* is in either of the *Equinoctial Points*, then in the Morning they caſt their *Shadow* towards the *Weſt*; in the *Afternoon* they caſt it towards the *Eaſt*; but at *Noon* their *Shadow* is Perpendicular or right down. Alſo when the *Sun* is in the *Northern Signs*, then they caſt their *Shadows* towards the *South*, and when he is in the *Southern*, they caſt them towards the *North*. Nevertheleſs, their *Day* is always 12 *Hours* without variation, becauſe they dwell in a *Right Sphere*. Finally, they ſee all the *Stars* towards the *South Pole*, and about the ſame, which are unknown to us, yea they may ſee both the *Poles* at one inſtant, and therefore may behold all the *Stars* in the *Firmament* to *Riſe* and *Set*, as by the *Globe* or *Sphere* may appear: therefore *Lucan* ſaith,

Tunc furor extremos movit Romanus Horeſtas
Carmanáſque duces, quorum jam flexus in Auſtrum
Æther non totam, mergi tamen aſpicit Arcton,
Lucet & exigua velox ubi nocte Boötes.

But in our ſituation theſe *Stars* never Set; therefore *Virgil*,

Hic Vertex nobis ſemper ſublimis, at illum
Sub pedibus ſtyx atra videt, manéſque profundi.

And *Lucan* in another place:

Axis in Occiduus gemina clariſſimus Arcto.

2. *Of them who dwell between the* Equinoctial *and* Tropick *of* Cancer.

Thoſe whoſe *Zenith* is between the *Equinoctial* and *Tropick* of *Canter*, have the *Sun* paſſing by their *Zenith* twice in one Year, which may thus appear: let there be imagined a *Parallel Circle* to the *Equinoctial*, paſſing by their *Zenith*; this *Circle* will cut the *Zodiac* in two places equally diſtant from the beginning of *Cancer*: therefore the *Sun* being in theſe two Points, paſſeth by their *Zenith*; ſo that they have *two Summers*, and *two Winters*, and *four Shadows*, as they have under the *Equinoctial*. Some ſay *Arabia* is ſituate under this Clime; whereupon *Lucan* ſpeaking of the *Arabians* coming to *Rome* to help *Pompey*,

Ignatum vobis Arabes veniſtis in orbem,
Umbras mirati nemorum non ire ſiniſtras.

Becauſe, that in their own Country, their *Shadows* were ſometimes on the *Right Hand*, ſometimes on the *Left*, ſometimes *Perpendicular*, ſometimes towards the *Eaſt*, and ſometimes towards the *Weſt*: but when they come to *Rome*, which is on this ſide the *Tropick* of *Cancer*, then they had always their *Shadows* at *Noon tide* toward the *North*.

3. *Of thoſe who live right under the* Tropick *of* Cancer.

Thoſe People, whoſe *Zenith* is in the *Tropick* of *Cancer*, have the *Sun* but once in the *Year* in their *Zenith*, which cometh to paſs when he is in the firſt Point of the *Dodecatemory*, *Cancer* (the *Sun* being then in his *Solſtice*,) for their *Shadow* at *Sun Riſing* tendeth towards the *Weſt*; at *Night* when the *Sun* goeth down, it tendeth towards the *Eaſt*; but at *Noon* it is *Perpendicular*, inclining on neither ſide, but falling right

right down: and under this *Climate* is the *City* of *Syene* situated, so that *Lucan*, speaking of the *Zones*,

——— Calida medius mihi cognitus axis
Ægypto atque umbras nusquam flectente Syene.

4. *Of those who dwell between the* Tropick *of* Cancer, *and the* Circle Arctick.

Those People whose *Zenith* is between the *Tropick* of *Cancer*, and the *Circle Arctick*, have the *Sun* never *Vertical* over their Heads, because he is no sooner attained to his greatest *Altitude*, but immediately he returneth again *Southward*, and their *Shadow* at *Noon* always tends toward the *North*; and in this *Parallel* are we situated, and the greatest part of the inhabitable *World*: yet it is to be noted that *Ethiopia*, or some part thereof, is in the *Tropick* of *Cancer*, according to *Lucan* in his third Book,

Æthiopúmque solum quod non premeretur ab ulla
Signiferi regione poli, ni poplite lapso
Ultima curvati procederit ungula Tauri.

5. *Of those that live right under the* Circle Arctick.

They that have their *Zenith* in the *Circle Arctick*, have also, at all times of the *Year*, the *Zenith* to be the *Pole* of the *Zodiack*, and have the *Zodiack* or *Ecliptick Line* for their *Horizon*; so that the one half in an instant riseth above the *Horizon*, and the other part in the same instant goeth down, therefore *Alfranganus* well observes: *Quod ibi occidunt repentè sex signa, & reliqua sex oriuntur cum toto Æquinoctiali.* And seeing that the *Eclyptick* is their *Horizon*, the *Tropick* of *Cancer* must be all above the *Horizon*, and the *Tropick* of *Capricorn* all under the *Horizon*: so that the *Sun* being in the first Point of *Cancer*, their *Artificial Day* is just 24 *Hours*, and their *Night* but a moment; for in an instant the *Sun* passeth that *Horizon*, and forthwith begins to ascend from the *North* Point towards the *East*: so contrariwise it is to them, when the *Sun* is in the first Point of *Capricorn*, for then their *Night* is 24 *Hours*, and their *Day* but a moment.

6. *Of those that dwell between the* Circle Arctick, *and the* Pole *of the* World.

Those People that inhabit between the *Circle Arctick* and the *Pole Arctick*, have their *Horizon* cutting the *Zodiack* in two Points, equally distant from the beginning of *Cancer*, and by the revolution of the *Firmament*, it happeneth that this Portion of the *Zodiack* intercepted, remains always above the *Horizon*: whereby it appears, that when the *Sun* is in this part of the *Zodiack*, they have *continual Day*, and *no Night*, their *Day* is a *Month long*, without a *Night*. If the said Portion contains *two Signs*, then their *Day* is *two Months* long, and so the farther *North* the longer *Day*, till you come under the *Pole Arctick*, where it is *six Months* long. Also to them it doth so fall out, that the Portion of the *Zodiack*, intercepted of those two Points, *equidistant* from the beginning of *Capricorn*, is always remaining under the *Horizon*; so that when the *Sun* is in this Position intercepted, the *Night* is *short* or *long*, according to the quantity of the intercepted Portion; but the other *Signs* which *Rise* and *Set* to them, *Rise* and *Set* preposterously or backward, as when *Taurus* Riseth before *Aries*, *Aries* before *Pisces*, *Pisces* before *Aquarius*, &c. Nevertheless the *Signs* opposite to these *Rise* in right Order, and *Set* preposterously: as, *Scorpio* before *Libra*; *Libra* before *Virgo*; and yet notwithstanding the *Signs* opposite to these *Set directly*, to wit, those which did *Rise* preposterously, or backward, as *Taurus*, *Aries*, *Pisces*, &c.

7. *Of those that dwell right under the* Pole Arctick.

Those People (if there be any) have the *Pole* for their *Zenith*, and the *Equinoctial* for their *Horizon*; wherefore seeing that the *Equinoctial* divideth the *Zodiack* exactly into two equal parts, it is thereby manifest, that six *Signs* (which is the one half of the *Zodiack*) is always above the *Horizon*, and that six *Signs* (which is the other half of the *Zodiack*) is always under their *Horizon*; so that when the *Sun* runneth through the half, which is from the beginning of *Aries* to the end of *Virgo*, it is continual *Day*, without *Night*; and when the *Sun* runneth through the other half, which is from the beginning of *Libra* to the end of *Pisces*, it is continual *Night*, without; but yet it cannot be so dark there, as elsewhere, by reason the *Sun* is never depressed under their *Horizon* above 23 degrees, and 31 minutes; which is, when he enters *Capricorn*, wherefore the one half of their *Year* is their *Artificial Day*, and the other half of their *Year* is their *Artificial Night*: from whence it is, that their whole *Year* is but a *Natural Day*,

but yet it is as long as 365 *Days* of ours: and albeit that the *Sun* is half a *Year* together above their *Horizon*, yet he is never elevated above 23 *deg.* 31 *min.* whereby the *Day* is intolerable *Cold* at all times of the *Year*; and the Air is *Groſs*, *Cold*, and *Cloudy*, by reaſon that the Beams of the *Sun* are there but weak, and not able to exhauſt and conſume the *groſs Vapours* which there abound.

SECT. XII.

Of the various Noon Shadows, *that the ſeveral Inhabitants of the* Earth *have, in reſpect of their* Poſition *in* Zone *and* Latitude.

THE Inhabitants of a Place, in reſpect of the *Shadows*, are either *Amphiſcii*, *Heteroſcii*, or *Periſcii*.

I. *Of the* Amphiſcii.

Amphiſcii is a *Greek* Word, and ſignifies *People of a double Shadow*, and ſuch are they that inhabit between the *Æquator* and the *Tropicks*, and whoſe *Latitude* exceedeth not 23 *deg.* and a half. Theſe Men have the *Sun* twice every *Year* in their *Zenith*, and then they make no *Noon Shadow* at all; but when the *Sun* paſſeth from their *Vertical Point* towards the *Northern Signs*, then at *Noon* it will caſt the *Shadow* towards the *South Coaſt*, but contrariwiſe coming from the *Zenith*, towards the *Southern Signs*, the *Shadow* will be caſt towards the *North*.

II. *Of the* Heteroſcii.

Heteroſcii alſo is a *Greek* Word; and ſignifies *People* that have their *Shadow caſt but one way*. Theſe Nations inhabit in a *Temperate Zone*, betwixt the *Tropicks* and *Polar Circles*, whoſe *Latitude* is never leſs than 23 *deg.* 30 *min.* and never exceedeth 66 *deg.* 30 *min.* So that thoſe that dwell in this *Zone*, on the *North ſide*, have their *Noon Shadow* caſt towards the *North*, and thoſe on the *South ſide* of the *Equinoctial* towards the *South*.

III. *Of the* Periſcii.

Periſcii alſo is a *Greek* Word, and ſignifies Men that have their *Shadows caſt round about them*. Such is their Habitation who are included in the *Frigid Zone*, circumſcribed between the *Polar Circles*, and the *Poles* of the *World*, whoſe *Latitude* is never leſs than 66 *deg.* 30 *min.* but extendeth thoſe degrees of *Latitude*, even to 90 *deg.* where the *Pole* is in the *Zenith*: here the *Sun* never cometh to be over their Heads, but moveth about them on one ſide or other, and ſo cauſeth their *Shadows* to doe the like. Of theſe *Periſcii* there are two ſorts, the one comprehended between the *Arctick Circle* and the *North Pole*, the other between the *Antarctick Circle* and the *South Pole*: of both which we have as yet little Diſcovery.

SECT. XIII.

Of the Poſition *of the* Inhabitants *of the* Earth, *compared one with another.*

THE Inhabitants of the Earth, compared one with another, in reſpect of their *Poſition*, are *Periœci*, *Antiœci*, and *Antipodes*.

I. *Of the* Periœci.

The Word *Periœci*, in the *Greek*, ſignifies ſuch People that dwell in two oppoſite Points of the ſame *Parallel*; the one (in reſpect of the other) being *Eaſtward*, and the other *Weſtward*, and being diſtant one from the other 180 *deg.* or a *Semicircle*, but not of the *Equinoctial*, or any other *Great Circle*, but of ſuch a *Circle* as they are placed in, which *Circles* are greater or leſſer, as they are nearer, or farther diſtant from the *Poles*.

II. *Of the* Antiœci.

The *Antiœci* (as the name imports) are ſuch as dwell *one againſt another*, having one and the ſelf-ſame *Meridian*, and an equal diſtance or *Latitude* from the *Æquator*, one being ſeated in the *Northern*, the other in the *Southern Hemiſphere*.

III. *Of*

III. *Of the* Antipodes.

Antipodes (as the Word ſignifies) are ſuch as dwell *Feet* to *Feet* one to another. But *Antipodes* are ſuch People as inhabit under one and the ſame *Meridian*, but under two *Parallels*, both *equidiſtant* from the *Æquator*, and in two oppoſite Points of thoſe *Parallels* alſo: ſo that a Right *Line* being drawn from one to the other, will paſs by the *Centre* of the *Earth*, whence they are exactly diſtant the one from the other 180 *deg.* of a *Great Circle.*

Corollary.

Thoſe which are to us *Periæci*, are the *Antiæci* to our *Antipodes.* Our *Antiæci* are the *Periæci* to our *Antipodes.* And our *Periæci* are the *Antipodes* to our *Antiæci.*

SECT. XIV.

How the Periœci, Antiœci, *and* Antipodes *are diverſly diſtinguiſhed, in reſpect of the* Cœleſtial Appearances.

I. *Of the* Periœci.

THE Properties of the *Periæci*, in reſpect of *Cœleſtial Appearances*, are chiefly Four.

1. They have the ſame *Elevation* of the *Pole*, and therefore the ſame *Temper* of the *Year*, and the ſame length of *Days* and *Nights.*
2. They dwell *Eaſt* and *Weſt* of each other.
3. They have contrary times of *Day* and *Night*; for when one hath his Noon, the other injoys his *Mid-Night*, and when with the one the *Sun Riſeth*, with the other it *Setteth.*
4. They have the ſame *Zone*, *Climate*, and *Parallel*, but differ in *Longitude* 180 *deg.*

II. *Of the* Antiœci.

The Properties of the *Antiæci* are Five.

1. They inhabit the like *Zones*, but in different *Hemiſpheres.*
2. They have the ſame *Elevation* of the *Pole*, but of the contrary *Kind*, the one beholds the *North*, the other the *South*, equally *Elevated.*
3. They have *Noon* and *Mid-Night* juſt at the ſame times.
4. They enjoy the ſame *Temper* of the *Heavens.*
5. They have the *Seaſons* of the *Year* contrary; For when the *Southern Antiæci* have their *Summer*, the *Northern* have their *Winter*, and the contrary.

III. *Of the* Antipodes.

To the *Antipodes* are allowed Three Properties.

1. They have the ſame *Elevation* of the *Pole*, but not of the ſame *Pole.*
2. They have the ſame *Temper* of the *Year*, and the ſame quantity of *Days*, but diverſly oppoſite to each other.
3. They have all the other *Accidents* contrary; for when one hath *Night*, the other hath *Day*; when one *Winter*, the other *Summer*; when one *Autumn*, the other *Spring.* They on the *North ſide* ſee not thoſe *Stars* towards the *South Pole*, nor thoſe on the *South ſide* thoſe about the *North Pole.* Thoſe *Stars* which never *Riſe* to the *Northern*, are always above the *Horizon* to the *Southern*; with divers the like *Accidents.*

SECT. XV.

Of Years, Months, Days, *&c.*

I. *Of* YEARS.

A *YEAR* is that principal and ordinary part of *Time*, whereby the *Beginnings*, *Progreſs*, *Duration*, and *Intervals* of all manner of *Actions* in the *World* are accounted; and is thus defined. *Annus eſt Revolutio periodica, &c.* It is a *Periodical Revolution*, or a *Great Circle* of *Months* and *Days*, in which the *Four Seaſons*, *Spring*, *Summer*, *Autumn*, and *Winter*, are, after one Revolution of the *Sun*, ordained to return in their Courſes.

By the *Latins* it is called *Annus*, from *Annulus* a *Ring*, for that a *Motion* in a *Ring* finiſhed, beginneth again without end, wherefore *Virgil*,

Atque in ſe ſua per veſtigia volvitur Annus.

Or it may be *Annus*, from *Anguis* a *Snake*, for that the *Egyptians*, before they had the Uſe of *Letters*, repreſented it by a *Serpent* or *Snake*, biting her own *Tail*. Or, (as ſome) *Annus*, *ab Innovatione*; becauſe the *Virtues* and *Strength* of all *Vegetables* are renewed and paſſed over by the Courſe of *Time*.

The *Year* is twofold, *Solar* and *Lunar*: the *Solar Year* is that ſpace of *Time*, wherein the *Sun* departing from ſome certain Point in the *Heavens* returns to the ſame again.

This *Solar Year* is alſo of two Kinds,

Natural and *Siderial*.

The *Natural Year* (called of the *Greeks* Ἔτος, ἀ τείνω, *quaſi vertus*) contains that Space of *Time* in which the *Sun* is going from ſome certain and determinate Point of the *Zodiack*, to his return to the ſame Point again; which Interval, or Space of *Time*, contains 365 *Days*, 5 *Hours*, 49 *Minutes*, and 4 *Seconds*.

The *Siderial Year* (which the *Greeks* call ἀστρολικὸν) is the Space of *Time* in which the *Sun* returneth to the ſame *Fixed Star*, from whence he laſt departed in the former *Year*.

The *Lunar Year* is of two Kinds alſo, *viz.* *Annus Communis*, and *Annus Embaliſmalis*. The *Annus Communis*, contains 12 *Synodical Lunations*, or 354 *Days*.

The *Annus Embaliſmalis*, is the Space of 13 *Moons* or *Lunations*, &c.

The *Political* or *Civil Years*, is that wherein the People (in all places) uſe to diſtinguiſh their *Times* and *Seaſons*; which *Year* is threefold, as having reſpect either to the Motion of

1. The *Sun* only.
2. The *Moon* only.
3. The *Sun* and *Moon* joyntly.

The *Egyptians* making the *Periodical* Motion of the *Sun* the *Meaſure* of *Time*, do conſtitute their *Year* to be 360 *Days* exactly; every *Year* having in it 12 *Months*, and every *Month* 30 *Days*, and to make up the former Number of 365 *Days*, they add 5 Intercalate *Days* at the end of their *Year*. And ſuch *Years* the *Ancient Hebrews* obſerved before the *Alexandrian Æra*, yet ſo, as that they intercalated one whole *Month* of 30 *Days* in every 120 *Years*.

The *Roman Year* is greater than that of the *Egyptians*, and Ancient *Perſians*: for, according to the Conſtitution of *Julius Cæſar*, they make it 365 *Days*, and 6 *Hours*, which 6 Hours every fourth *Year*, make the *Biſſextile* or *Leap-Year*, to contain 366 *Days*, whereas the other three *Years* contain only 365 —— This *Year* is retained, and generally uſed by the *Syrians*, *Syrogrecians*, *Æthiopians*, *Muſcovites*, and *Britains*.

The *Arabians*, *Turks*, and *Indians*, having reſpect to the *Moon* from their *Year* of 12 *Synodical Lunations*: But the ancient *Hebrews* have reſpect to both *Luminaries*; and although the *Atticks* and *Greeks* chiefly deſcribe and limit the *Year* by the Motion of the *Moon*, making it to conſiſt of 12 *Lunary Months*; yet they perceiving the Revolution of the *Sun* not to agree with the *Lunar Year*, they intercalate a whole *Month*; ſometimes the third *Year*, ſometimes the ſecond *Year*, according to the *Calippick Periods*, &c.

The *Siderial Year* is greater in quantity than the *Tropical Year*; for, as that contains the Space of *Time* that the *Sun* is going from one Point of the *Zodiack*, and continues till his return to the ſame again; ſo this *Siderial Year* (*à Sidere dictus*) is the return of the *Sun* to the ſame *Fixed Star*, with whom he was conjoyned in the *Year* laſt paſt (as was ſaid before) and containeth 365 *Days*, 6 *Hours*, 49 *Minutes*, 4 *Seconds*.

The *Egyptians* begin the *Year* from the *Meridian* of the firſt *Day* of the *Month Toth*, which (according to their *Calendar*-Account) is fixed neither to the *Equinoxes* nor *Solſtices*, but wandereth throughout all the *Months* of our *Year*; which ariſeth through the omiſſion of the 6 odd *Hours* computed in our *Calendar*, of which is made the *Biſſextile* or *Leap-Year*.

The *Romans*, from the *Ordination* of *Romulus*, did at firſt place the beginning of the *Year* about the *Vernal Equinox* in the *Month March*; but afterwards, they began it at the *Solſtice*, according to the conſtitution and appointment of *Numa*, approved of by *Julius Cæſar*. So *Ovid*,

Bruma

Bruma Anni prima eſt, veteriſque noviſſima Solis,
Principium capiunt Phœbus & Annus idem.

But foraſmuch as the *Hybernal Solſtice* in the *Reformed Calendar* differs 9 days, and in the *Old Calendar* 19 days from the firſt day of the firſt Month *January*; therefore the *Roman Year*, as it is at this day eſtabliſhed, agrees not with its firſt *Ordination*.

The *Moſcovites* begin their year at the beginning of *September*; but in the names of their *Months* and *Feſtivals*, they differ not much from the *Julian Account*.

The *Venetians* begin their year from the firſt of *March*; others from the 25 of *December*, being the Day of the *Nativity* of our *Saviour*: But we (and moſt others) account our year to begin the firſt of *January*, and ſo do all *Aſtronomers*.

The *Syrians* and *Syrogrecians* account the beginning of their *Year* from the firſt of *October*, according to the *Julian Account*, which is after the *Autumnal Æquinox*.

The *Coptites*, *Ethiopians*, and *Alexandrians*, begin their *Year* before the *Autumnal Æquinox*, upon the 4 *Cal.* of *September*, which is on the 29*th.* of *Auguſt*, according to the Old Account in the irreformed *Calendar*.

The *Atticks* begin their *Year* at the *Summer Solſtice*, or from the *New Moon* next unto the ſame.

The *Hebrews* have a double beginning of their *Year*. The *Eccleſiaſtical Year* they begin from the *New Moon*, next to the *Vernal Æquinox*; and the *Civil Year*, from the *New Moon* next to the *Autumnal Æquinox*. —— Therefore in things *Politick*, the *Year* of the *Jews* is deduced from the *Month Tiſri*; but in *Eccleſiaſtical* things, from the Month *Niſan*.

II. *Of* Months.

THE *Months* by which the *Years* are meaſured, are twofold, *viz.*
Aſtronomical and *Political*.

The *Aſtronomical Months*, (which are called *Natural*,) have relation to the Motion of the two *Luminaries*; and they are either *Solar* or *Lunar*.

The *Solar Months* are 12 in number, which principally reſpect the ingreſs of the *Sun* into the *Dodecatemories* of the *Zodiac*. ——

The *Lunar Months*, which are referred to the Motion of the *Moon* are threefold, *viz.*

1. *Periodical.* 2. *Synodical.* 3. *Illumination.*

The *Periodical Month* is that interval of time, in which the *Moon* returneth to the ſame place in the *Zodiack* from whence ſhe departed.

The *Synodical* is the interval or ſpace from and *Conjunction* of the *Sun* and *Moon*, to the next following.

The *Month* of *Illumination*, or *Appearance*, contains the time from the firſt day of the *Moon's appearing* after the *Change*, to the laſt Day of her *Viſibility*, before her next *Synod*, which may be *Greater* or *Leſſer*, according to her *Motion*, *Declination*, and her *Right* or *Oblique Aſcending* or *Deſcending* the *Horizon*.

The *Months Politick*, *Civil*, or *Uſual*, are thoſe whereby every Nation diſtribute their Year.

The Egyptian Months *are theſe*,

1 *Thoth.*	5 *Tybi.*	9 *Pachon.*
2 *Phophy.*	6 *Mechir.*	10 *Payni.*
3 *Athyr.*	7 *Phamenoth.*	11 *Epephi.*
4 *Chœac.*	8 *Pharmuthi.*	12 *Moſori.*

Every *Month* whereof contains 30 days, to which to make up their *Year* of 365 days, they add at the end of the *Year* 5 *Intercalarian* days.

The Perſian Months *are*,

1 *Pharavardin.*	5 *Mardad.*	9 *Adar.*
2 *Adarpabaſcht.*	6 *Echeberiz.*	10 *Di.*
3 *Cardad.*	7 *Mehar.*	11 *Behemen.*
4 *Thir.*	8 *Aban.*	12 *Alphander.*

Every of theſe *Months* have 30 days, to which in the *Common Year*, they add 5 days, and 6 in the *Leap Year*, which they call *Muſeraca*.

The Turkish Months *are,*

	Days.			*Days.*
1 *Mabarram.*	30	8	*Sahabeu.*	29
2 *Sephar.*	29	9	*Ramadhau.*	30
3 *Rabie* I.	30	10	*Scheval.*	29
4 *Rabie* II.	29	11	*Dulkadati.*	30
5 *Guimadi* I.	30	12	*Dulhagati.* / *Dulhittsche.*	30
6 *Guimadi* II.	29			
7 *Regeb.*	30	*In Ann. Abundanti*		355

The Usual *Month* is a Number of Days, according as they are placed in our *Calendar*: But at first *Romulus*, Builder of *Rome*, and the first King thereof, distinguished the Year into Ten *Months*, as *Ovid* hath it,

Tempora digereret cum conditor Urbis in Anno
Constituit Menses Quinque bis esse suo.

Wherein they included 304 Days.

The First of which was *March*, which *Romulus* so named, *à Marte*, from the God *Mars*; or, for that he would have the *Roman Nation* Warlike.

The Second *April*, ab *Aperiendo*, because then the Pores of the *Earth* are opened; or *ab ἀφρὸς, unde ἀφροδίτη Venus nomen habet, διὰ τὸν ἐκ τῆς ἀφρῆς γένεσιν, quoniam ex spumæ Maris nata fuisse fingitur.*

The Third *May*, a *Majoribus*, or *à Majo*, i.e. *Jupiter*; or *à Madefactione*, because of the Wet or Moisture then caused by the *Pleiades* and *Hyades*.

The Fourth *June*, *à Junioribus*, or from *Juno* the Wife of *Jupiter*.

The other *Months* were called *Quintilis*; *Sextilis*; *September*; *October*; *November*; *December*; to wit, *Quintilis* as being 5 *Months* from *March*; *Sextilis*, because Six *Months* distant; *September*, because 7 *Months* distant, *&c.* and as appears by that of *Ovid*,

Martis *erat primus Mensis*, Veneriscq; *Secundus*;
Hæc generis princeps, ipsius ille pater:
Tertius à Senibus, Juvenum de nomine Quartus,
Quæ sequitur numero turba notata fuit.

Numa Pompilius, Second King of the *Romans*, seeing the *Year* of his Predecessor *Romulus* to be too concise, added two *Months* more; namely, *January* and *February*; whence *Ovid*,

At Numa nec Janam *nec avitas præterit umbras,*
Mensibus antiquis addidit ille duos.

January seems to take its Name, either from *Janua* a *Gate* or *Entrance*, because through *January* there is made an entrance into the *Year*, as there is through a *Gate* into a *House*. Or from *Janus* having *two Faces*, the one looking to the *year* past, and the other to that to come.

February comes *à Februo*, that is *Sacrifico*; because then the *Romans* used to *Sacrifice* to *Pluto*, and other Infernal Gods, for the Souls of their Ancestors.

Afterwards *Julius Cæsar*, the first *Monarch* of the *Romans*, (being instructed by the best *Mathematicians* then living in *Alexandri*,) found that the true Solar year, yet wanted of its completion 10 days and about a quarter; so that by the advice of *Sosigenes*, (an excellent *Mathematician*,) after he had conquered *Pompey*, (being called out of *Egypt* to *Rome*,) he added these *Ten days*; and so made the *Months* to contain some 30, some 31 days apiece, which was afterwards a little altered by *Augustus Cæsar*; and the *Months Quintilis* and *Sextilis*, in honour of their Names, were afterwards called *July* and *August*. Hence the Civil *Roman Months*, are numbered in this manner,

1 *January*	31	5 *May*	31	9 *September*	30
2 *February*	28	6 *June*	30	10 *October*	31
3 *March*	31	7 *July*	31	11 *November*	30
4 *April*	30	8 *August*	31	12 *December*	31

Which *Numbers*, for the help of Memory, are comprehended in these Verses,

April *ter denos*, June, September*q*; November;
Uno plus reliqui, Viginti *Februus* Octo:
At si Bissextus *fuerit*, *superadditur* Unus.

Thus

Thus Englished,

Thirty Days hath *September*,
April, *June*, and *November*,
And all the rest have *Thirty* and *One*,
Excepting *February* alone:
Which unto *Twenty Eight days* we confine,
But in th' *Bissextile* year 'thath twenty nine.

III. *Of* DAYS.

THE *Natural Day* contains the Time of an integral *Revolution* of the *Earth* about the *Sun*, which comprehendeth *Day* and *Night*.

The *Artificial Day* is the whole interval of Time from the *Sun's Rising* to his *Setting*: And the *Artificial Night* is the time comprehended between the time of the *Sun's Setting* in the Evening, till its appearing the next Morning.

The *Caldeans* and *Ancient Hebrews* began their *Day* at the *Rising* of the *Sun*, and ended it again at his next *Rising*.

The *Romans* begin it at *Midnight*, and end it the next *Midnight* after.

The *Egyptians* begin their *Day* at *Noon*, and end it the *Noon* following: And according to this Account do all *Astronomers* frame their Computations.

The *Italians* begin their *Day* at *Sun Setting*, and end it the next *Sun Setting*.

The *Babylonians* begin their *Day* at *Sun Rising*, and end it at *Sun Setting*, and their *Night* from *Sun Setting* till its next *Rising*.

The *Jews* begin their *Day* at the time of *Sun Rising*, and end it at *Sun Setting*; and they begin their *Night* at *Sun Setting*, and end it at the next *Sun Rising*: But in the division of their *Hours* they differ from all others; for the *Hours*, as well of the *Day Natural* as *Artificial*, are always equal, and every hour is the exact 24th part of the *Natural Day*: But these divide the Time between *Sun Rising* and *Sun Setting* into 12 equal Parts, and the *Night*, from *Sun Setting* to the *Sun's Rising* again, they divide also into 12 equal Parts; so that the *Hours* of their *Day* and of their *Night*, are never equal one to the other, nor to those of other *Countries*; but when the *Sun* is in the *Equinoctial*, about the 10 or 11 of *March*, and the 11 or 12 of *September*: For all Inhabitants on the *North side* of the *Equinoctial*, have their *Days* longer than their *Nights* from the 10*th* of *March*, till the 12*th* of *September*, and all on the *South* Side of the *Equinoctial*, have their *Days* longer than their *Nights*, from the 12*th* of *September* till the 10*th* of *March*, notwithstanding, the *whole Natural Day* is equally divided into 24 Parts, each Part being an Hour, whether of the *Artificial Day* or *Night*; But the *Jewish* account of *Hours* differs from all these; for, on the *North* Side of the *Equinoctial*, from the 10*th* of *March* to the 12*th* of *September*, the *Hours* of their *Day* are longer than the *Hours* of their *Night*: And all the time between the 12*th* of *September*, and the 10*th* of *March*, the *Hours* of their *Night*, are longer than those of their *Day*: And our 12 at *Noon*, or *Mid-day*, is their 6*th Hour* of the *Day*, and our 12 at *Mid-night*, is their 6*th Hour* of their *Night*: So that when our *Artificial Day* is 15 hours between *Sun Rising* and *Sun Setting*, then doth the *Jewish Day-Hour* contain one *hour and a quarter* of our hour in length; and their *Night-hour* but *three quarters* of an *hour*: Also when our *Artificial Day* is but 9 *hours* long (as about the 27*th* of *January*) then is their *Day-hour* but 45 min. or 3 *quarters of our hour* long, and their *Night-hour* is 1 *hour* and 15 min. long: And so they vary all the year, and are never equal with our *Hours* (or the rest of the World,) but when the *Sun* is in the *Equinoctial*.

And from hence it is observable, that those People that Inhabit under the *Equinoctial*, have their *Days* and *Nights* all the year long of an equal Length, *viz.* 12 *hours*; But those that live either towards the *North* or *South Poles*, have both longer and shorter *Days*, according to the *Obliquity* of their *Sphere*: Of this see more in this Book in the Treatise of the *Sphere* or *Globe*.

the 12 of *Alexander*, and the 311 before *Christ*. The Year 1657, was the 1968 current year of this *Æra*, but from *October* began the 1969.

VIII. *Of the* Julian Epocha.

In this Year *Julius Cæsar* corrected the *Calendar*, began in the year 45 before our Saviour's Incarnation, in the 4668 of the *Julian Period*, the 3934 of the *World*, the 709 of the *City*, and the 732 *Olympiad*, or the 4*th* year of the 185 *Olympiad*. Wherefore the Year 1657 was the 1702, from the *Julian Emendation*.

JESUS CHRIST, the Son of GOD, and *Saviour* of the *World* was Incarnated in the 4713 year of the *Julian Period*, the 3949 of the *Creation*, the 4*th* of the 194 *Olympiad*, the 753 current year of the *Roman Foundation*, and in the 748 year Current of *Nabonnassar*.

IX. *Of the* Æra *of Martyrs*.

The *Æra* of the *Ethiopians*, or *Abyssins* (called *Diocletian*, the *Æra of Martyrs*, or the *Æra* of the *Cophti Martyrs*, for by all these Names it is called) used by the *Alexandrians*, and *Egyptians* at this day; began in the 4997 of the *Julian Period*, the 4233 of the *World*, the 4*th* year of the 265 *Olympiad*, the 1033 of *Nabonnassar*, the 18 day of the Month *Athir*, but with the 284 of *Christ*: And therefore the year 1657, is the 1373 compleat, but the 1374 Current year of the *Diocletian Æra*, beginning *August* 29.

X. *Of the* Turkish Æra.

This hath its rise from *Hegira*, or the *Flight* of *Mahomet* from *Mecha*, began in the 622 current Year of *Christ*, *July* 16. The Years are *Lunar*, repeding towards the Heads of the *Julian Years*. And the Year 1657 commenceth the 1067 of *Hegira*; but late *Turks* do reckon 11000 years till the present; because 537 *Arabian*, or *Turkish* years, make scarce 527 *Julian*: Or (as *Scaliger*) 235 *Arabian* Years, but 228 *Julian* wanting one Day.

This *Mahomet* (or *Machumed*) is the *Turks* great *Prophet*, or rather, *Grand Impostor*, whose Law is the *Alcoran*. *Arabia* was the *Nest* that bred and fostered that unclean *Bird*: *Medina* the place of his *Birth*, *Mecha* his *Burial*; both which therefore are had in great *Veneration*.

XI. *Of the* Persian Æra.

This *Æra* is two-fold, *Jesdagirdick* and *Gegalæan*. The *Jesdagirdick*, or Years from the Death of *Jesdagirda*, began the 623 year of *Our Saviour June* 16. And the Year of 1657 began the 1026 of that *Æra*.

This *Jesdagirda* was the last King of the *Persians*, whom *Othoman* the *Sarazen* Emperor overthrew, and at once deprived him both of *Life* and *Kingdom*.

The *Sultan* or *Gegalæan Æra* began the 1079 current year of our *Redemption*, *March* 14, in the 448 of the *Jesdargirdick*, on the 18*th* day of the Month *Pharavardim*, or *Pheurdim*: And the year 1675 began the 579 of this *Æra*.

This *Sultan Gelal* (so called by the *Persians*) was Emperor of *Choresan*, and *Mesopotamia*, who by the help of Eight *Persian Mathematicians*, then corrected the *Calendar* of that Nation.

XII. *Of the* Epocha *of the New* Roman *Account*.

This Account began in the Time of Pope *Gregory* XIII. *Anno Christi* 1582.

When *Christopherus Clavius*, the two *Italian* Brothers, (*Antonius* and *Aloysius Lilius*,) with some other *Mathematicians*, corrected the *Julian Calendar*, which was (and still is) called *Gregorean*, *Pontifician*, *Clavian*, or *Lilian*. The year 1657 was the 75 of this *Æra*, which takes beginning *October* 5 of the *Julian Year*, hereby made the 15 of the *Gregorean*.

XIII. *The Correction of the* Calendar *was thus*.

They substracted 10 days (from the 4*th* of *October*, unto the 15*th* of the same *Month* exclusively) in the Year 1582, that thereby they might make the *Vernal Æquinox* (on which the *Moveable Feasts* depend) agree to the 21 of *March*, as it was by the *Nicene* Council established, *Anno* 324. And (to retain the same for future times, invariably upon the same day,) they appointed, that of 400 years in the First, Second, and Third Hundred Years, the *Leap Day* (which in the *Julian Year* happeneth) should be

be omitted, and not intercalated, but in the 400 year, that the *Leap-Day* should not be omitted, but intercalated. For *Example*, over and besides the 10 days substracted as aforesaid, in the year 1700, by omitting the *Leap-Day*, the *Gregorian* year shall be 11 days shorter than our *Julian Year*; and so in the year 1800, it shall be 12 days shorter; and lastly in 1900, it shall be 13 days shorter; but in the year 2000, by keeping and intercalating the *Leap Day*, it shall still be but 13 days shorter, and so forward. But yet this *Correction* wants of exactness, and hath need of another *Amendment*.

A TABLE *of the* Anticipation *of the* Gregorian Calendar.

A. 5 Octob.	*Do*	*Anno Dom.*	*Do*	*Anno Dom.*	*Do*	*Anno Dom.*	*Do*	*Anno Dom.*
	10	1582						
	11	1700	14	2100	17	2500	20	2900
A. 24 Febr.	12	1800	15	2200	18	2600	21	3000
	13	1900	16	2300	19	2700	22	3100

XIV. *How to Reduce the Years drawn from other* Epochæ, *to that of our Saviour's:*

Learn first, what year before or after *Christ*, any other of the more notable *Epochæ* takes beginning; it being a task very easie (where the Fixed *Solar year* consisting of 365 days, 6 hours *ferè* are used) to reduce them to the Series of years respecting the *Birth* of our *Saviour*: For if they be drawn from an *Æra* before that of *Christ*, let the Number of years by which such *Epocha* is described to happen before *Christ*, be substracted from the Years of the *Epocha* given; and the remainder shall shew what year of our *Lord* agreeth thereto.

But when the given years of the *Epocha* proposed are less than the interval of years, by which it precedes *Christ*'s *Æra*, and so Substraction cannot be made as before; let the given years of the *Epocha* be substracted from the interval, and the residue will declare the current years thereof before *Christ*.

Moreover, if the given years be deduced from an *Æra* succeeding that of *Christ*, then add the interval of *this Epocha* from the *Radix* of *Christ*, to the years proposed, and the aggregate gives the number of years from the *Birth* of our *Saviour*, agreeable to the given Years of the *Epocha* proposed. As for Example,

I desire to know in what Year of Christ *the* Arabian Æra *commenceth, which* Arzachel *the Astronomer referreth to the* 932 *year of* Dhilkarnain.

Now, because the *Æra* of *Dhilkarnain* beginneth *Anno* 311 before *Christ*, substract 311 years from 933, and the remainder 622 shews the current Year of *Christ*, in which the *Arabian Æra* commenceth.

Again, *I would know how many years the* 452 *year of the City is before* Christ.

Here (because the *Æra* of the *City* falls out 753 years before *Christ*, which cannot be substracted from the proposed Year, being greater than it) I deduce 452 from 753, and the remainder 301 is the number of Years that the 452 of the *City* precedes our *Saviour*'s *Æra*.

Lastly, *I demand what Year of* Christ *corresponds to the* 1373 *year of* Diocletian.

The *Diocletian Æra* (as already I told you) began *Anno Christi* 284. Now by adding 284 (the interval of this *Æra* from *Christ*) to 1373, (the Year proposed,) I find that the Aggregate 1657 is the *Number* of Years from the *Birth* of our *Saviour*, agreeable to the 1373 year of *Diocletian*.

XV. *How to Reduce the* Tetraeterides *of the* Olympiads *to the Year of* Christ.

To Reduce the *Olympiads* proposed to the Year of *Christ*. First, Substract an Unite from the number of the *Olympiads* given, and multiply the Remainder by 4. To the Product add the current Year of the *Tetraeterial* proposed, the Summ is the elapsed Years from the first *Olympiad*.

Now if this Summ exceed 776, (for so many years their *Radix* preceded that of *Christ*,) deduct 776 from thence, and the residue shews the current Year of *Christ*: But if it be less, substract the same from 776, and what remains gives you the Year before *Christ*.

For

For Example.

I would know what Year of Christ *agrees to the first Year of the* 609 Olympiad.

First, Substract 1 from 609, the remainder is 608, which multiplied by 4, the Product is 2432; unto which add 1 (for the Year of the current *Tetraeterial*) and the Summ is 2433. Lastly, Substract 776 from 2433, and the remainder 1657 agrees to the proposed first year of the 609 *Olympiad.* Again,

I would know what Year of our Saviour *corresponds to the* 3 *Year of the* 93 Olympiad, *in which the* Moon (*according to* Zenophon) *is said to have been* Eclipsed.

First, Substract 1 from 93, and there remains 92, which multiplied by 4 produceth 368 years; to which add 3 (the current Years of the given *Tetraeterid*) and they make 371, which deducted from (becaufe they are lefs than) 776, there remains 405, which is the Year of *Christ*, agreeable to the propofed 3 year of 93 *Olympiad.*

XVI. *How to convert the wandering and uncertain Years of the* Egyptians *from* Nabannaffar, *to the Years of* CHRIST.

To change the *Egyptian Account*, Inchoated from *Nabannaffar* in the decurrent Years of *Christ*, is the defign of the following *Table.*

A TABLE *for Converting the* Egyptian Years *from* Nabannaffar *into* Julian.

Anni Neb.	*Ann. ante Christum.*	*Prim. Toth cadit in*
1	747	26 *Febr.*
4	744	25 *Febr.*
100	748	1 *Febr.*
220	524	1 *Jan.*
228	521	31 *Decem.*
348	401	1 *Decem.*
468	281	1 *Novem.*
592	157	1 *Octob.*
712	37	1 *Septem.*
748	1	23 *Aug.*
749	1	23 *Aug.*
752	4	22 *Aug.*

Anni Neb.	*Anni post Christum.*	*Prim. Toth. cadit in*
836	88	1 *Aug.*
960	212	1 *July*
1080	332	1 *June*
1204	456	1 *May*
1324	576	1 *April*
1448	700	1 *March*
1452	704	29 *Febr.*
1453	705	28 *Febr.*
1456	708	27 *Febr.*
1460	712	26 *Febr.*
1462	714	26 *Febr.*
2923	2174	26 *Febr.*
4384	3634	26 *Febr.*

Mens Egypt.	*Dies.*
Toth	30
Paophi	60
Athyr	90
Clioclac	120
Tybi.	150
Mechyr	180

Mens Egypt.	*Dies.*
Phamenoth	210
Pharmuthi	240
Pacon	270
Payni	300
Epephi	330
Mesori	360

Et post Mesori Epagomenæ quinque 365

The Use of this Table.

SEek in it the number of years from *Nabonnaffar*, which you desire to turn into *Julian*: Or (when you find them not precisely) the next greater. From which Number so found deduct the proposed Years, and the remainder added to, (if the time

time proposed be before *Christ*,) or, if after, substracted from the Year of *Christ*, (which stands on the right hand the *Table*, over against the Number of *Egyptian Years*,) leaves the Year to be reckoned *before* or *after Christ*, as the Title directeth.

This done, Take (in the same part of the *Table*) the *Day* of the *Julian Year* agreeable to the first of the Month *Toth*, and then divide the remainder of Years (after the first Substraction) by 4, for if the Quotient (which must ever be augmented by 1, when ought remaineth after Division, otherwise not at all) be added to that day of the *Julian Year*, you have the Seat of the *Egyptian Month Toth* in the *Julian Calendar*; and (the Head of the Year being found) it is quickly seen how the *Days* of the *Egyptian Months* agree.

For, take but (in the Canon of *Egyptian Months*) the elapsed Days from the Head of the Year, to the Day proposed, and add the Sum to the Day of the Month in the *Julian Year*, (unto which you have found that the first of *Toth* agreeth,) and you have the Day of the Month in the *Julian Year*, according to the *Egyptian* proposed.

But if the Years proposed be from *Alexander*, and following the *Egyptian Ordination*, then (because there are just 424 years between the *Æra* of the King's, and that of *Nabannassar*) add but 424 years to the given years from *Alexander*, and with the Sum, as you did with that of the Years of *Nabannassar*, find out the corresponding of the *Julian Year*; for it will be the same with that which would be found with the Years from *Alexander*. Lastly, The Day of the *Julian Year* thus gotten, you cannot be ignorant of the *Gregorean*, by adding but to the *Julian* the difference of the days betwixt them for the Century proposed, you have the Day in the *Gregorean*.

Example.

Theon, (*in his Commentaries upon the Six Books of* Ptolomy's Syntax) *makes mention of a* Solar Eclipse *observed by him at* Alexandria *in the* 1112 *year of* Nabannassar, *the* 22*th day of the Month* Payni; *I desire to know what day of the* Julian *year agrees thereunto.*

The next greater number of years from *Nabannassar*, are by the Table foregoing 1204, and the Years of *Christ* answerable, on the right hand 456 after *Christ*, together with *May* 1. ——— Therefore I substract 1112 from 1204, and there remains 92 years, which 92 deducted from the found years after *Christ*, leaveth 364, for the *Current years* after our *Saviour*. ——— This done, I divide the first Remainder 92 years by 4, and the Quotient is 23, and nothing remaining, which added to *May* 1, gives *May* 24, for the day unto which the first of *Toth* agreeth: —— Now seeing that from the Series of the *Egyptian* Months, 210 days are compleatly elapsed with the Month *Pachon*, and that the proposed day is the 22 *current* of the following *Payni*, together making 292, I add 292 to *May* 24, (unto which the first of *Toth* answereth,) and it produceth *March* 13; therefore the Year of *Christ* 365, *March* 13, *Post Christum*, in the *Julian Year*, agreeth to the proposed Time from *Nabannassar*.

XVII. *How to Convert the* Turkish *and* Arabick *Years from* Hegira, *into the Inchoate, or Current Years of our* Saviour.

A TABLE *of Days in the* Turkish *Years.*

Anni.	*Dies.*	*Anni.*	*Dies.*	*Anni.*	*Dies.*
1	354	16	5670	30 . 0	10631 . 0
2	709	17	6024	60 . 0	11262 . 0
3	1063	18	6378	90 . 0	31893 . 0
4	1417	19	6733	120 . 0	42524 . 0
5	1772	20	7087	150 . 0	53155 . 0
6	2126	21	7442	180 . 0	63786 . 0
7	2480	22	7796	210 . 0	74417 . 0
8	2853	23	8150	240 . 0	85048 . 0
9	3189	24	8505	270 . 0	95679 . 0
10	3543	25	8895	300 . 0	106310 . 0
11	3898	26	9213		
12	4252	27	9568		
13	4607	28	9922		
14	4761	29	10276		
15	5315	30			

Days

Days in the Turkiſh *Months.*

Mabarram	30	*Sahabeu*	236
Sephar	59	*Ramadhan*	266
Rabie I.	89	*Schevall*	295
Rabie II.	118	*Dulkadati*	325
Guimadi I.	148	*Dulhajati* }	355
Guimadi II.	177	*Dulbittſche Turc* }	
Regeb	207	*In Ann. abundanti*	355

Days in the Julian *Years.*

1	365 . 25		6	2191 . 50
10	3652 . 50		60	21915 . 00
100	36525 . 00		600	219150 . 00
1000	365250 . 00		6000	2191500 . 00
2	730 . 50		7	2556 . 75
20	7305 . 00		70	25567 . 50
200	73050 . 00		700	255675 . 00
2000	730500 . 00		7000	2556750 . 00
3	1095 . 75		8	2902 . 00
30	10957 . 50		80	29020 . 00
300	109575 . 00		800	290200 . 00
3000	1095750 . 00		8000	2902000 . 00
4	1461 . 00		9	3287 . 25
40	14610 . 00		90	32872 . 50
400	146100 . 00		900	328725 . 00
4000	1461000 . 00		9000	3287250 . 00
5	1826 . 25			
50	18262 . 50			
500	182625 . 00			
5000	1826250 . 00			

Days in the Julian *Months.*

January	31	31	*May*	151	152	*September*	273	274
February	59	60	*June*	181	182	*October*	304	305
March	90	91	*July*	212	213	*November*	335	336
April	120	121	*Auguſt*	243	244	*December*	365	366
Months,	*Com*	*Biſ.*	Months,	*Com*	*Biſ.*	Months,	*Com*	*Biſ.*

First, by the Table of Days in the *Julian Year*, reſolve the Interval of this *Epocha*, from that of *Chriſt*, into compleat Days.

Next, the time propoſed in the *Turkiſh* Account, into days, by the *Table* of days in the *Turkiſh Years*, for they being all collected into one ſumm, the ſame will give you the number of days, wherewith, by the *Table* of days in the *Julian Year*, you may find out the Month and Day deſigned from *Chriſt*, as followeth —

Seek out in the *Table* of days in the *Julian Year* the number of (if you find not the ſame exactly) the next leſſer than the Sum of days before found, and take the oppoſite Years on the left hand. Then from the Sum of days firſt Collected, ſubſtract this next leſſer Number, and with the remainder, (if leſs than 365) find out in the *Table* of days in the *Julian Month*, the Month and Day of the *Julian Year*, and ſo you will have the Year, Month, and Day from *Chriſt*, congruent to the propoſed, from *Hegira*.—

But

But if the remainder exceed 365 days, you must therewith re-enter the *Table*, and thence take the next less number of days (as you did before) together with the opposite Years on the left hand, which add to the former; so continuing your work, till fewer than 365 days do remain; that you may collect the *Julian Years* as aforesaid.

Example.

I desire to know what Day of the Julian *Year agrees to the* 1057 *from* Hegira, *the* 7*th Day of the Month* Saphar.

First then, the interval of the *Turkish Epocha* from that of *Christ*, is 621 years, *July* 15 compleat, which thus are resolved into Days, multiplying all by 365.25 or $365\frac{1}{4}$. But by the *Table* thus,

600	*Julian Years* give	219150 Days
20	Years give	7305
1	Year gives	365
621	*June* Compleat	181
	July Current	15
	Total Sum of days in the Interval.	227016 Days

For the *Arabick Years*, by the *Table*.

900	*Arabick Years* give	318930 Days
150	Years give	53155
6	Years give	2126
1056	*Maharram* Compleat	30
	Saphar Current	7
	The Aggregate of days in both	601264 Days

Now to Convert the Aggregate of Days into Julian Years.

The whole number of days	601264	Years
Number for 1000 Years	365250	1000
Remainder	236014	
The Number next less	219150	600
Remains	16864	
The Number next less	14610	40
Remains	2254	
The Number next less	2191	6
Remains	63	

Here it is evident that 1646 *Julian Years* are answerable to 601201 Days, which with 63 Days remaining, makes 601264, the whole number of Days: Now for the 63 days remaining, 59 of them are elapsed with *February*, and the 4 then remaining are to be accounted in *March*: Wherefore I conclude, that upon the 4*th* of *March* 1647 *Current* (after the *Julian Computation*, but the 14*th* of *March* in the *Gregorian*) agrees to 1057 from *Hegira*, the 7*th* day of the Month *Saphar*.

SECT. XVII.

Of the Common *or* Vulgar Notes *of the* Julian *and* Gregorian *Year, and how to find them.*

I. *Of the* Lunar Cycle, *commonly called, the* Golden Number, *or* Prime.

THE *Golden Number* is a Revolution of 19 years, in which time all the Lunations and Aspects between the *Sun* and *Moon*, do nearly return to the same places they were 19 years before.——To find it, Add one to the year of *Christ* (for at his Nativity the *Golden Number* was 1) and divide by 19, the Quotient tells you how many Revolutions the Circle hath made since the Incarnation, and the Remainder is the *Golden Number* for the year sought.

Example: For the year 1687, add 1, it is 1688; divide by 19, the Quotient is 88; and so many Revolutions hath the *Circle* made, and the Remainder 16, is the *Golden Number* for the year 1687.

II. *Of*

II. *Of the* Solar Cycle, *or* Cycle *of the* Sun.

This *Circle* is a Revolution of 28 years, and the chief use of it is thereby to find out the *Dominical* or *Sunday Letter* for any year: It is found by adding 9 (for so far was the *Circle* gone at the Birth of *Christ*) to the year of our *Lord*, and dividing the Product by 28, and the remainder is the *Circle* for the year; but if nothing remains, then 28 is the *Cycle*.

Example: For the year 1687. to which add 9, the Sum is 1696, which divided by 28, the Quotient is 60, and 16 Remaining; so that the *Sun's Cycle* for the year 1687, is 16.

III. *Of the* Epact.

This Number is contrived to adequate the difference between the *Solar* and the *Lunar years*.——To find which, multiply the *Golden Number* for the year by 11, and divide the Product by 30, the remainder is the *Epact* for that year.

Example: For the year 1687, the *Golden Number* for that year was found to be 16, which multiplied by 11, the Product is 176, which divided by 30, the Quotient is 5, and 26 remaining, which is the *Epact* for the year 1687.

IV. *Of the* Roman Indiction.

This Revolution was invented for the Paying of Tribute to *Augustus Cæsar*, and is now out of use. It is a *Circle* of 15 years, and is thus found.——Add to 3 the year of *Christ*, and divide the Sum by 15, the Remainder after division is the *Number* of the *Indiction*.

Example: The year 1687, to which add 3, it makes 1690, which divided by 15, the Quotient is 112, and the Remainder 10 is the *Indiction* for that year.

Tables for the finding of these Common Notes both in the Julian *and* Gregorian *Accounts.*

Yea. of *Chr.*	☉ Cycl.	Dominical Letter. *Jul.*	*Greg.*
1672	1	G F	C B
1673	2	E	A
1674	3	D	G
1675	4	C	F
1676	5	B A	E D
1677	6	G	C
1678	7	F	B
1679	8	E	A
1680	9	D C	G F
1681	10	B	E
1682	11	A	D
1683	12	G	C
1684	13	F E	B A
1685	14	D	G
1686	15	C	F
1687	16	B	E
1688	17	A G	D C
1689	18	F	B
1690	19	E	A
1691	20	D	G
1692	21	C B	F E
1693	22	A	D
1694	23	G	C
1695	24	F	B
1696	25	E D	A G
1697	26	C	F
1698	27	B	E
1699	28	A	D

Yea. of *Chr.*	Go. Nu.	Epac. *Jul.*	*Greg.*
1681	10	20	10
1682	11	1	21
1683	12	12	2
1684	13	23	13
1685	14	4	24
1686	15	15	5
1687	16	26	16
1688	17	7	27
1689	18	18	8
1690	19	29	19
1691	11	11	1
1692	2	22	12
1693	3	3	23
1694	4	14	4
1695	5	25	15
1696	6	6	26
1697	7	17	7
1698	8	28	18
1699	8	9	29

Yea. of *Chr.*	Indict.
1685	8
1686	9
1687	10
1688	11
1689	12
1690	13
1691	14
1692	15
1693	1
1694	2
1695	3
1696	4
1697	5
1698	6
1699	7

A Table

A Table shewing the Feast *of* Easter.

Golden Number for the *Julian* Account.	A	B	C	D	E	F	G	Golden Number for the *Gregorian* Account.
1	*April* 9	10	11	12	6	7	8	7
2	*Mar.* 26	26	28	29	30	31	*April* 1	8
3	*Apr.* 16	17	18	19	20	14	15	9
4	*Apr.* 9	3	4	5	6	7	8	10
5	*Mar.* 26	27	28	29	23	24	25	11
6	*Apr.* 16	17	11	12	13	14	15	12
7	*Apr.* 2	3	4	5	6	*Mar.* 31	*April* 1	13
8	*Apr.* 23	24	25	19	20	21	22	14
9	*Apr.* 9	10	11	12	13	14	8	15
10	*Apr.* 2	3	*Mar.* 28	29	30	31	*April* 1	16
11	*Apr.* 16	17	18	19	20	21	22	17
12	*Apr.* 9	10	11	5	6	7	8	18
13	*Mar.* 26	27	28	29	30	31	25	19
14	*Apr.* 16	17	18	19	13	14	15	1
15	*Apr.* 2	3	4	5	6	7	8	2
16	*Mar.* 26	27	28	22	23	24	25	3
17	*Apr.* 16	10	11	12	13	14	15	4
18	*Apr.* 2	3	4	5	*Mar.* 30	31	*April* 1	5
19	*Apr.* 23	24	18	19	20	21	22	6

SECT. XVIII.

Of the Reason of the Difference in Computation between the Old *and* New Accompts.

A Year (as hath been said before) is that space of time in which the *Sun* goeth through the whole Circle of the *Zodiack*, as from the *Tropick* of *Cancer* to the *Tropick* of *Capricorn*, and so to the *Tropick* of *Cancer* again; or from the *Equinoctial* to the *Equinoctial*; or from any other Point of the *Zodiack* to the same again. Now because of the unequal Motion of the *Sun* (concerning which, recourse may be had to the IX. Book hereof), it ever was, and yet is, a very hard matter to determin exactly; insomuch, that *Censorinus*, *De Die Natali*, said, that the Year consisted of so many Days, and how much more or less no body knows.

This uncertainty brought so much Confusion upon the Old *Roman Calendar*, that Time with them was grown a Commodity, and bought and sold at a price. Their Priests, who had to do with this Affair, having in their power to make any Year longer or shorter at their pleasure, which the Emperor *Julius Cæsar* looking upon as a matter no way below his greatest consideration, advised with some *Ægyptian Mathematicians* about it, by whose Instructions, he found that the *Sun*'s yearly Motion in the *Zodiack*, was performed in the space of 365 days and 6 hours.

The 6 odd hours he caused to be reserved in store till every fourth Year, till they made 24 hours, or one whole day; so accounting, that the 3 first Years should consist of 365 days, and the 4th. of 366, one day more, and every 4th. Year was therefore (as still it is) called the *Leap-Year*, and the thing it self *Intercalation*, or putting in betwixt the *Calendar*.

This was very much which the Emperor did, but he left as much to do; for though it cannot yet be found out exactly in what space of time the *Sun* goeth its yearly Course, yet thus much is made good by infallible experience, that the Emperor's Mathematicians allotted too much; for the number of Days they were in the right; for it is certain no Year can consist of more than 365, but for the odd Hours, it is as certain that they cannot be fewer than 5, nor so much as 6, so that the doubt is upon the Minutes, 60 whereof go to the making up of an Hour; a small matter one would think, and yet how great in the recess and consequence we shall see.

Julius Cæsar allotted 365 Days and 6 Hours to this Revolution, but the *Sun* goeth about in less time, that is (according to more exact account) in 365 Days, 5 Hours, and 49 Minutes, and a little more; so that the *Emperor's Year* is much about 10 Min.

48 Seconds greater than the *Sun*'s, which muſt of neceſſity breed a difference of ſo many Minutes and Seconds every Year, betwixt the Year which the *Sun* it ſelf deſcribes in the *Zodiack*, and that which it reckoned on in the *Calendar*; which though for a Year or two may paſs inſenſibly, yet in the ſpace of 134 Years it will riſe to one whole Day, that is, the beginning of the Year in the *Calendar* muſt be ſet one Day back. As for *Example*; Let the Year begin at the *Vernal Equinox*, or *Spring*; In the Emperor's time that fell out to be at the 24th. of *March*, but now this Year it fell out upon the 10th. of *March*, 13 Days backwards, and ſomewhat more, and ſo, if it be let alone, will go back to the Firſt of *March*, and Firſt of *February*, till *Eaſter* come to be on *Chriſtmas Day*, and ſo infinitely.

To reform this difference in the Account, ſome of the late *Roman* Biſhops earneſtly endeavoured; and the thing was brought to that perfection it now ſtands in (ſo much as it is) by *Gregory* the 13th. in the Year 1582. His *Mathematicians* (whereof *Lillius* was the chief) adviſed him thus: That conſidering there had been an Agitation in the Council of *Nice* ſomewhat concerned in this matter, upon the motion of that Queſtion about the *Celebration of Eaſter*; and that the Fathers of that Aſſembly, after due deliberation with the *Aſtronomers* of that time, had fixed the *Vernal Equinox* at the 21ſt. of *March*, and conſidering alſo, that ſince that time a difference of 10 whole days had been paſt over in the *Calendar*; that is, that the *Vernal Equinox*, or *Spring*, which begun upon the 21ſt. of *March*, had prevented ſo much as to begin in *Gregory*'s days, at the 10th. of the ſame, 10 Days difference, or thereabouts; they adviſed that 10 Days ſhould be cut off from the *Calendar*, which was done, and the 10 Days taken out of *October* of that Year 1582. as being the Month of that Year in which that *Pope* was born; ſo that when they came to the 5th. of that Month, they reckoned the 15th. and ſo the *Equinox* was come up to its place again, and happened upon the 21ſt. of *March*, as at the Council of *Nice*. But that *Lillius* ſhould bring back the beginning of the Year to the time of the *Nicene* Councils, and no farther, is to be marvelled at; he ſhould have brought it back to the *Emperor*'s own time, where the miſtake was firſt entered, and, inſtead of 10, cut off 13 Days: However this is the reaſon why theſe two *Calendars* differ the ſpace of 10 Days one from the other. And as the *Old Account* was called the *Julian*, from the *Emperor*; ſo the *New* is called from *Gregory* the *Pope*; and *Lillius* the chief Agent, the *Gregorian*, or *Lillian Account*.

For the Third *Calendar* there needs not much be ſaid, though it be more abſolute than the Second; for it reduceth the beginnings of the Year to the Emperor's own time, and ſo leaveth the *Old Style* 13 Days behind, as it ought to do.

SECT. XIX.

The Uſe of the Terreſtrial Globe *in* Geography *and* Navigation.

WHat the *Terreſtrial Globe* is, and what is deſcribed upon it, as alſo the *Circles*, *Lines*, and *Points* anſwering to thoſe in the *Heavens*, and to what Uſes they ſerve, is largely diſcourſed of in Book III. Chap. I. Of *Practical Aſtronomy*, whereunto I refer you for Satisfaction: I ſhall here only ſhew the Uſe of the Globes in the Solution of ſuch *Problems* as concern *Geography* and *Navigation*.

GEOGRAPHICAL PROBLEMS.

PROB. I. *To find the* Longitude *of a* Place.

Definition.] LOngitude is the diſtance of a place from the firſt *Meridian* reckoned in the degrees of the *Æquator*, beginning, as was ſaid, in this *New Terreſtrial Globe*, at S. *Michael*'s Iſland in the *Azores*.

Practice.] Bring the place (that is, the mark of the place) ſuppoſe *London*, to the *Brazen Meridian*; then count how many degrees of the *Æquator* are contained between the firſt *Meridian* and that of *London*, cut by the *Brazen Meridian*, which you will find to be 28 deg. which is the *Longitude* required. And in this manner you ſhall find

London

		deg.	min.			deg.	min.	
London	to be diſtant from the firſt *Meridian* by the *New Globe.*	28	0	By other *Globes* and *Maps*		26	0	reckoned from the ſame *Meridian.*
Jeruſalem		66	30			73	30	
Jedo in *Japan*		167	0			178	0	
Rio de la plata		32	0			21	0	
Mexico		75	0			86	0	
Charlton Iſle		51	30			65	50	

PROB. II. *To find the* Latitude *of a* Place.

Deſinition.] THE *Latitude* of a place, is the Diſtance of the *Æquator* from the Parallel of that place, reckoned in the degrees of the *Great Meridian*; and is either *North* or *South*, according as it lies between the *North* or *South Poles* of the *Æquator.*

Practice.] To find the *Latitude*, bring the mark of the place; for *Example*, Suppoſe *London* to the *Brazen Meridian*, then count the number of degrees upon the *Meridian*, contained between the *Æquator* and the place given. Thus you ſhall find the *Latitude* of

	deg.	min.		deg.	min.
London to be	51	30	By other *Globes* and *Maps*	51	30
Lahor in the *Moguls* Country to be	31	30		23	30
The *South-part* of the *Caſpian* Sea to be	37	0		41	0
Aſtracan, on the *North-part* of the *Caſpian* Sea to be	46	0		49	0
The *North-part* of *China* to be	42	0		52	0
Delli in *India* to be	28	0		21	0

PROB. III. *The* Longitude *and* Latitude *of any place being known, to find the true Situation of it, though not expreſſed upon the* Globe.

Practice.] BRing the degree of the *Æquator* that anſwereth to the *Longitude* of the place to the *Meridian*, and then reckon the *Latitude* of the place upon the degrees of the *Meridian* towards either *Pole*, according as it is either in *North* or *South Latitude*; and right under that degree and minute upon the *Meridian*, is the true Situation of the place enquired after.

PROB. IV. *To find what Time or Hour of the* Day *or* Night *it is in any Part of the Earth.*

Deſinition.] BY reaſon of the *Earth*'s diurnal Motion round the *Sun* in 24 hours, the *Sun* enlightening but one half of it at the ſame time, it comes to paſs, that when it is Morning in one place, it is Noon in a ſecond, Night in a third, and Midnight in a fourth, according to their ſeveral Situations in reſpect of *Eaſt* and *Weſt* one from the other. This difference of time is known by the number of degrees contained in the *Æquator* between any two places propoſed, converted into hours and minutes, reckoning 15 deg. to an hour, *&c.* but more readily by the *Globe* thus:

Practice.] Suppoſe at *London*, at 12 of the Clock at Noon, you would know what a Clock it is at *Mexico* in the *Weſt Indies*; bring *London* to the *Meridian*, and ſet the *Index* of the Hour-circle to 12. then turn the *Globe Eaſtward*, becauſe *London* is *Eaſt* of *Mexico*, till you bring *Mexico* to the *Meridian*; then ſee what hour the *Index* points at, for that is the hour then at *Mexico*. Thus you will find,

		h.	m.	
When it is 12 a Clock at *London*, it is at	*Mexico*	5	10	a Clock before Noon.
	Charlton Iſland	6	45	
	Rio de la plata	8	10	
	Jeruſalem	2	35	a Clock after Noon.
	Surrat	5	15	
	Jedo	9	18	

And thus by knowing what difference of time there is between place and place at 12 a Clock, the like difference is to be underſtood of all other hours.

PROB. V. *To find the Distance of any two places upon the* Globe *one from another.*

Practice.] LAY the *Quadrant of Altitude* upon both the places required; then count the number of degrees of the *Quadrant of Altitude* contained between the two places; which being found, multiply them by 60, gives the distance in *English* Miles. Thus you will find,

		deg.	m.		deg.	m.	
The distance of *London* from	*Mexico*	81	30	by the new size of *Globes.*	91	30	by the great *Dutch Globes* and *English.*
	Jerusalem	33	30		38	0	
	Surrat	65	50		71	40	
	Jedo	85	10		92	30	
	Rio de la plata	100	20		106	0	
	Charlton Island	46	20		54	0	

If you find (as you needs must) that the proportion of Miles upon these new *Globes* do very much differ from those distances set down by other Authors, you are desired not to think much; for the *Longitudes* are not yet exactly agreed on: The perfection is not one Man's, nor one Age's Work, and must be waited for. Where you find the places upon this *Globe* to agree with others, you have cause to suspect they have lain upon the lees of time, not as yet enquired into: Where you find them to disagree, you may conclude that they have been brought to a truer correction and amendment.

PROB. VI. *To find the* Position, *or what Point of the* Compass *any two places are Situate one from another.*

Definition.] THE Position is an *Angle* which is made by the meeting of the *Meridian* of one place, with the *Vertical Circle* of another.

Practice.] To find this out, you are to elevate the *Pole* to the *Latitude* of one of the places, suppose *London*; then bring it to the *Meridian*, and it will fall out to be directly in the *Zenith*; for the Elevation is always equal to the *Latitude*; then fasten the *Quadrant of Altitude* to the *Zenith*, and turn it about till it fall upon the other place, suppose the Isle of *Tenerif*, and the end of the *Quadrant* where it toucheth the *Horizon* will shew that the Isle of *Tenerif* beareth from *London* SSW: So also the bearing of *Barbadoes* from the *Lizard* to be SW, half a Point *Westerly*; and the opposite Point NE, half a Point *Easterly*, the bearing of the *Lizard* from the *Barbadoes.*

PROB. VII. *To know at any time in what place of the Earth the* Sun *is in their* Zenith.

This must be to such Inhabitants of the Earth only that inhabit in the *Torrid Zone* between the *Tropicks.*

Practice.] BRing the place you are in, suppose *London*, to the *Meridian*, and the *Index* to the hour 12; then consider the time of the day, which suppose to be half an hour after 5 in the Afternoon, the *Sun* having then 10 deg. of *North* Declination; then because it is Afternoon, turn the body of the *Globe Eastward*, till the *Index* hath passed 5 hours and an half from 12, that will be to 7 a Clock and an half, and there stay the *Globe*; then see what place or Country is under the *Meridian* that cuts 10 deg. of *North* Declination, and you will find *Nombre di dios* upon the *Isthmus* of *Panama* in the *West-Indies.* But if it were required the same day at half an hour after 6 in the Morning, then you should have turned the *Globe Westward* till the *Index* had passed 6 hours and an half: And then under the *Meridian*, and upon the Parallel of 10 deg. *North* Declination, you will find it near 4 or 5 little Islands close by the *West-side* of *Mallacca* in the *East-Indies*, where the *Sun* will be in the *Zenith* at that time.

Having found in what place of the Earth the *Sun* is in the *Zenith*, elevate the *Globe* to the *Latitude* of the place, either *North* or *South*; then bring that place to the *Meridian*, so shall all places cut by the *Horizon* have the *Sun* in their *Horizon*; those to the *Westward* shall have the *Sun rising* in their *Horizon*; those to the *Eastward* shall have the *Sun setting.* In those Countries that are above the *Horizon* it is day-light, and in those but 18 deg. below the *Horizon*, it is twilight; but in those Countries farther below the *Horizon*, it is at that time midnight.

PROB.

PROB. VIII. *The Difference of* Longitude *being known, to find what Degree of the* Ecliptick *culminates at any other place at any time proposed.*

March the 10th at 10 of the Clock before Noon here at *London*, I would know what degree of the *Ecliptick* culminates then at *Jerusalem*.

Practice. ELevate the *Globe* to the *Latitude* of your place, *viz.* 51 *deg.* 30 *min.* then bring the *Sun's* place for that day, *viz.* ♈ 0 *deg.* to the *Meridian*, and the *Index* to 12, then turn the *Globe Eastward* till the *Index* point at the given hour, *viz.* 10 of the Clock, and you will find the 28 *deg.* of ♒ then culminating here at *London*. Next turn about the *Globe Westward* until 33 *deg.* 30 *min.* of the *Æquator* be passed through the *Meridian*, or till the *Index* of the hour *Circle* be moved 2 hours 35 *min.* which is the difference of *Longitude* given, and then you will find the *Meridian* cut the *Ecliptick* at almost 4 *deg.* of ♈: so that I say, ♈ 4 *deg.* is the point of the *Ecliptick* that is then culminating at *Jerusalem*.

PROB. IX. *To find out the several* Positions *of the Inhabitants of the* Earth, *the distinction of Shadows, the Different Habitations*, &c.

THE *Longitude* and *Latitude* of a place once resolved on, the *Position* of the *Sphere* you cannot miss of; for if the place you try for have no *Latitude*, it must of necessity lie under the *Æquator*; and therefore in a Right *Position*: if it have less or more *Latitude*, the *Position* is *Oblique*; if the place have 90 *deg.* of *Latitude*, the *Position* is Parallel: the Reasons were told before, and may evidently be discerned upon the *Globe*.

For the *Climes* and *Parallels*, and consequently the length of the longest day, the fore-knowledg of the *Latitude* leadeth you directly; for they who are under the *Æquator*, have their Day always 12 hours, and their Night 12 hours long. Now as each Country declines from the *Æquator* towards either of the *Poles*, so the Days vary their length in *Summer*, and the Nights theirs in *Winter*: according therefore to the different lengthning of their Days, the Ancients did distinguish the Earth into several portions or parts, which they called *Climates* and *Parallels*; every *Clime* contains two *Parallels*; so that where the longest Days are half an hour longer than at the *Æquator*, the first *Climate* begins; and where they are encreased an hour longer than at the *Æquator*, the second *Climate* begins, which you will find to be at 8 *deg.* 34 *min.* where the second begins; and so for any of the rest, as by the Table in the X. Section of this Part.

The *Tropicks* and *Polar Circles* divide the *Surface* of the *Globe* into five parts, or spaces, which are called *Zones*, whereof one is contained within the *Arctick Circle*, another compassed by the *Antarctick Circle*, and are called the *Frigid Zones*; the other two lying between the *Arctick Circle* and *Tropick* of *Cancer*, and between the *Antarctick Circle* and the *Tropick* of *Capricorn*, are called the *Temperate Zones*; and the other lying between the two *Tropicks*, is called the *Torrid* or *Mid-Zone*.

Knowing these, you may easily conclude upon the distinction of Shadows: for those of the *Frigid Zones* are termed *Periscii*, because there their Shadows have a *Circular* motion. Those of the *Temperate Zone* are called *Heteroscii*, because their *Meridian* Shadows bend towards either *Pole*; towards the *North* to those that dwell within the *Tropick* of *Cancer*, and the *Arctick Circle*; towards the *South* to those that dwell within the *Tropick* of *Capricorn* and the *Antarctick Circle*. The Inhabitants of the *Torrid Zone* were called *Amphiscii*, because the Noon Shadows according to the time of the year do sometimes fall towards the *North* when the *Sun* is in the *Southern Signs*, and sometimes towards the *South* when the *Sun* is in the *Northern Signs*.

To find out the other distinction of Habitation, viz. Antiœci, Periœci, Antipodes.

Let *London* be the place; bring it to the *Meridian*, where you find it to be 51 *deg.* 30 *min.* elevated above the *Æquator*, account so many degrees of *Southern Latitude* below the *Æquator*, and you meet with the *Antiœci* (if any be.) Remove *London* from the *Meridian* 180 *deg.* and you shall find your *Periœci* under the *Meridian* where *London* was before, and your *Antipodes* is in the place where their *Antiœci* stood before.

NAUTICAL PROBLEMS.

THere be four things upon which the practice of *Navigation* is principally grounded, *viz.* 1. *Longitude.* 2. *Latitude.* 3. *Course.* 4. *Distance.*

All which I shall handle in the Seventh Book, which treateth wholly of *NAVIGATION*; and therefore in this place I shall only shew you the Use of the *Globes* (both *Cœlestial* and *Terrestrial*) in Working of such *Problems* as are of frequent use in that Art.

PROB. I. *To find the* Latitude.

I. *By the* Meridian Altitude *of a known* Star (suppose *Algol.*)

BRing *Algol* to the *Meridian*, and from the Centre of the *Star*, downwards, count his *Altitude* 62 *deg.* and mark that point upon the *Meridian*: then bring that point to the *South* part of the *Horizon*; and you shall find the *North Pole* to be elevated 67 *deg.* 39 *min.* which is the *Latitude* you are then in. In like manner, if you should observe

		d.	*m.*		*d.*	*m.*	
The *Bull's Eye*,	Upon the *South* part	51	0	You would	54	48	*N.*
Spica Virginis,	of the *Meridian*, ha-	32	0	be in *La-*	67	31	*S.*
The *Great Dog*,	ving *Altitude*	72	0	*titude*	34	14	*N.*

II. *By observing of two* Stars, *one being upon the* Meridian, *and the other* Rising *or* Setting.

Bring the *Star* which you see upon the *Meridian* unto the *Meridian*, and there holding the *Globe* fast, move the *Meridian* in the *Horizon*, till you see the other *Star* on the *East* or *West*, *Rising* or *Setting*, as you observed it, and then shall the *Globe* stand at the *Latitude* you are in.

So if you should see *Regulus* upon the *Meridian*, and *Lyra* rising towards the *East*, the *Latitude* would be found 37 *deg.* 50 *min.*

III. *By the* Altitude *of two known* Stars, *being both of them upon the same* Azimuth *or point of the* Compass.

Lay the *Quadrant of Altitude*, or rather your thin *Brass Semicircle* to both the *Stars*, at the proper degrees of *Altitude*, as you observed them to be in the *Heavens*; (for the difference of their *Altitudes* is equal to their distance.) Then turn the *Globe* about in the *Horizon*, till the *Quadrant* or thin *Plate* of Brass do touch the *Horizon* in that *Azimuth* (or point of the *Compass*) on which you observed the *Stars* to be; so shall the *Globe* rest at the *Latitude* you are in.

So *Capella* and *Scheder*, *Capella* being 20 *deg.* high, and *Scheder* 66 *deg.* high, and both of them upon the *North-East* point of the *Compass*, you will find your self to be by this observation in the *Latitude* of 40 *deg.* 0 *min. North.*

IV. *By the* Altitude, Azimuth, *and* Declination *of the* Sun, *or of a known* Star.

Suppose a fixed *Star*, as that in the Right Knee of *Hercules*, having 47 *deg.* 9 *min.* of *Declination*, should be observed at Sea to have 122 *deg.* of *Azimuth* from the *North* part of the *Meridian*, and to be 60 *deg.* high; and from hence the *Latitude* were required:

Elevate the *Globe* to the *Star's Altitude* 60 *deg.* and from the *Pole* count the Complement thereof 30 *deg.* to which screw the *Quadrant of Altitude*: also count 122 *deg.* upon the *Horizon* from the *North* part of the *Meridian*, and to those degrees bring the *Quadrant of Altitude*, and there keep it; then turn the *Globe* about, till 47 *deg.* 9 *min.* the *Star's Declination* (counted upon the *Equinoctial Colure* from the *Equinoctial*) do cut the *Quadrant of Altitude*, and those degrees will cut the *Quadrant of Altitude* in 71 *deg.* 13 *min.* and that is the *Latitude* in which you then are.

V. *By*

V. *By the* Sun's, *or a* Star's Declination *and* Amplitude.

Let the *Sun* or a *Star* have 10 *deg.* of *North Declination*, and let the *Amplitude* thereof at its *Riſing* or *Setting* be obſerved to be 57 *deg.* from the *North*.

Elevate the *Globe* to 33 *deg.* (the Complement of the *Amplitude*) and count the *Amplitude* it ſelf 57 *deg.* upon the *Meridian* from the *Pole* forward, and thereto ſcrew the *Quadrant of Altitude*, and bring the other end thereof to the *Eaſt* or *Weſt* Points of the *Horizon*; then count 80 *deg.* (the Complement of the *Star's* or *Sun's Declination*) from the *Pole* upon the *Equinoctial Colure*, and bring thoſe degrees to the *Quadrant of Altitude*; ſo ſhall 80 *deg.* of the *Colure* cut 71 *deg.* 24 *min.* of the *Quadrant*, and that is the *Latitude* you are then in.

VI. *By the* Sun's *Aſcenſional Difference, and* Amplitude.

If the *Aſcenſional Difference* be 27 *deg.* 7 *min.* and the *Amplitude* 33 *deg.* 20 *min.* and the *Latitude* were required;

Elevate the *Globe* to 27 *deg.* 7 *min.* the *Aſcenſional Difference*, and from the *Pole* count the Complement thereof 62 *deg.* 53 *min.* and thereto ſcrew the *Quadrant of Altitude*; then bring the *Equinoctial Colure* to the *Meridian*, and count upon the *Quadrant of Altitude* upwards, the Complement of the *Amplitude* 57 *deg.* 40 min. which degrees bring to the *Equinoctial*, and then ſhall the *Quadrant of Altitude* cut upon the *Horizon* 51 *deg.* 30 *min.* counted from the *Eaſt* or *Weſt*, which is the *Latitude* of the place.

And if you count the degrees of the *Equinoctial* comprehended between the *Meridian* and the *Quadrant of Altitude*, you ſhall find them to be 20 *deg.* 5 *min.* which is the *Sun's Northerly Declination* at that time.

VII. *By the* Sun's *or a* Star's Declination, *and the Time that he is upon the* Eaſt *or* Weſt *Points of the* Compaſs.

Let the *Sun* or a *Star's Declination* be 15 *deg. North*, and let the Time that the *Sun* is upon the *Eaſt* or *Weſt* Points of the *Compaſs* be 59 *deg.* 45 *min.* which in time is 3 *hours* 56 *min.* and hence let the *Latitude* be required.

Elevate the *Globe* to 15 *deg.* the *Declination*, and ſcrew the *Quadrant of Altitude* to 75 *deg.* the *Zenith Point*; then count 59 *deg.* 45 *min.* or 3 *hours* 56 *min.* (the hour) upon the *Horizon* from the *South*, *Eaſtward* or *Weſtward*; and thereto bring the *Quadrant of Altitude*: Then look what degrees of the *Quadrant* are cut by the *Equinoctial*, and you ſhall find 28 *d.* counted from the *Zenith*, and that is the *Latitude* ſought.

And thus have you ſeveral ways both by the *Sun* and *Stars* to find the *Latitude* at any time. I will now proceed to ſome other *Problems* for finding the *Rumb* and Diſtance.

PROB. II. *Any two places given, to find their* Rumb.

Definition.] THoſe Lines which a Ship following the direction of the *Magnetical Needle* deſcribeth on the ſurface of the Sea, are called *Rumbs*, and are deſcribed upon the *Terreſtrial Globe* by certain *Spiral Lines*; for the better underſtanding whereof, I ſhall premiſe theſe few Propoſitions:

Firſt, The *Needle* touched with the *Load-Stone* pointeth out the common Interſection of the *Horizon* and *Meridian*; the one end reſpecting the *North*, the other the *South*, as aforeſaid.

Secondly, a *Circle* drawn through the *Vertex* of any place that is diſtant from the *Æquator*, cannot cut divers *Meridians* at equal *Angles*.

Thirdly, A great *Circle* drawn through the *Vertical Point* of any place, and inclining to the *Meridian*, maketh greater *Angles* with all other *Meridians*, than with that from whence it was drawn.

Fourthly, If we Sail upon any Point of the *Compaſs* except *North* or *South*, we often change our *Horizon* and *Meridian*.

Fifthly, the ſame *Rumb* cutteth all *Meridians* of all places at equal *Angles*, and reſpecteth the ſame Quarters of the World in every *Horizon*.

Sixthly, The portions of the ſame *Rumb* intercepted between any two *Parallels* whoſe difference of *Latitude* is the ſame, are alſo equal to each other: therefore an equal *Segment* of the ſame *Rumb* equally changeth the difference of *Latitude* in all places; ſo that in an equal ſpace paſſed in one and the ſame *Rumb*, one of the *Poles* is equally elevated, and the other depreſſed.

Seventhly, *Rumbs* though never ſo far continued, do not paſs through the *Poles*, but wind about the *Poles* until they looſe themſelves.

Hence you may underſtand if your Ship be directed under the *North* or *South Rumb*, your Courſe will be always under the ſame *Meridian*; if under the *Eaſt* or *Weſt Rumb*, you will either deſcribe the *Æquator*, or a *Circle* Parallel to it: If your *Vertical* Point be under the *Æquator*, your Ship will deſcribe an *Arch* or *Segment* of the *Æquator*; but if your *Zenith* or *Vertical Point* be diſtant from the *Æquator* either *North* or *South*, your Courſe will then deſcribe a *Parallel* as far diſtant from the *Æquator*, as the *Latitude* of the place is whence you firſt ſet forwards: But if your Voyage be to be made under the *Rumb* which inclineth to the *Meridian*, your Courſe will then be neither in a greater or leſſer *Circle*, but your Ship will deſcribe a kind of a crooked *Spiral Line*.

Practice.] Find the two places upon the *Globe*, and when you have found them, ſee what one *Rumb Line* paſſeth through both of them, and that is the *Rumb* or Point of bearing of thoſe two places one from the other. So *C. del Gade* on the Coaſt of *Zanguebar* and *C. Cormorin* are both of them found upon the *W S W* and *E N E Rumb*, and that is their Point of bearing, or *Rumb* required.

If you can find no one *Rumb* that paſſeth through both your places, then you muſt look what *Rumb Line* upon the *Globe* runneth moſt Parallel to both the given places, and conclude that to be the Point or *Rumb* of thoſe two places bearing one from another. ——So if the two places were *Seirra Liona* in *Africa*, and the Iſland S. *Hellena*, if you look upon the *Globe*, you ſhall find no one *Rumb Line* to paſs through both the places, but that *Rumb* to which the places lie moſt Parallel, is the *N N W* and *S S E Rumb*; and ſo *Sierra Liona* beareth from S. *Hellena N N W*, and on the contrary S. *Hellena* is ſituate from *Sierra Liona S S E*, which is the *Rumb* required.

PROB. III. *Having the diſtance Sailed, and the* Rumb *you have Sailed upon given, to find the Difference of the two places both in* Longitude *and* Latitude.

HAving found the *Rumb* upon which you made your Courſe, make a ſmall mark thereupon, for the place you departed from: then from the *Equinoctial* take the number of Miles or Leagues you have Sailed upon that *Rumb* (allowing 20 Leagues for a *deg.*) and ſet that diſtance upon the *Rumb* from the former Point made; and at the termination of your number of Miles or Leagues, make a ſecond mark upon the *Rumb*: then bringing the place, or Point you departed from, to the *Meridian*, you ſhall there find the *Latitude* of that place or Point, and the *Meridian* cutting the *Equinoctial* will ſhew you the *Longitude* of that place or Point. —— Do ſo by bringing the ſecond Point or place to the *Meridian*, and there ſhall you find the *Latitude*, and upon the *Equinoctial* the *Longitude* of that place or Point. Now if you ſubtract the leſſer *Latitude* from the greater, you have the difference of *Latitude*; and the leſſer *Longitude* ſubtracted from the greater, gives the difference of *Longitude*.

PROB. IV. *The* Latitude *of two places, and the* Rumb *that the two places bear each from other given, to find the Difference of* Longitude *of thoſe two places, and alſo their Diſtance upon the* Rumb.

Practice.] FIrſt, find the *Rumb* upon the *Globe*, and turn the *Globe* about till that *Rumb* doth cut the *Meridian* in the *Latitude* of the firſt place from whence you departed, and there make a mark or Point upon the *Rumb*, and at the ſame time ſee alſo what degrees of the *Equinoctial* are cut by the *Meridian*; for that is the *Longitude* of the firſt point.

Secondly, Turn the *Globe* about, till the ſame *Rumb* does cut the *Meridian* in the *Latitude* of the ſecond place, and there make another mark upon the *Rumb*; and alſo ſee what degrees of the *Equinoctial* are cut by the *Meridian*, which degrees are the *Longitude* of the ſecond Point or place: and the leſſer *Longitude* being ſubtracted from the greater, gives the Difference of *Longitude* of the two places or Points.

Then for the Diſtance upon the *Rumb*, the Diſtance between the two points before made, being meaſured upon the *Equinoctial*, and reduced to Miles or Leagues, ſhall give the diſtance upon the *Rumb*.

¶ Here

¶ Here note, *That the Diſtance upon the* Rumb *being intirely taken and applied to the* Equinoctial, *will give the diſtance in the* Arch *of a great* Circle, *and not really in the* Rumb, *for the diſtance upon the* Rumb *will be always greater than the great Circular diſtance: wherefore the better way will be, to take in a pair of* Compaſſes, *one, two, three (or ſome ſmall number of) degrees of the* Equinoctial, *and run that diſtance over upon the* Rumb Line *from point to point; and the number of all thoſe returns of the* Compaſſes *(reduced to Miles or Leagues) ſhall be the near Diſtance of the two places upon the* Rumb.

SECT. XX.

Of MAPS *and* CHARTS: *General and Particular. Their Deſcription, Conſtruction and Uſe.*

OF *Maps* or *Charts*, there are pricipally two Sorts, *Univerſal* and *Particular*. The *Univerſal* is that which containeth the whole Surface of the *Terreſtrial Globe*, Land and Water, upon a *Plain Superficies*, and this kind of deſcription is properly called **Geography** *in Plain.*

The Deſcription of any great Part of the *Earth*, as of *Great Britain, Ireland, France, Germany*, &c. is moſt properly called **Chorography.**

The Deſcription of any ſmall Parcels of the *Earth*, as of *Counties, Shires, Hamlets*, &c. is properly termed **Topography.**

I. *Of* Geographical Maps *or* Charts.

The *Geographical Map* or *Chart*, containing the *whole Terreſtrial Globe*, is deſcribed in Plain after two manner of ways, *viz.*

By { *Planiſphere.* / *Parallelogram.*

The way by *Planiſphere* is moſt natural, and that is of two kinds, *viz.*

Meridional.
Equinoctial.

And either of theſe require two *Hemiſpheres* to contain the *Whole.*

I. *Of the* Meridional, Geographical Planiſphere, *or* Projection.

In this Projection the outermoſt *Circle* repreſents the *General Meridian*, from which the beginning of *Longitude* is counted; and alſo they repreſent the two *Solſtitial Colures.* The *Centre* of one *Hemiſphere* repreſents the Point of the beginning of the *Vernal*, the other of the *Autumnal Equinox.* And the Right *Line* paſſing through theſe two *Centres* repreſents the *Equinoctial Circle*; and if (in each *Hemiſphere*) a Right *Line* be drawn through the *Centre*, at Right *Angles* to the *Equinoctial*, theſe two Right *Lines* ſhall be the two *Equinoctial Colures*; the one the *Vernal*, the other the *Autumnal*, cutting the *Equinoctial* in *Aries* and *Libra*; as the *Meridian Circles* doth it in the *Solſtitial Colures*; namely in *Cancer* and *Capricorn.*

In this *Projection* the *General Meridian* are two perfect *Circles*; and the *Equinoctial*, and the two *Equinoctial Colures*, are ſtreight *Lines*; and all the other *Meridians* are *Circular* alſo: but the nearer they approach from the outermoſt *Meridian* towards the *Colures*, the leſs Arching they appear: for imagin the *Globe* of the *Earth* to be placed in its *Horizon*, ſo that the *Pole* be in the *Zenith*, then will the *Equinoctial* be in the *Horizon*, and let the two *Equinoctial* Points of *Aries* and *Libra* be under the *Braſs Meridian*, then if the *Globe* (in this *Poſition*) were preſſed or flatted together, the *Colures* will become ſtreight *Lines* drawn from *Pole* to *Pole*, and all the other *Meridians* which are drawn on the *Globe* at every 10. or 15. degree, will be *Circular Arches*, all of them concurring or meeting in the two *Poles.* And the *Equinoctial* alſo will be a ſtreight *Line*, as hath been ſaid before; but the two *Tropicks*, the two *Polar Circles*, as alſo the other intermediate *Parallels*, drawn at every 5. or 10. degree from the *Equinoctial* towards either *Pole*, are *Circular*, and grow leſs and leſs, as they approach nearer to the *Poles* of the *World.*

The *Meridians*, which meet together in the two *Poles*, are called *Circles of Longitude*, for that by them the *Longitudes* of Places are counted by the degrees numbered upon the *Equinoctial Line*, or *Circle.*

The *Circles* which are *Parallel* to the *Equinoctial*, are called *Parallels of Latitude*, for that by them the *Latitudes* of *Places* are accounted.

The

The *Tropicks* and the *Polar Circles* limit out the *Zones*, the Spaces between the *North* and *South Poles*, and the *Arctick* and *Antarctick Circles* compriſe the two *Frigid Zones*. The Space between the two *Tropicks*, comprehends the two *Torrid Zones*; and the *Spaces* between the *Tropicks*, and the *Arctick* and *Antarctick Circles*, contain the two *Temperate Zones*. The *Ecliptick* lies Obliquely between the two *Tropicks*, cutting the *Tropicks* of *Cancer* and *Capricorn*, and the *Equinoctial* in the Points of *Aries* and *Libra*; of this *Circle* there is no uſe in *Geography*. And thus have you the deſcription of the *Meridional Planiſphere*.

II. *Of the* Equinoctial, Geographical Planiſphere, *or* Projection.

In theſe *Projections* the outermoſt *Circles* repreſent the *Equinoctial*, and the other *Circles* deſcribed upon the ſame *Centre*, are the *Tropicks*, the two *Polar Circles*, and the *Circles* or *Parallels* of *Latitude*, by which the *Latitude of Places* are accounted. And the Right *Lines* drawn from the *Centres* (which are the *Poles* of the *World*) are *Circles of Longitude*, by which the *Longitudes* of *Places* are accounted: this *Projection* is much more eaſie to *Project* upon a *Plain*, than is the other before treated of, for that it conſiſts of perfect *Circles*, and ſtreight *Lines*: for if the *Terreſtrial Globe* were ſet as before, the *Pole* in the *Zenith*, and the *Equinoctial* in the *Horizon*, and the *Pole* of the *Globe* were preſſed, or flatted down into the *Equinoctial*, the *Equinoctial* will ſtill retain being a *Circle*, and the Compreſſed *Pole* the *Centre* thereof, and all the other *Parallels of Latitude Concentrick* thereunto:, but the *Meridians* (or *Circles of Longitude* will become ſtreight *Lines*, meeting together in the *Poles*. And let this ſuffice to be ſpoken of theſe two *Geographical Projections* in *Plain* by *Planiſphere*. Concerning the *Projection* of which two *Planiſpheres*, you may ſee more in the Second Part of the Third Book hereof, which Treats of *Spherical Projection*, to which I refer you, both for their *Projection* and there *Uſe*: and now I will proceed to the Deſcription of the other, *viz.*

III. *Of the* Geographical Projection *by Parallelogram*.

This *Projection* conſiſts all of Streight and Parallel *Lines*, both as to the *Meridians* of *Longitude*, and *Parallels* of *Latitude*, but not after the old rude (and ſtill too much practiſed) way of *Projecting*, imagining the *Terreſtrial Globe* of the *Earth* and *Water* to be a real *Flat*, and ſo the *Parallels of Latitude* and *Meridians of Longitude* to be in all parts of the *World equidiſtant*. It is true, the *Meridians* in all places are *equidiſtant*, but the *Parallels of Latitude* are not; for we know, that upon the *Globe* it ſelf, that the *Meridians* about the *Equinoctial* are *equidiſtant*; but as they approach nearer to the *Poles*, their diſtance diminiſh proportionally, till they come to a Concurrence in the *Pole* it ſelf: anſwerably the *Parallels*, as they are deeper in *Latitude*, ſo they grow leſs and leſs with the *Sphere*; ſo that at 60 *deg.* of *Latitude*, that *Parallel* is but half what a *Parallel* at the *Equinoctial* is, and ſo proportional leſs and leſs as the *Parallels* do approach nearer to the *Poles*: from hence it will follow; that if the *Earth* and *Water* ſhould be deſcribed upon a *Parallelogram*, whoſe *Meridians* and *Parallels* ſhould be in all places equally diſtant, the *Parallel* of 60 *deg.* ſhall be as great as one at the *Equinoctial* it ſelf; and he that ſhall *Coaſt* about the *World* in the *Parallel* of 60 *deg.* of *Latitude*, ſhall have as far to go (according to ſuch an erroneous *Chart*) as he that Coaſteth about in the *Equinoctial Circle*, although it be but half ſo long; for the *Longitude* of the *Earth* in the *Equinoctial Circle* it ſelf, is 21600; but in the *Parallel* of 60 *deg.* but 10800 Miles.

This *Error* hath been complained of by the *Geographers* and *Navigators* of *Old*, and hath been attempted by many to be reformed, as by *Mercator*, and others, who cauſed the *Parallels* of *Latitude* to increaſe as they approached nearer to the *Poles*. Although (I ſay) this *Reformation* hath been often attempted, it was never brought to perfection, till about one hundred years ſince, by a learned *Mathematician* and *Navigator* of our Country, Mr. *Edward Wright*, who in his *Detection of Errors in Navigation*, hath laid down a familiar demonſtration, whereby the *Meridians* and *Parallels* may be laid down by ſtreight *Lines* upon a *Parallelogram*, to receive the *Longitudes* and *Latitudes* of *Places* agreeable to the *Globe* it ſelf: which demonſtration cannot be rendred in better terms than in his own words, which take as follow.

'Suppoſe (ſaith he) a *Spherical Superficies*, with *Meridians*, *Parallels*, *Rumbs*, &c. 'to be inſcribed into a *Concave Cylinder*, their *Axes* agreeing in *One*. Let this *Spheri*-'*cal Superficies* ſwell like a *Bladder*, while it is in blowing, equally all ways in every 'part thereof, (that is, as much in *Longitude* as *Latitude*) till it apply, and joyn it ſelf '(round about, and all along alſo, towards either *Pole*) unto the *Concave Superficies* of the *Cylinder*,

'*Cylinder*, each *Parallel* upon this *Spherical Superficies* increasing successively from 'the *Equinoctial* towards either *Pole*, until it come to be of equal *Diameter* with the '*Cylinder*: and consequently, the *Meridians* still widening themselves till they come to 'be so far distant every where, each from other, as they are at the *Equinoctial*. And 'thus it may most easily be understood how a *Spherical Superficies* may by *Extension* 'be made a *Cylindrical*, and consequently a *Plain Parallelogram Superficies*, because the '*Superficies* of a *Cylinder*, is nothing else but a *Plain Parallelogram* round about two 'equal *equidistant Circles*, that have one common *Axis* perpendicular upon the *Centres* 'of them both, and the *Peripheries* of each of them equal to the length of the *Paralle-* '*logram*, as the distance between those *Circles* or height of the *Cylinder* is equal to the 'breadth thereof.

In this *Parallelogram*, thus conceived to be made, all Places must needs be situated in the same *Longitudes*, *Latitudes*, *Direction*, or *Courses*, and upon the same *Meridians*, *Parallels*, and *Rumbs*, that they were in the *Globe*; because, that at every Point between the *Equinoctial* and the *Pole*, we understand the *Spherical Superficies* to swell equally in *Longitude* as in *Latitude*, till it joyn it self to the Concavity of the *Cylinder*, so as hereby no part is any way distorted, or displaced out of his true and Natural Situation upon his *Meridian*, *Parallel*, or *Rumb*, but only dilated, and enlarged: the *Meridians*, *Parallels*, and *Rumbs*, also dilating and enlarging themselves likewise, at every Point of *Latitude* in the same Proportion.

Let this suffice for the Reason of the truth of this continuance in the *Parallelogram*, and for the *Map* described upon it: it is divided in the midst by a *Line* drawn from *North* to *South*, for the *General Meridian*; cross unto this, and at Right *Angles*, another *Line* is drawn from *East* to *West* for the *Æquator*; and Parallel to that, two other Right *Lines*, to compleat the *Parallelogram*. Now this way of *Projection*, though not so natural, yet hath it been followed (and still is) by as able *Geographers*, and especially *Navigators*, as this Age affords.

These are the three Principal ways of *Projecting Geographical Maps* or *Charts* of the *Whole World*: Figures of all which I have here (as Precedents) inserted. As,

For the manner how they are Projected,

I shall need say nothing thereof in this place, I having done that at large elsewhere in this *Book*; as for the *Meridional* and *Equinoctial Planispheres*, the *Projection* of them is handled at large in the second Part of the third Book, Entituled, *Of Spherical Projection*, Chap. III. and Chap. V. And for the *Projection* by *Parallelogram*, both in *Whole* and in *Part*, that I have also shewed how to perform in the second Part of the VII. Book hereof, of *Navigation*: *Of Mercator's Sailing*; and for these I refer my Reader, to the two forecited Places.

IV. *Of the Furniture of Universal* Maps *or* Charts.

Having thus briefly given you a Description and Sight of the several *Geographical Projections*; it resteth now to say something concerning the furnishing them with such things for which they were principally intended: namely the several *Countries*, *Seas*, &c. to perform which, it is requisite to know the true *Longitudes* and *Latitudes* of such *Countries*, *Seas*, &c.

The degrees of *Latitude* are (in all *Maps*) counted from the *Equinoctial*, towards either the *North* or *South Pole*, according as the Place is Situate: the *Longitude* is counted from some certain *Meridian* towards the *East* and *West*: now, which to call the *First* or *General Meridian*, I cannot tell, some beginning their *Longitude* at one *Place*, some at another, some at the *Fortunate Islands*, some at the *Canaries*, some at the *Azores*, some at S. *Michael*, some at *Corvo*, some at the *Pike* of *Teneriffe*: some of which *Places* differ from other 7, 8, or 10 degrees *East* or *West*, so that it is impossible to find out at what *Meridian* they begin their account of *Longitude*; yet Mr. *Richardson* in his State of *Europe* seems to give some satisfaction herein, when he tells you, that the *Longitudes* in his *Book* shall be taken from that *Meridian* which passeth through the *Azores*: but whether he means that of St. *Michael*, or that of *Corvo*, he setteth not down, and yet the difference of *Longitude* between those two, is about seven degrees. Now in this Confusion the only way I know for reconciliation is this, to let all Persons who make new Descriptions either of the *Whole*, or in *Parts*, not fail to set down what *Meridian* they mean to go by.

Now therefore, to avoid this mistake, I have in this *Geography* begun the *Longitude* at St. *Michaels*, and according to that the *Longitudes* are reckoned in the late new

Terrestrial

Terreſtrial Globes, and ſeveral other *Particular Maps*, ſet forth by Mr. *Robert Morden*. And thus much for the *Projection*, and furniſhing of *Geographical Maps* or *Charts* of the *Whole*.

V. Of Chorographical Maps or Charts.

I call thoſe *Chorographical Maps* or *Charts*, which (as I ſaid before) do contain ſome large *Part*, as *Province*, *Empire*, *Kingdom*, &c. all which are but *Limbs* or *Members* of the *Whole* or *Univerſal*: and being ſuch, they may be *Projected* in parts, either of the *Planiſphere* or *Parallelogram*: but by which ſoever of the *Projections* they are deſcribed, they are always drawn into a *Parallelogram*; and becauſe no *Empire*, *Kingdom*, or *Province* is exactly *Square*, ſo much of the bordering *Territories* are uſually (in the vacancies) thruſt in, which do not only ſhew the bounds of the deſigned *Province*, but fill up the vacant *Corners* or parts of *Sides* of the *Parallelogram* in which it is Deſcribed: and theſe being *Parts* or *Members* ſeparated from the *Whole*, ought to retain as great a reſemblance, and to be deſcribed with the ſame integrity as was the *Whole*.

For the *Projecting* of any *Country*, the firſt thing to be conſidered is the Extenſion of the intended *Part*, in *Longitude* from *Eaſt* to *Weſt*, and in *Latitude* from *North* to *South*: which two extreme *Parallels* of *Longitude* and *Latitude* are firſt to be deſcribed in *Plain*, in their due places as they are in the *Globe* it ſelf, and they being deſcribed, the other (every 5 or 10. degree,) of the intermediate *Parallels* of *Longitude* and *Latitude* are to be drawn quite over the *Parallelogram*, with *Circular Parallels* of *Longitude* and *Latitude*, if it be part of the *Meridional Projection*; or with *Circular Parallels* of *Latitude*, and ſtreight *Lines* for *Longitude*, if it be part of the *Equinoctial Planiſphere* or *Projection*: or both *Parallels* of *Longitude* and *Latitude*, ſtreight *Lines*, if it be part of the *Geographick Chart* by *Parallelogram*.

And now by way of Advice, I would wiſh that ſuch as deſcribe or *project Maps* of any conſiderable part of the *Whole*, that they would conſider what *Projection* is fitteſt to make choice of for the *Part* he intends; for my own part, I ſhould do as followeth, *viz.*

1. If the *Part* to be deſcribed ſhould lie between the *Tropicks*, not exceeding 23 or 24 *deg.* in *Latitude* on either ſide the *Equinoctial*, and all on the *North*, or all on the *South* ſide thereof: I would make uſe of the *Equinoctial Planiſphere* or *Projection*; becauſe in that both the *Parallels* of *Longitude* and *Latitude* are at their greateſt diſtance, and ſo the *Countries* may be the more exactly inſerted according to their true *Longitudes* and *Latitudes*.

2. If the *Part* to be *Projected* do lie, one part thereof on the *North*, and the other part on the *South* ſide of the *Equinoctial*, I would then uſe the *Meridional Projection* or *Planiſphere*, becauſe the *Parallels* of *Longitude* and *Latitude* are there at their greateſt diſtance.

3. If the *Part* lie between the *Tropicks* and the *Arctick* or *Antarctick Circles*, I would then make uſe of the *Meridional Planiſphere*, or *Mercator*'s, (*alias*, Mr. *Edward Wright*'s *Projection*) by *Parallelogram*. And,

4. If the *Part* lie between the *Arctick* or *Antarctick Circles*, and the *Poles*, then the foreſaid way of laying down by *Parallelogram* is of all other to be embraced.

VI. Of Chorographical Maps or Charts.

Suppoſe I would Deſcribe a *Chorographical Map* of *Europe*, upon part of the *Meridional Projection*; I firſt conſider that the neareſt *Latitude* of any part of *Europe* towards the *Equinoctial* is about 29 *deg.* and the greateſt *Latitude* is 71 *deg.* and the difference of *Longitude* between the moſt *Eaſterly*, and moſt *Weſterly* Parts of *Europe*, is 70 *deg.* alſo. Wherefore I deſcribe thoſe *Parallels* of *Latitude*, and *Meridians* of *Longitude* as they are (according to the Laws of *Projection*) to be deſcribed, which done, incloſe the ſaid *Meridians* and *Parallels* within a *Parallelogram*, as is the *Parallelogram* A B C D, extending the *Meridians* of *Longitude* from ſide to ſide of the *Parallelogram*, writing *North* (or *Septentrio*) at the top, and *South* (or *Meridies*) at the bottom, and number them, both at top and bottom, with ſuch degrees of *Longitude* as they are from (St. *Michaels* or) the *General Meridian*.

Likewiſe deſcribe the *Parallels* of *Latitude* from ſide to ſide of the *Parallelogram*, writing *Oriens* (or *Eaſt*) on the right ſide, and *Occidens* (or *Weſt*) on the left ſide, numbering them on both ſides with 30, 35, 40, 45, 50, 55, 60, 65, 70, ſo far as your greateſt *Northern Latitude* doth extend.

Then

Then set in the Principal *Kingdoms*, and other eminent *Places*, according to their true *Longitudes* and *Latitudes*, incompassing them with *Seas*, *Rivers*, *Bays*, &c. and also all *Islands*, *Capes*, &c. as in the Figure.

VII. *Of* Topographical Maps *or* Charts.

For *Topographical Maps* or *Charts*, they may very well be described in *Plano*, by laying down the places in them contained, in equal Parts both in *Longitude* and *Latitude*, in Miles and parts of Miles, as in this *Topographical Map* of *Middlesex*; here inserted.

VIII. *Con-*

VIII. *Concerning the measuring of the true Distance of Places upon* Maps.

The difference of Miles in ſeveral *Countries* is very great; but let it be enough to know, that the *Engliſh* and *Italian* Miles are reckoned as all one: Four *Engliſh* make one *German Mile*; Two a *French League*; Three and ſomewhat more, a *Spaniſh League*: The *Sueditſh* or *Daniſh* Mile contains five *Engliſh Miles*, and ſomewhat more.

And as the *Miles* of ſeveral *Countries* do very much differ, ſo thoſe of the ſame *Country* do not very well agree, and therefore the *Scales* which they commonly put into *Maps* are threefold, and written upon with *Magna*, *Mediocria*, *Parva*, to ſhew the difference; and the meaning is this, that you ſhould meaſure the *Milliaria magna* upon the longeſt *Scale*, the *Parva* upon the uppermoſt, and the *Mediocria* upon the middlemoſt; but the beſt (or all) of theſe are very uncertain.

And therefore, laying aſide this, and the like uncertainties, I ſhall here ſhew a way whereby you may know the true diſtance between any two Places whoſe *Latitudes* and *Longitudes* (or difference of *Longitudes* rather) are known, let the *Map* be *projected* by any of the formentioned ways, or any other, but not by taking the diſtance between them in your *Compaſſes*, as is uſual, but by another *Artifice*, which with a very ſmall trouble by one *Univerſal Scale*, and a plain *Semicircle* (undivided) ſhall give the true diſtance between any two Places howſoever ſituate upon the whole *Terreſtrial Globe*, according to the *Arch* of a *Great Circle*; which of all other ways is the moſt conſentaneous to Truth, and therefore moſt exact.

How to perform this upon the *Globe* it ſelf, is before ſhewed in the **V.** *Propoſition* of the *Uſe of the Terreſtrial Globe*; but the way here is much different: and the *Scale* may be put into ſome ſpare place of any *Map*, or one that hath much occaſion for this Work, may have one made in *Wood* or *Braſs*, (or may make one upon good *Paſtboard* himſelf) to lie conſtantly by him for that purpoſe: for the ſame *Scale* will ſerve all *Maps*.

But before I ſhew you the Deſcription and Uſe of the *Scale*, take notice of theſe Rules following.

I. If

I. If the two Places be both of them situate under the *Equinoctial*, then the difference of *Longitude*, converted into *Miles* (by multiplying them by 60) is the distance between the two Places.

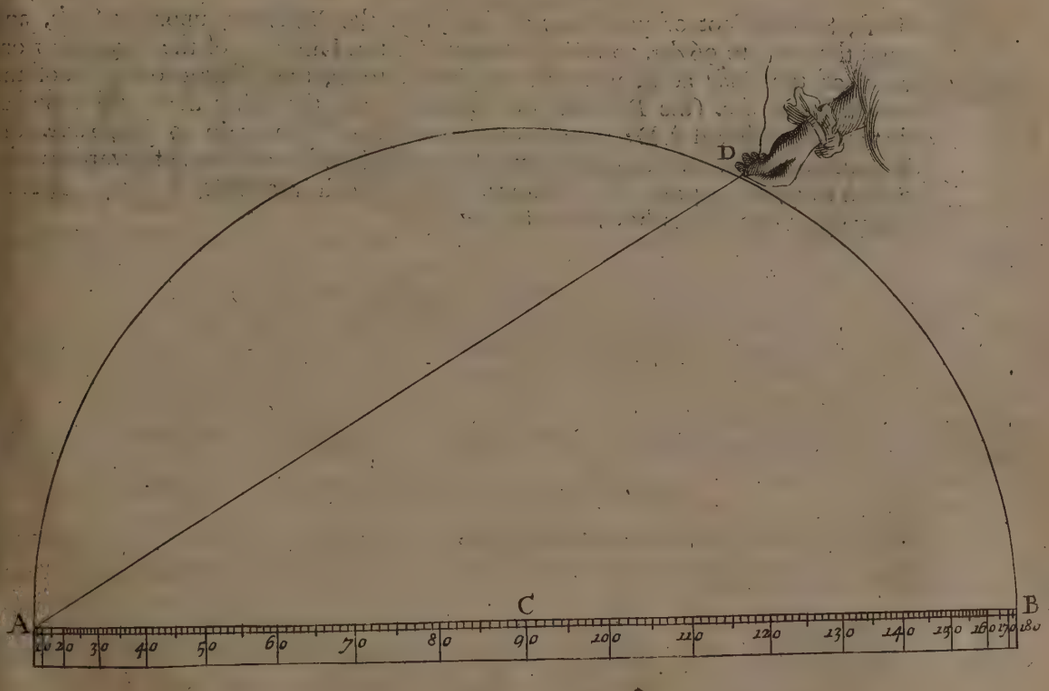
II. If the two Places lie in the same *Latitude*, and on the same side of the *Equinoctial*, the degrees contained between the two *Meridians* which pass through the two Places, multiplied by 60, is the distance.

III. If the two Places lie under the same *Meridian*, and both on the same side of the *Equinoctial*, the difference between the two *Latitudes*, multiplied by 60, is the distance in *Miles*.

IV. If the two Places lie under the same *Meridian*, and on different sides of the *Equinoctial*, the one on the *North*, and the other on the *South*, the summ of the two *Latitudes*, multiplied by 60, is their distance in *Miles*.

The Description of the Metrical Scale.

The *Scale* is no other than a *Scale of Natural verſed Sines*, as A B, numbered from A towards B, by 10, 20, 30, &c. to 180, at B, (for greater difference of *Longitude* can no two Places have) upon the middle whereof at 90 *deg.* upon the Point C, describe the *Semicircle* A D B, at the end of the *Scale* at A, let there be fixed a small Silk or Thred: and this is all the *Instrument* or what is to be used with it besides the *Compasses*: Thus much for the Description



The USE *of the* Metrical Scale.

LET the two Places proposed be *London* and *Jerusalem*, both lying in *North Latitude*, but the difference of *Longitude* between them is 38 *deg.* 30 *min.* Now by knowing the two *Latitudes*, and the difference of *Longitude*, their true Distance (according to the *Arch* of a *Great Circle*) may be known by this General

RULE.

RULE.

FInd the Summ *and* Difference *of the* Complements *of the* Latitudes *of the two given Places, and with your* Compasses *take the distance between them upon the* Scale, *set that distance upon the* Semicircle *from the end of the* Scale; *and to that Point lay the Thread, keeping it there by laying your Finger upon it: Then set one Foot of the* Compasses *in the* Difference *of* Longitude, *and with the other take the neareſt diſtance to the Thread, then ſetting one Foot of that diſtance upon the* Scale *at the* Difference *of the* Complements *of the two* Latitudes, *the other Foot will reach forwarder, upon the* Scale, *to the degrees of the diſtance between the two Places.*

Example: Let the two Places be *London* and *Jeruſalem.*

		d.	*m.*		*d.*	*m.*
The *Latitude* of	*London* is	51	30	*Complement*	38	30
	Jeruſalem is	32	00		58	00
				The *Summ*	96	30
The *Difference* of *Longitude* 47 *deg.*				The *Difference*	19	30

First, Set one Foot of your *Compaſſes* in 19 *deg.* the *Difference*, upon the *Scale*, and extend the other to 96 *deg.* 30 *min.* the *Summ*, ſet this Diſtance of the *Compaſſes* from the end of the *Scale* at B, upon the *Semicircle*, to D, then ſetting one Foot of the *Compaſſes* in 47 *deg.* (the *Difference* of *Longitude*) turn the other about till it do but only touch the Thread A D; then ſet one Foot of that Diſtance in 19 *deg.* 30 *min.* (the Difference of the *Complements* of the two *Latitudes*) the other will reach forwarder upon the *Scale* to 39 *deg.* 14 *min.* the Diſtance, which turned into Miles is 2354. And ſo for any other two Places howſoever ſituated.

The End of the Third Book.

CURSUS MATHEMATIUS:

BOOK V.

OF

TRIGONOMETRY.

Containing,

The Doctrine of the Dimension of *Plain* and *Spherical* TRIANGLES.

In Three PARTS.

I. ARITHMETICALLY: By the *Triangular* CANON of *Sines*, *Tangents*, and *Logarithms*.

II. GEOMETRICALLY: By *Scale* and *Compasses*. And,

III. INSTRUMENTALLY: (1) By *Logarithmical Scales*. (2) By the *Globe*. (3) By the *Catholick Planisphere*, or ANALEMMA.

By *WILLIAM LEYBOURN*, Philomath.

LONDON:

Printed *Anno Dom.* M DC XC.

ELEMENTS:

TRIGONOMETRY.

Cursus Mathematicus.

Book V.

ARITHMETICAL TRIGONOMETRY.

Part I.

CHAP. I.

Of Definitions and Affections of Triangles.

OF *Triangles* there are two kinds, *Plain*, and *Spherical*; either of which do consist of *Six Parts*, namely, of Three *Sides*, and as many *Angles*.

1. *A Plain (or right lined) Triangle, consisteth of three Sides, and as many Angles*, and such are the two *Figures* ABC, and DCB in which, in the first, AB, BC, and CA are the three Sides of the *Triangle* ABC: And CAB, ACB, and CBA, are the three *Angles* of the same *Triangle* ABC. *And here you are to Note, that an* Angle, *in any case, is noted with three Letters, the middlemost whereof respecteth the Angular Point.* So in the *Triangle* ABC, If I would express the *Angle* at C, I would say the *Angle* ACB, or BCA. — Also in the Second *Triangle* DBC, the Lines DC, DB, and CB, are the Sides of the *Triangle* DBC. Also the *Angles* DCB, CBD, and BDC, are the *Angles* of the same *Triangle* DBC. *Fig.* I.

2. *Any two Sides of a Triangle, are called the Sides of the Angle contained by* (or comprehended between) *them*, as the Sides BC and CA are the Sides containing the *Angle* BCA.

3. *Every Side of a Triangle is the subtending Side of that Angle, which is opposite unto it.* As in the *Triangle* ABC, BC is the subtending Side of the *Angle* CAB; the Side BA subtends the *Angle* BCA; and the Side CA subtends the *Angle* CBA. *The greatest Side always subtendeth the greatest Angle; the lesser Side the lesser Angle; and equal Sides subtend equal Angles.*

4. *The Measure of an Angle is the Arch of a Circle, described upon the angular point, and intercepted between the two Sides containing the Angle; (increasing the Sides if need be.)* As in the *Triangle* ABC, the measure of the *Angle* CBA is the Arch OP, or CF.

5. *Every Circle is divided into 360 degrees, and every degree into 60 minutes, &c. which degrees are so much the greater, by how much the Circle is greater; and those Arches which contain the same number of degrees in equal Circles are equal; but in unequal Circles they are termed like Arches.* As the Arches CF and DE are equal, being equal parts of the same *Circle*: *But the Arches* CF *and* OP *are like Arches.* For Example, As CF is 40 parts in the great *Circle* DEFC, so is OP 40 parts in the lesser *Circle* POG.

6. *A Quadrant* (or Quarter) *of a Circle, is an Arch of 90 degrees*, as is the *Quadrant* or *Arch* HF.

7. *The Complement of an Arch less than a Quadrant is, so much as that Arch wanteth*

of 90 *degrees.* As the Complement of the *Arch* CF 40 degrees, is the *Arch* CH 50 deg.

8. *The exceſs of an Arch greater than a* Quadrant, *is ſo many degrees as that Arch exceedeth* 90 *deg.* As the *Arch* DHC being 140 deg. is the *Arch* HC, 50 degrees, more than a *Quadrant.*

9. *A Semicircle is an Arch of* 180 *degrees.*

10. *The Complement to a Semicircle, of an Arch leſs than a Semicircle, is ſo much as that Arch wanteth of* 180 *degrees*; as the Complement of the Arch DHC 140, is the Arch CF 40 degrees.

11. *The oppoſite Angles made by the croſſing of two Lines are equal.* As the Angles CBF, and DBE are equal, and ſo alſo are the Angles EBF, and DBC. *Euclid.* Lib. 1. Pr. 15.

12. *An Angle is either Right or Oblique.*

13. *A Right Angle is that whoſe Meaſure is a Quadrant or* 90 *degrees, as* CAB.

14. *An Oblique Angle is either Accute or Obtuſe.*

15. *An Accute Angle is that whoſe meaſure is leſs than a Quadrant,* as the Angles BCA and CBA; in the *Triangle* ABC.

16. *An Obtuſe Angle is that whoſe meaſure is more than a Quadrant.* As the Angle DBC.

17. *The Complements of Angles are the ſame as the Complements of Arches.*

18. *All Angles coming* (or meeting) *together upon any right Line, all of them being taken together are equal to two Right Angles.* As the Angles HBD, HBC, and CBE, meeting all together in the Point B, upon the *Line* DF, are equal to the two *Right Angles,* DBH and HBF.

19. *A Triangle hath ſome of its Sides equal, or elſe they be all unequal.*

20. *A Triangle of ſome equal Sides, is either Equicrural, or Equilateral.*

21. *An Equicrural Triangle, is that which hath only two equal Sides.* As the *Triangle* DBE; the Sides BD and BE being equal.

22. *An Equinoctial Triangle is Equiangled at the Baſe:* And if in ſuch a *Triangle,* a Perpendicular be let fall from the meeting of the equal Sides, it cutteth the *Baſe,* and the Angle oppoſite to the *Baſe* into two equal parts. As in the *Equicrural Triangle* DLC (*Fig.* 1.) the Perpendicular DA cutteth the *Baſe* CL in two equal Parts in A, and alſo biſecteth the Angle CDL, (from whence it is let fall.)

23. *An Equilateral Triangle is that whoſe Sides are all equal; and whoſe Angles contain each of them* 60 *degrees.* As the *Triangle* BEK; whoſe Sides BE, BK, and EK, are all equal, and whoſe Angles EBK, EKB and BEK do contain each of them 60 deg.

24. *A Triangle is Right Angled or Oblique angled.*

25. *A Right angled Triangle is that which hath one Right Angle,* as the *Triangle* CBA, Right angled at A.

26. *An Oblique Angled Triangle, is that which hath all its Angles Oblique.* As the *Triangle* DBC.

27. *An Oblique angled Triangle is either Acute angled, or Obtuſe angled.*

28. *An Accute angled Triangle is that which hath all its Angles Accute.* As the *Triangle* BDE.

29. *An Obtuſe Angled Triangle is that which hath but one Obtuſe Angle.* As the *Triangle* DCB, *Obtuſe angled* at B.

CHAP. II.

Of certain Geometrical Theorems, *for the better underſtanding of the Doctrine of Plain* (or Right Lined) Triangles.

Fig. II. I. *If a* Right Line *do fall upon two* Parallel Right Lines, *it maketh the alternate* Angles *equal.*

AS the *Right Line* PQ, falling upon the two *Parallel Lines* LM and NO, doth make the alternate *Angles* equal. As the *Angle* PRM equal to the *Angle* NSQ; And PRL equal to QSO. *Eucl.* L. 1. P. 29.

2. *If*

II. *If divers* Right Lines *be cut by divers other* Right Lines, *which are* Parallel *one to the other, the* Segments *are proportional.* *Fig.* III.

Let the two *Lines* R S and R T be cut by the four *Parallel Right Lines*, V, X, Y, Z, I ſay then the Interſegments R *a* and R *b*, as alſo *a* T and *b* S, are proportional one to the other. For if R *a* be one third part of R T, R *b* ſhall alſo be one third part of R S. *&c.* Becauſe, the *Right Line a* X cutteth off one third part of the Space ÆVTZ, and therefore it cutteth off a third part from every *Line* drawn within that ſpace.

III. *If two* Right Lines *be multiplied into one another, there is made of them a* Right Lined Parallelogram.

Let the two Sides to be multiplyed be A 7 and B 9. If of the *Line* A and B *a* rectangled *Parallelogram* be made, it will be ſuch a *Figure* as C D E F: Or if A 7 and B 9 be multiplyed each by other, the Product will be 63. and ſo many little *Parallelograms* are there contained in the Larger *Parallelogram* C D E F. *Fig.* IV.

IV. *If two* Rectangled Figures *be made of one of the Sides of that* Figure, *and of any two* Segments *of the other Side of the ſame* Figure, *thoſe two* Parallelograms *added together, ſhall be equal to that* Figure.

So if the *Parallelogram* G L I M, be made of the Side B 9, and of a Segment of the other Side, namely of 4, that *Parallelogram* ſhall produce 36, for 9 times 4 is 36. And if another *Parallelogram* ſhall be made of the whole Side A 9, and the other Segment of the other Side B 7, namely 3, they will produce the other *Parallelogram*, L H M K 27, for three times 9 is 27. And theſe two *Parallelograms* 36 and 27 added together, do make 63 equal to the firſt.

V. *If four* Right Lines *be proportional,* (that is, as the firſt is to the ſecond, ſo is the third to the fourth,) *the* Right Angled Figure *made of the two Means* (or middle terms) *ſhall be equal to the* Rectangle Figure *made of the two Extreme terms.*

Let there be 4 Proportionals, as N 4, O 6, P 8, Q 12. I ſay that the *Right angled Figure* R S T V, made of the two Means O 6 and P 8, ſhall be equal to the *Rectangle Figure* X Y Z Æ, made of N 4, and Q 12, the two Extremes: For as 6 times 8 is 48; ſo is 4 times 12 48: And from hence will follow this. *Fig.* V.

CONSECTARY. 1.

If four Right Lines (or Numbers) *be proportional, if three of them be given, the fourth is alſo given*: For the *Rectangle Figure*, made of the two Means, being divided by one of the Extremes, the *Quotient* ſhall be the other Extreme: —— As the *Rectangle Figure* made of the two Means, 6 and 8, is 48; this divided by 4 the leſſer Extreme, ſhall give you 12 the greater Extreme; — or if the proportional terms given had been (by tranſpoſition) 4, 8, 6, and 12. The *Rectangle* made of the Means 8 and 12 would be 48; which divided by 12, the *Quotient* will be 4, the other Extreme.

CONSECTARY. 2.

Hence alſo it followeth, *That equal* Right angled Figures *have their Sides reciprocally proportional*; that is, as the leſſer Side of the firſt *Figure*, to the leſſer Side of the ſecond; ſo is the greater Side of the ſecond, to the greater Side of the firſt, *& contra.* As in the *equi-rectangled Figures* R S T U, and X Y Z Æ appeareth. For,

As R T	: is to	X Y	:: So is	X Z	: to	R S
6		4	Or	12		8
As X Y	: is to	R T	:: So is	R S	: to	X Z
4		6		8		12

VI. *If three* Lines *or* Numbers *be proportional,* (that is, as the firſt is to the ſecond, ſo ſhall the ſecond be to a fourth,) *the* Square *made of the Means, is equal to the* Oblong *made of the Extremes.*

As let the Proportional numbers be 4 — 8 — 8 — the Proportion will be
As 4 is to 8 :: ſo 8 to 16.
So the Square of the two Means 8 is 64, ſo alſo is the Oblong made of the Extremes; for 4 times 16 is 64 alſo.

Fig. III. VII. *In a* Plain Triangle, *a* Line *drawn* Parallel *to the* Baſe *cutteth the Sides thereof proportionally.*

As in the *Plain Triangle* R T S, if *c d* be parallel to the Baſe T S, it cutteth off from the Side R S one third part: And alſo it cutteth from the Side R T one third part alſo; by the 2 *of this Sect*: And ſo they ſhall be proportional.

For As R T : to R S :: ſo is R *c* : to R *d*. Alſo,
As R *c* : to *c* T :: ſo is R *d* : to *d* S. Alſo,
As R *c* : to R *d* :: ſo is *c* T : to *d* S.

Fig. VI. VIII. *If divers* Plain Triangles *be compared together*: *All* Equiangled Triangles *have their Sides about* (or containing) *the equal* Angles *proportional*; and the contrary. *Eucl. Lib.* 6. *Pr.* 4.

For Illuſtration.

(1) Let A B C and A D E be two *Plain* and *Equiangled Triangles*; ſo as the *Angles* at B and D; at A and A; and alſo at C and E be equal one to another, each to its correſpondent: I ſay their Sides about the equal *Angles* are Proportional. For,

1 As A B : is to B C :: ſo is A D : to D E.
2 As A B : is to A C :: ſo is A D : to A E.
3 As A C : is to C B :: ſo is A E : to E D.

For Demonſtration hereof.

2. Becauſe the *Angles* B A C, and D A E are equal, therefore if A B be applied to A D: then A C ſhall of neceſſity fall in A E; and by ſuch application ſhall ſuch a *Figure* be made.

In which *Figure*, becauſe that A B and A C do meet together; and alſo the *Angles* at B and D, and at C and E are equal, therefore the other Sides B D and D E ſhall be *Parallel*, (by the 1. of this Section:) But in a *Plain Triangle, a* Right Line Parallel *to the Baſe cutteth the Sides proportionally*, (by the 7 of this Sect.) Therefore, in the *Triangle* A D E, the *Right Line* B C, being *Parallel* to the Baſe D E, cutteth the Sides A D and A E proportionally; and thence it follows, that

As A B : to A D :: ſo A C : to A E.

(3) Again, by the point B, let the *Right Line* B F be drawn, *Parallel* to the Baſe A E, and it ſhall cut the other two Sides D E and D A proportionally, in the points B and F, (by the laſt) and the proportion will be,

As A B : to A D :: ſo is F E : to D E, or,
As A B : to A D :: ſo is B C : to D E.

For F E and B C are equal (by the 2 of this Section.) And ſince it is, that

As A B : is to A D :: ſo is A C : to A E,
And ſo B C to D E; they ſhall alſo be
As A C : is to A E :: ſo is B C : to B E.

For thoſe two things which are agreeable to a third thing, are agreeable one to another.

Therefore it generally follows,

1 As A B : is to A D :: ſo is B C : to D E.
2 As A B : is to A D :: ſo is A C : to A E.
3 As A C : is to A E :: ſo is B C : to D E.

Or

Or by tranſpoſition of the ſecond and third terms of the proportion, thus

1 As A B : is to B C :: ſo is A D : to D E.
2 As A B : is to A C :: ſo is A D : to A E.
3 As A B : is to B C :: ſo is A E : to D E.

And thus it is demonſtrated, that all *Plain Equiangled Triangles* (as theſe A B C and A D E are) have their Side comprehending the equal *Angles* Equal.

IX. *If ſeveral* Plain Triangles (how many ſoever) *be compounded, and be cut with* Parallel Right Lines ; *the* Interſegments *are proportional.* As thus,

If the two *Triangles* H F I and I F K be compounded, and be cut with the *Parallel Lines* G L M and H I K; their Interſegments are proportional. For,

As G L : is to H I :: ſo is L M : to I K. Or,

As G L : is to L M :: ſo is H I : to I K. (by the 2 or 7 of this Sect.)

For the *Triangles* F G L are F H I are *Equiangled* (by the 1 Sect.) becauſe G L and H I are *Parallels*: Therefore,

As F L to F I :: ſo is G L : to H I. (by the 7 of this Sect.) But (by the ſame.)

As F L : to F I :: ſo is L M : to I K. And thoſe that are agreeable to a third are alſo agreeable between themſelves. Therefore,

As G L : is to H I :: ſo is L M : to I K. *&c.*

X. *If any one Side of a* Plain Triangle *be continued, the outward* Angle (made by that continuation) *is equal to the two inward and oppoſite* Angles *of the ſame* Triangle.

If in the *Plain Triangle* N O P, the Side N P be continued to Q: The outward *Angle* O P Q ſhall be equal to the two inward *Angles* O N P, and N O P: For,

If from the point P a *Right Line*, P R, be drawn *Parallel* to N O, the outward *Angle* O P Q ſhall be compounded of the *Angles* O P R and R P Q. But the *Angles* R P Q and R P O are equal to the two inward oppoſite *Angles* O N P, and N O P; that is to ſay, the *Angle* R P Q to the *Angle* O N P, and the *Angle* O P R to the *Angle* N O P, (by the 1 of this Section,) becauſe of the *Parallels* N O and P R: And therefore the *Angle* O P Q is equal to the two inward and oppoſite *Angles*, O N P and N O P: *Which was to be demonſtrated.*

XI. *The Three* Angles *of every* Plain (*or* Right Lined) Triangle, *are equal to two* Right Angles.

As in the *Plain Triangle* N O P (*Fig.* VIII.) I ſay the three *Angles*, N O P, P N O, and O P N, are together equal to two *Right Angles*. Fi. VIII.

For the *Angles* meeting in one point, in one and the ſame *Line*, are equal to two *Right Angles*, (by the 18 of Sect. 1.)

But the three *Angles*, N O P, O P N and O N P, are equal to the three *Angles*, meeting in the Point P upon the ſame *Line* N Q; ——— For the *Angle* O P N is common to both, and the *Angles* R P Q and R P O are equal to the *Angles* O N P, and N O P, (by the laſt;) therefore the three *Angles*, N O P, O P N, and O N P, are equal to two *Right Angles*.

And from hence will follow theſe *Corollaries.*

1. *That there can be but one* Right, *or one* Obtuſe Angle *in any* Plain Triangle.
2. *And if one* Angle *be either* Right *or* Obtuſe, *the other two ſhall be* Acute.
3. *That the third* Angle *of any* Triangle, *is the complement of the other two, to two* Right Angles.
4. *That if two* Triangles *be* Equiangled *in any two of their* Angles, *they are wholly* Equiangled.

XII. *In every* Right Angled Plain Triangle, *the* Square *made of the Side which ſubtendeth* (or is oppoſite to) *the* Right Angle, *ſhall be equal to both the* Squares *which are made of the two Sides which ſubtend the* Right Angle.

Fig. IX. As in the *Plain Right Angled Triangle* SVT, *Right Angled* at T; I ſay the Sides ST and TV including the *Right Angle* STV, are equal in power to the *Hypotenuſe* VS: that is, the Squares of the Sides ST and VT, namely, the Squares STWÆ, and VTBC, added together, are equal to the Square of the *Hypotenuſe* VS, to wit, the Square YXVS.

For, if from the *Right Angle* at T, be let fall the Perpendicular TAZ, then out of the Square ZYVS is made two *rectangle Figures* ASYZ and AZXV; the *Rectangle* (or *Oblong*) ASYZ equal to the Square SWTÆ, and the *Rectangle* AVXZ equal to the Square VTBC; —— and therefore the Square VXYZ compounded of thoſe two *Oblongs*, is equal to the two Squares VTBC, and SWTÆ.

But that the two *Oblongs* AZYS and AVXZ are equal to the two Squares SWTÆ, and VTBC ſhall thus be proved.

If three Right Lines *be proportional* (as by the 6 of Sect. 2.) *The Square of the Means is equal to the* Oblong *made of the two Extreams*: But the three *Right Lines*, SY, (equal to SV,) ST, and SA, are proportional; that is,

As SY (equal to SV) is to ST :: ſo is ST : to SA; therefore the Square of ST is equal to the *Oblong* made of SY (equal to SV) and SA; for the *Triangles* STV and TAS are *Equiangled*, becauſe of the common *Angle* at S, and the two *Right Angles* T and A: Therefore (by the 8 of Sect. 2.)

As SV : is to ST :: ſo is ST : to SA.

In the ſame manner it is alſo proved, that the *Oblong* VXZA is equal to the Square VTBC; for the *Triangles* TSA and ATV are *Equiangled*, becauſe of their common *Angle* at V, and the two *Right Angles* at T and A: Therefore,

As SV : is to TV :: ſo is TV : to AV.

And ſo (by the 6 of Sect. 2.) the Square of TV is equal to the *Oblong* XZVA: *Which was to be demonſtrated.*

Hence it followeth: That,

If in a *Right angled plain Triangle*, any of the two Sides be given, the third may be ſaid to be given alſo;

As, if the two Sides including the *Right Angle* TV 8 and TS 6, their Squares 64 and 36 being added together make 100, the Square Root whereof is 10 for the third Side (or *Hypotenuſe*) VS. —— On the contrary if the *Hypotenuſe* 10, and one of the containing Sides ST 6 be given, ſubtract the Square of 6 *viz.* 36, from the Square of 10. *viz.* 100, the remainder will be 64, whoſe Square Root is 8 for the other containing Side VT.

XIII. *In a* Plain Right Angled Triangle, *the one of the* Accute Angles *is the* Complement *of the other* (by the 11 of Sect. 2.) *and may thus be proved.*

In the *Plain Triangle* BAC (*Fig.* 1.) *Right Angled* at A, the one of the *accute Angles* ACB, is equal to the *Angle* CBH (by the 1 of Sect. 2.) becauſe of the *Parallels* HB and CA. But the *Angle* HBC is the complement of the *Angle* CBA by the Work, therefore the *Angle* BCA, is the *Complement* of the *Angle* CBA, which was *&c.*

XIV. *If a* Plain Triangle *be inſcribed in a* Circle, *the* Angles *oppoſite to the Circumference, are half as much as that part of the Circumference, which is oppoſite to thoſe* Angles; which may be thus demonſtrated.

In the *Triangle* CDL (in *Figure* I.) from the *Angle* at D, let the *Diameter* DBF be drawn. —— And from the *Centre* B, to the Circumference, let the two *Radii* BC and BL be drawn. I ſay, the divided *Angles* LDF and CDF are the half of the *Angles*

Angles divided LBF and FBC: For the *Angles* LDB and DLB are equal, (by the 3 of Sect. 1.) And the *Angle* LBF is equal to the *Angles* LDB and BLD added together, (by the 10*th* of Sect. 2.) Therefore the *Angle* LBF is double to the *Angle* LDF. Also,

The *Angles* BDC and BCD are equal, (by the 3 of Sect. 1.) and to both these together, is the *Angle* CBF equal (by the 10*th* of Sect. 2.) Therefore the *Angle* FBC is double to the *Angle* FDC.

Now, because the parts of the *Angle* LBC are double to the parts of the *Angle* LDC, therefore also the whole *Angle* LBC is double to the whole *Angle* LDC, and thereupon the *Angle* LDC is half of the *Angle* LBC, and consequently of half the Arch LFC, which is the measure of the *Angle* LBC: *Which was to be demonstrated.* Fig. I.

CHAP. III.

Concerning Sines, Tangents *and* Secants, *by which the* Angles *of* Right Lined, *and the Sides and* Angles *of* Spherical Triangles *are Measured.*

1. *THE Mensuration of* Triangles *is by knowing any three of the Six parts of which a* Triangle *consists* (as of three Sides, and as many *Angles* (by the 1 of the 1 Sect.) *whether Sides or* Angles, *or both,* to find out any of the other parts of the *Triangle,* whether Sides or *Angles.*

2. *And this is performed by the Rule of Proportion, (commonly and deservedly) called the* Golden Rule *of* Arithmetick) *which teacheth of Four Numbers proportional one to another, any three of them being given, to find out the fourth.*

3. *So that for the Measuring* (which is also called the *Resolving*) *of* Triangles, *there must be certain Proportions of all the parts of a* Triangle one to another, and those Proportions explained in Numbers. But,

4. *The certain knowledge of the proportions that all the parts of a* Triangle *have one to another, is not yet known; for that every crooked* Line *in a* Triangle (*as the Measures of the* Angles *in all* Right lined, *and the Measures of Sides and* Angles *in a* Spherical Triangle *are*) *must be reduced to a* Right Line, *a Proportion for which was never yet found.* But,

5. *Crooked* Lines *are nearly reduced to* Right Lines *by the definition of the quantity which* Right Lines *applied to a* Circle, *have in respect of the* Radius (*or* Semidiameter) *of the* Circle.

6. *The* Right Lines *applied to a* Circle *are* Subtenses (*or* Chords) Sines, Tangents, *and* Secants.

7. *A* Subtense (or Chord) *is a* Right Line *inscribed in a* Circle, *dividing the whole* Circle *into two Parts, and so subtendeth both the Segments.*

8. Subtense *in a* Circle *is either the Greatest, or not the Greatest.*

9. *The Greatest* Subtense *is that which divideth the whole* Circle *into two equal Parts or Segments* (*which* Subtense *doth always pass through the* Centre *of the* Circle) *and so subtendeth both the* Segments *or* Semicircles.

10. *A* Subtense *not the Greatest, is that which divideth the* Circle *into two unequal parts,* (or which passeth not through the *Centre* of the *Circle;*) *and so on the one Side subtendeth an* Arch *less than a* Semicircle, *and on the other Side an* Arch *more than a* Semicircle. As in Fig. I.

The *Right Line* DF, divideth the *Circle* into the two *Semicircles* DHF and DKF, and this *Subtense* DF is called the *Diameter* of the *Circle.*

And the *Right Line* DC, which on the one Side subtendeth the *Arch* DHC, less than a *Semicircle,* and on the other Side the *Arch* DEKFC, Greater than a *Semicircle.*

11. *A* Sine *is either* Right, *or* Versed.

12. *A*

12. *A* Right Sine *is the one half of the* Subtenſe *of the double* Arch: —— As the *Right Sine* of the *Arches* CF or CHD, is the *Right Line* CA, it being the half of the *Subtenſe* of the double *Arches* of CF and CHD; that is, the half of the *Right Line* CAL, which ſubtendeth the *Arches* CFL, and CDL: —— So the *Right Sine* of the *Arches* CH or CK, is the *Right Line* CM; that is, the half of the *Right Line* CMN.

From hence it will follow, that

1. *The* Right Sine *of an* Arch, *more or leſs than a* Quadrant, *and leſs than a* Semicircle, *is one and the ſame*: —— As the *Sine* of the *Arches* CF and CD, is the ſame *Right Line* CA, for that it is the half of the *Right Line* CAL, which ſubtendeth as well the *Arch* CDL as the *Arch* CFL.

2. Hence it is, *that whenſoever the* Right Sine *is called the* Sine *of the Complement, it is underſtood only the* Sine *of the Complement of an* Arch *leſs than a* Quadrant: —— As the *Right Sine* of the Complement of CF, that is, of the *Arch* CH, is the *Right Line* CM.

3. *Every* Right Sine *is perpendicular to the* Diameter *drawn from one term of the* Arch *given*: —— For, becauſe in the *Triangle* BCL, conſiſting of two equal Sides, BC and BL, the *Semidiameter* BF, drawn from the concourſe of the equal Sides, cutteth the Baſe CL in A.

4. *The* Right Sine *of the Complement is equal to the Segment of the* Diameter *or* Radius, *intercepted between the* Right Sine *of the* Arch, *and the* Circles Centre: —— As the *Right Sine* of the Complement CH, *viz.* CM, is equal to the *Right Line* AB.

13. *The* Verſed Sine *is the Segment of the* Diameter, *intercepted between the* Right Sine *and the Circumference*: —— As the *Verſed Sine* of the *Arch* CF is the Segment of the *Diameter* AF; and the *Verſed Sine* of the *Arch* CD, is the Segment of the *Diameter* AD: Therefore,

14. *Of* Verſed Sines, *ſome are Greater, and ſome Leſſer.*

15. *A Greater* Verſed Sine *is the* Verſed Sine *of an* Arch, *greater than a* Quadrant; *and a leſſer* Verſed Sine *is the* Verſed Sine *of an* Arch, *leſs than a* Quadrant: —— As DA is the *Verſed Sine* of the *Arch* DHC, being greater than a *Quadrant*, and AF is the *Verſed Sine* of the *Arch* CF, leſs than a *Quadrant.*

16. *A* Tangent *is a* Right Line *drawn perpendicularly on the extremity of the* Diameter *of the* Circle, *and paſſing by the other end of the* Arch: —— As FO is the *Tangent* of the *Arch* CF.

17. *A* Secant is *a* Right Line *drawn by the one end of the* Arch, *till it meet with the* Tangent: —— As the *Secant* of the *Arch* CF, is the *Right Line* BO.

18. *The Definition of the Quantity which* Right Lines *have, being applied to* Circles, *is the making of the* Tables *of* Sines, Tangents, *and* Secants.

19. *The* Tables *of* Sines, Tangents, *and* Secants, *are called the* Canon *of* Triangles.

20. *The* Tables *of* Sines, Tangents, *and* Secants, *are extended no farther than to a* Quadrant, or 90 deg. *For the* Right Sines *of* Arches *more or leſs than a* Quadrant, *are the ſame,* (by the 10*th* of the 1 Sect.) *and there can be no* Tangents *and* Secants *of* Arches *greater than a* Quadrant, (by the 15 and 16 of Sect. 1.)

Note, The *Tables* of *Sines, Tangents,* and *Secants* here mentioned, are to be underſtood of *Natural Sines, Tangents,* and *Secants,* whoſe uſe in the Solution of *Triangles,* was performed by *Multiplication* and *Diviſion*; But the *Canon* of *Triangles* now in uſe are theſe which were invented by *John* Lord *Neprie, Baron* of *Marchiſton* in *Scotland,* about the Year 1621, which *Canon* conſiſteth of *Sines* and *Tangents Artificial,* or rather *Logarithmical Sines* and *Tangents,* which performeth by *Addition* and *Subtraction,* that which the *Natural Canon* did by *Multiplication* and *Diviſion*; ſo that by theſe more *Triangles* may be reſolved in one hour, than by the *Natural* could be in *Six:* Theſe *Canons* as alſo the *Ten Chiliads* of *Logarithms* of the ſame Authors, are every where to be had; and ſuch are theſe at the end of this Book. So I ſhall ſay no more of them in this place, but come to ſhew the *Uſe* of them in the *Solution* of *Triangles* both *Plain* and *Spherical,* and (having laid the foregoing Foundations therein,) uſe all the brevity (which may conſiſt with perſpecuity) that poſſible I can.

CHAP.

CHAP. IV.

Of the Dimension *of* Right Angled Plain Triangles.

IN the Solution of *Right Angled Plain Triangles*, there are 7 *Cases*, the resolving of which depend upon this following

AXIOME.

In every Plain Right angled Triangle, *any of the Three Sides may be put as* Radius, *and then will the other Sides be as* Sines, Tangents, *or* Secants.

THus, in the three *Triangles*, A, B, C.

1. If you put the Side BC subtending the *Right Angle*, for *Radius*, the Sides CA, and BA including the *Right Angle*, are *Sines* of the *acute Angles* B and C, opposite to them.

2. If you put for *Radius* the Greater of the Sides including the *Right Angle*, as BA, then the lesser of the Sides including the *Right Angle* CA is the *Tangent*, and the subtending Side BC is the *Secant* of the lesser *Acute Angle* at B.

3. If you put the Lesser of the two Sides including the *Right Angle* CA for *Radius*, the Greater Side including the *Right Angle* BA is the *Tangent*, and the Side subtending the *Right Angle* BC is the *Secant* of the Greater *Acute Angle* at C.

Now from this Axiome *followeth this* Consectary.

That, *In all* Right angled Plain Triangles, *the* Angles *being given, the Reasons of the Sides are also given three ways.* And consequently,

If one Side be given, besides the three Angles, *every of the other Sides are given by a Threefold Proportion, according as you put,* This, That, *or the* Other *Side for* Radius.

Now for the Solution of the VII *Cases* of *Right Angled Plain Triangles*, I shall make use of the same *Triangles*, noted with CBA in *Fig.* X.

Wherein,

BA is the *Base.*	and contains	287.00 Miles.
CA is the *Perpendicular* or *Cathetus.*		211.08 Miles.
BC is the *Hypotenuse.*		356.26 Miles.
A is the *Right Angle.*		90 deg. 00 min.
B is the *Angle* at the *Base.*		36 deg. 20 min.
C is the *Angle* at the *Perpendicular.*		53 deg. 40 min.

Now, for the avoiding of Trouble in turning over the *Canon* of *Tables*, I will set down the *Sine*, *Secant*, and *Tangent*, of every *Angle*; and the *Logarithm* of every *Side* of this *Exemplary Triangle*: As followeth,

The

2. *By making the Perpendicular* C A Radius.

As the Perpendicular C A, 211.08	2.3244471
Is to the *Base* B A, 287 Miles	12.4578819
So is *Radius* 90 deg.	10.
To the *Tangent* of the *Angle* C, 53 deg. 40 min.	10.1334348

The *Complement* whereof is 36 deg. 20 min. the other *Acute Angle* at B.

CASE V.

The Base B A, *and the* Hypotenuse B C *being given, to find the other* Acute Angles B *and* C.

1. *By making* B C *the* Hypotenuse Radius.

As the *Hypotenuse* B C, 356.26 Miles	2.5517670
Is to the *Base* B A, 287 Miles	12.4578819
So is *Radius* 90 deg.	10.
To the *Tangent* of the *Angle* C, 53 deg. 40 min.	9.9061149

Whose *Complement* 36 deg. 20 min. is the other *Acute Angle* at B.

2. *By making the* Base B A Radius.

As the *Base* B A, 287 Miles	2.4578819
Is to the *Hypotenuse* B C, 356.26	12.5517670
So is *Radius* 90 deg.	10.
To the *Secant* of the *Angle* at B 36 deg. 20 min.	10.0936851

CASE VI.

The Base B A, *and the Perpendicular* C A *being given, to find* Hypotenuse B C.

This and the following *Case* do require two *Operations*, the first to find the *Acute Angles*, and the second (from thence) to find the third side:

1. *Operation.*

As the Perpendicular C A, 211.08	2.3244471
Is to the *Base* B A, 287	12.4578819
So is *Radius* 90 deg.	10.
To the *Tangent* of the *Angle* C, 53 deg. 40 m.	10.1334348

2. *Operation.*

As the *Sine* of the *Angle* at C, 53 deg. 40 min.	9.9061107
Is to the *Base* B A, 287	12.4578819
So is *Radius* 90 deg.	10.
To the *Hypotenuse* B C, 356.26	2.5517712

CASE VII.

The Base B A, *and the* Hypotenuse B C *being given, to find the Perpendicular.*

1. *Operation.*

As the *Hypotenuse* B C, 356.26	2.5517670
Is to the *Base* B A, 287	12.4578819
So is *Radius* 90 deg.	10.
To the *Sine* of the *Angle* at C, 53 deg. 40 min.	9.9061149

Whose *Complement* 36 deg. 20 m. is the *Angle* at the *Base* B.

2. *Operation.*

As *Radius* 90 deg.	10.
Is to the *Hypotenuse* B C, 356.26	2.5517670
So is the *Sine* of the *Angle* B, 36 deg. 20 min.	9.7726751
To the Perpendicular C A, 211.08	12.3244421

A more compendious way to work this last Case, *by* Logarithms *only.*

1. Find the Summ, and the Difference of the *Base*, and the *Hypotenuse*, and add the *Logarithms* of them together, half that *Logarithm* shall be the *Logarithm* of the Perpendicular: As

The *Hypotenuse* is	356.26	
The *Base* is	287.	
Their Summ	634.26	*Log.* 2.80838655
Their Difference	69.26	*Log.* 1.84048249
Their Summ		4.64886904
The half Summ		2.32443452
And this is the *Logarithm* of the Perpendicular	211.08	

CHAP. V.

Of the Dimension of Oblique Angled *Plain Triangles.*

IN the solution of *Oblique Angled* Plain Triangles there are but V. *Cases*, and the resolving of them dependeth upon the Three following *Axioms.*

The first of which serveth for the resolving of the *first* and *second Cases.*—The second for the resolving of the *third* and *fourth Cases*,—And the third is appropriate only to the *fifth* and *last Case.*—And in the same manner as I did proceed in the solution of the several *Cases* of *right Angled* Plain Triangles, so I shall in this of *Oblique Angled*; namely, First, lay down (and demonstrate) the *Axiom*, and such *Consectaries* as follow therefrom, and then proceed to the *Cases* resolved thereby: Wherefore,

AXIOME I.

In every plain Triangle (*as well* Right *as* Oblique Angled) *the sides are in proportion one to the other, as are the Sines of the Angles opposite to those sides*, & contra.

Construction.] LET the *Triangle* A B C be inscribed in a *Circle*, and from the *Centre* D, let be drawn the several *Radii* D E, D F, D G, perpendicular to the respective sides of the *Triangle*, so will they bisect as well the several *Peripheries* A E C, C F B, and B G A, as their *Subtenses* A C, C B and B A: And let there be also drawn the *Radius* D C. *Fig.* XI.

Demonstration.] Now, because the *Angle* at the *Centre* E D C, is equal to the *Angle* in the *Periphery* A B C, and C D F at the *Centre*, equal to C A B, (by the 20 P. *Eucl. Lib.* 3.) therefore shall the halves of the sides be as *Sines*: And what proportion the side C A hath to the side C B, the same shall the *Sine* C H have to the *Sine* C K; for what proportion the whole hath to the whole, the same proportion hath the half to the half: *Which was to be demonstrated.*

And from this *Axiome* follow these *Consectaries*: that,

1. *If the Angle of a Triangle be given, the Reason of the Sides is also given.*

And Consequently,

If one side be given besides the Angles, both the other sides are also given.

2. *If two sides of a Triangle be given, with an Angle opposite to one of them, the Angle opposite to the other of them is also given.*

Now for the solution of the two *Cases* of an *Oblique Spherical Triangle* deduced from this *Axiome*, I shall make use of the *Oblique Angled Triangle* A C B (in *Fig.* XI.) wherein

The *Side*	A B	is 478.83	Leagues, whose *Logarithm* is		2.6801813
	B C	is 361.56			2.5581038
	C A	is 234.00			2.3692158
The *Angle*	C A B	is 46 deg. 50 min.	whose *Sine* is		9.8629460
	A B C	is 28 deg. 10 min.			9.6739769
	B C A	is 105 or 75 deg.			9.9849438

CASE

CASE I.

The Angles *and one Side being given, to find the other two Sides.*

Let the *Angles* CAB 46 deg. 50 min. and CBA 28 deg. 10 min. And the Side AC 234 Leagues be given, and let it be required to find the Side BC.

As the *Sine* of the *Angle* CBA 28 d. 10 m.	9.6739769
Is to the Side AC 234 Leagues.	2.3692158
So is the *Sine* of the *Angle* CAB 46 d. 50 m.	9.8629460
	12.2321618
To the Side CB 361.56 Leagues.	2.5581849

And for the Side BA.

As the *Sine* of the *Angle* CAB 46 d. 50 m.	9.8629460
Is to the Side CB 361.56 Leag.	2.5581038
So is the *Sine* of the *Angle* ACB 105 d. (or 75 d.)	9.9849438
	12.5430476
To the Side AB 478.83 Leagues	2.6801016

CASE II.

Two Sides, with an Angle *oppoſite to one of them, being given, to find the* Angle *oppoſite to the other of them.*

Let the two Sides AC and BC, and the *Angle* CAB oppoſite to the Side CB be given, and let the *Angle* CB (oppoſite to the Side AC) be required.

As the Side BC 361.56.	2.5581038
Is to the *Sine* of CAB 46 d. 50 m.	9.8629460
So is the Side AC 234.	2.3692158
	12.2321618
To the *Sine* of CBA 28 d. 10 m.	9.6740580

Or if the two Sides AB and BC, and the *Angle* A oppoſite to CB had been given, and the *Angle* ACB had been required, Then

As the Side CB 361 . 56.	2.5581038
To the *Sine* of CAB 46 d. 50 m.	9.8629460
So is the Side AB 478.83.	2.6801813
	12.5431273
To the *Sine* of ACB 75 d. (or 105 d.)	9.9850235

Note, *In this* Caſe, *there may a doubt ariſe, for if two Sides be given, whereof one of them is the greateſt Side, together with the* Angle *oppoſite to the leſſer of the two given Sides, and the* Angle *oppoſite to the greater of the two given Sides be demanded (as in this ſecond Caſe) it will be doubtful whether the* Angle *found be* Acute *or* Obtuſe; *for that the ſame* Sine *found in the* Canon, *doth anſwer to both: Nor can this doubt be cleared better than by a true delineating of the* Triangle.

AXIOME II.

In all Plain Triangles, *As the Summ of the Sides is to the Difference of the Sides, ſo is the* Tangent *of the half Summ of the oppoſite* Angles, *to the* Tangent *of half their Difference.*

Fig. XII. *Conſtruction.* IN the *Oblique Angled Triangle* ABC (*Fig.* XII.) Let the two known Sides be AB and BC, and the *Angle* comprehended between them, namely, the *Angle* ABC, (which is an *Obtuſe Angle* in the upper, but an *Acute Angle* in the lower Diagram.) Let the Side AB be continued to H, ſo that BH be made equal to BC, and draw the *Right Line* CH. Then make BI equal to the other given Side AB, and

and through the Points B and I, draw the *Right Lines* BD and IG, Parallel to the unknown Side AC.

Demonstration, The exterior *Angle* CBH is equal to the two interior *Angles* BAC and ACB, oppoſite thereto, (by the 10*th* of Sect. 2.) and by the 32 of the 1 of *Eucl.*) for the *Angle* CBD is equal to the *Angle* ACB; and the *Angle* DBH is equal to the *Angle* BAC. Moreover, from the Point B let fall the Perpendicular BE, which ſhall biſect CH in E, and upon the Angular Point B, with the *Radius* BE, deſcribe the *Periphery* or *Arch* MEL; ſo ſhall CE be the *Tangent* of half the Summ of the oppoſite *Angles* CAB, and ACB; and DE (equal to FE) the *Tangent* of half their difference. Now becauſe AC, BD, and IG are *Parallels,* therefore ſhall CD, DG, and FH be equal, as alſo DF and GH: Therefore I ſay,

As AH, is to IH :: ſo is CH, to GH, or by equality of Proportion;
As AH, is to IH :: ſo is CE, to DE; that is,
As AH, *the Summ of the Sides,* is to IH, *the Difference of the Sides.*
So is CE, the Tangent *of half the* Angles CAB *and* ACB, *to* DE, *the* Tangent *of half the Difference of theſe* Angles: Which was to be demonſtrated.

And from this Demonſtration *followeth this* Conſectary.

That, If in a Plain Oblique angled Triangle, *two Sides be given, with the* Angle *comprehended between them, the other two* Angles *are alſo given.*

Now for the Solution of the two *Caſes* of *Oblique angled Plain Triangles* deduced from this *Axiome,* I ſhall make uſe of the *Triangle* in the two foregoing *Caſes*; where the *Sides* and *Angles* are the ſame, as in this *Figure* XII. *Fig.* XII.

CASE III.

Two Sides, with the Angle *comprehended by them being given, to find the other two* Angles.

In the *Triangle* ACB (*Fig.* XII. in the uppermoſt Diagram) let there be given the Side BC 361.56, the Side AB 234. with the *Obtuſe Angle* ABC (comprehended between them) 105 deg. And let the two other *Angles* CAB and ACB be required.

The Summ of the Sides AB and BC is 595.56.
The Difference of theſe Sides is 127.56.
The Summ of the unknown *Angles* at A and C is 75 deg.
The half Summ of the *Angles* is 37 d. 30 m.

Being thus prepared, ſay,

As the Summ of the Sides AB and BC 595.56.	2.7749255
Is to the Difference of thoſe Sides 127.56.	2.1057145
So is the *Tangent* of half the Summ of the unknown *Angles* ACB and CAB, 37 deg. 30 m.	9.8849805
	11.9906950
To the *Tangent* of 9 deg. 20 min.	9.2157695

Which is the *Tangent* of half the Difference of the two *Angles* ACB, and CAB, which 9 deg. 20 min. added to 37 deg. 30 m. (the half Summ of the unknown *Angles*) giveth 46 deg. 50 m. for the greater *Angle* CAB; and ſubtracted from the half Summ, giveth 28 deg. 10 min: for the leſſer *Angle* ACB.

Again in the ſame and the lower Diagram.

Let the two Sides AB 234, and BC 478.83, and the *Acute Angle* CBA, comprehended between them, be given, and let the other two *Angles* CAB, and BCA be required.

The Summ of the Sides AB and CB is 712.83.
The Difference of thoſe Sides is 244.83.
The Summ of the unknown *Angles* A and C is 133 d. 10 m.
The half Summ of thoſe *Angles* is 66 deg. 35 m.

Being thus prepared, say,

As the Summ of the Sides AB and BC 712.83.	2.8529859
Is to the Difference of those Sides 244.83.	2.3888646
So is the *Tangent* of half the Summ of the two unknown *Angles*, ACB and CAB, 66 d. 30 m.	10.3616981
	12.7505627
To the *Tangent* of 38 deg. 18 m.	9.8975768

Which added to the half Summ 66 deg. 30 min. giveth 104 d. 48 m. for the greater *Angle*, and subtracted leaveth 28 deg. 12 m. for the lesser, the one *Angle* greater and the other lesser than by the former Work.

Note, *This difference ariseth from some Seconds omitted in the two* Acute Angles, *for the* Angle A *is really but* 46 *deg.* 49 *min.* 48 *Seconds, and the other* Acute Angle 28 *d.* 10 *m.* 12 *Seconds, but I here omitted the Seconds for ease in the Work, and the rather, because the manner how to find out these minute Parts and Fractions, is taught in the Use of the* Triangular Canons.

CASE IV.

Two Sides, and the Angle *comprehended between them being given, to find the third Side.*

In the *Triangle* ACB (*Fig.* XII. and the upper Diagram) let there be given the Sides AB 234 and BC 361.56, with the *Angle* ABC comprehended between them 105 degrees, and let the third Side AC be required.

First find the *Angle* A or C by the last Case,

As the Summ of the Sides AB and BC 595.56.	2.7749255
Is to their Difference 127.5.	2.1057145
So is the *Tangent* of half A and C 37 d. 30 m.	9.8849805
	11.9906950
To the *Tangent* of 9 deg. 20 m.	9.2157695

Which subtracted from 37 deg. 30 min. (the half Summ of the *Angles* at A and C,) leaves 28 deg. 10 min. for the *Angle* at C.

Then say again by the First Case:

As the Sine of the *Angle* ACB 28 de. 10 m.	9.6739769
Is to the *Sine* of the Side AB 234	2.3692158
So is the *Sine* of the *Angle* ABC 105 (or 75 d.)	9.9849438
	12.3541596
To the Side AC 478.83.	2.6801827

AXIOME III.

In all Plain Triangles. *As the greatest Side is in proportion to the Summ of the two other Sides, so is the Difference of those Sides, to a Segment or part of the Greatest Side, which being subtracted from the Greater Side, a Perpendicular shall fall in the middle of the Remainer.*

Fi. XIII. *Construction:* In the *Triangle* ABC (*Fig.* XIII.) the Greatest Side is BC, the least Side is AC.

Upon A as a *Centre* at the distance of the shortest Side AC, describe the *Circle* CDEF, and continue the Side BA to D, then shall BD be the Summ of the Sides BA and AC, for AC and AD are equal, (by the Construction;) and then shall BE be the Difference between the Sides AC and AB, and AC and AE are equal: Then I say,

As CB : is to BD :: so is EB : to BF. Then that the Perpendicular AG, shall bisect the Segment FC, we are to prove.

Demonstration: First, Equal Right angled Figures *have their Sides Reciprocally proportional,* (by the 2 Consectary of the 5*th* of Sect. 2.) And the *Oblongs* made of BD and BE

BE, and also of BC and BF are *Equal Right angled Figures*: Therefore their Sides are Reciprocally proportional; For,

As BC: to BD :: so is BE: to BF.

And because the *Triangle* FAC hath the two Sides AF and AC equal by Construction, therefore the Perpendicular AG bisecteth the *Base* FC, (by the 22 of Sect. 1.) Therefore,

In an Oblique angled Triangle, *as the greatest Side is to the Summ of the other two, so is the difference of those two, to a Segment of the greater Side*, which subtracted from the greater Side, a Perpendicular shall fall in half the Remainer; which was to be demonstrated.

And from this Demonstration *ariseth this* Consectary.

That, *If three Sides of a* Plain Oblique angled Triangle *be given, either of the Segments of the Greater Side is also given, in which Segment, from the greatest* Angle *a Perpendicular shall fall.* And,

So we will proceed to the Solution of the *Case* of *Oblique Plain Triangles* deduced from the former *Axiome.*

CASE V.

The Three Sides of an Oblique Angled Triangle *being given, to find the* Angles.

In the *Oblique Angled Triangle* ABC (*Fig.* XIII.) let the Side AB 361.56. AC 234. and BC 478.83 be given, and let the *Angles* be required. *Fi.* XIII.

The Greatest Side BC is —— 478.83.
The Summ of the two Lesser Sides AB and AC is 595.56.
The Difference of them is 127.56.

Being thus prepared say:

As the greatest Side BC 478.83.	2.6801813
Is to the Summ of the other two Sides AB and AC 595.56.	2.7749255
So is the Difference of those Sides 127.56.	2.1057145
	4.8806400
To the Segment BF 158.65.	2.2004587

Which being subtracted from the greatest Side BC 478.83, the remainer will be 320.18 for the other Segment of the greatest Side BC, namely FC, in the middle whereof, namely at G the Perpendicular AG will fall. And,

By this means, the *Oblique Angled Plain Triangle* ABC, is reduced into two *Right Angled Plain Triangles, viz.* ABG and ACG, in either of which the *Hypotenuse* and *Base* is given, so that the *Angles* may be found by the V or VII *Case* of *Right Angled Plain Triangles.*

Another way to resolve this Case, *and that at one Operation.*

Mr. *Henry Briggs* in the 18*th* Chapter of his *Arithmetica Logarithmica*, hath another way whereby to resolve this *Case*, shewing, how to find any *Angle*, by having the three Sides given, from whence this following *Precept* (which effecteth the same) was deduced.

From half the Summ of the Sides of the Triangle *subtract each particular Side: And let the Summ of the* Logarithms *of the half Summ, and difference of the Side subtending the enquired* Angle, *be subtracted from the Summ of the* Logarithms *of the other differences, and the double* Radius: *And half the Remainer shall be the* Logarithm *of a* Tangent, *whose degrees doubled shall be equal to the enquired* Angle.

Illuſtration:

In the forementioned *Triangle* ABC (*Fig.* XIII.)
Let the three Sides AB, BC and AC be given, and let the *Angle* BAC be enquired.

Half Summ —— 537.19

The Side { AB is —— 361.56 / BC is —— 478.83 / AC is —— 234.00

The Difference of the half Summ and { BC is —— 58.36 / BA is —— 175.63 / AC is —— 303.19

Their Summ is —— 1074.39
The half Summ is —— 537.19

Being thus prepared:

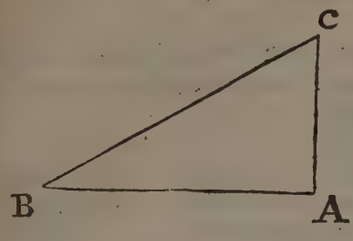
The half Summ 537.19.	Log.	2.73012792
The Difference of the half Summ and BC 58.36.	Log.	1.76611528
The Summ of the Log. of the half Summ and BC.		4.49624320
The Difference of the half Summ are { AB 175.63	Log.	2.24459870
AC 303.19	Log.	2.48171487
The double *Radius*		20.00000000
The Summ		24.72631357
The Summ of the Log. of the half Summ and BC. Sub.		4.49624320
The Remainer.		20.23007037
Half the Remainer.		10. 1503518

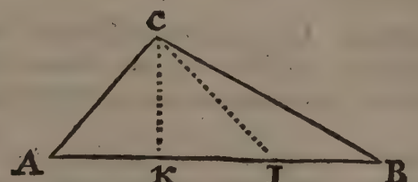
Which is the *Logarithm Tangent* of 52 deg. 30 min. the double whereof is 105 deg. for the quantity of the *Angle* BAC, which was required.

And thus have you the ſeven *Caſes* of *Right Angled*, and the *Five Caſes* of *Oblique Angled Plain Triangles* ſufficiently demonſtrated and illuſtrated by *Examples:* I ſhall now proceed to ſome *Problems Trigonometrical*, which do not fall within the Limits of the forementioned *Caſes*, and yet theſe or ſuch like may ſometimes be propoſed.

A Synop-

A Synopsis *of the* Doctrine *of* Plain Triangles.





Right Angled.

Given	Req.	Proportions.	Cas.
AB & B	CA	1. sC:BA::sB:CA 2. R:BA::tB:CA 3. tC:R::BA:CA	I
AB & C	BC	1. sC:BA::R:BC 2. R:BA::seB:BC 3. tC:BA::seC:BC	II
BC & B	BA	1. R:BC::sC:BA 2. seB:BC::R:BA 3. seC:tC::BC:BA	III
AB & AC	B & C	1. BA:CA::R:tB whose Complem. is C 2. CA:BA::R:tC whose Complem. is B	IV
BC & CA	B & C	1. BC:BA::R:tC whose Complem. is B 2. BA:BC::R:seB whose Complem. is C	V
AB & AC	BC	1. CA:BA::R:tC Then again, 2. sC:BA::R:BC	VI
BA & BC	AC	1. BC:BA::R:sC whose Complem. is B Then again, 2. R:BC::sB:CA	VII

Oblique Angled.

Given	Req.	Proportions.	Cas.
AC A & B	AB	sB:CA::sC:AB	I
AC AB & B	A	CA:sB::CB:sA	II
AC CB & C	A & B	CB+CA:CB−CA:: ::+½A+B:t the diff. wch. diff. {added to / subtr. from} the half sum, gives the {Greater / Lesser} Angle.	III
CA CB & C	AB	Find the *Angl.* A and B by the last *Case*, then by the First *Case* you may find the Side AB.	IV
AB AC & CB	A B & C	AB:CB+AC::CB−CA::BI. Then AB−BI = AI. ½ AI is AK. Thus is the *Oblique Angled Triangle* resolved into 2 *Right Angled*, in either of which the *Hypotenuse* is known:	V
		In this *Synopsis*. s signifies *Sine*. t *Tangent*. s e *Secant*. (ded. + 2 *Lines* or *Angles* ad- − 2 *Lines* or *Angles* subtracted.	

Chap.

CHAP. VII.

Problems *or* Cases *Extraordinary.*

PROB. I. *One side of an* Oblique Angled Plain Triangle, *the* Angle *opposite to that Side, and the Summ of the other two Sides being given, to find the other two Sides and* Angles *severally.*

Fi. XIII. IN the *Oblique Angled Plain Triangle* A B C. (*Fig.* XIII.) let the Side B C 478.83: the *Angle* B A C opposite to it 105 *deg.* And the other two Sides B A and A C in one Summ 595.56 be given, and let the *Angles* C and B be required.

Construction, Extend the Side B A, of the *Triangle* A B C, to D, making A D equal to A C, and draw the *Line* D C; so shall you have constituted two other *Oblique Angled Triangles* B D C, and A C B. In the *Triangle* A B C, you have given the *Angle* B A C 105 *deg.* and consequently in the *Triangle* A C D you have given the *Angle* D A C, 75 *deg.* it being the *Complement* of the other to 180 *deg.* (by the 10. of Sect. 2.) also the *Triangle* A D C is *Equicrural,* by the Construction, and therefore the *Angles* at the *Base* A D C, and A C D are equal, (by the 22. of Sect. 1.) namely, either of them equal to half the given *Angle* B A C, 52 *deg.* 30 *min.* Now in the *Oblique Angled Triangle* B D C, there is given,

1. The Side B C 478.83.
2. The Side B D 595.56.
3. The *Angle* B D C 52 *deg.* 30 *min.*

From whence you may find the *Angle* D C B, by the third *Case* of *Oblique angled Triangles.* For,

As the Side B C 478.83	2.6801813
To the *Sine* of B D C 52 *deg.* 30 *min.*	9.8994667
So is the Side D B 595.56	2.7749255
	12.6743922
To the *Sine* of the whole *Angle* D C B 99 *deg.* 20 *min.*	9.9942109

From which the *Angle* A C D, equal to A D C, 52 *deg.* 30 *min.* being Subtracted, the remainder 46 *deg.* 50 *min.* is the *Angle* A C B.

Which being known, the other *Angle* A B C is known, and by them may be found the Sides A B 234, and A C 361.56 severally, which was required to be done.

PROB. II. *One Side of an* Oblique Angled Plain Triangle, *the* Angle *opposite thereto, and the Difference of the other two Sides, being given, to find the other two* Angles, *and the two Sides severally.*

Fi. XIV. IN the *Oblique Angled Triangle* A B C (*Fig.* XIV.) let the Side B C 478.83, and the *Angle* B A C 105 (or 75 *deg.*) and the Difference of the two Sides A B and A C, (*viz.* B E 127.56) be given: and let the *Angles* A C B, and A B C be required, as also the two Sides A B, and A C severally.

Construction, Make A E equal to A C, and draw the *Line* E C, constituting the *Triangle* A E C, which is *Equicrural*; and the *Angle* E A C being 105 *deg.* the *Complement* thereof to 180 *deg.* is 75 *deg.* for the two *Angles* A E C and A C E, which being equal one to the other, are therefore either of them equal to half 75 *deg. viz.* 37 *deg.* 30 *min.* and by drawing the former *Line* E C, there is also made another *Oblique Angled Triangle* E B C; in which is given,

1. The Side B C, 478.83.
2. The Side B E, 127.56 the Difference of A B and A C.
3. The *Angle* B E C, 142 *deg.* 30 *min.* it being the *Complement* of the *Angle* A E C 37 *deg.* 30 *min.* to 180 *deg.*

And from these things given you may find the *Angles:* A C B and A B C, as also the Sides A B and A C severally, For,

By

By Case III.

As the Side B C 478.83	2.6801813
Is to the *Sine* of A E C 37 *deg.* 30 *min.* whose *Complement* to 180 *deg.* is E C B 142 *deg.* 30 *min.*	9.7844471
So is the Side B E 127.56.	2.1057145
	11.8901616
To the *Sine* of the *Angle* E C B 9 *deg.* 20 *min.*	9.2099803

Which being added to the *Angle* A C E 37 *deg.* 30 *min.* giveth 46 *deg.* 50 *min.* for the whole *Angle* A C B, and ſubtracted therefrom, leaveth 28 *deg.* 10 *min.* for the *Angle* A B C.

Having thus found all the *Angles*, and the Side B C being known, the other two Sides may eaſily be found (by the 1. Caſe of *Oblique Angled Spherical Triangles*) which was the thing required.

THE DOCTRINE Of the DIMENSION of Spherical TRIANGLES.

CHAP. I.

Of the Affections of Spherical Triangles, *and of ſuch* Definitions *and* Theorems *as are neceſſary to the right underſtanding of the* Dimenſion *of them.*

THat a *Spherical Triangle*, as well as a *Plain* or *Right Lined Triangle* doth conſiſt of *Six Parts*, namely of three *Sides*, and as many *Angles* hath been already declared: we now come to ſhew the *Affections* of *Spherical Triangles*, which ſhall be contained under theſe twenty Heads following.

1. *The three Sides of every* Spherical Triangle, *are the* Arches *of three Great* Circles *of the* Sphere, *every one of them being leſs than a* Semicircle; or conſiſting of fewer degrees than 180. which is the Meaſure of a *Semicircle* (by the 9. of Sect. 1. of *Plain Triangles*.).

2. *A Great* Circle *of the* Sphere, *is ſuch a* Circle *as divideth the whole* Sphere *or* Globe, *into two equal Parts or* Hemiſpheres, *and ſo is every where diſtant from his* Poles *by a* Quadrant 90 deg. *of a Great* Circle.

3. *If one Great* Circle *if the* Sphere, *doth paſs by the* Poles *of another Great* Circle, *thoſe two Great* Circles *do interſect, or cut each other at* Right Angles, *and the contrary.* —— Thus:

Let A E C be a Great *Circle* of the *Sphere*, (*and let it repreſent the* Horizon *of any place*,) whoſe *Poles* let be B and D; (*the* Zenith *and* Nadyr *of the ſame place*, equidiſtant *from the* Horizon A E C 90 deg.) by which *Poles* B and D, let another Great *Circle* paſs, namely B E D, (*one of the* Colures, *or an* Azimuth *or* Vertical Circle:) Now I ſay, that the Great *Circle* B E D cutteth the Great *Circle* A E C at *Right Angles* in *Fig.* I.

the Points E and F. For, if upon the Pole E or F, another great *Circle*, ABCD, be deſcribed, it is manifeſt AB, BC, CD, and DA, ſhall be the meaſures of the *Angles* at E and F, (*by the* 4 *of* Sect. 1. *Of Plain Triangles.*) But the *Arches* AB, BC, CD, and DA, are Quadrants, (*by the third hereof*;) and therefore the *Angles* at E and F are *Right Angles*, (*by the* 13*th. of* Sect. 1. *Of Plain Triangles:*) *Which was to be demonſtrated.*

4. *The meaſure of a Spherical Angle is the Arch of a great Circle deſcribed from the Angle, and intercepted between the two ſides, they being continued out till they be Quadrants,* (by the firſt hereof). So,

Fig. II. In the *Spherical Triangle* ABC, the meaſure of the *Spherical Angle* at A, is not the *Arch* BC, but the *Arch* ED, intercepted between the two ſides AB and AC, continued till they be *Quadrants*, that is, to the Points E and D, becauſe the *Arch* BC is not deſcribed upon the *angular* Point A, but the *Arch* ED is (*by the Firſt hereof*) therefore the *Arch* BC cannot be the meaſure of the *Angle* BAC, (*by the* 4*th. of* Sect. 1. *Of Plain Triangles.*)

5. *The ſides of a Spherical Triangle, being continued till they meet together, do make two Semicircles, and at their Interſection do comprehend an Angle equal and oppoſite to the firſt Angle.*—— Thus,

Fig. III. In the *Triangle* ABC, the ſides AB and AC of the *Angle* BAC, being continued to D, do make the *Semicircles* ABD and ACD, which do comprehend the *Angle* BDC equal to the *Angle* BAC; becauſe the ſame *Arch* GH (being diſtant both from A and D 90 deg.) meaſureth both thoſe *Angles*, (*by the* 4*th. hereof.*)

6. *Every Spherical Triangle hath from every Angle thereof another Triangle oppoſite thereunto, whoſe Baſe and Angle oppoſite to the Baſe are the ſame; and the other parts of that Triangle, are the Complements of the parts of the other Triangle.* — Thus (*Fig.* III.)

The former *Triangle* ABC hath another *Triangle* BDC oppoſite thereunto, whoſe *Baſe* BC, and *Angle* oppoſite thereto, BDA, are the ſame, (*by the* 5*th. hereof.*) And the ſides BD and CD, are the *Complements* of the ſides BA and CA to a *Semicircle*: And the *Angles* DBC and DCB are the *Complements* of the *Angles* ABC and BCA to two *Right Angles*, or 180 deg. (*by the* 18*th. of* Sect. 1. *Of Plain Triangles*).

7. *The ſides of a Spherical Triangle may be changed into Angles, and the contrary: The Complement of the greateſt ſide or greateſt Angle to a Semicircle, being taken for the greateſt ſide or greateſt Angle:*——As in

The *Spherical Triangle* ABC, obtuſe *angled* at B: Let DE be the meaſure of the *Angle* at A, and let FG be the meaſure of the *accute Angle* at B, (being the *Complement* of the obtuſe *Angle* at B, the greateſt *Angle* in the *Triangle*,) and let HI be the meaſure of the *Angle* at C: Now

KL, LM, KM	is equal to	DE, FG, HI	becauſe	KD, LG, KI	and	LE, FM, MH	are *Quadrants*, and their common *Complement* is	LD, LF, KH

Therefore, The ſides of the *Triangle* KLM, are equal to the *Angles* of the *Triangle* ABC, taking for the greateſt *Angle* ABC, the *Complement* thereof FBG.

It may alſo be demonſtrated, That the ſides of the *Triangle* ABC, are equal to the *Angles* of the *Triangle* KLM, (*by the Converſe of the former:*) For,

The Side AB, BC, AC	is equal to	OP, FH, DI	the meaſure of the *Angle*	MLK, LMK, DKI	which is the *Complement* of the obtuſe *Angle* MKL.

For that,

AD, AP, BF	and	CI, OB, CH	are *Quadrants*, and their common *Complement* is	CD, AO, CF

Therefore, The ſides may be turned into *Angles*, and the contrary: *Which was to be demonſtrated.*

8. *A Right Angled Spherical Triangle hath one Right Angle, or more than one.*

9. *In* Fig. III. *Suppoſing the Angles at* A *and* D, viz. BAC, *and* BDC, *to be Right Angles.*

The *Triangle* BAC, BDC, CDE	hath	One *Right Angle*, and two *Acute*. Two *Obtuſe Angles*, and one *Right*. One *Obtuſe*, and two *Acute*.

10. *A*

10. *A* Right Angled Spherical Triangle, *with two* Acute Angles, *hath from the* Right Angle *a* Right Angled Triangle *oppoſite thereunto, with two* Obtuſe Angles: *& contra*, as in the *Triangles* B A C, and B D C.

11. *The Sides of a* Right Angled Spherical Triangle, *with two* Acute Angles, *are each of them leſs than* Quadrants: As the Sides of the *Triangle* A B C, are all of them leſs than *Quadrants*.

12. *The two Sides of a* Right Angled Spherical Triangle, *with two* Obtuſe Angles, *are greater than* Quadrants, *and the third Side is leſs than a* Quadrant: As in the *Triangle* B D C, *Obtuſe Angled* at B and C, the Sides B D and C D are greater than *Quadrants*, and the Side B C Leſſer.

13. *A* Right Angled Spherical Triangle, *with two* Acute Angles, *is* (*from the* Acute Angle) *oppoſite to a* Right Angled Spherical Triangle, *with one* Acute *and one* Obtuſe Angle: —— As in the *Right Angled Triangle* E D F, having the *Angles* at E and F *Acute*, is oppoſite to the *Right Angled Triangle* C D E, with the *Acute Angle* E C D, and the *Obtuſe Angle* C E D.

14. *The Sides ſubtending the* Right Angles *of a* Spherical Triangle, *having divers* Right Angles *are* Quadrants. —— As in the *Triangle* A G H, If the two Great *Circles* A G and A H, do cut the Great *Circle* G H at *Right Angles*, in the Points G and H. A is the *Pole* of the Great *Circle* G H, (*by the* 3 *hereof.*) and A G and A H are *Quadrants* (*by the* 2 *hereof.*) But if the *Angle* at A be alſo a *Right Angle*, then G H is alſo a *Quadrant*, (*by the* 13 *of Sect.* 1. *of* Plain Triangles, *and by the* 4 *hereof.*)

15. *A* Spherical Triangle *having divers* Right Angles, *hath either two or three* Right Angles; *and ſo of the Sides it hath two or three of them* Quadrants. As if the *Angle* at A be put for a *Right Angle*, the *Spherical Triangle* A G H ſhall have three *Right Angles* at A, G, and H; and therefore the three Sides alſo, A G, A H, and G H, ſhall be *Quadrants*: —— But if you put the *Angle* at A for an *Acute Angle*, then the *Spherical Triangle* A G H ſhall have two *Right Angles* at G and H, and thereupon the two oppoſite Sides A G and A H *Quadrants*.

16. *If the third* Angle *of a* Spherical Triangle, *having two* Right Angles, *be* Acute, *the third Side is leſs than a* Quadrant; *but if* Obtuſe, *then is it greater than a* Quadrant: As in the *Spherical Triangle* G H I, *Acute Angled* at G, the third Side H I is leſs than a *Quadrant*. — But in the *Spherical Triangle* A G I, *Obtuſe Angled* at G, the third Side A I is more than a *Quadrant*.

17. *An* Oblique Spherical Triangle *conſiſteth ſimply of* Acute *or* Obtuſe Angles, *or of both of them*.

18. *A* Spherical Triangle, *with two* Obtuſe Angles *and one* Acute Angle, *is oppoſite to a* Spherical Triangle *ſimply* Acute Angled, *& contra*: —— As if the *Angles* at A and D be ſuppoſed *Acute*, then the *Triangle* B D C, with two *Obtuſe Angles* at B and C, and one *Acute Angle* at D, is oppoſite to the *Simple Acute Angled Triangle* A B C.

19. *A* Spherical Triangle, *with two* Acute Angles *and one* Obtuſe, *is oppoſite to a* Spherical Triangle ſimply Obtuſe Angled, *& contra*: —— As if the *Angles* at A and D be ſuppoſed *Obtuſe*, then the *Triangle* A B C, with two *Acute Angles* at B and C, and one *Obtuſe Angle* at A, is oppoſite to the *ſimply Obtuſe Angled Triangle* B D C.

20. *The three* Angles *of every* Spherical Triangle, *are greater than two* Right Angles.

This is manifeſt in *Spherical Triangles* having more *Right* or *Obtuſe Angles* than One: But in *Acute Angled Triangles*, it may be thus Demonſtrated.

In the *Right Angled Spherical Triangle* A B C *Right Angled* at C, and *Acute Angled* at A and B. *Fig.* IV.

The meaſure of the *Acute Angle.* { B A C / A B C / or / D B E } is the *Arch* { E F / H I } (*by the* 4 *hereof.*)

But the *Arches* E F and E D together, are equal to a *Quadrant*; therefore the *Arches* F E and H I added together, are more than a *Quadrant*, and conſequently the *Angles* anſwering to theſe *Arches*, namely, the *Angles* B A C and A B C together, are more than a *Quadrant*. But the *Angle* A C B is a *Right Angle*, by the Propoſition; therefore, in the *Spherical Triangle* A B C, conſiſting of two *Acute Angles*, the three *Angles* together are greater than two *Right Angles*.

Again,

Fig. V. Again, In the *Acute Angled Spherical Triangle* K L M.

The Measure of the { K L M / M K L / L M K } *Acute Angle* is the *Arch* { N O / V X / R Q }

But these three *Arches*, N O, V X, and R Q together, are more than two *Quadrants*: For P Q and P V, (being the *Complements* of the two *Arches* Q R and V X,) added together, are less than the *Arch* N O, *by the Proposition*: Therefore the *Arch* N O, being the Measure of the third *Angle*, is more than the *Complements* of the other two *Angles* added together; and consequently, the third *Angle* is greater than the *Complements* of the other two *Angles*. And therefore, *in* Acute Angled Spherical Triangles, *the three* Angles *are greater than two* Right Angles: *Which was to be demonstrated.*

CHAP. II.

Of the Four Fundamental AXIOMS, *laid down by* Pitiscus *and others for the* Solution *of* Spherical Triangles; *with their Demonstrations.*

AXIOME I.

In Right Angled Spherical Triangles, *having the same* Acute Angles *at the* Base: *The* Sines *of the* Hypotenuses, *and of the* Perpendiculars *are Proportional one to the other,* & contra.

Fig. VI. *Construction.* LET D F A K M be a *Sphere*, in which let K M F A be the *Horizon*; let D be the *Zenith*, (or *Pole* of the *Horizon*;) let F D K and C D R be two *Azimuth* or *Vertical Circles*, passing through D the *Pole* of the *Horizon*, and cutting the *Horizon* at *Right Angles* in the Points K, R, F, and C, which they will do, (*by the* 3 *of Sect.* 1. *hereof.*) Also, Let M E A be a Great *Circle* of the *Sphere*, cutting the *Horizon* obliquely, but the *Vertical Circle* K D F at *Right Angles* at E, for that it passeth by the *Poles* thereof M and A, (*by the* 3 *of Sect.* 1. *hereof.*) And is also cut by it into two *Quadrants* M E and A E, (*by the* 2 *of Sect.* 1. *hereof.*)

In this disposing of these *Circles* of the *Sphere*, there are (among others) constituted two *Right Angled Spherical Triangles*, *viz.* A B C and A E F; In which,

The { *Hypotenuses* are { A E and A B } and the { I E and G B
Perpendiculars are { E F and B C } *Sines* the { E N and B O
Bases are { A F and A C } *Right Lines* }

And the *Acute Angle* at the *Bases*, A F and A C, let be the same *Angles*, E A F and B A C.

Now I say, that these *Sines* of these *Hypotenuses*, and *Perpendiculars*, are all proportional one to the other. And so, any three of them being given, the fourth may be found;

For,
As I E to E N, so is G B to B O.
And likewise,
As G B to B O, so is I E to E N.
And contrarily,
As N E to I E, so is O B to B G.
Also, if you change the middle terms, it will be,
As I E to G B, so is E N to B O.
And in like manner,
As G B to I E, so is B O to E N.

And

And contrarily,

As E N to O B :: ſo is I E to B G *(by the* 5 *of Sect.* 1. *Of Plain Triangles.*

Demonſtration. If the *Sines* G B and B O, be joyned together by the *Right Line* G O, ſo that thereby be made the *Triangle* G B O; it is manifeſt that the *Triangles* G B O and I E N are *Equiangled.* For firſt, for that the *Right Lines* E N and O B, do fall perpendicularly upon the *Plain* M F C, they do make *Right Angles* alſo with all the *Lines* drawn in the ſame *Plain*; and ſo, the *Angles* I N E and G O B are *Right Angles.* Again, ſecondly, becauſe the *Right Lines* I E and G B are *Parallel (by the* 1 *of Sect.* 2. *of Plain Triangles)* being they are drawn alike upon the ſame *Right Line* I A: And (becauſe the whole *Plain* M E A, is every where inclined with the ſame *Right Angle* to the *Plain* M F A,) therefore alſo the *Parallels* drawn therein I E and G B, are inclined with the ſame *Angles* to the *Parallels* I N and G O under them, in the *Plain* M E A; And ſo the *Angles* I E N and G B O are equal; and conſequently in the *Triangles* I E N and G B O; and where two *Angles* are equal to two, there alſo the third is equal to the third, *(by the* 11 *of Sect.* 2. *of Plain Triangles)* and ſo the *Triangles* I E N, and G B O are *Equiangled*; and being they are *Equiangled*, they have their Sides about the equal *Angles* proportional *(by the* 8 *of Sect.* 2. *of Plain Triangles)* And ſo,

As I E : to E N :: ſo is G B : to B O. Which was to be demonſtrated.

AXIOME II.

In Right Angled Spherical Triangles, *having the ſame* Acute Angles *at the* Baſe: *The* Sines *of the* Baſes, *and the* Tangents *of the* Perpendiculars, *are Proportional,* & contra.

Conſtruction. IN the two *Triangles* A E F and A B C in the former Diagram, in which *Fig.* VI.
The *Sines* of the *Baſes* A F and A C are I F and H C. But,
The *Tangents* of the *Perpendiculars* E F and B C, are L F and P C.

I ſay then, that thoſe *Sines* I F and H C; and thoſe *Tangents* L F and P C, are proportional; ſo that *any three of them being given, the fourth may be diſcovered.*

For,

As I F : to F L : : ſo is H C : to P C.

And likewiſe,

As H C : to C P :: ſo is I F : to F L.

And contrarily,

As F L : is to F I : : ſo is P C : to H C.

And by changing the middle terms it will be,

As I F : to H C :: ſo is F L : to P C.

And likewiſe,

As H C : to I F : : ſo is C P : to F L.

And contrarily,

As F L : : to P C : : ſo is F I : to C H, *(by the* 5 *of Sect.* 1. *of Plain Triangles.)*

Demonſtration. Having drawn the *Right Lines* I L and H P, and conſtituted the *Triangles* I L F and H P C, thoſe *Triangles* ſhall be *Equiangled*; and conſequently proportional in their Sides; they ſhall be *Equiangled* becauſe of the *Right Angles* at F and C, and their equal *Angles* at I and H, and therefore alſo at L and P.

Again, the *Angles* I F L and H C P are *Right Angles*, becauſe the *Tangents* of the *Perpendicular Arches* E F and B C, namely, L F and P C, are perpendicular to the whole *Plain* of the *Circle* M F A, (by the *Conſtruction,*) and therefore alſo to the *Lines* I F and H C drawn in that *Plain.*

Laſtly, The *Angles* L I F and P H C are equal, becauſe the *Right Lines* I L and H P, drawn by the ſame *Plain*, are *Parallel* one to another, and to the *Plain* of the inclining *Circle* D F: Therefore are they inclined with equal *Angles* to the ſubjected *Parallels* I F and H C, which two are *Parallel*, for that they are drawn (both of them alike) upon the ſame *Right Line* I A. —— Alſo the *Right Lines* I L and H P are *Parallels*, they being the extreams of the two *Triangles* I L F and H P C; and being erected perpendicularly upon the *Parallel Baſes* I F and H C; becauſe of the *Perpendicular Tangents* C P and F L. —— And alſo, the *Right Lines* I L and H P are drawn

by

by the same *Plain* of the *Semicircle* M E A; because the *Secant* I L, when it cutteth the *Circle* M E A, in the Point E, cannot fall but upon the *Plain* of that *Circle*.

In like manner, the *Secant* I P, when it cutteth the same *Circle* M E A in the Point B, cannot fall but upon the *Plain* of the same *Circle*; which *Plain*, if it should be extended, according to the *Right Line* I P, it should fall upon the *Tangent* P C, in the Point P; and so the Point P should be in the *Plain* of the *Circle* M E A so extended: But the Point H is appointed in the same *Plain*; therefore the *Right Line* P H is a *Line* falling betwixt two Points in the same *Plain*, and thereupon drawn by the same *Plain*: *All which was to be demonstrated.*

AXIOME III.

In all Spherical Triangles, *the* Sines *of their Sides are directly proportional to the* Sines *of their opposite* Angles.

Fig. VII. *Construction.* LET A B C be a *Spherical Triangle Right Angled* at C: And let the Sides thereof be continued till they make the *Quadrants* A E, A F, and C D. Also from the *Pole* of the *Quadrant* A F, *viz.* from D, let be two other *Quadrants* drawn down to H and F; so shall there be made three new *Triangles*, B D E, and G D E *Right Angled*, and B D G *Oblique Angled*. Now I say, in the *Right Angled Spherical Triangle* A B C,

As A C B is to A B, so is A B C to A C. And so is B A C to B C.
Or by changing the middle terms,
As A C B is to A B C, so is A B to A C, *&c.*

Likewise, in the *Oblique Angled Spherical Triangle* B D G, I say, That
As B D G is to B G, so is B G D to B D; and so is D B G to D G, *&c.*

Demonstration. First, as to the *Right Angled Triangle* A B C.

Wherein the *Angle* { A C B, B A C, A B C } and the measure thereof { A E, or N D; E F; O P } are of the same quantity; *(by the 3 of Sect. 1. hereof.)*

Now, it is all one if I shall say,
As A C B is to A B, so is B A C to B C. Or,
As A E is to A B, so is E F to B C. *(by the First Axiome hereof,)*
In like manner, it is all one if I say,
As A C B is to A B, so is A B C to A C. Or,
As O B is to A B, so is O P to A C, *(By the First Axiome hereof also.)*

But by the *Rules* already demonstrated,

As A C B is to A B, so is A B C to A C; And so is B A C to B C.
Therefore,
As A B C is to A C, so is B A C to B C.

Secondly: As to the *Oblique Angled Triangle* B D G, because *(by the Demonstration of Right Angled Triangles,)*

As B D is D E B, so is D E to D B E. And,
As D G is to D E G, so is D E to D G E, or to D G B.
Also,
As D G is to D B, so is D B E (or D B G) to D G B, *&c.*
And if from B, a *Perpendicular* be let fall to S.
Because then,
As B D is to B S D, so is B S to B D S. And,
As B G is to B S G, so is B S to B G S, or to B G D.
Therefore also,
As B G is to B D, so is B D S (or B D G) to D G B, *&c.*

For if thus,

As { 4, 2, 2 } to { 12, 12, 4 } So is { 1, 1, 3 } to { 3 And, 6 Then, 6 *Which was to be demonstrated.*

Axiome

AXIOME IV.

In all Spherical Triangles; *if first you add two Sides, either of them less than a* Quadrant, *together, and then add the lesser Side to the* Complement *of the greater Side: And if you subtract the* Sine *of the* Complement *of the former composed* Arch, *from the* Sine *of the latter composed* Arch; *Or if you add it to the* Sine *of the excess: Then,*

AS the *Radius* is to that half of the *Right Line*, so made, (either by Addition or Subtraction;)
So is the *Versed Sine* of the *Angle* comprehended between the two said Sides,
To a *Right Line*,
Which subtracted from the *Sine* of the latter *Composed Arch*, leaveth the *Sine* of the *Complement* of the third Side; — Or from whence, the *Sine* of the latter *Composed Arch* subtracted, leaveth the *Sine* of the Excess of the Third Side.
Or contrarily,
As the half of that *Right Line*,
Is to the *Radius*.
So is the *Right Line* made of the *Sine* of the latter *Composed Arch*, (either by Subtraction of the *Sine* of the *Complement* of the third Side; or by adding of the *Sine* of the excess of the same third Side,)
To the *Versed Sine* of the *Angle* comprehended between the other two Sides.

This is the Fourth *Axiome* laid down by *Pitiscus*, as it is rendred in *English* by Mr. *Ralph Handson*, which *Axiome Pitiscus* demonstrates by *Right Sines*, and maketh several *Cases* thereof: Now this *Fourth Axiome*, being most ingeniously laid down, and demonstrated by *Versed Sines*, by Mr. *Michael Dary* (a late painful and industrious *Mathematician*) I will here insert, instead of this of *Pitiscus*, it being more brief and general, and I shall give it you in his own words, as many Years since I received it from him.

The following *Axiome* being demonstrated by the *Versed Sines*, I shall first give you

The Definition of a Versed Sine.

The Versed Sine *of an* Arch *is the Segment of the* Diameter, *drawn from one end of that* Arch *till it be cut by a* Perpendicular *from the other end of the same* Arch.

Now according to this *Definition*, [I say of the *Versed Sines*, as *Pitiscus* of the *Right Sines*,] *The* Versed Sine *of an* Arch, *more or less than a* Semicircle, *is one and the same* Right Line: As in *Fig.* VIII.

I say the *Right Line* B S is the *Versed Sine* of the *Arch* B G, by letting fall the *Perpendicular* G S. *Fi.* VIII. Also I say, the same *Right Line* B S, is the *Versed Sine* of the *Arch* B G D R, which doth as much exceed the *Semicircle* B G D, as the *Arch* B G doth fall short of it.

I thought good to premise this, lest when there had hapned an Arch *more than* 180 *d. in the Use of the succeeding* Axiome, *a beginner might have been put to a* Non-plus.

Now

Now I shall deliver the *Axiome* in these Words.

AXIOME. IV.

In all Spherical Triangles,
As the Difference of the Versed Sines, *of the Summ, and of the Difference of any two Sides including an* Angle,
Is to the Diameter:
So is the Difference of the Versed Sines *of the third Side, and of the Difference of the two including Sides,*
To the Versed Sine *of the* Angle *comprehended between those two Sides.*
Or Contrarily,
As the Diameter,
Is to the Difference of the Versed Sines *of the Summ, and of the Difference of any two Sides including an* Angle:
So is the Versed Sine *of the* Angle *comprehended by those two Sides,*
To the Difference of the Versed Sines *of the Third Side, and of the Difference of the two including Sides.*

WHich difference being *added* to the *Versed Sine* of the difference of the two including Sides, you have the *Versed Sine* of the third Side.

Now (according to this *Axiome*) I shall shew you the *Proportion Geometrically*, by laying down a *Projection* of the *Sphere*.

Fi. VIII. *Construction.* Let there be a *Circle* described, as A B E D A (in *Figure* VIII.) and let it represent the *Meridian*; Also let A represent the *Zenith*, E the *Nadir*, A E the *Prime Vertical Circle* of *East* and *West*: Let B represent the *Pole Arctick*, and D the *Pole Antarctick*, and B D the *Hour Circle of Six*, or the *Axis* of the *Sphere*: So shall A B be the *Complement of the Latitude of the Place*, (the *Sphere* being thus rectified.) Also let A G, A C, and A H, each of them be *Complements* of the *Sun's Altitude*, or of his *distance from the Zenith* A. And let B K, B C and B L, be *Complements* of the *Sun's Declination*, or of his distance from the *Pole* B. These things thus laid down, I say,

Demonstration. Let the *Triangle* A B C be given, whose Sides A B, A C, and C B, are known; and let the *Angle* C A B be required. I now say, that B G is the Summ of the Sides A B and A C; for A G (equal to A C) is put to A B; (*but if equal things be put to equal things, their total shall be equal*;) therefore I say, the *Arch* B G is the Summ of the two including Sides A B and A C.

Now the *Versed Sine* of this *Arch* B G, is the *Right Line* B S (*by the Definition of a Versed Sine*) made by the falling of the *Perpendicular* G S R, upon the *Diameter* B D (*by the* 13*th of* 2. *Petis.*)

Again, I say, B H is the difference of the Sides A B and A C, (equal to A C) A B taken away from it, (*but if equal things be taken from equal things, the remainers shall be equal*;) Therefore I say the *Arch* B H, is the *Remainer* or *Difference*, of the two including Sides A B and A C: Now the *Versed Sine* of this *Arch* B H, is the *Right Line* B M (by the *Definition* of a *Versed Sine*) made by the falling of the *Perpendicular* H M V, upon the *Diameter* B D (*by the said* 13 *of the* 2. *of Pitisc.*) Also the *Versed Sine* of the *Arch* B C, the third Side is the *Right Line* B P, made by letting fall the *Perpendicular* K P L upon the *Diameter* B D.

Now the *Right Lines* H V, L K, and R G are *Parallels* (*by the* 38 *of the* 1 *of Pitiscus, Or by the* 1 *of Sect.* 2. *of Plain Triangles*, and also *by the Construction of the Figure*) Then (*by the* 39 *of the* 1 *of* Pitiscus, *or by the* 2 *of Sect.* 2. *of* Plain Triangles. *Or by the* 28 *of the* 5. *of Ramus*) *If* Right Lines *be cut jointly by many* Parallel Right Lines, *their Intersections are proportional one to another.* But the *Right Line* M S (*the difference of the* Versed Sines *of the Summ and of the difference of the two including Sides* A B *and* A C,) as also G 6 H the *Parallel of Altitude*; [which may be put to represent the *Diameter* of the *Circle* given, (though it be the *Diameter* of the *Almicanter*) which if it be

be the Point C, divideth it into two *Verſed Sines, viz.* C G and H C] are jointly cut by the foreſaid *Parallels*. Therefore I ſay, the *Interſegments* of the *Right Lines* M S and G H are proportional one to another; That is,

As M S, *the Interſegment of the Parallels* U H *and* G R, [which is alſo the difference of the *Verſed Sines* of the Summ, and of the difference of the two inclining *Sides* A B and B C,]

Is to G H, *the other Interſegment of the ſame Parallels* U H *and* G R, [which is alſo the *Diameter*;]

So is M P, *the Interſegment of the Parallels* V H *and* K L, [which is the difference of the *Verſed Sines* of the third Side C B, and of B H, the difference of the two including Sides A B and A C,]

To H C, *the other Interſegment of the ſame Parallels*, [which is the *Verſed Sine* of the *Angle* C A B.]

Which was, &c.

The Work (*according to the* Axiome) *illuſtrated by Numbers.*

In the *Spherical Triangle* A B C, let the three Sides be

A B 30 deg.
A C 50 deg. } And let it be required to find the *Angle* C A B.
B C 70 deg.

The Summ of A B and A C, *viz.* B G. 80 d. the *Verſed Sine* B S	82635
The third Side C B 70 deg. The *Verſed Sine* P B	65798
The Difference of A B and A C, *viz.* B H 20 d. the *Verſed Sine* B M	06031
The Difference of B M and B S, is M S	76604
The Difference of B M and B P is M P	59767
As M S 76604 is to G H 200000, ſo is M P 59767 to C H	156041

Which 156041 is the *Line* C H, and is the *Verſed Sine* of the *Angle* C A B 124 d. 5 min. the *Sun's Azimuth* from the *North* part of the *Meridian*. And thus is the firſt part of the *Axiome* laid down ſufficiently proved: And for the ſecond part, it is nothing but the *Converſe* of the Firſt; and the Proportion is changed thus,

As the Second Term, is to the Firſt,
So is the Fourth Term to the Third.

The Lord *NEIPER'S*

INSTITUTION

OF

Spherical Trigonometry,

Fitted and applyed to Practice.

THE following *Inſtitution* of the Right Honourable *John Lord Neiper*, Baron of *Merchiſton* in *Scotland*, thus fitted and applyed to *Practice*, I received (many Years ſince) in a *Latin Manuſcript*, from the Hands of the Right Worſhipful (and my ever Honoured Friend) Sir *Charles Scarborough* M.D. comprehended in 23 Sections, which I have faithfully Tranſlated, and retained in the ſame Number, but have taken the boldneſs a little to *inlarge* ſome of them, and to *exemplifie others*, wherein I apprehended any doubt or difficulty might ariſe to the Young *Tyro* in this *Doctrine*; for which, if the more knowing herein ſhall blame me, yet the more unlearned will excuſe, my principal aim (not only in this *Tractate*, but in all the others of this *Miſcellany*) being to render all things therein obvious to the meaneſt capacity.

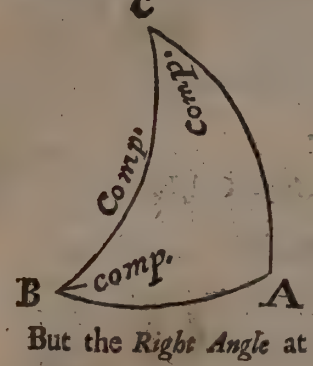
CHAP.

CHAP. I.

Of Rectangled Spherical Triangles.

SECT. I.

IN a *Rectangled Spherical Triangle*, there are (beſides the *Right Angle*) Five other Parts; whereof thoſe three which are more remote from the *Right Angle*, *Neiper* changed into their *Complements*.



As in this *Triangle* A B C, *Right Angled* at A; for the three remote Parts, to wit, the *Angles* B and C, and the Side B C, he takes their *Complements*. Theſe three *Complements*, with the Sides A B and A C, do make *Five Parts*; which by an Artificial Term, he called *Circular*.

viz. The { Side A B. / Side A C. / *Complement* of the *Angle* at B. / *Complement* of the *Angle* at C. / *Complement* of the Side B C.

But the *Right Angle* at A, is ſet aſide from being any of the *Circular Parts*.

SECT. II.

Now in the Reſolution of a *Rectangled Spherical Triangle*, there be two other *Terms* given (beſides the *Right Angle*) to find out a *Third*.

SECT. III.

Theſe Three *Terms* (namely the two that are given, and the third which is required) muſt be firſt looked upon according to their *Circular Parts*.

SECT. IV.

Of which, one is named, the *Middle* (or *Mean*) *Part*; the other two are called the *Extreme Parts*, borrowing their Appellation from the Situation of the *Terms* themſelves; For, of three *Terms*, one muſt be neceſſarily in the *Middle*, and the other two in the *Extremes*; therefore, the *Circular Part* of the *Middle Term*, is called the *Middle Part*; And the *Circular Parts* of the *Extreme Terms*, are called, the *Extreme Parts*.

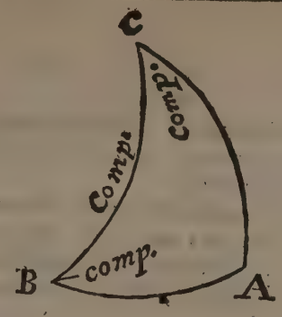
SECT. V.

But the *Extreme Parts*, may be two-fold, either *Conjunct*, or *Disjunct*: For thoſe three *Terms* (beſides the *Right Angle*) do come in queſtion according as the two *Extremes* are from either Part immediatly joined to a third *Mean*, or are *Disjoyned* from the ſame, by a *Side* or *Angle* interpoſed on both Sides: So are their *Circular Parts* named *Extremes Conjunct*, or *Extremes Disjunct*.

SECT. VI.

But of thoſe three *Terms* which may fall in queſtion, we will ſubject all their *Varieties* in their *Circular Parts* according as every one of them ought (in reſpect of each other) to be called the *Middle Part*; or the *Extremes Conjunct* or *Diſtinct*; as in the following *Analyſis* or *Table* is fully Demonſtrated.

If



If the { *Side* AB, *Side* AC, *Compl.* B, *Compl.* C, *Compl.* BC } be the middle part, then are the { *Sides* AC, and *Com.* B; *Sides* AB, and *Com.* C; *Sides* AB, and *Com.* BC; *Sides* AC, and *Com.* BC; *Comp.* B, and *Com.* C } Extremes conjoyn'd, } And { *Com.* BC, & *Com.* C; *Com.* BC, & *Com.* B; *Side* AC, & *Com.* C; *Side* AB, & *Com.* B; *Side* AB, & *Com.* AC } Extrem. disjoyn.

☞ *But here it is to be noted, that the Sides* A B *and* A C, *are ſuppoſed to be joyned together (as one entire part) becauſe the* Right Angle *at* A *is not reckoned amongſt the Circular Parts.*

SECT. VII.

Therefore in the Reſolution of a *Right angled Spherical Triangle*, to know the *Mean* and *Extreme Parts*, you muſt obſerve, that,

1. If one of the three *Terms* (which, beſides the *Right Angle*, come in queſtion) doth ſtand alone by it ſelf, ſevered from the other two on both Sides, (as the Side B C, from the Sides C A and B A, by the *Angles* B and C interpoſed,) that ſhall be the *middle Term*, and ſo its *Circular Part* ſhall be called the *middle Part*: And the other two *Circular Parts* are the *Extremes disjoyned*: But,

2. If the three *Terms* do immediately adhere together, the *middle Term* doth eaſily ſhew the *middle Part*; and the *Extreme Terms* the *Extreme Parts conjunct*.

Theſe things being all rightly underſtood, the whole *Trigonometry of Sphericals* will be abſolved by this one *Propoſition*, which therefore we will call *Catholick*, or *Univerſal*.

SECT. VIII.

Propoſition Univerſal.

The *Sine* of the *Middle Part*, and the *Radius*, are *Reciprocally Proportional* with the *Tangents* of the *Extreme Parts Conjunct*, and with the *Co-Sines* (or *Sines Complements*) of the *Extremes Disjunct*: That is,

As the *Radius*,
To the *Tangent* of one of the *Extremes Conjunct*,
So is the *Tangent* of the other *Extreme Conjunct*,
To the *Sine* of the *Middle Part*; *& contra*. Then alſo,
As *Radius*
To the *Co-Sine* of one of the *Extremes Disjunct*,
So the *Co-Sine* of the other *Extreme Disjunct*,
To the *Sine* of the *Middle Part*; *& contra*.

Corollary.

Wherefore, 1. If the *Middle Part* be ſought, the *Radius* ſhall be in the *firſt Place* of the *Proportion*; but if one of the *Extreme Parts* be ſought, then the other *Extreme* ſhall be in the *firſt Place*. Of the *ſecond* and *third Places* it mattereth nothing how they be diſpoſed. Alſo,

2. If the *Extremes* in any *Proportion* be *disjunct* from the *Middle Part*, the *Proportion* will be performed by *Sines* only: But if the *Extremes* be *conjunct* to the *Middle Part*, it muſt be performed by *Sines* and *Tangents* joyntly.

The *Demonſtration* of the *Univerſal Propoſition* is obvious enough, for where the *Extremes* are *Disjunct*, the *Proportions* differ nothing from the common ones; and in the *Extremes Conjunct*, where it is commonly ſaid

As *Radius*
To the *Tangent*.

Here we say,
As the *Co-Tangent*
Is to *Radius*.

And likewise Inversely and Contrarily, which is plainly the same thing: because *The* Radius, *is a* Mean Proportional *between the* Tangent *of an* Arch, *and the* Tangent Complement *of the same* Arch.

But here Note, *That when a* Complement *in any Proportion, do chance to concur with a* Complement *in the* Circular Parts; *you must then always take the* Sine *it self*; *or the* Tangent *it self*; *instead of the* Co-Sine *or* Co-Tangent *in the* Circular Parts: *because the* Co-Sine *of the* Co-Sine, *is the* Sine; *or the* Co-Tangent *of the* Co-Tangent, *is the* Tangent *it self*.

As in the IV. *Case* following, where A B and C A are given, and the *Angle* at B is required: here C A is the *Middle Part*, and C B and B are the *Extremes distinct*; wherefore (by the second part of the foregoing *Corollary*) the Proportion will be wrought by *Sines* only. And (by this) because the two *Extremes* C B and B fall upon *Complements* in the *Circular Parts*; therefore instead of *Co-Sine* B C, and *Co-Sine* B; you must use *Sine* B C, and *Sine* B.

To give Examples of all these in the Solution of the XVI. *Cases* of *Right Angled Spherical Triangles*, will not be unprofitable. And

For the performance hereof I shall make use of the foregoing *Right Angled Spherical Triangle* A B C, the quantity of whose *Sides* and *Angles* the Table following doth plainly exhibit. And in every *Case* I shall express the two *Terms Given* (besides the *Right Angle*) by marking the *Side* or *Angle* given with a dash through the same thus |, and the *Term Required*, I shall mark with (o). Also, to avoid the often turning over of the Tables, I have here set down the *Sines* and *Tangents*, and the *Co-Sines*, and *Co-Tangents*, of all the *Sides* and *Angles* of this *Exemplary Triangle*, as followeth.

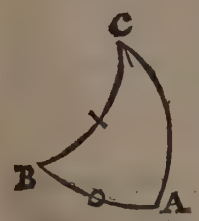
			Sine.	*Co-Sine.*	*Tangent.*	*Co-Tang.*	*Compl.*
		de. m.					*de. mi.*
Side	A B	50.10	9.8853109	9.8065575	10.0787534	9.9212466	39.50
	A C	51.30	9.8935444	9.7941496	10.0993948	9.9006052	38.30
	B C	66.30	9.9623978	9.6006997	10.3616981	9.6383019	23.30
Angle	B	58.35	9.9311522	9.7170526	10.2140996	9.7859004	31.25
	C	56.52	9.9229334	9.7376611	10.1852723	9.8147277	33.08

SECT. IX.

The XVI. Cases *of Right Angled Spherical* Triangles.

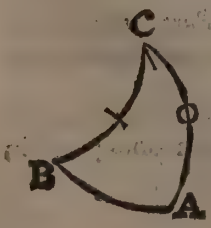
The Hypotenuse B C, *and the Angle* C *given, To find*

CASE I. *The opposite Side* A B, *the* Middle Part.



As *Radius*, 90 deg.	10,0000000
To *Sine* C 56 deg. 52 m.	9.9229334
So is *Sine* B C 66 deg. 30 m.	9.9623978
To *Sine* B A 50 deg. 10 m.	9.8853312

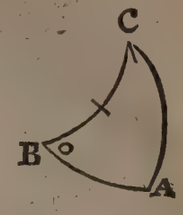
CASE II. *The Side adjacent* A C: Extreme Conjunct.



As *Co-Tangent* B C 23 deg. 30 m.	9.6383019
To *Radius* 90 deg.	10.
So *Co-Sine* C 33 deg. 8 m.	19.7376611
To *Tangent* C A 51 deg. 30 m.	10.0993592

CASE

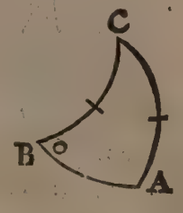
CASE III. *The other Angle* B. Extreme Conjunct.



As *Co-Tangent* C 33 deg. 8 m.	9.8147277
To *Radius* 90 deg.	10.
So *Co-Sine* BC 23 deg. 30 m.	19.6006997
To Co-Tangent B 31 deg. 25 m.	9.7859770

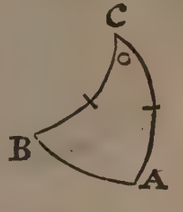
The Hypotenuse BC, *and Side* AC, *to find*

CASE IV. *The Opposite Angle at* B, Extreme Disjunct.



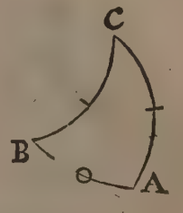
As *Sine* BC 66 deg. 30 m.	9.9623978
To *Radius* 90 deg.	10.
So is *Sine* CA 51 deg. 30 m.	19.8935444
To *Sine* B 58 deg. 35 min.	9.9311466

CASE V. *The adjacent Angle* C, Middle Part.



As *Radius*, 90 deg.	10.
To *Tangent* CA 51 deg. 30 m.	10.0993948
So *Co-Tangent* BC 23 deg. 30 m.	9.6383019
To *Co-Sine* C 33 deg. 8 m.	*x*9.7376967

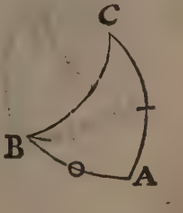
CASE VI. *The other side* AB. Extreme Disjunct.



As *Co-Sine* CA 38 deg. 30 m.	9.7941496
To *Radius* 90 deg.	10.
So *Co-Sine* CB 23 deg. 30 m.	19.6006997
To *Co-Sine* BA 39 deg. 50 m.	9.8065501

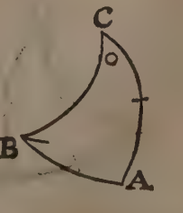
The Side AC, *and the Angle opposite thereto* B, *being given, To find*

CASE VII. *The other Side* AB. Middle Part.

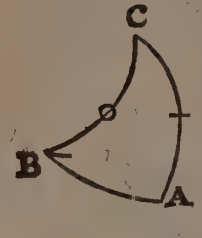


As *Radius* 90 deg.	10.
To *Tangent* CA 51 deg. 30 m.	10.0993948
So *Co-Tangent* B 31 deg. 25 m.	9.7859004
To *Sine* BA 50 deg. 10 m.	*x*9.8852952

CASE VIII. *The other Angle at* C. Extreme Disjunct.



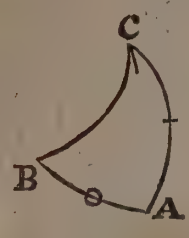
As *Co-Sine* CA 38 deg. 30 m.	9.7941496
To *Radius* 90 deg.	10.
So *Co-Sine* B 31 deg. 25 min.	19.7170526
To *Sine* C 56 deg. 52 m.	9.9229030



CASE IX. *The Hypotenuse* B C. Extreme Disjunct.

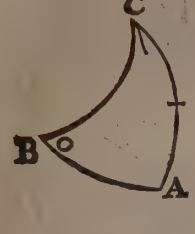
As *Sine* B 58 deg. 35 m.	9.9311522
To *Radius* 90 deg.	10.
So *Sine* C A 51 deg. 30 m.	19.8935444
To *Sine* B C 66 deg. 30 m.	9.9623922

The Side A C, *and the Angle adjacent thereto* C, *being given, To find*



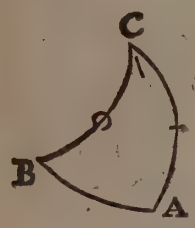
CASE X. *The other side* A B. Extreme Conjunct.

As *Co-Tangent* C 33 deg. 8 m.	9.8147277
To *Radius* 90 deg.	10.
So is *Sine* C A 51 deg. 30 min.	19.8935444
To *Tangent* B A 50 deg. 10 m.	10.0788167



CASE XI. *The other Angle* B. Middle Part.

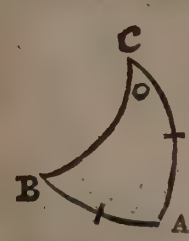
As *Radius* 90 deg.	10.
To *Sine* C 56 deg. 52 m.	9.9229334
So *Co-Sine* C A 38 deg. 30 m.	9.7941496
To *Co-Sine* B 31 deg. 25 m.	9.7170830



CASE XII. *The Hypotenuse* B C. Extreme Conjunct.

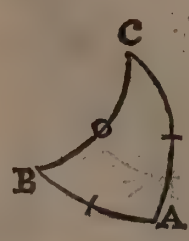
As *Tangent* C A 51 deg. 30 m.	10.0923948
To *Radius* 90 deg.	10.
So *Co-Sine* C 33 deg. 8 m.	19.7376611
To *Co Tangent* B C 23 deg. 30 min.	9.6382663

The two Sides A C *and* A B *being given, To find*



CASE XIII. *Either Angle, as* C. Extreme Conjunct.

As *Tangent* B A 50 deg. 10 m.	10.0787534
To *Radius* 90 deg.	10.
So *Sine* C A 66 deg. 30 m.	19.8935444
To *Co-Tangent* C 33 deg. 8 m.	9.8147910



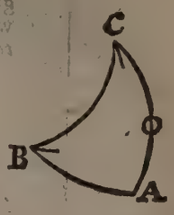
CASE XIV. *The Hypotenuse* C B. Middle Part.

As *Radius* 90 deg.	10.
To *Co-Sine* A C 38 deg. 30 m.	9.7941496
So *Co-Sine* B A 39 deg. 50 m.	9.8065575
To *Co-Sine* C B 23 deg. 30 m.	19.6007071

The

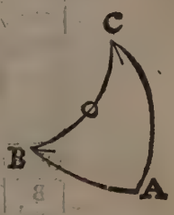
The Two Angles B and C given. To find

CASE XV. *Either of the Sides*, viz. A C. Ext. Disjunct.



As *Sine* C 56 deg. 52 m.	9.9229334
To *Radius* 90 deg.	10.
So *Co-Sine* B 31 deg. 25 m.	19.7170526
To *Co-Sine* C A 38 deg. 30 m.	9.7941192

CASE XVI. *The Hypotenuse* B C. Middle Part.



As *Radius* 90 deg.	10.
To *Co-Tangent* C 33 deg. 8 m.	9.8147277
So *Co-Tangent* B 31 deg. 25 m.	9.7859004
To *Co-Sine* B C 23 deg. 30 m.	*x*9.6006281

Note, These are the *Proportions* according to the *Universal Proposition*, yet may (many of them) be varied; so that the *Radius* may be brought in the *first place*; and that by the latter part of the foregoing *Corrollary*, which saith, *Radius is a mean Proportional between the Tangent of an Arch, and the Tangent Complement of the same Arch*: So that (in the XIII. *Case*) where it is said,

As *Tangent* B A
Is to *Radius*;
So is *Sine* C A
To *Co-Tangent* C:

It is all one, as if you should say,

As *Radius* 90 deg.	10.
To the *Co-Tangent* of B A 39 deg. 50 m.	9.9212466
So is *Sine* of C A 51 deg. 30 m.	9.8935444
To *Co-Tangent* C 33 deg. 8 m.	*x*9.8147910

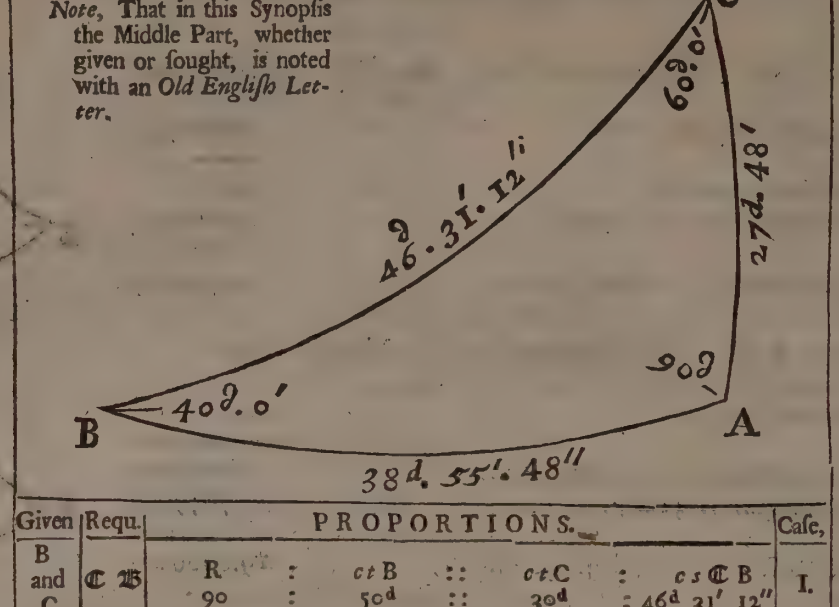
And likewise the same course may be taken in the *Second*, *Third*, *Tenth*, and *Twelfth Cases:*

And thus have you the whole *Doctrine* of *Right Angled Spherical Triangles*, performed by help of this one *Catholick Proposition.*

And as in *Right Lined Triangles*, I gave you in a *Synopsis* a brief view of the several Workings of all the *Cases*; I shall here do the like in *Right Angled Spherical Triangles*, wherein I have not exactly followed the *Catholick* or *Universal Proposition*, but have (according to this last *Note*) varied them, to bring the *Radius* in the *First Place* of the *Proportion* as often as it might.

A SYNOPSIS *of the* XVI. Cases *of Right Angled Spherical* Triangles.

Note, That in this Synopsis the Middle Part, whether given or sought, is noted with an *Old English Letter*.



Given	Requ.	PROPORTIONS.				Case,
B and C	ℭ 𝔅	R : 90	:: *ct* B : 50d	:: *ct* C : 30d	:: *cs* ℭ B : 46d 31′ 12″	I.
B and ℭ	B A	*s* B : 40d	:: R : 90d	:: *cs* ℭ : 30d	:: *cs* B A : 51d 4′ 12″	II.
B and 𝔅 ℭ	C	R : 90d	:: *cs* 𝔅 ℭ : 43d 28′ 48″	:: *t* B : 40d	:: *ct* C : 30d	III.
B and B C	ℭ A	R : 90d	:: *s* C B : 46d 31′ 12″	:: *s* B : 40d	:: *s* C A : 27d 48′	IV.
𝔅 and B C	B A	R : 90d	:: *cs* 𝔅 : 50d	:: *t* C B : 46d 31′ 12	:: *t* B A : 38d 55′ 48″	V.
B and A B	ℭ	R : 90d	:: *cs* B A : 51d 4′ 12″	:: *s* B : 40d	:: *cs* ℭ : 30d	VI.
𝔅 and A B	B C	R : 90d	:: *cs* 𝔅 : 50d	:: *ct* B A : 51d 4′ 12″	:: *ct* C B : 43d 28′ 48″	VII.
B and 𝔅 A	C A	R : 90d	:: *s* 𝔅 A : 38d 55′ 48″	:: *t* B : 40d	:: *t* C A : 27d 48′	VIII.
𝔅 and C A	C	*cs* C A : 62d 12′	:: R : 90d	:: *cs* 𝔅 : 50d	:: *s* C : 60d	IX.
B and ℭ A	B C	*s* B : 40d	:: R : 90d	:: *s* ℭ A : 27d 48′	:: *s* C B : 46d 31′ 12″	X.
B and C A	𝔅 A	R : 90d	:: *ct* B : 50d	:: *t* C A : 27d 48′	:: *s* 𝔅 A : 38d 55′ 48″	XI.
C B and 𝔅 A	C	*s* C B : 46d 31′ 12″	:: R : 90d	:: *s* 𝔅 A : 38d 55′ 48″	:: *s* C : 60d	XII.
B C and B A	𝔅	R : 90d	:: *t* B A : 38d 55′ 48″	:: *ct* C B : 43d 28′ 48″	:: *cs* 𝔅 : 50d	XIII.
𝔅 ℭ and C A	C A	*cs* B A : 51d 4′ 12″	:: R : 90d	:: *cs* ℭ 𝔅 : 43d 28′ 48″	:: *cs* C A : 62d 12′	XIV.
𝔅 A and C A	B	R : 90d	:: *s* 𝔅 A : 38d 55′ 48″	:: *ct* C A : 62d 12′	:: *ct* B : 50d	XV.
B A and C A	ℭ 𝔅	R : 90d	:: *cs* C A : 62d 12′	:: *cs* B A : 51d 4′ 12″	:: *cs* C B : 43d 28′ 48″	XVI.

CHAP.

CHAP. II.

Of Oblique Angled Spherical Triangles.

SECT. X.

IN *Oblique angled Spherical Triangles*; if the things *Given* and *Required* be opposite, they are solved after the old manner, by the usual Proposition, which is the same with *Axiome* III. beforegoing, namely, that *in all Spherical Triangles*; the *Sines* of their Sides are directly Proportional to the *Sines* of their opposite *Angles*, *which is there Demonstrated.*

And as it will appear in the two following *Cases*, in the *Oblique angled Triangle* B C D.

CASE I.

Two Sides, and an Angle *opposite to one of them being given, to find the* Angle *opposite to the other.*

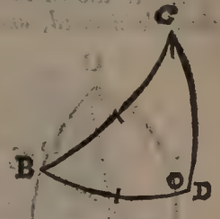
Let the Sides B D and B C, and the *Angle* C (opposite to the Side B D) be given, to find the *Angle* at D, (opposite to the other Side C B.)

As the *Sine* of B D 48 d. 47 m. —— 9.5078428

Is to the *Sine* of C 23 d. 30 m. —— 9.6006997
So is the *Sine* of C B 42 d. 51 m. —— 9.8325609

19.4332606

To the *Sine* of D 57 d. 22 m. —— 9.9254178



Note here, in the use of this *Axiome*, the same doubt may arise in *Spherical Oblique Triangles*, as did in *Plain Triangles*, namely to know whether the *Angle* or *Side* inquired, be *more* or *less* than a *Quadrant*, therefore circumspection ought to be had, lest in such case you be deceived in finding an *Acute Angle* for an *Obtuse*, and the contrary. And therefore, besides the bare inquiring of the *Angle*, the *Species* of it ought also to be given, namely whether it be *Acute* or *Obtuse*: As in this *Example*, the inquired *Angle* D, was 122 d. 38 m. the *Complement* of 57 d. 24 m. (the *Angle* found) to 180 deg.

CASE II.

Two Angles, *and a Side opposite to one of them being given, to find the Side opposite to the other of them.*

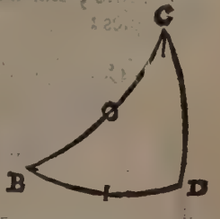
Let the two *Angles* C and D, and the Side B D (opposite to the *Angle* C,) be given, to find the Side C B (opposite to the *Angle* D.)

As the *Sine* of C 23 d. 30 m. —— 9.6006997

Is to *Sine* B D 18 d. 47 m. —— 9.5078428
So is *Sine* D 122 d. 38 m. (or 57 d. 22 m.) —— 9.9253037

19.4332265

To the *Sine* of C B 42 d. 51 m. —— 9.8325268



And thus are the Two first Cases in *Oblique angled Spherical Triangles* solved by help of the III *Axiome* before cited.

SECT. XI.

There be Ten other *Cases*, in *Oblique angled Spherical Triangles*, which may be resolved by the *Catholick Proposition*, (but by two operations at the least) and that most fitly, by reducing of the *Oblique Triangle* into two *Right angled Triangles*, by letting fall a *Perpendicular*.

First therefore, we must consider of the manner of letting the *Perpendicular* fall.

Secondly, What is to be inquired out in the first Operation. And,

Thirdly, How the *Question* is to be solved by the Second.

SECT. XII.

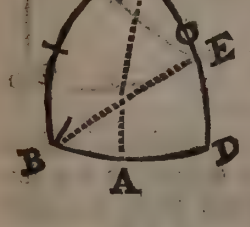
The *Perpendicular* must be let fall in such manner as that the *Rectangle Triangle* may be resolved from the three things given in the *Oblique angled Triangle*, and that by two Operations only, even as this *Rule* teacheth.

SECT. XIII.

General Rules for letting fall the Perpendicular.

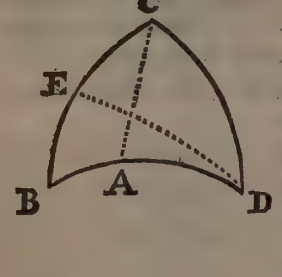
1. *Let the* Perpendicular *fall from the extremity of a given Side, and let it subtend an adjacent* Angle *given.* And besides,

2. (If the three things given be conterminal) *let it also fall from the extremity of the Side inquired, and let it subtend the* Angle *inquired.*



In the *Triangle* B C D, Let the *Angles* C and B, and the Side C B, be given, and the Side C D required.

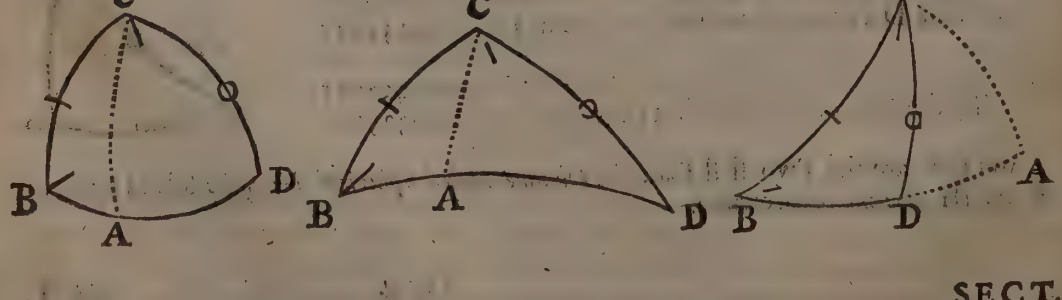
The *Perpendicular* C A, is let fall from the extremity of the given Side B C, and subtendeth the adjacent *Angle* B: And the three terms given being conterminal (in this Example) it also falleth from the extremity of the inquired Side C D, and subtendeth the inquired *Angle* at D.



But if the *Perpendiculars* B E and D E, be constituted according to the first part of the *Rule*, truly then the *Triangle* may be resolved, but not at two Operations, for which one things sake, the second part of the *Rule* comes to be observed, and that in those two *Cases* only; for in the other, the first part of the *Rule* alone sufficeth.

SECT. XIV.

The Perpendicular *falleth within the* Triangle, *when the* Angles *at the* Base (or Side upon which the *Perpendicular* falleth) *are both of one* Species, viz. *either both* Obtuse, *or both* Acute; *But without the* Triangle, *if one be* Obtuse *and the other* Acute. As in these Examples:



SECT.

SECT. XV.

The *Perpendicular* being rightly let fall, the first Operation will be in the first *Triangle, viz.* in that wherein three things are given, and that will be either for the finding of the *Vertical Angle* or the *Base*, and that by the *Universal Proposition.*

SECT. XVI.

First, If an *Angle* be sought, then first let the *Vertical Angle* be found, (unless the *Perpendicular* doth fall upon a known Side.)

Secondly, If a *Side* be sought, let first the *Base* be found (unless the *Perpendicular* fall from a known *Angle*) for then the contrary must be observed every way.

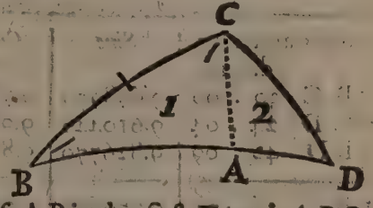
SECT. XVII.

The Second Operation consists in comparing the two *Homogenial Terms* of both *Triangles* between themselves; *viz.* Of the *Hypotenuses* with the *Vertical Angles*, or with the *Bases*: And also of the *Angles* at the *Bases* with the same.

SECT. XVIII.

For the *Homogenial Terms* of each *Triangle*, are proportional; and *Homogenial* in the *Sines* and *Tangents* of the *Circular Parts.*

As in the *Oblique angled Spherical Triangles* B C D, reduced into the two *Right angled Spherical Triangles* A B C and A C D, by the *Perpendicular* C A.



I say, that in the *Sines* and *Tangents* of the *Circular Parts* it will be,

As the *Base* A B: To the *Angle at the Base* B.
So the *Base* A D: To the *Angle at the Base* D.

For the *Perpendicular* C A being assumed, the *Base* A B in the first *Triangle* A B C, will be the *middle Part*, and then the *Complement of the Angle* B, and the *Perpendicular* A C, are the *Extreme Parts Conjunct*; Therefore, by the *Universal Proposition*,

As the *Tangent* of A C,
Is to *Radius*;
So is the *Sine* of A B,
To the *Co-Tangent* of B.

In like manner, in the second *Triangle* A D C; the *Base* A D, will be the *Middle Part*; than the *Complement* of the *Angle* at D, and the *Perpendicular* A C, are the *Extremes Conjunct*; Wherefore, by the *Universal Proposition*,

As *Tangent* A C,
Is to *Radius*;
So is *Sine* A D,
To *Co-tangent* D.

Therefore (by the 11. P. 5. *Euch.*)

As the *Sine* of A B,
To *Co-tangent* of B;
So is the *Sine* of A D,
To the *Co-tangent* of D.

And there is the like reason in *Demonstrating* in the other *Parts*; for, the *Perpendicular* shall be always one (and the same) of the *Extremes* in both *Triangles.*

SECT. XIX.

Wherefore to determine a Proportion of two *Homogenial Terms* of both *Triangles* betwixt themselves; we must imagine the *Perpendicular* to be compared both with the *Term* given, and inquired in the second *Triangle*, that it may be understood which of those may be made the *Middle Part*, and which the one *Extreme*, and of what kind it

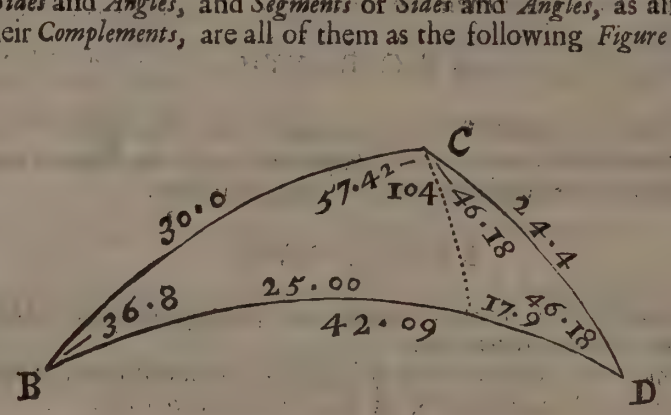
is;

is; whereupon *Sines* or *Tangents* may be fitted to them according to the *Propoſition Univerſal*, to which do anſwer in all things the *Homogenial Parts* of the firſt *Triangle*: Then the *Perpendicular* being rejected, together with the *Radius*, we compare the *Middle Part* and one of the *Extremes* of the firſt *Triangle* with the *Middle Part*, and one of the *Extremes* of the Second, as is done before, and as will be manifeſt in the eight following *Caſes*.

But we ſuppoſe the *Firſt Operation*, for finding the *Vertical Angle*, or the *Baſe*, (according as the Queſtion requires,) to be performed by *Section* VIII and Section XV.)

Now what hath hitherto been delivered concerning the Solution of *Oblique angled Spherical Triangles*, I will here illuſtrate in the eight following *Caſes*: And that all difficulties may be removed, I will work the ſeveral Operations at large.

The *Triangle* that I ſhall make uſe of, ſhall be the *Oblique angled Spherical Triangle* B C D. The *Sides* and *Angles*, and *Segments* of *Sides* and *Angles*, as alſo the *Sines* and *Tangents* and their *Complements*, are all of them as the following *Figure* and *Table* doth exhibit.

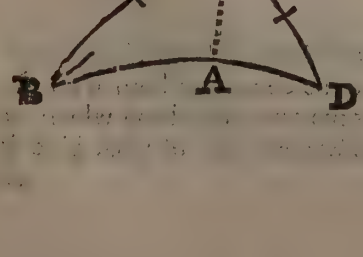


Sides and Ang.		Sine	Co-Sine		Tangent	Co-Tangent	Com.
	d. m.						d. m.
B C	30 . 00	9.698970	9.937531		9.761439	10.238561	60. 0
C D	24 . 04	9.610446	9.960505		9.649942	10.350058	65.56
B D	42 . 09	9.826770	9.870047		9.956723	10.043277	47.51
B	36 . 08	9.770606	9.907222		9.863385	10.136615	54.52
C	104 . 00	9.986904	9.383675		10.603229	9.396771	76. 0
D	46 . 18	9.859119	9.839404		10.019714	9.980285	43.42
B A	25 . 00	9.625948	9.957276		9.668672	10.331327	65. 0
A D	17 . 09	9.469637	9.980247		9.489390	10.510610	72.51
B C A	57 . 42	9.926991	9.727827		10.199163	9.800836	32.18
A C D	46 . 18	9.859119	9.839404		10.019714	9.980285	43.42
C A	17 . 9	9.469637	9.980247		9.489390	10.510610	72.51

The Eight following Caſes *of* Oblique Spherical Triangles, *Exemplified.*

CASE III.

Two Sides B C ***and*** C D, ***and an*** Angle ***oppoſite to one of them*** B ***being given; To find the*** Angle C, ***comprehended between the two given Sides.***



In the *Oblique angled Triangle* B C D, let there be given the two Sides C B and C D, and the *Angle* B oppoſite to the Side C D; and let the *Vertical Angle* at C be required.

Firſt, for the *Perpendicular* C A *(by the firſt part of the Rule Sect.* 13.*)* it is let fall from the extremity of the given Side B C, and it ſubtendeth the

the given *Angle* B. And (*by the* 14 *Sect.*) it falls within the *Triangle*, becauſe the *Angles* at the *Baſe* be both *Acute*.

And thus is the *Oblique angled Triangle* reduced into the two *Right angled Triangles*, BCA and DCA, both of them *Right angled* at A, in the firſt of which *Triangles*, namely in BAC, there is given (beſides the *Right Angle*) the Side CB, and the *Angle* at B, to find the *Vertical Angle* BCA, which may be found by the I. *Caſe* of *Right angled Spherical Triangles*; thus,

As the *Co-tangent* of B 54 d. 52 m.	10.136615
Is to *Radius* 90 d.	10.
So is the *Co-Sine* of BC 60 d.	19.937531
To the *Co-tangent* of BCA, 57 d. 42 m.	9.800916

Having thus found the *Vertical Angle* BCA, in the firſt *Triangle* ABC, you muſt conſider the *Homogeneal* Parts in both *Triangles* (*by the* 18*th Section.*) which will be the Side CB, and *Angle* BCA in the firſt, and CD and the *Angle* ACD in the Second. And then,

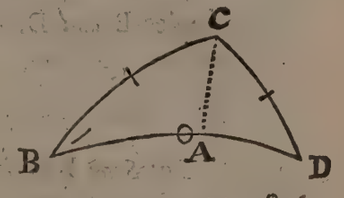
As the *Co-tangent* of BC 60 d.	10.238561
To the *Co-ſine* BCA 32 d. 18 m.	9.727828
So is *Co-tangent* CD 65 d. 56	10.350058
	20.077886
To the *Co-ſine* ACD 43 d. 42	9.839325

Whoſe *Complement* 46 deg. 18 min. is the *Angle* ACD, which added to 57 deg. 42 m. the former found *Angle* BCA, maketh 104 deg. for the whole *Angle* BCD.

CASE IV.

To find the Side BD *adjacent to the given* Angle B.

The *Perpendicular* being let fall, the *Triangle* is reduced into two *Rectangled Triangles* ABC and ACD, in the firſt whereof is given (beſides the *Right Angle*) the Side BC, and the *Angle* at B, to find the Side BA, (*By the* II. *Caſe* of *Right Angled Spherical Triangles.*)



As *Co-tangent* BC 60 d.	10.238561
To *Radius* 90 d.	10.
So is *Co-ſine* B 54 d. 52 m.	19.907222
To the *Tangent* of BA 25 d.	9.668661

Which being found, the *Homogeneal* Terms in the two *Triangles* are the Sides CB and CD, and the *Segments* of the *Baſe* AB and AD. Wherefore,

As *Co-ſine* BC 60 deg.	9.937531
To *Co-ſine* CD 65 deg. 56	9.960505
So *Co-ſine* AB 65 d.	9.957276
	19.917781
To *Co-ſine* AD 72 d. 51	9.980250

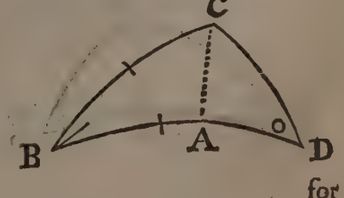
Whoſe *Complement* 17 d. 9 m. Added to BA 25 deg. maketh 42 deg. 9 min. for the whole Side BD.

The two Sides BC *and* BD, *and the* Angle B *comprehended between them being given,* to find,

CASE V.

Either of the Oppoſite Angles, viz. DB.

The *Perpendicular* being let fall, in the *Right angled Triangle* ABC, there is given CB and B to find the *Segment* BA, (by the II. *Caſe* of *Right angled Spherical Triangles,*) as in the laſt *Caſe*, it was found to be 25 deg. which found and ſubtracted from BD 42 deg. 9 m. leaves 17 deg. 9 m.



for

for the *Segment* A D. And now the *Homogeneal* Terms are the Sides A B and A D, and the *Angles* B and D, wherefore,

As the *Sine* of A B 25 deg.	9.625948
Is to the *Co-tangent* of B 54 d. 52 m.	10.136615
So is the *Sine* A D 17 d. 9 m.	9.469637
	19.606252
To the *Co-tangent* of D 43 d. 42 m.	9.980304

Whose *Complement* 46 d. 18 min. is the quantity of the *Angle* at D.

CASE VI.

The Third Side, C D.

You must first find the *Base* B A in the first *Triangle*, 25 deg. which subtracted from B D 42 d. 9 m. leaveth 17 deg. 9 m. for the *Base* A D in the Second *Triangle*. And the *Homogeneal* Terms will be the *Bases* A B and A D, and the *Hypotenuses* C B and C D. Wherefore,

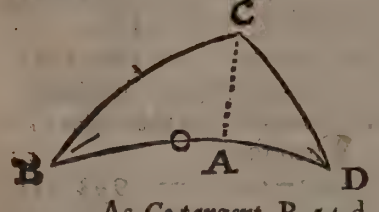
As the *Co-Sine* of A B 65 deg.	9.957276
To *Co-sine* B C 60 d.	9.937531
So *Co-sine* A D 72 d. 5 m.	9.980247
	19.917778
To *Co-sine* D C 65 d. 56 m.	9.960502

Whose *Complement* 24 deg. 4 min. is the third Side C D.

Two Angles B *and* D, *and a Side opposite to one of them* B C *being given*, to find,

CASE VII.

The Side B D *included between the two Given* Angles.



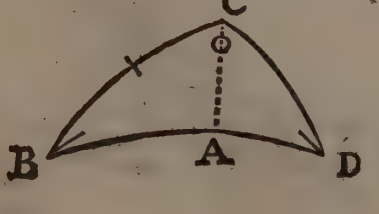
The *Segments* B A of the *Base* B D being found as before, the *Homogeneal* Terms are the two *Segments* of the *Base* A B and A D, and the two *Angles* B and D. Wherefore,

As *Co-tangent* B 54 d. 52 m.	10.136615
To *Sine* A B 25 deg.	9.625948
So is *Co-tangent* D 43 d. 42 m.	9.980285
	19.606233
To *Sine* A D 17 d. 9 m.	9.469618

Which added to A B 25 deg. giveth 42 deg. 9 m. for the whole required Side B D.

CASE VIII.

The Third Angle *at* C.



You must first find the *Angle* B C A, (by the III *Case* of *Right angled Spherical Triangles*) which you will find to be 57 deg. 42 m. Which found the *Homogeneal* Terms in the two *Triangles* will be the two *Angles* B and D, and the Parts or *Segments* of the *Angle* at C. Wherefore,

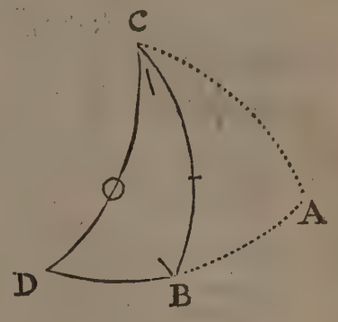
As

As the *Co-sine* of B 54 d. 52 m. —— 9.907222
To *Sine* A C B 57 d. 42 m. —— 9.926991
So *Co-Sine* D 43 d. 42 m. —— 9.839404
19.766395
To *Sine* A C D 46 d. 18 m. —— 9.859173

Which added to B C A 57 d. 42 m. giveth 104 deg. for the whole *Angle*, B C D as was required.

The Two Angles B *and* C, *and the Side included between them* C B *being given*, to find,

CASE IX. *An Oppoſite Side*, viz. C D.



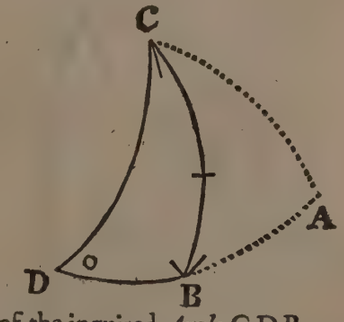
In the *Oblique angled Spherical Triangle* C B D, let there be given, the *Angle* D C B 36 d. 8 min. The *Angle* C B D 104 deg. and the Side included between them C B 30 deg. And let the Side C D, (oppoſite to B) be required.

Here the *Perpendicular* is let fall from the extremity of the known Side C B, and it is adjacent to the known *Angle* C B D, (or rather its *Complement* C B A:) And becauſe the *Angles* at the *Baſe* are of different *Species*, *viz.* one *Acute*, the other *Obtuſe*, it therefore falleth without the *Triangle*, and ſo the *Oblique angled Triangle* is reduced into the two *Rectangles* A B C, and A C D; in which the *Homogeneal* Terms are the two *Angles* A C B and A C D, and the two Sides C B and C D. Wherefore, in the *Triangle* A B C, we have given the Side B C 30 deg. and the *Angle* C B A 76 deg. (it being the *Complement* of the *Angle* C B D 104 d. to 180,) to find the *Angle* B C A; which (by the III *Caſe* of *Right angled Spherical Triangles*) you will find to be 16 deg. 4 m. which added to the given *Angle* B C D 36 deg. 8 m. the Summ will be 52 deg. 12 m. for the whole *Angle* D C A. Now by the *Homogeneal* Parts in both *Triangles*, ſay,

As the *Co-ſine* of A C B 73 d. 56 m. —— 9.982696
Is to *Co tangent* B C 60 deg. —— 10.238560
So is *Co-ſine* A C D 38 de. 48 m. —— 9.787394
19.025954
To the *Co-tangent* of D C 47 d. 51 m. —— 9.043258

Whoſe *Complement* 42 deg. 9 min. is the quantity of the inquired Side C D.

CASE X. *The Third* Angle *at* D.



You muſt firſt find the *Angle* A C B, and by conſequence the whole *Angle* A C D, as in the laſt *Caſe*, and then the *Homogeneal* Terms in both *Triangles* will be the *Angles* A C B and A C D, and the *Angles* B and D. Wherefore,

0.557903
As *Sine* A C B 16 d. 4 m. —— 9.442096
To *Co-ſine* B 14 deg. —— 9.383675
So *Sine* A C D 52 d. 12 m. —— 9.897712
19.281387
To *Co-ſine* D 43 d. 42 m. —— 9.839291

Whoſe *Complement* 46 deg. 18 min. is the quantity of the inquired *Angle* C D B.

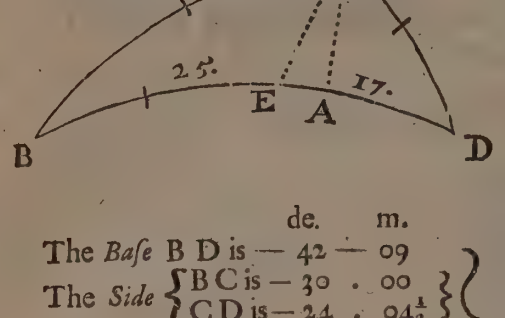
SECT. XX.

In the following *Caſes* the *Perpendicular* falls oppoſite to the *Angle* inquired, then the whole difficulty conſiſts in the firſt Operation, for the finding of the *Baſes*, and it is in manner following.

The

The Three Sides BC, CD, *and* BD *being given, To find*

CASE XI. *The Angle* D.



In this *Case* the *Perpendicular* A falls opposite to the *Angle* inquired, and the whole difficulty consists in the first Operation, for the finding of the *Segments* of the *Base*, which may be effected after this manner.

First find the half Summ of the *Base* B D.

Secondly, The half Summ of the Sides B C and D C.

Thirdly, The half difference of the Sides B C and C D.

	de.	m.			
The *Base* B D is	42	09			
The *Side* B C is	30	00			
The *Side* C D is	24	04½			
Their Summ is	54	04½	The half Summ	27	02¼
Their differ.	5	55½		2	57½

Then will the *Analogy* or *Proportion* be,

As the *Tangent* of half the Summ of the *Base* B D. 21 d. 4½ m. —— 9.585886

To the *Tangent* of half the Summ of the Sides B C and C D 27 d. 2¼ m. — 9.707790

So is the *Tangent* of half the difference of the Sides B C and C D, 2 d. 57½ m. 8.712083

18.419873

To the *Tang.* of half the difference of the *Segments* of the *Base* A E 3 d. 54½ m. 8.835987

Which half difference 3 d. 54½ m. being *added* to the half *Base* 21 d. 4½ m. gives 25 deg. for the Greater *Segment* of the *Base* A B, and being subtracted therefrom, leaves 17 d. 9 m. for the lesser *Segment* A D.

And now for the finding of the inquired *Angle* at D. You may say (*by the* V Case *of* Right angled Spherical Triangles,)

As *Radius* 90 deg. —— 10.

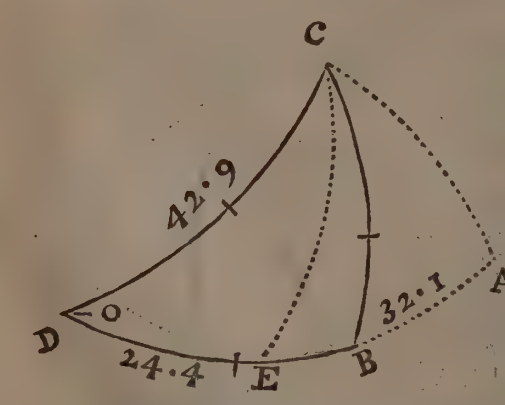
To the *Tangent* of A D 17 de. 9 m. —— 9.489390

So *Co tangent* of C D 65 d. 56 m. —— 10.350058

To the *Co-sine* of C D A 43 d. 42 m. —— 9.839448

Whose *Complement* 46 de. 18 min. is the quantity of the *Angle* C D B which was required to be found.

Another Example in this Case.



In the *Triangle* B C D let the three *Sides* D B, 24 d. 4 m. B C 30 de. and D C, 42 d. 9 m. be given, and let the *Angle* at D be required.

The *Base* B D is 24 deg. 4 m. the half 12 deg. 2 min.

The Sides C B and C D 72 d. 9 m. the half 36 d. 4½ m.

The Difference of C B and C D 12 d. 9 m. the half 6 d. 4½ m.

Then say,

As *Tangent* half the *Base* B D 12 deg. 2 min. —— 9.3287153

To *Tangent* half C B and C D 36 deg. 4½ m. —— 9.8624233

So *Tangent* of half differ. of C B and C D 6 deg. 4½ m. —— 9.0270548

18.8894781

To the *Tangent* of half the difference of the *Bases* E A 9 deg. 59 m. — 9.5607628

Which

Which 19 d. 59 m. being added to the half *Base* 12 deg. 2 m. giveth 32 deg. 1 m. for A E.

Then for finding the *Angle* at D, (by the V *Case* of *Right angled Spherical Triangles*) say,

As *Radius* 90 deg. —— 10.

To the *Tangent* of B A 32 d. 1 m. —— 9.796070

So the *Co-tangent* of B C 47 deg. 51 m. —— 10.043277

To the *Co-sine* of B 43 d. 42 m. —— 9.839347

Whose *Complement* 46 deg. 18 min. is the quantity of the *Angle* C D B which was required.

SECT. XXI.

If the three *Angles* of a *Triangle* be given, and a Side be sought, let the *Angles* be changed into the opposite Sides, and contrariwise the Sides into *Angles*, taking for the greatest Side and greatest *Angle* their *Complements* to a *Semicircle*; by which *Conversion* the *Question* is solved by the work of the two former *Examples*;

The *Demonstration* of this *Conversion* of *Angles* into Sides, and Sides into *Angles*, is shewed at large in the 7*th* of Section I. of *Spherical Triangles*.

A more compendious way of finding an *Angle* by having the three Sides of an *Oblique Spherical* given;

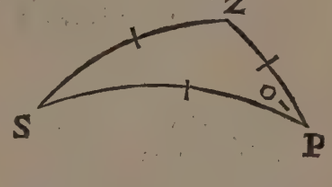
In the *Triangle* Z P S, let there be given,

The Side { Z P —— 40 de. 00 m. / Z S —— 46 44 / S P —— 72 14 } And let the *Angle* at P be required.

The RULE.

Take half the difference between the two Sides containing *Angle* required, which add to half the Side opposite to the inquired; and also subtract it from the same; noting the Summ and the remainder. Then the *Complements Arithmetical* of the Sides containing *Angle*, and of the *Sines* of the forefound *Summ* and remainder; being added together, half the Summ of them shall be the *Sine* of half the *Angle* required.

	de.	m.
The Side Z P is	40	00
S P is	72	14
Their difference	32	14
The half difference	16	07



	de.	m.	de.	m.
Z S opposite to A, is ——	46	44	the half	23 — 22
The half difference ——			added	16 — 07
Their Summ is ——				39 — 29
Their Difference is ——				7 — 15

	de.	m.		
Z P ——	40	00	*Co-Ari* —	0.1919325
S P ——	72	13		0.0212635
The Summ —	39	29	*Sine* —	9.8033572
The Remainder	7	15		9.1010558

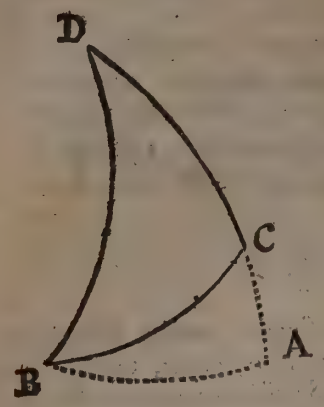
The Summ 19.1176070

The half Summ 9.5588035

Which is the *Sine* of 21 deg. 14 m. the double whereof is 42 deg. 28 m. for the quantity of the *Angle* at P. required.

Sect.

SECT. XXII.



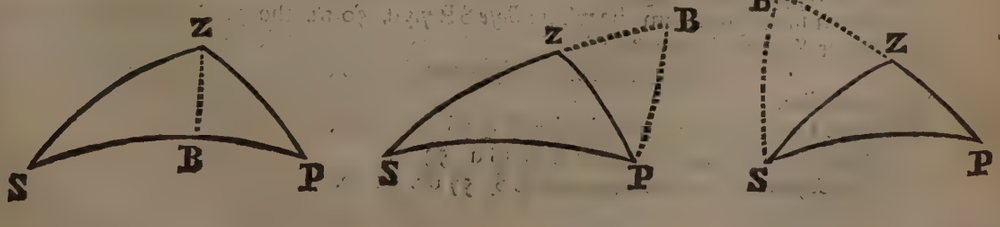
If in an *Oblique angled Spherical Triangle* as B C D, if one Side B D be a *Quadrant*, let one of the other Sides D C be produced to a *Quadrant*; and from the *Pole* B, let be inſcribed the *Arch* B A, the *Meaſure* of the *Angle* at D, therefore ſhall the *Triangle* A B C be *Right angled*: For the Reſolution whereof, the *Oblique angled Triangle* B D C is withal reſolved: For the one of them conſiſts of the *Complements* of the other, or of the ſame Parts. This Section (among divers others neceſſary to be known to the *Trigonometer*) is largely demonſtrated in the forementioned firſt Section of *Spherical Trigonometry*.

SECT. XXIII.

Laſtly, It appears from theſe things, that in every *Rectangled*, and alſo *Oblique angled Triangle*, of which one only Side is a *Quadrant*; there may be XVI *Caſes*, and XXX *Problems*: And in other *Oblique angled Spherical Triangles* XII *Caſes* and LX *Problems*: All truly, and very eaſily to be reſolved by what is delivered in this foregoing *Epitomy*.

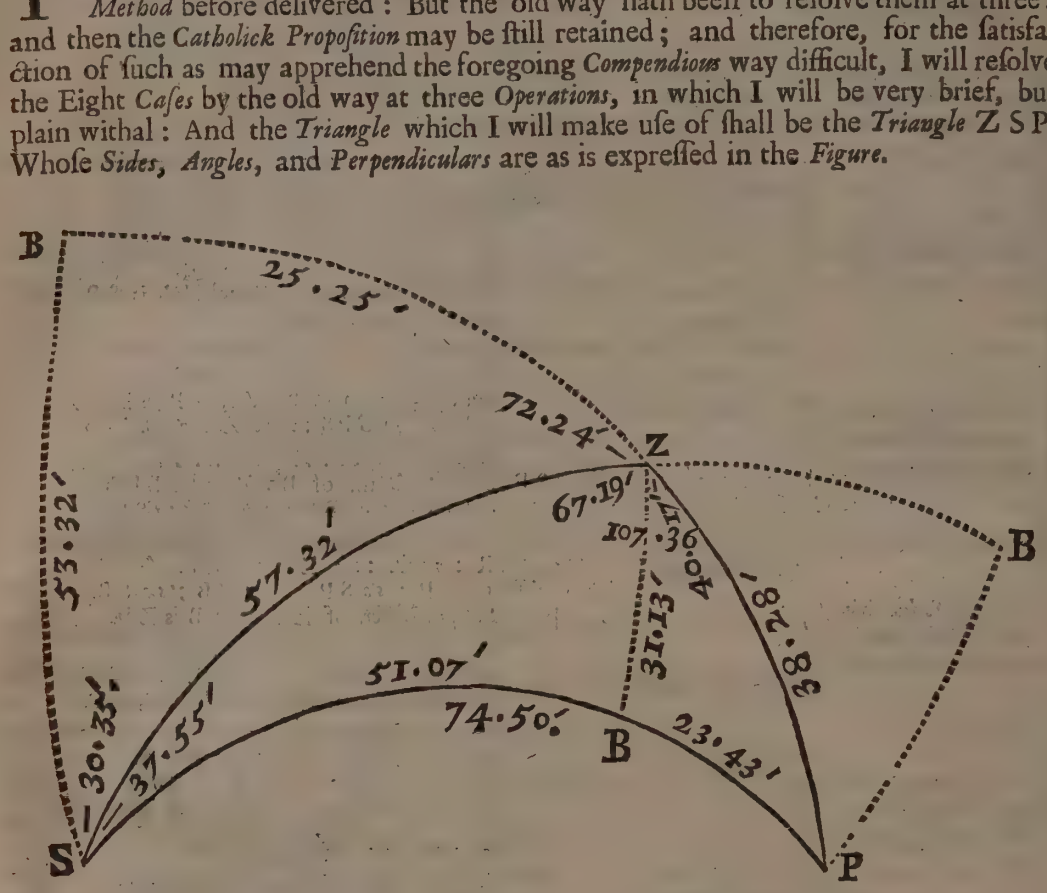
A Synopsis *of the* XII Cases *of* Oblique Angled Spherical Triangles.

	Given	Req.	The Proportionality.	Case.
Two *Angles*, and a *Side* opposite to one of them given; to find the *Side* opposite to the other.	Z S ZP	SP	*s*S : *s*ZP :: *s*Z : *s*SP.	I
Two *Sides*, with an *Angle* opposite to one of them given; to find the *Angle* opposite to the other.	ZP Z SP	S	*s*SP : *s*Z :: *s*ZP : *s*S.	II
Two *Sides* with their contained *Angle*, given, to find — The third *Side*	ZP Z	SP	R : *cs*Z :: *t*. ZP : *t*ZB. Sum or rem. of ZB & ZS is SB. *cs*ZB : *cs*SB :: *cs*ZP : *cs*SP.	III
Two *Sides* with their contained *Angle*, given, to find — One of the other *Angles*.	ZP ZS	S	R : *cs*Z :: *t*ZP : *t*ZB. Sum or Remain of ZB and ZS, is SB. *s*SB : ZB :: *t*Z : *t*S.	IV
Two *Angles*, and the *Side* between them given; to find — The third *Angle*	ZP Z	S	R : *cs*ZP :: *t*Z : *tc*BPZ. Sum or Remain of BPZ and P, is BPS. *s*BPZ : *s*BPS :: *cs*Z : *cs*S.	V
Two *Angles*, and the *Side* between them given; to find — One of the other Sides.	P	SP	R : *cs*ZP :: *t*Z : *tc*BPZ. Sum or Rem. of BPZ and PBPS. *cs*BPS : *cs*BPZ :: *t*ZP : *t*SP.	VI
Two *Sides*, with one of their opposite *Angles* given; to find — The third *Side*	ZP Z	ZP	R : *cs*Z :: *t*ZD : *t*ZB. *sc*ZP : *sc*SP :: *sc*ZB : *sc*SB. Sum or Differ. of ZB and SB is ZP.	VII
Two *Sides*, with one of their opposite *Angles* given; to find — Their contained *Angle*.	SP	P	R : *cs*ZP :: *t*Z : *ct*BPZ. *tc*ZP : *tc*SP :: *sc*BPZ : *sc*BPS. Sum or Differ. of BPZ, and BPS, is P.	VIII
Two *Angles*, with one of their opposite *Sides* given; to find — The third *Angle*	ZP Z	P	R : *cs*ZP :: *t*Z : *ct*BPA. *sc*Z : *sc*S :: *s*BPZ : *s*BPS. Sum or Difference of BPZ and BPS is P.	IX
Two *Angles*, with one of their opposite *Sides* given; to find — The *Side* between them	S	ZS	R : *cs*Z :: *t*ZP : *t*ZB. *t*S : *t*Z :: *s*ZB : *s*SB. Sum or Differ. of ZB and SB, is ZS.	X
The three *Sides* ZS, ZP, SP, given, to find the *Angle* Z. — ZS 78.00 ZP 38.30 SP 70 The Summ SP Subst. Difference	the half 39.00 19.15 35.00 93.15 70.00 23.15	Z	(1) R : *s*ZS :: *s*ZP : *Sine* 90 78 d.0′ 38.30′ 37.31 (2) Sin : *s*Sum :: *s*Differ : *Sine* 37.31′ 93.15 23.15 9.81099 To which add *Radius*, it is 19.81099 deg. The half is 9.90549.853.33′ The *Complement* is 36.27 That doubled is 72.54 The quantity of the *Angle* Z.	XI
The three *Angles* given to find a *Side*.	Z S P	SP	Turn the *Angles* into Sides, and work as in the XI *Case*.	XII



APPENDIX. *Containing the laſt Eight* Caſes *of* Oblique Angled Spherical Triangles, *Reſolved at Three* Operations.

THE Eight *Caſes* beforegoing, have been reſolved at two *Operations* by the *General Method* before delivered: But the old way hath been to reſolve them at three; and then the *Catholick Propoſition* may be ſtill retained; and therefore, for the ſatisfaction of ſuch as may apprehend the foregoing *Compendious* way difficult, I will reſolve the Eight *Caſes* by the old way at three *Operations*, in which I will be very brief, but plain withal: And the *Triangle* which I will make uſe of ſhall be the *Triangle* Z S P. Whoſe *Sides*, *Angles*, and *Perpendiculars* are as is expreſſed in the *Figure*.



CASE I. *Two Sides,* Z P *and* S P, *and their contained* Angle *at* P, *given, to find the third Side* Z S.

Here the *Perpendicular* is let fall from Z, the end of the Side given Z P, and adjacent to the *Angle* given P, upon the Side S P, reſolving the *Oblique Triangle* into two *Right angled Triangles* Z B P and Z B S. In the firſt of which is given (1) Z P the *Hypotenuſe*, (2) the *Angle* at the *Baſe* P, by which you may find the *Perpendicular* Z B.

As *Radius*,
To the *Sine* of the *Hypotenuſe* Z P, 38 deg. 28 m.
So is the *Sine* of the *Angle* at 16 *Baſe* P, 56 d. 26 m.
To the *Sine* of the *Perpendicular* Z B 31 d. 13 m.

Secondly, for the *Baſe* B P.

As *Radius* 90 deg.
Is to the *Tangent* of the *Hypotenuſe* 38 d. 28 m. Z P.
So is the *Co-Sine* of the *Angle* at the *Baſe* P. 32 d. 34 m.
To the *Tangent* of the *Segment* of the *Baſe* B P, 23 d. 43 m.
Which ſubtracted from the whole *Baſe* S P 74 d. 50 m. there remains 51 de. 7 m. for the other *Segment* of the *Baſe* S B.

Thirdly, for the Side required Z S.

As *Radius*,
Is to the *Co-ſine* of the *Perpendicular* Z B 58 de. 47 m.
So is the *Co-ſine* of the *Segment* F B 48 d. 53 m.
To the *Co-ſine* of the Side required Z S, 57 de. 32 m.

Caſe

CASE II. *Two Sides* Z S *and* Z P, *and their contained* Angle *at* Z, *given, to find one of the other* Angles, *as that at* P.

In this *Case* the *Perpendicular* falls from S, upon the Side P Z, extended to B, and so is oppoſite to the *Angle* given Z, and to the *Angle* required P.

Firſt, for the *Perpendicular* S B.

As *Radius*,
Is to the *Sine* of the *Hypotenuſe* Z S, 57 d. 32 m.
So is the *Sine* of the *Angle* at G 107, 36 m. (or 72 de. 24 m.) B Z S.
To the *Sine* of B S 53 d. 32 m.

Secondly, for the *Baſe* B G.

As the *Radius*,
Is to the *Tangent* of the *Hypotenuſe* Z S 57 d. 32 m.
So is the *Co-ſine* of B Z S 17 de. 36
To the *Tangent* of B Z 25 deg. 25.
Which added to Z P, 38 deg. 28 m. the Summ will be 63 de. 53 m. for the whole Side B Z P.

Thirdly, for the *Angle* H,

As the *Radius*,
Is to the *Co-tangent* of S B 36 de. 28 m.
So is the *Sine* of B P 63 de. 53 m.
To the *Co-tangent* of P 33 de. 34 m. whoſe *Complement* 56 de. 26 m. is the *Angle* at P.

CASE III. *Two* Angles P *and* Z, *and the Side between them* Z P, *given; to find the third* Angle *at* S.

In this *Caſe*, the *Perpendicular* may fall from Z or P, as here from Z, upon the *Side* S P.

Firſt, for the *Perpendicular* B Z.

As *Radius*,
Is to the *Sine* of Z P, 38 de. 28 m.
So is the *Sine* of the *Angle* at P, 56 d. 26 m.
To the *Sine* of the Z B, 31 d. 13 m.

Secondly, for the *Angles* at the *Perpendicular*, Z S B, and B Z P.

As the *Radius*,
Is to the *Tangent* of P, 56 d. 26 m.
So is the *Co-ſine* of Z P, 51 de. 32 m.
To the *Co-tangent* of B Z P 40 de. 17 m.
This 40 deg. 17 min. ſubtracted from 107 deg. 36 min. leaves 67 de. 19 m. for the *Angle* B Z S.

Thirdly, for the *Angle* at S.

As *Radius*,
To the *Sine* of B Z S 67 de. 19 m.
So is the *Co-ſine* of B Z 58 deg. 47 m.
To the *Co-ſine* of S, 37 de. 55 m.

CASE IV. *Two* Angles, P *and* Z, *and the Side between them* Z P, *being given, to find one of the other Sides, as* Z S.

Let the *Perpendicular* fall from Z, upon the Side S P, for ſo it falls from the end of the given Side H Z, and oppoſite to its adjacent *Angle* given P; and alſo oppoſite to the Side required Z S, then

Firſt, for the *Perpendicular* Z B.

As the *Radius*,
Is to the *Sine* of the *Hypotenuſe* Z P, 38 de. 28 m.
So is the *Sine* of the *Angle* at the *Baſe* P, 56 d. 26 m.
To the *Sine* of the *Perpendicular* Z B, 31 d. 13 m.

Secondly, for the *Angle* B Z P.

As *Radius*,
To the *Tangent* of the *Angle* at P, 56 d. 26 m.
So is the *Co-ſine* of Z P, 51 d. 32 m.
To the *Co-tangent* of B Z P, 40 d. 17 m.

Which, 40 de. 17 m. ſubtracted from SZP 107 d. 36 m. the Remainder is 67 d. 19 m. for the *Angle* SZB.

Thirdly, for the Side ZS.

As the *Radius*,
Is to the *Co-tangent* of ZB, 58 d. 47 m.
So is the *Co-ſine* of BZS 22 d. 41 m.
To the *Co-tangent* of ZS 32 deg. 28 min. whoſe *Complement* is 57 de. 32 m. for the Side ZS.

CASE V. *Two Sides*, SZ *and* SP, *with the* Angle SZP, *oppoſite to one of them* SP, *being given, to find the third Side* ZP.

The *Perpendicular* is in this *Caſe*, let fall from S, upon the Side ZP extended to B.

Firſt, for the *Perpendicular* SB.

As *Radius*,
Is to the *Sine* of ZS, 57 de. 32 m.
So is the *Sine* of BZS, 72 de. 24 m. the *Complement* of the given *Angle* SZP 107 de. 36 m. to 180 deg.
To the *Sine* of the *Perpendicular* SB 53 d. 32 m.

Secondly, for the *Baſe* BZ.

As *Radius*,
To the *Tangent* of SZ, 57 d. 32 m.
So is the *Co-ſine* of SZB 17 d. 36 m.
To the *Co-tangent* of BGH 26 d. 7 m.
To the *Tangent* of BZ, 25 d. 25 m.

Thirdly, for the inquired Side ZP.

As the *Co-ſine* of DB 36 d. 28 m.
Is to the *Radius*;
So is the *Co-ſine* of SP 15 d. 10 min.
To the *Co-ſine* of the Side BZP 26 d. 7 m. whoſe *Complement* 63 de. 53 m. is the whole Side BZP, from which BZ 25 d. 25 m. being ſubtracted, there remains 38 d. 28 m. for the inquired Side, ZP.

CASE VI. *Two Sides* ZS, *and* ZP, *with the* Angle *at* P, *oppoſite to the Side* ZS, *given, to find their contained* Angle *at* Z.

Let fall the *Perpendicular* from Z, upon the Side SP. Then,

Firſt, for the *Perpendicular* ZB,

As the *Radius*,
Is to the *Sine* of the Side ZP, 38 de. 28 m.
So is the *Sine* of the *Angle* at P, 56 de. 28 m.
To the *Sine* of ZB, 31 de. 13 m.

Secondly, for the *Angle* BZP,

As *Radius*,
Is to the *Tangent* of the *Angle* at P, 56 de. 26 m.
So is the *Sine Complement* of the Side ZP, 51 de. 32 m.
To the *Co-tangent* of BZP 49 de. 43 m. whoſe *Complement* is 40 de. 7 m. for BZP.

Thirdly, for the inquired *Angle* at Z.

As *Radius*,
To the *Tangent* of ZB, 31 de. 13 m.
So is the *Co-tangent* of SP, 15 de. 10 m.
To the *Co-ſine* of BZS, 22 de. 41 m. whoſe *Complement* 67 deg. 19 m. is the *Angle* SZB, which added to 40 de. 17 m. the *Angle* BZP before found, the *Sun* will be 107 de. 36 m. for the whole *Angle* ſought SZP.

CASE VII. *Two* Angles, SZP, *and* ZPS, *with the Side* SZ *oppoſite to* P, *given, to find the third* Angle *at* S.

Let the *Perpendicular* fall from S, upon the *Line* ZP extended to B. Then,

Firſt, for the *Perpendicular* SB.

As *Radius*,
To the *Sine* of ZS, 57 de. 32 m.
So is the *Sine* of Z, 107 de. 36 m. (or 72 d. 24 m.)
To the *Sine* of SB, 53 d. 32 m.

Secondly,

Secondly, for the *Angle* B S Z,
As *Radius*,
To the *Tangent* of the *Angle* at Z, 72 d. 24 m.
So is the *Co-sine* of S Z, 32 de. 28 m.
To the *Co-tangent* of B S Z 30 d. 35 m.
Thirdly, for the inquired *Angle* at S.
As *Radius*,
To the *Co-sine* of S B, 36 d. 28 m.
So is the *Co-sine* of the *Angle* at P, 33 d. 34 m.
To the *Sine* of B S P 68 d. 29 m.
From which 68 d. 29 m. subtract B S Z 30 de. 35 m. and the Remainder will be 37 de. 54 m. for the inquired *Angle* B S Z.

CASE VIII. *Two* Angles Z P S *and* S Z P, *and the Side* S Z *opposite to the* Angle P, *being given, to find the Side between them* Z P.

In this *Case* the *Perpendicular* must fall from S, upon the Side Z P extended to B. Then,
First, for the *Perpendicular* S B.
As *Radius*,
Is to the *Sine* of the *Hypotenuse* Z S, 57 d. 32 m.
So is the *Sine* of the *Angle* B Z P, 72 de. 24 m. (the Comp. of the *Obtuse Angle* S Z P, to a *Semicircle*.)
To the *Sine* of the *Perpendicular* S B, 53 d. 32 m.
Secondly, For the *Base* B Z,
As *Radius*,
Is to the *Tangent* of the *Hypotenuse* S Z, 57 de. 32 m.
So is the *Co-sine* of B Z S, 17 de. 36 m.
To the *Tangent* of the *Base* B Z, 25 de. 25 m.
Thirdly, for the whole Side B Z P.
As the *Radius*,
Is to the *Tangent* of the *Perpendicular* B S, 53 d. 32 m.
So is the *Co-tangent* of Z P S, 33 de. 34 m.
To the *Sine* of B P, 63 de. 53 m.
From which 63 de. 53 m. subtract the *Segment* of the Side B Z 25 de. 25 m. the Remainder will be 38 de. 28 m. for the inquired Side Z P.
Thus have you the 3, 4, 5, 6, 7, 8, 9, and 10*th Cases* of *Oblique angled Spherical Triangles* Resolved at three Operations, which to some may give better satisfaction, it being more natural, and not so burthensome to the Memory; for that the *Catholick Proposition* does the whole Work.

The End of the First Part.

GEOMETRICAL TRIGONOMETRY.

PART II.

The Doctrine of *Plain* and *Spherical Triangles*, *Geometrically* performed, by *Scale* and *Compass*.

OF the Doctrine of the *Dimension* of *Plain* and *Spherical Triangles*, we have already largely treated, and how the same is to be performed by the *Logarithms*, and *Canons* of *Sines* and *Tangents*. I shall now (in this little Tractate) shew how the same may be performed by the *Scales* upon the Ruler before described. Book III. Part II. hereof.

CHAP. I.

Of Plain, *or* Right Lined Triangles.

IN the resolving of *Right-Lined Plain Triangles*, if they be *Right-angled*, I call the Side opposite to the *Right Angle* the *Hypotenuse*; and of the other Sides, comprehending the *Right Angle*, I call the longer of them the *Base*, and the shorter the *Perpendicular*. But in the Solution of *Oblique angled plain Triangles*, I call the longest Side of the *Triangle* the *Base*; and the other two the two Sides, without any other distinction of Denomination or Name. These things being premised, I come now to the Solution of *plain Triangles*, both *right* and *Oblique-angled*. And,

I. *Of* Right Angled Plain Triangles.

Fig. I. The *Triangle* which I shall make use of in the several *Cases* belonging to a *Right angled plain Triangle* shall be this following, CAB, in which

		parts.
AB, the *Base*,	contains	180
CA, the *Perpendicular*,		135
CB, the *Hypotenuse*,		225
And		deg. min.
A, the *Right Angle*,	contains	90 — 00
C, the *Angle* at the Per.		53 — 07
B, the *Angle* at the *Base*,		36 — 53

CASE I. *The* Base BA 180, *and the* Perpendicular CA 135, *being given, to find the* Angles B *and* C.

Draw a *Line* AB, and from your *Scale* of equal parts take 180, and set them from A to B; then on the Point A raise the *Perpendicular* AC, and because it contains 135, take 135 parts from your *Scale* of equal parts, and set them from A to C; then draw the *Line* CB; which three *Lines* will constitute the *Triangle* CAB.

Now to find the *Angles* C and B, take in your Compasses 60 degrees of your *Line* of *Chords*, and setting one foot in C, describe the *Arch f g*; also setting one foot in B, describe the *Arch d e*: then take in your Compasses the distance from *f* to *g*, which measured upon your *Line* of *Chords*, will reach from the beginning thereof to 53 deg. 7 min. and such is the quantity of the *Angle* at C. In like manner take the distance between *d* and *e* in your Compasses, that distance applied to your *Line* of *Chords*, will reach to 36 deg. 53 min. — *Or, when you had found the quantity of the* Angle C *to be* 53 *deg.* 7 *min. if you had subtracted that from* 180 *degrees, the remainder would have been* 36 *deg.* 53 *min. the quantity of the* Angle *at* B, *without drawing of the* Arch *d e, and measuring it upon your* Chord.

CASE II. *The* Hypotenuse CB 225, *and the* Base AB 180, *being given, to find the* Angles B *and* C.

Draw a *Right Line* AB containing 180 parts of your *Scale* of equal parts, also out of the same *Scale* of equal parts take 225, your *Hypotenuse*, and setting one foot of your Compasses in B, with the other describe the obscure *Arch h k*; then on the Point A raise the *Perpendicular* AC, which will cut the obscure *Arch h k* in C; then draw the *Line* CB, so have you the *Triangle* CAB; then may you measure the quantity of the *Angles* at C and B as in the last *Case*. And so will C be 53 deg. 7 min. and B 36 deg. 53 min.

CASE III. *The* Base AB 180, *the* Angle C 53 *deg.* 7 *min. and the* Angle B 36 *deg.* 53 *min. being given, to find the* Perpendicular CA.

Draw a *Right Line* AB containing 180 parts of your *Scale*, for the *Base* of your *Triangle*; then taking 60 degrees from your *Line* of *Chords*, on the Point B describe the

the *Arch d e*, and (becauſe the *Angle* at B contains 36 deg. 53 min.) take 36 degrees 53 min. from your *Chord*, and ſet it from *d* to *e*, and from B, through the Point *e*, draw the *Line* B C. Alſo upon the Point A erect the *Perpendicular* A C, croſſing the *Line* B C in C. So have you formed the *Triangle* C A B. Laſtly, take the length of the *Line* A C in your Compaſſes, and meaſuring it upon your *Line* of equal parts, you ſhall find it to contain 135. And that is the length of the *Perpendicular* C A.

CASE IV. *The* Hypotenuſe C B 225, *the* Angle C 53 *deg.* 7 *min. and the* Angle *at* B 36 *deg.* 53 *min. given, to find the* Baſe B A, *and the* Perpendicular C A.

Draw a *Right Line* C B containing 225 of your *Line* of equal parts; then taking 60 degrees, out of your *Line* of *Chords*, ſet one foot of the Compaſſes in B, and with the other deſcribe the *Arch e d*; alſo (the Compaſſes continuing at the ſame diſtance) place one foot in C, and with the other deſcribe the *Arch g f*. Then from the Point B, and through the Point *d*, draw a *Right Line*; alſo from the Point C, and through the Point *f*, draw another *Right Line*; theſe two *Lines* will interſect or croſs each other in the Point A, forming the *Triangle* C A B. Laſtly, take the *Line* A B in your Compaſſes, and applying it to your *Scale* of equal parts, you ſhall find it to contain 180; and that is the length of the *Baſe* A B. Likewiſe A C being taken in the Compaſſes, and meaſured upon the *Line* of equal parts, will be found to contain 135, which is the length of the *Perpendicular* C A.

CASE V. *The* Hypotenuſe C B 225, *and the* Baſe A B 180, *being given, to find the* Perpendicular C A.

Draw a *Right Line* A B, containing 180 of your *Scale* of equal parts, and upon the end A erect a *Perpendicular* A C. Then take out of your *Scale* of equal parts 225, (the length of your *Hypotenuſe* given,) and ſetting one foot of the Compaſſes in B, with the other deſcribe the *Arch h k*, cutting the *Perpendicular* A C in C, then draw the *Line* C B; ſo have you conſtituted the *Triangle* C A B. Laſtly, take in your Compaſſes the length of the *Line* A C, and apply it to your *Line* of equal parts, where you ſhall find that it will contain 135; and that is the length of the *Perpendicular* C A.

CASE VI. *The* Baſe A B 180, *the* Angle C 53 *deg.* 7 *min. and the* Angle B 36 *deg.* 53 *min. being given, to find the* Hypotenuſe C B.

Draw a *Right Line* A B, containing 180 parts of your *Scale*, for the *Baſe* of your *Triangle*, and on the end A erect a *Perpendicular* A C. Then take 60 deg. out of your *Line* of *Chords*, and upon the Point B, with that diſtance, deſcribe the *Arch d e*; and (becauſe the *Angle* at B is 36 deg. 53 min.) take 36 deg. 53 min. from your *Line* of *Chords*, and ſet it upon the *Arch* from *d* to *e*. Then from B, through the Point *e*, draw a *Right Line*, till it meet with the *Perpendicular* before drawn, which it will do in the Point C. And thus have you protracted your *Triangle* C A B. Laſtly, take in your Compaſſes the length of the *Hypotenuſe* C B, and meaſure it upon your *Scale* of equal parts, and you ſhall find it to contain 225.

CASE VII. *The* Baſe A B 180, *and the* Perpendicular C A 135, *being given, to find the* Hypotenuſe C A.

Draw a *Right Line* A B, containing 180 equal parts, and upon the end A erect the *Perpendicular* A C, and out of your *Scale* of equal parts take the length thereof 135, which ſet from A to C, and draw the *Line* C B, which conſtitutes the *Triangle* C A B. Laſtly, take the length of the *Hypotenuſe* C B in your Compaſſes, and meaſuring it upon your *Line* of equal parts, you ſhall find it to contain 225.

Theſe are the ſeveral Varieties or Caſes that can at any time fall out in the Solution of Right angled Plain Triangles, *wherefore we will now proceed to the Solution of* Oblique Plain Triangles.

II. *Of* Oblique Angled Plain Triangles.

THE *Triangle* which I shall make use of in the Solution of the several *Cases* appertaining to an *Oblique angled plain Triangle*, shall be this following, C D B, in which

		parts
D B, the *Base*,	contains	335
C B, the longer Side,		271
D C, the shorter Side,		100
And		deg. m.
C, the *Obtuse Angle*,	contain	122—00
D, B, the 2 *Acute Angles*,		43—20 / 14—40

CASE I. *Two* Sides, *as the* Base D B 335, *and the* Side C B 271, *and the* Angle D 43 *deg.* 20 *min. opposite to* C B, *to find the* Angle *at* C, *opposite to the* Base D B.

Draw a *Right Line* D B representing the *Base* of your *Triangle*, which, by help of your *Scale* of equal parts, make to contain 335. Then upon the Point D, with the distance of 60 degrees of your *Line* of *Chords*, describe the *Arch k l*, and from your *Chords* take 43 deg. 20 min. the quantity of the *Angle* at D, and set it upon the *Arch-line* from *l* to *k*, drawing the *Line* C D. And because your other given Side B C contains 271 parts, take 271 out of your *Line* of equal parts, and setting one foot in B, with the other describe the *Arch m n*, crossing the former *Arch k l* in the Point C; then draw the *Line* C B. So shall you have constituted the *Triangle* C D B. Lastly, because it is the *Angle* at C that is required, take 60 degrees of your *Chords*, and upon C describe the *Arch g h*, and taking the distance between *g* and *h*, apply it to your *Line* of *Chords*, and you shall find it to reach from the beginning thereof beyond the end of the *Line*; wherefore take 90 degrees the whole *Line*, and set that distance from *g* to *o*; then take the remainder of the *Arch o h*, and measure that upon your *Chord*, and you shall find it to contain 32 degrees, which added to 90 deg. make 122 degrees, and that is the quantity of the *Angle* at C, which was required.

CASE II. *The* Base D B 335, *and the* Side D C 100, *with the* Angle D, 43 *deg.* 20 *min. contained between them, to find either of the other* Angles *at* B *and* C.

Draw a *Right Line*, as D B, containing 335 of your *Scale* of equal parts, which shall be the *Base* of your *Triangle*. Then with 60 degrees of your *Line* of *Chords*, upon the Point D describe the *Arch k l*; and because the given *Angle* at D contains 43 deg. 20 min. take 43 deg. 20 min. from your *Line* of *Chords*, and set it from *l* to *k*, drawing the *Line* D *k*. Again, because the given Side D C contains 100, set 100 of your *Line* of equal parts from D to C; then drawing a *Right Line* from C to B, you shall by that means find the *Oblique angled Triangle* C D B. Lastly, being the other two *Angles* at B and C are to be found, with 60 degrees of your *Chord* on the Point B describe the *Arch e f*; also upon the Point C describe the *Arch g o h*. Then if you take the distance between *e* and *f* in your Compasses, and measure it upon your *Line* of *Chords*, you shall find it to contain 14 deg. 40 min. And that is the quantity of the *Angle* at B. Then being the *Angle* at C, which is also required, is *Obtuse*, and contains above 90 deg. take 90 deg. out of your *Line* of *Chords*, and set that distance upon the *Arch g h*, from *g* to *o*: and taking the other part of the *Arch o h* in your Compasses, measure that upon your *Chord*, and you shall find it to contain 32 deg. which added to 90 deg. makes in all 122 deg. And such is the quantity of the other inquired *Angle* at C.

CASE III. *The three* Sides, D B 335, C B 271, *and* D C 100, *being given, to find any of the* Angles, *as* B.

Draw a *Right Line* C D, containing 100 of your *Line* of equal parts. Then the other Side of the *Triangle* being 271, take 271 out of your *Scale* of equal parts, and setting

setting one foot of the Compasses in C, with the other describe the *Arch r s*. Also take the length of your *Base* 335 out of your *Line* of equal parts, and setting one foot of the Compasses in D, with the other describe the *Arch p q*, crossing the former *Arch r s* in the Point B. Then draw the *Lines* C B and D B, and they shall form your *Triangle* C D B. Lastly, because the *Angle* at B is sought, take 60 degrees of your *Line* of *Chords*, and setting one foot of the Compasses in B, with the other describe the *Arch e f*. Then take the distance between *e* and *f*, and measure it upon the *Line* of *Chords*, and you shall find it to contain 14 deg. 40 min. which is the quantity of the *Angle* at B. And in the same manner might any of the other *Angles* at C or D have been found.

CASE IV. *The three* Angles, C 122 *deg.* D 43 *deg.* 20 *min. and* B 14 *d.* 40 *min. being given, to find any of the* Sides, *as* B C.

In this *Case*, where the three *Angles* are given, and none of the Sides, you are to take notice, that the Sides cannot be absolutely found themselves, but the Proportionality of them. Wherefore,

Draw a *Line*, as D B, of any length, and taking 60 degrees of your *Line* of *Chords*, set one foot of the Compasses upon D, and with the other describe the *Arch l k*; also set one foot in B, and with the other describe the *Arch e f*. Then, because the *Angle* at D contains 43 deg. 20 min. take 43 deg. 20 min. from your *Line* of *Chords*, and set them from *l* to *k*. Also the *Angle* at B being 14 deg. 40 min. take them likewise from your *Line* of *Chords*, and set them from *e* to *f*. This done, draw the *Lines* B *f* and D *k*, extending them till they meet one with another, which they will do in the Point C. So have you constituted the *Triangle* C D B, the Sides whereof will be in proportion the one to the other as the Sides of this *Triangle* are.

CASE V. *The two* Sides D C 100, *and* C B 271, *with the* Angle *at* C 122 *degrees, being given, to find the* Base D B.

Draw a *Right Line*, as C B, containing 271 of your *Line* of equal parts, and on the Point C, with 60 degrees of your *Line* of *Chords*, describe the *Arch g o h*. Then, because the given *Angle* C contains 122 deg. it being 32 deg. above 90 deg. first take 90 degrees from off your *Line* of *Chords*, and set it upon the *Arch* from *g* to *o*; and then take 32 degrees from your *Chord* also, and set them upon the same *Arch* from *o* to *h*, and draw the *Line* C *h*; then take 100, the length of the other given Side, and set it from C to D, and draw the *Line* D B, which will contain 335 of your Scale of equal parts. And that is the length of the *Base* D B, which was required.

THE *Solution* of SPHERICAL TRIANGLES by *Projection*.

IN the Resolving of *Spherical Triangles* by the *Canons* of *Sines* and *Tangents*, as is before taught, in the First Part, there are 28 *Cases*, viz. *Sixteen* for the resolving of *Right Angled*; and *Twelve* for *Oblique Angled*: All which (by this *Projective* way) will be reduced to *Eleven*; for the 16 *Cases* of *Right Angled*, will be comprehended in *Five*; and the 12 of *Oblique Angled* in *Six*.

This way of resolving *Spherical Triangles* by *Projection*, will give great light to the *Calculation* of *Spherical Triangles* by the *Canons* of *Sines* and *Tangents*, or other *Instruments* for that purpose. For here you shall lay down the *Triangle* that your *Question* is to be resolved by, very exactly; and shall also *measure* the *Sides* and *Angles* thereof upon the *Scheme* of your *Projection*, as easie as if they were *Streight Lines*, by help of the *Scales* of *Chords*, *Sines*, *Tangents*, *Secants*, and *Half Tangents* upon the *Ruler* (or *Sector* rather) before described. I shall give you *Examples* in all the *Cases* both of *Right* and *Oblique Triangles*: And here you are to note, that divers *Arches* of Great *Circles*, promiscuously drawn within one *Primitive Circle* of a *Sphere*, being either in a *Right* or *Oblique Position*, do by their Intersections one with another, constitute several *Spherical Triangles*; some *Right angled*, some *Quadrantal*, and others *Oblique angled*: All which *Circles* have their respective *Centres* and *Poles*; which two things must be first found, before the Great *Circles* (which constitute the *Triangle* that concerns your *Question*) can be described; or the *Sides* or *Angles* of it *measured*: And therefore I shall shew,

I. *How*

I. *How within a* Primitive Circle *given, to describe the* Arch *of another Great* Circle, *which shall make an* Angle *with the* Primitive Circle *equal to any number of Degrees and Minutes given.*

Fig. III. Let the *Primitive Circle* ZHNO represent the *Meridian*, and HO the *Horizon* of any Place, (suppose in the *Latitude* of 65 deg. 40 min.) in which *Latitude* I have given,

1. The *Latitude* 65 deg. 40 min.

2. The *Hour* from *Midnight*, or from the *North* part of the *Meridian* 47 deg. 29 m. or 3 hours 10 min.

And let the *Sun's Amplitude* at his *Rising* and *Setting* be required.

The *Primitive Circle* being drawn, it will represent the *Meridian* of the Place, and *Solsticial Colure*; and crossed at *Right Angles* with the *Diameters* ZN and HO; Z will represent the *Zenith* of the Place, N the *Nadir*, HO the *Horizon*, H the *South*, and O the *North* part thereof; and forasmuch as the *Latitude* is 65 deg. 40 m. take it out of the *Scale* of *Chords*, and set it from O to P, from H to S, from Z to Æ, and from N to *æ*, and draw the *Line* PS for the *Axis* of the *World*, P representing the *North*, and S the *South Pole*, and the *Line* Æ *æ* the *Equinoctial Circle*; and ZDN the *Equinoctial Colure*, the *Prime Vertical Circle*, or *Azimuth* of *East* and *West*.

Then the *Hour* from Midnight, being 47 deg. 29 min. take 42 deg. 31 min. (the *Complement* thereof,) out of the *Scale* of *Half Tangents*, and set them from D to M; so have you three Points, PM and S, through which the *Hour Circle* of 3 hours 10 m. or, 47 d. 29 m. from the *North* part of the *Meridian* must pass.

II. *To find the* Centre *of any Great* Circle, *so described.*

The *Hour Circle* to be described PMS being 47 de. 29 min. from the *Meridian*, take 47 de. 29 m. out of the *Scale* of *Secants*, and set them upon the *Equinoctial Circle* (extended if need be) from M to A, and it will fall near unto Æ, so shall A be the *Centre* of the *Hour Circle* PMS. —— Or, the *Tangent* of 47 de. 29 m. set from D will reach, upon the *Equinoctial* to A, the *Centre* as before.

Note, That the *Centres* of all *hour Circles* will fall in the *Equinoctial Circle*, (extended if need be.

III. *To find the* Pole *of any Great* Circle *described within a* Primitive Circle.

As the *Centres* of all *Hour Circles* are in the *Equinoctial Circle*, (extended if need require,) so the *Poles* of them also are in the same *Circle*; but as the *Centres* fall sometimes within, and sometimes without the *Primitive Circle*, the *Poles* always fall within; and the *Pole* of every Great *Circle* is 90 deg. or a *Quadrant* of a *Circle*, distant from the *Circle* it self; and may be thus found,

Example. Let it be required to find the Pole *of the* Hour Circle PMS.

Lay a *Ruler* from P to M, and it will cut the *Primitive Circle* in *e*, take 90 deg. of the *Chords*, and set them from *e* to *d*, so a *Ruler* laid from P to D, will cut the *Equinoctial Circle* in F; so shall F be the *Pole* of the *Hour Circle* PMS. —— Or the *Half Tangent* of 42 de. 31 m. set from D upon the *Equinoctial Circle*, will reach to F the *Pole* of the *Hour Circle* PMS also. And here,

Note 1. That the *Primitive Circle* it self is a Great *Circle*, and the *Pole* thereof is O the *Centre*.

Note 2. That the *Streight Lines* ZN, PS, HO and Æ *æ* are Great *Circles* of the same *Sphere*, and the *Poles* of them (being 90 de. distant from the *Circles* themselves) are always in the *Periphery* of the *Primitive Circle*, in that Point, where a *Line* drawn from the *Centre*, at *Right Angles* to the *Streight Line* representing the Great *Circle*; As,

The *Poles* of the *Circle*
- ZN; The *Prime Vertical Circle*, are H and O, the *South* and *North* Points of the *Horizon*.
- PS, The *Axis* of the World, are Æ and *æ*, the *South* and *North* Points of the *Equinoctial*.
- Æ *æ*, The *Equinoctial Circle*, are P and S, the *North* and *South Poles*.

By the Intersections of these three Great *Circles*, *viz.* ZHNO the *Meridian*; PMS an *Hour Circle*, and HDO the *Horizon*, there is constituted the *Spherical Triangle* POR, *Right angled* at O; which *Triangle* being thus constituted, the next thing to be done, is to shew how,

IV. *To*

IV. *To measure the* Sides *and* Angles *of a* Spherical Triangle, *Projected within a* Primitive Circle.

1. *For the* Sides.

Let it be required to measure the Sides of the *Spherical Triangle* P O R.

1. For the Side P O:—— Take in your Compasses the distance from O to P, which distance measured upon the *Scale* of *Chords* you shall find to contain 65 de. 40 m. and that is the quantity of the Side P O.

2. For the Side R O: —— R O upon the *Horizon* is the *Measure* of that Side of which D R is the *Complement*; wherefore D R taken in your Compasses, and applyed to the *Scale* of *Half Tangents*, will reach to 45 deg. 11 min. whose *Complement*, 44 d. 49 min. is the quantity of the Side R O.

3. For the Side P R:—— the Side P R is measured upon the *Primitive Circle*, by laying a *Ruler* to the *Pole* thereof at F, and the extreme Points of the Side; so a *Ruler* laid to F, (the *Pole* of the *Hour Circle* P R S) to P, it will cut the *Primitive Circle* in P; and laid from F to R, it will cut the same *Circle* in *f*; the distance P *f* measured upon the *Scale* of *Chords*, will be found to be 73 degrees, which is the quantity of the Side P R. And here,

Note 1. That all *Arches* of the *Primitive Circle*, are measured by the *Scale* of *Chords*.

Note 2. That all *Circles* represented by *Streight Lines* in the *Projection*, are measured by the *Scale* of *Half Tangents*.

Note 3. That all *Oblique Arches*, as *Hour Circles*, *Azimuth Circles*, *&c.* are measured from their *Poles* to the *Primitive Circle*, and the distance from their extreme ends measured upon the *Scale of Chords* will give their *Quantities*.

Note 4. That the *Centres* of all Great *Circles* are found by setting the *Tangents* of their *Quantities* from the *Centre* of the *Primitive Circle*; or the *Secants* of their *Quantities* from the *Circles* themselves, upon that *Streight Line* which cutteth the Great *Circular Arch* at *Right Angles*.

Thus having shewed how a *Spherical Triangle* is *Projected*, and how the *Sides* and *Angles* thereof are measured; it resteth now to shew how the Five several *Cases*, and the variety of *Questions* that will arise out of every *Spherical Triangle*, are to be resolved.

Therefore in the *Right angled Spherical Triangle* P O R.

The *Side* { P O is the *Latitude* of the Place.
R O is the *Sun's Amplitude* from the *North*.
P R is the *Sun's* distance from the *Pole*, or the *Complement* of his *Declination*.

The *Angle* { R P O is the *Hour* from *Midnight*, or from the *North* part of the *Meridian*.
P R O is the *Angle* of the *Sun's Position* at the time of the *Question*.
P O R is the *Right Angle*.

The Parts of the Triangle *being declared, and of what* Circles *of the* Sphere *the* Sides *do consist; I will now come to the* Cases, *which are* 5 *in every* Right angled Triangle. *So that any two parts of the* Triangle *(besides the* Right Angle*) being given, I will shew in every of the* 5 Cases *what parts may be found.*

In a Right Angled Spherical Triangle.

CASE I. *The* Base R O *and* Perpendicular P O *being given, to find the other parts of the* Triangle.

In the *Triangle* P R O here is given P O, the *Latitude*, 65 de. 40 min. and R O, the *Amplitude* from the *North* part of the *Meridian*; 47 deg. 49 min. by which you may find,

3. { 1. P R, the *Complement* of the *Sun's Declination*, 73 deg.
2. R P O, the *Hour* from *Midnight*, 47 deg. 29 m.
3. P R O, the *Angle* of the *Sun's Position*, 72 de. 20 m.

Case

CASE II. *The* Hypotenuse PR *and* Perpendicular PO *being given, to find the other parts.*

In the *Triangle* PRO here is given PO, the *Latitude*, and PR, the *Complement* of the *Sun's Declination*; by which may be found,

1. RO, the *Amplitude* from the *North*, 44 d. 49 m.
2. OPR, the *Hour* from *Midnight*, 47 d. 29 m.
3. PRO, the *Angle* of the *Sun's Position*, 72 d. 20 min.

And if instead of the *Perpendicular* there had been given the *Base* RO, you might then find, 6.

4. PO, the *Latitude*.
5. RPO, the *Hour* from *Midnight*.
6. PRO, the *Angle* of the *Sun's Position*.

CASE III. *The* Hypotenuse PR *and one* Angle PRO, *or* RPO *being given, to find the other Parts.*

Here is given in the *Triangle* PRO, the *Complement* of the *Sun's Declination*, PR, and RPO, the *Hour* from *Midnight*; by which may be found,

1. RO, the *Amplitude* from the *North*.
2. PO, the *Latitude*.
3. PRO, the *Angle*, of the *Sun's Position*.

And if in lieu of the *Angle* at P, the *Angle* at R had been taken, then you might have found, 6.

4. RO, the *Amplitude* from the *North*.
5. PO, the *Latitude*.
6. RPO, the *Hour* from *Midnight*.

CASE IV. *The* Perpendicular PO, *or* Base RO, *and either of the* Angles RPO *or* PRO, *given, to find the other Parts.*

In the *Triangle* PRO let there be given the *Amplitude*, RO, and the *Angle* of the *Sun's Position*, PRO; by which you may find,

1. PO, the *Latitude*.
2. RPO, the *Hour* from *Midnight*.
3. PR, the distance of the *Sun* from the *Pole*.

But if the Side given had been RO, and the *Angle* given RPO, then you might have found,

4. PR, the *Complement* of the *Sun's Declination*.
5. PO, the *Latitude*.
6. PRO the *Angle* of the *Sun's Position*.

But again, if RO and RPO had been given, then might be found, 12

7. RP, the *Complement* of the *Sun's Declination*.
8. RO, the *Amplitude* from the *North*.
9. PRO, the *Angle* of the *Sun's Position*.

And again, if PO and ORP had been given, we might then have also found,

10. RP, the distance of the *Sun* from the *Pole*.
11. RO, the *Amplitude* from the *North*.
12. RPO, the *Hour* from *Midnight*.

CASE V. *The* Angles PRO, OPR, *being given, to find the other Parts.*

If the two *Angles* PRO and OPR be given, there may be found, 3.

1. PO, the *Latitude*.
2. RO, the *Amplitude* from the *North*.
3. PR, the *Complement* of the *Sun's Declination*.

Thus you see, that in this one Right angled Spherical Triangle, *by the several Parts given in these* Five Cases, *there are* 30 Spherical Problems *resolved*; *and so many are resolvable in every* Right angled Spherical Triangle.

And

And thus have I done with a *Right angled Spherical Triangle*, I ſhall now uſe the ſame method in the Projecting and meaſuring of the *Sides* and *Angles* of an *Oblique angled Spherical Triangle*; and the variety of *Queſtions* reſolvable thereby.

Suppoſe therefore, in the ſame *Latitude* of 65 deg. 40 min. there were given,

1. The *Latitude* 65 de. 40 m. or its *Complement* 24 deg. 20 min.

2. The *Hour* from *Noon* 30 de. or 2 *hours*.

3. The *Sun's Azimuth* from the *South* 37 de. 3 min. and let the *Sun's Altitude* be required.

I. *How to project the* Triangle, *and to find the* Poles *and* Centres *of the ſeveral* Circles.

1. The *Latitude* given being 65 deg. 40 m. which ſet from O to P, the *Complement* thereof is P Z 24 deg. 20 min.

2. The *Hour* from *Noon* being 30 de. or 2 *hours*, take 60 de. (the *Complement* thereof) out of the *Half Tangent Scale*, and ſet them upon the *Equinoctial Circle* from D to L; ſo have you three Points P L and S, whereby to deſcribe the *Hour Circle* of 30 de. from the *Meridian*, (which is the *Hour Circle* of 10 or 2 of the Clock,) and to find the *Centre* thereof, take the *Tangent* of 30 de. and ſet it (upon the *Equinoctial Circle*) from D; or the *Secant* of 30 de. and ſet it from L, either of which will reach to B, the *Centre* of the *Hour Circle* P L S: —— Alſo, take the *Half Tangent* of 30 deg. and ſet it upon the *Equinoctial Circle* from D to E, ſo ſhall E be the *Pole* of the *Hour Circle* P L S.

3. The *Sun's Azimuth* from the *South* being 37 de. 3 min. take the *Complement* thereof 52 deg. 57 m. out of the *Half Tangent Scale*, and ſet them upon the *Horizontal Circle* from D to M, ſo have you three Points Z, M, and N, whereby to deſcribe the *Vertical* (or *Azimuth*) *Circle* Z M N: And to find the *Centre* thereof, take 37 deg. 3 m. from the *Tangent Scale*, and ſet them from D, or the *Secant* of 37 deg. 3 m. and ſet them from M (upon the *Horizontal Circle*,) either of which will terminate in G, which is the *Centre* of the *Vertical* (or *Azimuth*) *Circle* Z M N. —— And to find the *Pole* thereof, take the *Half Tangent* of 37 de. 3 min. and ſet them upon the *Horizon* from D to C; ſo ſhall C be the *Pole* of the *Vertical* (or *Azimuth*) *Circle* Z M N.

Now by the *Arch* of the *Primitive Circle* Z P, the *Arch* of the *Hour Circle* P L N, interſecting the *Azimuth Circle* Z M N in ⊙, there is conſtituted the *Oblique angled Spherical Triangle* Z ⊙ P. The *Triangle* being *Projected*, let us now proceed,

I. *To* Meaſure *the* Sides *of the* Triangle Z ⊙ P.

1. For the Side Z P: — A *Ruler* laid to D, the *Pole* of the *Primitive Circle*, will cut the *Primitive Circle* in Z and P; ſo Z P meaſured upon the *Scale* of *Chords* will give 24 deg. 20 m. for the Side Z P.

2. For the Side Z ⊙: —— A *Ruler* laid to C (the *Pole* of the *Vertical* (or *Azimuth*) *Circle* Z F N) to Z, will cut the *Primitive Circle* in Z, and laid from C (the *Pole*) to ⊙, it will cut the *Primitive Circle* in *h*; ſo *h* Z meaſured upon the *Scale* of *Chords*, will give 52 deg. 34 min. for the Side Z ⊙.

3. For the Side P ⊙: —— A *Ruler* laid from E, (the *Pole* of the *Hour Circle* P L S) to P, will cut the *Primitive Circle* in P, and a *Ruler* laid from E to ⊙, will cut the *Primitive Circle* in *k*. And *k* P meaſured upon the *Scale* of *Chords*, ſhall give 73 de. for the Side ⊙ P. And thus are the three Sides of the *Spherical Triangle* Z ⊙ P, meaſured.

II. *To meaſure the* Angles *of the* Triangle Z ⊙ P.

1. For the *Angle* Z P ⊙: —— The *Meaſure* of the *Angle* Z P ⊙, is the *Arch* of the *Equinoctial Circle* Æ L, the *Complement* whereof is L D, wherefore take the diſtance L D, and meaſure it upon the *Scale* of *Half Tangents*, and you will find it to contain 60 deg. the *Complement* whereof 30 deg. is the *Meaſure* of the *Angle* Z P ⊙.

2. For the *Angle* ⊙ Z P: — The *Angle* ⊙ Z P being an *Obtuſe Angle*, contains above 90 degrees, and the *Meaſure* of it is counted upon the *Horizontal Circle* from M to O; wherefore D O being 90 deg. of the *Half Tangents*, D M muſt be the ſupplement of the *Angle* ⊙ Z P above 90 deg. therefore take D M in your *Compaſſes*, and *meaſure* it upon the *Half Tangent Scale*, and you ſhall find it to contain 52 deg. 17 m. which added to 90 deg. make 142 deg. 57 m. for the quantity of the *Angle* ⊙ Z P.

3. For the *Angle* Z ⊙ P, to which the *Angle* S ⊙ N is equal; — *Meaſure* 90 degrees upon the *Azimuth Circle* Z M N from ⊙ to Q, and 90 deg. upon the *Hour Circle* P L S, from ⊙ to T; which to perform doe thus: A *Ruler* laid from C (the *Pole* of the

the *Azimuth Circle*) to ⊙, will cut the *Primitive Circle* in *h*, and 90 deg. set upon the *Primitive Circle* from *h*, will reach to *n*; a *Ruler* laid from C to *n* will cut the *Azimuth Circle* in Q; so is ⊙ Q 90 deg. Also a *Ruler* laid from E, (the *Pole* of the *Hour Circle* P L S) to ⊙, will cut the *Primitive Circle* in *l*, and 90 deg. set from *l* upon the *Primitive Circle*, will reach to *m*; and a *Ruler* laid from E to *m*, will cut the *Hour Circle* P L S in T; so is ⊙ T 90 deg This done; a *Ruler* laid from ⊙ to Q, will cut the *Primitive Circle* in *o*, and a *Ruler* laid from ⊙ to T, will cut the *Primitive Circle* in *q*, and the distance *o q* measured upon the *Scale of Chords* will be found to be 15 deg. 3 m. for the quantity of the *Angle* S ⊙ N, to which the *Angle* Z ⊙ P is equal.

Having thus plainly exhibited the manner how an *Oblique angled Spherical Triangle* is to be *projected*, and the *Sides* and *Angles* thereof *measured*, it resteth now, that I shew how the *Six* several *Cases*, and the Variety of *Questions* that will arise out of every *Oblique angled Spherical Triangle*, are to be Resolved.

Therefore,

In the Oblique angled Spherical Triangle Z ⊙ P.

The *Side*
- Z P is the *Complement* of the *Latitude*, 24 deg. 20 m.
- P ⊙ is the *Complement* of the *Sun's Declination*, or his distance from the *North Pole*, 73 degrees.
- Z ⊙ is the *Complement* of the *Sun's Altitude*, 52 deg. 34 m.

The *Angle*
- ⊙ Z P is the *Sun's Azimuth* from the *North* part of the *Meridian*, 142 degrees 57 min.
- Z P ⊙ is the *Hour* from Noon, 30 deg. or 2 *Hours*.
- Z ⊙ P is the *Angle* of the *Sun's Position*, 15 deg. 3 min.

CASE I. *The three Sides* Z P, Z ⊙ *and* ⊙ P *being given, to find an* Angle.

In the *Triangle* Z ⊙ P if there be given the Side ⊙ Z, the *Complement* of the *Sun's Altitude*, Z P, the *Complement* of the *Latitude*, and ⊙ P, the *Sun's* distance from the *Pole*, or the *Complement* of his *Declination*, you may find,

3.
1. ⊙ Z P, the *Sun's Azimuth* from the *North*.
2. Z ⊙ P, the *Angle* of the *Sun's Position*.
3. Z P ⊙, the *Hour* from *Noon*.

CASE II. *Two Sides* ⊙ Z *and* Z P, *and the* Angle *comprehended by them* ⊙ Z P *being given, to find the other Parts of the* Triangle.

If in the *Triangle* Z ⊙ P there be given the Side ⊙ Z and Z P, and the *Angle* between them ⊙ Z P, you may find,

9.
1. Z ⊙ P, the *Angle* of the *Sun's Position*.
2. Z P ⊙, the *Hour* from the *South*, or *Noon*.
3. ⊙ P, the *Sun's* distance from the *Pole*.

But if the Sides Z P and P ⊙, and the *Angle* Z P ⊙ between them, had been given, then might have been found,

4. P ⊙ Z, the *Angle* of the *Sun's* Position.
5. ⊙ Z, the *Complement* of the *Sun's Altitude*.
6. ⊙ Z P, the *Azimuth* of the *Sun* from the *North*.

And if the Sides Z ⊙ and P E, with the *Angle* Z ⊙ P contained by them, had been given, there might be found,

7. ⊙ Z P, the *Sun's Azimuth* from the *North*.
8. Z P, the *Complement* of the *Latitude*.
9. Z P ⊙ the *Hour* from *Noon*.

CASE III. *Two* Angles ⊙ Z P, *and* Z P ⊙, *and a* Side *contained by them* Z P, *being given, to find the other Parts.*

If in the *Triangle* Z ⊙ P the *Angle* ⊙ Z P and the *Angle* Z P ⊙, with the Side contained between them, Z P, be given, we may find,

3.
1. Z ⊙, the *Complement* of the *Sun's Altitude*.
2. Z ⊙ P, the *Angle* of the *Sun's Position*.
3. ⊙ P, the *Complement* of the *Sun's Declination*.

But if the Side ⊙ P, and the *Angles* Z ⊙ P and Z P ⊙, had been given, then might be found,

4. ⊙ Z,

4. ⊙Z, the *Complement* of the *Sun's Altitude.*
5. ⊙ZP, the *Azimuth* from the *North.*
6. ZP, the *Complement* of the *Latitude.*

And if the Side Z⊙, and the *Angles* PZ⊙ and P⊙Z, had been given, then you might find,

7. ZP, the *Complement* of the *Latitude.*
8. ZP⊙, the *Hour* from *Noon.*
9. P⊙, the *Sun's* distance from the *Pole.*

CASE IV. *Two* Sides, ZP and ⊙P, *with an* Angle *oppoſite to one of them* Z⊙P, *being given, to find the other Parts.*

If there be given the Side ZP, the Side ⊙P, and the *Angle* Z⊙P, there may be found,

1. ⊙Z, the *Complement* of the *Sun's Altitude.*
2. ⊙ZP, the *Sun's Azimuth* from the *North.*
3. ZP⊙, the *Hour* from the *South.*

But if the Side ZP, and the Side ⊙P, with the *Angle* ⊙ZP, had been given, then might be found,

4. ⊙Z, the *Complement* of the *Sun's Altitude.*
5. Z⊙P, the *Angle* of the *Sun's Poſition.*
6. ZP⊙, the *Hour* from *Noon.*

And if there had been given ⊙P, ⊙Z, and ⊙ZP, then might be found,

7. ZP, the *Complement* of the *Latitude.*
8. ZP⊙, the *Hour* from *Noon.*
9. Z⊙P, the *Angle* of the *Sun's Poſition.*

In like manner, if the Sides P⊙ and ⊙Z, with the *Angle* ZP⊙, had been given, then might be found, 18

10. ZP, the *Complement* of the *Latitude.*
11. ⊙ZP, the *Azimuth* from the *North.*
12. Z⊙P, the *Angle* of the *Sun's Poſition.*

Again, if the Sides ⊙Z and ZP, with the *Angle* ZP⊙, had been given, then would be found,

13. P⊙, the *Sun's* diſtance from the *Pole.*
14. Z⊙P, the *Angle* of the *Sun's Poſition.*
15. ⊙ZP, the *Sun's Azimuth* from the *North.*

Laſtly, if the Sides ⊙Z and ZP, with the *Angle* Z⊙P, had been given, then you might find,

16. P⊙, the *Complement* of the *Sun's Declination.*
17. ZP⊙, the *Hour* from *Noon.*
18. ⊙ZP, the *Azimuth* from the *North.*

CASE V. *Two* Angles, ⊙ZP *and* ZP⊙, *and a* Side *oppoſite to one of them* P⊙, *being given, to find the other Parts of the* Triangle.

In the *Triangle* Z⊙P, if there be given the *Angles* ⊙ZP, and ZP⊙, with the Side P⊙, there may be found,

1. ZP, the *Complement* of the *Latitude.*
2. Z⊙, the *Complement* of the *Sun's Altitude.*
3. Z⊙P, the *Angle* of the *Sun's Poſition.*

But if there were given ⊙ZP and ZP⊙, with the Side Z⊙, then might be found,

4. ZP, the *Complement* of the *Latitude.*
5. P⊙, the *Sun's* Diſtance from the *Pole.*
6. Z⊙P, the *Angle* of the *Sun's Poſition.*

And if the *Angles* ZPE and ZEP, with the Side EP, were given, then might be found,

7. ZP, the *Complement* of the *Latitude.*
8. P⊙, the *Hour* from *Noon.*
9. ⊙ZP, the *Sun's Azimuth* from the *North.*

Again, if there were given the *Angles* ZP⊙ and Z⊙P, with the Side ZP, you might then find, 18

10. Z⊙,

10. Z ⊙, the *Complement* of the *Sun's Altitude.*
11. P ⊙, the *Complement* of the *Sun's Declination.*
12. E Z P, the *Sun's Azimuth* from the *North.*

Alſo if there were given the *Angles* Z ⊙ P and ⊙ Z P, with the Side Z P, you might find,

13. Z ⊙, the *Complement* of the *Sun's Altitude.*
14. P ⊙, the *Complement* of the *Sun's Declination.*
15. Z P ⊙, the *Hour* from *Noon.*

And laſtly, if there were given the *Angles* Z ⊙ P and E Z P, with the Side P E, then might be found,

16. Z ⊙, the *Complement* of the *Sun's Altitude.*
17. Z P, the *Complement* of the *Latitude.*
18. Z P ⊙, the *Hour* from *Noon.*

CASE VI. *The three* Angles ⊙ Z P, Z ⊙ P *and* Z P ⊙, *being given, to find the other Parts.*

In the *Triangle* Z ⊙ P, if the three *Angles* ⊙ Z P, Z P ⊙, and P ⊙ Z, be given, there may be found,

3. { 1. Z P, the *Complement* of the *Latitude.*
   { 2. P ⊙, the *Sun's* Diſtance from the *Pole.*
   { 3. Z ⊙, the *Complement* of the *Sun's Altitude.*

Thus have you in theſe Six Caſes *all the Varieties that will ariſe out of an* Oblique angled Spherical Triangle, *in the Converſion of which* Caſes *you may obſerve* 60 Queſtions *of the* Sphere *to be reſolved; and ſo many are reſolvable in every* Oblique angled Spherical Triangle, *and* 30 *in every* Right angled: *So that in theſe two* Triangles 90 Queſtions *are reſolved. For,*

In the Right angled Spherical Triangle, P R O.

By the	First	*Caſe* are reſolved	3	*Spherical Queſtions.*
	Second		6	
	Third		6	
	Fourth		12	
	Fifth		3	

In all 30.

In the Oblique angled Spherical Triangle, Z ⊙ P.

By the	First	*Caſe* are reſolved	3	*Spherical Problems.*
	Second		9	
	Third		9	
	Fourth		18	
	Fifth		18	
	Sixth		3	

In all 60.

Inſtru-

INSTRUMENTAL TRIGONOMETRY.

PART III.

Three ſeveral ways Performed.

The First Way. *By the* Scales *of Artificial* Sines, Tangents *and* Logarithms.

THE Third Way of *Inſtrumental Arithmetick* beforegoing Book I. Part IV. hereof, is performed by *Artificial*, or *Logarithmical Scales* put upon a Two foot *Ruler*, and the manner of their diſpoſal upon the *Ruler* is there ſhewed; and as the *Logarithms* or *Artificial Numbers* were transferred from the *Table* of *Logarithms*, ſo are theſe *Scales* of *Artificial Sines* and *Tangents*, transferred from the *Tables* or *Canons* of *Artificial Sines* and *Tangents* at the end of this Book; and the manner how to tranſfer them is ſo well known to all that are makers of *Mathematical Inſtruments*, that I ſhall ſay nothing concerning it in this Place, but ſhew how they are diſpoſed on the other Side of the Two foot *Ruler* before-mentioned: And ſo refer you to the *Figure* of it in the fore-mentioned Firſt Book. Part IV. of *Inſtrumental Arithmetick*, where you have the *Figure* of both Sides of the *Ruler* with the ſeveral *Scales* upon them diſpoſed as is there and here directed.

How the Scales *of* Sines, Tangents, *and* Logarithms *are to be diſpoſed on the backſide of the forementioned* Ruler.

I. The firſt *Scale* is a *Scale* of *Artificial Sines*, the whole length of the *Ruler*, divided into 90 unequal *Parts* or *Degrees*, and numbered by 1, 2, 3, *&c.* to 10, and from thence by 20, 30, 40, *&c.* to 90 degrees at the end; each degree being divided (or ſuppoſed ſo to be) into 60 parts called *Minutes*.

II. The Second *Scale*, is a *Scale* of *Artificial Tangents*, of the ſame length with the *Sines*, divided unequally, and numbered upwards by 1, 2, 3, *&c.* to 10, and from thence by 10, 20, 30, *&c.* to 45 degrees, and back again downwards by 45, 50, 60, *&c.* towards 90 deg. ſo far as the *Ruler* will permit.

Theſe two *Scales* of *Sines* and *Tangents*, are to be uſed with the *Double Scale* of *Logarithms* on the other Side of the *Ruler*: But for the more convenience in working, in many *Caſes*, the ſame *Double Scale* is again repeated on this Side of the *Ruler*.

III. The Third *Scale*, is a *Scale* of *Artificial Sines* of the ſame nature with the former, but it is but half the length thereof; it having 90 de. in the middle of the *Ruler*; from whence the diviſions are continued upwards beyond 90, and numbered by 10, 20, 30, *&c.* towards 90 repreſenting *Secants*; but it would be better to number them by 90, 100, 110, *&c.* towards 180 degrees, and ſo it will reach at the end of the *Ruler* to 174 deg. 10 m.

IV. The Fourth is a *Scale* of *Artificial Tangents*, having the *Radius* (or *Tangent* of 45 deg.) in the middle of the *Scale*, juſt againſt 90 deg. of the *Sines*, and is continued upwards beyond 45 deg. to the end of the *Ruler*, where it terminates at 84 degrees, 10 min.

These two last *Double Scales* (for so I shall hereafter call them) of *Sines* and *Tangents*, are also to be used with the forementioned *Double Scale* of *Logarithms* (or *Numbers*) and therefore, it were best to place it between the *Double*, and the *Single Scales* of *Sines* and *Tangents*, as in the *Figure* of the *Ruler* it is. These *Double Scales* will be of excellent Use in *Trigonometrical Calculation*, when *Angles* or *Sides* do exceed 90 de. of *Sines*, or 45 degrees of *Tangents*, as will appear in the *Cases* following.

And thus much for the Description and Disposal of the *Scales* upon the *Ruler*, I shall now proceed to shew the *Uses* of these *Scales*, in the *Solution* of *Triangles* both *Plain* and *Spherical*.

Advertisement.

In the *Solution* of *Plain* and *Spherical Triangles* by the *Proportional Scales*; it will be necessary to use sometimes the *Single Scale* of *Sines*, and sometimes the *Double Scale*; but always the *Double Scales* of *Tangents* and *Logarithms*. And to know when to use one, and when the other, observe the following *Rules*.

I. If your *Proportion* be in *Sines* alone, or in *Sines* and *Logarithms* jointly use the } *Single* }
II. If your *Proportion* be in *Sines* and *Tangents* jointly, always } *Double* } *Scales*.
use the

SECT. I.

Of Right Angled Plain Triangles.

Fig. I. The *Triangle* which I shall make use of in the resolving of the several *Cases* belonging to *Right angled Plain Triangles*, shall be the same, which I made use of in *Geometrical Trigonometry* noted with C A B *Fig.* I.

In which *Triangle Parts*,

A B The *Base*
C A The *Perpendicular* } contains { 180 / 135 / 225 } of any *Measure*:
C B The *Hypotenuse*

And,

		d.	m.
A The *Right Angle*	contains	90	00
C The *Angle* at the *Perpendicular*		53	07
B The *Angle* at the *Base*		36	53

CASE I. *The* Base A B, *and* Perpendicular C A, *given, to find the* Angles *at* B *and* C.

The *Proportion* is,

Log. A B (180) : Log. C A (135) :: R (45 d.) : *t* B (36 d. 53′)

Extend the Compasses from 180 downwards to 135, (upon the *Log. Scale*,) the same extent will reach from 45 degrees downwards, upon the *Scale* of *Tangents*, to 36 d. 53 m. for the *Angle* at B, and also to 53 d. 7 min. (its *Complement*) for the *Angle* at C.

CASE II. *The* Hypotenuse C B, *and the* Base A B *given, to find the* Angles B *and* C.

The *Proportion* is,

Log. B C (225) : Log. B A (180) :: R (90 d.) : *s* C (53 d. 7′)

Extend the Compasses from 225 downwards to 180, the same extent will reach from the *Sine* of 90 downwards to 53 d. 7 m. for the *Angle* at C; whose *Complement* 36 d. 53 m. is the *Angle* at B.

Case

CASE III. *The* Base A B, *the* Angles C *and* B *given, to find the* Perpendicular C A.

The *Proportion* is,

	s C	:	s B	::	Log. A B	:	Log. C A
	53 d. 7′		36 d. 53′		180		135

The extent of the Compasses from the *Sine* of 53 d. 7 m. downwards to 36 d. 53 m. will reach from the Log. of 180 downwards to 135, for the *Perpendicular* C A.

CASE IV. *The* Hypotenuse C B, *and the* Angles C *and* B *given, to find the* Perpendicular.

The *Proportion* is,

	R	:	s B	::	Log. B C	:	Log. C A
	90 d.		36 d. 53′		228		135

Extend the Compasses from the *Sine* of 90 downwards to the *Sine* of 36 d. 53 m. the same extent will reach from the Log. of 228 downwards to 135, the length of the *Perpendicular* C A.

CASE V. *The* Base A B, *and the* Perpendicular A C *given, to find the* Hypotenuse.

The *Proportions* are,

(1)	Log. B A	:	Log. C B	::	s R	:	s C	
	180		228		45 d.		53 d. 7′	
(2)	s B	:	R	::	Log. C A	:	Log. C B	
	36 d. 53′		90		135		228	

1. Extend the Compasses from the Log. of 180 upwards to 228, the same extent will reach from the *Tangent* of 45 d. (on the double *Scale*) upwards to the *Tangent* of 53 d. 7 m. the *Angle* at C.

2. The extent of the Compasses from the *Sine* of 36 d. 53 m. upwards to 90 deg. will reach from the Log. of 135 upwards to 228, the *Hypotenuse* B C.

CASE VI. *The* Hypotenuse C B, *and the* Base A B *given, to find the* Perpendicular C A.

The *Proportions* are,

(1)	Log. C B	:	Log. A B	::	R	:	s C	
	228		180		90 d.		53 d. 7′	
(2)	R	:	s B	::	Log. C B	:	Log. C A.	
	90 d.		36 d. 53′		228		135	

1. Extend the Compasses from 228 downwards to 180, the same extent will reach from 90 deg. downwards to 53 d. 7 m. the quantity of the *Angle* at C.

2. The extent from 90 downwards to the *Sine* 36 d. 53 m. will reach from 228 downwards to the Log. of 135, the *Perpendicular* C A.

CASE VII. *The* Base A B, *and the* Angles C *and* B, *given, to find the* Hypotenuse C B.

The *Proportion* is,

	s C	:	R	::	Log. A B	:	Log. C B
	53 d. 7′		90 d.		180		228

The extent of the Compasses from the *Sine* of 53 d. 7 m. upwards to 90 de. will reach from 180 upwards to 228, the Log. of C B, the *Hypotenuse*.

SECT. II.

Of Oblique Angled Plain Triangles.

Fig. II. The *Triangle*, which I ſhall make uſe of in the Solution of *Oblique Angled Plain Triangles*, ſhall be the *Triangle* C D B: In which

		Parts;	
D B the *Baſe*	contains	335	Of any *Meaſure.*
C B the longer *Side*		271	
D C the ſhorter Side		100	

		deg.	min.
G the *Obtuſe Angle*	contains	122	00
D the 2 *Acute Angles*		43	20
B the 2 *Acute Angles*		14	40

In the Solution of which *Oblique Triangle*, I call the longeſt *Side* (always) the *Baſe*; and the other two I call the *Sides* (or *Legs*,) without any other appellation or diſtinction of *Baſe* or *Perpendicular.*

CASE I. *The* Baſe B D, *the* Side C B, *and the* Angle *at* D, (*oppoſite to the* Side C D) *being given, to find the* Angle *at* C, *oppoſite to the* Baſe B D.

The Proportion is,

Log. B C	:	ſ D	::	Log. B D	:	ſ C
271		43 d. 20′		335		122

Extend the Compaſſes from the Log. of 271 to the Log. of 335, the ſame extent will reach from the *Sine* of 43 deg. 20 min. (upon the *Double Scale of Sines*) to 58 deg. whoſe *Complement* to 180 deg. is 122 deg. for the *Angle* at C.

CASE II. *The* Baſe B D, *the* Side D C, *with the* Angle D (*comprehended between them*) *being given, to find the other two* Angles B *and* C.

To reſolve this *Caſe*, you muſt firſt get the Summ and the Difference of the two given *Sides*, and alſo the half Summ of the two unknown *Angles*, at B and C, in this manner:

The two given *Sides* are,	D B	335
	D C	100
Their Summ		435
Their Difference		235
Their half Difference		117.5
The *Angle* given is D		43 d. 20 m.
Which ſubtracted from		180 . 00 m.
Leaves		136 . 40 m.
Which is the Summ of the two *Angles* at C and B,		
The half whereof is,		68 . 20 m.

Being thus prepared,

The Proportion is,

Log. C D and D B	:	Log. differ.	::	$t.\frac{1}{2}$ C and B	:	$t.\frac{1}{2}$ Dif. C and B.
435		235		68 d. 20 m.		53 d. 40′

Extend the Compaſſes from 435 (the Sum of the *Sides*) downwards to 235 (the Difference of the *Sides*,) the ſame extent will reach (*upon the double Scale of Tangents*) from the *Tangent* of 68 d. 20 m. (the half Summ of the *Angles* C and B) downwards to the *Tangent* of 53 de. 40 m.

This 53 d. 40 m. being added to 68 d. 20 m. gives 122 d. for the greater *Angle* at C; and being ſubtracted therefrom, leaves 14 d. 40 m. for the leſſer *Angle* at B.

Caſe

CASE III. *The three* Sides BD, DC, and BC *being given, to find any of the* Angles, *viz.* B.

For the Solution of this *Case* you must get the Summ, and the Difference of the two *Sides*, containing the *Obtuse Angle*; Thus,

The *Side* { CD / BC } is { 100 / 271 }

Their Summ	371
Their Difference	171
The *Base* BD is	335

These things thus found, the Proportion is,

Log. BD : Log. CD and BC :: Differ. CD and BC : Fourth.
335 371 171 189.4

Extend the Compasses from 335 upwards, to 371; the same extent will reach, the same way, from 171, to 189.4. This *Fourth Summ*, being subtracted from the *Base* BD, 335, will leave 145.6; the half whereof is 72.8: And that is the length of the *Base* from D to E; in which Point, a *Perpendicular* let fall from the *Obtuse Angle* at C, will fall: And so the *Obtuse angled Triangle* BCD, will be reduced into two *Right angled Triangles* CED, and CBE; and any of the *Parts* (*Sides*, or *Angles*) of any of them may be found, by the respective *Cases* of *Right angled Plain Triangles*, as in the First Section hereof.

CASE IV. *The three* Angles B, C, *and* D, *being given, to find any of the* Sides.

For the Solving of this *Case*, no absolute Proportion can be prescribed; for the *Sides* themselves cannot absolutely be found; but their Proportion one to another may be obtained: For the *Three Angles* of one *Triangle*, may be equal to the *Three Angles* of another *Triangle*; although their *Sides* be altogether unequal.

CASE V. *Two* Sides BC, *and* CD; *with the* Angle *at* C, *contained between them*, *being given, to find the* Base BD; *opposite to the given* Angle *at* C.

By the two foregoing *Cases*, you must find the two *Angles* at B and D; then make choice of one of the containing *Sides* (as CD) to work by: And then,

The Proportion is,

sB : sC to 180 :: Log. CD : Log. BD.
14 d. 40′ 58 d. 100 335

Extend the Compasses, from the *Sine* of 14 de. 40 min. upwards to 58 de. the same extent will reach from the Log. of 100, the same way; to 335, for the *Base* BD.

CHAP. II.

The Solution of Spherical Triangles; *By the* Scales *of* Artificial Sines *and* Tangents.

SECT. I.

Of Right Angled Spherical Triangles.

Fig. IV. IN the Solution of the X *Cases* of *Right angled Spherical Triangles*, by the Proportional *Lines*, I shall make use of the *Triangle* BAC, *Right angled* at A: The quantity of the *Sides* and *Angles* whereof, with their *Complements*, are here exhibited: *viz.*

1. For the *Sides*.

	de.	m.	de.	m.
The *Hypotenuse* CB	30	00	60	00
The *Base* BA	27	54	62	06
The *Perpendicular* CA	11	30	78	30

2. For the *Angles*.

The *Angle* at the *Base* B	23	30	66	30
The *Angle* at the *Perpendicular* C	69	22	20	38

CASE I. *The* Hypotenuse BC, *and the* Angle *at the* Base B, *given, to find the* Perpendicular CA.

The Proportion is,

R	:	*s* BC	::	*s* B	:	*s* CA
90 d.		30 d.		23 d. 30′		11 d. 30′

Extend the Compasses from the *Sine* of 90 d. (in the Single *Scale*) downwards to 30 deg. the same extent will reach (the same way) from 23 d. 30 m. to 11 d. 30 m. the *Perpendicular*.

CASE II. *The* Hypotenuse CA, *and* Perpendicular CA *given, to find the* Base BA.

The Proportion is,

cs CA	:	R	::	*cs* CB	:	*cs* BA
78 d. 30′		90 d.		60 d.		62 d. 6′

Extend the Compasses, from the *Sine* of 78 d. 30 m. up to 90 deg. the same extent will reach from 60 deg. upwards to 62 deg. 6 m. whose *Complement* 27 de. 54 m. is the *Base* BA. —— Or, the extent of the Compasses from 78 d. 30 m. downwards to 60 de. will reach from 90 de. down to 62 de. 6 min. for the *Complement* of the *Base* BA.

CASE III. *The* Angles *at the* Base *and* Perpendicular B *and* C, *given, to find the* Perpendicular CA.

The Proportion is,

s C	:	*cs* B	::	R	:	*cs* CA
69 d. 22′		66 d. 30′		90 d.		79 d. 30′

The extent of the Compasses from 69 d. 22 m. down to 66 de. 30 m. will reach from 90 de. down to 79 d. 30 m. whose *Complement* is 11 de. 30 m. for the *Perpendicular* CA: —— Or, the extent from 69 d. 36 m. up to 90 d. will reach from 66 d. 30 m. up to 79 de. 30 m. as before.

Case

CASE IV. *The* Base B A, *and the* Angle *at the* Base B, *being given, to find the* Perpendicular C A.

The Proportion is,

R	:	*s* B A	::	*t* B	:	*t* C A
90 d.		27 d. 54′		23 d. 30′		11 d. 30′

The extent of the Compasses, from the *Sine* of 90 (upon the double *Scale*) down to 27 de. 54 m. will reach (upon the *Tangent Scale*) from 23 de. 30 m. downwards to 11 de. 30 m. for the *Perpendicular* C A.

CASE V. *The* Perpendicular C A, *and the* Angle *at the* Base B, *given, to find the* Base, B A.

The Proportion is,

t B	:	*t* C A	::	R	:	*s* B A
23 d. 30′		11 d. 30′		90 d.		27 d. 54′

Extend the Compasses from the *Tangent* of 23 d. 30 m. down to 11 d. 30 min. the same extent will reach from the *Sine* of 90 d. (upon the *double Scale*) down to 27 d. 54 m. for the *Base* B A.

CASE VI. *The* Hypotenuse C B, *and* Angle *at the* Perpendicular C, *given, to find the* Perpendicular C A.

The Proportion is,

R	:	*cs* C	::	*t* B C	:	*t* C A
90 d.		20 d. 24′		30 d. 0′		11 d. 30′

Extend the Compasses from the *Sine* of 90 d. downwards (upon the *double Scale*) to 20 d. 24 m. the same extent will reach the same way from the *Tangent* of 30 de. to the *Tangent* of 11 de. 30 m. for the *Perpendicular*, C A.

CASE VII. *The* Angles *at the* Base *and* Perpendicular B *and* C, *given, to find the* Hypotenuse C B.

The Proportion is,

t C	:	*ct* B	::	R	:	*cs* C B
69 d. 22′		66 d. 30′		90 d.		60 d.

Extend the Compasses from the *Tangent* of 69 d. 22 m. down to 66 deg. 30 min. the same extent will reach the same way from the *Sine* of 90 deg. to the *Sine* of 60 deg. Whose *Complement* 30 deg. is the *Hypotenuse* C B.

CASE VIII. *The* Base B A, *and* Perpendicular C A, *given, to find the* Hypotenuse B C.

The Proportion is,

R	:	*cs* B A	::	*cs* C A	:	*cs* C B
90 d.		62 d. 6′		78 d. 30′		60 d.

The extent of the Compasses from the *Sine* of 90 deg. to the *Sine* of 62 d. 6 m. (upon the Single *Scale*) will reach from the *Sine* of 78 de. 30 m. to the *Sine* of 60 d. for the *Complement* of the *Hypotenuse* C B 30 d.

CASE IX. *The* Perpendicular C A, *and the* Angle *at the* Base B, *given, to find the* Hypotenuse B C.

The Proportion is,

s B	:	R	::	*s* C A	:	*s* C B
23 d. 30′		90 d.		11 d. 30′		30 d. 0′

Extend

Extend the Compaſſes from the *Sine* of 23 d. 30 m. up to 90; the ſame extent will reach the ſame way from 11 d. 30 m. to 30 d. for the *Hypotenuſe* B C.

CASE X. *The* Baſe B A, *and* Angle *at the* Baſe B, *given, to find the* Hypotenuſe B C.

The Proportion is,

R	:	*cs* B	::	*ct* B A	:	*ct* B C
90 d.		66 d. 30 m.		62 d. 6′		60 d. 0′

Extend the Compaſſes from the *Sine* of 90 de. to the *Sine* of 66 de. 30 m. the ſame extent will reach from the *Tangent* of 62 de. 6 m. the ſame way, to the *Tangent* of 60 d. whoſe *Complement* 30 d. is the *Hypotenuſe* B C.

CASE XI. *The* Perpendicular C A, *and the* Angle *at the* Perpendicular C, *given, to find the* Angle *at the* Baſe B.

The Proportion is,

R	:	*s* C	::	*cs* C A	:	*cs* B
90 d.		69 d. 22′		78 d. 30′		66 d. 30′

The extent of the Compaſſes, from the *Sine* of 90 de. (upon the ſingle *Scale*) downwards to 69 de. 22 m. will reach from 78 d. 30 m. to 66 d. 30 m. whoſe *Complement* 23 d. 30 m. is the *Angle* at the *Baſe* B.

CASE XII. *The* Perpendicular C A, *and the* Angle *at the* Baſe B, *given, to find the* Angle *at the* Perpendicular C.

The Proportion is,

cs C A	:	*cs* R	::	R	:	*s* C
78 d. 30′		66 d. 30′		90 d.		69 d. 22′

Extend the Compaſſes upon the ſingle *Scale*, from the *Sine* of 78 d. 30 m. to 66 d. 30 m. the ſame extent will reach from the *Sine* of 90 d. to the *Sine* of 69 d. 22 m. for the *Angle* at the *Perpendicular* C.

CASE XIII. *The* Hypotenuſe B C, *and the* Perpendicular C A, *given, to find the* Angle *at the* Baſe B.

The Proportion is,

s B C	:	R	::	*s* C A	:	*s* B
30 d.		90 d.		11 d. 30 m.		23 d. 30

The extent of the Compaſſes from the *Sine* of 30 de. up to 90 de. will reach from 11 de. 30 m. up to 23 de. 30 m. for the *Angle* at the *Baſe* B.

CASE XIV. *The* Baſe B A, *and* Perpendicular C A, *given, to find the* Angle *at the* Baſe B.

The Proportion is,

s B A	:	R	::	*t* C A	:	*t* B
27 d. 54′		90 d.		11 d. 30′		23 d. 30′

Extend the Compaſſes from the *Sine* of 27 de. 54 m. up to 90 d. the ſame extent will reach from the *Tangent* of 11 d. 30 m. up to 23 d. 30 m. for the *Angle* at B.

CASE XV. *The* Hypotenuſe B C, *and* Perpendicular C A, *given, to find the* Angle *at the* Perpendicular C.

The Proportion is,

t B C	:	*t* C A	::	R	:	*cs* C
30 d. 0′		11 d. 30′		90 d.		20. 24

Extend

Extend the Compaſſes from the *Tangent* of 30 de. down to the *Tangent* of 11 de. 30 m. the ſame extent will reach from the *Sine* of 90 deg. to the *Sine* of 20 d. 24 m. whoſe *Complement* 69 d. 36 m. is the *Angle* at the *Perpendicular*.

CASE XVI. *The* Hypotenuſe B C, *and* Angle *at the* Perpendicular C, *given, to find the* Angle *at the* Baſe B.

The Proportion is,

R	:	*cs* C B	::	*t* C	:	*ct* B
90 d.		60 d.		69 d. 22′		66 d. 30

Extend the Compaſſes from the *Sine* of 90 d. down to 60 d. the ſame extent will reach from the *Tangent* of 69 d. 22 m. to the *Tangent* of 66 d. 30 m. Whoſe *Complement* is 23 d. 3 m. for the *Angle* at B.

SECT. II.

Of Oblique angled Spherical Triangles.

IN the Solution of the XII *Caſes* of *Oblique angled Spherical Triangles* by the *Scales*, I ſhall make uſe of the *Triangle* Z P S, *Obtuſe angled* at Z, whoſe *Sides* and *Angles*, with their *Complements*, are as is here exhibited; *viz.*

		De.	M.		De.	M.
The *Side*	Z P	38	30	Whoſe *Complement* is	51	30
	Z S	40	00		50	00
	S P	79	00		20	00
The *Angle*	Z	130	08		49	52
	S	30	24		59	36
	P	31	32		58	28

The *Angle* { S Z R —— 65.49 / P Z R —— 64.19 } whoſe *Complement* is { 24 —— 11 / 25 —— 41

The *Perpendicular* Z R 18.59

		De.	M.		D.	M.
The *Segments* of the *Baſe*	S R	35	54	*Complement*	54	06
	P R	34	06		55	54

CASE I. *Two* Sides Z P, Z S, *and the* Angle S, *oppoſite to the* Side Z P, *given, to find the* Angle P, *oppoſite to the other* Side Z S.

The Proportion is,

s Z P	:	*s* S	::	*s* S Z	:	*s* P
38 d. 30′		30 d. 24′		40 d. 0′		31 d. 32′

Extend the Compaſſes from the *Sine* of 38 d. 30 m. to the *Sine* of 30 d. 24 m. the ſame extent will reach (the ſame way) from 40 d. to 31 d. 32 m. for the *Angle* at P.

CASE II. *Two* Angles Z *and* P, *with the* Side S P, *oppoſite to* Z, *given, to find the* Side S Z, *oppoſite to the* Angle *at* P.

The Proportion is,

s Z	:	*s* S P	::	*s* P	:	*s* S Z
130 d. 8′		70 d. 0′		31 d. 32′		40 d. 00′

The extent of the Compaſſes from the *Sine* of 49 de. 52 m. (the *Complement* of 130 d. 8 m. the *Angle* at Z, to 180 d.) to 70 d. will reach the ſame way, from 31 d. 32 m. to 40 d. for the *Side* Z S.

In the Solution of the following *Ten Caſes*, it will be neceſſary to *reduce* the *Oblique Triangle* into *Two Right angled Triangles*, by letting fall a *Perpendicular* from an *Angle* upon a *Side* oppoſite thereunto: For the letting fall of which obſerve the following *Directions*:

1. The

1. The *Perpendicular* muſt be let fall from an unknown *Angle*, upon a *Side* oppoſite thereunto; extended if need require.

2. By the *Perpendicular* Z R, ſo let fall, the *Oblique angled Triangle* Z S P is reduced into two *Right angled Triangles* Z R S, and Z R P, both *Right angled* at R.

3. This *Perpendicular* falls ſometimes *Within*, and ſometimes *Without* the *Oblique Triangle*: For,

4. When both the *Angles* at the ends of that *Side* upon which the *Perpendicular* is to fall, are either both *Acute*, or both *Obtuſe*, the *Perpendicular* falls *Within* the *Triangle*: As in our *Triangle*, the *Perpendicular* falls upon the *Side* S P, whoſe *Angles* at S and P are both *Acute*. —— But: when one of the *Angles* is *Acute*, and the other *Obtuſe*, the *Perpendicular* will fall *Without* the *Triangle* upon a *Side* extended.

5. That *Side* muſt be always called the *Baſe* upon which the *Perpendicular* falls.

Theſe things premiſed, I proceed to

CASE III. *Two* Sides S Z *and* S P, *with the* Angle S *contained between them, given, to find the oppoſite* Angle *at* P.

In this *Caſe*, the *Baſe* is that given *Side* which is adjacent to the *Angle* ſought, namely S P.

The Proportions are,

(1)	R	:	*cs* S	::	*t* Z S	:	*t* S R
	90 d.		59 d. 36′		40 d. 0′		35 d. 54′
(2)	*s* S R	:	*s* R P	::	*t* S	:	*t* P
	35 d. 54′		34 d. 6′		30 d. 24′	:	31 d. 32′

(1) Extend the Compaſſes from the *Sine* of 90 de. to the *Sine* of 59 d. 36 m. the ſame extent will reach from the *Tangent* of 40 deg. to the *Tangent* of 35 deg. 54 m. for S R.

(2) Extend the Compaſſes from the *Sine* of 35 d. 54 m. to 34 deg. 6 m. the ſame extent will reach from the *Tangent* of 30 deg. 24 m. to the *Tangent* of 31 d. 32 m. for the *Angle* at P.

CASE IV. *Two* Sides Z P *and* P S, *with the* Angle *at* P, *included between them, given, to find the* Side Z S *oppoſite to the given* Angle *at* P.

In this *Caſe* the *Baſe* is one of the given *Sides*: And,

The Proportions are,

(1)	R	:	*cs* P	::	*t* Z P	:	*t* P R
	90 d.		58 d. 28′		38 d. 30′		34 d. 6′
(2)	*cs* P R	:	*cs* S R	::	*cs* Z P	:	*cs* Z S
	55 d. 54′		54 d. 6′		51 d. 30′		50 d.

(1) Extend the Compaſſes from the *Sine* of 90 d. to the *Sine* of 58 d. 28 min. the ſame extent will reach from the *Tangent* of 38 d. 30 m. to the *Tangent* of 34 d. 6 m.

(2) Extend the Compaſſes from the *Sine* of 55 d. 54 m. to the *Sine* of 54 d. 6 m. the ſame extent will reach from the *Sine* of 51 d. 30 m. to the *Sine* of 50 d. whoſe *Complement* 40 d. is the *Side* Z S.

CASE V. *Two* Sides Z P *and* Z S, *with the* Angle P *contained between them, given, to find the* Angle S, *oppoſite to* Z P.

The Proportions are,

(1)	R	:	*cs* P	::	*t* Z P	:	*t* R P
	90 d.		58 d. 28′		38 d. 30′		34 d. 6′
(2)	*s* P R	:	*s* S R	::	*t* P	:	*t* S
	34 d. 6′		35 d. 54′		31 d. 32′		30 d. 24

(1) Extend the Compaſſes from the *Sine* of 90 d. to the *Sine* 58 d. 28 m. the ſame will reach from the *Tangent* of 38 d. 30 m. to the *Tangent* of 34 d. 6 m.

(2) The extent of the Compaſſes from the *Sine* of 34 d. 6 m. to 35 d. 54 m. will reach from the *Tangent* of 31 d. 32 m. to 30 d. 24 m. for *Angle* at S.

Caſe

CASE VI. *Two* Sides Z P *and* Z S, *with the* Angle P, *opposite to* S Z, *given, to find the* Angle Z, *contained between them.*

In this *Case*, the *Base* is always the *Side* unknown. And,

The Proportions are,

(1)	R	:	*cs* Z P	::	*t* P	:	*t* R Z P
	90 d.		51 d. 30′		31 d. 32′		25 d. 38′
(2)	*t* Z S	:	*t* Z P	::	*cs* R Z P	:	*cs* S Z R
	40 d.		38 d. 30′		25 d. 41′		24 d. 11′

(1) Extend the Compaſſes from the *Sine* of 90 d. to the *Sine* of 51 d. 30 m. that extent will reach from the *Tangent* of 31 d. 32 m. to the *Tangent* of 25 d. 38 m. whoſe *Complement* 64 d. 19 m. is the quantity of the *Angle* R Z P.

(2) The extent of the Compaſſes from the *Tangent* of 40 d. to the *Tangent* of 38 d. 30 m. will reach from the *Sine* of 25 d. 41 m. to the *Sine* of 24 d. 11 m. whoſe *Complement* 65 d. 49 m. is the *Angle* S Z R; which added to 64 d. 19 m. the *Angle* R Z P, makes 130 de. 08 m. for the whole *Angle* S Z P.

CASE VII. *Two* Sides Z S *and* Z P, *with the* Angle S, *oppoſite to* Z P, *given, to find the* Side S P, *adjacent to the given* Angle S.

The Proportions are,

(1)	R	:	*cs* S	::	*t* Z S	:	*t* S R
	90 d.		59 d. 36′		40 d. 0′		35 d. 54′
(2)	*cs* Z S	:	*cs* Z P	::	*cs* S R	:	*cs* P R
	50 d. 0′		51 d. 30′		54 d. 6′		55 d. 54′

(1) Extend the Compaſſes from the *Sine* of 90 d. to 59 d. 36 m. the ſame extent will reach from the *Tangent* of 40 d. to the *Tangent* of 35 d. 54 m. for S R.

(2) Extend the Compaſſes from the *Sine* of 50 d. to the *Sine* of 51 d. 30 m. the ſame will reach from the *Sine* of 54 d. 6 m. to the *Sine* of 55 d. 54 m. whoſe *Complement* 34 d. 6 m. is R P, which added to S R 35 d. 54 m. makes 70 d. for the whole *Side* S P.

CASE VIII. *Two* Angles S *and* Z, *with the* Side S Z, *comprehended between them, being given, to find the* Angle *at* P, *oppoſite to the given* Side Z S.

In this *Case*, the *Base* may be either of the unknown Sides; And

The Proportions are,

(1)	R	:	*ct* S Z	::	*t* S	:	*ct* R Z S
	90 d.		50 d. 0′		30 d. 24′		24 d. 11′
(2)	*s* R Z S	:	*s* R Z P	::	*cs* S		*cs* P
	65 d. 49′		64 d. 19′		59 d. 36′		58 d. 28′

(1) Extend the Compaſſes from the *Sine* of 90 d. to the *Sine* of 50 d. the ſame extent will reach from the *Tangent* of 30 d. 24 m. to the *Tangent* of 24 d. 11′, whoſe *Complement* 65 d. 49 m. is the *Angle* R Z S.

(2) The extent of the Compaſſes from the *Sine* of 65 d. 49 m. to the *Sine* of 64 d. 19 m. will reach from the *Sine* of 59 d. 36 m. to 58 d. 28 m. whoſe *Complement* 31 d. 32 m. is the *Angle* at P.

CASE IX. *Two* Angles Z *and* P, *with the* Side Z P, *comprehended between them, being given, to find the* Side Z S, *oppoſite to the given* Angle *at* P.

In this *Case*, the *Base* is the *Side* S P, which is neither given nor ſought. And,

The Proportions are,

(1)	R	:	*cs* Z P	::	*t* P	:	*ct* R Z P
	90 d.		51 d. 30′		31 d. 32′		25 d. 41′
(2)	*cs* R Z S	:	*cs* R Z P	::	*t* Z P	:	*t* Z S
	24 d. 11′		25 d. 41′		38 d. 30 m.		40 d. 0′

(1) Extend

(1) Extend the Compasses from the *Sine* of 90 d. to the *Sine* of 51 d. 30 m. the same extent will reach from the *Tangent* of 31 d. 32 m. to the *Tangent* of 25 d. 41 m. whose *Complement* 64 d. 19 m. is the *Angle* PZR.

(2) Extend the Compasses from the *Sine* of 24 d. 11 m. to the *Sine* of 25 d. 41 m. the same extent will reach from the *Tangent* of 38 d. 30 m. to the *Tangent* of 40 deg. for the *Side* SZ.

CASE X. *Two* Angles S *and* P, *with a* Side *opposite to one of them*, SZ *being given, to find the third* Angle *at* Z.

In this *Case*, the *Base* is the *Side* opposite to the *Angle* sought, *viz.* SP. And

The Proportions are,

(1)	R	:	*cs* ZS	::	*t* S	:	*ct* SZR
	90 d.		50 d. 0′		30 d. 24′		24 d. 11′
(2)	*cs* S	:	*cs* P	::	*s* SZR	:	*s* RZP
	59 d. 36′		58 d. 28′		65 d. 49′		64 d. 19′

(1) Extend the Compasses from the *Sine* of 90 d. to the *Sine* of 50 d. the same will reach from the *Tangent* of 30 d. 24 m. to the *Tangent* of 24 d. 11 m. whose *Complement* 65 d. 49 m. is the quantity of the *Angle* SZR.

(2) Extend the Compasses from the *Sine* of 59 d. 36 m. to the *Sine* of 58 d. 28 m. the same will reach from the *Sine* of 65 d. 49 m. to the *Sine* of 64 d. 19 m. for the *Angle* RZP; which added to 65 d. 19 m. the *Angle* SZR, makes 130 d. 8 m. for the whole *Angle* at Z.

CASE XI. *Three* Sides SZ, ZP *and* SP, *being given, to find an* Angle, *viz. the* Angle *at* Z.

In this *Case*, the *Side* opposite to the enquired *Angle* is the *Base*; and before the *Triangle* can be resolved, you must,

First, Add all the three *Sides* together, and so have you the Summ of them.

Secondly, Take half that Summ.

Thirdly, Take the Difference between the half Summ and the *Side* opposite to the inquired *Angle*. Thus,

The *Side*	SZ	40	00
	ZP	38	30
	SP	70	00
Their Summ		148	30
The half Summ		74	15
The Difference between the *Base* SP 70 d. and the half Summ 74 deg. 15 m. is		04	15

Having made this provision,

The Proportions will be,

(1)	R	:	*s* SZ	::	*s* ZP	:	*Sine*
	90 d.		40 d. 0′		38 d. 30′		23 d. 35′
(2)	*Sine*	:	*s* half Summ	::	*s* Difference	:	*Sine*
	23 d. 35′		74 d. 15′		4 d. 15′		10 d. 17′

(1) Extend the Compasses from the *Sine* of 90 d. to the *Sine* of 40 d. the same extent will reach from the *Sine* of 38 d. 30 m. to the *Sine* of 23 d. 35 m.

(2) The extent of the Compasses from the *Sine* of 23 d. 35 m. to the *Sine* of 74 d. 15 m. (the half Summ) will reach from the *Sine* of 4 d. 15 m. (the Difference) to the *Sine* of 10 d. 17 m.

(3) Divide the Space (upon the *Scale* of *Sines*) between 10 deg. 17 min. last found and 90 deg. and the Compass point will rest upon 24 deg. 56 min. whose *Complement* is 65 deg. 4 min. and that doubled makes 130 deg. 8 min. for the enquired *Angle* at Z.

Case

CASE XII. *The Three* Angles Z, S, *and* P, *given, to find any of the other* Sides.

This is but the converſe of the former *Caſe*, and may be reſolved in the ſame manner, for if either of the *Angles* adjacent to the required *Side* be turned into its *Complement* to 180 deg. thoſe *Angles* will be turned into *Sides*; and the *Sides* into *Angles*; and ſo the *Triangle* may be reſolved as in the laſt *Caſe*.

INSTRUMENTAL TRIGONOMETRY.

The Second WAY.

By the Cœleſtial GLOBE.

Spherical Triangles, *are beſt repreſented by the* Circles *upon, and thoſe appendent to the* Globes; *upon which the* Sides *and* Angles *of all* Spherical Triangles *are naturally repreſented, and moſt expeditiouſly* Meaſured.

IN the Solution of *Spherical Triangles* by the *Globe*; in *Right angled Triangles*, I call that *Side* which is oppoſite to the *Right Angle* the *Hypotenuſe*; and the other two *Sides* which comprehend the *Right Angle*, I call the *Sides* or *Legs*, without any diſtinct denomination of *Baſe* or *Perpendicular*, as in the other ways of reſolving of them I do, and that for ſeveral reaſons, which will hereafter appear; And,

In *Oblique Angled Spherical Triangles*, I call the *Sides Sides*, and the *Angles Angles*, without any other diſtinction.

In the Solution of *Right angled Spherical Triangles*, there are uſually 16 *Caſes*, which will all be reduced to Five; for that by the *Globe* three things are at once found. And in *Oblique angled Spherical Triangles*, there are uſual 12 *Caſes*, which by the *Globe* will be reduced to 6; the *Globe* anſwering two at one Poſition.

In the *Scheme* or *Figure* of the *Sphere* or *Globe*, it being in an *Oblique* Poſition, *viz.* elevated to the *Latitude* of *London*, 51 deg. 30 min. you have upon the Superficies thereof divers *Spherical Lines* and *Circles*, by the interſections whereof are conſtituted divers *Spherical Triangles*; ſome whereof are *Right angled*, and others *Oblique angled*: Of which I ſhall make uſe only of *Two*; one for the *Five Caſes* of *Right angled Triangles*, the other for the *Six Caſes* of *Oblique angled Triangles*. *Fig.* VI.

The Triangle *which I ſhall make uſe of for the* Five Caſes *of* Right angled Triangles, *ſhall be the* Triangle P O ⊙, Right angled *at* O, *and noted in the middle thereof with* I.

In which *Right angled Spherical Triangle* P O ⊙,

The *Side*
- P ⊙, being an *Arch* of the *Braſs Meridian*, we ſuppoſe to be the *Latitude* of *London*, 51 d. 30 min.
- ⊙ O, being an *Arch* of the *Horizon*, we ſuppoſe to be the *Amplitude* of the *Sun*'s Riſing or Setting, from the *North* part of the *Meridian* 56 d. 40 min.
- P ⊙, being the *Arch* of another *Meridian*, or *Hour Circle*, (which the *Equinoctial Colure* will beſt ſupply or repreſent) we ſuppoſe to be the *Sun*'s diſtance from the *Pole*, or the *Complement* of his *Declination* 70 d. 00 min.

The *Angle*
- ⊙ P O, is the *Hour* from Midnight, whoſe *Meaſure* is to be reckoned upon the *Equinoctial*, between the *Equinoctial Colure*, and the *Braſs Meridian*, and will be 62 d. 45 m. or in time 4 hours 11 m.
- P ⊙ O, is the *Angle* of the *Sun*'s *Poſition* at the time of the Queſtion, whoſe *Meaſure* is the *Arch* of a Great *Circle*, comprehended between the *Complement* of the *Side* ⊙ O, on the *Horizon*, and the *Complement* of the *Side* ⊙ P, on the *Equinoctial Colure*; both thoſe *Sides* being continued to *Quadrants*, on the contrary *Side* of the *Braſs Meridian*, and will contain 56 d. 39 min.
- P O ⊙, is the *Right Angle*, whoſe *Meaſure* is the deg. of the *Quadrant* of *Altitude*, contained between the *Eaſt* or *Weſt* points of the *Horizon* and the *Zenith*, which is 90 deg.

The ſeveral parts of the *Triangle* being thus declared, let us now proceed to the ſeveral *Caſes*, which ariſe out of this, and every *Right angled Spherical Triangle*.

SECT. I.

Of Right angled Spherical Triangles.

CASE I. *The two* Sides (*or* Legs) P O 51 *deg.* 30 *min. and* ⊙ O 56 *d.* 40 *min. being given, to find the* Hypotenuſe P ⊙, *and the* Angles ⊙ P O, *and* P ⊙ O.

SEeing that the *Side* P O is an *Arch* of the *Braſs Meridian*, and contains 51 deg. 30 min. count thereupon from the *Pole*, 51 deg. 30 min. and bring thoſe degrees to the *Horizon*. Then the *Side* ⊙ O being an *Arch* of the *Horizon*, and contains 56 deg. 40 min. count upon the *Horizon*, from the *Meridian*, 56 deg. 40 min. and turn the *Globe* about till the *Equinoctial Colure* do touch thoſe degrees of the *Horizon*; ſo ſhall you have the true *Triangle* perfectly deſcribed upon the *Globe* it ſelf: For from the *Pole* to the *Horizon*, upon the *Meridian*, is equal to P O; from the *Horizon* to the *Pole*, upon the *Colure*, is equal to P ⊙; And from the *Colure* to the *Meridian*, upon the *Horizon* is equal to ⊙ O. Now (1.) P O, being 51 d. 30 m. and O ⊙, 56 deg. 40 min. the degrees of the *Colure* counted from the *Pole* to the *Horizon*, will be found 70 degrees; for the *Hypotenuſe* P ⊙, which was required. (2.) For the *Angle* ⊙ P O, count the degrees of the *Equinoctial* which are contained between the *Colure* and the *Braſs Meridian*, and you ſhall find them to be 62 d. 45 min. or four hours eleven min. for the quantity of the *Angle* ⊙ P O, (3.) For the *Angle* P ⊙ O, count the *Complement* of ⊙ P 20 degrees upon the *Equinoctial Colure*, on the other *Side* of the *Meridian*; (ſo ſhall that *Point* be diſtant from the *Point* ⊙ 90 d.) Alſo count the *Complement* of the *Side* ⊙ O, *viz.* 33 d. 20 m. upon the *Horizon*, on the other *Side* of the *Meridian*, (which *Point* will be 90 d. diſtant from the *Point* ⊙, upon the *Horizon*;) ſo ſhall the diſtance between theſe two *Points*, (meaſured by the *Quadrant* of *Altitude*, or by Compaſſes,) contain 56 deg. 39 min. for the *Angle* P ⊙ O.

And now by this one *Data* you have reſolved three *Problems* of the *Sphere*; for you have found,

1. The *Side* P ⊙, the *Complement* of the *Sun*'s *Declination* 70 degrees.
2. The *Angle* ⊙ P O, the hour from *Midnight* 4 hours 11 minutes.
3. The *Angle* P ⊙ O, the *Angle* of the *Sun*'s *Poſition* 56 degrees 39 minutes.

Caſe

CASE II. *The* Hypotenuse P ⊙, 70 *degrees, and the* Side (*or* Leg) PO, 51 *deg.* 30 *min. being given, to find the other* Leg, O ⊙, *and the* Angles ⊙ PO, *and* O ⊙ P.

This *Case* differeth little from the former; for PO being an *Arch* of the *Meridian*, and containing 51 deg. 30 min. the *Meridian* being set thereto in the *Horizon*, turn the *Globe* about till 70 deg. of the *Equinoctial Colure* do touch the *Horizon*. So shall you find the degrees of the *Horizon* intercepted between the *Colure* and the *Meridian* to be 56 d. 40 min. for the *Side* ⊙ O. The *Angles* at P and ⊙, are to be found in all respects by their measures, as in the former *Case*.

And by this one *Data*, you have resolved three other *Problems* of the *Sphere*, for by it you have found,

1. ⊙ O. The *Sun's Amplitude* from the *North*, 56 deg. 40 min.
2. ⊙ PO. The hour from *Midnight* 62 deg. 45 min. or 4 hours 11 min.
3. P ⊙ O. The *Angle* of the *Sun's Position*, 56 deg. 39 min.
6. And if instead of the *Leg* PO, there had been given the *Leg* ⊙ O, you might have then found, [by bringing 70 degrees of the *Equinoctial Colure*, to meet with 56 d. 40 m. of the *Horizon*,]
4. PO. The *Latitude* 51 d. 30 min.
5. ⊙ PO. The hour from *Midnight*, 4 hours 11 min.
6. P ⊙ O. The *Angle* of the *Sun's Position* 56 d. 39 min.

CASE III. *The* Hypotenuse P ⊙, 70 *deg. and the* Angle P, 62 *deg.* 45 *min. being given, to find the two* Legs, PO *and* ⊙ O, *and the* Angle P ⊙ O.

Count the *Angle* P, 62 deg. 45 min. upon the *Equinoctial* from the *Colure*, and bring that *Point* to the *Brass Meridian*, and there keep the *Globe* fast in the *Meridian*; then move the *Brass Meridian* upwards or downwards in the *Horizon*, till 70 deg. of the *Equinoctial Colure* do justly touch the *Horizon*, then shall the *Brass Meridian* rest in the *Horizon*, at 51 deg. 30 min. for the *Side* PO, and the degrees of the *Horizon* intercepted between the *Equinoctial Colure*, and the *Meridian* will be 56 deg. 40 min. for the *Side* ⊙ O; and the distance between 33 deg. 20 min. of the *Horizon*, and 20 degrees of the *Equinoctial Colure* shall be 56 deg. 30 min. for the *Angle* P ⊙ O.

So that by this single *Data*, there are also three other *Problems* resolved, *viz.* there is found,

1. The *Side* PO, the *Latitude* 51 deg. 30 min.
2. The *Side* ⊙ O, the *Amplitude* from the *North* 56 d. 40 min.
3. The *Angle* P ⊙ O, the *Angle* of the *Sun's Position*.
6. And if instead of the *Angle* at P, the *Angle* at ⊙ had been given, you might then have found, [by counting 20 deg. upon the *Equinoctial Colure*, and to that *Point* apply 56 deg. 39 min. of the *Quadrant of Altitude*, (or a pair of Compasses opened to that distance;) then turn the *Meridian* in the *Horizon*, and the *Globe* about his *Axis*, till 70 deg. of the *Colure* and 00 deg. of the *Quadrant of Altitude* do touch the *Horizon* on either *Side* of the *Meridian*; for then will the *Meridian* rest in the *Horizon*, at 51 deg. 30 min.]
4. The *Side* ⊙ O, the *Amplitude* from the *North* 56 deg. 40 min.
5. The *Side* PO, the *Latitude* 51 d. 30 m.
6. The *Angle* ⊙ PO, the hour from *Midnight* 62 deg. 45 min.

CASE IV. *One* Leg ☉O, 56 *deg.* 40 *min. and one* Angle P☉O, 56 *d.* 39 *min. being given, to find the* Side PO, *the* Hypotenuse P☉, *and the* Angle ☉PO.

Count upon the *Horizon* from the *Brass Meridian* 56 deg. 40 min. the given *Side* (or *Leg*) ☉O, also from the *Brass Meridian*, on the contrary *Side* thereof, count 33 deg. 20 min. the *Complement* of ☉O, to which degrees apply 00 deg. of the *Quadrant of Altitude*. Then move the *Brass Meridian* up and down in the *Horizon*, till the *Equinoctial Colure* cuts 56 deg. 40 min. and 56 deg. 39 min. of the *Quadrant of Altitude* do intersect the *Colure*. So will the *Brass Meridian* rest in the *Horizon* at 51 deg. 30 min. For the *Side* PO, the *Horizon*, will cut the *Equinoctial Colure* in 70 deg. For the *Hypotenuse* P☉, and the degrees of the *Equinoctial* intercepted between the *Equinoctial Colure* and the *Brass Meridian*, will be 62 d. 45 m. for the quantity of the *Angle* ☉PO; which was required.

And by this *Data*, there are three *Problems* resolved; for there is found,

1. The *Side* PO, the *Latitude* 51 d. 30 m.
2. The *Angle* ☉PO, the hour from *Midnight* 62 d. 45 min.
3. The *Side* P☉, the *Sun*'s distance from the *Pole* 79 deg.

But if the given *Leg* had been ☉O, and the given *Angle* ☉PO, [by counting the *Angle* P upon the *Equinoctial*, and bringing that *Point* to the *Meridian*, and moving the *Meridian* up and down in the *Horizon*, till the *Equinoctial Colure* did touch 56 d. 40 min. of the *Horizon*,] you should then have found,

4. The *Hypotenuse* P☉, the *Sun*'s distance from the *Pole* 70 deg.
5. The *Side* PO, the *Latitude* 51 d. 30 m.
6. The *Angle* P☉O, the *Angle* of the *Sun*'s *Position* 56 deg. 39 min.

Also, if the *Hypotenuse* P☉, and the *Angle* ☉PO, had been given, [by counting the *Angle* ☉PO upon the *Equinoctial*, and bringing it to the *Meridian*, and moving the *Meridian* in the *Horizon*, till 70 deg. of the *Equinoctial Colure* did touch the *Horizon*,] you might then find,

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7. The *Side* ☉O, the *Sun*'s *Amplitude* from the *North* 56 deg. 40 min.
8. The *Side* ☉P, the *Sun*'s distance from the *Pole* 70 d.
9. The *Angle* P☉O, the *Angle* of the *Sun*'s *Position* 56 d. 39 m.

And again, if the given *Side* (or *Leg*) had been PO, and the *Angle* O☉P, [by setting the *Brass Meridian* to 51 deg. 30 m. in the *Horizon*, and turning the Body of the *Globe* about till the *Equinoctial Colure*, and the *Horizon* do make an *Angle* of 56 d. 39 min.] you might then find,

10. The *Hypotenuse* P☉, the *Sun*'s distance from the *Pole* 70 deg.
11. The *Side* ☉O, the *Sun*'s *Amplitude* from the *North* 56 deg. 40 min.
12. The *Angle* ☉PO, the hour from *Midnight* 62 deg. 45 min.

CASE V. *The two* Acute Angles ☉PO, 62 *deg.* 45 *min. and* P☉O, 56 *deg.* 39 *min. being given, to find the* Side PO, P☉ *and* ☉O.

Count the quantity of the *Angle* ☉PO, 62 deg. 45 min. upon the *Equinoctial*, from the *Equinoctial Colure*, bringing those degrees to the *Meridian*, and there fix the *Globe*; then move the *Meridian* upward or downward in the *Horizon*, till the intersection of the *Equinoctial* with the *Horizon* do make an *Angle* of 56 d. 39 m. So shall the *Side* PO be found to be 51 d. 30 m. the *Leg* ☉P 56 d. 40 m. and the *Hypotenuse* P☉ 70 deg.

And

And from this *Data*, three *Problems* are resolved; for you have found,

3 { 1. The *Side* P O, the *Latitude* 51 d. 30 m.
2. The *Side* ⊙ P, the *Amplitude* from the *North* 56 d. 40 min.
3. The *Hypotenuse* P ⊙, the *Sun*'s distance from the *Pole* 70 deg.

This last *Case* may best be resolved by changing of the *Angles* into *Sides*, as shall be hereafter taught.

These are the Five Cases *of* Right angled Spherical Triangles: *And here you see that in this one* Right angled Spherical Triangle, *by the several Parts given in these* Five Cases, *there are* 30 Spherical Problems *resolved: Namely*, 3 *by the* First Case, 6 *by the* Second Case, 6 *by the* Third Case, 12 *by the* Fourth Case, *and* 3 *by the* Fifth Case. *And so many are resolvable in every* Right angled Spherical Triangle.

SECT. II.

Of Oblique angled Spherical Triangles.

THE *Triangle*, which I shall make use of for the resolving of the *Six Cases* of *Oblique angled Spherical Triangles*, shall be the *Oblique Triangle* Z P E, *Obtuse angled* at Z; having 2 in the middle of it. *Fig.* VI.

In which *Triangle*,

The *Side*:
- Z P, being an *Arch* of the *Brass Meridian*, we will suppose to be the *Complement of the Latitude of London*, containing 38 d. 30 min.
- P E, being an *Arch* of a *Meridian* (or *Hour Circle*) we will suppose to be the *Sun*'s distance from the *Pole*, or the *Complement of the Sun*'s *Declination Northward* 70 deg.
- Z E, being an *Arch* of an *Azimuth* (or *Vertical Circle*) let be the *Complement* of the *Sun*'s, or a *Star*'s *Altitude*, and to contain 47 d. 12 min.

The *Angle*:
- Z P E, is the *hour from Noon*, whose *Measure* is to be reckoned upon the *Equinoctial*, between the *Equinoctial Colure* and the *Brass Meridian*, and will be found to be 45 deg. or 3 hours in time.
- E Z P, is the *Sun*'s *Azimuth* from the *North* part of the *Meridian*, and is to be *measured* upon the *Horizon* between the *North* part of the *Brass Meridian*, and the *Quadrant of Altitude*, and will be found to be 115 d. 6 m.
- Z E P, is the *Angle of the Sun*'s *Position* at the time of the Question, and may be *measured* either by turning of the *Quadrant of Altitude* to the contrary *Side* of the *Meridian*, and counting thereupon the *Complement* of the *Side* E Z 42 deg. 48 min. and the *Complement* of the *Side* Z P 20 deg. upon the *Colure*; and the distance of these two *Points* shall be the *Measure* of the *Angle* Z E P, which will be found to contain 36 deg. 52 min. Or you may turn the *Triangle*, and place Z E, in the place where before Z P was placed, and then may that *Angle* be *measured* upon the *Equinoctial* between the *Colure* and the *Brass Meridian*, as the *Angle* Z P E was.

The several *Sides* and *Angles* of this *Triangle* being discovered, I will now come to shew the several *Cases* which will arise out of this, and every *Oblique Spherical Triangle*; but first let me shew you,

How

How to expreſs the Triangle *upon the* Globe.

Elevate the *Pole* of the *Globe* to the *Complement* of any of the *Sides* of the *Triangle*, as in this Example, to 51 deg. 30 min. (which is the *Complement* of the *Side* Z P,) then count the *Side* Z P 38 deg. 30 min. from the *Pole*, and thereto ſcrew the *Quadrant of Altitude*. This done, count the *Side* E P 70 deg. upon the *Equinoctial Colure* from the *Pole*, and the *Side* Z E upon the *Quadrant of Altitude* from the *Zenith* downward, and ſo move the *Globe* and *Quadrant of Altitude* together, till the numbers counted upon both of them concur in one *Point*; and ſo ſhall you have your *Triangle* exactly delineated upon the *Globe*; which being done, I proceed to the *Six Caſes* of *Oblique angled Spherical Triangles*, and to ſhew the *Variety of Problems* that will naturally ariſe out of the ſolving of every ſuch *Oblique Triangle*.

CASE I. *The* Side Z P 38 *d.* 30 *m. The* Side E P 70 *d. and the* Side Z E 47 *d.* 12 *m. being given, to find the* Angles.

Elevate the *Globe* to the 51 deg. 30 min. the *Complement* of Z P, and count the *Side* Z P, the *Complement* of the *Latitude* 38 deg. 30 min. from P to Z, and there faſten the *Quadrant of Altitude*. Then count the *Side* E P, the *Complement* of the *Sun's Declination* 70 deg. upon the *Colure* from the *Pole* downwards: Alſo count the *Complement* of the *Sun's Altitude* 47 deg. 12 min. The *Side* Z E, upon the *Quadrant of Altitude* downwards; and move the *Globe* and *Quadrant* together till 70 d. of the *Colure*, and 47 d. 12 m. of the *Quadrant* do meet: So is your *Triangle* repreſented upon the *Globe*.

Now to find the ſeveral *Angles*. (1.) For the *Angle* at Z, count the number of degrees of the *Horizon*, which are contained between the *North* part of the *Meridian*, and the *Quadrant of Altitude*; and you ſhall find them to be 115 de. 6 m. and that is the quantity of the *Angle* E Z P, and is the *Sun's Azimuth* from the *North* part of the *Meridian*. (2.) For the *Angle* Z P E, count the number of degrees of the *Equinoctial*, which are contained between the *South* part of the *Meridian* and the *Colure*; which you will find to be 45 deg. and that is the quantity of the *Angle* Z P E, which is the hour from *Noon*, namely Nine in the Morning, or Three in the Afternoon. (3.) For the *Angle* Z E P, (either change the *Triangle* by elevating the *Globe* anew, or) count 42 deg. 48 min. the *Complement* of Z E, upon the *Quadrant of Altitude*, (it being brought to the other *Side* of the *Meridian*,) and alſo count the 20 deg. the *Complement* of E P, upon the *Colure*, (on the other *Side* of the *Meridian*,) the diſtance between theſe two *Points*, *meaſured* by Compaſſes or otherwiſe, will be found to contain 36 deg. 52 min. equal to the *Angle* Z E P, which is the *Angle* of the *Sun's* or *Star's Poſition* at the time of the Queſtion.

And thus by this one *Data*, you have reſolved three *Problems*: For you have found,

3. { 1. The *Angle* E Z P, the *Sun's Azimuth* 115 d. 6 m.
   { 2. The *Angle* Z P E, the hour 45 d. or 9, or 3 a Clock.
   { 3. The *Angle* Z E P, the *Sun's Angle* of *Poſition* 36 d. 52 m.

Caſe

CASE II. *The two* Sides E Z 47 *deg.* 12 *min.* Z P 38 *deg.* 30 *min. and the* Angle *comprehended between them* E Z P, 15 *d.* 6 *m. being given, to find the other Parts of the* Triangle.

Count P Z 38 deg. 30 min. (the *Globe* being elevated to the *Complement* thereof) upon the *Meridian*, from P to Z, and there fix the *Quadrant of Altitude*: Then from the *North* part of the *Meridian* upon the *Horizon*, count the quantity of the *Angle* E Z P, 115 deg. 6 min. and thereto bring the *Quadrant of Altitude*: Then count the other given *Side* E Z 47 deg. 12 min. upon the *Quadrant of Altitude* downwards; and turn the *Globe* about till the *Equinoctial Colure* cut the *Quadrant of Altitude* in 47 deg. 12 min. So is your *Triangle* delineated upon the *Globe*.

Now to find the several Parts. (1.) The degrees of the *Colure* contained between the *Pole* and the *Quadrant* being 70, is the *Side* E P. (2.) The degrees of the *Horizon* between the *Meridian* and *Quadrant*, being 115 deg. 6 min. is the *Angle* E Z P, and (3.) The distance between the *Complement* of E Z, upon the *Quadrant of Altitude*, and the *Complement* of E P upon the *Colure* (they being both continued to *Quadrants*, on the other *Side* of the *Meridian*) will be found 36 degrees 52 minutes, which is the *Angle* Z E P.

Thus by this *Data* you have found,

1. E P, The *Complement* of the *Sun's Declination.*
2. E Z P, The *Sun's Azimuth* from the *North.*
3. Z E P, The *Angle* of the *Sun's* or *Star's Position.*

But if the *Sides* Z P, P E, and *Angle* Z P E, had been given, there would then have been found,

4. P E Z, The *Angle* of *Position.*
5. E Z, The *Complement* of the *Sun's Altitude.*
6. E Z P, The *Azimuth* from the *North.*

Or if the *Sides* Z E, P E, and *Angle* Z E P, had been given, there would then be found,

7. E Z P, The *Sun's Azimuth* from the *North.*
8. Z P, The *Complement* of the *Latitude.*
9. Z P E, The *Hour* from *Noon.*

CASE III. *The two* Angles E Z P, 15 *deg.* 6 *min. and* Z P E, 45 *d. with the* Side Z P 38 *d.* 30 *m. comprehended between them, given, to find the other parts of the* Triangle.

Count 38 deg. 30 min. the *Side* Z P, upon the *Meridian* from the *Pole*, and thereto screw the *Quadrant of Altitude*; then count 45 deg. the *Angle* Z P E, upon the *Equinoctial*, (beginning at the *Colure*) and bring those 45 degrees to the *South* part of the *Brass Meridian*. Again, count 115 deg. 6 min. the *Angle* E Z P, upon the *Horizon*, from the *North* part of the *Meridian*, and thereto bring the *Quadrant of Altitude*. And so have you represented your *Triangle* upon the *Globe*.

Now to find the other *Sides*, and *Angle*. (1.) The degrees contained between the *Pole* and the *Colure* upon the *Quadrant*; namely, 47 deg. 12 min. will be the *Side* Z E. (2.) And the degrees between the *Pole* and the *Quadrant* upon the *Colure*, *viz.* 70, will be the *Side* E P. (3.) And the distance between the *Complements* of the *Sides* E Z, and E P, being continued to *Quadrants* on the other *Side* of the *Meridian*; namely, 36 d. 52 min. will be the *Angle* Z E P.

And

And by this *Data*, you have found,

1. Z E, The *Complement* of the *Sun*'s *Altitude*.
2. E P, The distance of the *Sun* from the *Pole*.
3. Z E P, The *Angle* of the *Sun*'s *Position*.

But if the *Side* E P, and the *Angles* Z E P, and Z P O, had been given, there would have been found,

4. E Z, The *Complement* of the *Sun* or *Star*'s *Altitude*.
5. E Z P, The *Sun*'s or *Star*'s *Azimuth* from the *North*.
6. Z P, The *Complement* of the *Latitude*.

9.

And if the *Side* Z E, and the *Angles* P Z E, and P E Z, had been given, you might then find,

7. Z P, The *Complement* of the *Latitude*.
8. Z P E, The *Hour* from *Noon*.
9. P E, The *Complement* of the *Sun*'s or *Star*'s *Declination*.

CASE IV. *The two* Sides Z P, 38 *deg.* 30 *min. and* E P, 70 *deg. with the* Angle Z E P, 36 *deg.* 52 *min. being given, to find the other* Side *and* Angles.

To resolve this *Case*, you may (if you please) revert the *Triangle*, by elevating the *Globe* to 20 deg. the *Complement* of the given *Side* P E, then counting 70 d. the *Side* E P, upon the *Meridian* from the *Pole*, there screw the *Quadrant of Altitude*. Then upon the *Horizon*, count 36 deg. 52 min. the given *Angle* E, and to them bring the *Quadrant of Altitude*. So have you fixed the *Angle* Z E P. Then turn the *Globe* about, till 38 deg. 30 min. the other given *Side* P Z, do touch the *Quadrant*, which it will do in 47 deg. 12 min. for the *Side* E Z. And for the *Angle* P, the degrees of the *Equinoctial* between the *Meridian* and the *Colure*, *viz.* 45, is the *Angle* at P, and for the *Angle* at Z, it may be found by reverting of the *Triangle* again, or by the *Complements* of the *Sides* extended on the other *Side* of the *Meridian*, as hath been before taught. Lastly, for the *Side* E Z, you have it upon the *Quadrant* 47 deg. 12 min.

The *Triangle* being thus delineated, you have found (1.) the *Side* E Z 47 d. 12 m. (2.) the *Angle* P, 45 d. (3.) the *Angle* E Z P, 115 d. 6 min.

¶ I thought good in this place to insert this manner of Change, not only for variety, but because in this and the next *Case* the *Triangle* may be more readily delineated upon the *Globe*.

And in this one *Data* you have found,

1. E Z, The *Complement* of the *Sun*'s *Altitude*.
2. Z P E, The *Hour* from *Noon*.
3. E Z P, The *Azimuth* of the *Sun* or a *Star* from the *North*.

But if the *Sides* Z P, and E P, and the *Angle* E Z P, had been given, you would then have found,

4. E Z, The *Complement* of the *Sun*'s or *Star*'s *Altitude*.
5. Z E P, The *Angle* of the *Sun*'s or *Star*'s *Position*.
6. Z P E, The *hour* from *Noon*.

And if there had been given E P, E Z, and E Z P, then would have been found,

7. Z P, The *Complement* of the *Latitude*.
8. Z P E, The *Hour* from *Noon*.
9. Z E P, The *Angle of Position*.

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In like manner, if P E, E Z, and Z P E, had been given, you would have found,

10. Z P, The *Complement* of the *Latitude*.
11. E Z P, The *Azimuth* from the *North*.
12. Z E P, The *Angle* of *Position*.

Again,

Again, if there had been given E Z, Z P, and Z P E, you might then find,

13. P E, The *Sun*'s diſtance from the *Pole*.
14. Z E P, The *Angle* of *Poſition*.
15. E Z P, The *Azimuth* from the *North*.

Laſtly, If E Z, Z P, and the *Angle* Z E P, had been given, there would have been found,

16. P E, The *Complement* of the *Sun*'s or *Star*'s *Declination*.
17. Z P E, The *Hour* from *Noon*.
18. E Z P, The *Azimuth* from the *North*.

CASE V. *The two* Angles E Z P, 115 *deg.* 6 *min.* Z P E, 45 *deg. with the* Side P E, 70 *deg. being given, to find the reſt of the* Triangle.

Elevate the *Globe* to 20 deg. the *Complement* of the *Side* E P. Then count the given *Side* E P, 70 deg. upon the *Meridian*, from P to Z, and there fix the *Quadrant*. Then the given *Angle* E P Z, being 45 deg. count 45 degrees upon the *Equinoctial* from the *Colure*, and bring that *Point* under the *Meridian*: So have you conſtituted the *Side* E P, and the *Angle* E P Z. Now for the *Angle* E Z P, you muſt either revert the *Triangle* again, or find it by the *Complements* of the *Sides* Z E, and Z P, extended to *Quadrants* on the other *Side* of the *Meridian*, which you ſhall find to be 115 deg. 6 min.

The *Triangle* thus conſtituted, (1.) For the *Side* Z P, the degrees of the *Colure* between the *Pole* and the *Quadrant* being 38 deg. 30 min. is that *Side*. (2.) The degrees of the *Quadrant of Altitude*, between the *Zenith* and the *Colure*, being 47 deg. 12 min. is the *Side* E Z. (3.) The degrees of the *Horizon* between the *Meridian* and the *Quadrant of Altitude* being 36 deg. 52 min. is the quantity of the *Angle* Z E P.

By which ſingle *Data*, you have found,

1. Z P, The *Complement* of the *Latitude*.
2. E Z, The *Complement* of the *Altitude*.
3. Z E P, The *Angle* of *Poſition*.

But if the *Angles* E Z P, Z P E, and the *Side* Z E, had been given, then might you find,

4. Z P, The *Complement* of the *Latitude*.
5. P E, The *Sun*'s diſtance from the *Pole*.
6. Z E P, The *Angle* of *Poſition*.

Or had there been given Z P E, Z E P, and the *Side* E P, you might from thence find,

7. Z P, The *Complement* of the *Latitude*.
8. E P, The *Complement* of the *Sun* or *Star*'s *Declination*.
9. E Z P, The *Sun*'s *Azimuth* from the *North*.

And if the *Angles* Z P E, Z E P, with the *Side* Z P, had been given, then would have been found,

10. Z E, The *Complement* of the *Sun*'s *Altitude*.
11. P E, The *Sun*'s diſtance from the *Pole*.
12. E Z P, The *Sun*'s *Azimuth* from the *North*.

But if the *Angles* Z E P, E Z P, and the *Side* Z P, had been given, then you might find,

13. Z E, The *Sun*'s diſtance from the *Zenith*.
14. P E, The *Sun*'s diſtance from the *Pole*.
15. Z P E, The *Hour* from *Noon*.

And laſtly, had there been given the *Angles* Z E P, E Z P, and the *Side* P E, then might be found,

16. Z E, The *Complements* of the *Sun*'s or *Star*'s *Altitude*.
17. Z P, The *Complement* of the *Latitude*.
18. Z P E, The *Hour* from *Noon*.

CASE VI. *The Three* Angles E Z P, 115 *deg.* 6 *min. the* Angle E P Z, 45 *deg. and the* Angle Z E P, 36 *deg.* 52 *min. being given, to find the three* Sides.

This *Case* may beſt be reſolved by turning the *Angles* of the *Triangle* into *Sides*, as ſhall be taught by and by; and ſo by this *Data*, you may find,

1. Z E, The *Complement* of the *Sun's Altitude.*
2. Z P, The *Complement* of the *Latitude.*
3. E P, The *Complement* of the *Sun's Declination.*

And thus have you out of this one Oblique Spherical Triangle, *by the variety that theſe* Six Caſes *afford, no leſs than Threeſcore* Problems *reſolved, viz.* 3 *in the* Firſt Caſe, 9 *in the* Second Caſe, 9 *in the* Third Caſe, 18 *in the* Fourth Caſe, 18 *in the* Fifth Caſe, *and* 3 *in the* Sixth Caſe; *in all* 60. *And ſo many* Varieties *or* Changes *are there in every* Oblique angled Spherical Triangle. *And beſides theſe* Varieties, *this* Triangle Z E P, *is not peculiar only to the appellations that I have here given them, but to other purpoſes alſo. For,*

This *Oblique angled Triangle* is not capable only of reſolving the forementioned *Aſtronomical Queſtions*, but may be applied to *Geographical* or *Nautical Queſtions* alſo. For,

1. The *Side* Z P, may repreſent the *Complement* of the *Latitude* of that *Town* or *City* whoſe *Zenith Point* is Z.
2. The *Side* E Z, may repreſent the diſtance between thoſe two *Cities* or *Towns.*
3. The *Side* P E will be the *Complement* of the *Latitude* of that *City* or *Town* at E.
4. The *Angle* E P Z, is the Difference of *Longitude* between the two places E and Z.
5. The *Angle* P Z E, the *Point* of the *Compaſs* leading from Z to E.
6. The *Angle* P E Z, the *Point* of the *Compaſs* leading from E to Z.

And in this *Triangle* the ſame things being given, they may be varied as before, and afford as many *Queſtions* in *Geography* or *Navigation* as in *Aſtronomy*: Namely 60.

Moreover, in the ſame *Triangle*, if you imagine the *Side* P Z to be 23 deg. 30 min. the diſtance of the *Pole* of the *World*, from the *Pole* of the *Ecliptick*: Then will,

1. The *Side* Z P, be the diſtance of the *Pole* of the World from the *Pole* of the *Ecliptick.*
2. The *Side* P E will be the *Complement* of the *Declination* of a *Star* at E.
3. The *Side* Z E, will be the *Northern Latitude* of the *Star* at E.
4. The *Angle* E P Z, will be the *Complement* of the *Star's Right Aſcenſion.*
5. The *Angle* P Z E, is the quantity of the *Star's Longitude.*
6. The *Angle* P E Z, is the *Angle* of the *Star's Poſition.*

And this way it will afford 60 Varieties more, as is before intimated.

Theor.

THEOREM. *The* Sides *of any* Spherical Triangle, *may be turned into* Angles; & contra, *the* Complement *of the Greateſt* Side, *or Greateſt* Angle (*to a* Semicircle) *being taken for the Greateſt* Side, *or Greateſt* Angle.

Demonſtration.

LET A B C, be a *Spherical Triangle Obtuſe angled* at B. And let

D E, H I, G F } be the *Meaſure* of the *Angle* { A, C, B *viz.* of its *Complement*, F B G, it being the *Angle* of the *Triangle*.

Now,

K L, L M, K M } is equal to { D E, F G, H I

Becauſe

K D, L G, K I } and { L E, F M, M H } are *Quadrants*, and their Common *Complement* is { L D, L F, K H.

Therefore, The *Sides* of the *Triangle* K L M, are equal to the *Angles* of the *Triangle* A B C, taking for the Greateſt *Angle* A B C, the *Complement* thereof F B G.

It may alſo be *demonſtrated*, that the *Sides* of the *Triangle* A B C, are equal to the *Angles* of the *Triangle* K L M, by the converſe of the former. For,

The *ſide* { A B, B C, A C } is equal to { O P, F H, D I } the *Meaſure* of the *Angle* { M L K. L M K: D K I. } Of the *Complement* of the *Obtuſe Angle* M K L.

For,

A D, A P, B F } and { C I, O B, C H } are *Quadrants*, and their Common *Complement* is { C D, A O, C F.

Therefore, The *Sides* may be turned into *Angles*, and the contrary; which was to be demonſtrated. And by this Converſion, may the *Fifth Caſe* of *Right*, and the *Sixth Caſe* of *Oblique Spherical Triangles* be reſolved.

The *Demonſtration* of this *Theorem* is handled at large, in the affections of *Spherical Triangles*, Chap. I. *Paragraph* 7, of *Spherical Triangles*. Pag.

How a Perpendicular *is to be let fall in any* Oblique Spherical Triangle, *thereby dividing it into two* Right angled Triangles.

THis *Problem* is not of any Uſe in the Solution of *Triangles* by the *Globe*, as is evident by what hath been already delivered concerning the ſolving of them: But for that in *Trigonometrical Calculations*, there is a neceſſity for ſo doing, and the doing of it no leſs difficult to conceive or imagine; and ſeeing how naturally, and lively it is repreſented upon the *Globe*, I will therefore here inſert it; and the *Demonſtration* of it depends upon this *Theorem*.

If two Great Circles *of the* Sphere *ſhall paſs through each others* Poles, *thoſe two* Circles *ſhall cut each the other at* Right Angles.

In the *Oblique angled Triangle* Z E P, before made uſe of, let there be given, (1.) the *Side* P Z. (2.) The *Side* E Z, and (3.) The *Angle* E P Z, and let it

be required to find the *Side* P E. This *Problem* is done by the *Globes*, by the *Fourth Case*, as you may there see, without the help of any *Perpendicular*; but in *Calculation* it is wholly necessary, and therefore may well deserve the place of a *Problem* here.

The *Triangle* being described upon the *Globe*, let it be required to let fall a *Perpendicular* from the *Angle* Z, upon the *Side* E P: The *Measure* of the *Given Angle* P, being upon the *Equinoctial*, see upon the *Globe* where the *Side* P E (being extended) cuts the *Equinoctial*; and from that *Point* count 90 degrees upon the *Equinoctial*, and that *Point* shall be the *Pole* of the *Circle* (P E.) (*For the Poles of all Great* Circles *are a* Quadrant, *or* 90 *degrees distant from their Peripheries:*) wherefore the *Quadrant of Altitude* being fixed in Z, bring it to this *Point* found in the *Equinoctial*, and then will it cut the *Side* E P in the *Point* where the *Perpendicular* must fall, which will be at K, 29 degrees 21 minutes distant from P.

This *Theorem* is also *Demonstrated* in the forementioned Chap. I.

CONCLUSION.

IN the foregoing *Precepts*, I have made use only of two *Triangles* for the Solution of the *Five Cases* of *Right*, and the *Six Cases* of *Oblique angled Spherical Triangles*; namely, of the *Triangle* P O ⊙ for *Right angled*, and Z E P for *Oblique angled*. Yet in the *Figure* of the *Globe* by the Intersection of the several *Circles* thereof, there are divers other *Triangles* (both *Right* and *Oblique*) constituted, all which the foregoing *Rules* and *Cases* will sufficiently *solve*. A few of the Principal I will mark in the *Globe* by numerical *Figures*, for distinction, and give you an account of what *Circles* they are composed, and what *Questions* of the *Globe* are *Resolvable* thereby.

The *Triangle* that I have made use of for *Right angled Triangles*, *viz.* P ⊙ O, I have marked with the *Figure* 1. And the *Oblique Triangle* Z E P, with the *Figure* 2; and shall say no more of them, having sufficiently dealt with them already; but come to give you an account of such other as I have marked in the *Figure*. And,

The first that I shall take notice of is, the *Triangle* A K M, marked with the *Figure* 3. *Right Angled* at M, which is constituted of,

A K an *Arch* of the *Equinoctial*.
A M an *Arch* of the *Ecliptick*, and
K M an *Arch* of a *Circle of Longitude* passing through the *Poles* of the *Ecliptick*.

In this *Triangle*,

The *Side* A M, is the *Sun*'s or a *Star*'s *Longitude*, or distance from the *Equinoctial* Point A.

The *Side*
- K M is the *South Latitude* of a *Star* at M, or the *Suns South Declination*.
- A K is the *Right Ascension*.

The *Angle*
- K A M is the *Sun*'s *greatest Declination*.
- A K M, the *Angle* the *Circle of Longitude* makes with the *Equinoctial*.
- A M K is a *Right Angle*.

The second *Triangle* that I shall take notice of is A ⊙ B, *Right angled* at B, and marked with the *Figure* 4; and which is constituted of

A ⊙, an *Arch* of the *Horizon*.
A B, an *Arch* of the *Equinoctial*, and
B ⊙, an *Arch* of a *Meridian* or *Hour-circle*.

In

In this *Triangle*,

The *Side* { A ☉ is the *Amplitude* of the *Sun's Rising* or *Setting* from A, the *East* or *West Points* of the *Horizon*.
B ☉ is the *Sun's Declination North*.
A B is the *Ascensional Difference*.

The *Angle* { ☉ B A is a *Right Angle*.
☉ A B is the *Complement* of the *Latitude*.
A ☉ B is the *Angle* of the *Sun's Position* at his *Rising*.

The third *Triangle* that I here take notice of, is the *Triangle* A L F, *Right angled* at F, and marked with the *Figure* 5. The which is constituted of

A L, an *Arch* of the *Equinoctial Colure*.
A F, an *Arch* of the *Horizon*, and
L F, an *Arch* of an *Azimuth* or *Vertical Circle*.

In this *Triangle*,

The *Side* { A F is the *Sun's Azimuth* being *East* or *West*.
A L is the *Sun's Declination North*.
L F is the *Sun's Altitude* at Six a Clock.

The *Angle* { L F A is a *Right Angle*.
L A F is the *Latitude*.
A L F is the *Angle of Position*.

The Fourth and Last *Triangle* that I shall mention, is A D C, *Right angled* at C, and marked with the *Figure* 6, and is composed of

A D, an *Arch* of the Prime *Vertical Circle*, or *Azimuth* of *East* or *West*.
A C, an *Arch* of the *Equinoctial*.
D C, an *Arch* of a *Meridian* or *Hour Circle*.

In which *Triangle*,

The *Side* { A D is the *Sun's Altitude* when he is due *East* or *West*.
A C is the *Ascensional difference*.
D C is the *Sun's Declination North*.

The *Angle* { A D C is a *Right Angle*.
C D A an *Angle of Position*.
D A C is the *Latitude*.

Thus have you an account of Four other *Right angled Spherical Triangles*, with the *Affections* or *Natures* of their Respective *Sides* and *Angles*; out of each of which, by the Varieties that will arise from the *Five Cases*, may be deduced 30 *Problems*, and in all the *Four Triangles* 120 *Problems*, all which may be performed according to the Directions of the *Five Cases* of *Right angled Spherical Triangles*; the Exercising whereof I leave to the Practitioner. There are divers other *Triangles* may be found both *Right* and *Oblique angled*, but these as the Principal I commend to the *Practice* of the young *Tyro*.

Instru-

INSTRUMENTAL TRIGONOMETRY.

The Third WAY.

By the Planisphere ANALEMMA.

SECT. I.

Of Right Angled Triangles.

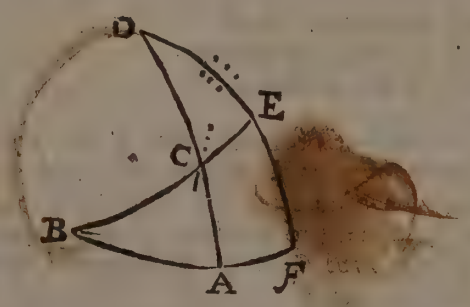
(1) THE *Hypotenuse* is represented always upon the *Index*.

(2) The *Right Angle* at the Section of any *Meridian* and the *Equinoctial*.

(3) The one *Leg* in the *Equinoctial*, and the other in a *Meridian*.

(4) One *Angle* only entering the *Question*, is represented at the *Centre*, between the *Equinoctial* and the *Index*, and is numbered in the Limb.

(5) But two *Angles* entering the *Question*, you must turn the *Case*, into an *Angle*, and its opposite Leg; for in the *Triangle* A B C, whose *Right Angle* is A, the other two *Angles* A B C and A C B, both entering the Question, you must lengthen the *Hypotenuse* to a full *Quadrant*, to E, as also the Legs adjacent to that



extremity of the *Hypotenuse* from which it was lengthened, as in this *Scheme*, the *Leg* A C adjacent to C, from which extremity the *Hypotenuse* was lengthened: And so in the *Triangle* C D E *Right angled* at E. The *Angle* D C E is equal to B C A, and the *Leg*, D E is equal to the *Complement* of the other *Angle* at B.

SECT. II.

Of Oblique Angled Spherical Triangles.

(1) *Three* Sides *being given, to find an* Angle *opposite to any of them.*

Reckon the greatest *Leg* from the *Pole* upon the Limb, and where it endeth apply the *Index*; upon this Index reckon from the Limb, the *Base*, (that is, the *Side* opposite to the *Angle* sought,) and to the *Point* of the Index where this *Base* endeth apply the *Square*. Then look where this *Square* cutteth the *Parallel* of the lesser *Leg*, to be reckoned from the *Poles*, and observe what *Meridian* passeth by this

Section

Section of the *Square* and *Parallel*, for where this *Meridian* cutteth the *Equinoctial*, there you have the *Measure* of the *Angle* ſought, to be reckoned from thence to the Limb.

(2) *Three* Angles *given*, —— turn them into *Sides*, and do with them as with the *Sides*.

(3) *The parts given and ſought being altogether oppoſite.*

Reckon the greater of the two firſt terms upon the Index, and where this numeration endeth upon the Index, apply that *Point* to the leſſer of the ſaid two firſt terms, which *Parallel* is to be reckoned from the *Equinoctial*, then order the terms, reckoning the firſt and third both upon the *Index*, or both upon the *Parallels*, and ſo likewiſe do with the ſecond and fourth terms.

(4) *Two* Sides *with an* Angle *between them given, to find the third* Side.

Reckon the greater *Side* given from the *Pole* upon the Limb, and to the end of it apply the Index: Reckon the other *Side* given upon the *Parallels* from the *Pole*: and the *Angle* given upon the *Equinoctial* from the Limb, and the *Meridian* it comes to, purſue till you come to the *Parallel* of the leſſer *Leg*, and to this Section of the ſaid *Meridian* and *Parallel* apply the *Square* or *Curſor*, ſo it may juſtly lye on the Index, and mark what *Point* of the Index it pointeth out, for this *Point* of the Index counted from the Limb, is the third *Side* required. After the third *Side* is found, you may find either of the unknown *Angles* by the *Rule* of oppoſite parts.

(5) *Two* Sides *and an* Angle *oppoſite to the Leſſer of them given, to find the third* Side.

Reckon the Difference of the given *Sides*, and the Summ of them, (one and the ſame way) from the *Pole*, and mark the *Points* where both of them do end. Count alſo the *Angle* given upon the *Equinoctial* from the *Limb*, and mark what *Meridian* it cometh to. Then extending a Streight *Line* between the two *Points* marked in the Limb, it will cut the ſaid *Meridian* in two places; ſo you are to obſerve the *Parallels*, where the Streight *Line* cutteth the ſaid *Meridian*, for theſe being reckoned from the *Pole*, will give you the quantity of the third *Side* required; for this ſame third *Side* may be of a two-fold quantity; the leſſer it is, when the *Angle* oppoſite to the greater given Side is *Obtuſe*: And the Greater it is, when the ſaid *Angle* (oppoſite to the greater given *Side*) is *Acute*.

(6) *Two* Sides *and an* Angle *oppoſite to the greater of them given, to find the third* Side.

Reckon the Difference of the given *Sides* from the *Pole* one way, and the Summ of the ſame the other way. Count alſo the given *Angle* upon the *Equinoctial*, and extend the Streight *Line* cutting the *Meridian* as in the foregoing Section, for now it will cut but once, and ſo the third *Side* admitteth only of one ſingle Anſwer.

FINIS.

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I. In *Lon*
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I. In *Pla*
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II. In *M*
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metrical

By

CURSUS MATHEMATICUS.

BOOK VI.

Containing the

DESCRIPTION and USE

OF

Geometrical Instruments:

AND THE

DOCTRINE

OF

PLAIN (or Right-Lined) TRIANGLES,

Applied to Practice.

IN THREE PARTS

I. In *Longimetria*; or, The Measuring of Heights and Distances; and in the Describing of Cities, Towns, &c.

II. In *Planometria*; or, The Surveying of Land divers ways, and finding the Content or Quantity of the same.

III. In *Military Architect*, or *Fortification*; and to find the Lengths and Quantities of all the Lines, Angles, and other Members of a Fort, and the Out-Works belonging to the same, by *Trigonometrical Calculation*.

By *WILLIAM LEYBOURN*, Philomat.

LONDON,
Printed ANNO DOM. MDCXC.

THE DESCRIPTION and USE OF *GEOMETRICAL INSTRUMENTS*, And the DOCTRINE of PLAIN (or Right-Lined) TRIANGLES, Applied to PRACTICE IN LONGIMETRIA, Or in the Measuring of Heights and Distances.

PART I.

CHAP. I.

A Description of such Instruments *as are necessary for the Performance of the Works following.*

BEfore I come to apply the *Doctrine* of Right-lined *Triangles* to Practice, in the taking of *Heights*, *Depths*, *Distances*, *&c.* and in *Surveying* of *Land*; it will be neceſſary to deſcribe what *Inſtruments* are moſt proper for thoſe purpoſes, and the general Uſes of them.

The Inſtruments are,

1. *Compaſſes.*
2. A *Ruler* of Braſs, or Wood, with a *Diagonal Scale*, and divers *Plain Scales* of *Equal Parts*, and two or three Scales of *Chords.*
3. A *Protractor* and *Protracting Pin.*
4. A *Quadrant* for *Altitudes* or *Deſcents.*
5. A *Theodolite*, with its Appurtenances, for *Diſtances*, and *Surveying* of *Land.*
6. *Meaſures* to Meaſure withal.

SECT. I. *Of the Compaſſes.*

COmpaſſes are ſo well known to all Men, that it may ſeem ſuperfluous to ſay any thing concerning them: But the *Compaſſes* fit for theſe and ſuch like Purpoſes, are of two ſorts, both of them of *Braſs*, with *Steel Points*, but one Pair to have three Movable *Points*, to take in and out of a Nut in one of the Legs for that purpoſe provided, and a Screw to keep the Point faſt in.—— One of the Movable Points is in all reſpects anſwerable to the other Fixed one; but the other two are different; the one being hollow, to put a Black-lead Pencil in; and the other is a Steel Pen, to be uſed with Ink. And ſo much concerning *Compaſſes*: I need ſay nothing concerning the Uſe of them, it being ſufficiently known to all; but ſee that your Points be of good Steel, and ſharp; and that the Joint move evenly, and not by Starts.

SECT. II. *Of the Scales, their Deſcription and Uſe.*

The Deſcription.

Fig. I. YOur *Scale* may be made either of *Silver*, *Braſs*, *Ivory*, or *Box*, of what length or breadth you pleaſe; but a convenient length will be eight or nine Inches, and in breadth one Inch and a quarter.

On one ſide thereof let be placed divers *Scales* of *Equal Parts*, as of 10, 11, 12, 16, 20, 24, and 30 in an Inch.

¶ *Note*, That when I ſay, a Scale of 12 in an Inch, you are to underſtand a Part of a Line divided into 10 Equal Parts, 12 of which Parts will make a juſt Inch; and the like of 16, 24, *&c.*

On the ſame Side of the Ruler let be placed two or three *Scales* of *Chords*, of ſeveral lengths, extended up to 90 *deg.*

On the other ſide of the Ruler let there be two *Diagonal Scales*, one of an *Inch*, the other of *Half an Inch*; out of which you may very accurately take the hundredth Part of an Inch or half an Inch.

The Uſe of the Scales.

The Uſe of theſe Scales is two fold, *viz.* (1.) To *lay down* any *Meaſure* taken; or, (2.) A Line being *laid down*, To find how much of that *Meaſure* that Line containeth.

I. *A* Diſtance *meaſured, how to lay down the ſame upon* Paper *according to any* Scale.

Fig. II. Suppoſe that meaſuring upon the Ground, you have meaſured 62 Foot, Yards, Perches, or any other Meaſure whatſoever; and you would lay this Line down upon Paper, by the Scale of 12 in an Inch. Firſt, draw a Right Line, as A B, and then with your Compaſſes ſet one Foot upon 6, (in the Scale of 12,) which now repreſents 60, and extend the other Foot to two of the ſmaller Diviſions in the uppermoſt ſubdivided Part, which Diſtance ſet upon the Line A B from A to C: So ſhall the Line A C repreſent 62 Foot, (or any other Meaſure,) it being laid down by the *Scale* of 12.

But if you would have your Line ſhorter, and yet to contain 62; then uſe ſome other of your *Scales*, as that of 16 in the ſame manner; and then 62 will reach from A to D: And if you would have it yet ſhorter, take it from the *Scale* of 24, and it will reach to E; or from the *Scale* of 30, and it will reach to F.

II. *A Right Line (of any Meaſure) being laid down, to find how much it containeth of that Meaſure, according to any Scale.*

Suppſe A C were a Line laid down, and it were required to know how many Feet it contained according to the *Scale* of 12.

Take the Length of the Line A C in your Compaſſes, and applying it to the *Scale* of 12, you ſhall find it to reach from 6 (or 60) to two of the ſmall Diviſions in the uppermoſt

uppermoſt ſub-divided Part, which makes it 62: And ſo much doth the Line A C contain by the *Scale* of 12. Alſo,

The Line { A D / A E / A F } will be found to contain 62, being applied to the Scale of { 16. / 24. / 30. }

Again, If it were required to know how many Yards the Line A B will contain, if meaſured by the *Scale* of 12.

Take the Line A B in your Compaſſes, and applying that Diſtance to the *Scale* of 12, you ſhall find it to reach from 6 (or 60) to 9 and a half of the ſmall Diviſions, ſhewing that the Line A B contains 69 Yards and a half, according to the *Scale* of 12.—— And if the ſame Line were meaſured by the *Scale* of 16, it would be found to contain 92 and a half.——Or by the *Scale* of 24, it would be 139 Yards, *&c.*

Concerning the *Diagonal Scales* on the other ſide of the Ruler, they ſerve to the ſame Uſe as the other *Plain Scales* do, only you may take off accurately the hundredth part of any Meaſure.—— *Example.* Suppoſe you were to take off 284 of any Meaſure,—— Count the 2 upon the great Figures on the Left-hand ſide of the *Ruler*, and the 8 on the ſmaller Figures on the ſame Left ſide, where the *Diagonal Lines* are drawn, and the 4, from 2 of the upright Lines on the Left ſide; and that Diſtance will contain 284 Feet, Yards, or any other *Meaſure.* And this Diſtance being taken from the *Diagonal Scale*, will reach upon the former Line A B, from A to S.

Of the Scales of Chords.

Upon the ſame ſide of the Ruler where the *Plain Scales* are, there are two or three *Scales* of *Chords*; the Deſcription and Making whereof you may find in Book II. Part II. Chap. II. And the Uſes of theſe *Scales* of *Chords* are principally two, *viz.* (1.) To lay down an *Angle* containing any *Quantity*: Or, (2.) An *Angle* being laid down, to find the Quantity thereof.

I. *To lay down upon Paper an* Angle *containing any Number of Degrees and Minutes required.*

Draw a Line at pleaſure, as A B, and upon the Point A, let it be required to *Protract* an *Angle* to contain 32 *deg.* 30 *min.* or 32½ Degrees. *Fig.* III.

Firſt, Place one Foot of your Compaſſes upon the beginning of any of your *Scales* of *Chords*, and extend the other Foot, always, to 60 Degrees: Then ſetting one Foot of the Compaſſes upon the given Point A, with the other deſcribe the Arch *e d*: Then ſetting one Foot of the Compaſſes in the beginning of the *Scale* of *Chords*, extend the other to 32 *deg.* 30 *m.* and ſet that Diſtance upon the Arch from *e* to *d*: Then from the Point A draw the Line A C, through the Point *d*: So ſhall the *Angle* D A C contain 32 *deg.* 30 *min.*

II. *An* Angle *being already laid down, to find the* Quantity *thereof.*

Let the *Angle* already laid down be the Angle C A B whoſe Quantity is required. *Fig.* III.

Open your Compaſſes from the beginning of any of your *Scales* of *Chords*, to 60 Degrees; then ſetting one Foot in the Angular Point A, with the other deſcribe the Arch *e d*, (between the two Sides containing the *Angle*:) Then take the Diſtance between *e* and *d*, which meaſure upon the ſame *Scale* of *Chords*, from the beginning of it, and it will reach to 32 *deg.* 30 *m.* which is the Quantity of the *Angle* C A B.

If an *Angle* given, or required, ſhall exceed 90 *deg.* you muſt ſet it off upon the Arch-line at twice: As if you were to lay down an *Angle* of 159 *deg.* Having deſcribed your Arch line far enough, your *Compaſſes* ſtanding at 60 *deg.* ſet that diſtance twice upon the Arch-Line, from E to *f*, and from *f* to *g*, ſo there will be 120 *deg.* then from your Scale take the remaining Degrees; namely 39, and ſet them on farther upon the Arch-Line from *g* to *h*, and through *h* draw the Line A D: So ſhall the *Angle* B A D contain 159 *deg.*

SECT. III. *Of the Description and Use of the* Protractor.

I. *Its Description.*

Fig. IV. THIS Instrument consisteth of two Parts; the one a *Semicircle* divided into Degrees, the other a *Parallelogram*, with *Scales* upon it. The *Instrument* ought to be made of a piece of thin Brass, well polished, and the Edges thereof very smooth; for the bigness, it may be about 4 or 5 Inches, and the breadth of the *Parallelogram* about an Inch and half. —— For the *Semicircle*, let it be divided into two *Quadrants* in the Point G, by the Perpendicular H G. Divide each *Quadrant* into 90 deg. and so the *Semicircle* into 180 deg. which let be numbred from E by 10, 20, 30, &c. to 180, ending at F. And the same way also (in another Margine) from 180 to 360, as you see done in *Figure* IV. so that there needs no farther Description of this Instrument.

II. *Its Use.*

The Uses of this Instrument may be performed by the *Scales* of *Chords* last treated of; but where many *Angles* are to be laid down, (as in *Surveying of Land*,) this *Protractor* is far more convenient: And its Uses are principally two, *viz.* (1.) To lay down an *Angle* of any assigned quantity of Degrees: Or, (2.) An *Angle* being *Protracted*, to find the quantity of Degrees that it containeth.

I. *How to* Protract *(or lay down upon Paper) an* Angle *containing any number of Degrees required.*

Let a Line be drawn at pleasure, as the Line B A E, and upon the Point A let it be required to Protract two *Angles*, from the Line B A; one to contain 32 *deg.* 30 *min.* and the other 159 *deg.*

First, Put your *Protracting Pin* in the given *Point* A, to which Point also bring the Centre of your *Protractor* H, turning it about upon the Centre-pin, till the Line E F of the *Protractor* do lie directly upon the Line B A E, the Centre H upon the Point A.

Secondly, Keeping the *Protractor* in this position, fast down to the Paper, with your left Hand, take out your *Protracting Pin*; and, because the two *Angles* to be protracted are 32 *deg.* 30. *m.* and 159 *deg.* count 32. *deg.* 30 *m.* upon the Limb (or Edge of your *Protractor*) from E, and you shall find it to end at *k*. Against which make a small mark upon the Paper at *k*. Also the other *Angle* being 159 *deg.* count them from E as before, and they shall end in *l*. where make a mark upon the Paper.

Lastly, Take away the *Protractor*, and from the Point A, and through the mark *k* draw the Line A *k* C.—— Also from the Point A, through the mark *l*, draw the Line A *l* D; so shall the *Angle* C A B contain 32 *deg.* 30 *m.* and the *Angle* B A D 159 Degrees.

II. *An* Angle *already laid down, or* Protracted, *to find the Quantity thereof.*

Let C A B, and B A D be two *Angles Protracted*, and let the quantities of each of them be sought by the *Protractor*. Apply the Centre H of the *Protractor* to the Angular Point A, and the Line E F upon the Line B A E; then shall you find the Line C A, to lie under 32 *deg.* 30 *min.* and the Line A D under 159 *deg.* and those are the quantities of the two *Angles* B A C 52 *deg.* 30 *min.* and B A D 159. *deg.*

But if it were required to find the quantity of the *Angle* C A D, apply the Centre of the *Protractor* H to the Point A, and the Line E F thereof upon the Line C A; so shall the Line A D lie just under 126 *deg.* 30 *min.* and that is the quantity of the *Angle* C A D.

SECT.

SECT. IV. *The Description of a* Quadrant, *for the taking of* Altitudes *or* Descents.

THIS *Instrument* may be made of Brass or Wood, and of what bigness you please, about 16 or 18 Inches is a competent size: the Limb whereof must be divided into 90 equal Parts or Degrees; and those again subdivided into as many Parts as the bigness of the *Quadrant* will permit, each Part containing 5, 6, or 10 *min.* Or, the Limb may be made broad, and so divided by *Diagonals* to express every single *minute*. —— On the back-side of the *Quadrant* may be a *Ball-Socket* to set the *Quadrant* either *Horizontal* or *Vertical*: And upon the upper Edge are placed two Sights, through which to look to behold any *Object*; and in the Centre is fixed a small Silk String, with a *Plummet* at the end thereof: And these are all the *Members* or *Parts* belonging to this *Instrument*. *Fig.* V.

For its *Use*, it is principally to take *Angles* of *Altitude*, as of the *Sun*, *Moon*, *Stars*, or of any *Tower*, *Steeple*, &c. upon the *Land*, as by *Examples* following shall appear. In the mean time view the *Figure* of the *Instrument*, with all its Parts put together; with the manner how it is to be used.

SECT. V. *Of the* Theodolite, *and its Appurtenances; the Description thereof, and to what* Uses *it principally serveth.*

THIS *Instrument* consisteth of several *Parts* or *Members*; As,

The first and principal part is a *Circle* of *Brass* or *Wood*, divided first into four *Quadrants*, representing the four Cardinal Points of the Campass *East*, *West*, *North*, and *South*, and therefore are noted with the Letters N. S. W. E. —— Each of these *Quadrants* is divided into 90 *deg.* and sub-divided according as the bigness will permit; or by *Diagonals*, as the *Quadrant* before. —— These four *Quadrants* are to be numbered by 10, 20, 30, &c. both ways, beginning at the *North* and *South* Points, and ending with 90 at the *East* and *West* Points. *Fig.* VI.

The second *Part* or *Member* is a *Box* and *Needle*, so conveniently contrived, to stand upon the *Centre* of the *Circle*; upon which *Centre*, the *Instrument*, the *Index*, with its *Sights*, must be made to turn about, and yet both the *Instrument*, and the *Box*, and *Needle* remain fixed —— At the bottom of the *Box* there must be a *Card*, or *Mariners Compass* fixed, answerable to the Letters E. W. N. S. upon the *Instrument*.

The Third *Member* is a *Socket* on the back-side (either Plain or with a Ball) to be put upon the head of a Three Legged Staff.

The Fourth *Member* or *Part* is a *Staff* to set the *Instrument* upon; the Neck, at the Head whereof, must be made to go into the *Socket* on the back-side of the *Instrument*: For better satisfaction view well the *Figure* of the *Instrument*, with all its Parts put together, and the manner how it is to be handled.

SECT. VI. *Of* Measures, *to measure withal.*

MEasures are many; but some are more commodious for one purpose than others: therefore I will only name some few, and for what Uses they are fittest to be applied.

1. The first and shortest is a *Rod of Ten Foot* in length, and each Foot divided into 10 Parts. And such a Rod is most convenient to measure any Height or Distance under 100 Foot.
2. A *Chain of* 100 *Foot* long, each *Link* being *one Foot* in length; and at each 10 Foot let there be a *Plate* of Brass, with a *Figure* engraven thereon, to shew readily how many Feet are from the beginning of the *Chain*: And for the more ease in accounting, let there be a *Brass Ring* at every five Links, that is, one between every two *Plates*. This *Chain* is most commodious for measuring of *Large Distances* or *Lengths*.

3. A *Chain* of 16 Foot and a half in length, and to contain 100 Links, with *Rings* at every tenth Link: Which will be good to measure small *Garden-grounds*, or *Orchards*, by *Pearch* or *Pole* measure.
4. A *Chain* of *Four Pole* or *Perches* in length, which is 66 Foot; which *Chain* let consist also of 100 Links, with *Plates* and *Rings*, as in the other before, for ease in accounting. This *Chain* of all others is the most commodious for *Land-measure*, as will appear hereafter.

CHAP. II.

Of Altimetria, *or the taking* Heights.

THE Object whose Altitude you desire to measure, may be either *Accessible* or *Inaccessible*: It may stand upon *Level Ground*, or upon a *Hill*; or be otherwise seated. Examples of several kinds.

SECT. I. *To take an* Accessible Altitude *at one* Station.

Fig. VII. LEt A be a *Tower* or other *Edifice*, standing upright, upon Level Ground: And let the Height thereof be required to be known, you standing at B.

1. Set your *Quadrant*, it being upon its Staff, at your Place of Standing B, and turn it in the Socket, so that the Thred, with the Plummet at the end thereof, may play freely by the side of the *Quadrant*: Then elevate the *Quadrant* upwards or downwards (as occasion requires) till through the *Sights* of the *Quadrant* you see the very top of the Battlements at S; and there keeping the *Quadrant* fast, (by means of a Screw for that purpose,) observe what Degrees and Minutes on the *Quadrant's* Limb are cut by the Thred, (suppose 35 *deg.* 20 *min.*) which note down.

2. Then (the *Quadrant* still being in this Position) extend a Line (or Cord) from the *Quadrants* side where the Sights stand, till it touch the Ground at C, and there make a Mark.

3. Measure with your *Rod* (or *Chain*) the Distance from C to T, the Foot of the Tower, which suppose to be 103 Foot and 4 tenth parts of a Foot; which also set down: And now you are prepared to find the Altitude of the *Tower* S T, by *Trigonometrical Calculation*.

4. For by *Figure* VII. you may perceive, that the *Tower* S T, standing Perpendicular upon the Plain Ground T C, and the *Visual Line* C S, made by observing the top of the Building at S, do make a *Right-angled Triangle* S T C; in which you have given,

1. The Angle at C, 35 *deg.* 20 *min.* which were the Degrees of the Quadrant cut by the Thred at C.
2. The Side T C, measured by the *Rod* or *Chain*, 103.4 Foot.
3. The Right Angle at T, for that the *Building* is perpendicular to the *Ground*.

Wherefore, by the I. *Case* of Right-angled Plain Triangles, say,

As c s C, *viz.*	S	:	T C	: :	s C	: S T.
$54^d\ 40^m$		:	103.4	: :	$35^d\ 20^m$	: 73.3.

Which 73.3 Foot is the Height of the *Tower* required.

In like manner might the *Hypotenuse* be found, by the II. *Case*. For,

As T S C		:	T C	: :	*Rad.*	: S C.
$54^d\ 40^m$		:	103.4	: :	90^d	: 126.7.

SECT.

SECT. II. *How to take an Inacceſſible* Altitude *at two Stations.*

LEt A B be a *Church Steeple*, and let it be required to find the Height thereof, you being at C, between which Place and A, the bottom of the Steeple, there is a River, ſo that it is inacceſſible. *Fig.* VIII.

Firſt, Set up your *Quadrant* at C, and move it up and down, till through the Sights you ſee the top of the Spire at B, noting what Degrees are cut by the Thred upon the Quadrant, which ſuppoſe 27 *deg.* 30 *min.*

Secondly, Meaſure from C towards A, ſo far as you can in a Right Line, as ſuppoſe to D, 92.7 Foot.

Thirdly, At D ſet up your Quadrant again, and move it up and down, till you ſee the top of the Spire at B, and note what Degrees are then cut by the Thred upon the Quadrant, which ſuppoſe to be 52 *deg.* 30 *min.*

Fourthly, By theſe two Obſervations at C and D, and the meaſured Diſtance C D, the Altitude of the Steeple may be found: For the two Viſual Lines through the Sights at C and D, together with the meaſured Diſtance D C, do make the Oblique-angled Triangle A D C; in which is given,

1. The Angle A C D, 27 *deg.* 30 *min.*
2. The Side C D, 92.7 Foot. And,
3. By the Angle obſerved at D, 52 *deg.* 30 *min.* alſo you have the Angle B D C, 127 *deg.* 30 *min.* it being the Complement of 52 *deg.* 30 *m.* to 180 *deg.* And,
4. Having the two Angles B D C, 127 *deg.* 30 *min.* and the Angle B C A, 27 *deg.* 30 *min.* their Sum is 155. You have alſo the Angle D B C given, 25 *deg.* it being the Complement of 155 to 180.

And thus in the Triangle A D C, having the three Angles, and one Side D C given, you may find the Side B D, by *Caſe* I. of Oblique-angled Plain Triangles. For,

As s D A C : D C : : s A C D : A D.
$25^{d}\,0^{m}$: 92.7 : : $27^{d}\,30^{m}$: 101.3.

Then again in the Triangle A B D, Right-angled at B, you have given the *Hypotenuſe* A D 101.3, and the Angle at the Baſe B D A, 52 *deg.* 30 *min.* to find the *Perpendicular* A B: Which by the II. *Caſe* of *Right-angled Triangles*, may be thus found.

As R : A D : : s B D A : B A.
90 : 101.3 : : $52^{d}\,30^{m}$: 80.3.

SECT. III. *How to take the Altitude of a* Tower, Steeple, Beacon, *or the like, which ſtandeth upon a* Hill.

LEt H K be a *Beacon* ſtanding upon a Hill, and you, ſtanding at L, deſire to know the Height thereof above the *Hill.* *Fig.* IX.

Firſt, Place your *Quadrant* at L; then looking through the *Sights* till you ſee the top of the *Beacon* at H, you find the Thred to cut 40 *deg.* of the Quadrant: And at the ſame Station you look through the Sights to K, the Foot of the Beacon, and find the Thred there to cut 22 *deg.* 3 *min.*

Secondly, Removing your Quadrant to O, and there looking through the Sights, till you ſee the top at H, you find the Thred to cut 60 *deg.* and looking to K the bottom, you find it to cut 39 *deg.* 54 *min.*

Thirdly, Meaſure the *Diſtance* O L, which you find 132 Foot, all which note down So in the Triangle H O L you have given,

1. The Angle H L O, 40 *deg.*
2. The Angle H O L, 120 *deg.*
3. The Diſtance O L, 132 Foot, by which the Side H O may be found 248, by the I. *Caſe* of *Oblique-angled*, and the whole Height H M, 214, by the I. *Caſe* of *Right-angled Triangles*; in all reſpects as the Height B A was found in the laſt *Section* before-going, and will be found to be 214 Foot.

Having thus found the whole Height H M 214, you have in the Right-angled Triangle H M O, given,

1. The

1. The Perpendicular H M, 214 Foot.
2. The Angle at the Base, H O M, 60 *deg.*

To find the Base M O. Which may be found by the IV. or V. *Cases* of *Right-angled Triangles*, thus.

As s H O M : H M : : s M H O : M O
60 d : 214 : : 30 d : 124.

And thus have you the whole Height M H 214, and the Distance O M 124. And so in the Right-angled Triangle K M O, you have given,

1. The Distance M O, 124 Foot.
2. The Angle K O M, observed at O, 39 *deg.* 54 *min.* whose Complement 50 *deg.* 6 *min.* is the Angle M K O.

By which you may find K M, by the I. *Case* of *Right-angled Triangles.*

As s M K O : M O : : s K O M : K M
50 d 6 m : 124 : : 39 d 54 m : 103.7.

Which 103.7 is the Perpendicular Height of the Hill; and being subtracted from 214, the whole Height H M, leaves 110.3, for the Height of the Beacon H K above the Hill.

SECT. IV. *How to find the* Horizontal *Line of a* Hill.

Fig. X. THis is a right necessary thing to be known, especially in *Surveying* of Land, where the Length of the Sides of the Hill is not required, but the Base Line upon which the Hill standeth.

In your Survey, suppose you should meet with such a *Hill* or *Mountain* as this, noted with M.

First, Place your *Quadrant* upon its Staff at the Foot of the Hill at D; then let one go to the top of the Hill at C, and there set up a Mark, which must be so much above the top of the Hill, as the top of the *Quadrant* is from the Ground: Then move the *Quadrant* up and down, till you see the top of the Mark at C, and note what Degrees of the *Quadrant* are cut by the Thred; which suppose 47 *deg.* Then measure up the *Side* of the *Hill*, from D to C, which let be 235 Foot.

Secondly, Remove your *Quadrant* from D to C, and there set it up, as before. Now (because there is a Level at the top of the Hill, from C to B,) measure the Length thereof, which suppose to be 122 Perches, and at B let a Mark be set up, of such height from the Ground as your *Quadrant* is.

Thirdly, Plant your *Quadrant* at B, and let one go to the Foot of the Hill at A, and there erect a Mark of equal height with your *Quadrant* at B; and through the Sights look down till you see the top of the Mark at A, observing what Degrees of the Quadrant are cut by the Thred, which suppose 46 *deg.*

Fourthly, Measure down the Descent of the Hill from B to A, which let be 253 Foot. Now by these two *Observations* made by your *Quadrant*, and the three Distances measured, you may lay them down upon Paper, and so shall you have constituted Two *Right-angled Plain Triangles*, and a *Parallelogram*; *viz.* the Triangles C F D, and B E A, and the Parallelogram B C F E; and from what is given in these *Triangles* and the *Parallelogram*, you may find the *Base Line* of the *Hill* A D: For,

First, In the Triangle C F D, you have given, (1.) The *Angle* C D F (observed by your *Quadrant* at D) 47 *deg.* (2.) The *Side* of the Hill C D measured 235 *Perches.* And (3.) The *Angle* F C D (the complement C D F) 43 *deg.* To find the *Base* F D.

By the III. *Case* of the Right-angled Triangles.

As R : C D : : s F C D : F D.
90 d : 235 : : 43 d : 160.

Then in the *Parallelogram*, you have given the top of the Hill measured from B to C 122 *Perches*: to which E F is equal.

And

And in the *Triangle* A E B, you have given, (1.) The *Angle* A B E (obſerved by your *Quadrant* at B) 46 *deg.* (2.) The *Hypotenuſe* (or ſide of the *Hill* meaſured B A) 253 *Perches*. To find the *Baſe* A E.

By the III. *Caſe* of Right-angled Triangles.

As	R	:	A B	: :	∠A B E	: A E.
	90^d	:	253	: :	46^d	: 182.

Which three Numbers 160, 122, and 182 added together, do make 464 for the whole *Baſe* of the *Hill* A D.

CHAP. III.

Of Longimetria, *or the taking of* Diſtances.

AS the *Quadrant* was the moſt convenient *Inſtrument* for the taking of *Altitudes*, ſo the *Theodolite* is the beſt for taking of *Diſtances* of *Trees*, *Steeples*, *Towers*, &c. either of *one* or *many* together: Of which here ſhall follow ſeveral Examples.

SECT. I. *To Meaſure one ſingle* Diſtance.

LET C be a *Tree*, ſtanding in a *Field* by it ſelf, and you being at A; (either in that *Field* or ſome other, it mattereth not, ſo you be but within convenient ſight of it) and you are deſired to know the Diſtance to that *Tree*, without approaching nearer to it than you are at A. *Fig.* XI.

Firſt, Set up your *Theodolite* upon its *Staff* at A, as Level (or Horizontal) as you can, laying the *Index* with *Sights* upon it on the *North* and *South Diametre* of the *Inſtrument*.

Secondly, Turn the *Inſtrument* about the *Socket*, (the Index and Sights ſtill lying upon the Diametre,) till through the Sights you ſee the *Tree* at C; which when you do, fix the *Inſtrument* to the Socket with the Screw.

Thirdly, From the Foot of your *Inſtrument* at A, Meaſure with your Rod or Chain, any number of Feet, Yards, or the like, any way; as from A to B 100 Foot: and there cauſe a Mark or Staff to be ſet up.

Fourthly, Turn the *Index* and *Sights* about upon the *Theodolite*, till through them you ſee the *Mark* ſet up at B; and note what Number of *deg.* and *min.* the *Index* cuts; which let be 120 *deg.* 10. *min.* which ſet down.

Fifthly, Then ſet up ſome viſible *Mark* at A, where the *Inſtrument* ſtood, and remove the *Theodolite* to B, and then ſetting it up, with the *Index*, upon the North and South Diametre, as before, turn the *Inſtrument* about, till through the *Sights* you ſee the *Mark* ſet up at A; and there fix it.

Sixthly, Turn the *Index* about till through the *Sights* you ſee the *Tree*, and note what Degrees the *Index* cuts upon the *Inſtrument* from the North and South Diametre, which let be 50 Degrees: which note down, and ſo have you done your work in the Field.

And now to find the diſtance from A, or from B to the *Tree*, you may ſee that the *Tree* at C, and the two Stations or Mark at A and B, do make an *Oblique Angled Triangle* A B C, in which is given,

1. The Angle C A B 120 *deg.* 10 *min.* obſerved at the firſt Station A.
2. The Diſtance Meaſured A B 100 Foot.
3. The Angle C B A, obſerved at the Second Station at B, 50 *deg.*

By which you may find the *Diſtances* A C and B C, by the I. and II. *Caſes* of *Oblique angled Plain Triangles*. For,

Having the two Angles at A 120 *deg.* 10 *min.* and at B 50 *deg.* their Summ is 170 *deg.* 10 *min.* which taken from 180 *deg.* there remains 9 *deg.* 50 *min.* for the Angle at C.

Then

Then by *Case* I.

As sBCA : AB :: sCBA : AC
$9^d\,50^m$: 100 :: $50^d\,0^m$: 448.6

And by *Case* II.

As CBA : CA :: *comp.* CAB : CB.
$50^d\,0^m$: 448.6 :: $59^d\,50$: 506.2

So the *Distance* from A to C is 443.6 Foot; and from B to C 506.2 Foot.

SECT. II. *How to take the* Distances *of divers things remote from you; As* Churches *and* Publick Halls *in a City: A* Camp *upon a* Heath, *a* Squadron *of* Ships *at Sea, or such like. And to make a* Map *or Draught of the same: With a* Scale *to Measure the* Distance *of any* Place *to any other therein described.*

Fig. XII. SUppose that A, B, C, D, E, and F, were a *Squadron* of *Ships* lying at *Anchor*, and you being on shore, and desired to make a Draught of them, representing their Situation and true Distance one from another.

I. *For the Observation made upon the Shore.*

Seek out two convenient places upon the Shore, from either of which you may see all the Ships at one view, and make those two Places your two Stations for Observation, as N and S. Then,

First, Set up your *Theodolite* at N upon its Staff, very firm and level.

Secondly, Turn it about upon the Socket, till the Needle hang directly over the *Meridian Line* of the Card, in the bottom of the *Box*, the North end of the Needle, over the *Flower-de-luce*: and then screw the Instrument fast. Your *Instrument* being thus fixed at N, call that your *First Station*; and turn the *Index* about, till through the Sights, you see the first *Ship* A, and note what Degrees of the *Theodolite* the *Index* cuts, as 60 *deg.* which note down —— Then turn the *Index* about, till through the *Sights* you see the Ship at B, and mark what Degrees are cut by the *Index*, as 74 *deg.* 30 *min.* and note them down. —— Doe thus with all of them, be there never so many.

Then Measure the Distance between N, your *First Station*, and S, your *Second Station*; which is 220 *Perches*: And bring your *Theodolite* from N to S, and there set it up, firm and level, as before at N; and laying the *Index* upon the North and South Diametre, turn it about till through the Sights you see your *First Station* at N: and then fix it.

Then turn the *Index* about, till through the Sights you see the first Ship at A, and note what Degrees the *Index* cutteth; which suppose 31 *deg.* 30 *min.* which note down —— Then turn the Index about to B, C, D, *&c.* noting the Degrees cut by the *Index* at every moving; and set them down in a *Table* ruled for that purpose, as is here done in this Example.

	At the First Station, the *Index* cut			At the Second Station, the *Index* cut	
	De.	M.		De.	M.
A	60	00		31	30
B	74	30	Stationary Di-	38	40
At C	84	30	stance, 220	43	20
D	104	50	Perches.	54	00
E	117	00		68	10
F	145	10		97	00

II. *How*

II. *How to* Protract *or lay down upon* Paper, *the* Obſervations *laſt taken.*

Upon a Sheet of large Paper or Parchment draw a Line at length, as O P; upon which aſſume any Point (as N) for your firſt *Station*; unto which Point apply the Centre of your *Protractor* H, and lay the Line E F of the *Protractor* juſt upon the Line O P. *Fig.* XII.

Then keeping the *Protractor* in that Poſition, lay your *Table* of *Obſervations* before you, count 60 deg. (the firſt *Obſervation* you made to the Ship A) upon the edge of the *Protractor*; againſt them make a mark, with your *Protracting-Pin*, upon the Paper: then count 74 deg. 30 min. upon the *Protractor*, and againſt them make a mark with your Pin, as before: Alſo do the like with 84 deg. 30 min. 104 deg. 50 min. *&c.* till you have pricked down all your *Obſervations* made at your *firſt Station* at N.

Then taking off your *Protractor*, draw obſcure Lines from N, through thoſe marks, as the Lines N A, N B, N C, *&c.*

Then out of ſome *Scale* of equal Parts (anſwerable to the bigneſs you intend your *Plot*,) take off 220 *Perches*, and ſet them upon the Line O P, from N to S.

Unto the Point S apply the Centre of your *Protractor*, laying the Diametre E F thereof upon the Line O P; where keeping the *Protractor* in that Poſition, lay your *Table* of *Obſervations* at your ſecond *Station* before you; and with your *Protracting-Pin* make marks at 31 deg. 30 min. your firſt *Obſervation*, and at 38 deg. 40 min. your *ſecond*, at 43 deg. 20 min. the *third*, &c. till you have pricked down all your *Obſervations* made at your *ſecond Station* at S.

Then taking away your *Protractor*, draw obſcure Lines from S, through the ſeveral marks made upon your Paper, as the Lines S A, S B, S C, *&c.* cutting the Lines before drawn N A, N B, N C, *&c.* in the Points A, B, C, D, E, and F; which Points ſhall repreſent the ſeveral *Ships* as they lie at *Anchor*, every one at his true *Diſtance* and *Poſition* from another, as they do in the *Sea*, *River*, or *Harbour*: as alſo from either of your *Stations* N or S. And now,

III. *To make a* Scale *to Meaſure any* Diſtance *upon this* Plot.

Foraſmuch as the diſtance between N and S was meaſured 220 Perches, divide a Line in ſuch ſort into Parts, as the *Scale* was divided from whence you laid down the diſtance N S 220; and ſuch is the *Scale* L M in the Figure: for if with a Pair of Compaſſes you take the diſtance between N and S, if you ſet one Foot of that diſtance upon the *Scale* in the Point noted 200, the other ſhall reach, from thence, upon the *Scale* to 20 of the ſmall Diviſions, in the firſt part of the *Scale*: And the *Scale* being thus divided and numbered, you may Meaſure any Diſtance or Diſtances upon the *Plot*. And ſo you ſhall find the Diſtance

From { N to A / N to B / A to B / A to S } to be { 241.9 / 236.7 / 61.0 / 399.6 } Perches.

SECT. III. *To Meaſure any of the* Diſtances, *thus laid down, by* Trigonometrical Calculation.

BY the Interſection of the ſeveral viſual Lines made at both *Stations*, and conſtituted ſeveral *Right-lined Triangles*; in either of which you will have enough given to find whatſoever diſtance you ſhall require. *Fig.* XII.

Example 1. *Let it be required to know the diſtance from your Station* N, *to the firſt Ship at* A.

By the Viſual Line N A, made at your firſt *Station*, and the Viſual Line S A, at your ſecond *Station*, interſecting each other in A; is conſtituted the *Right-lined Triangle* A N S; in which is given,

1. The Angle A N S, 120 deg. it being the Complement of the Angle obſerved to A, at your firſt *Station* (*viz.* 60 deg.) to 180 deg.

2. The Angle N S A, 31 deg. 30 min. the Angle observed to A, at the second *Station*; and
3. The Side comprehended between them, *viz.* the *Stationary* distance N S, 220 Perches.

To find N A.

Having the Angle A S N 31 deg. 30 min. and the Angle A N S 120 deg. you have also the Angle N A S 28 deg. 30 min. given, it being the Complement of the other two, to 180 deg. Then,

By the I. *Case* of Oblique-angled Triangles.

As s N A S	:	N S	: :	N S A	:	A N.
$23^d\ 30^m$	:	220 P.	: :	$31^d\ 30^m$	:	240.9 P.

Example 2. *For the distance from your second Station* S, *to the first Ship at* A, *making the Triangle* N A S.

By the foregoing I. *Case.*

As s N A S	:	N S	: :	s A N S	:	A S
$23^d\ 30^m$	:	220 P.	: :	60^d	:	399.6 P.

Example 3. *For the distance from* N *to* B.

In the Triangle N B S, there is given,

1. The Angle B N S 105 deg. 30 min. the Complement of the Angle made to B at the first *Station* at N, 74 deg. 30 min.
2. The Angle B S N 38 deg. 40 min. the Angle observed at your second Station to B.
3. The Stationary distance N S 220 P.

To find N B.

Having the Angle B N S 105 deg. 30 min. and the Angle N S B 38 deg. 40 min. you have also the Angle N B S 35 deg. 30 min.

By the I. *Case* of Oblique-angled Triangles.

As s N B S	:	N S	: :	s B S N	:	N B
$35^d\ 30^m$	:	220	: :	$38^d\ 40^m$	:	236.7

Example 4. *For the distance* A B.

In the Triangle A N B there is given,

1. The side A N, 241.9.
2. The side B N 236.7.
3. The Angle A N B 14 deg. 30 min. it being the difference between the two Angles made at the two Observations from N to A and B.

Then by the IV. *Case* of Oblique-angled Triangles.

As the summ of the sides A N and B N, 478.6.
Is to their *Difference* 5.2.
So is the *Tangent* of half the Summ of the Angles N A B and N B A 82 deg. 45 min.
To the *Tangent* of 4 deg. 53 min.
Which added to 82 deg. 45 min. makes 87 deg. 38 min. for the Angle A B N, and subtracted from 82 deg. 45 min. leaves 757 deg. 52 min. for the Angle B A N.

Then by the I. *Case.*

As s A B N	:	A N	: :	s A N B	:	A B.
$57^d\ 51^m$	:	241.9 P.	: :	$4^d\ 30^m$	:	61 P.
s 87. 38.	:		: :			

And in this manner may *Maps* or *Plots* of all eminent *Places* in *Cities*, *Towns*, &c. be taken. But having been so large in the explaining of this, I shall be the more brief in those which follow.

SECT.

SECT. IV. *How to take the* Distances *of the most Remarkable* Places *in a Town or City.*

LEt A, B, C, D, E, F, G, H, K, L, M, be some eminent Places in a City, such as are nominated in the Table following; of which make choice of two, from either of which all the other may be seen, as L, *St. Giles*'s, and M, *St. Hellen*'s: Then upon *St. Giles*'s, at L, set up your *Theodolite*, laying the *Index* upon the North and South Diametre thereof; and then turn the *Theodolite* about, till through the Sights you see *St. Hellen*'s at M, and there fix it. *Fig.* XIII.

Then move the *Index*, till through the Sights you see the *North Gate*, the *Index* cutting 37 deg. 30 min. Then direct it to the *School-house*, the *Index* cutting 77 deg. and so to all the Remarkable Places about the Town, noting down what Degrees the *Index* cut at every Place, which suppose to be such as are exhibited in the second Column of the following Table, under *Station* I. *St. Giles*'s.

Then removing from *St. Giles*'s to *St. Hellen*'s, there set up your *Theodolite*, laying the *Index* upon the North and South Diametre thereof, (as before,) and turn the *Theodolite* about, till through the Sights you see *St. Giles*'s, and there fix it.

Then turning the *Index* about, direct your Sights to the *North Gate*, the *Index* cutting 16 deg. 30 min. which note down; then to the *School-house*, the *Index* cutting 36 deg. 45 min. and so to all the remarkable Places, as before; so shall you find the Degrees cut by the *Index* at the several Directions, to be such as are set down in the last Column of this Table, under *Station* II. *Hellen*'s.

The Table of Observations.

The Names of the Places you observe.	Station I. St. *Giles*'s.		Station II. St. *Hellen*'s.	
	deg.	min.	deg.	min.
A *North Gate*	37	30	16	30
B *School-house*	77	00	36	45
C *Christ-Church*	115	20	27	45
D *Prison-house*	133	45	60	00
E *St. Bennet's*	168	00	140	15
F *South Gate*	187	30	197	30
G *The Hospital*	210	45	269	00
H *The Town-Hall*	216	30	248	45
K *Old Tower*	317	30	342	30

Having thus finished your *Table of Observations*, the next thing that you are to doe, is to find the Distance between L, *St. Giles*'s, and M, *St. Hellen*'s; which to doe, (if you connot conveniently measure it within the Town,) you may go out thereof, into some near adjoyning Field, or other open Place, where you may conveniently see both the Places; as, suppose, in a Field at O and R; where,

1. Set up your *Theodolite* at O, the *Index* lying on the North and South Diametre thereof, and there turn the *Theodolite* about, till through the Sights you see *St. Giles*'s at L; and then fix the Instrument.

2. The Instrument thus fixed, direct the Sights to any Mark set up at a distance, as at R, the *Index* there cutting 106 deg. and then to *St. Hellen*'s, the *Index* cutting 40 deg.

3. Measure the Distance between O and R, which let be 470 Geometrical Paces, (of 5 Foot to a Pace,) or 2350 Foot.

4. Set up your *Theodolite* at R, the *Index* lying upon the Diametre, as before, turning it about till through the Sights you see a Mark set up at your former Station at O, and there fix it: Then move the *Index*, till through the Sights you see *St. Giles*'s at L, the *Index* cutting 36 deg. Then move it forwarder, till you see *St. Hellen*'s at H, the *Index* cutting 65 deg. 30 min. And by these two Observations the Distance between *St. Giles*'s and *St. Hellen*'s may be obtained by the Directions in the foregoing *Section*. For,

In the Triangle L O R, you have given, (1.) The Distance O R, 2350 Foot. (2.) The Angle L O R, 106 deg. (3.) The Angle L R O, 36 deg. And having the Angles L O R, 106 deg. and L R O, 36 deg. the Summ of them is 142 deg. whose Complement to 180 deg. is 38 deg. for the Angle O L R, to find L O: Then by the *First Case* of *Oblique angled Triangles*,

As	s O L R	:	O R	: :	s L R O	: L O.
	38 d	:	2350	: :	36 d.	: 2244.

And by the forementioned I. *Case* may L R be found. For,

As	s O L R	:	O R	: :	s L O R	: L R.
	38 d.	:	2350	: :	106. or 74 d.	: 3669.

And again, In the Triangle O M R, there is given, (1.) O R, 2350. (2.) O R M, 65 deg. 30 min. (3.) The Angle M O R, 66 deg. and also the Angle O M R, 48 deg. 30 min. to find M R; which by the forementioned *Case* I. may be thus found.

As	s O M R	:	O R	: :	s M O R	: M R.
	48 d. 30 m.	:	2350	: :	66 d.	: 2866.

Then for the Distance L M, in the Triangle L R M, there is given, (1.) The Side L R, 3669. (2.) The Side M R, 2866. (3.) The Angle L R M, 29 d. 30 m. comprehended between L R and M R, to find the Side L M. By the IV. *Case* of *Oblique-angled Triangles*,

As the Summ of the Side L R and M R, 6535,
Is to the Difference of the same Sides, 803;
So is the Tangent of half the Summ of the two Angles R L M, and R M L, 75 d. 15 m.
To the Tangent of 25 deg.

Which 25 deg. added to the half Summ, 75 deg. 15 min. makes 100 deg. 15 min. for the Angle L M R; and being subtracted therefrom, leaves 50 deg. 15 min. for the Angle M L R.

Then by the I. *Case*,

As	s M L R	:	M R	: :	s L R M	: L M.
	$50^{d}\ 15^{m}$	:	2866	: :	$29^{d}\ 30^{m}$	: 1832.

Thus having found the Distance between *St. Giles*'s at L, and *St. Hellen*'s at M, to be 470 Paces, or (rather) 2350 Foot, you may lay down all the other Places according to their true Distances and Situations one from another, by the Directions of the second Section before-going, and as you see done in *Figure* XIII.

The End of the First Part.

THE DOCTRINE OF RIGHT-LINED TRIANGLES, Applied to PRACTICE IN PLANOMETRIA, OR Surveying of Land.

PART II.

THE *Instrument* which I shall principally make use of in this Tractate of *Surveying* of *Land*, shall be the *Theodolite* before described, *Chap.* I. *Sect.* V. of *Longimetria*: And the *Chain*, such as is described in the Fourth *Paragraph* of the VI *Section* of the said *Chap.* I. And the *Scales* and *Protractor* the same as are also there described in the II. and III. *Sections*. The Uses of all which are in the forementioned I. *Chapter* sufficiently taught: Wherefore I shall say nothing more of them here, but enter the *Field* to *Survey*.

CHAP. I.

How to take the Plot *of a* Field *or small* Inclosure *at one Station, taken in any Place within the same, from whence all the* Angles *may be seen.*

WHen you have entred your *Field*, and made choice of a convenient Place to fix your *Instrument*, cause *Marks* (or *Whites*) to be set, or held up, at every *Angle* of the *Field*; to which *Marks* direct the *Sights*, and note what Degrees the *Index* cuts, *&c.* *Fig.* I.

Let A B C D E F be a small Inclosure to be Measured, having made choice of a Place about the middle thereof, as at ⊙, from whence you may see all your *Marks* at A B C D E and F; fix there your *Theodolite*, the *Needle* hanging directly over the *Meridian-line* in the *Card*. Then,

First, Direct your *Sights* to A, the *Index* cutting 36 deg. 45 min. which note down in your *Field Book* in the Second Column thereof: and Measure the Distance from

⊙ to A, containing 8 *Chains* 10 *Links*: which ſet down in the Third Column of your *Field Book*.

Secondly, Direct the *Sights* to B, the *Index* cutting 99 deg. 15 min. and the Diſtance from ⊙ to B 8 *Chains* 75 *Links*: both which ſet down in your *Field Book* as before.

Doe the like for the reſt of the *Angles* from ⊙ to C, D, E and F: and ſet them down as you obſerve them, with their Diſtances meaſured as you ſee done in the following *Table*.

	D.	M.	Cha.	Lin.
A	36	45	8	10
B	99	15	8	75
C	163	15	10	65
D	212	00	8	53
E	287	15	8	15
F	342	00	9	55

Having thus finiſhed your Work in the *Field*, the next thing to be done is to *Protract* the ſame; that is, to lay down a *Plot* thereof upon Paper or Parchment: Which ſhall be the Work of the following *Chapter*.

CHAP. II.

How to Protract *any* Obſervations *taken according to the Directions of the laſt* Chapter.

DRaw a Line upon Paper or Parchment, as N S, repreſenting the *Meridian Line*. Then in ſome part of that Line, as at ⊙, make a Point, repreſenting your Place of Standing in the Field: Upon this Point place the Centre of your *Protractor*, ſo that the *Diametre* thereof, E F, may lie directly upon the Line N S of this *Figure* I.

Then laying your *Field Book* before you, and ſeeing that at the firſt Obſervation to A, the *Index* cut 36 deg. 45 min. therefore againſt thoſe Degrees make a prick by the Side of your *Protractor*.

2. At the Second Obſervation to B, the *Index* cut 99 deg. 15 min. againſt which make a prick as before.

3. At the Third Obſervation to C, the *Index* cut 99 deg. 15 min. againſt which alſo make a prick.

4. The Degrees cut by the *Index* at the Fourth Obſervation to D, were 212 Degrees; (Which being above 180 deg. you muſt turn the *Semicircle* of the *Protractor* downwards; yet ſo that the Centre of the *Protractor* may lie juſt upon the Station-Point ⊙, and E F upon the Line N S, as before it did;) againſt which Degrees make a prick upon your Paper.

5. The Degrees cut at the Fifth Obſervation to E, were 287 deg. 15 min. againſt them alſo make a prick. And,

6. The Degrees cut at the laſt Obſervation to F, were 342 deg. againſt them alſo make a mark.

Having thus made Marks upon your Paper againſt the ſeveral Degrees that were cut by the *Index*: Take away the *Protractor*, and laying a *Ruler* to the Station-Point ⊙, draw obſcure Lines from thence through thoſe Points: Then the Line at your firſt Obſervation from ⊙ to A being meaſured, 8 Chains 10 Links, take 8 Chains 10 Links from your Scale, and ſet them upon the firſt obſcure Line from ⊙ to A; which ſhall give you the angular Point A in the *Field*.

In

	Cha.	Lin.		
In like manner, set	8	75	upon the obscure Lines, from ⊙ to	B
	10	65		C
	8	53		D
	8	15		E
	9	55		F,

So shall the Points A, B, C, D, E, F, upon your Paper, represent the several Angles in the Field. And,

Lastly, Lines drawn from one Point to another, as the Lines A B, B C, C D, D E, E F, and F A, shall include the true Figure of the Field.

CHAP. III.

How to take the true Plot *of a* Field *at Two Stations, taken in any Two Parts thereof, from whence the Angles may be seen.*

THis manner of Working is chiefly to be used in such Fields whose Irregularity will not admit of any one Place from whence all the Angles may be seen. The manner of Work in this, as also the Protraction, will be the same with that in the two preceding Chapters hereof; only in this the Instrument must be twice placed, (and sometimes thrice,) whereas in the other it was but once.

Suppose therefore, that A B C D E F G H I K L and M were such an irregular Field: You must first find out two Places, as ⊙ and Q, for two Stations, so that at both of them all the Angles may be seen, though not at once from any one of them, but at both; for those Angles that cannot be seen from ⊙, may be seen from Q; and those that cannot be seen from Q, may be seen from ⊙. *Fig.* II.

Your two Stations, ⊙ and Q, being chosen, set up your *Theodolite* at ⊙, and turn it about till the Needle hang over the Meridian Line; and there fixing it, direct the Sights to A, the *Index* cutting 19 deg. 10 min. And the Line ⊙ A measured 7 Chains 46 Links, place the 19 deg. 10 min. in the second, and the 7 Cha. 46 Lin. in the third Column of your *Field-Book*.

Then direct the Sights to B, the Index cutting 53 deg. 30 min. and the Line ⊙ B measured 7 Ch. 18 Lin. which note down as before.

In this manner proceed with the rest of your Lines and Angles, namely, so many of them as you can conveniently see from your first Station at ⊙, as those at A, B, C, D, K, L, and M.

This done, direct your Sights to your second Station at Q, the Index cutting 161 deg. 45 min. (or rather 18 deg. 15 m. below 180 deg.) which note down in your *Field-Book* by it self; and also the Distance between the two Stations ⊙ and Q, 8 Cha. 89 Lin.

Having thus finished one part of the Field, remove your Instrument to the other Station at Q, and laying the *Index* upon 18 deg. 15 min. thereof, (which is the Inclination or Difference of the Meridians of the two Stations,) turn it about, till through the Sights you see your first Station at ⊙, and there fix it; then will the Needle hang over the Meridian Line, and the Instrument truly situate.

Then direct the Sights to E, the *Index* cutting 52 deg. 15 min. and the Line Q E measured 5 Chains 10 Links, which must be noted in your Field-Book, in all respects as before.

In this manner make Observation of the Degrees cut at E, F, G, H, and I, and of the Lengths of the Lines; all which being collected into your Field-Book, will stand as you see in the following Table.

The

The Field-Book.

	D.	M.	Ch.	Li.	
A	19	10	7	46	
B	53	13	7	18	
C	95	15	7	21	
D	132	00	6	33	At the first Sta-
E	166	30	5	57	Station ⊙.
K	251	30	7	83	
L	282	00	9	95	
M	304	30	8	05	

The Stationary Distance is 8 Chains, 89 Links; and the Angle ⊙ Q N, 18 d. 15 m. the Inclination of the two Meridians.

E	52	15	5	10	
F	99	30	7	64	
G	148	30	6	40	At the second
H	232	30	5	33	Station Q.
I	275	00	6	95	
K	321	20	7	61	

CHAP. IV.

How to Protract the Observations taken according to the Work of the last Chapter.

Fig. II. UPon your Paper draw a Meridian Line, N ⊙ S, the Point ⊙ representing your first Station. Upon this Point ⊙ place the Centre of the *Protractor*, laying the Line E F thereof directly upon the Meridian Line hereof, N S: Then laying the Field-Book before you, observe the Degrees there noted, and the Lines measured, namely,

(1.) At A, 19 deg. 10 min. and the Line ⊙ A, 7 Chains 46 Links.
(2.) At B, 53 deg. 30 min. and the Line ⊙ B, 7 Chains 18 Links.
(3.) At C, 95 deg. 15 min. and the Line ⊙ C, 7 Chains 21 Links.

and so of the rest; against which Degrees and Minutes make Marks by the edge of your *Protractor*, and draw Lines from ⊙ through these Marks, as ⊙ A, ⊙ B, ⊙ C, ⊙ D, ⊙ E, ⊙ K, ⊙ L, ⊙ M; and upon these Lines set off their several Lengths from ⊙, as you find them in your Field-Book.

Having thus Protracted the Observations at your First Station, (before you move your *Protractor*,) make a Mark against 18 deg. 15 min. and draw th Line ⊙ Q, setting off upon it 8 Chains 89 Links, the Length or Distance between the two Stations ⊙ and Q.

Then upon Q place the Centre of the *Protractor*, as before, moving it up and down, till the Line ⊙ Q lies just under 18 deg. 15 min. of the *Protractor*; and holding it there, lay your Field-Book before you, and prick down by the Edge thereof the several Degrees and Minutes, as by your Instrument you observed them, together with the Lengths of the Lines, as they were measured, drawing Lines through those Points also, as the Lines Q E, Q F, Q G, Q H, Q I, Q K.

Lastly, Draw the Lines A B, B C, C D, D E, E F, *&c.* So shall you have upon your Paper the exact Plots of your Field; in which (if there be no Error in the Work) the Line M A being drawn, will close exactly with the Point B A, in the Point A.

CHAP.

CHAP. V.

How to take the Plot *of a* Field *at Two Stations, taken in any Two Places thereof, from either of which all the Angles in the Field may be seen, and by measuring only the Stationary Distance.*

THE Work of this Chapter, both for the *Observation* and *Protraction*, differeth nothing from that in the Second, Third, Fourth, and Fifth Sections of the Second Chapter of *Longimetria* beforegoing; and therefore in it I will be very brief. *Fig.* III.

Let the Field to be Plotted be A B C D E F G H: Having made choice of two Places for your two Stations, as ⊙ and Q, place your *Instrument* at ⊙, and turn it about till the *Needle* hang over the *Meridian Line* in the *Card*, and there fix it. Then,

				Deg.	Min.	
First,	Direct the Sights to	A	The *Index* cutting	21	30	At the First Station at ⊙.
Secondly,		B		69	15	
Thirdly,		C		124	45	
Fourthly,		D		168	10	
Fifthly,		E		202	30	
Sixthly,		F		237	30	
Seventhly,		G		307	00	
Eighthly,		H		328	30	

This done, measure the Distance between your two Stations ⊙ and Q, which is 7 Chains, and remove your *Theodolite* to Q, turning it about till the *Needle* hang directly over the *Meridian Line* of the *Card*, as before, and there fix it. Then,

				Deg.	Min.	
First,	Direct the Sights to	A	The *Index* cutting	11	00	At the second Station at Q.
Secondly,		B		35	30	
Thirdly,		C		79	45	
Fourthly,		D		153	15	
Fifthly,		E		224	30	
Sixthly,		F		279	30	
Seventhly,		G		329	00	
Eighthly,		H		347	30	

Thus having made your Observations to all the Angles round about the Field, at both Stations, and set them down in your Field-Book, and also the Distance between your two Stations, as you see here done, you may proceed to Protract your Work, as is directed in the following Chapter.

CHAP. VI.

How to Protract *the* Observations *taken by the last* Chapter.

DRaw a Right-Line, as N S, representing the *Meridian:* upon which Line assume any Point at pleasure, as ⊙ for your first Station; unto which Point apply the Centre of your *Protractor*, with the Diametre E F thereof, upon the *Meridian Line* N S before drawn. Then look into your *Field-Book* for the Degrees and Minutes observed at the first Station at ⊙, and against every of them make Marks *Fig.* III.

by

by the Edge of your *Protractor* upon your Paper: And when you have marked them all, draw obſcure Lines from the Station ⊙, through every of them; as the Lines ⊙ A, ⊙ B, ⊙ C, *&c.*

This done, take 7 *Chains*, (your Stationary Diſtance,) and ſet it upon the *Meridian Line* N S, from ⊙ to Q.

Upon the Point Q (your ſecond Station) apply the Centre of your *Protractor*, with the Diametre E F thereof, upon the Line N S, as before: Then from your *Field Book* take all the Obſervations you made at Q, and againſt them make Marks by the Edge of the *Protractor*, upon your Paper; and through thoſe Points, from the Point Q, draw the Right-Lines Q A, Q B, Q C, *&c.*

The Line	Q A	Interſecting or Croſſing the Line	⊙ A	in the Point	A
	Q B		⊙ B		B
	Q C		⊙ C		C
	Q D		⊙ D		D
	Q E		⊙ E		E
	Q F		⊙ F		F
	Q G		⊙ G		G
	Q H		⊙ H		H

Laſtly, If you draw the Right Lines, from Point to Point, of theſe Interſections; as the Lines A B, B C, C D, D E, E F, F G, G H, and H A, they will give you the true *Plot* or *Figure* of the *Field* ſo taken: as in the foregoing *Chapter*.

CHAP. VII.

How to take the Plot *of a* Park, Common, *or Large* Champion Plain, *by going round about the ſame, and making Obſervation at every* Angle *thereof.*

Fig. IV. FIrſt place your *Inſtrument* at the *Angle* A, and lay the *Index* on the Diametre thereof, turning the whole *Inſtrument* about, till through the Sights you ſee the ſecond *Angle* at B, and there fix it: Then turn the *Index* about backwards till you ſee the *Angle* at G, the *Index* cutting 97 deg. which is the quantity of the *Angle* G A B: and meaſure the Line A B, containing 12 *Chains* 5 *Links*; which ſet down in your *Field Book*, as formerly.

2. Remove the *Inſtrument* to B, the *Index* lying upon the Diametre, and turn the whole *Inſtrument* about, till by the Sights you ſee the third *Angle* at C, and there faſten it; then turn the *Index* backwards, till through the Sights you ſee the *Angle* at A, the Degrees cut by the *Index* being 120 deg. 30 min. and the Line B C 4 *Chains* 45 *Links*; which note down.

3. Remove the *Theodolite* to C, the *Index* lying upon the Diametre, turning the *Inſtrument* about, till you ſee the fourth *Angle* at D, and there fix it; then turn the *Index* backwards till you ſee the *Angle* at B, the *Index* cutting 132 deg. 30 min. and the Line C D 8 *Chains* 85 *Links*.

4. Bring the *Inſtrument* to D, turning it about till you ſee the fifth *Angle* at E, and there fix it; then turn the *Index* backwards, till through the Sights you ſee the *Angle* at C, the *Index* cutting 125 deg. and the Line D E 13 *Chains* 4 *Links*.

5. Remove the *Theodolite* to E, turning it about till you ſee the ſixth *Angle* at F, and there fix it; turning the *Index* backwards to D, it cutting 121 deg. 30 min. and the Line E F 7 *Chains* 70 *Links*.

6. Remove the *Theodolite* to F, turning it about till you ſee the ſeventh *Angle* at G, and there fix it; then turn the *Index* backwards, till through the Sights you ſee the *Angle* at E, the *Index* cutting 80 deg. and the length of the Line F G 5 *Chains* 67 *Links*.

7. *Laſtly*, Place the *Inſtrument* at G, and the *Index* lying upon the Diametre, turn the whole *Theodolite* about, till through the Sights you ſee the *Angle* at A, where you began; and there fixing it, direct the Sights backwards to F, the *Index* cutting 224 deg. and the Line G A 7 *Chains* 87 *Links*. Having

Having thus made *Observation* at every *Angle* of the *Field*, and measured the Length of every Line, and entered them into a *Field Book*, you will find them to stand as in this *Table*.

	D.	M.	Cha.	Lin.
A	97	00	12	5
B	120	30	4	45
C	132	00	8	85
D	125	00	13	4
E	121	30	7	70
F	80	00	5	67
G	224	00	7	87
	900	00		

CHAP. VIII.

How to Protract *the* Observation *taken in the last* Chapter.

1. DRaw a Line, as A G; then apply the Centre of your *Protractor* to the Point A, and (because the Degrees cut at your first *Observation* were 97 deg.) turn the *Protractor* about, till the Line A G lie directly under 97 deg. and then at the beginning of the Degrees of the *Protractor* (marked with the Letter E) make a Mark, and through it draw the Line A B; setting 12 *Chains* 5 *Links* upon it from A to B. *Fig.* IV.

2. Apply the Centre of the *Protractor* to the Point B, and turn it about, till 120 deg. 30 min. (the Degrees cut by the *Index* at B) do lie just over the Line A B: and at the beginning of the Degrees make a Point upon your Paper; through which draw the Line B C, containing 4 *Chains* 45 *Links*.

In this manner must you deal with all the rest of the *Angles*: And when you come to *Protract* the *Angle* at F, and have drawn the Line F G, you shall find it to cut the Line A G, first drawn, in the Point G, leaving the Line A G to contain 7 *Chains* 85 *Links*, and the Line F G, 5 *Chains* 67 *Links*.

In the performance hereof, Practice is better than many Words, and the Sight of the *Figure* more than Information; in which you may see the *Protractor* lie in its true Position at every *Angle*.

CHAP. IX.

How to Discover whether the Angles *made at your several* Stations *be truly taken or not.*

THis way of Surveying, (which is inferior to none, although not altogether so expeditious as some ways which follow,) hath a preheminence above the rest; for by help of your *Field Book* you may (before you go out of the *Field*) know whether your *Plot* will close at the end of your *Protraction*, before you begin: And for to effect it, this is the

RULE.

Multiply 180 *deg. by a Number less by two than the Number of the Sides or Angles in your* Plot, *the Product of that Multiplication shall be equal to the Summ of all the Angles observed.*

Example. In the *Plot* of the *Field* last taken, [*Fig.* IV.] the Number of the *Sides* or *Angles* thereof are *Seven*, two less is *Five*: wherefore Multiply 180 by 5, and the Product will be 900. Then add all *Angles* of your several Observations together,

as

as they ſtand in your *Field Book*, and they make 900 alſo; which demonſtrates the *Angles* to be truly taken: for if they had differed, there had been ſome error committed in your Obſervations.

Note, That the quantities of all *Angles*, are to be reckoned in the inſide of the *Plot*, and not without, as in the foregoing *Plot*, the *Angle* at G being 224 deg. Theſe are counted within the *Field*, but the outward *Angle* thereof is but 136 deg. by which *Angle* it muſt be Protracted, as in the *Figure* is evidently ſeen.

CHAP. X.

A more Compendious way of taking the Plot *of any* Wood, Park, *or other large* Champion Plain, *by the* Theodolite; *by going about the ſame, and making Obſervation at every* Angle *of the ſame.*

Fig. V. LET ABCDEFGHK be a large *Common Field*, or the like, to be Plotted.

In this way of Surveying, you muſt always obſerve this Method; *viz.* (1.) At every Station, to ſet the *Inſtrument* ſo that the *Needle* may hang directly over the *Meridian Line* in the *Card*. And (2.) to keep one end of the *Index* next your Eye, and not ſometimes one and ſometimes another, for that will cauſe confuſion in your Work. Then (3.) always count the Degrees which are cut by that end of the *Index* which is fartheſt from you. Obſerving theſe *Cautions*, I will proceed to the *Practice*; in which, as alſo in the *Protraction*, I will be very brief.

				D.	M.			Ch.	Lin.
Place the *Theodolite*, the *Needle*, being over the *Meridian Line* of the *Card*, at	A	and direct the *Sights* to	B	191	00	And the Line	A B	10	75
	B		C	279	00		B C	6	83
	C		D	216	30		C D	7	82
	D		E	325	00		D E	6	96
	E		F	12	30		E F	9	71
	F		G	324	30		F G	7	54
	G		H	98	30		G H	7	54
	H		K	71	00		H K	7	74
	K		A	161	30		K A	8	22

The *Index* cutting — containing

Having gone round about the *Field* in this manner, and collected the Degrees cut by the *Index*, and the Lines meaſured by the Chain into a *Field Book*, they will ſtand as above: By help whereof you may *Protract* and draw a *Plot* thereof, as ſhall be directed in the following *Chapter*, very much differing from all the former ways of Protraction.

CHAP. XI.

How to Protract *the* Obſervations *taken according to the Directions of the laſt* Chapter.

Fig. V. UPon a Sheet of Paper, or Skin of Velom, as L M N O; draw ſeveral Lines parallel one to another, as in the *Figure*, but let the Diſtance of thoſe Parallels be leſs than the Breadth of the Square part of the *Protractor*. Theſe Parallel Lines are called *Meridians*: and upon (or parallel to) one of theſe, muſt the *Meridian Line* of your *Protractor*, noted with E F, always be laid.

Your Paper or Velom thus prepared, aſſign a Point upon any *Meridian* (as A;) upon this Point A place the *Centre* of the *Protractor*, and the *Meridian Line* thereof,

E F,

E F, upon the *Meridian* of your Paper: Then by your *Field-Book* you find that the Degrees cut by the *Index* at A, were 191 deg. which being more than 180, lay the Semicircle of the *Protractor* downwards, and againſt 191 deg. make a Mark with your *Protracting-Pin*; then from A, through this Point, draw the Line A B, to contain 10 Chains 75 Links.

2. Apply the Centre of the *Protractor* to the Point B, with the *Meridian Line* E F thereof, parallel to one of the pricked *Meridians* upon the Paper: And ſeeing the Degrees cut at B are more than 180, *viz.* 279, lay the *Protractor* downwards, (as before,) and make a Mark againſt 279 deg. through which draw the Line B C, containing 6 Chains 83 Links.

3. Apply the *Protractor* to C, and the *Meridian* of it parallel to the *Meridians* on the Paper, and the Degrees cut at C being more than 180, *viz.* 216 deg. 30 min. lay the *Protractor* downwards, and make a Mark againſt 216; through which, from C, draw the Line C D, containing 7 Chains 82 Links.

4. Lay the Centre of the *Protractor* upon D, the Semicircle downwards, and againſt 325 deg. (which were cut by the *Index* at D,) make a Mark, through which draw a Line from D to E, to contain 6 Chains 96 Links.

5. Apply the Centre of the *Protractor* to the Point E, and the *Meridian Line* thereof upon (or parallel to) one of the *Meridians* upon the Paper: Now (becauſe the Degrees cut by the *Index* at this Obſervation made at E, were leſs than 180, *viz.* but 12 deg. 30 min.) therefore lay the Semicircle of the *Protractor* upwards, and make a mark or prick againſt 12 deg. 30 min. through which, from the Point E, draw the Line E F, to contain 9 Chains 71 Links.

In this manner muſt you *Protract* all the other *Angles* at F, G, H, and K; and more, if more had been: Remembring always, that if the Degrees to be Protracted be leſs than 180, the Semicircle of the *Protractor* muſt be placed upwards; but if above 180 deg. downwards.

CHAP. XII.

How to take the Plot *of a* Lordſhip, *or* Mannor, *conſiſting of divers Severals, as* Paſture, Arable *and* Wood-Land.

I Need not uſe many Words to illuſtrate what I intend to teach in this *Chapter*, having already ſeveral ways declared the manner how this Work, or the like, may be performed: Yet I will here ſhew one other way, ſomewhat (though not much) differing from the laſt way in *Chapter* X: and in the Explanation thereof I ſhall be very brief, giving you only the Sight of a *Field-Book*, in which the Obſervations were entred that were taken in the ſeveral *Fields*: the *Sight* whereof, compared with the *Figure*, (obſerving the Directions in the two foregoing *Chapters*,) will give more Light than a large *Chapter* of Information or Precepts.

The Form and Order of the Field-Book.

	C.	L.				
			⊙ 1			
	C.	L.	0d 0′	C.	L.	
	0	10	At ⊙	0	10	
			1	1	50	1 60 and a Gate
a Gate	0	52	2	2	20	
	0	15	2	2	45	
			2	2	83	0 45 and a Hedge
a Hedge	0	50	4	4	0	
			4	4	77	0 0
			5	5	00	0 60
	0	30	5	5	81	
			6	6	80	0 60
	0	45	7	7	0	
	0	07	7	7	20	
			7	7	60	0 20
			8	8	25	

	0	30	⊙ 2		0	40
			151 0		<	
a Hedge and Stile	0	25	At		9	52 a Gate
			1	1	38	
	0	10	1	1	58	
			1	1	94	

	0	25	⊙ 3		0	16
		>	161 0			
			At			
	0	10	1	1	10	
Lanes end	0	30	2	2	43	
			2	2	60	0 42 Lanes End(
			2	2	85	

	0	50	⊙ 4			
			100d 30′		<	
			At			
			0	2	0	0 60 Lanes End
	0	60	1	2	0	
			1	2	45	0 15
	0	35	1	3	95	
			2	3	30	0 65
			2	3	70	0 20
			3	4	55	0 45 Lanes End
			3	4	80	

			⊙ 5			
			79d 0′		<	
			At			
Lanes Corner	0	15	0	0	20	0 25
			0	0	50	Corner of
			1	1	40	the Lane

		>	⊙ 6			
	0	20	129d 0′		0	25
	0	20	0	0	73	0 25

	0	20	⊙ 7		0	25
			118d 0′		<	
			At			
	0	40	1	1	10	
			1	1	40	0 0
			2	2	30	0 60
	0	20	2	2	40	
			2	2	60	0 45

	C.	L.				
			⊙ 8			
		>	139½d			
			At			
			0	7	0	0 15 and 1 C.
	0	15	1	3	5	0 L. more.
	0	20	2	4	0	0 30
			2	8	0	The Station's

end: But ſet forwards 0 C. 60 L. to againſt a Hedge, and ſet off to the Right 30 L. for the Hedge Corner at *a*.

			⊙ 9			
			80d 0′		<	
			At			
	0	35	0	0	30	I croſs a
			1	1	10	0 60 Hedge.
	0	97	1	1	88	
			2	2	15	0 20
	2	30	3	3	05	

			⊙ 10			
			131d 6′		<	
			At			
			0	0	32	0 73
			1	1	05	1 25
	1	95	1	1	25	
			1	1	70	0 55
	1	08	2	2	40	
			2	2	82	0 75
			4	4	00	The Station's

end, and the Wood encloſed.

Having thus far proceeded, and incloſed the Wood in your Protraction, according to the Notes in your *Field-Book*, from the Points ſet off in your *Field-Book* to the left Hand in your 9th and 10th Stations, which will be the Points *a*, *b*, *c*, *d*, and *e*.— If you now draw the Lines *ab*, *bc*, *cd*, *de*, *ef* and *fg*, you ſhall have incloſed *Weſt Field* alſo.

Then going to your *Inſtrument*, ſtanding all this while at your 10th Station, direct it to your 11th Station, and enter your Notes in your *Field-Book* as followeth.

			⊙ 10			Making an *Angle*
		>	128½d			with ⊙ 9
			At			of 128½d
			3	3	35	1 90
	0	00	3	3	70	
			4	4	50	1 22
	0	45	5	5	20	

			⊙ 11			
			100d 0′		<	
			At			
	0	20	0	0	00	
	0	65	1	1	35	
			1	1	75	I croſs a
			4	4	60	Hedge.

I am come now to my Firſt Station, where I firſt began.. *Obſerve*

Observe well the Field-Book *foregoing, and compare it with* Figure VI.

YOur *Instrument* being placed at Station 1, [thus marked in the *Figure* ⊙1,] and your Sights directed to ⊙ 2, your Visual Line was ⊙ 1, ⊙ 2. — In the measuring of which Line I find, by my *Field-Book*, that at ⊙ 1 I was distant from the Hedge corners on either side 10 Links of a Chain: wherefore upon a Sheet of Paper I draw a Right Line, as ⊙ 1, ⊙ 2, and on either side thereof I set 10 Links from ⊙ 1 to *n* on the Right-hand, and from ⊙ 1 to *f* on the Left-hand. — Then I find by my *Field-Book*, that at the end of 1 Chain and 15 Links, my Visual Line is distant from the bow of the Hedge on the Right-hand 16 Links, which I set from my Visual Line to *o*, and draw *n o*. Also, at the end of 2 Chains 45 Links, I find that I am distant from a bow in the Hedge and a Gate there 52 Links: wherefore I set 52 Links from my Visual Line to *t*, and draw the Line *f t*. And in this manner I proceed (as my *Field-Book* directs me) till I have gone quite round the several parcels to be Surveyed, (which are here Six:) and proceeding in this manner, you shall have the true *Plot* of all your Severals, as in the *Figure*; which you may cast up and order according to the directions following.

And now having finished your Field-work of these Four other Closes, and by your Protraction found the Points *h, i, k, l, m, n, o, p,* and *q*, if you draw the Lines *c h, b i, i k*, and *k d*, you shall have inclosed the *Upper Fall*. —— And if you draw the Lines *k p, p r*, and *r f*, you shall inclose *Brill Mead*. — Also, if you draw the Lines *i q, q n, n o*, and *o p*, you will inclose the *Lower Fall*. —— *Lastly*, If you draw the Lines *b l, l m*, and *m p*, you will inclose the *East Picle*.

CHAP. XIII.

The Construction, Description, and Use of a Decimal Table, *Shewing the Difference of* Longitude *and* Latitude *of any* Line *at every Degree of Intersection with the* Meridian, *for the ready finding the* Easting *and* Westing, Northing *and* Southing *of any Measured* Line, *having its* Bearings, *and also the* Perpendicular Altitude *and* Horizontal Line *of any* Hill *or* Mountain.

The Decimal Table.

D	10	20	30	40	50	60	70	80	90	100
01	0017	0035	0052	0070	0087	0104	0121	0139	0156	0174
02	0035	0070	0105	0140	0174	0210	0244	0280	0314	0349
03	0052	0105	0157	0209	0262	0310	0364	0416	0468	0523
04	0070	0140	0209	0279	0349	0420	0488	0558	0628	0698
05	0087	0174	0261	0349	0436	0523	0610	0697	0784	0872
06	0105	0209	0314	0418	0523	0628	0732	0837	0941	1045
07	0122	0244	0366	0487	0609	0731	0853	0975	1097	1219
08	0139	0278	0418	0557	0696	0835	0974	1114	1253	1392
09	0156	0313	0469	0629	0782	0938	1095	1251	1408	1564
10	0174	0347	0521	0694	0868	1042	1215	1389	1562	1736
11	0191	0382	0572	0763	0954	1145	1336	1526	1717	1908
12	0208	0416	0624	0832	1040	1248	1456	1664	1872	2079
13	0225	0450	0675	0900	1125	1350	1575	1800	2025	2249
14	0242	0484	0726	0968	1210	1452	1694	1936	2288	2419
15	0259	0518	0776	1035	1294	1553	1812	2070	2329	2588
16	0276	0551	0827	1102	1378	1654	1930	2205	2481	2756
17	0292	0585	0877	1169	1462	1754	2046	2339	2631	2924
18	0309	0618	0927	1236	1545	1854	2163	2472	2781	3090
19	0326	0651	0977	1303	1628	1954	2280	2505	2831	3256
20	0342	0684	1026	1368	1710	2052	2394	2736	3087	3420
21	0358	0717	1075	1433	1792	2150	2509	2867	3226	3584
22	0375	0749	1124	1498	1873	2248	2622	2997	3371	3746
23	03 1	0781	1172	1563	1954	2344	2735	3126	3516	3907
24	0407	0813	1220	1627	2034	2441	2847	3254	3661	4067
25	0423	0845	1268	1690	2113	2536	2959	3381	3804	4226
26	0438	0877	1315	1753	2192	2630	3069	3507	3946	4384
27	0454	0908	1362	1816	2270	2724	3178	3632	4086	4540
28	0469	0939	1408	1878	2347	2817	3286	3756	4225	4695
29	0485	0970	1454	1939	2424	2909	3394	3878	4363	4848
30	0500	1000	1500	2000	2500	3000	3500	4000	4500	5000
31	0515	1030	1545	2060	2575	3090	3605	4120	4635	5150
32	0530	1060	1590	2120	2650	3179	3709	4239	4769	5299
33	0545	1089	1634	2178	2723	3268	3812	4357	4901	5446
34	0559	1118	1678	2237	2796	3355	3915	4474	5033	5592
35	0574	1147	1721	2294	2868	3442	4015	4589	5162	5736
36	0588	1176	1763	2351	2939	3527	4114	4702	5290	5878
37	0602	1204	1805	2407	3009	3611	4212	4814	5416	6018
38	0616	1231	1847	2463	3078	3695	4310	4926	5542	6157
39	0629	1259	1888	2517	3147	3775	4405	5034	5663	6293
40	0643	1286	1928	2571	3214	3857	4499	5142	5785	6428
41	0656	1312	1968	2624	3280	3937	4593	5249	5905	6561
42	0669	1338	2007	2677	3346	4015	4684	5353	6022	6691
43	0682	1364	2046	2728	3410	4092	4774	5456	6138	6820
44	0695	1389	2084	2779	3473	4168	4863	5558	6252	6947
45	0707	1414	2121	2828	3536	4243	4950	5656	6365	7071

The Decimal Table.

D	10	20	30	40	50	60	70	80	90	100
89	0999	2000	3000	3999	4999	5998	6998	7998	8998	9998
88	0999	1999	2998	3998	4997	5996	6996	7994	8994	9994
87	0999	1997	3996	3995	4993	5989	6988	7987	8986	9986
86	0998	1995	2993	3990	4988	5986	6984	7983	8972	9976
85	0996	1992	2989	3985	4981	5977	6973	7970	8966	9962
84	0995	1989	2983	3978	4973	5962	6957	7952	8947	9945
83	0993	1985	2978	3970	4963	5953	6946	7939	8932	9925
82	0990	1981	2971	3961	4951	5940	6931	7922	8912	9903
81	0988	1975	2963	3951	4938	5925	6913	7901	8892	9877
80	0982	1969	2954	3939	4924	5909	6894	7878	8863	9848
79	0982	1963	3945	3926	4908	5890	6871	7853	8834	9816
78	0988	1956	2934	3913	4891	5869	6847	7825	8803	9781
77	0975	1949	2923	3897	4872	5846	6821	7795	8779	9744
76	0970	1941	2911	3881	4851	5821	6792	7762	8773	9703
75	0966	1932	2898	3864	4830	5796	6762	7728	8694	9659
74	0961	1923	2884	3845	4806	5768	6729	7690	8652	9613
73	0256	1913	2869	3825	4782	5737	6694	7650	8607	9563
72	0951	1902	2853	3804	4755	5706	6657	7608	8559	9511
71	0946	1891	2836	3782	4727	5673	6618	7563	8510	9455
70	0240	1878	2818	3758	4697	5638	6578	7518	8457	9397
69	0934	1867	2801	3734	4668	5600	6533	7467	8401	9336
68	0927	1854	2782	3709	4636	5564	6489	7416	8344	9272
67	0921	1841	2761	3682	4602	5520	6441	7362	8284	9205
66	0914	1827	2741	3654	4568	5478	6392	7306	8221	9135
65	0906	1813	2719	3625	4531	5434	6341	7248	8156	9063
64	0899	1798	2696	3595	4494	5390	6290	7189	8088	8988
63	0891	1782	2673	3564	4455	5343	6235	7126	8018	8910
62	0883	1766	2649	3532	4414	5295	6179	7062	7946	8829
61	0875	1749	2624	3498	4373	5247	6122	6997	7872	8746
60	0866	1732	2598	3464	4330	5196	6062	6928	7794	8660
59	0857	1714	2572	3429	4286	5142	5999	6857	7714	8572
58	0848	1696	2544	3392	4240	5086	5934	6783	7631	8480
57	0839	1677	2516	3355	4193	5029	5869	6708	7548	8387
56	0829	1658	2487	3316	4145	4971	5801	6630	7460	8290
55	0819	1638	2457	3276	4095	4912	5732	6552	7372	8192
54	0809	1618	2427	3236	4045	4851	5661	6470	7280	8090
53	0799	1597	2396	3194	3993	4779	5576	6372	7169	7986
52	0788	1576	2364	3152	3940	4726	5514	6303	7091	7880
51	0777	1554	2331	3109	3886	4661	5439	6216	6994	7771
50	0766	1532	2298	3064	3830	4596	5362	6128	6894	7660
49	0755	1509	2264	3019	3774	4528	5283	6038	6792	7547
48	0743	1486	2229	2973	3716	4461	5203	5946	6688	7431
47	0731	1463	2194	2925	3657	4392	5122	5853	6581	7314
46	0719	1439	2158	2877	3597	4348	5072	5796	6522	7193
45	0707	1414	2121	2828	3536	4243	4950	5656	6365	7071

The Description, Use, and Construction of the foregoing Decimal Table, *for the finding of the* Easting *and* Westing, Northing *and* Southing *of any Line, the Length and Bearing being given. And also the* Horizontal *and* Perpendicular Line *of any Hill*, &c.

I. *The Description.*

THIS *Table* is so ordered, as to be in two Pages, the one facing the other, so that both Pages may be seen at one view; the Left-hand Page having the first 45 Degrees of the *Quadrant* in a little Column on the Left-side thereof, and beginneth at the top, and is numbred downward, beginning with 1, 2, 3, *&c.* and ending with 45. And this Page hath also ten Columns more towards the Right-hand, which are noted at the Head with 10, 20, 30, *&c.* to 100, over the last Column. And as the Left-hand Page is noted at the Head, so is the Right-hand Page also; and hath a little Column also towards the Left-hand, which is numbred upwards, as the other was numbred downwards; and it hath the Degrees answering to the other half of the *Quadrant*, *viz.* from 45 to 90 Degrees, accounted upwards, for the readier finding the Complement of any Number of Degrees to 90, for the convenience of taking out the Numbers answering to any Degrees, and the Complement of the same Degrees, both at one view. The Use of this *Table* will more plainly appear by the Examples following, than by using of many Words for the explaining of it.

II. *The Use of the* Table.

Example 1. The following *Figure* VII. is the *Plot* of a large Wood near *Hornsey*, and adjoining to a place called *Copt-Hall*, which is called by the name of *Brown's Wood.*

Fig. VII. The Lines of the Sides are noted with A B C D E F G, as you see the pricked Lines of the outside of the *Plot* between Letter and Letter; and the black Lines within the pricked ones are the Bounds of the Wood; and the short pricked Lines which touch the other pricked at Right Angles, are Perpendicular Sets off from the Chain Line to the outside of the Wood.

There are two other pricked Lines; the one noted with *N. S.* which signifieth the *Meridian*, or *North* and *South*; the other noted with *E. W.* which sheweth the *East* and *West* parts of the *Plot* of the Wood: and these Lines intersect or meet one another in the Point A. And if a Line be drawn parallel to *N. S.* through the Point B, that will also be a *Meridian* Line, and shew the *North* and *South* parts of the *Plot*: and then from the Line *East*, *West*, draw also a Line parallel, cutting the Point B; and that will shew the *East* and *West* sides of the *Plot*. And in the like manner you may continue the drawing of Parallel-Lines from the *North* and *South*, and *East* and *West*, to all the *Angles* of the *Plot*, or any other, if you list to take that pains.

Then having provided a *Field-Book*, in form of that which followeth, divided and ruled into 8 Columns, you may proceed to the Field-Work in manner following, *viz.*

First, Placing your *Instrument* at A, and the *Flower-de-luce* towards you, direct your Sights to B, and observe what Degrees the *South* end of the Needle cutteth, which I find to be 37 Degrees, and beareth from the *North* towards the *West* 37 Degrees, as by the *Plot* and the Line of *North* and *West* from the Point A is discovered. And you may very easily discern that the Line A B lieth nearer to the *North* than the *West*, and therefore the *North* Column must have the bigger Number; and for the bigger Number you must always use the bigger *Angle*; which in this case is the Complement of 37 Degrees to 90 Degrees, which is 53 Degrees. Then measure the distance of the *Instrument* from the corner of the Wood, it being here Perpendicular to the corner: but if the *Instrument* had been set up before you come to the Perpendicular let fall from the corner of the Wood upon the measured Line, then you must measure so far of the measured Line, till there will a Perpendicular fall from the

the corner of the Wood or Field upon the measured Line. Then count how much of the measured Line cometh to that Perpendicular, and note that down in the Column provided for the Chain Line; and then measure the distance from the place the Chain was accounted from the *Instrument*, and set that down in another Column fitted for that purpose, for Sets off, either on the Left or Right-hand. And as in this *Plot* all the Sets off are to the Right-hand, the distance from the *Instrument* to the corner of the Wood at A is 1 Chain and 60 Links, which I set down in a Column on the Right hand of the Chain-Line Column, and Cyphers in the room of Chains and Links; signifying there was no part of the Line A B measured where that Perpendicular fell. Then measure from A to the next Perpendicular at Number 1, which is 7 Chains 00 Links: this must be put in the Chain-Column under the Cyphers. Then measure the distance from the Chain to the Bounds of the Wood, 1 Chain 73 Links, and put that in the next Column under 1 Chan 60 Links. Then measure from 1 to the Number 2, which is from A 10 Chains and 60 Links, which must be placed under 7 Chains 00; and there the distance from the Wood is nothing; therefore place Cyphers in the next Column, under 1 73. Then from A to 3 is 14 Chains 00 Links, which place under 10 60, and measure the distance from thence to the Bounds of the Wood, which is 80 Links, which put in the next Column under 0 00. Then from A to 4 is 16 Chains 80 Links, which place under 14 00, and at that place the distance of the Bounds of the Wood, which is 88 Links; place 88 under 80 in the next Column. And from A to 5 is 19 Chains 60 Links; then put 19 60 under 16 80, and there take the distance of the Bounds of the Wood, which is 1 Chain 06, which place under 0 88, and proceed to the Number 6. The distance from A to B is 29 Chains 40, which place under 19 60, and find the shortest distance of the Bounds of the Wood, which is 1 Chain, which place in the next Column: and then put 29 40 under 29 40, and take the greatest distance, which is 1 Chain 92: which place under 1 00, and by that means, when you come to Plot the Wood, you will have that break at Number 6. (But if the break had been the contrary way, then you should have taken the greatest distance first, and set down the number of Chains and Links from A; and then afterward taken the shortest distance, and against it in the Chain-Line set down the distance from A: and by using of this method you will not be subject to mistake, whether the break standeth from you or cometh towards you.) Then having finished my work at Number 6, I measure forward to the Beacon placed at B, and I find the distance from A to B is 35 Chains and 20 Links, which I place under 29 40; and then measure the distance from the Beacon in a Perpendicular Line to the Bounds of the Wood, which is 1 Chain and 96 Links, which must be placed under 1 92: then is the Line A B finished.

Then take down the Beacon and set it up at C, and place the *Instrument* at B, where the Beacon stood; and putting the *Flower-de-luce* towards you, direct the Sights to C, and you will find that the *South* end of the *Needle* will stand over 71 Degrees from the *South* towards the *West*; and therefore the *West* Column must have a great deal the bigger Number, as the proportion of the Sine of 19, the Complement of 71, and the Sine of 71 require. Then place down this 71 Degrees in its proper Column, and *S. W.* with it, to signifie that the Line is *South west* 71 Degrees. Then measure the distance from the *Instrument* to the Bounds of the Wood in a Perpendicular Line to B C, and note it down as you did in the Line A B. Then, as you measure towards C, take notice of the Perpendiculars as you pass by, and set them down in the same order as you did in the Line A B: and having set them down in order in your *Field-book*, and measured to the Beacon at C, which is 10 Chains 20 Links, and the Perpendicular nothing; then is the Line B C finished.

Then take down the Beacon at C, and set it up at D, and place the *Instrument* where the Beacon stood at C, and bring the *Flower-de-luce* towards you, and direct the Sights to D; the *Instrument* being set level, you will find that the *South* end of the *Needle* will rest over 18 Degrees *Westward* of the *North*; which must be noted in your *Field-Book* thus, 18 D. *N. W.* And here the *North* Column must have the bigger Number; the Line C D being nearer the *North* than the *West*, must be the bigger Number, and longer Line. Then measure from C to D, which is 17 Chains and 20 Links; taking notice of the Perpendicular Sets off as you go along, as you were directed in the Line A B, and enter it into your *Field-Book* accordingly.

Then

Then remove the *Inſtrument* to D, and place it where the Beacon ſtood, and ſet up the Beacon at E; then turn the *Flower-de-luce* towards you, and direct your Sights to E, and the *South* end of the *Needle* will cut 52 Degrees *Eaſtward* from the *North*: therefore you muſt enter in your *Field-Book* 52 D. *N. E.* And here the *Eaſt* Column muſt have the bigger Number; for 52 being more than 45, the Line D E is more *Eaſt* than *North*: for if the Degrees of bearing be juſt 45, the Number in both Columns is alike.

Then meaſure from D towards E, until you come againſt the Perpendicular of the corner of the Bounds of the Wood; and in this *Plot* we go forward 88 Links before we come to the Perpendicular of the Wood-corner: then ſet down 00 88 in the Chain-Column, and meaſure how far it is in that place to the Chain-Line, which will be found to be 50 Links; which muſt be put into the next Column, as you did in the Line A B.

Then meaſure to the Beacon at E, and as you paſs by take an account of the Length of the Line to that corner of the Wood, which is 20 80, and the diſtance there from the corner of the Wood 50 Links: ſet them both down as you did the other, and meaſure to the Beacon at E, which is 21 10; which enter in the *Field-Book*, and the Line D E is finiſhed.

Then remove the *Inſtrument* to E, and the Beacon to F, and turning the *Flower-de-luce* towards you, direct the Sights to F, and the *South* end of the *Needle* will cut 38 Degrees, which is from the *South Eaſtward*, and muſt be noted down in your *Field-Book* thus, 38 D. *S. E.* or thus, *S. E.* 38. This 38 being leſs than 45 to the *Eaſtward*, the Line E F will be nearer to the *South* than to the *Eaſt*, and the *South* Column muſt have the bigger Number. Then meaſure from E to the corner of the Wood 50 Links, and the diſtance is alſo 50 from the corner, as you may plainly ſee in the *Figure* of the *Field-Book* following. So going forward, and taking an account of all the Perpendicular Sets off from thence to F, as they are noted in the *Field Book*.

Then remove the *Inſtrument* to F, and the Beacon to G, and bring the *Flower-de-luce* towards you, and direct the Sights to G, and the *South* end of the *Needle* will cut *South-Eaſt* 22 Degrees: which place in your *Field-Book* as before, and meaſure the Line F G as you did the others, and put it in the *Field-Book*.

Then remove the *Inſtrument* to G, and the Beacon to A, and the *Flower-de-luce* ſtanding towards you, and the Sights directed to A, the *South end* of the *Needle* will ſtand *South-Weſt* 50 Degrees; which note in your *Field-Book* thus, *S. W.* 50 D. Then meaſure the Line G A, and take P. the Perpendicular Sets off, as you go along; and enter this Line in your *Field-Book*, as you did in the other; and they will ſtand as in the *Figure* of the *Field-Book* is expreſſed.

This *Field Book* hath Eight Columns. The Firſt of them towards the Left-hand is for the Bearing of any Line from the *North* or *South*, *Eaſtward* or *Weſtward*, and the number of Degrees that the Line differs from the *Meridian*; as in the firſt Line of the *Figure*, A B is *Weſtward* from the *North* 37 Degrees, as you ſee noted in the upper part of the firſt Column. The Second Column is noted at the Head with Per. S. which ſignifie Perpendicular Sets off towards the Left-hand, which will many times happen, although in this Example all the Sets off are to the Right-hand; but many times in meaſuring through Lanes, it will be convenient to take Sets off on both ſides, and therefore have a Column for both hands. The Third Column is noted at the Head with C. L. which ſignifies Chains and Links: and in that Column is ſet down in Chains and Links the diſtance from the *Inſtrument* to any Break, or Perpendicular Set off, in all the Lines that encloſe the Wood, or any other parcel of Ground. And the Fourth Column is for the ſame uſe as the Second, only the difference of Left and Right-hand Sets off, as is before declared in the meaſuring of the Line A B. The other Four Columns are noted at Head in this order, *North. South. Eaſt. Weſt.* and are for no other uſe, but to ſet down how far any Line goeth upon any of thoſe Points.

Having finiſhed your Work in the *Field*, lay your *Field-Book* before you; wherein you ſhall find your firſt Bearing to be *N. W.* 37 Degrees, and the Length of your firſt Stationary Line, A B, to be 35 Chains 20 Links.

Now repair to your *Decimal Table*, and look for 37 Degrees in the firſt Column of the Left-hand Page; againſt which (and under 30, for 30 of your Chains) you ſhall find 1805, and in the ſame Line under 50, (for the 5 Chains) you ſhall find 3009, which ſet under 1805 one place backwards: Then for the 20 Links, look

in

The Figure of the Field Book.

Bearing	D.	Per.	S.	C.	L.	Per.	S.	North.	South.	East.	West.
N. W.	37			00	00	1	60				
				07	00	1	73				
				10	60	0	00	23.96			18.05
				14	00	0	80	3.993			3.009
				16	80	0	88	1597			1204
				19	60	1	06				
				29	40	1	00				
				29	40	1	92				
				35	20	1	96				
S. W.	71			00	00	0	48		03.256		09.495
				02	00	0	00		0651		0651
				06	80	1	20				
				09	00	0	80				
				10	20						
N. W.	18			00	00	0	00	09.511			03.090
				04	40	1	40	6.657			2.163
				13	20	0	00	1902			0618
				16	75	0	60				
				17	20	0					
N. E.	52			00	00	0	00	12.31		15.76	
				00	88	0	50	0616		0788	
				20	80	0	50	0616		0788	
				21	10						
S. E.	38			00	00	0	00				
				00	50	0	50				
				02	40	0	00		23.64	18.47	
				06	00	0	80		2364	1847	
				13	70	0	40		3152	2463	
				17	60	0	50				
				21	40	1	00				
				26	80	2	80				
				33	40	0	40				
S. E.	22			00	00	0	40				
				03	70	0	00		18.54	07.49	
				15	00	0	90		0.927	0.375	
				21	40	0	40		5564	2248	
				21	60						
S. W.	50			00	00	0	00				
				00	30	0	20		06 43		07.66
				02	40	0	00		0.643		0766
				05	60	0	30		4499		5362
				10	40	0	00				
				11	70						
The Summ is								57.3585	57.1866	45.2799	45 0165

in the ſame Line (of 37 deg.) under 20, and there you ſhall have this Number 1204, which ſet under the other, ſtill one place backward: All which Numbers muſt be ſet in this Order, under the Weſt Column of the Field-Book; and ſo will they ſtand there as you ſee them ſtand in the Margin, and their Summ is 21.1794.

	18.05
	3.009
	.1204
	21.1794

Alſo look in the Right-hand Page of the Decimal Table, for 53 deg. (the Complement of 37 d.) and againſt 53, and under 30, (for 30 Chains,) you ſhall find 2396; and in the ſame Line, under 50, (for the 5 Links,) 3993, which ſet under the other; and in the ſame Line, under 20, (for the 20 Links,) you ſhall find 1597, which ſet under the two other, which are to be placed in the North Column, as in the Margin, whoſe Summ is 28.1127. And theſe Numbers you ſhall find ſtand in the North and Weſt Columns of your Field-Book, becauſe the Bearing was North-Weſt: And thus muſt you doe with all the Bearings, as is done in the Figure of the Field-Book. Which done, add up each Column; and if the North and South are both equal, and the Eaſt and Weſt alſo equal, then you may be ſure that the Work is exactly done, and the Degrees of Bearing truly taken, and the Lines alſo truly meaſured. But in this Example there is a quarter of a Chain difference between the Eaſt and Weſt, and 17 Links difference betwixt the North and South; which in this large Wood is not very material, by reaſon we uſe all whole Degrees. And this ſmall Error may very eaſily be mended in the plotting of the Chain-Lines. But if you uſe an Inſtrument that you can diſcern Halfs or Quarters of a Degree, or Tenth parts of a Degree, if poſſible, then if you make uſe of this Table, you muſt make proportion for the Minutes; but if you make uſe of a Table of Natural Sines, the Work may be done by one Multiplication for each Column, which will plainly appear as followeth.

23.96	
3.993	
.1597	
28.1127	

Example II. *How by a Table of Natural Sines, to ſupply the Uſe of the foregoing Decimal Table.*

If the Line A B had been N. W. 37 Degrees 15 Minutes, or one quarter of a Degree, the Natural Sine thereof is 605294; which being multiplied by the Length of the Line 3520, produceth 21,30634880, which is 21 Chains 30 Links, and a little more than half, for the Weſt Column. Then to find the Number for the North Column, ſubtract 37 Degrees 15 Minutes from 90 Degrees, and the Remainder is 52. 45: Then look the Natural Sine thereof, which is 7960, (the four firſt Places thereof:) then multiply 7960 by 3520, and the Product will be 28.019200, which is 28 Chains and almoſt 2 Links for the North Column. And by this you ſee how far the Line goeth North, and how far Weſt. But by reaſon all Perſons may not be furniſhed with a Canon of Natural Sines, it may be as well done by two Additions in the Sines and Logarithms in the Third Book, where you may be furniſhed with every 10 Minute, and the Logarithms to 1000, which will very well ſerve for ſhort Lines, or ſmall Incloſures; but the Line in the laſt Example is fitter to be done by a larger Canon, as in the next Example.

Example III. *How to perform the Work of the foregoing Example by the Tables of Artificial Sines and Logarithms.*

Firſt, take out the Sine and Co-ſine of the obſerved Angle, and ſet them down as followeth.

Sine 37 Degrees 15 Minutes,	9,7819664.	Co-ſine,	9,9009142.
Logarithm of 35. 20.	1,5465426.	Log.	1,5465426.
Number 21. 31.	1,3285090.	28. 02.	1,4474568.

Having thus ſet down the Sine and Co-ſine, then look the Logarithm of 35 Chains and 20 Links, and ſet it down under the Sine and Co-ſine, and add the Logarithm to the Sine and Co-ſine: Then look into the Tables of Logarithms for the neareſt Logarithm that you can find to theſe Numbers, and the Numbers anſwering to thoſe Logarithms is the Number in Chains and Links; for the North Column is 28. 02, and the Number for the Weſt Column is 21. 31. And ſo you may readily find how far

far any Line goeth North or South, East or West, by Addition only, as you see by this Example.

Example IV. *How to find the Horizontal Line of a Hill by this Decimal Table.*

Let the pricked Lines A H and B H in *Figure* VII. represent some Hill whose Altitude is 37 Degrees, and the Length of the Line A H be 28 Chains, and the Angle A B H 53 Degrees, and the Line B H 21 Chains 36 Links: Then by the foregoing Decimal Table find the Line A O by the Complement of the Angle at A, *viz.* 53 Degrees, and answering to the Line A H, 28 Chains. First, against 53, and under 20, I find 1597; and against 53, and under 80, which I tak for 8, I find 6372: Place them as you see in the Margin, and add them together, and they will stand thus: So part of the Horizontal Line of the Hill, from A to O, is 22 Chains and 34 Links; the other part will be as easily found by the Complement at B, which is 37 Degrees, and the length of the Line B H, 21 Chains and 36 Links. *Fig.* VII.

1597
6372
22.342

First, find 37 Degrees in the little Column on the Left-hand Page, and under 20 at Head you may find 1204; and in the same Line, under 10, which I make use of for 1, you may find 0602; and in the same Line, under 30, may be found 1805; and farther in the same Line, under 60, which is used for 6, may be found 3611; which you must place all in order, one place farther towards the Right-hand, each Number as they stand in order in the Margin; then add them together, and their Summ will be, as you see, 12 Chains and almost 86 Links; and to this part add the former, which you found for A O, and their Summ will be 35 Chains and 20 Links, which is the whole Base-line of the supposed Hill, or the true Length of the Line A B, as it was first measured. And by the same way of working, the Horizontal Lines of any Hill that can be measured over, although you have 10 or 20 Stations in going up, and as many in coming down, yet by this Table all the Horizontal Lines may be found by Addition, and the whole Length of the Base gained by many, as well as this of two Stations: The Line that goeth over the Hill supposed, is 49 Chains and 36 Links, and the Horizontal or Base-line thereof is but 35 Chains and 20 Links. I hope you may as well understand, by the supposing of these Lines to be a Hill, as if they had been a Real Hill.

1204
0602
1805
3611
12.85861
22.342
35.20061

Example V. *How by this Table you may find the Perpendicular Line of any Hill that you can measure up or over, as is supposed in* Figure VII. *before treated of, marked with* H O.

For the finding the Perpendicular Height of any Hill, the Altitude being taken, and the Line to the top measured, as in the former Figure, the Angle at A being 37, and the Line A H 28 Chains; First, against 37 Degrees, and under 20, may be found 1204; and under 80, in stead of 8, in the same Line is 4814; which being placed as in the Margin, will stand thus; and being added together, will make 16 Chains 85 Links and almost a half; and so much is the Perpendicular Line O H, which is the Height of the supposed Hill. And after the same manner the Perpendicular Height of any Hill may be obtained.

1204
4814
16.854

I Thought here to have concluded with *Surveying*; but forasmuch as there is an Ancient Instrument, called the *Plain Table*, much used by some, whose Plainness makes it so common, (but he that uses it, shall find it very tedious, and not at all times fit to be used,) I shall exemplifie the Use thereof in one Example, according to the Method used by my very loving Friend Mr. *Vincent Wing*, an able Astronomer and Surveyor, and the greatest Lover of this Instrument I ever met with; and therefore I shall here insert an Example of his own.

CHAP.

CHAP. XIV.

How to take the Plot of a Field by measuring from one Angle to another, round about the same.

Fig. VIII. Suppose this Irregular Figure A B C D E F G H I K L S to represent a Field, into which when I first enter, I cause Whites to be set up in every Corner of the same; which done, I make choice of the most convenient Angles thereof for my Stations, not regarding their number, whether I make three, four, five, or six Stations, yet the fewer the better: but you must always take so many, as you may clearly see from one Station to another. Thus in the former Figure I make choice of my first Station at X, as a Place most fit for my purpose, in regard that from thence I can see the Point S; and therefore I send one with a Beacon to S, so as when I go on in the Line X S, I may take a Tangent at B: Then planting my Table at X, I direct my *Index* to D, and strike the Line X D, which I measure, finding it $5^{p}\ 30^{l}$; and this I set down from X to D, and so make a Prick at D. This done, I direct the *Index* to S, and strike the Line X S; then I measure on in the said Stationary Line, till I come to Z, $47^{p}\ 30^{l}$ from X, where laying my *Index* upon the Line S Z X, I direct the Sights to X, and there fix the Table with the Screw-pin. Then turning the *Index* upon the Point Z, till through the Sights I espie the White at C, I strike the Line Z C, and finding the Distance $13^{p}\ 50$, I set it down from Z to C, where I make a Point, and draw the Line D C. Then I go on with my former Measure, till I come to B $68^{p}\ 40$, and there I make a Point, and draw the Line C B; which done, I go on carefully with my former Measure, till I come to the place of my second Station at S, finding the Stationary Distance X S $95^{p}\ 10$, which from my Scale I apply from X to S, where I make for my second Station such a Mark as this, (⊙). Then planting my Table there, I direct my Sights to A ♌ and L, and I find S A $13^{p}\ 0$, S ♌ $9^{p}\ 0$, and S L $3^{p}\ 0$, which set down severally upon their respective Lines, I have the Points A, ♌, and L, upon the Paper. Then drawing the Lines A B, A ♌, and ♌ L, we have one Side of the Field described.

Secondly, I plant my Table at S, and laying the Fiducial Line of the *Index* upon the Line S X, I move the Table, till through the Sights I espie the Point X; then fixing my Table, I turn my *Index* about upon the Point S, till through the Sights I see the Beacon at P, and by the Edge of the *Index* I draw the Line S P; which done, I proceed, and measure on the Stationary Distance; but as I go on, and am come to T, 35^{p} from my second Station, I observe I am in a manner against the Angle K; wherefore I plant my Table there, and direct the Sights to K. Then I measure the Distance, finding it $7^{p}\ 50$, which I set down from T to K, and draw the Line L K; which done, I continue my former Measure S T to P, and finding it $52^{p}\ 0$, I set it down from S to P, where again I make such a Mark (⊙).

Thirdly, I plant my Table there, and laying my *Index* upon the Line ♈ P S R, I turn the Table about, till through the Sights I see the Place of my second Station S: Then (the Table being there fixed) I direct the Sights of my *Index* from P to I, and finding it (by the Chain) to contain $4^{p}\ 60$, I set it down from P to I; which done, I describe the Line K I with the Point of my Black-lead Pencil, as before. Afterwards, from P, I direct the Sights to my next Station at G, and draw the occult Line P G: Then I measure on with my Chain from P towards G, and when I come to N $37^{p}\ 10$, (where I make a Prick with the Point of my Compasses,) I perceive I am almost against the Angle H; then laying my *Index* upon the Line P G, I turn the Table, till I espie G with the Fore-sight, and P with the Back-sight, (by which you may know whether you be exactly in the Line or no:) Which done, fix your Table, and direct your Sights from N to H, pricking down the Distance $7^{p}\ 40$, and so with your Pencil describe the Line I H: And after this is done, continue on your former Measure begun from P, till you come to G, (your next Station,) which you shall find to be $85^{p}\ 0$, which set down to G, and mark it again thus, (⊙).

Lastly, At G plant your Table, and lay the *Index* exactly upon the Line Y G P ♈: Then turn the Table gently, (for fear of shaking the *Index* off of the Line,) till through the Sights you espie the Beacon at P; which done, fix your Table, and remove the *Index*, directing the Sights from G to the place of your first Station at X,

where

where you began; and if it cut the Mark, it is very probable that your Plot is exact. Then I measure on towards X, and when I have gone to O, 18° 60 from G, I describe the Angle F, drawing the Line G F; and measuring still on, I find the Tangent Point E, 39° from G, which I note with the Point of my Compass, drawing the Lines F E and E D; which done, I continue on my former Measure in the Line G X, finding it to be exactly 65° 90: Then, because the same Extent taken from the Scale fals upon the Plot precisely in the Point X where I first began, it is an infallible Sign that the Plot of the Field is exactly taken.

CHAP. XV.

The Plot of a Field being taken by any of the foremen-tioned Ways, to find the Content thereof in Acres, Roods, and Perches.

IN the Treatise of the Mensuration of Geometrical Figures, there is taught how to measure all sorts of Plain Figures, both Regular and Irregular; so that to say any thing of that in this place, were superfluous: Yet (because Land is measured by a Chain particularly designed for Land-measure, such an one as is described before, in the Fourth *Paragraph* of the VI. *Section* of *Chap.* I. of *Longimetria*,) I will give you One Example thereof.

I. *How to Multiply Chains and Links by Chains and Links.*

Multiply the Numbers one by another, as is largely taught in *Decimal Arithmetick*; but in this, although you make a Point or Prick to separate the whole Chains from the Links, yet you are not to cut off just so many Figures of your Product as there are Figures behind the Points in the *Multiplicand* and *Multiplier*, which for the most part will be four Figures, but (always) five Figures must be cut off from the *Product*; and if there be not so many Figures as five in the *Product*, you must then make them up five, by adding Cyphers before them. I will here give you Examples in all Cases ready wrought, which will give more Light to the Thing intended, than many Words.

Example I. Multiply 27 Chains 53 Links by 8 Chains 25 Links.

	Ch.	*Lin.*
	27.	53
	8.	25
	13765	
	5506	
	22024	
Here the Product is 22 Acres and .71225 parts of an Acre.	22.	71225

Example 2. Multiply 12 Chains 9 Links by 7 Chains 15 Links.

	Ch.	*Lin.*
	12.	09
	7.	15
	6045	
	1209	
	8463	
Here the Product is 8 Acres and .64435 parts of an Acre.	8.	64435

Example 3. Multiply 23 Chains 80 Links by 4 Chains.

	Ch.	*Lin.*
	23.	80
	4.	00
Here the Product is 9 Acres and .52000 parts of an Acre.	9.	52000

	Ch. Lin.
Example 4. Multiply 1 Chain 25 Links by 0 Chains 15 Links.	1. 25 0. 15
	625 125
Here the Product is .01875 parts of an Acre.	.01875

	Ch. Lin.
Example 5. Multiply 27 Links by 8 Links.	0. 27 0. 08
Here the Product is .00216 parts of an Acre.	.00216

These are all the varieties that can at any time happen; and (always) the Figure or Figures to the Left-hand are whole Acres, and the Figures to the Right-hand of the Point are $\frac{327\,94}{100000}$ parts of an Acre, which must be reduced into Roods and Perches, as followeth.

II. *How to reduce the Fraction Parts of an Acre into Roods and Perches.*

You are to understand, that in an Acre there are 160 Square Perches, each Perch containing 16.5 Foot in length; so that in one Square Perch there are 272.25 Square Feet, and so in an Acre 43560 Square Feet: Also in one Rood is contained 40 Perches; so that a Rood contains 10890 Square Feet. And now to reduce the Decimal Fraction Parts of an Acre into Roods and Perches, this is the

RULE.

Multiply the Fraction Part (always) by 40, *and from that Product cut off (always) five Figures; the Figures to the Left-hand of the Point are Whole Roods, and the five Figures to the Right-hand are Decimal Parts of a Rood; which multiply by* 40, *and from the Product cut off five Figures; so shall the Figures to the Left-hand be Whole Perches, and the Figures to the Right-hand Parts of a Perch.*

Example. In the first Example beforegoing, which was 27 Chains 53 Links, multiplied by 8 Chains 25 Links, the Product whereof was 22.71225, which is 22 Acres .71225 Parts, which multiplied by 4, the Product will be 2.84900, that is, 2 Rood and .84900 Parts of a Rood, which multiplied by 40, the Product will be 33.96000, that is, 33 Perches and .96000 parts of a Perch; which may well be called 34 Perches, because .960000 is very near 1.00000: And so may you always doe, when the first Figure of the Fraction remaining is either 7, 8, or 9. And thus if this were a Piece of Land lying in a long Square, and being 27 Chains 53 Links in Length, and 8 Chains 25 Links in Breadth, the Content thereof will be 22 Acres, 2 Roods, and 34 Perches, as is evident by the Work.

	Ch. Lin.
	27. 53 8. 25
	13765 5506 22024
Acres	22.71225 4
Roods	2.84900 40
Perches	33.96000

III. *The Plot of any Piece of Land being taken, according to any of the ways before prescribed, to find the Content thereof in Acres, Roods, and Perches.*

Fig. IX. Suppose the Irregular Plot A B C D E F G to be a Field Surveyed and Plotted by some of the former Directions, and the Content thereof were required: You must first by the Doctrine of the *Mensuration* of *Geometrical Figures*, reduce the

the Irregular Plot into *Trapezias* and *Triangles*, as is there taught; so shall this Plot, consisting of seven Sides, be reduced into two *Trapezias* and one *Triangle*, *viz.*

(1.) The *Triangle* A G F, noted with K.
(2.) The *Trapezia* A B E F, noted with L.
(3.) The *Trapezia* B C D E, noted with M.

The Lengths of the several Bases and Perpendiculars whereof, let be such as the Numbers set to them do contain in Chains and Links. Then shall,

(1.) Half the Base, A F, of the *Triangle* A G F, *viz.* 24 Chains 46 Links, multiplied by 10 Chains 65 Links, the Perpendicular G H, produce for the Content of the *Triangle* K	26.04990
(2.) Half the Common Base of the *Trapezia* A B E F, *viz.* B F, 28 Chains 03 Links, multiplied by 40 Chains 92 Links, the Summ of the two Perpendiculars A N and E O, produce for the Content of the *Trapezia* L	14.69876
(3.) Half the Common Base, C E, of the *Trapezia* A B E F, 25 Chains, multiplied by 32 Chains 50 Links, the Summ of the two Perpendiculars B P and D Q, produce for the Content of the *Trapezia* M	81.25000
	221.99766

Which 221.99766 being reduced by the Directions of the two foregoing Sections, will be 221 Acres, 3 Roods, 39 Perches, and .62560 parts of a Perch, which may well be called 222 Acres, it wanting but the $\frac{3744}{10000}$ part of a Perch thereof.

IV. *A Better and more Exact Way of Casting up of any large Irregular Ground, being Plotted, than the foregoing Way of Reducing the Plot into* Triangles.

In going about your Field, and making Observation at every material Angle thereof, you measure your Station-Line from one Station to another, and as you pass by any small Bow or Bending in the Hedge, you, (1.) Note against what Number of Chains and Links such a Bow or Bend is, and set that Number down in your Field-Book. (2.) You measure from your Chain up to that Bow or Bending, and note that Distance down in your Field-Book, in a Column by the side of the other Column, on the Right-hand, if your Hedge be on your Right-hand; or on the Left, if it be on the Left-hand. Then when you come to Plot your Field, you first lay down the Lines you measured in the Field, which will bring the greatest part of the Body of the Field into a large *Parallelogram*, or a Four, Five, or Six-sided Figure, consisting all of Right-lines, they being the same that you measured with your Chain; and this large Right-lined Figure must be cast up by the Rules last delivered, by dividing it into *Triangles* or *Trapezias*: But for the Casting up of the other Parts, which lie between the Chain-lines and the Hedges, if you reduce them into *Triangles*, they will be a great many in number, and being Lines not measured by your Chain, in taking the Bases and Perpendiculars with your Compasses, you shall very much mistake, both in laying them down first, and in taking them off afterwards, especially if the Scale you protract or lay down by be very small, where 10 or 12 Links of a Chain, (which is half a Rod,) is barely to be estimated, though your Scale be well divided, and the Points of your Compasses very small. For the remedying of this Inconvenience, I shall here shew you a Way, (not commonly used by Mercenary Surveyors,) whereby you shall cast up the Quantities of those Out-lets without reducing of them into *Triangles*, (and so taking the Bases of them with your Compasses,) but by the Lines you really measured with your Chain, and then, let the Scale you Plot by be great or small, you shall have the true Quantity of those Out-lets as exact as any other part of the Field. An Example will make plain what I have here delivered.

Suppose that you have measured in the inside of a Field, by a Hedge lying on your Left-hand, a Chain-line containing 8 Chains 12 Links, which Line let be A B, and the Hedge is distant towards my Left-hand, from A, 25 Links of my Chain, which I note down in my Field-Book; and as I measure from A towards B, at 1 Chain 20 Links, I find a Bow in the Hedge, from which I measure with my Chain, and find the Distance thereof from my Chain to be 0 Chains 90 Links, which I set down, and going on forwarder, till I have measured 2 Chains 30 Links *Fig.* X.

towards B, I meet with another Bow in the Hedge, which is distant from my Chain, 1 Chain 05 Links, this I note down in my *Field-Book* also, and so all my other Lengths and Distances, as I find them in my passage from A to B: and when I come to B, my Observations as I made by the way, will be as in this *Table*; by which I Protract my Line A B (and consequently my Hedge) as followeth.

Ch.	Lin.	Ch.	Lin.	Ch.	Lin.
		Station			
0	25	A			
0	90	1	10		
1	05	2	30		
0	70	3	25		
1	50	5	30		
0	75	7	45		
0	25	8	12		

V. *To Protract this Observation.*

First, Out of any Scale, take 8 Chains 12 Links, and lay that distance down from A to B.

Secondly, Take 15 Links out of your Scale, and set it from A to C.

Thirdly, Take 1 Chain 20 Links, and set them from A to *b*; and from thence set .90 Links to D.

Fourthly, Take 2 Chains 30 Links, and set them from A to *c*; and from thence set 1 Chain 05 Links to E.

Fifthly, Take 3 Chains 25 Links and set them from A to *d*; and from thence set 70 Links to F.

Sixthly, Take 5 Chains 30 Links, and set them from A to *o*; and from thence set 1 Chain 50 Links to G.

Seventhly, Take 7 Chains 45 Links, and set them from A to *f*; and from thence set .75 Links to H.

Lastly, From B, at the end of 8 Chains 12 Links, set 25 Links from B to K.

Then draw the Line C D E F G H K, which shall represent the Hedge: This done, the next work will be,

VI. *To find the true* Area *or* Superficial Content *of this irregular Figure* A C D E F G H K B, *in Acres, Roods, and Perches.*

1. The Perpendicular A C is 25 Links, and the Perpendicular *b* D is .90 Links; which added together make 1 Chain .15 Links: the half whereof is 57 Links, which multiplied by 1 Chain 20 Links, the Product is .06840, for the Content of the *Trapezia* L.

2. The Perpendicular *b* D is .90 Links, and the Perpendicular *c* E is 1 Chain 05 Links; which added together make 1 Chain 95 Links; the half whereof is .97 Links: Then subtract 1 Chain 20 Links from 2 Chains 30 Links, the remainder will be 1 Chain 10 Links; which multiplied by 97 Links, the Product will be .10670 for the Content of the *Trapezia* M.

3. The Perpendicular *c* E is 1 Chain 05 Links, and the Perpendicular *d* F .70 Links, which added together make 1 Chain 75 Links: the half whereof is .87 Links: Then subtract *b c* 2 Chains 30 Links from *c d* 2 Chains 25 Links, the remainder will be .95 Links; which multiplied by .87 Links, the Product is .08265, for the Content of the *Trapezia* N.

4. The Perpendicular *d* F is 70 Links, and the Perpendicular *e* G is 1 Chain 50 Links, which added, make 2 Chains 20 Links; whose half is 1 Chain 10 Links: Then subtract *c d* 3 Chains 25 Links from 5 Chains 30 Links, the remainder is 2 Chains 05 Links; and that multiplied by 1 Chain 10 Links produceth .22550, for the Content of the *Trapezia* O.

5. The

5. The Perpendicular *e* G is 1 Chain 50 Links, and the Perpendicular *f* H is .75 Links; their ſumm is 2 Chains 25 Links, and the half 1 Chain 12 Links: Then ſubtract 5 Chains 30 Links from 7 Chains 45 Links, the remainder is 2 Chains 15 Links; which multiplied by 1 Chain 12 Links, the Product .24080, is the Content of the *Trapezia* P.

6. The Perpendicular *f* H is .75 Links, and the Perpendicular B K is 25 Links, their Summ is 1 Chain; the half whereof is 50 Links. Subtract 7 Chains 45 Links from 8 Chains 12 Links, there remains .67 Links; which multiplied by .50 Links, the Product is .03350, for the Content of the *Trapezia* Q.

Add all theſe Products together, as here done, and the Summ of them is .75755. which reduced, is 0 Acres, 3 Rood, and 1 Perch; and ſo much is contained in the irregular Piece A C D E F G H K B.

L—	.06840
M—	.10670
N—	.08265
O—	.22550
P—	.24080
Q—	.03350
	.75755

And this way of Caſting up theſe irregular Off-ſets is the moſt exact of any other.

The End of the Second Part.

THE DOCTRINE OF RIGHT-LINED TRIANGLES, Applied to PRACTICE IN Military Architect, OR FORTIFICATION.

PART III.

CHAP. I.

Of a Fort, *or* Fortification; *and of ſeveral* Axioms *to be obſerved therein.*

Firſt, A Fort is made to the intent that a few Men might be able to defend themſelves, and the Place, againſt a greater Number. (2.) Therefore the Place is environed with a Rampire or Wall, and a Ditch, of ſufficient height, breadth, and depth, to impeach the Aſſaults of an Enemy. (3. And becauſe the Sides thus incloſing a Fort are not of themſelves ſufficient, they have therefore Flanks to defend them; which Flanks are alſo themſelves Flanked by the Curtains or Sides. (4.) And for the better defence of each Curtain, it is requiſite that every Side of a Fort ſhould have two Flanks; and if the Curtain be very long, it may have four, ſix, or more. (5.) And ſo there will be two Flanks placed near together, the one ſcowring the Side towards the Right hand, and the other towards the Left, either Flank ſtanding Perpendicular to the Curtain that they Flank. (6.) And ſeeing the Curtains and Fronts of a Fort are eſpecially defended, therefore the greater the Flanks and the Gorge between them are, the better they are. (7.) And foraſmuch as the Front of a Bulwark needs the moſt defence, it ought ſo to be drawn, that it may be defended by Shot from as great a part of the Curtain as may be, which part of the Curtain is called the Second Flank. (8) The outward Flanking Angle ought never to exceed 150 Degrees; for, by how much the leſſer, ſo much the better it is: Neither ſhould the Outward or Diamond-point of a Bulwark be greater than 90 Degrees, nor leſſer than 60 Degrees. (9) And as the inward Flanking Angle, and the Angle of the Shoulder of the Bulwark, increaſe and decreaſe together, one exceeding the other 90 Degrees; therefore, as the inward Flanking Angle ſhould never be leſs than 15 Degrees, ſo the Angle of the Shoulder ſhould never be leſs than

than 105 Degrees. (10.) The longeſt Line of Defence drawn from the Angle of the Flank to the outward Angle of the Bulwark, ſhould never exceed 720 Foot, for that it ought not to be without Musket-ſhot. (11.) A Regular Fort is ſuch a Fort as conſiſts of Equal Sides and Angles: And, (12.) Such a Fort doth encloſe a greater quantity of Ground, and the Force is in all Parts alike; therefore a Regular Fort, (if the Ground or Place will admit, is better than an Irregular of the like number of Sides and Angles. (13.) And by what hath been already ſaid, in the Eighth before-going, it is evident, that a Fort of Three Sides and Angles is of no moment, nor one of Four of any great value; but the more Sides and Angles a Fort hath, the better and ſtronger it is. Wherefore, (14.) If the Fixed Line of Defence be 720 Foot, or (as in the Tables following) 72.00 Rod, then may the Curtain be about 42 Rod, the Front of the Bulwark about 28 Rod, and the Flank to the Gorge as 6 is to 7, and the Angle forming the Flank about 40 Degrees.

CHAP. II.

Explaining the Terms of Art *uſed in* Fortificatoin, *eſpecially of the* Parts *or* Members *of a* Fort.

I. A *Fort* is a Piece of Ground environed with a Rampire or Wall, and a Ditch, to impeach the Aſſaults of an Enemy. *Fig.* I.

II. A *Fortreſs* is a ſmall Fort, Caſtle, or Sconce.

III. A *Rampire* is a Wall of Earth encloſing the Place Fortified, the Foot or Foundation whereof is (in the Figure below) noted with *a b*.

IV. A *Curtain*, O N.

V. A *Bulwark*, N F G H E.

VI. The *Front* of a Bulwark, F G, or K L.

VII. A *Flank*, N F, or O L.

VIII. The *Gorge* of a Bulwark, or the Space between two Flanks, as N E.

IX. The *Gorge-Line*, N C.

X. The *Head-Line*, C G.

XI. The *Shoulder*, F or L.

XII. The *Diamond-Point* of the Bulwark; or ſometimes, the *Flanked Angle* of the Bulwark.

XIII. The *Second Flank*, O *i*.

XIV. The *Fixing-Fixed*, or *Longeſt Line of Defence*, O G.

XV. The *Shorteſt Line of Defence*, *i* G.

XVI. The *Inward Flanking Angle*, F *i* N.

XVII. The *Outward Flanking Angle*, K P G.

XVIII. The *Fauſſe-bray*, the Breadth whereof is noted with B C.

XIX. The *Ditch*, the Breadth whereof is noted with *d e*.

XX. The *Covert-way*, the Breadth whereof is noted with *e f*.

XXI. A *Caſemate*.

XXII. The *Parapet*, *viz.* of the Rampire, Fauſſe-bray, and Covert-way.

XXIII. The *Walk* on the Rampire.

XXIV. The *Scarpe* inward and outward, *viz.* of the Rampire, Parapets, and Ditch.

XXV. *Palizadoes*.

XXVI. A *Bank* or *Foot-pace*.

XXVII. The *Brim* of the Ditch.

XXVIII. The *Counterſcarpe*.

XXIX. A *Ravelin*.

XXX. An *Half-Moon*.

XXXI. An *Horn-work*.

XXXII. A *Trench*.

XXXIII. *Gabions*.

XXXIV. A *Breach*.

XXXV. A *Mine*.

XXXVI. A *Counter-mine*.

There

There are other *Terms* used in this Art, which will be better understood when there shall be occasion to make use of them.

Having hitherto delivered several *Definitions* and *Axioms* necessary to be known, I shall now come to apply the Doctrine of *Right-lined Plain Triangles* to Practice in this Art.

CHAP. III.

To find the Quantity of the Angles *in all Parts of a* Fort, *of any number of Sides proposed.*

BY the Thirteenth *Axiom* before-going, a Fort is to consist at least of Four Sides; and by the Eighth *Axiom*, the Flanked Angle of a Bulwark ought to be (at the least) 60 Degrees; therefore in a Regular Fort of Four Sides, the Flanked Angle of each Bulwark ought to be 60 Degrees; and therefore (by the Ninth *Axiom*) the Outward Flanking Angle (exceeding the Inward Flanking Angle by 90 Degrees) must needs be 150 Degrees.

Fig. II. In this Figure let B C be one Side of a Square, fortified with four Bulwarks, one of which let be N F G H T: And seeing the Flanked Angle of this Bulwark F G H is 60 Degrees, therefore the Half thereof, F G C, is 30 Degrees; and I G C, (being equal to D C A, namely, half the Angle of the Angle of the *Tetragon* or Square,) 45 Degrees, therefore S G F is 15 Degrees, and the Complement thereof, S F G, 75 Degrees, whereto is equal the Angle I M G, which is the half of K M G: Therefore the Outward Flanking Angle K M G is 150 Degrees, which was to be proved.

And thus seeing that in a Quadrangular Fort, the Flanked Angle is 60 *Degrees*, and the Outward Flanked Angle 150 Degrees, what these Angles will be in other Forts, consisting of more Sides than Four, we may find by help of these in manner following; for which this is the

RULE.

Subtract the Angle of the Square, 90 *Degrees, from the Angle of the* Polygon *proposed; half the Remainder add to the Flanked Angle of the Square,* (viz. 60 *Degrees*;) *so have you the Flanked Angle of the* Polygon *proposed.*—— *Also, Subtract the Half-remainder from the Flanking Angle of the Square,* (viz. 150 *Degrees,*) *and the Remainder is the Flanking Angle of the* Polygon *proposed.*

Example I. Of a *Pentagon.*		*Degr.*
The Angle at the *Perimetre* of the *Pentagon* is		108
The Angle of the Square,	Subtract	90
The Remainder		18
The Half whereof is		9
To which add the Flanked Angle of the Square		60
Gives the Flanked Angle of the *Pentagon*		69
From the Flanked Angle of the Square		150
Subtract the aforesaid Half-remainder		9
There remains the Flanked Angle of the *Pentagon*		141

Example II. Of a *Hexagon.*		*Degr.*
From the Angle of the *Hexagon*, being		120
Subtract the Angle of the Square		90
And there remains		30
The Half whereof		15
Added to the Flanked Angle of the Square		60
Gives the Flanked Angle of the *Hexagon*		75
And from the Flanked Angle of the Square		150
Subtract the foresaid Half-remainder		15
There remains the Flanked Angle of the *Hexagon*		135

And proceeding in this manner, you shall find, that the Flanked Angle will not be 90 Degrees, till you come to a Fort of Twelve Sides. And, Now

Now the Flanked Angle of a Bulwark being given, you may thereby come to the knowledge of all the other Angles requisite to be known.

Fig. III.

In the *Figure*, let B C be the side of a *Pentagon*, whose Angle at the Centre is		D.	M.
	BAC	72	00
The half whereof is	CAD	36	00
The Complement thereof to 90 Degrees is	DCA	54	00
Now, admit the Angle at the Bulwark	FGH	69	00
The half thereof	FGC	34	30
Subtracted from S G C, equal to	DCA	54	00
Remains the inward Flanked Angle	SGF	19	30
Equal to F P N, the Complement of either	SFG	70	30
Subtracted from two Right Angles		180	00
Leaves the Angle of the Shoulder	NFG	109	30
Again, The same Angle S F G, or	IMG	70	33
Doubled, gives the outer Flanked Angle.	KMG	141	00
Lastly, From two Right Angles		180	00
Subtract half the Angle of the *Polygon*	BCA	54	00
There remains the Angle	DCG	126	00

And thus, from any Flanked Angle proposed, you may find the Quantities of every of the other Angles.

But for any *Polygon* proposed, you may more compendiously set down the Angles of the Bulwark, and all the other Angles after the form of this *Example* following: remembring, that if the *Polygon* have more than 12 Sides, you make the Angle at the Bulwark a Right-Angle.

		D.	M.
To half the Angle of the *Polygon*	BCA	54	00
Add always		15	00
The Summ is the Flanked Angle	FGH	69	00
The half whereof	FGC	34	30
Subtract from half the Angle of the *Polygon*	BCA	54	00
There remains the inward Flanking Angle	SGF	19	30
Whose Complement is	SFG	70	30
Which subtracted from two Right Angles		180	00
Leaves the Angle of the Shoulder	GFN	109	30
And the same Complement S F G, or	IMG	70	30
Doubled, is the outward Flanking Angle	KMG	141	00

For the Angle forming the Flank, namely, the Angle F C N, it may be always about 40 Degrees. And according to this Rule is the following *Table* made.

A TABLE of the Dimensions of the Angles observed in in Fortifying any regular Polygon, *from the Square, to a* Figure *of* XII. *Sides: increasing, so that the Flanked Angle thereof is a Right Angle.*

The Number of the Sides of the *POLYGONS.*		IV.		V.		VI.		VII.		VIII.		IX.		X.		XI.		XII.	
		deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.
Angle at the Centre	BAC	90	0	72	0	60	0	51	17	45	0	40	0	36	0	32	43	30	0
Angle of the *Polygon*	BCE	90	0	108	0	120	0	128	43	135	0	140	0	144	0	147	22	150	0
Half the Angle of the *Polygon*	BCA	45	0	54	0	60	0	64	17	67	30	70	0	72	0	73	37	75	0
To which (always) add		15	0	15	0	15	0	15	0	15	0	15	0	15	0	15	0	15	0
The Flanked Angle	FGH	60	0	69	0	75	0	79	17	82	30	85	0	87	0	88	38	90	0
Half the Flanked Angle	FGC	30	0	34	30	37	30	39	38	41	15	42	30	43	30	44	19	45	0
Inward Flanking Ang. F P N, or S G F		15	0	19	30	22	30	24	38	26	15	27	30	28	30	29	19	30	0
Which added to a Right Angle		90	0	90	0	90	0	90	0	90	0	90	0	90	0	90	0	90	0
Angle of the Shoulder	NFG	105	0	109	30	112	30	114	38	116	15	117	30	118	30	119	37	120	0
Angle opposite to the Head Li.	GFC	55	0	59	30	62	30	64	38	66	15	67	30	68	30	69	43	70	0
Angle opposite to the Front	FCG	95	0	86	0	80	0	75	43	72	30	70	0	68	0	66	21	65	0
Complement of S G F, *viz.*	SFG	75	0	70	30	67	30	65	42	63	45	62	30	61	30	60	44	60	0
Outward Flanking Angle	KMG	150	0	141	0	135	0	130	42	127	30	125	0	123	0	121	21	120	0
Angle fronting the Flank	FCN	40	0	40	0	40	0	40	0	40	0	40	0	40	0	40	0	40	0

But if you would have the Flanked Angle of the Bulwark ſo to increaſe that for an *Octagon* it may be a Right Angle, then make the Flanked Angle two Third parts of the Angle of the *Polygon* propoſed; as is done in the *Table* following: But for any *Polygon* of above Eight Sides, let the Flanked Angle be a Right Angle.

A TABLE of the Dimenſions of the Angles obſerved in Fortifying any regular Polygon, *from the Square to a* Figure *of* VIII. *Sides, ſo increaſing, that the Flanked Angle of the* Octagon *is a Right Angle.*

The Number of the Sides of the *POLYGONS.*		IV.		V.		VI.		VII.		VIII.	
		deg.	m.	deg.	m.	deg.	m.	deg.	m.	deg.	m.
The Angle at the Centre	B A C	90	0	72	0	60	0	51	26	45	00
Half the Angle at the Centre	I A G	45	0	36	0	30	0	25	43	22	30
The Angle of the *Polygon*	B C E	90	0	108	0	120	0	128	34	135	0
The Flanked Angle	F G H	60	0	72	0	80	0	85	43	90	0
Half the Angle of the *Poylgon*	B C A	45	0	54	0	60	0	64	17	67	30
Half the Flanking Angle	F G C	30	0	36	0	40	0	42	51	45	0
The inward Flanking Angle	S G F	15	0	18	0	20	0	21	26	22	30
To which add a Right Angle		90	0	90	0	90	0	90	0	90	0
The Angle of the Shoulder	N F G	105	0	108	0	110	0	111	26	112	30
Angle oppoſite to the Head Line	G F C	55	0	58	0	60	0	61	26	62	30
The oppoſite Angle to the Front	F C G	95	0	86	0	80	0	75	41	72	30
The Complement of S G F, *viz.*	S F G	75	0	72	0	70	0	68	34	67	30
The outward Flanking Angle	K M G	50	0	144	0	140	0	137	08	135	0
The Angle forming the Flank	F C N	40	0	40	0	40	0	40	0	40	0

CHAP. IV.

Of the Quantity of the Curtains, Flanks, Fronts, Gorges, *and other* Lines *and* Sides, *in* Regular Forts *of any determined number of* Sides.

THere is no neceſſity that the Angles in Forts ſhould be exactly ſuch as are form'd and ſet down by the foregoing Rules, but they may be ſomething more or leſs, as the place or other occaſions ſhall require: But, firſt ſuppoſing them to be ſuch, I will ſhew how to determine the quantity of the Sides and Lines of a Fort accordingly: Both by *Trigonometrical Calculation*, and by *Tables* alſo.

PROB. I.

The Length of the Curtain, and of the Front of the Bulwark given, to find what the other Sides and Lines ſhould be.

AS in the regular *Pentagonal* Fort, *Fig.* III. and ſo in others, to the intent the Line of defence may be about 72 Rod, the Curtain 42, and the Front about 28; as is before noted in the 14th *Axiom*, *Chap.* I. and that the Proportion of the Flank to the Gorge be as 6 to 7: and let the Angle forming the Flank be 40 deg.

Thus then

The Curtain is	O N 420 Foot.
The Front of the Bulwark is	F G 280 Foot.
The Angle forming the Flank	F C N 40 deg.
And let the Flanked Angle be	T G H 69 deg.

Then

Then will the other Angles be found, by the firſt Rule of the foregoing Chapter, to be ſuch as are expreſſed in the Firſt of the two Tables; but for the Sides, we will find them by *Trigonometrical Calculation*, as followeth.

Example I. In the *Right-angled Triangle* S G F; By *Caſe* III. of *Right-angled Triangles*.

As *Radius*, 90 deg.	10.00000
To the Front of the Bulwark F G, 280 Foot	2.44715
So is the Sine of the Inner Flanking Angle S G F, 19 d. 30 m.	9.52350
To the Line S F, 93 d. 47 m.	1.97065

Again, By the ſame *Caſe* I.

As *Radius*, 90 deg.	10.00000
To the Front of the Bulwark F G, 280 foot,	2.44715
So Co-ſine the Inward Flanking Angle, 70 d. 30 m.	9.97435
To the Line S G, 263.94 foot	
Half the Curtain S I, add 210.00	
The Summ is 473.94 for the Line I G; which doubled, is the Side of the Outward *Polygon*, or the Diſtance of the Diamond Points of the Bulwark K G, 947.88.	

In the *Triangle* I A G, by *Caſe* II. of *Right-angled Triangles*.

As the Sine of half the Angle at the Centre I A G, 36 d. 0 m. Co. Ar.	0.23078
To half the Side of the Outward *Pentagon* I G, 473.94.	2.67572
So is the *Radius*, 90 d.	10.
To the Semidiametre of the Outward *Pentagon* A G, 806.31.	*x*2.90650

In the ſame *Triangle*, by the *Caſe* I. of *Right-angled Triangles*.

As the Sine of the Angle at the Centre I A G, 36 d. 00 m. Co. Ar.	0.23078
Is to half the Side of the Outward *Pentagon* I G, 47396.	2.67572
So is the Co-ſine of half the Angle at the Centre I A G, 54 d. 00 m.	9.90796
To the Perpendicular of the Outward *Pentagon* A I, 652.32.	*x*2.81446

In the *Triangle* F C G, by *Caſe* I. of *Obliquæ-angled Triangles*.

As the Sine of the Angle F C G, 86 d. 00 m. Co. Ar.	0.00106
Is to the Front F G, 280.00.	2.44715
So is the Sine of half the Flanked Angle F G C, 59 d. 30 m.	9.93532
To the Line F C, 158.98.	*x*2.38353

In the ſame *Triangle*, and by the ſame *Caſe*.

As the Sine of the Angle F C G, 86 d. 00 m. Co. Ar.	1.00106
Is to the Front F G, 280.00	2.44715
So is the Sine of the Angle G F C, 59 d. 30 m.	9.93532
To the Head Line C G, 241.44	3.38353
Which ſubtracted from A G, 806.31 the Semidiametre of the Inner *Pentagon*,	
There remains 564.87	

In the *Triangle* F C N, by *Caſe* III. of *Right-angled Triangles*.

As *Radius*, 90 deg.	10.00000
To the Line F C, 158.98, before found,	2.20134
So is the Sine of the Angle forming the Flank F C N, 40 d. 00 m.	9.80807
To the Flank F N, 102.19	*x*2.00941
The Flank F N being 102.19	
Add to it the Line firſt found, S F 93.47	
The Summ is the Difference of the *Pentagons* I O, 195.66	
Which ſubtracted from the Perpendicular A I, 652.32	
There remains 456.66	
for A D, the Perpendicular of the Inner *Pentagon*.	

In

In the Triangle F N C, by *Case* III. of *Right-angled Triangles.*

As the *Radius*, 90 deg.		10.00000
To the Line before found, F C, 158.98		2.20134
So is the Co-sine of the Angle forming the Flank F C N, 50 deg,		9.88425
To the Gorge-Line N O, 121.78.		*r*2.08559
To the Gorge-Line, N O	121.78	
Add half the Curtain D N	210.00	
The Summ is the Line D C	331.78	
Which doubled is B C	663.56	
for the Side of the Inward *Pentagon.*		

In the *Triangle* F P N, by *Case* I. of *Right-angled Triangles.*

As the Sine of the Inner Flanking Angle F P N, 16 d. 30 m.		Co. Ar. 0.47650
Is to the Flank F N, 162.19		2.00941
So is Co-sine of the Inner Flanking Angle F P N, 70 d. 30 m.		9.97435
To the Line P N	288.58	*r*2.46026
Which subtracted from the Curtain O N,	420.00	
The Remainder is the Second Flank O P,	131.42	

In the *Triangle* R O G, by *Case* IV. of *Right-angled Triangles.*

To the Line before found, S G,	263.94
Add the Curtain O N	420.00
The Summ is the Line R G	683.94

Then,

First, As the Line R O, or I D, 195.66		Co. Ar. 7.70850
Is to the Line R G, 603.94		2.83502
So is *Radius*, 45 deg.		10.00000
To the Tangent of the Angle R O G, 74 d. 02 m.		10.54352
Secondly, As the Sine of the Angle R O G, 74 d. 02 m.		Co. Ar. 0.01709
Is to the Line R G, 683.94		2.83502
So is the *Radius*, 90 deg.		10.
To the Length of the Line of Defence, O G, 711.40		*r*2.85211

And in this manner the Distances D M, P M, *&c.* might be found.

Example II. In the same *Pentagonal* Fort, *Fig.* III. let the Parts be as before, namely,

The	Curtain O N	420.00 Foot.
	Front of the Bulwark F G	289.00 Foot.
	Angle forming the Flank F G N	40 Degrees.

And let the Flanked Angle of the Bulwark be F G H 72 Degrees.

Then will the other Angles be found by the Second Rule of the Chapter before-going, to be such as are there expressed in the latter of the two Tables: And for the Sides, we will find them as before.

In the *Triangle* S G F.

As the *Radius*, 90 deg.		10.00000
Is to the Front of the Bulwark F G, 280.00 Foot		2.44715
So is the Sine of the Inward Flanking Angle S G F, 18 d. 00 m.		9.48998
To the Line S F, 86.52 Foot		*r*2.33713

In the same *Triangle* S G F.

As the *Radius*, 90 deg.		10.
To the Front of the Bulwark F G, 280.00		2.44715
So is the Co-sine of the Inward Flanking Angle, S G F. 72 d. 00 m.		9.97821
To the Line S G, 266.29		*r*2.42536
To which add half the Curtain S I,	210.00	
The Summ is the Line I G,	476.29	
The Double whereof is	952.58, and is the Distance K G, between the two Angular Points of the Bulwarks.	

In

In the *Triangle* I A G.

As the Sine of half the *Angle* at the Centre A G, 36 d. 00 m. Co. Ar. 0.23078
To half the Side of the outward *Pentagon* I G, 476.29. 2.67787
So is *Radius* 90 d. 10.
To the Semidiametre of the outward *Pentagon* A G, 810.31. *x*2 90865

In the same *Triangle*.

As the Sine of half the *Angle* at the Centre I A G, 36 d. 00 m. Co. Ar. 0.23078
To half the Side of the outward *Pentagon* I G, 476.29. 2.67787
So is the Co-Sine of half the *Angle* at the Centre, I A G 54 d. 00 m. 9.90796
To the greater Perpendicular A I, 655.56 *x*2.81661

In the *Triangle* F C G.

As the Sine of the *Angle* F C G, 86 d. 00. Co. Ar. 0.00106
Is to the Front F G, 280.00. 2.44715
So is the Sine of half the Flanked *Angle* F G C, 36 d. 06 m. 9.76922
To the Line F C, 164.98. *x*2.21743

In the same *Triangle* F C G.

As the Sine of the *Angle* F C G, 86 d. 00 m. Co. Ar. 0.00106
Is to the Front F G, 280.00. 2.44715
So is the Sine of the *Angle* G F C, 58 d. 00 m. 9.92842
To the Head Line C G, 238.03 *x*1.37663
Which subtracted from the Semidiametre A G, 810.31
There remains 572.28, for the Semidiametre of the inner *Pentagon*.

In the *Triangle* F C N.

As *Radius* 90 d. 10.
To the Line before found, F C, 164.98 2.21743
So is the Sine of the *Angle* forming the Flank F C N, 40 d. 00 m. 9.80807
To the Flank F N, 106.05 F N, 106.05 *x*2.02550
To which add the Line first found S F 86.52
The summ is the distance of the *Pentagons* S N or I D 192.57
And this subtracted from the Perpendicular A I 655.56
Leaves the Perpendicular of the inner *Pentagon* A D 462.99

In the *Triangle* F N C.

As the *Radius*, 90 deg. 10.
Is to the Line last found F C, 164.98. 2.21743
So is the Co-sine of the *Angle* forming the Flank F C N, 50. d. 0 m. 9.88425
To the Gorge Line N C 126.38 *x*2.10168
To which add half the Curtain D N 210.00
The summ is the Line D C 336.38
Which doubled, is the side of the inward *Pentagon* B C 672.70

In the *Triangle* F P N.

As the Sine of the inward Flanking *Angle* F P N, 18 d. 00 m. Co. Ar. 0.51001
Is to the Flank F N, 106.05 2.02550
So is the Co-Sine of the inward Flanking *Angle* F P N, 72. d. 00 m. 9.97821
To the Line P N 326.39 *x*2.51372
Which subtracted from the Curtain O N 420.00
There remains the second Flank O P 93.67

In the *Triangle* R O G.

To the Line before found S G 266.29
Add the Curtain O N 420.00
Their summ is the Line R G 686.29

Then,

First, As the Line R O, or I D, 192.57	Co. Ar.	7.71541
Is to the Line R G, 686.29		2.8365.1
So is the *Radius*, 45 d.		10.
To the Tangent of the *Angle* R O G, 74 d. 20 m.		10.55192
Secondly, As the Sine of this *Angle* R O G, 74 d. 20 m.		9.98356
Is to the Line R G, 686.29.		2.83651
So *Radius* 90 d.		10.
To the longeſt Line of defence O G, 712.80		2.85295

Fig. II. *Example* III. In the Quadrangular Fort, *Fig.* II. Let the Parts be as followeth.

The { Length of the Curtain O N 420.00 Foot. / Front of the Bulwark F G 280.00 Foot. / *Angle* forming the Flank F C N 40 d. 00 m. / Flanked *Angle* of the Bulwark F G H 60 d. 00 m.

In the *Triangle* S G F.

As *Radius*, 90 d.		10.
To the Front of the Bulwark F G 280.00		2.44715
So is the Sine of the inward Flanking *Angle* S G F, 15 d. 00 m.		9.41300
To the Line S F, 72.47.		*r* 1.86015

In the ſame *Triangle* S G F.

As *Radius* 90 d.		10.
Is to the Front of the Bulwark F G, 280.00		2.44715
So is the Co-Sine of the inward Flanking *Angle* S G F, 75 d. 00 m.		9.98494
To the Line S G 270.45		*r* 2.43209
To which add half the Curtain S I 210.00		
The ſumm is the Line I G 480.45		
The double whereof is 960.90		

In the *Triangle* I A G.

As the Sine of half the *Angle* at the Centre I A G, 45 d. 00 m.	Co. Ar.	0.15051
To half the ſide of the outward Square I G, 480.00		2.68165
So *Radius* 90 d.		10.
To the Semidiametre of the outward Square A G, 679.46		*r* 2.83216

In the ſame *Triangle*.

As the Sine of half the *Angle* at the Centre I A G, 45 d. 00 m.	Co. Ar.	0.15051
To half the ſide of the outward Square I G, 480.45		2.68165
So is the Co-Sine of half the *Angle* at the Centre I A G 45 d. 00 m.		9.84949
To the greater Perpendicular A I, 480. 45		*r* 2.68165

In the *Triangle* C G F.

As the Sine of the *Angle* F C G, 95 d. 00 m.	Co. Ar.	0.00166
Is to the Front F G 280.00		2.44715
So is the Sine of half the Flanked *Angle* F G C 30 d. 00 m.		9.69897
To the Line F C 140 53		*r* 2.14778

In the ſame *Triangle* C G F.

As the Sine of the *Angle* F C G 95 d. 00 m.	Co. Ar.	0.00166
Is to the Front F G 280.00		2.44715
So is the Sine of the *Angle* G F C 55 d. 00 m.		9.91336
To the Head Line C G, 230.23		*r* 2.36217
Which taken from the greater Semidiametre A G, 679.46		
There remains the leſſer Semidiametre A C, 449.23		

In

In the *Triangle* F C N.

As the *Radius*, 90 d. — 10.
To the Line before form'd F C, 140.53 — 2.14778
So is the Sine of the *Angle* forming the Flank, F C N, 40 d. 00 m. — 9.80807
To the Flank F N 90.33 — *r*1.95585
To which adding the Line first found S F 72.47
You have the distance of the two Squares, K G and B C 162.80
Which subtracted from the Perpendicular A I 480.45
There remains the Perpendicular of the inward Square A D 317.65

In the *Triangle* F N C

As the *Radius* 90 d. — 10.
To the Line before found F C 140.53 — 2.14778
So is the Co-Sine of the Angle forming the Flank F C N 50. d. 00 m. — 9.88425
To the Gorge-Line N C 107.66 — *r*2.03203
To which add half the Curtain D N 210.00
So have you the Line D C 317.66
Which doubled, gives the Side of the inward Square B C 635.32

In the *Triangle* P F N.

As the Sine of the inward Flanking Angle F P N 15 d. 00 m. — Co. Ar. 0.58700
Is to the Flank F N 90.88 — 1.95585
So is the Co-Sine of the inner Flanking *Angle* F P N 75 d. 00 m. — 9.98494
To the Line P N 337.13 — *r*2.52779
Which subtracted from the Curtain O N 420.00
The remainder is the second Flank O P 82.87

In the *Triangle* R O G.

To the Line before found S G 270.45
Add the Curtain O N 420.00
The summ is the Line 690.45

Then

First, As the Line R O, or I D, 162.80 — Co. Ar. 7.78835
Is to the Line R G 690.45 — 2.83913
So is the *Radius* 90 d. — 10.
To the Tangent of the *Angle* R O G 76 d. 44 m. — 10.62748

Secondly, As the Sine of the *Angle* R O G 76 d. 44 m. — Co. Ar. 0.01175
Is to the Line G R 690.45 — 2.83915
So is the *Radius* 90 d. — 10.
To the fixed (or longest) Line of Defence O G 709.42 — *r*2.85090

Example IV. Let there be a *Heptagon*, or a *Figure* of Seven Sides, given to be Fortified with Bulwarks, *&c.* *Fig.* IV.

Let the Length of the Curtain be O N 420.00 Foot.
The Front of the Bulwark F G 280.00 Foot.
The *Angle* of the Bulwark F G H 85 d. 43 m.

Then will the other *Angles* be according to the Second Rule, and Second Table of *Chapter* III. and for finding of the Sides and other Lines, proceed as followeth.

In the *Triangle* S G F.

As the *Radius*, 90 deg. — 10.
Is to the Front of the Bulwark F G, 280.00 — 2.44715
So is the Sine of the Inward Flanking *Angle* S G F, 21 d. 26 m. — 9.56279
To the Line S F, 102.31 — *r*2.00994

In the same *Triangle* S G F.

As the *Radius*, 90 deg.		10.
Is to the Front of the Bulwark F G, 280.00		2.44715
So is the Co sine of the Inward Flanking *Angle*, S G F, 63 d. 34 m.		9.96888
To the Line	S G, 260.63	✗2.41603
To which add half the Curtain	S I, 210.00	
The Summ is the Line	I G, 470.63	
Whose Double is the Side of the Outward *Heptagon*	941.26	

In the *Triangle* I A G.

As the Sine of half the *Angle* at the Centre, I A G, 25 d. 43 m.	Co. Ar.	0.36259
Is to half the Side of the Outward *Heptagon* I G, 470.63		2.67268
So is the *Radius*, 90 deg.		10.
To the Semidiametre of the Outward *Heptagon* A G, 1084.61		✗3.03527

In the same *Triangle* I A G.

As the Sine of half the *Angle* at the Centre I A G, 25 d. 43 m.	Co. Ar.	0.36259
To half the Side of the Outward *Heptagon* I G, 470.63		2.67268
So is the Co-sine of half the *Angle* at the Centre, I A G, 64 d. 17 m.		9.95470
To the greater Perpendicular A I, 977.17		✗2.98997

In the *Triangle* F C G.

As the Sine of the *Angle* F C G, 75 d. 43 m.	Co. Ar.	0.01364
Is to the Front F G, 280.00		2.44715
So is half the Flanked *Angle* F G C, 42 d. 51½ m.		9.83261
To the Line F C, 196.52		✗2.29340

In the same *Triangle* F C G.

As the Sine of the *Angle* F C G, 75 d. 43 m.	Co. Ar.	0.01364
Is to the Front F G, 280.00		2.44715
So is the Sine of the *Angle* G F C, 61 d. 26 m.		9.94362
To the Head-Line	G O, 253.75	✗2.40441
Which taken from the Greater Semidiametre	A G, 1084.61	
There remains the Semidiametre of the Inward *Heptagon*	A C, 830.86	

In the *Triangle* F C N.

As the *Radius*, 90 deg.		10.
To the Line before found, F C, 196.52		2.29340
So is the Sine of the *Angle* forming the Flank, F C N, 40 d. 00 m.		9.80807
To the Flank	F N, 126.32	✗2.10147
To which add the Line first found,	S F, 102.31	
The Summ is the Distance of the *Heptagons*	I D, 228.63	
Which subtracted from the Perpendicular	A I, 977.18	
There remains the Perpendicular of the Inward *Heptagon*,	748.55	

In the *Triangle* F N C.

As the *Radius*, 90 deg.		10.
To the Line before found, F C, 196.52		2.29340
So is the Co-sine of the *Angle* forming the Flank, F C N, 50 d. 00 m.		9.88425
To the Gorge-Line	N C, 150.54	✗2.17765
To which add half the Curtain	D N, 210.00	
The Summ is the Line	D C, 360.54	
Whose Double is the Side of the Inward *Heptagon*	B C, 721.08	

In

In the *Triangle* F P N.

As the Sine of the Inward Flanking *Angle* F P N, 21 d. 26 m.	Co. Ar.	0.43721
Is to the Flank F N, 126.32		2.10147
So is the Co-sine of the Inward Flanking *Angle*, F P N, 68 d. 34 m.		9.96888
To the Line P N, 321.78		*r*2.50756
Which subtracted from the Curtain O N, 420.00		
The Remainder is the Second Flank O P, 98.22		

In the *Triangle* R O G.

To the Line before found,	S G, 260.63
Add the Curtain	O N, 420.00
The Summ is the Line	R G, 680.63

Then,

First, As the Line R O, or I D, 228.63	Co. Ar.	7.64087
Is to that Line R G, 680.63		2.83291
So is the *Radius*, 90 deg.		10.
To the *Tangent* of the *Angle* R O G, 72 d. 26 m.		10.47378
Secondly, As the Sine of that *Angle* R O G, 72 d. 26 m.	Co. Ar.	0.02321
Is to the Line R G, 680.63		2.83291
So is the *Radius*, 90 deg.		10.
To the Fixed (or Longest) Line of Defence, O G, 718.00		*r*2.85612

And according to the Form of these Examples, the Quantities of the Sides and Lines of Forts (of any other Number of Sides, under or above XII.) may be determined.

Example V. In a *Quindecagon*, or Fort of Fifteen Equal Sides.

In which let be,

Curtain	O N, 420.00 Foot.
Front of the Bulwark	F G, 280.00 Foot.
Angle forming the Flank	F C N, 40 d. 00 m.
Flanked *Angle* of the Bulwark	F G H, 90 d. 00 m.

Then will the other *Angles* be as followeth.

		Deg.	*Min.*
The *Angle* at the Centre of the *Polygon*	B A C	25	00
Half the *Angle* at the Centre is	D A C	12	00
Whose Complement is half the *Angle* of the *Polygon*	B C A	78	00
Which doubled, is the *Angle* of the *Polygon*	B C E	156	00
And seeing the Flanked *Angle* of the Bulwark is	F G H	90	00
Half the Flanked *Angle* is	F G C	45	00
Taken from half the *Angle* of the *Polygon*	B C A	78	00
Leaves the Inward Flanking *Angle*	S G F	33	00
To which add a *Right Angle*		90	00
The Summ is the *Angle* of the Shoulder	N F G	123	00
From which take the Centre of the *Angle* forming the Flank	N F C	50	00
There rests the *Angle* opposite to the Head-Line	G F C	73	00
To which adding half the Flanked *Angle*	F G C	45	00
The Summ is		118	00
Which subtracted from two *Right Angles*		180	00
There remains the *Angle* opposite to the Front	F G C	62	00
Also the Centre of the Inward Flanking *Angle* is	S F G	57	00
Which doubled is the Outward Flanking *Angle*	K M G	114	00

Having thus set down the *Angles*, the Sides and other Lines may be found as in the foregoing Four Examples, which in this Example I purposely omit; but will (in two Tables) exhibit the Lines in a Fort of any number of Sides, from the *Tetragon*, (or Square,) to the *Dodecagon*, (or Figure of XII Sides,) according to the *Angles* found by either of the Two Rules delivered in *Chap.* III. before-going.

A TABLE of the Dimensions of any Regular Fortification, from the Tetragon, *(or Figure of* IV. *Sides,) to the* Dodecagon, *(or Figure of* XII. *Sides,) the Flanking Angle being* 15 *Degrees more than Half the Angle of the* Polygon.

The Number of the Sides of the *POLYGONS.*		IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
		deg. m.	deg. m.	deg. m.	deg. m	deg. m.	deg. m.	deg. m.	deg. m.	deg. m.
The Angle of the *Polygon*	B C E	90 0	108 0	120 0	128 34	135 0	140 0	144 0	147 16	150 0
Flanking Ang. of the Bulwark	F G H	60 0	69 0	75 0	79 17	82 30	85 0	87 0	88 38	90 0
The Angle of the Shoulder	N F G	105 0	109 30	112 30	114 38	116 15	117 30	118 30	119 19	120 0
The Inward Flanking Angle	S G F	15 0	19 30	22 30	24 38	26 15	27 30	28 30	29 19	30 0
The Outward Flanking Angle	K M G	150 0	141 0	135 0	130 43	127 30	125 0	123 0	121 21	120 0
The Angle fronting the Flank	F C N	40 0	40 0	40 0	40 0	40 0	40 0	40 0	40 0	40 0
		R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.
The Curtain	O N	42 00	42 00	42 00	42 00	42 00	42 00	42 00	42 00	42 00
The Front of the Bulwark	F G	28 00	21 00	28 00	28 00	28 00	28 00	28 00	28 00	28 00
The Gorge-Line	N C	10 77	12 18	13 26	14 12	14 83	15 42	15 92	16 36	16 73
The Semidia. of the Inner *Polyg.*	A C	44 92	56 45	68 52	80 94	93 63	106 49	119 49	132 57	145 80
The Side of the Inner *Polygon*	B C	63 53	66 36	68 52	70 24	71 66	72 84	73 85	74 71	75 47
The Perpend. of the Inner *Polyg.*	A D	31 76	45 67	59 34	72 90	86 50	100 06	113 64	127 20	140 83
The Semid. of the Outer *Polygon*	A G	67 95	80 63	93 74	107 05	120 50	124 02	147 59	161 16	174 83
The Side of the Outer *Polygon*	K G	96 09	94 79	93 74	92 90	92 22	91 67	91 21	90 83	90 50
The Perpend. of the Outer *Polygon*	A I	48 04	65 23	81 18	96 42	111 32	125 93	140 37	154 63	168 87
The Distance of the *Polygons*	D I	16 28	19 57	21 84	23 52	24 83	25 87	26 72	27 43	28 04
The Flank	F N	9 03	10 22	11 13	11 85	12 44	12 94	13 36	13 72	14 04
The Head-Line	C G	23 02	24 18	22 22	26 11	26 87	27 53	28 10	28 59	29 03
The Shoulder from the Centre	F C	14 05	15 90	17 31	18 43	19 36	20 13	20 79	21 35	21 85
The Second Flank	O P	8 29	13 12	15 13	16 14	16 77	17 14	17 39	17 56	17 68
The longeſt Line of Defence	O G	70 94	71 14	71 30	71 43	71 55	71 67	71 77	71 86	71 94

A TABLE of the Dimensions of any Regular Fortification, from the Tetragon, *(or Figure of* IV. *Sides,) to the* Octogon, *(or Figure of* VIII. *Sides,) the Flanked Angle being Two third parts of the Angle of the* Polygon:

The Number of the Sides of the *POLYGONS.*		IV.	V.	VI.	VII.	VIII.
		D. 10 par.	D. 10 par.	D. 10 par.	D. 10 par.	D. 10 par.
The Angle of the *Polygon*	B C E	90 0	108 0	120 0	128 34	135 0
The Flanked Angle of the Bulwark	F G H	60 0	72 0	80 0	85 41	90 0
The Angle of the Shoulder	N F G	105 0	108 0	100 0	111 25	112 30
The inward Flanking Angle	S G F	15 0	18 0	20 0	21 25	22 30
The outward Flanking Angle	K M G	150 0	144 0	140 0	137 08	135 0
The Angle forming the Flank	F C N	40 0	40 0	40 0	40 00	40 0
		R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.	R. 100 par.
The Curtain	O N	42 00	42 00	42 00	42 00	42 00
The Front of the Bulwark	F G	28 00	28 00	28 00	28 00	28 00
The Gorge Line	N C	10 77	12 64	14 00	15 05	15 90
The Semidiametre of the Inner *Polygon*	A C	44 92	57 23	70 00	83 06	96 43
The Side of the Inner *Polygon*	B C	63 53	67 28	70 00	72 11	73 80
The Perpendicular of the Inner *Polygon*	A D	31 76	46 30	60 57	74 86	89 09
The Semidiametre of the Outer *Polygon*	A G	67 95	81 03	94 62	108 46	112 47
The Side of the Outer *Polygon*	K G	96 09	95 26	94 62	94 13	93 74
The Perpendicular of the Outer *Polygon*	A I	48 04	65 56	81 94	97 72	113 15
The Distance of the *Polygons*	D I	16 28	19 26	21 32	22 86	24 06
The Flank	F N	9 03	10 60	11 75	12 63	13 34
The Head-line	C G	23 02	23 80	24 62	25 37	26 04
The Shoulder from the Centre	F C	14 05	16 50	18 28	19 15	20 76
The Second Flank	O P	8 29	9 36	9 72	9 82	9 79
The Longeſt Line of Defence	O G	70 94	71 28	71 56	71 80	72 00

Thus have you the Manner how to find the *Sides*, *Lines*, and *Angles* of any *Regular Fortification*, by *Trigonometrical* Calculation, by which the Excellency of that Doctrine doth appear. And now concerning *Irregular Fortification*, there might be propoſed an infinite number of different Examples; but of *Irregular Fortification* in general, the *Axioms* ſet down in the Firſt Chapter, and the Examples before given in *Regular* Forts, may be ſufficient: Only obſerve, That the Figure propoſed to be fortified being *Irregular*, it ought (the Plot thereof being taken by an *Inſtrument*, as is directed in *Surveying*) to be reduced to as much *Regularity* as the Place will permit, taking in and leaving out here and there a little, to make ſome near Equality of *Sides* and *Angles*. And if any *Angle* of your Figure be leſs than 90 deg. you are not to ſet a Bulwark at that *Angle*, but rather to make that *Angle* to be the *Flanked Angle* of a Bulwark: And for the other *Angles*, you are to fit Bulwarks ſo, as the ſtanding *Angle* of the Bulwark may be anſwerable to the *Angle* of the *Polygon* whereon it ſtands, according to either of the Two Rules before-given in *Chap.* III. But my Intent here, is not to write a Treatiſe of *Fortification*, but to ſhew the Uſe of *Plain Triangles* in that Particular: Yet before I end, I will ſhew you how to delineate the *Ground-work* of a *Fort* upon Paper, and then ſay ſomething concerning the *Works* that are to be raiſed on ſuch a *Ground-work*.

CHAP. V.

How to delineate a Fort *upon* Paper *or* Vellom.

ADmit it were required to draw the Platform of a Fort of Six Sides or Bulwarks, according to ſome aſſigned Proportion: You muſt firſt find, as hath been already taught, the Angles, Sides, and other Lines in ſuch a Fort, requiſite to be known; which admit to be ſuch as this following Table doth exhibit in Rods, Feet, and Tenth parts of a Foot.

	Rods.	Feet.	10 Parts.
The Semidiametre of the Outward *Hexagon*	93	7	4
The Side of the Outward *Hexagon*	93	7	4
The Head-Line, (the ſame,)	25	2	2
The Semidiametre of the Inward *Hexagon*	68	5	2
The Side of the Inward *Hexagon*, (the ſame,)	68	5	2
The Diſtance of the *Hexagons*	21	8	4
The Gorge-Line	13	2	6
The Flank	11	1	3
The Second Flank	15	1	3
The Fixing, or Longeſt Line of Defence	71	3	0
The Curtain	42	0	0
The Diſtance from the Centre of the Bulwark to the Shoulder	17	3	1
The Front of the Bulwark	28	0	0
The Angle at the Bulwark let be	75 Degrees.		

Being thus prepared, you may delineate the Fort as followeth.

1. From a Diagonal Scale take in your Compaſſes the Semidiametre of the Outward *Hexagon*, 93 Rod, 7 Foot, and 4 Tenths of a Foot, (or 93 Rod, 74 *Centeſms* of a Rod:) Then aſſuming the Point A upon your Paper for the Centre of the Fort, upon A, with the Diſtance 93.74, deſcribe a Circle; and (becauſe the Side of a *Hexagon* is equal to the Semidiametre,) ſet the ſame Meaſure, 93.74, from G to K; ſo is G the Diamond-Point of one Bulwark, and K of the other: And draw the Lines A G and A K. *Fig.* V.

2. Take the Semidiametre of the Inward *Hexagon*, 68.52, and ſet the ſame from A to B and C; ſo C and B are the Centres of the two Bulwarks; and drawing the Line B C, ſet down the Gorge-Line, 13.26, from B to O, and from C to N; the Reſidue of which Line, *viz.* O N, is the Curtain, to which, upon the Points O and N, raiſe the Perpendiculars N F and O L, for the Flanks, which muſt be ſet off according to their Lengths, *viz.* 11.13 Rods. Or otherwiſe, Set off in the Curtain from

from O to P, and from N to Q, 15.13 Rods for the ſecond Flanks; and drawing the ſhorteſt Lines of Defence, F G and Q K, they interſect the Perpendiculars raiſed for the firſt Flanks, in the Points L and F; ſo is N F the Flank, and F G the Front. And in this manner you muſt proceed with the other Sides of the Fort.

Otherwiſe,

Having drawn the Line K G, ſet down in the ſame the Side of the Outward *Hexagon* 95.74, from K to G, as before, which is the Diſtance of the Angular Points or Heads of the Bulwarks: Then to the Right Line K G, and to the Points in the ſame K and G, deſcribe the Angles B K G, and C G K, here in this Example, each 60 Degrees; and on the Lines K B and G C, ſet off from K and G 25.22 for the Head-Lines, ending at B and C, which Points B and C are the Centres of the two Bulwarks; then drawing the Line B C, proceed as before.

Otherwiſe,

Let K and G be the Angular Points of the Bulwarks: Draw the Line K G, and on the Points K and G deſcribe the Angles A K G and A G K, each in this Example 60 Degrees; and ſet off from K to B, and from G to C, the Head-Lines 25.22, drawing the Line B C, as before. Then to the Line B C, and to the Point in the ſame C, deſcribe the Angle F C N, of 40 Degrees; alſo to the Line G A, and to the Point in the ſame G, deſcribe half the Flanked Angle F G C, which is in this Example 37 deg. 30 min, and at the Concourſe of theſe Lines C F, and G F, *viz.* at F, is the Shoulder of half the Bulwark; from which letting a Perpendicular fall to the Curtain F N, that Line F N is the Flank, N C the Gorge-Line, N O the Curtain, F G the Front, *&c.* And thus are the moſt eſſential Parts of this Fort deſcribed three ſeveral ways.

I ſhould now proceed to the ſetting out of this or any other Fort upon the Ground; but for that I muſt refer you to the foregoing Tractate of *Surveying of Land*.

CHAP. VI.

Of the Works without the Fort, the Rampire, Parapets, *&c.*

IT is to be underſtood, that that which you have drawn by the Directions of the laſt *Chapter* in *Fig.* V. namely, the Lines K L, L O, O N, N F, F G, G H, and H T, is the outer Edge of the Rampire; which Rampire may be 7 Rods, or ſomewhat more or leſs, as occaſion requires; but in a Fort of 12 Sides or more, it may be 10 Rod thick; and in a Fort of Four Bulwarks, it being of leſs importance, it may be but 5 Rod; and in ſmall Sconces much leſs: This thickneſs is repreſented in *Fig.* I. by D V; ſo that the Line drawn by V, doth repreſent the inward Side of the Rampire, being in the Curtain, Flank, and Front, every where parallel to the outſide of the Rampire before deſcribed; ſo that ſometimes the Bulwarks are quite filled up, and in ſome caſes it is neceſſary they ſhould be ſo, becauſe Aſſaults are commonly made againſt them: but here we ſuppoſe the Middle parts to be empty.

Next, if you make a Walk or *Fauſſe-Bray*, then from the outer Edge of the Rampire before-deſcribed, meaſure 2 Rods for the Breadth thereof; and 2 Rods more for the Thickneſs of the *Parapet* of the *Fauſſe-Bray*; which Spaces are here repreſented by D P and P Q.

Next without this *Parapet*, from the Foot of it, to the Side of the Ditch may be left half a Rod to ſtrengthen the Foot of the *Parapet*.

All theſe Parts (or Members) are between the Rampire and the Ditch.

The Port or Ports are beſt to be made in the Middle of the Curtains, and to be placed as low as may be, and a Wooden Bridge, with Gates and Draw-Bridges over the Ditch in ſeveral places.

Then ſet out the Breadth of the Ditch, which may be 12 Rods, more or leſs, as occaſion requires: Then from the Front of the Bulwark F G, and from the angular Point thereof G, raiſe the Perpendicular G S; and being the *Fauſſe-Bray*, with the *Parapet*, is 4 Rod, and the Ditch 12 Rod Broad, therefore take 16 Rod from your Scale,

Scale, and set them from G to S, and by the Point S, draw the Line R S I, parallel to the Front of the Bulwark G F.

Next without the Ditch must be the *Coridor*, or *Covertway* of the *Counterscarp*, whose Breadth from the outside of the Ditch, may be two Rods, or thereabouts, represented by the Space S M: And without that Covert-way must be an *Argin* (or *Parapet*) 5 or 6 Rod Broad represented by M W.

CHAP. VII.

Of the Height of the Rampire *and* Parapet, *and of the Depth of the* Ditch.

THE *Ichnography* or *Ground-Plot* of the Fort being drawn upon Paper or Velom, and from thence the *Rampire*, *Fausse-Bray*, *Parapet*, *Ditch*, *Covert-way*, and *Counterscarp*, as in *Figure* I. I come now to speak of the Height of them. And,

1. The *Rampire* and *Parapets* are raised of Earth, taken out of the Ditch; and touching the Form of the Works, in Height, Depth, and Scarpings, that it may be the better conceived, draw a Line, as *a b c d e f g*, cross the Front of the Bulwark, Ditch, Counterscarp, *&c.* at Right Angles; upon which Line we may represent the Breadth, Height, Depth, and Scarpings of all the Works; which, that it may be the more sensible, I here draw a longer Line apart, as in *Figure* VI. A *b c d e f* G, and on this Line set down the Breadth of the Rampire from A to *b*, 70 Foot; the Breadth of the Fausse bray, *b c*, 20 Foot; the Breadth of the Parapet thereof, *c d*, 20 Foot; leaving 5 or 6 Foot without it, for the Brow of the Ditch, and from thence set off the Breadth of the Ditch to *e*, 120 Foot; and without that the Breadth of the *Argin* or *Parapet* thereof, F G, 60 Foot; And thus you have expressed in this Line A G the Breadth of all the Works to be made.

2. Between the Points A and *b* the Rampire is to be raised, which in Forts of Four Sides may be but 12 Foot, but in a Fort of 12 Sides, or more, 18 or 20 Foot. In this Example it is made 15 Foot high.

The Rampire is to be raised on every side Scarping; *viz.* on the Outside, for every 2 Foot that it riseth, it may scarp 1; but in this Example, for every 3 that it riseth, it scarps 2 Foot; so that the top of it being 15 Foot, it scarps 10 Foot: But the Inward Side of the Rampire next the Fort scarpes more, *viz.* for every Foot that it riseth, it scarps 1 Foot; and being raised to its full height of 15 Foot, it scarps also 15 Foot: And so, though the Bottom of the Rampire be 70 Foot, yet the upper Superficies of it is but 45 Foot broad.

3. Upon the Outside of the upper Superficies of the Rampire is raised a *Parapet*, sometimes 15, sometimes 24 Foot broad; this is made to 10 Foot broad below, and on the Inward Side 6 Foot high, with a Step or Foot-pace 3 Foot broad, but outwardly not above 4 Foot high. In like sort is raised the *Parapet* of the *Fausse-bray*, and also that of the Covert-way without the Ditch, only the Outside of the Covert-way is scarping about 60 Foot, till it be even with the Champion-ground about it.

4. Touching the Ditch, it is in this Example 120 Foot broad, and 10 Foot deep, either Side of it also scarping 10 Foot; all which doth sufficiently appear in *Fig.* VI. which drawn as here you see, is called the *Section*, or *Profile*.

The End of the Third Part, and of the Sixth Book.

CURSUS MATHEMATICUS.

BOOK VII.

OF NAVIGATION.

IN FOUR PARTS.

Containing

I. An Introduction to that *Art*; with the Deſcription and Uſe of ſuch INSTUMENTS, both for *Obſervation* and *Operation*, as are of daily Uſe at Sea.

II. and III. The Doctrine of *Right-Lined Triangles*, applied to Practice, both in PLAIN, and in MERCATOR's Sailing.

IV. The Doctrine of *Spherical Triangles*, applied to Practice in *Circular Sailing*; or, *Sailing* according to the ARCH OF A GREAT CIRCLE: And in the *Solution* of ſuch GEOGRAPHICAL and ASTRONOMICAL *Problems* as are of frequent Uſe in NAVIGATION.

All which

THREE WAYS of SAILING

Are *Arithmetically*, *Geometrically*, and *Inſtrumentally* Performed.

ALSO

A Large CONNECTION of ſuch RULES and TABLES, As are of continual Uſe in that ART.

By *WILLIAM LEYBOURN*, Philomat.

LONDON,
Printed ANNO DOM. MDCXC.

OF

NAVIGATION.

PART I.

THE

Proœme.

Of the Antiquity and Progress of it.

THE *first* Invention *of this most excellent* Art *can be ascribed to no other than* GOD *himself, who first taught the* Hebrews, *his chosen People, (and not the* Ægyptians *and* Phænicians, *as some have falsly imagined.) For we read in* Genesis, *that* Noah, *according to* God's Precept, *made an* Ark *for the preservation of himself, and other living Creatures from the* Deluge: *for before this we do not read of any Skill in* Navigation: *And of this we have many Reasons and Conjectures given by Ancient Writers.* 1. *Because in those times there was greater need of* Cities *than* Ships; *because* Cities *are not made for* Ships, *but* Ships *for the Trade or Defence of* Cities. 2. *Because, in those days, little Commodity could be reaped from other Countries, they lying then rude and uninhabited.* 3. *Some would have this to be a reason, why God revealed not this to the* Old Worldlings; *because, being ready to perish in the* Flood, *no man might have means to escape, which doubtless they would have attempted, had* Navigation (*or the use of* Boats *or* Ships) *been known. From what hath been said it is probable that the knowledge of* Ships (*and consequently of* Navigation) *was discovered to* Noah *at the time of the* Deluge; *whose* Ark *afterwards resting upon the Mountains of* Ararat, *gave a Precedent to other Nations, near bordering, in what nature* Ships *were to be formed. From whence it came to pass, that the first to whom this Skill was derived, next to the* Hebrews, *were the* Tyrians *and* Phænicians; *Nations, as well for the commodity of the* Place, *as other Inducements, inclining to* Navigation: *For* Tyre *being a principal* Mart Town *of* Phænicia, *bordering upon the Sea, and this knowledge being derived from them to other Nations, might give occasion to* Stratus *and* Strabo *to conjecture that they were the first Inventers of it. From the* Phænicians *it was derived to the* Ægyptians, *though but in a rude and unpolished manner, as may appear by* Pliny, *who testifies, that they began to Sail in a certain* Vessel *called* Ratis, (*which now signifies a* Ship, *but was then) made only of* Beams *joyned together, in which they are reported to have passed the* Mediterranean *Sea. From the* Ægyptians *it came unto the* Grecians, *and one* Danaus *of that Country made a Ship of a more ample form than formerly, for which he was celebrated by the* Grecians *as the first Inventer.* Strabo *gives the Invention of it to* Minos, *others (as* Diodorus Siculus) *to* Neptune, *for which he was afterwards translated into the number of the Gods. But this is certain, that amongst the* Grecians, *the* Cretensians *were the first that excelled in this faculty. Others ascribe the*

building of Ships *to* Dedalus, *a rare Woork-man in* Mechanical *Occupations. From the* Grecians *it was afterwards communicated to the* Italians, *amongst whom, the* Genevenſians *and* Venetians *moſt excelled. For the* Venetians *Skill in this matter, we read of no other argument then their great* Riches, *and Magnificent Power, eſpecially by Sea, which continues ſo to this day. After theſe aroſe the* Portugals, *who, under the conduct and direction of* Columbus, *an* Italian, *firſt diſcovered* America; *which gave example to other* Nations *to adventure farther: Amongſt which no Nation hath waded farther than the* Engliſh; *who, under* Drake *and* Candiſh, *have ſurrounded the World, and left an eternal Trophie of their immortal fame to all poſterity. And to conclude, I will not let the* Hollanders, Flemming, *and* Zelanders, *paſs without their juſt due in this particular; who, by their extraordinary Power at Sea, have kept (in deſpight of the Uſurping* Spaniards) *theſe* Provinces, *and increaſed the* Riches *of them to far greater abundance than they were when they began their* Wars. *And thus much may ſuffice for the Authors and firſt Inventers of NAVIGATION.*

CHAP. I.

Of the Art of Navigation *in general.*

THE Care and Skill of a Perfect *Sea-man*, is to guide the Ship at Sea to any place that ſhall be deſired; which cannot be done, unleſs he be able to find out in what place the Ship is at any time: So that the Practick Part of *Navigation* is principally grounded upon the knowledge of theſe Four things. 1. The *Longitude.* 2. The *Latitude.* 3. The *Courſe.* 4. The *Diſtance.*

SECT. I. *Of the* Longitude.

TOuching the *Longitude*; although it may be found by the other Three, yet there hath not hitherto been diſcovered any general Rule, true and practicable at Sea, whereby the *Longitude* of Places may immediately and ordinarily be found of themſelves, which if they could, it would be the *Maſter-piece of Nautical Science*; and the diſcovery whereof hath ſet on work the Wits and Inventions of many Men.—Some have laboured to find the reaſon thereof by the Variation of the *Magnetical Needle*; ſuppoſing certain Poles or Points unto which the ends of the *Needle* doth in all Places directly reſpect. But beſides that, the *Meridian* is difficult to be obtained with ſufficient preciſeneſs; eſpecially at Sea, where the chief Uſe of *Longitude* is: and the conceit is only imaginary, without the warrant of any Natural Principle.—Some conſidering the ſwiftneſs of the Motion of the *Moon*, (which is every day about 13 Degrees,) have ſuppoſed, that either by the true Place of the *Moon*, to be obſerved by exact *Inſtruments*, or elſe by the moment of the *Moon's* coming into the *Meridian*, the *Longitude* might be obtained: But neither the true Motion of the *Moon* is ſo exactly known, nor Obſervation at Sea can be ſo preciſely made, that any certain truth, in ſo ſubtile a buſineſs, may be argued thereby.—Some have thought to obſerve the *Longitude* by Artificial Motions of long continuance; but not without great error, and hallucination.—Some by Sand-Glaſſes or Water-Glaſſes, but both obnoxious to divers alterations and temperatures of the Air and Clime wherein they are; eſpecially thoſe of Sand, that of Water being more probable, eſpecially if the Liquor be ſome *Chimical Spirit*, which will neither freeſe nor decay. Theſe are the principal ways of keeping an Account of the *Time* as yet we know; and any, or ſome of them, which ſhall be found (by experience) moſt certain muſt be embraced; till it ſhall pleaſe God to open a more natural and proper way for the diſcovery of *Longitude*

SECT.

SECT. II. *To find the* Latitude *of the Place where the Ship is.*

THE *Latitude* of the Place wherein the Ship is, may be certainly found by the *Meridian Altitude* of the *Sun*, or of a known *Star*; or by the *Amplitude* of the *Sun*, or a known *Star*, at their Rising or Setting; as also by the *Pole Star*, &c. And divers such ways you shall find in several places of this *Book*, especially among the *Astronomical Problems* hereafter following.

SECT. III. *Of a Ship's Course.*

THE Course or Line by which the Ship must go. Now, the Course of a Ship upon the Sea, depends upon the Winds, and the designation of these upon the certain knowledge of one principal Wind; which (considering the condition and situation of the whole Sphere) ought in nature to be *North* or *South*: the *North* to us on the *North* side of the *Equinoctial*, the *South* to those on the other side of the Line: And in making this Observation, Men were to intend themselves towards one or other certain fixed part of the *Horizon*; and therefore, to one of these.

In the *Southern Hemisphere* there is not found any *Star* so notable, and of so near distance to the Pole, as to make any precise direction of that Wind; but in the *Northern Hemisphere* we have One of the Second Magnitude in the Tail of the lesser Bear, making so small, and insensible a Circle about the Pole, that it cometh all to one, as if it were the Pole it self.

This *Star* pointed out the *North*-Wind to the *Mariners* of old; and was therefore by them called the *Lead*, or *Load Star*; but this could be only in the Night season, and not always then neither: But it is now more constantly and surely shewed by the *Needle* touched with the *Magnet*, which is therefore now called the *Lead*, or *Load-Stone*, for the same reason of the Leading or Directing their Course to the *North* and *South* Parts of the Earth; not in all Parts directly, because in following the constitution of the *Great Magnet* of the whole Earth, it must needs be here and there led aside towards *East* or *West*, by the unequal temper of the *Terrestrial Globe*, consisting more of Earth than Water in some places, and more of Water than of Earth in others.

This Deviation of the *Needle* is called by the *Mariners*, the *North Easting* or *North-Westing* of the *Needle*, as it falls to be; but more Artificially, it is called the *Variation of the Compass*; and although it pretend uncertainty, yet it proveth to be one of the greatest helps that Sea-men have.

The *North* and *South* Winds being thus assured by the Motion of the *Needle*; the *Mariner* supposeth his Ship to be (as it always is) upon some *Horizon* or other; the Centre whereof is the Place of the Ship, represented by the Centre of the *Compass*, the Diametre whereof we will suppose to be the Line of *North* and *South*, which being crossed by another Line at Right Angles, sheweth the *East* and *West*; and so they have the *Four Cardinal Winds*: Again, cross these, and they have the *Eight Whole Winds*, as they call them: Another division of these make *Eight* more, which they call *Half Winds*: A third division make 16, which they call *Quarter* Winds: So that in all there are 32, which they call *Points* of the *Compass*. Thus, the *Four Cardinal Winds*, *North*, *South*, *East*, *West*, are called *Cardinal Points*; and the other intermediate *Points* are called *Rumbs*.

SECT. IV. *To find the Distance of the Ship's Way to the Place assigned.*

FOR the Distance: The finding of the Distance of a Ship's Way to any Place from which she hath departed, is the principal thing in the Practick Part of *Navigation*, which (if the *Longitude* could be easily and accurately found) would be very easie; but as hath been before delivered, that being not only intricate (or rather impossible) at all times to be put in Practice, especially at Sea, where there is most need thereof, we must content our selves with such ways as we may best manage: **And** this is most commonly done by a little Log fastned to a Line, and let down by the Ships side into the Water, which *Mariners* call the *Log-Line*.

SECT. V. *Of the Diviſion of the* Log-line, *and how thereby to reckon the Way of the Ship.*

BY the quantity of Line that the *Log* ſhall draw out of the Ship in ſome certain quantity of Time, is uſually reckoned the Diſtance that the Ship hath run in that Time, or may run for ſome ſmall portion of Time after: And although this be not an Infallible Rule, yet doth it ſtand the Mariner many times in good ſtead.

The Way of finding the Diſtance run, is grounded upon this Opinion, That 5 of our *Engliſh* Statute Feet do make a Pace, and that 1000 of ſuch Paces do make a Mile, and 60 ſuch Miles do make a Degree upon the Earth, (anſwering to a Degree in the Heavens;) and according to this, a Degree ſhould contain 300000 Feet: But this Error hath been long ſince detected by my loving Friend and Country-man Mr. *Richard Norwood*, who by the Experiments of others, as well as of his own, found it to contain a greater number of Feet. The Meaſure of a Degree aſcertained to us by his Experiment is 367200 of our *Engliſh* Feet; but in reſpect to the Rotundity of the Number, and becauſe it is ſafer to reckon with the leaſt, rather than too much, he doth ſuppoſe a Degree to contain only 360000 *Engliſh* Feet: And retaining ſtill the ſame Diviſion of a Degree into 60 Miles, or 20 Leagues, a Mile will contain 6000 Feet.

Now ſuppoſing the time of the running out of the *Log-line* to be meaſured by a Half-minute Glaſs; if you obſerve how many Foot ſhe runs in half a Minute, you may thereby find out her Way for an Hour, 4 Hours, or any other Time. As,

Admit that 50 Foot of the *Log-line* ſhould run out in half a Minute, or 30 Seconds of Time; it is then apparent, that the Ship runs a Mile an Hour, 100 Feet being the 60th part of a Mile: And therefore,

As 1 Minute, is to 60 Minutes;
So is 100 Feet, to 6000 Feet.
Or,
As 1 Minute, is to 60;
So is 200 Feet, to 12000 Feet or Miles.

Upon this Ground, If at about half a ſcore Fathom, or more, from the *Log*, you make a Mark or Knot, and beginning from thence, meaſure out 50 Foot, and there make a Mark, and 50 Foot farther 2 Knots, and 50 Foot farther 3 Knots, and ſo proceeding, look how many Knots are run out in half a Minute, ſo many Miles doth the Ship run in an Hours time; and every 5 Foot more beſides the Knots, is the tenth part of a Mile: As, if there were run out, in half a Minute, 6 Knots and 45 Foot, the Ship's Way is, after that rate, 6 Miles and $\frac{4}{10}$ of a Mile in an Hour, *&c.*

SECT. VI. *Of Currents, how to diſcover where they are, which way they ſet, and their Rate of Drift or Driving.*

WHen you have a ſmooth Sea, and not much Wind, heave out the Boat, and in her three or four Aſſiſtants, with a Log-line, Compaſs, a Half-minute Glaſs, and a Warp or Line of about 100 Fathom long: To the end of your Warp faſten a Triangular piece of Board, * and at one of the Angles faſten a Weight of Lead to ſink it. When you are off from the Ship, caſt over your Board, (or other Veſſel,) and let it ſink at leaſt 60 or 100 Fathom, or more, if you have Line enough; then belaying the Line faſt about the Stern, it will bring her up, and make her ride as if at an Anchor.

* *A Kettle, or other hollow Veſſel, tied to the Warp, may ſerve.*

Then caſt over your Log, turn up the Glaſs, and as you veer out the Log-line, ſet the Drift of the Log with your Compaſs; ſo ſhall you know whether there be any Current or not; and if any, how it ſets; and the Rate of Drift or Driving.

But you muſt remember, always, to add to the Drift, (if the Line ſhe ride by be 60 Fathom, one third part; if it be 80 Fathom, one fourth part; if 100 Fathom, one fifth part; and ſo more or leſs, according to the length of your Warp or Line,) more for the Drift of the Boat: For though ſhe ſeem to ride (as at Anchor,) or lie ſtill, yet it is found by Experience, that ſhe drives in the mean time. And note,

That

That the heavier the Board and Weight is, that the Boat is to be rid by, the leſs will her Drift be.

Some uſe to judge of Currents by Rippling of the Water, and driving of Froth along the Shore, *&c.* But theſe are great Uncertainties, and not at all to be relied upon; and if the foregoing Way be not ſo exact as might be wiſhed, yet it is found agreeable with Experience, and therefore ought to be embraced, till by farther Experience a better way be found out.

SECT. VII. *Of Lee-way.*

FOr the better judging of a Ship's Lee-way, always when you are by a Wind, and have Land in ſight juſt on Head, take notice, as you keep the Ship ſteered on the Point the Land bore on, how ſhe ſlides away to Lee-ward, and the Land appears more and more to Wind-ward; for by ſuch kind of Obſervations, taking notice of the Sail abroad, Streſs of Wind, Ship's Run, and Growth of the Sea, the Judgment may be ſo ſtrengthned, that you ſhall rarely fail of giving a very near Allowance.

But if the Land be not right on Head, yet, by taking notice what Point it was on at firſt, and how it Weathers in keeping the ſame Courſe, you may thereby attain to the ſame advantage.

CHAP. II.

Of Inſtruments *appertaining to* Navigation, *both for* Obſervation *and* Operation; *their* Deſcription *and* Uſe.

THe Principal Inſtruments appertaining to *Navigation*, (and all, or the moſt of them a Navigator cannot be well without, beſides *Books*, *Maps*, *Sea-Plats*, *Plain* and *Mercator's Charts*, *Pens*, *Ink*, *Paper*, *Slate*, *Scales* and *Compaſſes*, are,

I. A *Plain Compaſs.*
II. An *Azimuth Compaſs.*
III. An *Univerſal Ring-Dial.*
IV. A *Croſs-Staff*, or *Fore-Staff.*
V. A *Quadrant*, or *Back-Staff.*
VI. A *Nocturnal.*

The Deſcription and Uſe of moſt of which follows.

SECT. I. *Of the Plain Compaſs.*

THE Plain Compaſs is no other than a Circle divided into 32 equal Parts; the Four Pricipal are the *North*, *South*, *Eaſt*, and *Weſt*: On the North Point always there is a *Flower-de-lis*; and theſe Four are called the Four *Cardinal Points*, *Winds*, or *Rumbs*: The middle Point between any of theſe *Cardinal* ones are called *Half-Points*, *Winds*, or *Rumbs*, as are the *North-Eaſt*, *South-Weſt*, *North-Weſt*, and *South-Eaſt*: The other Eight are denominated as you ſee in *Fig.* I. and alſo in the two following Tables. *Fig.* I.

Points	*Between the North and East.*	*Between the North and West.*	*Between the South and East.*	*Between the South and West.*
	North	North	South	South
I	North by East	North by West	South by East	South by West
II	North North East	North North West	South South East	South South West
III	North E. by North	North W. by North	South E. by South	South W. by South
IV	North-East	North West	South East	South West
V	North East by East	North W. by West	South East by East	South W. by West
VI	East North East	West North West	East South East	West South West
VII	East by North	West by North	East by South	West by South
VIII	East.	West	East	West

And for ease in Counting, I have here inserted

A Table shewing the Angles which every Rumb or Point of the Compass make with the Meridian.

North	South	D.	M.	South	North	Points.
		02	49			
		05	38			
		08	26			
N *by* E	S *by* E	11	15	S *by* W	N *by* W	1
		14	04			
		16	53			
		19	41			
NNE	SSE	22	30	SSW	NNW	2
		25	19			
		28	08			
		30	36			
NE *by* N	SE *by* S	33	45	SW *by* S	NW *by* N	3
		36	34			
		39	23			
		42	11			
NE	SE	45	00	SW	NW	4
		47	49			
		50	37			
		53	26			
NE *by* E	SE *by* E	56	15	SW *by* W	NW *by* W	5
		59	04			
		61	52			
		64	41			
ENE	ESE	67	30	WSW	WNW	6
		70	19			
		73	07			
		75	56			
E *by* N	E *by* S	78	45	W *by* S	W *by* N	7
		81	34			
		84	22			
		87	11			
East	East	90	00	West	West	8

Now

Now to fit this Compaſs for Uſe at Sea, there are placed under the Card two Wyres, touched with a Load-ſtone, and paſted or glewed, ſo that the Ends of both the Wyres, being bended, meet together at both Ends, which Ends are placed directly under the North and South Points of the Card; and in the Centre of the Card is placed a Braſs Cap or Socket, which is to make the Card hang upon a Pin perpendicularly erected from the Centre of the Bottom of the Box; and a Glaſs over the Card, to prevent it and the Wyres from Wind and Wet.

This Box, with the Card and Wyres in it, is to be hanged in two Circles or Hoops of Braſs, and thoſe put into another Box; ſo that the innermoſt Box, and conſequently the Card, may hang Horizontally which way ſoever the Ship moveth. This Compaſs thus fitted the *Flower-de-luce* always hath reſpect to the North part of the Heavens, and conſequently all the reſt to the reſpective Points as they are denominated upon the Card; but not ſo directly, at all times, and in all places, but there is a Deflection or Deviation from the true Coaſt, which Mariners call the *Variation* of the Compaſs; of which we have already ſaid ſomething, and ſhall more hereafter.

SECT. II. *Of the* Azimuth *Compaſs.*

UPon the top of the Box wherein the Fly and Needle is faſtned, there is a broad Circle of Braſs, one half of the Limb wherof is divided into 90 deg. numbred from the middle thereof both ways by 10, 20, 30, *&c.* to 45 deg. and theſe Degrees again ſubdivided by Diagonals into 5, 10, or 15 min And the Diviſions of theſe Degrees and Minutes are drawn from the oppoſite part of the Limb whereon the *Index* moveth. Upon the *Index* is erected a Sight, to be let down or raiſed up perpendicularly at pleaſure; alſo, from the top of this Sight, down to the middle of the *Index*, is faſtned a String, to caſt its Shadow upon the middle Line of the *Index*. *Fig.* II.

This broad Circle of Braſs is croſſed at Right Angles with two Strings, and from the termination of thoſe Strings are drawn four ſmall black Lines down the Sides, on the inſide of the Box, for rectifying the Inſtrument in time of Obſervation.

This Compaſs being fitted in manner as aforeſaid, is hung in a ſtrong Braſs Ring, and thoſe alſo faſtned into a ſquare Wainſcot Box, fitted for that purpoſe. The Figure of this Compaſs, fitted as aforeſaid, in its Box, you may ſee in *Figuae* II.

The Uſe of the Azimuth *Compaſs.*

You muſt firſt rectifie the Braſs Limb, by the Needle and Fly within the Box; then if the Obſervation be in the {Forenoon / Afternoon} put the Centre of the *Index* upon the {Weſt / Eaſt} Point of the Card or Fly, and ſo that the four Lines on the inſide of the Box, and the four Lines on the Edge of the Card, do always agree.

The Compaſs thus fitted, turn the *Index* towards the Sun, until the Shadow of the ſloap Thred fall directly into the Slit of the Perpendicular Sight, and alſo upon a ſtreight Line in the middle of the *Index*; then at the ſame time will the inner Edge of the *Index* ſhew the *Magnetical Azimuth* from the North or South.

The Uſe of the Azimuth *Compaſs in taking of the Sun's Amplitude.*

If you obſerve the Sun's Amplitude in the {Morning / Evening} at the {Riſing / Setting} of the Sun, then turn the Centre of the *Index* right over the {Weſt / Eaſt} Point of the Fly, and rectifie the Compaſs by the Lines within the Box, and the Lines over the Fly.

Then at the time of the Sun's {Riſing, / Setting,} turn the *Index* towards the Sun, till you cut the Body of the Sun by your *Index* and Thred; then will the inner Edge of the *Index* give you the Degrees and Minutes of the *Magnetical Amplitude* upon the Limb of the Compaſs, from the {Eaſt, / Weſt,} towards either North or South.

The *Magnetical Azimuth* or *Amplitude* being thus obtained: by the *Compaſs*, for the ſame time, you muſt find the true *Azimuth* or *Amplitude*, as is formerly taught.

And

And if the observed *Azimuth* or *Amplitude* do agree with the *Magnetical Azimuth* or *Amplitude*, then hath your *Compass* no Variation; but if they differ, that difference is the Variation: and which way, towards either *East* or *West*, is taught how to discover by the VI^th *Chapter* hereof.

Or briefly thus, by the *Azimuth*:

1. In the Forenoon.

If the *Sun's Azimuth* from the *North* part of the *Meridian* by Calculation be {Greater / Lesser} than the *Magnetical Azimuth* by Observation; then is the the Variation {*Eastward.* / *Westward.*}

2. In the Afternoon.

If the *Sun's Azimuth* from the *North* part of the *Meridian* be {Greater / Lesser} than the *Magnetical Azimuth*, then is the Variation {*Westerly.* / *Easterly.*}

Briefly thus by the *Amplitude*:

1. *At the Sun's Rising.*

If the *Sun's Amplitude* be {nearer to / farther from} the *North* part of the *Meridian* than the *Magnetical*, then is the Variation {*Westerly.* / *Easterly.*}

2. *At Sun's Setting.*

If the *Sun's Amplitude* be {nearer to / farther from} the *North* part of the *Meridian* than the *Magnetical*, then is the Variation {*Easterly.* / *Westerly.*}

Of the Variation of the Compass, *and towards what Coast it Varieth.*

The two principal ways for finding the Variation of the *Compass*, are by the *Sun's Amplitude* at his Rising or Setting, and by his *Azimuth* at any other time of the Day.

1. *By the Sun's Amplitude.*

You are by your *Azimuth Compass* to Observe at the *Sun's* Rising or Setting, how many Degrees the *Sun* is from the *East* or *West* Points of the *Compass*; which number of Degrees, if they do exactly agree with the *Amplitude* found, by Calculation or otherwise, and be on the same Side of the *Meridian*, then hath the *Compass* no Variation; but if they differ, look how many Degrees that difference is, for so much is the Variation.

For Example, Admit that I find the *Suns* * *Amplitude* for some Day or other, to be 24 deg. 52 min. *Northerly:* Then I know that the *Sun* should Set that Evening at almost 25 deg. from the *West*, towards the *North*: But at *Sun* Setting, observing with my *Azimuth Compass*, I find that the *Sun* Sets but 19 Degrees from the *West* towards the *North:* from whence I gather, that the Variation is 5 deg. 52 min. (that is almost 6 deg.) And by this means you may Observe the Variation as often as opportunity will serve.

* *How to find the* Sun's Amplitude *any Day in the Year, in any* Latitude, *is shewed in several Places of this* Book, *and among the* Astronomical Problems *following: as also the* Sun's Azimuth, *&c.*

To know towards what Coast, East *or* West, *the* Compass *Varies.*

1. *By the* Amplitude.

If the Degrees of the *Compass*, which should directly respect the *Sun* at his Rising or Setting, (namely the Degrees of *Amplitude* found as before,) be more towards the Right-hand than the *Sun* Rising or Setting is, the Variation is *Easterly:* But if it be more towards the Left-hand, the Variation is *Westerly*; because, when you look with

with your Face towards the *North*, the *East* is on your Right-hand, and the *West* on your Left-hand.

As in the former *Example*, I find by the *Amplitude*, that the *Sun* ſhould Set almoſt 25 deg. from the *Weſt Northerly*; but Setting the *Sun* with my *Compaſs*, I find that 25 deg. of my *Compaſs* is more to the Right-hand than 19 deg. the Place of the *Sun* Setting: therefore I conclude the Variation to be almoſt 6 deg. *Eaſterly*; ſo that the *North* Point of the *Compaſs* ſhews not the true *North*, but points almoſt 6 deg. to the *Eaſtward* of the *North*: and conſequently, all the other Points of the *Compaſs* direct more towards the Right-hand than they ſhould do: And according to this Rule, the like in all Points is to be underſtood, if the Obſervation had been made at the *Sun's*-Riſing.

Note, It will be beſt to make theſe Obſervations when the *Sun* ſeems to be a little above the *Horizon*; namely, when the lower edge of the *Sun* ſeems almoſt to touch the *Horizon*: for then is the *Sun* in the *Horizon*, but by reaſon of his Refraction and Parallax he ſeems to be above it.

II. *By the Sun's* Azimuth.

The Variation of the *Compaſs*, by the *Sun's Azimuth*, may be found according to the Directions following, *viz.*

About the middle of the Forenoon or Afternoon (the farther from Noon the better) you muſt take the *Sun's Altitude* by ſome *Inſtrument* for that purpoſe hereafter mentioned; which being noted down, you are at the ſame time, as near as may be (or another for you at the ſame time) to ſet the *Sun* with your *Compaſs*, and note down likewiſe upon what Degree of the *Compaſs* (reckoning from the *North*) you found the *Sun* to be. Then knowing your *Latitude*, and (by your *Tables*, at the end of this *Book*) the *Sun's Declination*, you may find the *Sun's Azimuth* from the *North*, as hath been ſhewed how to do ſeveral ways, in divers places of this *Book*; which *Azimuth*, compared with the Degrees before found by the *Compaſs*, if both agree, the *Compaſs* hath no Variation; if there be any difference, that difference is the Variation: which Variation, whether it be *Eaſterly* or *Weſterly*, may be found by the Rule laſt delivered for that purpoſe.

Example, Admit that upon ſome Day in the Forenoon, I obſerved the Height of the *Sun* to be 32 deg. 28 min. and knowing my *Latitude*, and *Sun's Declination* for that time, I find the *Sun's Azimuth* from the *North* to be 107 deg. 36 m. At the ſame time as I took my *Altitude* in the Forenoon, I ſet the *Sun* by my *Azimuth Compaſs*, and found that the *Sun* then was 102 deg. the difference between the true obſerved *Azimuth*, and that by my *Compaſs* being 5 deg. 36 m. which is the Variation of the *Compaſs*.

Now to know whether this Variation be *Eaſterly* or *Weſterly*. I conſider that the *Sun's* true *Azimuth* (found by the *Aſtronomical Problems* in this *Book*) from the *North* 107 deg. 36 min. or from the *Eaſt* Point of the *Compaſs Southwards* 17 deg. 36 min. whereas ſetting the *Sun* with my *Compaſs* it was but 12 deg. from the *Eaſt Southwards*: So that the Degrees that the *Sun* ſhould have been on, was more towards the Righ-hand than the Degrees whereon it was: wherefore, I conclude the Variation to be *Eaſterly* 5 deg. 36 min.

SECT. III. *Of the Univerſal* Ring-Dial.

The Deſcription of the ſeveral parts of this Dial.

THE Dial conſiſteth of three Parts, *viz.* Of two great Circles of the Sphere. 1. The *Meridian*, which is the outermoſt Circle thereof. 2. The *Æquinoctial*, which is the innermoſt Circle thereof. And, 3. Of a *Bridge* or *Stile*, repreſenting the *Axis* of the World: And to theſe three General Parts is added this Particular; namely, a ſmall *Ring* and *Index*, to ſet it to the *Latitude* of the Place in what Part of the World ſoever you are; and alſo to hold in your Hand, when you would find either the *Latitude* of a Place, or the Hour of the Day. *Fig.* III.

How

How the ſeveral parts of the Dial *are divided.*

1. The *Meridian* on one ſide is firſt divided into Four Parts or *Quadrants*, and one of thoſe *Quadrants* divided into 90 equal Parts or Degrees; to one of which the ſmall Ring muſt be ſet when you would find the Hour in any *Latitude*.—— On the other-ſide of the *Meridian* is projected a *Quadrant* of *Altitude*, whereby to take the Height of the *Sun*; and ſo conſequently thereby to find the *Latitude* of an unknown Place, in what Part of the World ſoever you are.

2. The *Æquinoctial* Circle is divided in the inſide thereof; firſt, into 24 equal Parts or Hours, and thoſe again ſubdivided into Halves and Quarters, and numbred both ways, by I. II. III. *&c.* to XII. ſtanding at the *Meridian*. And although all the 24 Hours are neceſſary; Yet in ſmall *Inſtruments* they ſeldom do graduate more than from 3 in the Morning till 9 at Night.

3. The *Axis*, *Stile*, or *Bridge*, which paſſeth croſs the *Meridian* Circle from Pole to Pole, always cutting the *Æquinoctial* Circle at Right Angles, is made of a piece of thin (but broad) Braſs; in the middle whereof is a long ſlit, in which there moveth a *Curſor*, which hath in it a ſmall hole, which muſt be ſet to the Day of the Month, graduated on the *Axis*, when you would find the Hour of the Day. Or to the *Sun's* Declination, which is graduated on the other ſide thereof, when you would find the *Latitude* of any unknown Place. Hitherto of the Deſcription: Now ſhall follow

The Uſe of this Univerſal Dial.

The Uſes are principally theſe following. Although larger *Inſtruments* of this kind may be made to perform divers others.

I. The Day of the Month being known, you may find the *Sun's* Declination: Or,

II. The Declination being known, you may find the Day of the Month.

III. The *Latitude* of the Place you are in, and the *Sun's* Declination being known, you may find the *Meridian-Altitude*. Or,

IV. The *Meridian-Altitude* and Declination being known, you may find the *Latitude*.

V. The *Latitude* and Day of the Month (or Declination) known, you may find the Hour of the Day in any Part of the World.

1. *To find the Sun's Declination.*

Move the *Curſor* with your Thumb along the ſlit in the *Axis*, till the hole ſtand juſt againſt the Day of the Month, then turn to the other ſide of the Bridge, and the ſmall hole will ſtand juſt againſt the *Sun's* Declination.

			D.	M.	
So if you ſet the hole of the *Curſor* againſt the	10*th.* of *January*	It will ſhew on the other ſide of the Bridge	20	1	S.
	20*th.* of *February*		6	57	S.
	10*th.* of *March*		0	00	
	15*th.* of *May*		21	7	N.
	20*th.* of *Auguſt*		8	56	N.

And on the 12*th.* of *September* the Declination will be 00 deg. 00 min. again: And by this Example you may perceive that from the 10*th.* of *March*, to the 12*th.* of *September*, the *Sun* hath *North* Declination; and from the 12*th.* of *September*, to the 10*th.* of *March*, it hath *South* Declination.

2. *To find the Day of the Month.*

This is but the converſe of the former Propoſition: For if you

	D.	M.			
Set the *Curſor* to	20—	1 *South*	Declination, and on the other ſide of the Bridge you ſhall find	*January*	the 10*th.*
	6—	57 *South*		*February*	the 20*th.*
	0—	00		*March*	the 10*th.*
	21—	7 *North*		*May*	the 15*th.*
	8—	56 *North*		*Auguſt*	the 20*th.*

3. *To*

3. *To find the Sun's* Meridian Altitude.

If the *Sun* have {*North*, *South*} Declination {Add, Subtract} the Complement of the *Latitude* (which is ſo much as the *Latitude* wants of 90 deg.) {to, from} the *Sun's* Declination, and the {Summ, Remainer} will be the *Meridian Altitude* of the *Sun* that Day: Thus

	D.	M.		D.	M.			D.	M.	
The *Latitude* being	51	30	Complement	38	30	}	or {	38	30	}
And the Declination				16	20	*North*		16	20	*South*
The *Meridian Altitude* will be				54	50			22	10	

4. *To find the* Latitude.

This muſt be performed by Obſervation: and the Obſervation that you are to make is of the *Sun's Meridian Altitude*; to perform which ſtick a Pin in the hole which is on that ſide of the *Meridian* on which the *Quadrant* of *Altitude* is graduated. Then bring the ſmall *Hand-Ring* to the beginning of the Degrees on the other ſide of the *Quadrant*, and turn the Pin to the *Sun*, till it caſt its Shadow through the Degrees of the *Quadrant* of *Altitude*: then what Degree the Shadow of the Pin cuts, that is the *Sun's Altitude* at that time: So then to obſerve his *Meridian Altitude* (which is always at 12 a Clock) you muſt about Noon wait till you ſee the *Sun* at his greateſt *Altitude*, for that (in theſe temperate *Zones*) is the *Meridian Altitude*.

The Meridian Altitude *being obtained, the* Latitude *is thus found.*

Add the *Meridian Altitude* and the *Sun's* Declination together, if the Declination be *South*, (or ſubtract the Declination from the *Meridian Altitude*, if the Declination be *North*,) and the Summ or Difference ſhall be the Complement of the *Latitude* of the Place. Thus,

The *Meridian Altitude* being {54—50, 22—10} and the Declination {16 20 *North*, 16 20 *South*,} the Complement of the *Latitude* will be found to be 38 deg. 30 min. which taken from 90 deg. leaves 51 deg. 30 min. for the true *Latitude*.

5. *To find the Hour of the Day.*

Firſt, Set the *Hand-Ring* to the *Latitude* of the Place, and the hole in the *Curſor* to the Day of the Month, then open the *Equinoctial* Circle at Right Angles to the *Meridian* Circle, and turn the *Axis* towards the Hour of the Day you gueſs it to be: Then holding the *Hand-Ring* between your Forefinger and Thumb, guide the *Meridian* Circle, by help of your little Finger, to the *Sun*, ſo that the Beams may paſs through the hole of the *Curſor*, and pierce upon the middle of the *Equinoctial* Circle, where the Hours, Halves, and Quarters are graduated; ſo ſhall the little ſpot of Light there ſhew the Time and Hour of the Day.

SECT. IV. *Of the* Croſs-Staff, *or* Fore-ſtaff.

The Deſcription of the Croſs-Staff.

THE Staff is made in length about three Foot, and four Square, each Side being about half or three quarters of an Inch ſquare broad: Each of theſe Sides is divided into unequal Parts, as a Line of *Tangents* is, *Fig.* IV.

		D.	D.
The Diviſions of the	First	3	10
	Second	10	30
	Third	20	60
	Fourth	30	90

Side begin at and end at

To

To theſe Four Sides belong a ſeveral Croſs or Vane: As

To the Side of { D. 10, 30, 60, 90 } uſe the { D. 10, 30, 60, 90 } Vane, which ſerves for all *Altitudes* between { D. 0, 10, 30, 60 } and { D. 10, 30, 60, 90 }

The Uſe of this Staff.

It principally is uſed in taking of the *Altitude* of the *Sun*, *Moon*, or *Stars*: And according (as by eſtimation) you gueſs the *Altitude* of the *Sun* or *Star* to be, accordingly put upon your Staff the Tranſome proper for ſuch an *Altitude*.

How to make your Obſervation.

Hold the end of your Staff to the corner of your Eye, and there let it reſt upon the Eye-Bone, as near the Centre of your Eye as you can with convenience, ſo that it hinder not your Sight: Then look to the upper end of your Croſs, to the *Sun* or *Star*, and at the lower end of the Croſs for the *Horizon*; but if you cannot ſee the *Horizon* at the lower end, but you ſee all Sky and no Water, then draw your Croſs a little nearer to your Eye: but, on the contrary, if you ſee all Water and no Sky, then ſlide your Croſs a little from your Eye, and then make Obſervation again as before: Then if you ſee the Centre of the *Sun* or *Star* at the upper end of the Croſs, and the *Horizon* at the lower end, then your Croſs doth ſtand as it ought. Then wait till the *Sun* be upon the *Meridian*, and make Obſervation again as often as you think convenient; and as you find the *Sun* to riſe higher, draw your Croſs a little nearer to your Eye; but if the *Sun* be fallen, then you will not ſee the *Horizon*, for the Water will hinder; and in ſuch caſe you muſt not alter your Croſs, but look upon that Side of your Staff unto which your Croſs belongeth: and thoſe Degrees and Minutes that the Croſs doth ſtand at upon its proper Side of the Staff is the *Meridian Altitude* for that day.

SECT. V. *Of the* Quadrant, *or* Back-Staff.

The Deſcription of the Quadrant.

Fig. V. THIS *Quadrant* is the principal *Inſtrument* now in Uſe at Sea, for taking the *Meridian Altitude* of the *Sun*: It was the Invention of one Captain *Davis*, an *Engliſh* Man, and called *Davis's Quadrant*.

The Parts of this Inſtrument *are Two.*

(1.) Three *Vanes*; and (2.) Two *Arches*.

The three *Vanes* are, (1.) The *Sight-Vane*. (2.) The *Horizontal-Vanes*. (3.) The *Shade-Vane*.

It is called the *Shade-Vane*, becauſe in time of Obſervation it giveth the Shadow upon the *Horizontal-Vane*: And the *Sight-Vane* is ſo called, becauſe it is placed at the Eye in time of Obſervation.

The leſſer *Arch* is called the *Arch of* 60 deg. and the greater, the *Arch of* 30 deg. which together make 90 deg. and from thence it is denominated a *Quadrant*.

The 60 *Arch* may be divided into 60 deg. but every Fifth is ſufficient, and numbered downwards by 10, 20, 30, *&c.* to 60. —— The greater *Arch* is called the 30 *Arch*, and is divided into 30 deg. and by *Concentrick* Circles and *Diagonal* Lines ſub-divided into 5 or 10 min.

The Uſe of the Quadrant.

Firſt, Put the *Horizontal-Vane* upon the end of the *Quadrant*; and then the upper edge of the *Shade-Vane* upon the 60 *Arch*, at ſome even 5 or 10 deg. as you ſhall ſee

ſee cauſe; and laſtly, put the Sight-Vane upon the 30 Arch. Your *Quadrant* being thus fitted, when you begin to take the Altitude with it, you muſt turn your Back to the Sun, and hold the Handle of your *Quadrant* with the Sight-Vane to your Eye, as upright as you can.

Then look through the Sight-Vane upon the Horizon-Vane, and bring the upper part of the Shade to lie upon the upper part of the Edge of the Slit of the Horizon-Vane, and at the ſame time look for the Horizon through the Slit of the Horizon-Vane; and if you cannot ſee the Horizon, but all Sky and no Water, then put your Sight-Vane a little lower; but if you ſee all Water and no Sky, then put your Sight-Vane a little higher; and when you have done ſo, obſerve again; and then, if you ſee the Shade fall upon the upper part of the Slit of the Horizon-Vane, and at the ſame time you ſee the Horizon through the Sight Vane, it is all at preſent you have to do.

Then wait till the Sun be upon the Meridian, and make Obſervation as often as your Judgment ſhall think fit, until you find the Sun at the higheſt; for if the Sun be fallen, and you go to obſerve, you will not ſee the *Horizon*, for the Water will hide it from you: Then look what Degrees do ſtand by the upper Edge of your Sight-Vane, and add them to the Degrees to which you ſet your Shade Vane, and their Summ is the Diſtance of the upper Edge of the Sun from the *Zenith*; to which Summ if you add 16 min. for the Sun's Semidiametre, you will have the true Diſtance of the Sun's Centre from the *Zenith*, whoſe Complement is the Sun's Meridian Altitude: But if you obſerve by the lower part of the Shade, then it is the lower Limb of the Sun that gives the Shade: Therefore, if you obſerve by the upper part of the Shade, you muſt add 16 min. but if by the lower part of the Shade, you muſt add or ſubtract 16 min. to or from the obſerved Altitude, and the Summ or Remainder is the Diſtance of the Sun's Centre from the *Zenith*, or the Complement of the Meridian Altitude.

There are other Inſtruments for taking of the Altitude of the Sun and Stars; as the *Plough*, the *Aſtrolabe*, the *Demi-Croſs*, the *Bow*, *&c.* whoſe Deſcription I here purpoſely omit, as being, moſt of them, not made uſe of by our *Engliſh* Navigators.

SECT. VI. *Of the Nocturnal.*

THere are ſeveral ſorts of *Nocturnals*, of which ſome are Projections of the Sphere, as the two Hemiſpheres mentioned in the Second Part of the Third Book of *Practical Aſtronomy*; but with Seamen there are only Two, which they generally make uſe of, and the manner of uſing of either of them is the ſame. One of them is fitted to be uſed with the *North-Star*, and the firſt of the *Guards* in the *Little Bear*; the other for the *North-Star*, and the *Guards*, (or *Pointers*, as ſome call them,) of the *Great Bear*. *Fig.* VI.

The *Nocturnal* conſiſts of three Parts, or Piecees of Wood or Braſs, the largeſt of which hath a Handle to hold it by when you would obſerve, and oppoſite to the Handle a ſmall Tooth or Point, which (if it be made for the *Little Bear*) ſtands againſt the 25th. of *April*, but if for the *Great Bear*, againſt the 17th. of *February*, which are the times that thoſe Stars come to the Meridian at 12 a Clock at night. Upon this bigger Piece or Part are two Circles deſcribed, in the outermoſt of which are the Months and Days of the Month, and upon the innermoſt the 24 Hours of a natural Day; and upon the back ſide of this part alſo are deſcribed and marked with Letters the 32 Points of the Compaſs.

The ſecond Part of the *Nocturnal* hath two Circles deſcribed upon it; the outermoſt whereof is divided into 29 and a half equal Parts, for the Days of the Moons Age, and the innermoſt into 24 Hours; and at the beginning of the Days of the Moons Age, and at XII, there is a Tooth to be ſet to the Day of the Month in the Upper Part.

The third Part is an *Index* with a Fiducial Edge iſſuing from the Centre, and muſt be ſo long that a good part of it may extend beyond the outermoſt or biggeſt Piece. Theſe three Parts are ſo ordered, that by help of a ſmall hollow Braſs Socket they are made to move about the Centre of the Inſtrument; all which do plainly appear in the Figure thereof.

The Use of the Nocturnal.

I. *To find the Hour of the Night.*

Set the Tooth which is upon the middle Part to the Day of the Month; then looking towards the North, hold up the Instrument as upright as you can, the Fore-side being towards your Face: Then bow the upper part of the *Nocturnal* where the Tip is, so much towards you, that through the Hole which is in the Centre of the Instrument you may see the *North* or *Pole-Star*, and there hold it fast, turning the long *Index* about, till by the Edge thereof you can see the first of the *Guards* of the *Little Bear*, or the *Pointers* of the *Great Bear*, [according to which *Bear* the Instrument is made;] then shall the Edge of the *Index* or Ruler shew upon the innermost Circle of the Middle Part the Hour of the Night.

II. *To find upon what Point of the Compass the* Guards *are.*

When you have observed and found the Hour of the Night, as is before directed, let the Long Ruler or *Index* rest in its place that it was in when you observ'd the Hour, and then turn the Back-side of the Instrument, and upon what Point of the Compos the *Index* lies, upon that Point of the Compass were the *Guards* at the time of your Observation.

III. *To find at what Hour the Moon will be upon the South Part of the Meridian, at any Day of her Age.*

Seek the Days of the Moon's Age upon the outermost Circle of the Middle Piece, and right against it, in the innermost Circle of that Piece, is the Hour required.

Example. If upon the 18th or other Day of *May*, the Moon shall be found to be 11 days old, look for 11 in the outermost Circle, and against it in the innermost Circle you shall find VIII Hours and 48 min. at which time the Moon will be upon the Meridian when she is 11 days old.

CHAP. III.

Concerning the Finding of the Latitude *at Sea.*

IN the Fourth and Fifth *Sections* of the foregoing *Chapter* you have the Description and Use of the two principal Instruments for taking the Altitudes of the Sun or Stars, either upon the Meridian, or upon other Points of the Compass: It resteth now to shew, how these Observations are to be applied to Practice, in finding of the *Latitude*: For the obtaining of which, by the Meridian Altitude of the Sun or of a Star, mind these following

RULES.

1. *Note*, That so much as the *Zenith* (which is always the Complement of the Meridian Altitude to 90 deg.) is distant from the *Equinoctial* towards the North or South, so much is the respective (North or South) *Pole* elevated above the *Horizon*, and that is called *LATITUDE*.

2. If the Sun or Star have no Declination, the *Zenith* Distance (or Complement of its Meridian Altitude) is the *Latitude*: And then, If the Sun or Star be upon the North part of the Meridian, the *Latitude* is South; if upon the South part of the Meridian, the *Latitude* is North.

3. If the Sun or Star be in the *Zenith*, then the Declination of the Sun, or of that Star, is the *Latitude*: And if the Declination be Northerly, the *Latitude* is North; if Southerly, South.

4. In all other Cases, the Position of the *Zenith*, the *Equinoctial*, and the Sun or Star, are to be considered; as, (1.) Whether the *Zenith* be between the *Equinoctial* and the Star; or, (2.) The Star between the *Equinoctial* and the *Zenith*; or, (3.) The

(3.) The *Equinoctial* between the other two: For, according to these several Situations must the Rules for finding the *Latitude* be varied. But the Position of the *Zenith* is known, it being, always, the Complement of the Meridian Altitude to 90 deg. and by that, and the Declination, the *Latitude* may be obtained.

5. If the Declination of the Sun, or of a Star, be North or South, and the Meridian Altitude observed be the same way with the Declination, the Difference between the Declination and *Zenith* Distance is the *Latitude* towards that Coast, North or South, as the Declination is.

And that these Rules (here delivered) may be made obvious in all Cases, I shall give particular Examples of them in the following Schemes; in all which,

H Æ Z P O N represenleth the Meridian.
Æ æ the Equinoctial.
K A the Parallel of Declination.
H O the Horizon.
P S the *Axis* of the *Æquator* or World.
Z N the Prime Vertical Circle.
P the North Pole.
S the South Pole.
Z the *Zenith*.
N the *Nadir*.
H the South Point of the Horizon.
O the North Point thereof.
H K, or O K, the Meridian Altitude.
Z K the *Zenith* Distance.
Æ K the Sun or Stars Declination.
Z Æ, or O P, the *Latitude*.

Fig. I.

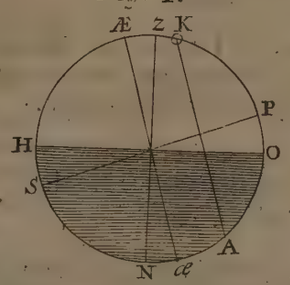


Fig. II.

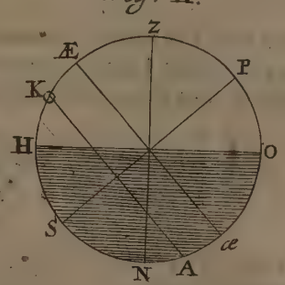


Fig. III.

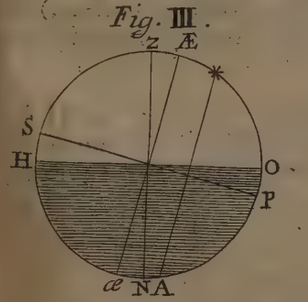


Fig. IV.

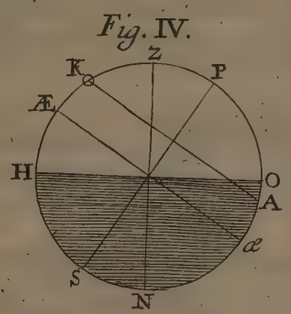


Fig. V.

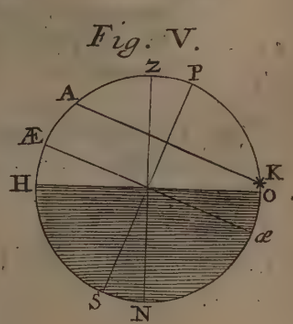


Fig. VI.

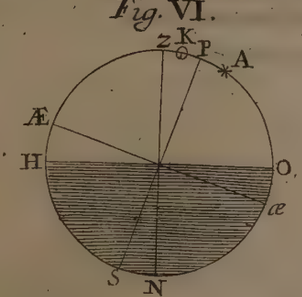
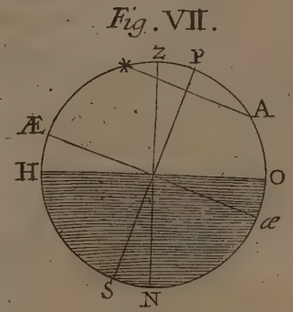


Fig. VII.



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		Deg.	min.	
Fig. I. The *Zenith* being between the *Equinoctial* and the Sun or Star,	The Declination North	23	30	Æ K.
	The *Zenith* Distance North	8	30	Z K.
	The Latitude North	15	00	Z Æ.

But if the Declination be leſs than the *Zenith* Diſtance, then the contrary Pole to the Declination is elevated.

Fig. II. The *Equinoctial* betwen the *Zenith* and the Sun or Star,

The *Zenith* Diſtance	71	30	Z K.
The Declination South	20	00	Æ K.
The Latitude North	51	30	P O.

Fig. III. The *Zenith* between the *Equinoctial* and the Sun or Star,

The *Zenith* Diſtance North	30	00	Z K.
The Declination North	20	00	Æ K.
The Latitude South	10	00	Z Æ.

But if the *Zenith* Diſtance be contrary to the Declination of the Sun or Star, the Summ of the *Zenith* Diſtance and Declination is the *Latitude* North or South, as the Declination is.

Fig. IV. The Summ between the *Zenith* and *Equinoctial*,

The *Zenith* Diſtance South	31	30	Z K.
The Delination North	20	00	Æ K.
The Latitude North	51	30	Z Æ.

Fig. V. The Summ between the *Zenith* and the *Equinoctial*,

Declination North its Comp.	66	30	P K.
Meridian Altitude North	8	30	O K.
Latitude North	75	00	O P.

But if the Meridian Altitude be more than the Declination, ſubtract the *Zenith* Diſtance from the Complement of the Declination, and the Remainder is the Height of the Pole, North or South, with the Declination.

Fig. VI. The *Zenith* between the *Equinoctial* and the Sun or Star,

The Declination North	80	00	Æ K.
The *Zenith* Diſtance North	5	00	Z K.
The *Latitude* North	75	00	Z Æ.

But here note, That which way ſoever the Meridian Altitude be, if the Meridian Altitude and twice the Complement of the Declination, (added together,) be leſs than 180 deg. the Sun or Star hath two Meridian Altitudes; and by the Second Rule you may have the *Latitude*.

Fig. VII. The Sun or Star between the *Equinoctial* and the *Zenith*,

The Meridian Altitude	77	00	H K.
Declination North 62 deg.	28	00	K P.
Its Complement twice	28	00	K A.
The Summ is	133	00	H A.

Which being leſs than 180 Degrees, therefore,

The *Zenith* Diſtance South	13	00	K Z.
The Declination North	62	00	Æ K.
The Latitude North	75	00	Z Æ.

CHAP.

CHAP. IV.

Concerning the Vulgar Notes *of the* Julian Year, *the Motion of the* Sun *and* Moon, *with brief Rules and Tables to find them for ever; and of their great Use in* Navigation.

OF ſome of theſe I have had occaſion to ſpeak in the Fourth Book of *COSMOGRAPHY*, and alſo in the Second Part thereof, and alſo at the end of this Seventh Book: Thoſe of the *Vulgar Notes* that I ſhall ſpeak of here, are theſe:

I. Of the *Prime* or *Golden Number*, whereby to find the *Conjunctions*, *Oppoſitions*, and other *Aſpects*, between the Sun and Moon.
II. The *Solar Circle*, (or *Cycle* of the *Sun*,) whereby to find the *Dominical Letter*.
III. The *Epact*, whereby the Difference between the *Solar* and the *Lunar Years* is equated.

SECT. I. *Of the* Prime, *or* Golden Number.

THis is a Circle or Revolution of 19 Years, in which time the Ancients thought that all the *Lunations* and *Aſpects* between the Sun and Moon did return to the ſame place they were at 19 Years before: But Experience hath found the contrary; for they will ſometimes differ 10, 12, or more Hours; notwithſtanding, for the Purpoſes here intended, they will be ſufficiently exact.

Meton (*th'* Athenian) *firſt this* Circle *found,*
Which Nineteen Civil Years devolveth round:
For all Lunations *make return therein,*
Nigh to the Place where firſt they did begin.

To find this Number for any Year paſt or to come, add 1 to the Year of our Lord, (for this Number was 1 at the Birth of our Saviour,) and divide by 19, the the Quotient will tell you how many Revolutions this Circle hath made ſince the Birth of our Saviour, and the Remainder is the *Prime* or *Golden Number* for the Year enquired.

So in the Year 1691 add 1 to it, it makes 1692; which divide by 19, the Quotient is 89; which ſhews, that ſince the Birth of *Chriſt*, this *Cycle* hath made 89 compleat Revolutions, and 1 more of the 90th. which 1 remaining, is the *Prime* or *Golden Number* for the Year 1691.

TABLE I.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
A larger TABLE, shewing the PRIME or *Golden Number* for ever.	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900
	2000	2100	2200	2300	2400	2500	2600	2700	2800	2900	3000	3100	3200	3300	3400	3500	3600	3700	3800
1 20 39 58 77 96	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2
2 21 40 59 78 97	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3
3 22 41 60 79 98	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4
4 23 42 61 80 99	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5
5 24 43 62 81	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6
6 25 44 63 82	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7
7 26 45 64 83	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8
8 27 46 65 84	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9
9 28 47 66 85	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10
10 29 48 67 86	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11
11 30 49 68 87	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12
12 31 50 69 88	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13
13 32 51 70 89	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14
14 33 52 71 90	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15
15 34 53 72 91	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16
16 35 54 73 92	3	8	13	18	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17
17 36 55 74 93	4	9	14	19	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18
18 37 56 75 94	5	10	15	1	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19
19 38 57 76 95	6	11	16	2	7	12	17	3	8	13	18	4	9	14	19	5	10	15	1

The Use of this Table.

In this Table you have, in six little Columns, all Numbers of Years, successively proceeding, from 1 to 100, by 1, 2, 3, 4, *&c.* to 99: — And in two Rows at the Head of the Table, you have all Numbers of Hundreds of Years, successively increasing, from 100 to 3800, in this order, 100, 200, 300, *&c.* to 3800.

Now the Use of this Table is plain: For if you find the Hundreds or Thousands of Years proposed in the Head of the Table, and the other odd Number of Years in any of the six little Columns on the Left-hand of the Table, in the common Angle, or meeting of those two, you have the *Prime* for that Year.

Example 1. *I would know what was the* Prime *or* Golden Number *in the Year of our Lord* 1529.— Look for 1500 in the Head of the Table, (which you will find in the Fifteenth Column,) and look for 29 on the Left-hand of the Table, (which you shall find in the second of the six little Columns,) and right against 29, and under 1500, you shall find 10; and such was the *Prime* or *Golden Number* for the Year 1529.

Example 2. *Let the* Prime *or* Golden Number *be required for the Year* 2672.— Look for 2600 in the Head of the Table, and for 72 in the Left-hand small Columns, and against 72, and under 2600, you shall find 13, which will be the *Prime* for the Year 2672, *&c.*

SECT.

SECT. II. *Of the* Solar Circle, *or* Cycle *of the* Sun.

THIS Circle maketh its periodical Revolution in 28 Years; and was invented, more to find thereby the true order of the *Dominical Letters*, than to shew any great Changes of the *Sun's* Motion therein.

Seven Hebdomaick *Letters used be,*
And those are A, B, C, D, E, F, G:
The Solar Circle *shews us which to call,*
The Sunday-Letter, *or* Dominical.

To find this Number for any time; add 9 to the Year given, and divide by 28, the Remainder, after the Division is ended, shews the *Cycle* of the *Sun* for that Year.

So in the Year 1691, add 9 to it, and it makes 1700; which divided by 28, gives 60 in the *Quotient*, and 20 remaining; which 20 is the *Cycle* of the *Sun* for the Year 1691.

TABLE II.

A TABLE *shewing the* Dominical Letter *from the First Year of* Christ *to* 3400.

1	2	3	4	5	6	7
000	100	200	300	400	500	600
700	800	900	1000	1100	1200	1300
1400	1500	1600	1700	1800	1900	2000
2100	2200	2300	2400	2500	2600	2700
2800	2900	3000	3100	3200	3300	3400

0	28	56	84	D	C	E	D	F	E	G	F	A	G
1	29	57	85	B		C		D		E		F	
2	30	58	86	A		B		C		D		E	
3	31	59	87	G		A		B		C		D	
4	32	60	88	F	E	G	F	A	G	B	A	C	B
5	33	61	89	D		E		F		G		A	
6	34	62	90	C		D		E		F		G	
7	35	63	91	B		C		D		E		F	
8	36	64	92	A	G	B	A	C	B	D	C	E	D
9	37	65	93	F		G		A		B		C	
10	38	66	94	E		F		G		A		B	
11	39	67	95	D		E		F		G		A	
12	40	68	96	C	B	D	C	E	D	F	E	G	F
13	41	69	97	A		B		C		D		E	
14	42	70	98	G		A		B		C		D	
15	43	71	99	F		G		A		B		C	
16	44	72		E	D	F	E	G	F	A	G	B	A
17	45	73		C		D		E		F		G	
18	46	74		B		C		D		E		F	
19	47	75		A		B		C		D		E	
20	48	76		G	F	A	G	B	A	C	B	D	C
21	49	77		E		F		G		A		B	
22	50	78		D		E		F		G		A	
23	51	79		C		D		E		F		G	
24	52	80		B	A	C	B	D	C	E	D	F	E
25	53	81		G		A		B		C		D	
26	54	82		F		G		A		B		C	
27	55	83		E		F		G		A		B	

G	F	A	G	B	A	C	B	D	C	E	D	F	E
E		F		G		A		B		C		D	
D		E		F		G		A		B		C	
C		D		E		F		G		A		B	
B	A	C	B	D	C	E	D	F	E	G	F	A	G
G		A		B		C		D		E		F	
F		G		A		B		C		D		E	
E		F		G		A		B		C		D	

The

The Use of this Table.

This *Table*, in its Uſe, differeth little from the former, for if you find the Hundreds, or Thouſands of Years in the Head of the *Table*, and the odd Tens of Years in any of the Four leſſer Columns to the Left-hand; in the common *Angle*, or meeting of theſe two, is found the *Dominical* or *Sunday-Letter* for that Year.

Example 1. *I would know what the* Dominical-Letter *was in the Year of our Lord* 1626— Look for 1600 in the Head of the *Table*, (which you will find in the Third Column,) and look for 26 in the Four ſmall Columns, (which will be found in the Firſt of the Four Columns,) and right againſt 26, and under 1600, you ſhall find A; which was the *Dominical-Letter* for the Year 1626.

Example 2. *What will the* Dominical-Letter *be in the Year* 1864.— Look for 1800 in the Head of the *Table*, and for 64 in the Four ſmall Columns, and againſt 64, and under 1800, you ſhall find E D, which are the *Dominical-Letters* for that Year. And becauſe it hath two Letters, it appeareth to be a *Leap-Year*: The firſt Letter E being in Uſe till the 24*th.* of *March*, and the latter D, to the end the Year.

SECT. III. *Of the* Epact.

THE *Epact* is a Number never exceeding 30, becauſe the *Moon* between Change and Change never exceeds 30 days, and thereby the common *Lunar Year*, conſiſting of 13 Months, is leſs than the *Solar Year* by 11 days; for to every Month are attributed but 29 days and a half: ſo that a *Lunar Year* containeth but 354 days; and the common *Solar Year* conſiſting of 365 days, exceedeth the other by 11 days: from whence the *Epact* taketh its Original.

Epactæ *from* ἐπάγειν, *is deriv'd,*
They'r adventitious Days fitly contriv'd
To adequate the Difference that appears,
Betwixt the Solar *and the* Lunar Years.

To find the *Epact* for any Year, you muſt by *Section* 1. find the *Golden Number* for that Year; which multiply by 11, and divide the Product by 30, the Remainder, after the Diviſion is ended, ſhall be the *Epact* for that Year.

So in the Year 1691, the *Golden Number* for that Year is 1; which multiplied by 11, the Product will be 11; which ſhould be divided by 30; but being leſs than 30, 11 is the *Epact* of the Year 1691.

Likewiſe in the Year 1694, the *Golden Number* for that Year is 4; which multiplied by 11, the Product is 44; and that divided by 30, the *Quotient* is 1, and 14, the Remainder, is the *Epact* for 1694.

The Uſe of this *Table*: Seek the Year of Chriſt in the Firſt Column of the *Table*, and right againſt it, in the Second, you ſhall have the *Epact*.

So in the Year 1691, I would know the *Epact* for that Year, ſeek for 1691 in the Firſt Column, and right againſt it ſtands 11; which is the *Epact* for the Year 1691.

If you would know the *Epact* for any Year after 1708, you muſt begin the Column of Years again, calling 1690 1709, *&c.* So ſhall the *Epact* for the Year 1712 be found to be 3, and of 1722 to be 23, *&c.*

And here note, That the *Golden Number*, the *Cycle* of the *Sun*, and the *Dominical Letter*, take their beginning every Year on the Firſt of *January*: but the *Epact* taketh its beginning not till the Firſt of *March*.

TABLE III.

A *Table* ſhewing the *Epact* for ever.

1690	29
1691	11
1692	12
1693	13
1964	14
1695	25
1696	16
1697	17
1698	18
1699	19
1700	10
1701	1
1702	12
1703	23
1704	4
1705	15
1706	26
1707	7
1708	18

TABLE

TABLE IV.

A TABLE *ſhewing in what Sign and Degree of the* Zodiack *the Sun is every Day in the Year at* 12 *at Noon.*

Months.	*January.*	*February.*	*March.*	*April.*	*May.*	*June.*	*July.*	*Auguſt.*	*September.*	*October.*	*November.*	*December.*
Days	♑	♒	♓	♈	♉	♊	♋	♌	♍	♎	♏	♐
1	22	23	21	22	21	21	19	19	19	18	19	20
2	23	24	22	23	22	22	20	20	20	19	20	21
3	24	25	23	24	23	23	21	21	21	20	21	22
4	25	26	24	25	24	24	22	22	22	21	22	23
5	26	27	25	26	25	25	23	23	23	22	23	24
6	27	28	26	27	26	26	24	24	24	23	24	25
7	28	29	27	28	27	27	25	25	25	24	25	26
8	29	♓	28	29	28	28	26	26	26	25	26	27
9	♒	1	29	♉	29	29	27	27	27	26	27	28
10	1	2	♈	1	♊	♋	28	28	28	27	28	29
11	2	3	1	2	1	1	29	29	29	28	29	♑
12	3	4	2	3	2	2	♌	♍	♎	29	♐	1
13	4	5	3	4	3	3	1	1	1	♏	1	2
14	5	6	4	5	4	4	2	2	2	1	2	3
15	6	7	5	6	5	5	3	3	3	2	3	4
16	7	8	6	7	6	6	4	3	4	3	4	5
17	8	9	7	8	7	7	5	4	5	4	6	6
18	9	10	8	9	8	7	6	5	6	5	7	7
19	10	11	9	10	9	8	7	6	7	6	8	8
20	11	12	10	11	10	9	8	7	8	7	9	9
21	12	13	11	12	11	10	8	8	8	8	10	10
22	13	14	12	13	12	11	9	9	9	9	11	11
23	14	15	13	13	13	12	10	10	10	10	12	12
24	15	16	14	14	14	13	11	11	11	11	13	13
25	16	17	15	15	15	14	12	12	12	12	14	14
26	17	18	16	16	16	15	13	13	13	13	15	15
27	18	19	17	17	17	16	14	14	14	14	16	16
28	19	20	18	18	18	17	15	15	15	15	17	17
29	20		19	19	19	18	16	16	16	16	18	18
30	21		20	20	20	19	17	17	17	17	19	19
31	22		21		21		18	18		18		20

The Uſe of this Table.

The Uſe of this *Table* is very eaſie, for if you find the Month in the Head of the *Table*, and the Day of the Month in the firſt Column, towards the Left-hand, in the common Angle of Meeting you have the Sign and Degree in which the *Sun* is that Day at Noon.

Example. On the 4*th.* of *February*, look for *February* at the Head of the *Table*, and 4 in the firſt Column; then under *February*, and againſt 4 in the firſt Column, you ſhall find 26, and the Character of ♓ *Piſces* at the Head, which ſhews the *Sun* is in 26 deg. of *Piſces* on the 4*th.* of *February* at Noon.— Alſo, upon the 1[illegible]*th.* of *October* the *Sun* will be found to be in 4 degrees of ♏ *Scorpio*, &c.

TAB.

TABLE V.

A TABLE shewing the Days, and Hour of the Day, upon which the Moon Changes in every Month, till the Year 1702; and may indifferently serve for the Purposes it is in this place intended, till the Year 1721.

Years of *Christ.*	*January*		*February*		*March*		*April*		*May*		*June*		*July*		*August*		*September*		*October*		*November*		*December*		Golden Num.
x	D.	H.	D.	H.	D.	H.	D.	H.	D.	H.	D.	H.	H.	D.	D.	H.	D.	H.	D.	H.	D.	H	D.	H.	
1682	28	4	26	21	28	21	26	22	26	7	24	14	23	22	22	4	20	14	20	2	18	16	18	8	11
1683	17	2	15	22	17	17	16	6	15	19	14	5	13	14	11	21	10	6	9	15	8	1	7	14	12
1684	6	5	4	23	5	15	4	8	4	1	2	14	2	3	$\frac{1}{29}$	$\frac{17}{22}$	28	7	27	17	26	2	25	15	13
1685	24	4	22	18	24	9	23	1	22	18	21	8	20	22	19	10	17	22	17	9	15	18	15	6	14
1686	13	19	12	3	13	15	12	4	11	19	10	10	10	1	8	15	7	6	6	20	5	10	4	21	15
1687	3	9	1	18	3	4	1	14	$\frac{1}{1}$	$\frac{0}{12}$	29	2	28	17	27	8	26	1	25	18	24	10	23	23	16
1688	22	10	20	20	21	6	19	14	18	22	17	8	16	20	15	10	14	1	13	20	12	13	12	7	17
1689	10	22	9	11	10	21	9	7	8	14	6	20	6	6	4	15	3	4	2	20	1	13	$\frac{1}{31}$	$\frac{8}{0}$	18
1690	29	20	28	0	29	22	28	7	27	15	25	20	25	5	23	13	22	1	21	15	20	7	20	2	19
1691	18	23	17	17	19	8	17	20	17	7	15	15	14	21	13	6	11	13	10	23	9	13	9	4	1
1692	7	21	6	17	7	11	6	4	5	18	4	4	3	13	$\frac{1}{31}$	$\frac{21}{0}$	29	14	29	0	27	11	27	2	2
1693	25	18	24	10	26	4	24	22	24	11	23	1	22	11	20	21	19	7	18	16	17	2	16	12	3
1694	15	1	13	14	15	6	13	22	13	14	12	4	11	19	10	8	8	20	8	8	7	17	6	5	4
1695	4	14	3	2	4	13	3	1	2	16	1	5	$\frac{1}{30}$	$\frac{0}{12}$	29	3	27	18	27	7	25	18	25	8	5
1696	23	16	22	3	22	12	20	23	20	10	18	23	18	13	17	4	15	21	15	14	14	6	13	20	6
1697	12	10	10	18	12	5	10	13	9	21	8	7	7	17	6	6	4	22	4	16	3	9	3	3	7
1698	$\frac{1}{31}$	$\frac{21}{0}$	4	0	$\frac{1}{31}$	$\frac{1}{0}$	29	13	28	20	27	5	26	12	25	1	23	16	23	9	22	3	21	23	8
1699	20	18	19	8	20	20	19	7	18	14	16	20	16	5	14	13	12	23	12	12	11	4	10	22	9
1700	9	18	8	13	9	6	7	17	7	6	5	14	4	21	3	5	1	13	$\frac{1}{30}$	$\frac{0}{12}$	29	0	28	18	10
1701	27	13	26	7	28	10	26	14	26	3	24	13	23	21	22	6	20	14	19	23	18	10	17	23	11
Differ. in 19 years.	0	15	0	14	0	11	0	8	0	4	0	1	0	1	0	2	0	0	0	3	0	6	0	5	

The Use of this TABLE.

IF you would know on what Day of the Month the Moon Changeth in any Year, look in the Table for the Year of our Lord in the first Column towards the Left-hand, and for the Month in the Head of the Table, and against the Year, and under the Month, you shall have your desire.

Example. I would know upon what Day of *March*, in the Year 1691, it will be New Moon. Look for 1691 in the first Column, and along that Line, under *March*, you shall find 19 Days 8 Hours, which shews, that the Moon will change upon the 19th Day of *March* at 8 a Clock at night, in the Year 1691.——— Again, I desire to know when it will be New Moon in the Month of *September*, in the Year 1698. Find the Year 1698 in the first Column of the Table, and in the same Line, under *September*, you shall find 23 16, which shews that the Moon will change upon the 23 Day, at 16 Hours after 12 at Noon; that is, the 24 Day at 4 in the Morning: And so of any other. For if the Hours be under 12, it is that Day at such an Hour in the Afternoon, or at Night: If the Hours be just 12, the Moon changeth at Midnight; if the Hours be above 12, take 12 from them, and the Remainder will be the time of the Moon's changing the next Day in the Morning, as in the last *Example*. And in this manner you shall find, that the Moon will change in the Month of *June*,

in

in the Year 1699, upon the 16th Day, and 20 Hours, from which take 12, and there remains 8 Hours; ſo that the Moon changeth in the Month of *June*, 1699. upon the 17th Day, at 8 in the Forenoon or Morning; and ſo of any other Year or Month in the Table.

This Table only ſhews you the Day and Hour on which the Moon changeth in any Month in any Year; but by it you may find the Day and Hour when it will be Full Moon, or when it enters into any of the Quarters, by help of this Little Table.

To the Time of the New Moon add		D.	H.
For the	First Quart.	7	9
	Full Moon	14	18
	Laſt Quart.	22	4
From Moon to Moon		29	12

So in the First Example, where in the Year 1691, in the Month of *March*, the Moon changeth upon the 19th Day, at 8 at Night; then 7 Days 9 Hours added thereto, makes 26 Days 17 Hours, which is the 27th Day, at 5 in the Morning; and then the Moon enters into the First Quarter in the Month of *March* 1691.—— Alſo, in the Year 1698, in the Month of *March*, the Moon changes on the Firſt Day at 8 at Night, to which if you add 7 Days 9 Hours, it makes 8 Days 17 Hours; that is, the 9th Day, 5 in the Morning, at which time the Moon enters into the Firſt Quarter: But if you add to 1 Day 8 Hours, 14 Days 18 Hours, it will make 15 Days 26 Hours; that is, the 16th Day at 2 in the Morning, at which time the Moon will be at Full: And if you add to 1 Day 8 Hours, 22 Days 4 Hours, it will make 23 Days 12 Hours, which is the 24th Day at Midnight, at which time the Moon will enter into the Laſt Quarter. And laſtly, If to 1 Day 8 Hours, you add 29 Days 12 Hours, the Summ will be 31 Days 21 Hours, at which time the Moon will change again that ſame Month of *March*, in the Year 1698.

SECT. IV. *To find the Age of the Moon.*

TO the Day of the Month in which you deſire to know the Moon's Age, add the *Epact* for that Year, (if the time propoſed be after the firſt of *March*; or the *Epact* for the Year before, if the time be in *January* or *February*,) and ſo many Days more as there are Months from *March* to the Month propoſed, (including both Months:) Which three Numbers added together, gives the Age of the Moon, if it be under 30; but if the Summ be above 30, caſt away 30, and the Remainder is the Moon's Age.—— This Rule is true when the Month hath 31 Days in it; but if the Month have but 30 Days in it, then from the Summ caſt away but 29, and the Remainder will be the Moon's Age.

Example 1. *I deſire to know the Age of the Moon upon the Ninth of* May *in the Year* 1691.

1691	The *Epact* for that Year is	11
	The Days of the Month *May*	9
	The Months from *March* to *May*	3
	The Moon is old	23 Days.

Example 2. *Let the Moon's Age be required upon the Eighteenth Day of* June *in the Year* 1700.

1700	The *Epact* for that Year is	20
	The Days of the Month *June* are	18
	The Month from *March* to *June* inclusive	4
	The Summ is	42
	From which ſubtract 29, (becauſe *June* hath hath but 30 days,)	29
	The Remainder 13 is the Moons Age	13 Days.

The Mon's Age may alſo be found by *Table* V. For if you look into that Table upon what Day the Moon changed next before the time propoſed, count how many Days

Days are contained between the Day of the Change and the Day propoſed, and thoſe are the Days of the Moons Age.

By the knowledge of the Moon's Age many neceſſary Concluſions, uſeful in *Navigation*, may be performed; as ſuch as follow.

SECT. V. *To find at what Hour the Moon will be South at any time.*

MUltiply the Moon's Age by 4, and divide the Product by 5, the Quotient will give you the Hours, and the Remainder multiplied by 12, the Minutes. This is when the Moon is leſs than 15 Days old; if ſhe be above 15 Days old, ſubtract 15 therefrom, and work with the Remainder, as before.

Example. *At what time upon the Ninth of* May, 1691. *will the Moon be upon the Meridian?*

Upon that Day the Moon is 23 Days old, from which take 15, the Remainder is 8, which multiplied by 4, makes 32, which divided by 5, the Quotient is 6 Hours, and the Remainder 2 multiplied by 12, gives 24 Minutes; ſo that the Moon will be South at 24 min. after 6, upon the Ninth of *May*, 1691.

Note, That if the Moon be leſs than 15 Days old, ſhe cometh to the Meridian before 12 at Night; if juſt 15 Days old, juſt at 12 at Night; if above 15 Days old, after 12 at Night.

SECT. VI. *To know how many Signs and Degrees the Moon is departed from the Sun ſince her laſt Conjunction with him.*

MUltiply the Days of the Moon's Age by 2, and the Product divide by 5, the Quotient ſhall be the Number of Signs; and the Remainder (if any be) every Unite is 6 Degrees.

Example. *Upon the Ninth of* May, 1691, *I deſire to know how many Signs the Moon is departed from the Sun ſince the laſt Conjunction or New Moon.*

Upon the Ninth of *May*, by the former Directions, I find the Moon is then 23 Days old, which multiplied by 2, (or doubled,) is 46; this divided by 5, gives in the Quotient 9 Signs and 1 remaining, which is 6 Degrees; ſo that the Moon is departed from the Sun ſince her laſt Conjunction with him 9 Signs and 6 Degrees.

SECT. VII. *To find the Time of the Moon's Shining.*

MUltiply the Moon's Age by 48, and divide the Product by 60, the Quotient ſhall give you the Hours, and the Remainder the Minutes at the Time of the Moon's Shining.——This is when the Moon is leſs than 15 Days old: If ſhe above 15 Days old, ſubtract the time of her Shining found, as before, from 24 Hours, and the Remainder will be the time of her Shining in the Morning.

Example 1. *How long will the Moon ſhine upon the Eighteenth of* June, 1670?

The Moon that Day is 13 Days old; that multipled by 48, the Product is 624, which divided by 60, the Quotient gives 10 Hours, and the Remainder 24 min. for the time of the Moon's Shining at Night.

Example 2. *How long will the Moon ſhine upon the Ninth of* May, 1691?

The Moon that Day is 23 Days old, which multiplied by 48, the Product is 1004, and that divided by 60, gives in the Quotient 18 Hours, and 24 min. remaining; which ſubtracted from 24 Hours, there remains 5 Hours 36 min. for the time of the Moon's Shining in the Morning.

TABLE

TABLE VI.

A TABLE *shewing at any Day of the Moon's Age, at what time the Moon cometh to the South:—— The time of her Shining:—— Her Distance from the Sun:— And time of High-water at* London-Bridge.

		Time of the Moon's coming to South.		*The time of the Moon's Shining.*		*The Dist. of the Moon from the Sun.*		*High-water at* Lond. *Bridg.*	
		H.	M.	H.	M.	Sig.	Deg.	H.	M.
From	0	12	0	0	0	☌ 0		3	0
	1	12	48	0	48	0	12	3	48
	2	1	36	1	36	0	24	4	36
In the	3	2	24	2	24	1	6	5	24
	4	3	12	3	12	1	18	6	12
	5	4	0	4	0	2 ✶	0	7	0
Change to	6	4	48	4	48	2	12	7	48
	7	5	36	5	36	2	24	8	36
After	8	6	24	6	24	3	6	9	24
	9	7	12	7	12	3 □	18	10	12
Age.	10	8	0	8	0	4	0	11	0
Full.	11	8	48	8	48	4	12	11	48
	12	9	36	9	36	4	24	12	36
noon.	13	10	24	10	24	5	6	1	24
	14	11	12	11	12	5	18	2	12
Moon's	✶ 15	12 ✶	0	12 ✶	0	6 ♂	0	3	0
	16	12	48	11	48	6	12	3	48
	17	1	36	10	36	6	24	4	36
	18	2	24	9	24	7	6	5	24
In the	19	3	12	8	12	7	18	6	12
From	20	4	0	8	0	8	0	7	0
	21	4	48	7	48	8	12	7	48
Morn	22	5	36	6	36	8 □	24	8	36
	23	6	24	5	24	9	6	9	24
	24	7	8	4	12	9	18	10	12
of the	25	8	0	4	0	10	0	11	0
	26	8	48	3	48	10	12	11	48
	27	9	36	2	36	10	24	12	36
ing.	28	10	24	1	24	11	6	1	24
	29	11	12	12	12	11	18	2	12
Days	30	12	0	12	0	12 ☌	0	3	0

The Use of this Table.

LOok in the first Column of the Table for the Moon's Age for the Time proposed; and against it, in the Second, Third, Fourth, and Fifth Columns, you shall find such things as are expressed at the top of each Column.

Example. *Upon the Ninth of* May, 1691, *I desire to know the time of the Moon's coming to the South,* &c.

Upon the 9*th.* of *May*, the *Moon* is 23 days old, look for 23 in the firſt Column, and right againſt it in the Second you ſhall find 6 hou. 24 min. at which time the *Moon* will be *South* in the Morning: — In the Third, 5 hou. 24 min. and ſo long will the *Moon* ſhine in the Morning. — In the Fourth, 9 Signs 6 Degrees; which ſhews that the *Moon* is gone ſo many Signs and Degrees from the *Sun*, ſince her laſt Conjunction with him. — In the Fifth, 9 hou. 24 min. at which time it will be High-Water at *London-Bridg*.

Again,

When the *Moon* is 7 days Old ſhe
- will be *South* at 5 h. 36 m. in the Afternoon.
- will ſhine 5 h. 36 m. in the Evening.
- will be 2 Signs 24 Degrees diſtant from the *Sun*.
- will make Full Sea, or High-Water at *London-Bridg*, at 8 hours 36 min.

SECT. VIII. *To find the Sign and Degree in which the* Moon *is.*

TABLE VII.

You are to note that the XII *Signs are thus named, charactred, and numbered.*

Names.	Characters.	Number.
Aries	♈	0
Taurus	♉	1
Gemini	♊	2
Cancer	♋	3
Leo	♌	4
Virgo	♍	5
Libra	♎	6
Scorpio	♏	7
Sagittarius	♐	8
Capricorn	♑	9
Aquarius	♒	10
Piſces	♓	11

Note alſo, That at, or upon the Day of the Change, at that Hour the *Sun* and the *Moon* are both in the ſame Sign and Degree.

Theſe things premiſed, you may proceed to find the Place of the *Moon* in the *Zodiack* at any time, in this manner.

A Table *of the* Moon's *Motion in Signs, Degrees, and Minutes, for every Day and Hour of her Age.*

The Days of the *Moon's* Age	S.	D.	M.	The Hours of the *Moon's* Age	S.	D.	M.
1	0	13	11	1	0	0	33
2	0	26	21	2	0	1	6
3	1	9	32	3	0	1	39
4	1	22	42	4	0	2	12
5	2	5	53	5	0	2	45
6	2	19	3	6	0	3	18
7	3	2	14	7	0	3	51
8	3	15	25	8	0	4	24
9	3	28	35	9	0	4	56
10	4	11	46	10	0	5	29
11	4	24	56	11	0	6	2
12	5	8	7	12	0	6	35
13	5	21	18	13	0	7	8
14	6	4	28	14	0	7	41
15	6	17	39	15	0	8	14
16	7	0	49	16	0	8	47
17	7	14	0	17	0	9	20
18	7	27	11	18	0	9	53
19	8	10	22	19	0	10	26
20	8	23	32	20	0	10	59
21	9	0	42	21	0	11	32
22	9	19	53	22	0	12	5
23	10	3	3	23	0	12	38
24	10	16	14	24	0	13	11
25	10	29	25				
26	11	12	35				
27	11	29	46				
28	0	8	56				
29	0	22	27				

Example.

Example. Let it be required to know in what Sign and Degree of the Zodiack *the* Moon *will be upon the 31ſt. Day of* May, *in the Year* 1691.

Firſt look into *Table* V, and ſee there what Day in *May* the *Moon* changed upon, which you will find to be upon tht 17*th* day at 7 at Night; on which day at Noon, by *Table* IV, you will find the *Sun* to be 7 deg. of *Gemini*, and in that *Sign* alſo is the *Moon* at that time; which ſet down as you ſee done in the *Example* following: Then between the 17*th*. Day 7 Hours, to the 31 Day at Noon, are contained 13 Days 17 Hours: Then out of this *Table* VII, take the *Moon's* Mean Motions anſwering to 13 Days and 17 Hours: Theſe three Numbers added together, will give the Signs and Degrees in which the *Moon* is.

The Operation.

	S.	D.	M.
1691. *Sun* and *Moon* in 7 deg of ♊	2	7	00
Moon's Mean Motion for 13 days	5	21	18
Moon's Motion for 17 hours is	0	9	20
The *Moon's* Place	8	7	38

That is, in 7 deg. 38 min. of ♐ *Sagittarius.*

Another Example *of this for the* Moon's *Place upon the* 20*th of* April 1700.

	S.	D.	M.
The *Moon* Changes in *April* the 7*th*. day, at 17 hours; at which time the *Sun* is in 29 deg. 29 min. of *Aries*: That is	0	29	29
The Difference between 7 days 17 hours and 20 days, is 12 days 7 hours			
The *Moon's* Mean Motion in 12 days	5	08	67
The Mean Motion in 7 hours	0	3	51
The *Moon's* Place	6	11	27

That is, in 11 deg. 27 min. of *Libra.*

But here Note, That if the Number of *Signs* do exceed 12, you muſt ſubtract 12 *Signs*, and the Remainder is the *Sign* in which the *Moon* is.

As in this *Example*, where the *Moon's* Age is 22 days 16 hours, and the *Sun's* Place at the preceding Change in 20 deg. 12 min. ♒.

	S.	D.	M.
Place of the *Sun* on the Change day	10	20	12
The *Moon's* Mean Motion in 22 days	9	19	53
In 16 hours	0	08	47
The Summ	20	18	52
Subtract	12	00	00
Moon's Place	8	18	52

That is, in ♏ 28 deg. 52 min. *Scorpio.*

CHAP. V.

Concerning the Tides.

THE true knowledge of the *Tides* in all places is a matter of great moment; yet nothing more neglected, or ſlightly paſſed over than that is: For the uſual way for finding of the *Tides* is not ſufficiently exact: For there is one onely General Rule uſed, which is true only under the *Poles*, where the *Equinoctial* and *Horizon* are the ſame: And it alſo ſuppoſes, that the departure of the *Moon* from the *Sun* were at all times equal: In both which reſpects the Rule is deficient.

For in the *Latitude* of *London* 51 deg. 30 min. where a *S.W.* or *N. E. Moon* makes a full Sea, and that is at 3 of the Clock; and this is true when the *Moon* is in *Aries* or *Libra*, (or near thereto, and hath no *Latitude*;) but, in the ſame place, the *Moon* being in *Cancer*, and having 5 deg. of *North Latitude*, it will be 30 min. paſt 10 of the Clock before the *Moon* will be *N. E*; which ſhould be at 9 a Clock, by the

vulgar Rule, too late by 1 hour 30 min. And it will be *S. W.* at 30 min. after 1, which is 1 hour 30 min. too ſoon. And if it be in leſſer *Latitude*, as in 20, 30, or 40 deg. the Error will be ſtill greater.

Now to correct this Error, the beſt way is to reckon the time of high Water by the time of the day, and not by the *Moon's* being upon ſuch a Point of the *Compaſs*: For the *Table* following ſhews, that at the New or Full a *S. W. Moon* makes a Full Sea at *London-Bridge*, and that is at 3 of the Clock: It is true, that upon the days of the Full or Change, it is High-Water at *London-Bridg* at 3 of the Clock: But if you ſhould ſet your *Compaſs* at 3 of the Clock, you ſhall not find the *Moon* to be always *S. W.*

As, take a New *Moon* in *June*, and a Full *Moon* in *December*, the *Moon* being then about the *Tropick* of *Cancer*, and you ſhall find the *Moon* (at *London*) to be *S. W.* at 3 quarters of an hour paſt One a Clock, and then the *Moon* will be *S S W*, which is two Points farther.

It is to be obſerved alſo, That there is a conſiderable difference between the *Spring* and *Neap-Tides*, in reſpect of Time: for the *Neap-Tides* will be ſometimes an hour or more ſooner than the the uſual *Tables* and *Inſtruments* ſhew them to be.

So the *Moon* being in the firſt Quarter, the foregoing *Table* VI ſhews the time of High-Water at *London* to be about 9 of the Clock; but if you obſerve the time of the *Tide* there, you ſhall find it to be High-Water before 8 of the Clock: And doubtleſs the like, or (it may be) greater difference is in other Places alſo: And therefore to know the true time of the *Tides*, make uſe of this little *Table* inſtead of that in the laſt Column of *Table* VI; wherein ſome Minutes are ſubtracted from the Time there found, which will render this *Table* far more exact: it being done by the long Obſervation and Experience of my loving Friend Mr. *Henry Phillips*, deceaſed, who lived for many Years upon the ſame *Bridg*, and made many Experiments about this matter.

TABLE VIII.

A Table, *ſhewing the time of High-Water at* London-Bridg, *corrected.*

Days of the *Moon's* Age.		High-Water at *London-Bridg.* H.	M.
1	16	3	48
2	17	4	31
3	18	5	14
4	19	5	52
5	20	6	30
6	21	7	3
7	22	7	36
8	23	8	24
9	24	9	27
10	25	10	30
11	26	11	28
12	27	12	26
13	28	1	19
14	29	2	12
15	30	3	0

A Tide-

A Tide-Table, *shewing upon what Point of the* Compass *the* Moon *being, and also at what Hour of the Day it will be Full Sea, upon the Full or Change-Day, in any of these Ports.*

Points of the *Compass* making full Sea.	Names of Places.	Time of Day. H.	M.
S *by* E	*Rye, Wilchelsey, Calshot.*	11	15
S *by* W	*Rochester, Maldon, Black-tail.*	0	45
S S E	*Yarmouth, Dover, Harwich.*	10	30
S S W	*Gravesend, Downs, Blackness, Silly.*	1	30
S E *by* S	*Needles, Orford, S. and N. Forelands.*	9	45
S W *by* S	*Dundee, S. Andrews, Lisbon, S. Lucas.*	2	15
S E	*Pool, Isle of Man, Dunbar, Diep.*	9	0
S W	*London, Tinmouth, Amsterdam, Rotterdam.*	3	0
S *by* W	*Portland, Hartflew, Dublin.*	8	15
S W *by* W	*Barwick, Flushing, Humborough.*	3	45
E S E	*Milford, Bridgwater, Land's-End.*	7	30
W S W	*Baltimore, Cork, Seavern, Calice.*	4	30
E *by* S	*Bristol, Start-Point, Waterford.*	6	45
W *by* S	*Falmouth, Humber, New-Castle.*	5	15
W *and* E	*Plimouth, Hull, Lyn, S. Davids.*	6	0
N *and* S	*Queenborough, Southampton, Portsmouth.*	12	0

The Sea hath fits, Alternate Course she keeps,
From Deep to Shoar, and from the Shoar to Deeps:
Whether it were, that at the first the Ocean
From God's own Hand receiv'd this double Motion;
Or, whether Cynthia, *that with Changeful Laws*
Commands moist Bodies, doth this Motion cause.

The End of the first Part.

OF NAVIGATION.

PART II.

OF Plain-Sailing.

WHEREIN

The DOCTRINE of RIGHT-LINED TRIANGLES is applied to PRACTICE, in the Solution of fundry *Nautical Problems* and *Queſtions* in *Plain-Sailing*.

All which are Performed

I. By *Protraction*, with *Scale* and *Compaſſes*.
II. By *Trigonometrical Calculation*, either by *Tables* or *Proportional Scales*.
III. Wrought upon the *Chart* it ſelf.

THE *Nautical Problems* which I intend in this Treatiſe the Solution of, both by *Protraction* and *Trigonometrical Calculation*, ſhall be ſuch as principally concern *Longitude*, *Latitude*, *Rumb*, (or *Courſe*,) and *Diſtance*. Now,

LONGITUDE is the *Diſtance* of a Place from ſome known *Meridian*, to that Place; and is always counted upon the *Equinoctial*, from the *North* or *South* towards the *Eaſt* or *Weſt*.

LATITUDE is the *Diſtance* of any Place from the *Equinoctial*, counted upon that *Meridian Circle* which paſſeth over that Place towards either of the *Poles*, either *North* or *South*; and is ſo denominated, *North* or *South Latitude*.

RUMB, (or *Courſe*,) is the *Angle* that a Ship in her Sailing makes with the *Meridian* of the Place in which the Ship is: But the *Complement* of the *Rumb* is the *Angle* that the Ship makes with any *Parallel*.

DISTANCE is the Number of *Leagues*, *Miles*, or *Centeſms*, *&c.* that a Ship hath ſailed upon any one *Rumb* or *Courſe*.

In Sailing,

If your *Rumb* or *Courſe* be directly *Eaſt* or *Weſt*, you alter not your *Latitude* at all. And,

If your *Courſe* be from the *Equinoctial Northward*, you continually raiſe the *North-Pole*; or if directly *South*, the *South-Pole*.

The *Raiſing* of the *Pole* is, when you ſail from a *Leſſer* to a *Greater Latitude*.

Depreſſing of the *Pole* is, when you ſail from a *Greater Latitude* to a *Leſſer*.

For the more eaſie underſtanding and reſolving the following *Problems*, it will be neceſſary to know that the *Rumb* or *Courſe* that a Ship ſteereth is diſcovered (in the general) by the *Magnetical Needle*, which always reſpects the *North*, and (though not directly, yet) its Variation being often obſerved, and the *Card* or *Compaſs* rectified thereby, is the beſt (if not only) Help that *Navigators* yet have to ſteer their Courſe by.

PROBLEMS

PROBLEMS in PLAIN-SAILING.

THE *Triangle* which I shall make Use of in the Solution of the Six following *Problems*, shall be the *Triangle* A B C, whose Sides and Angles are as followeth: *Fig.* I.

The Side	A B	81.2	Leagues
	B C	97.5	
	A C	45.3	

		deg.	*min.*		
The Angle	A	90	00	0	Points from the Meridian.
	B	33	45	3	
	C	56	15	5	

And forasmuch as both in *Plain* and *Mercator*'s Sailing there will be continnal Use of Laying down and finding the Quantities of *Lines* and *Angles* already laid down, and how that is to be performed, is supposed already known; but for such as do not, I would have them have recourse to the Three first Sections of the Fifth Book beforegoing, where they shall find ample Satisfaction in that Particular; and so I shall omit to say any thing thereof in this Place, but immediately proceed to Practice.

PROBL. I. *Sailing* 97.5 *Leagues upon the Fifth* Rumb *from the* Meridian, *how much shall I have altered my* Latitude?

I. By *Protraction.*

DRaw a Line at pleasure, as W E, and upon any Point therein, as B, protract an Angle of III Points, (the Complement of V Points, the *Rumb* given,) as the Angle C B A, and upon that Line, by your *Diagonal Scale*, prick down 97.5, (the Leagues sailed from B to C.) Then from the Point C let fall a Perpendicular upon the Line W E, which will fall in A: Then measure the Line C A upon the same *Diagonal Scale*, and you shall find it to contain 54.3 Leagues; and so much have you altered your *Latitude.* *Fig.* I.

II. By *Trigonometry, Case* III.

As the *Radius*, 90 deg.	10.
Is to the Leagues sailed, C B, 97.5	1.98900
So is the Co-sine of the *Rumb* B, 33 d. 45 m.	9.74474
To the *Difference* of *Latitude*, C A, 54.3 Leagues	*r*1.73374

II. *If I sail* 97.5 *Leagues upon the Fifth* Rumb *from the* Meridian, *how far am I departed from the* Meridian *from whence I came?*

I. By *Protraction.*

DRaw a Line at pleasure, as N S, and upon any Point thereof, as C, protract an Angle of Five Points or *Rumbs*, or 56 deg. 15 min. as *a* C *b*, upon which set the *Distance* sailed, 97.5 Leagues from C to B: Then from the Point B let fall a Perpendicular (or take the neareſt *Distance*) to the Line N S, as B A, which measured upon your *Scale*, will be found to contain 81.2 Leagues; and so far are you departed from the *Meridian* from whence you came.

II. By *Trigonometry, Case* III.

As the *Radius*, 90 deg.	10.
Is to C B, the *Distance* sailed, 97.5 Leagues	1.98900
So is the Sine of the *Rumb*, 56.15 m.	9.91985
To the *Departure* B A, 81.2 Leagues	*r*1.90885

III. *If*

III. *If I ſail upon the Fifth* Rumb *till I have altered my* Latitude 54.3 *Leagues, how many Leagues have I ſailed?*

I. By *Protraction.*

FIrſt, draw a line at pleaſure, as W E, and upon any Point thereof, as A, erect a Perpendicular, as A N; upon which Line ſet 54.3, (the Alteration of *Latitude* from A to C:) Then upon the Point C protract an Angle of Five *Rumbs*, or 56 d. 15 m. as *a* C *b*, and draw the Line G *b*, extending it till it cut the Line W E in B; ſo ſhall the Line C B be your *Diſtance* ſailed, which meaſured upon your Scale, will be found to be 97.5 Leagues.

I. By *Trigonometry, Caſe* II.

As the Co-ſine of the Fifth *Rumb*, 33 d. 45 m.	9.74474
Is to C A, the *Difference* of *Latitude*, 54.3 Leagues	1.73480
So is the *Radius*, 90 deg.	10.
To the *Diſtance* ſailed, C B, 97.5 Leagues	1.99006

IV. *If I ſail upon the Fifth* Rumb *till I have altered my* Latitude 54.3 *Leagues, how much am I departed from the* Meridian *from whence I came?*

I. By *Protraction.*

DRaw a Right Line at pleaſure, as W E, and upon any Point therein, (as at A,) erect the Perpendicular A N, and upon it ſet 54.3 Leagues from A to C: Then upon C protract an Angle of Five *Rumbs*, (or 56 deg. 15 min.) as *a* C *b*, and draw the Line C *b*, extending it till it cut the Line W E in B; ſo ſhall the Line A B, meaſured upon your Scale, be 81.2 Leagues; and ſo far are you departed from your firſt *Meridian.*

II. By *Trigonometry, Caſe* IV.

As the *Radius*, 90 deg.	10.
Is to the *Difference* of *Latitude*, C A, 54.3 Leagues	1.73480
So is the *Tangent* of the Fifth *Rumb*, 56 d. 15 m.	10.17511
To the *Departure* A B, 81.2 Leagues	1.90991

V. *Sailing upon ſome* Rumb *between the South and Weſt, 97.5 Leagues, and then finding that I have altered my* Latitude 54.3 *Leagues, I would know upon what* Rumb *I have ſteered.*

I. By *Protraction.*

DRaw a Right Line at pleaſure, as W E, and upon any Point thereof, as A, erect the Perpendicular A N, and upon it ſet 54.3 Leagues, the Alteration of *Latitude*, from A to C: Then take in your Compaſſes, from your Scale, 97.5, the Leagues ſailed, and with that *Diſtance*, ſetting one Foot in C, with the other deſcribe the ſmall Arch *c d*, cutting the Line W E in B. This done, by your *Scale* of *Chords* or *Protractor* ſeek the Quantity of the *Angle* A C B, which you ſhall find to contain 56 d. 15 m. or Five Points from the *Meridian*, *viz.* S W by W; and upon that *Rumb* have you ſteered.

II. By *Trigonometry, Caſe* V.

As the *Diſtance* ſailed, C B, 97.5 Leagues	1.98900
Is to the *Difference* of *Latitude*, C A, 54.3 Leagues	11.73480
So is the *Radius*	10.
To the Co-ſine of the *Rumb*, 33 d. 45 m.	9.74580

VI. *If*

VI. *If I Sail upon ſome* Rumb *between the South and the* Weſt 97.5 *Leagues, and then finding that I have parted from my firſt* Meridian 81.2 *Leagues; I demand how much I have alter'd my* Latitude.

I. By *Protraction.*

FIrſt draw a Right Line W E, and upon any Point therein, as at A, erect a Perpendicular A N: Then take 81.2 Leagues, your departure from your firſt *Meridian*, and ſet them from A to B: Then, for that you have ſailed 97.5 Leagues, take 97.5 out of your Scale, and ſetting one Foot in B, with the other deſcribe the ſmall Arch *e f* croſſing the Perpendicular A N in C. Laſtly, draw the Line C B, ſo ſhall you include two Angles, *viz.* A C B, containing Five *Rumbs*, or 56 deg. 15 min. and the other A B C three *Rumbs*, or 33 deg. 45 min. and the Line C A, meaſured upon your Scale, will be found to be 54.3 Leagues; and ſo much have you alter'd your *Latitude.*

II. By *Trigonometry, Caſe* VII.

(1.) As the Diſtance ſailed C B, 97.5 Leagues,	1.98900
Is to your Departure A B, 81.2 Leagues,	11.90955
So is the *Radius*, 90 deg.	10.
To the Sine of the Angle at C, 56 deg. 15 min.	9.92055
(2.) As *Radius*, 90 deg.	10.
Is to B C the Diſtance ſailed, 97.5 Leagues,	1.98900
So is the Sine of the Angle at B, 33 deg. 45 min.	9.74474
To the alteration of *Latitude* C A, 54.3 Leagues,	11.73374

Theſe *Problems* might be enlarged, but theſe are ſufficient in this Place, and therefore I will proceed to the Protracting and Calculating of a *Traverſe.*

Of a TRAVERSE.

A *Traverſe* is the variation or alteration of the Ship's Courſe or Way, upon the ſhifting of the Wind, or other interruptions of Rocks, Sands, or the like.

VII. *A Ship ſails from the* North *towards the* Weſt, *upon theſe following Points of the Compaſs,* viz.

	Leagues.	Points.
1. South Eaſt *by* Eaſt, half a Point Eaſterly	5	V½
2. Eaſt South Eaſt	4	VI
3. Eaſt *by* South	7	VII
4. Eaſt, half a Point Northerly	3	VII½
The Wind Changing ſhe ſteers		
5. South South Weſt	5	II
6. South *by* Weſt	6	I
7. South	4	o
8. South South Eaſt	7	II
9. South Eaſt *by* South	3	III

I demand, (1.) How much I am departed from my firſt *Meridian.* (2.) The Difference of *Latitude.* (3.) The Diſtance of the two Places upon the *Rumb.* (4.) The Bearing of the one Place from the other.

I. By *Protraction.*

Upon a Sheet of Paper or fine Paiſt-board, draw a Right-Line N S for your *Meridian*, and another at Right-Angles thereunto, as E W, for your Parallel: and let A, the *Fig.* II.

A, the Point of Intersection of these two Lines, represent the Place from whence you set Sail. Then,

1. The first Course being S E *by* E, $\frac{1}{2}$ a Point Easterly, that is $5\frac{1}{2}$ Points, or 61 deg. 52 min. from the *Meridian*, I Protract an Angle of that quantity, as the Angle *b* A *c*, and upon the Line A *b* I set my first distance sailed from A to B, five Leagues.

2. Through the Point B, draw a Line B *d*, parallel to the *Meridian* N S; and upon B, Protract an Angle of Six Points, or 67 deg. 30 min. *i. e.* E S E, as the Angle *e* B *d*, and draw the Line B *e*, setting thereupon 4 Leagues, your Second Distance sailed from to C.

3. Through the Point C, draw a Line C *f*, parallel to the *Meridian* N S; and upon C Protract an Angle of Seven Points, *viz.* E *by* S, or 78 deg. 45 min. as *g* C *f*, and draw the Line C *g*, setting thereupon your Third distance sailed, *viz.* 7 Leagues from C to D.

4. Through the Point D, draw a Line D *h*, parallel to the *Meridian* N S; and upon D Protract an Angle of $7\frac{1}{2}$ Points, (*i. e. East* $\frac{1}{2}$ a Point *Northerly*) your fourth Course, or 81 deg. 33 min. and draw the Line D *k*; upon which set 3 Leagues (your fourth distance sailed) from D to E.

The Wind now shifting *Northerly*.

5. Through the Point E, draw a Line E *l*, parallel to N S, and upon E Protract an Angle *l* E *m*, equal to your fifth Course S S W, or two Points, or 22 deg. 30 min. and draw the Line E *m*; upon which set 5 Leagues, your fifth distance sailed from E to F.

6. Through the Point F, draw the Line F *n*, parallel to the *Meridian* Line N S, and upon F Protract an Angle *n* F *o*, equal to your sixth Course S *by* W, one Point, or 11 deg. 15 min. and draw the Line F G, setting thereon your sixth distance sailed, *viz.* 6 Leagues from F to G.

7. Your seventh Course being directly *South* through G, draw a Line parallel to N S, and upon it set your seventh Course sailed, *viz.* 4 Leagues from G to H.

8. Through H draw a Line H *q*, parallel to N S, and upon H Protract an Angle *p* H *q* equal to your eight Course S S E, two Points, or 22 deg. 30 min. and draw the Line H *p*; upon which set your eighth distance sailed 7 Leagues from H to K.

9. Through K draw a Line K *s*, parallel to the *Meridian* N S, and upon the Point K Protract an Angle *v* K *s*, equal to your ninth Course S E *by* S, or three Points, or 33 deg. 45 min. and draw the Line K *v*; upon which set your ninth distance sailed from K to L, three Leagues; and so have you finished your *Traverse*.

The *Traverse* being thus finished, you may find,

1. The Departure from the first *Meridian* A P, to be 28.43 Leagues.
2. The Difference of *Latitude* P L, to be 18.62 Leagues.
3. The Distance of the two Places upon the *Rumb* A L, to be 34.00 Leagues.
4. The Bearing of the one Place from the other L A P, to be S E *by* S. *ferè*.

II. By *Trigonometry*.

(1.) In the First Course for the difference of *Latitude*.

As the *Radius*, Sine of 90 deg.	10.
Is to the distance sailed A B, 5.00 Leagues,	0.69897
So is the Co-Sine of the *Rumb b* A *c*, 61 deg. 52 min.	9.94539
To the difference of *Latitude* M A, 4.41.	10.64436

(2.) In the First Course for the departure.

As *Radius*, 90 deg.	10.
Is to the distance sailed A B, 5.00 Leagues,	0.69897
So is the Sine of the *Rumb b* M A B, 28 deg. 8 min.	9.67350
To the departure A O, 2.36 Leagues,	10.37247

And thus must you Work with all the other Courses and Distances, and collect them into a *Table*, as here following:

Courses

Conrſes	Numb. of Points.	Diſtance Run.	North.	South.	East.	Weſt.
S E *by* E ½ East	5½	5 . 03		2 36	4 41	
E S E	6	4 . 00		1 53	3 70	
E *by* S	7	7 . 00		1 37	6 86	
East ½ North	7½	3 . 00	0 29		2 98	
S S W	2	5 . 00		4 62		1 19
S *by* W	1	6 . 00		5 88		1 17
South	0	4 . 00		4 00		0 00
S S E	2	7 . 00		6 47	2 08	
S E *by* S	3	3 . 00		2 49	1 67	
The Summs			0 29	28 72	21 . 70	3 08
				0 29	3 08	
			Departure.	28 . 43	18 . 62	Dif. Lat.

This *Table* you ſee conſiſts of Seven Columns: the Firſt contains the Courſes or Points of the *Compaſs* you Steer upon.— The Second ſhews how many Points and Parts of a Point are contained between it and the *Meridian*.— The Third contains the Number of Leagues, Miles, *&c.* ſailed.— The Four other Columns contains the *Northing*, *Southing*, *Eaſting*, or *Weſting* of any Courſe.

Your *Table* being thus prepared, You ſee that in your *Traverſe*, the Firſt Courſe ſteered was S E *by* E ½ a Point *Eaſterly*; which I ſet down in the Firſt Column of the *Table*.— And becauſe it is 5½ Points from the *Meridian*, ſet 5½ in the Second Column.— And becauſe upon that Courſe, the diſtance run was 5.00 Leagues, I ſet them in the Third Column.— And then finding by my *Trigonometrical Calculation*, that my difference of *Latitude* was *Eaſterly* 4.41 Leagues, I ſet 4.41 in the Column under *Eaſt*: And my departure being 2.36 Leagues *Southward*, I ſet 2.36 in the Column under *South*.

Then for the Second Courſe, it being E S E (or 6 Points) and the diſtance ſailed 4.00 Leagues, I ſet them down in the Three firſt Columns: and for the difference of *Latitude* and departure, I go again to my *Trigonometrical* Work: Saying,

(1.) For the Difference of *Latitude*.

As *Radius*, 90 deg.	10.
Is to the Co-Sine of the *Rhumb* e B *d*, 67 deg. 30 min.	9.96562
So is the Diſtance ſailed B C, 4.00 Leagues,	0.60206
To the Difference of *Latitude*, 3.70 Leagues,	*r*0.56768

Which I ſet in the *Eaſt* Column: And ſay,

(2.) For the Departure.

As *Radius*, 90 deg.	10.
Is to the diſtance ſailed B C, 4.00 Leagues,	0.60206
So is the Sine of the *Rhumb*, 22 deg. 30 min.	9.58284
To the Departure, 1.53 Leagues,	*r*0.18490

Which muſt be ſet in the Column under *South*.

And proceeding in this manner with all your Courſes and Diſtances, collect them into a *Table*, and you ſhall find them as in the *Table* before is exhibited.

This done, add the Leagues in every Column together, and ſet their Summs at the bottom: Then compare the *North* and *South* Columns together, as 0.29 Leagues, and 28.72 Leagues, Subtract the leſſer from the greater, and the Remainder is the Departure, and is here 28.43 Leagues for A P.— Alſo compare the *Eaſt* and *Weſt* Columns together, and they are 21.70 Leagues, and 3.08 Leagues, the leſſer ſubtracted from the greater, leaves 18.62 Leagues for the difference of *Latitude*.

Then

Then for the *Rhumb* leading from the First Place to the Second, *viz.* from A to L, say,

By *Case* VI of *Plain Triangles.*

As A P (the Departure from the first *Meridian*,) 28.43 Leagues,	1.45377
Is to *Radius*, *Tangent* 45 deg.	10.
So is L P (the Difference of *Latitude*,) 18.62 Leagues,	11.26997
To the *Tangent* of the *Rhumb* L A P, 33 deg. 13 min. That is S E *by* S *fere.*	9.81620

And for the Distance A L,

As the Sine of the *Rhumb* L A B, 33 deg. 13 min.	9.73863
Is to the Difference of *Latitude* L P, 18.62 Leagues,	1.26997
So is *Radius* 90 deg.	10.
To the Distance upon the *Rhumb* A L, 34.00 Leagues,	1.53134

How to work a Traverse, *by help of the* Table *following, without* Trigonometrical *Calculation.*

VIII. *A Ship being in* 47 *deg.* 30 *min. of* North Latitude, *and counting her to be in* 00 *deg.* (*or the beginning*) *of* Longitude, *Sails*

	Leagues.	Points.
South East *by* South	16	III
South South West	13	II
East *by* North	18	VII
North *by* East, half a Point Easterly,	16	I½
North North west, half a Point Westerly,	15	II½
West North West	18	VI
South East *by* South	18	III
South South West	15	II
South West *by* South	12	III
West South West	18	VI
East South East	18	VI

And it is required to find the Latitude *the Ship is in, and the Departure from the* Meridian, *or Difference of* Longitude.

HAving prepared a blank *Table*, and divided it into Seven Columns, as in the former *Traverse*, and set down in the First Column the several Courses you have sailed upon; in the Second, the Distance you have sailed; in the Third, the Number of Points from the *Meridian*; and in the Four last, the *Easting*, *Westing*, *Northing*, and *Southing*, as you see here done.

Courses sailed.	Leagues	Points *à Mer.*	North.	South.	East.	West.
S E *by* S	16	III		13.30	08.89	
S S W	13	II		12.01		04.97
E *by* N	18	VII	03.51		17.65	
N *by* E ½ East	16	I½	15.31		04.64	
N N W ½ W	15	II½	13.23			07.07
W N W	18	VI	06.89			16.63
S E *by* S	18	III		14.97	10.00	
S S W	15	II		13.86		05.74
S W *by* S	12	III		09.98		06.67
W S W	18	VI		06.89		16.63
E S E	18	VI		06.89	16.63	
Summs of the Four last Col.			3.894	77.90	57.81	57.71
Subtract				38.94	57.71	
Rem. Difference of *Latitude.*				38.96	00.10	Depart.

Your

Your *Table* being thus prepared, you must have recourse to the following *Traverse Table*; and seeing that your first Course was S E *by* S, that is III Points from the *Meridian*, and the Leagues sailed were 16: Look for 3 Points in the Head of the *Table*, and under it, against 16, you shall find 13.30 Leagues for the *Southing*, and 08.89 Leagues for the *Easting*; which set under the *South* and *East* Columns, for that the Course was *South-Easterly*: And thus must you doe with all the rest of the Courses. As for the Fourth Course, which is N *by* E, half a Point *Easterly*; that is, 1 Point and a half from the *Meridian*; find 1½ Point in the Head of the *Table*, and 18 Leagues in the first Column to the Left-hand, and against it you shall find 15.31 Leagues, for the *Northing*, and 04.64 Leagues for the *Easting*; which set in the *North* and *East* Columns, because the Course was *North-Easterly*: and so of the rest; only, if your Number of Points be above IV from the *Meridian*, you must then find the Number of Points in the bottom of the *Table*, and the distance Sailed in the first or last Column of the *Table*, and in the common Meeting you shall find the *Easting*, *Westing*, *Northing*, and *Southing*, as before: As in the Sixth Course, which is W N W; that is, VI Points from the *Meridian*; find 6 Points in the Foot of the *Table*, and 18 the Leagues Sailed in the first or last Column, and in the Angle of Meeting you shall have 06.89 Leagues for your *Northing*, and 16.63 Leagues for your *Westing*; which set down in the *North* and *West* Columns. And when you have thus done with all your Courses and Distances, add up every Column severally, and subtract the Lesser *Northing* or *Southing* from the Greater, and the Remainder shall give you the difference of *Latitude*, in this *Example*, 38.96 Leagues: And the Lesser *Easting* or *Westing*, subtracted from the Greater, shall give you the Departure, which is only 00.10 Parts of a League; which shews the Course to be *South* 00.10 Parts of a League *Easterly*. And let these two *Examples* suffice for Traversing both by *Protraction*, *Calculation*, and by the *Traverse Table* following.

A

TRAVERSE TABLE,

TO

Every Point, Half Point, and Quarter Point,

OF THE

COMPASS:

GIVING

The Difference of *Longitude*, and Departure from the *Meridian* in Leagues or Miles, and Hundred Parts of the same, without *Trigonometrical Calculation*.

The TRAVERSE TABLE.

Leagues or Miles sailed.	0 Deg. 49 M. 0 Point. 1 Quar. N.	S.	E.	W.	5 Deg. 37 M. 0 Point. 2 Quar. N.	S.	E.	W.	8 Deg. 26 M. 0 Point. 3 Quar N.	S.	E.	W	11 Deg. 15 M. 1 Point. N.	S.	E.	W.	Leagues or Miles sailed.
1	01	00	00	05	01	00	00	10	00	98	00	14	00	98	00	20	1
2	02	00	00	10	01	99	00	20	01	97	00	29	01	96	00	39	2
3	03	00	00	15	02	98	00	29	02	96	00	44	02	94	00	58	3
4	04	00	00	20	03	98	00	39	02	95	00	58	03	92	00	78	4
5	04	09	00	25	04	97	00	49	04	94	00	73	04	90	00	98	5
6	05	09	00	29	05	97	00	59	05	93	00	88	05	88	01	17	6
7	06	09	00	34	06	97	00	69	06	92	01	02	06	86	01	37	7
8	07	09	00	39	07	96	00	78	07	91	01	17	07	85	01	56	8
9	08	09	00	44	08	96	00	88	08	90	01	32	08	83	01	76	9
10	09	09	00	49	09	95	00	98	09	89	01	46	09	81	01	95	10
11	10	98	00	54	10	95	01	08	10	88	01	61	10	79	02	15	11
12	11	98	00	59	11	94	01	18	11	87	01	76	11	77	02	34	12
13	12	98	00	63	12	94	01	27	12	86	01	91	12	75	02	54	13
14	13	98	00	68	13	93	01	37	13	85	02	05	13	73	02	37	14
15	14	98	00	73	14	93	01	47	14	84	02	20	14	71	02	19	15
16	15	98	00	78	15	92	01	57	15	83	02	34	15	69	03	12	16
17	16	98	00	83	16	92	01	67	16	82	02	49	16	67	03	32	17
18	17	97	00	88	17	91	01	76	17	80	02	64	17	65	03	51	18
19	18	97	00	93	18	91	01	86	18	79	02	79	18	64	03	71	19
20	19	97	00	98	19	90	01	96	19	78	02	93	19	62	03	90	20
21	20	97	01	03	20	90	02	06	20	77	03	08	20	60	04	10	21
22	21	97	01	08	21	89	02	16	21	76	03	22	21	58	04	29	22
23	22	97	01	13	22	89	02	25	22	75	03	37	22	56	04	49	23
24	23	97	01	17	23	88	02	35	23	74	03	52	23	54	04	68	24
25	24	97	01	22	24	88	02	45	24	73	03	66	24	52	04	88	25
26	25	96	01	27	25	87	02	55	25	71	03	81	25	50	05	07	26
27	26	96	01	32	26	87	02	65	26	70	03	96	26	48	05	27	27
28	27	96	01	36	27	86	02	75	27	69	04	10	27	46	05	46	28
29	28	96	01	42	28	86	02	84	28	68	04	25	28	44	05	66	29
30	29	96	01	47	29	86	02	94	29	67	04	40	29	42	05	85	30
31	30	96	01	52	30	85	03	04	30	66	04	55	30	40	06	05	31
32	31	96	01	57	31	85	03	14	31	65	04	69	31	38	06	24	32
33	32	96	01	61	32	85	03	23	32	64	04	84	32	37	06	44	33
34	33	95	01	66	33	84	03	33	33	63	04	98	33	35	06	53	34
35	34	95	01	71	34	83	03	43	34	62	05	13	34	33	06	83	35
36	35	95	01	75	35	83	03	53	35	61	05	28	35	31	07	02	36
37	36	95	01	81	36	82	03	63	36	60	05	42	36	29	07	22	37
38	37	95	01	86	37	82	03	73	37	59	05	57	37	27	07	41	38
39	38	95	01	91	38	81	03	82	38	58	05	72	38	25	07	61	39
40	39	95	01	96	39	81	03	92	39	57	05	87	39	23	07	80	40
41	40	95	02	01	40	80	04	02	40	55	06	02	40	21	08	00	41
42	41	95	02	06	41	80	04	12	41	54	06	16	41	19	08	19	42
43	42	95	02	11	42	79	04	21	42	53	06	31	42	17	08	39	43
44	43	94	02	15	43	79	04	31	43	52	06	45	43	15	08	58	44
45	44	94	02	20	44	78	04	41	44	51	06	60	44	14	08	78	45
46	45	94	02	25	45	78	04	51	45	50	06	75	45	12	08	77	46
47	46	94	02	30	46	77	04	61	46	49	06	89	46	10	09	17	47
48	47	94	02	35	47	77	04	70	47	48	07	04	47	08	09	36	48
49	48	94	02	40	48	76	04	80	48	47	07	20	48	06	09	56	49
50	49	94	02	45	49	76	04	90	49	46	07	33	49	04	09	75	50
60	59	92	02	54	59	71	05	88	59	35	08	80	58	85	11	70	60
70	69	91	03	43	69	66	06	86	69	24	10	27	68	65	13	65	70
80	79	90	03	92	79	61	07	84	79	13	11	73	78	46	15	60	80
90	89	89	04	41	89	56	08	82	89	02	13	20	88	27	17	55	90
100	99	87	04	90	99	51	09	80	98	91	14	67	98	08	19	50	100
200	199	76	09	80	199	02	19	90	197	82	29	34	196	16	39	00	200
	E.	W	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S	
	7 Points 3 Quar.				7 Points 2 Quar.				7 Points 1 Quar.				7 Points.				
	87 Deg. 11 M.				84 Deg. 22 M.				81 Deg. 34 M.				78 Deg. 45 M.				

The TRAVERSE TABLE.

Leagues or Miles sailed.	14 Deg. 04 M. 1 Point. 1 Quar. N.	S.	E.	W.	16 Deg. 52 M. 1 Point 2 Quar. N.	S.	E.	W.	19 Deg. 41 M. 1 Point 3 Quar. N.	S	E.	W.	22 Deg. 30 M. 2 Points N.	S	E.	W.	Leagues or Miles sailed.
1	00	97	00	24	00	96	00	29	00	94	00	33	00	92	00	38	1
2	01	94	00	48	01	91	00	58	01	88	00	67	01	85	00	76	2
3	02	91	00	72	02	87	00	87	02	82	01	01	02	77	01	75	3
4	03	88	00	97	03	83	01	16	03	77	01	34	03	70	01	53	4
5	04	85	01	21	04	78	01	45	04	71	01	68	04	62	01	91	5
6	05	82	01	45	05	74	01	74	05	65	02	02	05	54	02	30	6
7	06	79	01	70	06	70	02	03	06	59	02	35	06	47	02	69	7
8	07	76	01	94	07	66	02	32	07	53	02	70	07	39	03	16	8
9	08	73	02	18	08	61	02	61	08	47	03	03	08	31	03	44	9
10	09	70	02	43	09	57	02	90	09	41	03	37	09	20	03	83	10
11	10	67	02	67	10	53	03	19	10	36	03	71	10	16	04	21	11
12	11	64	02	91	11	48	03	48	11	30	04	04	11	09	04	59	12
13	12	61	03	15	12	44	03	79	12	24	04	36	12	01	04	97	13
14	13	58	03	40	13	40	04	06	13	18	04	72	12	93	05	36	14
15	14	55	03	64	14	35	04	35	14	12	05	05	13	36	05	74	15
16	15	52	03	88	15	31	04	64	15	06	05	39	14	78	06	12	16
17	16	49	04	13	16	27	04	93	16	00	05	73	15	71	06	51	17
18	17	46	04	37	17	22	05	22	16	95	06	06	16	53	06	89	18
19	18	43	04	61	18	18	05	51	17	89	06	40	17	55	07	27	19
20	19	40	04	86	19	14	05	81	18	83	06	74	18	48	07	65	20
21	20	37	05	10	20	10	06	10	19	77	07	08	19	40	08	04	21
22	21	34	05	34	21	05	06	39	20	71	07	41	20	32	08	42	22
23	22	31	05	48	22	01	06	68	21	66	07	75	21	25	08	80	23
24	23	28	05	83	22	97	06	97	22	60	08	08	22	17	09	18	24
25	24	25	06	07	23	91	07	26	23	54	08	42	23	10	09	57	25
26	25	22	06	31	24	88	07	55	24	48	08	76	24	02	09	95	26
27	26	19	06	56	25	84	07	84	25	46	09	10	24	94	10	33	27
28	27	16	06	80	26	79	08	13	26	36	09	43	25	87	10	71	28
29	28	13	07	04	27	75	08	42	27	30	09	77	26	79	11	10	29
30	29	10	07	28	28	71	08	71	28	25	10	11	27	72	11	48	30
31	30	07	07	53	29	66	09	00	29	19	10	44	28	64	11	86	31
32	31	04	07	77	30	62	09	29	30	13	10	74	29	56	12	25	32
33	32	01	08	01	31	58	09	58	31	07	11	12	30	49	12	63	33
34	32	98	08	26	32	54	09	87	32	01	11	45	31	44	13	01	34
35	33	95	08	50	33	49	10	16	32	95	11	79	32	34	13	39	35
36	34	92	08	74	34	45	10	45	33	89	12	13	33	26	13	78	36
37	35	89	08	99	35	41	10	74	34	84	12	47	34	18	14	16	37
38	36	86	09	23	36	36	11	03	35	78	12	80	35	11	14	54	38
39	37	83	09	47	37	32	11	32	36	72	13	14	36	03	14	92	39
40	38	80	09	71	38	28	11	61	37	66	13	48	36	96	15	31	40
41	39	77	09	96	39	23	11	90	38	60	13	81	37	88	15	69	41
42	40	74	10	20	40	19	12	19	39	54	14	15	38	80	16	07	42
43	41	71	10	44	41	15	12	45	40	49	14	49	39	73	16	45	43
44	42	68	10	69	42	12	12	77	41	43	14	72	40	65	16	84	44
45	43	65	10	83	43	9	13	06	42	37	15	16	41	57	17	22	45
46	44	62	11	17	44	02	13	35	43	31	15	50	42	50	17	60	46
47	45	59	11	42	44	98	13	64	44	25	15	83	43	44	17	99	47
48	46	56	11	66	45	93	13	93	45	19	16	17	44	35	18	37	48
49	47	53	11	90	46	89	14	22	46	13	16	37	45	27	18	75	49
50	48	50	12	14	47	85	14	51	47	08	16	85	46	19	19	13	50
60	58	20	14	57	57	42	17	42	56	49	20	21	55	43	22	58	60
70	67	90	17	00	66	90	20	31	65	90	23	58	64	67	26	78	70
80	77	60	19	43	76	55	23	22	75	32	26	95	73	91	30	61	80
90	87	30	21	80	86	10	26	12	84	73	30	39	83	14	34	44	90
100	97	00	24	29	95	69	29	02	94	15	33	68	92	38	38	36	100
200	194	00	48	58	191	38	58	04	188	30	67	36	184	76	76	52	200
	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	
	6 Points 3 Quar.				6 Points 2 Quar.				6 Points 1 Quar.				6 Points.				
	75 Deg. 56 M.				73 Deg. 07 M.				70 Deg. 1 M.				67 Deg. 20 M.				

The TRAVERSE TABLE.

Leagues or Miles sailed.	25 Deg. 19 M. 2 Points 1 Quar. N.	S.	E.	W.	28 Deg. 07 M. 2 Points 2 Quar. N.	S.	E.	W.	30 Deg. 56 M. 2 Point 3 Quar. N.	S	E.	W.	33 Deg. 45 M. 3 Points N.	S	E.	W.	Leagues or Miles sailed.
1	00	90	00	43	00	88	00	47	00	86	00	51	00	33	00	56	1
2	01	81	00	85	01	76	00	94	01	71	01	03	01	66	01	55	2
3	02	71	01	28	02	65	01	44	02	57	01	54	02	49	01	67	3
4	03	61	01	71	03	53	01	89	03	43	02	06	03	32	02	22	4
5	04	52	02	14	04	41	02	36	04	29	02	57	04	16	02	78	5
6	05	42	02	56	05	29	02	83	05	15	03	08	04	99	03	33	6
7	06	33	02	99	06	17	03	30	06	00	03	60	05	82	03	89	7
8	07	23	03	42	07	05	03	77	06	86	04	11	06	65	04	44	8
9	08	14	03	85	07	94	04	24	07	72	04	63	07	48	05	00	9
10	09	04	04	28	08	82	04	71	08	58	05	14	08	31	05	55	10
11	09	94	04	70	09	70	05	18	09	44	05	66	09	15	06	11	11
12	10	85	05	13	10	58	05	66	10	29	06	17	09	98	06	67	12
13	11	75	05	56	11	46	06	13	11	15	06	68	10	81	07	22	13
14	12	66	05	99	12	35	06	60	12	00	07	20	11	64	07	78	14
15	13	56	06	41	13	23	07	07	12	87	07	71	12	47	08	33	15
16	14	46	06	84	14	11	07	54	13	72	08	23	13	30	08	89	16
17	15	37	07	27	14	99	07	01	14	54	08	74	14	13	09	44	17
18	16	27	07	70	15	87	08	48	15	44	09	25	14	97	10	00	18
19	17	18	08	12	16	76	08	96	16	30	09	77	15	80	16	56	19
20	18	8	08	55	17	64	09	43	17	16	10	28	16	62	11	11	20
21	18	98	08	98	18	52	09	90	18	01	10	80	17	46	11	67	21
22	19	89	09	41	19	40	10	37	18	87	11	31	18	29	12	22	22
23	20	79	09	83	20	28	10	84	19	73	11	32	19	12	12	78	23
24	21	60	10	26	21	17	11	31	20	59	12	34	19	95	13	33	24
25	22	60	10	69	22	05	11	78	21	44	12	35	20	79	13	89	25
26	23	50	11	12	22	93	12	26	22	30	13	57	21	62	14	44	26
27	24	41	11	54	23	81	12	73	23	16	13	88	22	45	15	15	27
28	25	31	11	97	24	69	13	20	24	02	14	39	23	28	15	56	28
29	26	22	12	48	25	58	13	67	24	87	14	91	24	11	16	11	29
30	27	12	12	83	26	46	14	14	25	73	15	42	24	94	16	67	30
31	28	02	13	25	27	34	14	61	26	59	15	94	25	78	17	22	31
32	28	93	13	68	28	22	15	08	27	45	16	45	26	61	17	78	32
33	29	83	14	11	29	10	15	56	28	31	16	97	27	44	18	33	33
34	30	74	14	54	29	98	16	03	29	16	17	48	28	27	18	89	34
35	31	64	14	96	30	87	16	50	30	02	17	99	29	10	19	44	35
36	32	54	15	39	31	75	17	97	30	88	18	51	29	93	20	00	36
37	33	45	15	82	32	63	17	44	31	74	19	02	30	76	20	56	37
38	34	35	16	25	33	57	17	91	32	59	19	54	31	60	21	11	38
39	35	26	16	68	34	40	18	38	33	45	20	05	32	43	21	57	39
40	36	16	17	10	35	28	18	86	34	31	20	56	33	26	22	22	40
41	37	06	17	53	36	16	19	34	35	17	21	08	34	09	22	78	41
42	37	97	17	96	37	04	19	81	36	02	21	59	34	92	23	34	42
43	38	87	18	38	37	92	20	27	36	88	22	11	35	75	23	89	43
44	39	78	18	81	38	86	20	74	37	74	22	62	36	58	24	44	44
45	40	68	19	24	39	69	21	21	38	60	23	14	37	42	25	0	45
46	41	51	19	67	40	57	21	68	39	46	23	65	38	95	25	56	46
47	42	49	20	09	41	41	22	16	40	31	24	16	39	08	26	11	47
48	43	39	20	52	42	33	22	63	41	17	24	68	39	91	26	57	48
49	44	30	20	95	43	21	23	10	42	03	25	19	40	74	27	22	49
50	45	20	21	48	44	10	23	57	43	89	25	71	41	57	27	78	50
60	54	24	25	65	52	91	28	28	51	46	30	84	49	85	33	33	60
70	63	27	29	92	61	73	32	99	60	04	35	98	58	22	38	88	70
80	72	31	34	20	70	55	37	71	68	61	41	12	66	51	44	44	80
90	81	35	38	47	79	37	42	43	77	19	46	26	74	83	50	00	90
100	90	39	42	75	88	19	47	13	85	77	51	41	83	14	55	55	100
200	180	78	85	50	176	38	94	26	171	54	102	82	166	28	111	10	200
	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	E.	W	N.	S.	
	5 Points 3 Quar. 64 Deg. 42 M.				5 Points 2 Quar.				5 Points 1 Quar.				5 Points.				

The TRAVERSE TABLE.

Leagues or Miles sailed.	36 Deg. 34 M. 3 Points 1 Quar. N.	S.	E.	W.	39 Deg. 22 M. 3 Points 2 Quar. N.	S.	E.	W.	42 Deg. 11 M. 3 Points 3 Quar. N.	S	E.	W.	45 Deg. 00 M. 4 Points N.	S	E.	W.	Leagues or Miles sailed.
1	01	80	00	60	00	77	00	63	00	74	00	67	00	71	00	71	1
2	02	61	01	19	01	55	01	27	01	48	01	34	01	41	01	41	2
3	03	41	01	41	02	32	01	90	02	22	02	01	02	12	02	12	3
4	04	21	02	38	03	09	02	54	02	96	02	69	02	83	02	83	4
5	04	02	02	98	03	86	03	17	03	70	03	36	03	54	03	54	5
6	04	82	03	57	04	64	03	81	04	44	04	03	04	24	04	24	6
7	05	62	04	17	05	41	04	44	05	18	04	78	04	55	04	55	7
8	06	43	04	76	06	18	05	07	05	93	05	37	05	66	05	66	8
9	07	23	05	36	07	96	05	71	06	67	06	04	06	36	06	36	9
10	08	03	05	96	07	73	06	94	07	41	06	72	07	07	07	07	10
11	08	83	06	55	08	50	06	98	08	15	07	39	07	78	07	78	11
12	09	64	07	15	09	28	07	61	08	89	08	06	08	49	08	49	12
13	10	44	07	74	10	05	08	25	09	63	08	73	09	19	09	19	13
14	11	24	08	34	10	82	08	80	10	37	09	40	09	90	09	90	14
15	12	05	08	94	11	60	09	52	11	11	10	07	10	61	10	61	15
16	12	85	09	53	12	37	10	15	11	85	10	74	11	31	11	31	16
17	13	66	10	13	13	14	10	78	12	70	11	42	12	02	12	02	17
18	14	46	10	72	13	91	11	42	13	34	12	09	12	73	12	73	18
19	15	26	11	32	14	69	12	04	14	08	12	76	13	44	13	44	19
20	16	06	11	91	15	46	12	69	14	82	13	43	14	14	14	14	20
21	16	87	12	51	16	23	13	32	15	56	14	10	14	85	14	85	21
22	17	67	13	11	17	01	13	56	16	30	14	77	15	56	15	56	22
23	18	47	13	70	17	78	14	59	17	04	15	45	16	26	16	26	23
24	19	28	14	30	18	55	15	22	17	78	16	12	16	97	16	97	24
25	20	08	14	89	19	32	15	86	18	52	16	70	17	68	17	68	25
26	20	88	15	49	20	10	16	49	19	26	17	46	18	38	18	38	26
27	21	69	16	08	20	87	17	13	20	00	18	13	19	09	19	09	27
28	22	49	16	68	21	64	17	76	20	75	18	70	19	80	19	80	28
29	23	29	17	27	22	42	18	40	21	49	19	44	20	51	20	51	29
30	24	10	17	87	23	19	19	03	22	23	20	12	21	21	21	21	30
31	24	90	18	47	23	96	19	67	22	97	20	82	21	92	21	92	31
32	25	70	19	06	24	74	20	30	23	71	21	49	22	63	22	63	32
33	26	51	19	66	25	51	20	93	24	45	22	16	23	33	23	33	33
34	27	31	20	25	26	28	21	57	25	19	22	83	24	04	24	04	34
35	28	11	20	85	27	06	22	20	25	93	23	50	24	75	24	75	35
36	28	91	21	46	27	83	22	84	26	67	24	17	25	46	25	46	36
37	29	72	22	04	28	60	23	47	27	41	24	85	26	16	26	16	37
38	30	52	22	64	29	37	24	11	28	16	25	52	26	87	26	87	38
39	31	32	23	23	30	15	24	74	28	90	26	19	27	56	27	56	39
40	32	13	23	83	30	92	25	38	29	64	26	86	28	28	28	28	40
41	32	93	24	42	31	69	26	01	30	38	27	53	28	99	28	99	41
42	33	73	25	02	32	47	26	64	31	12	28	21	29	10	29	10	42
43	34	53	25	61	33	24	27	28	31	86	28	88	30	41	30	41	43
44	35	34	26	21	34	01	27	91	32	60	29	55	31	11	31	11	44
45	36	14	26	81	34	78	28	55	33	34	30	22	31	82	31	82	45
46	36	94	27	40	35	56	29	18	34	08	30	89	32	53	32	53	46
47	37	75	28	00	36	33	29	82	34	82	31	56	33	23	33	23	47
48	38	36	28	59	37	10	30	45	35	57	32	23	33	94	34	94	48
49	39	17	29	19	37	88	31	08	36	31	32	91	34	65	34	65	49
50	40	17	29	78	38	65	31	72	37	05	33	58	35	35	35	35	50
60	48	19	35	74	46	38	38	06	44	45	40	29	42	43	42	43	60
70	56	22	41	69	54	11	44	41	51	85	47	00	49	49	49	49	70
80	64	25	47	65	61	84	50	75	59	26	53	72	56	56	56	56	80
90	72	28	53	61	69	57	57	09	66	67	60	44	63	63	63	63	90
100	80	32	59	56	77	30	63	43	74	08	67	15	70	71	70	71	100
200	160	64	119	12	154	60	126	86	148	16	134	30	141	42	141	42	200
	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	E.	W.	N.	S.	
	4 Points 3 Quar. 53 Deg. 26 M.				4 Points 2 Quar. 50 Deg. 37 M.				4 Points 1 Quar. 47 Deg. 49 M.				4 Points. 45 Deg. 00 M.				

OTHER
NAUTICAL QUESTIONS,

Performed by *Protraction* and *Calculation*, either by the Tables of *Sines*, *Tangents*, and *Logarithms*, or by the *Proportional Scales*.

QUEST. I. *There are two Ships that ſet Sail from the Port* A, *the one ſails directly North*, 2941 *Leagues, to* C; *and the other directly Eaſt*, 33.26 *Leagues, to* B: *I demand how the two Ships bear one from the other, and alſo how far they are aſunder?*

By *Protraction.*

Fig. III. DRaw a Right Line, A B, and upon A erect the Perpendicular A C: Let A repreſent the Port from whence the Ships ſet Sail; then (becauſe the Firſt Ship ſailed 29.41 Leagues North) take from any Scale of Equal Parts 29.41 Leagues, and ſet them from A to C; ſo ſhall C be the place of the Firſt Ship. Now, (becauſe the other Ship ſailed directly Eaſt, which is one Quarter of the Compaſs diſtant from the North,) therefore the Angle at A muſt be a Right Angle: And becauſe the ſecond Ship ſailed Eaſt 33.26 Leagues, take 33.26 Leagues from the ſame Line of Equal Parts, and ſet them upon the Line A B, from A unto B; ſo ſhall B be the Place of the ſecond Ship.

Now to know how theſe two Ships bear one from another, draw the Line C B, and meaſure the Quantity of the Angle at B, which you ſhall find to be 33 deg. 45 min. which is 3 Points from the Weſt, Northerly, that is, the N. W. by W. Point of the Compaſs; and ſo doth the ſecond Ship B, bear from the firſt Ship C.—— Again, Find the Quantity of the Angle at C, which you ſhall find to be 56 deg. 15 min. which is 5 Points from the South, Eaſterly; that is, the S. E. by E. Point of the Compaſs; and ſo do the two Ships bear one from the other.—— Then from the Diſtance that the two Ships are from each other, take in your Compaſſes the Diſtance between B and C, which meaſure upon your Scale of Equal Parts, and you will find it to contain 40 Leagues; and ſo far aſunder are the two Ships B and C.

By *Calculation.*

The Bearing of the Ships one from the other is found by the IV. *Caſe* of *Right-angled Plain Triangles*, by this Analogy.

As the Diſtance that the firſt Ship ſailed, 29.41 Leagues,
Is to the Diſtance that the ſecond Ship ſailed, 33.26 Leagues;
So is the *Radius*,
To the *Tangent* of the *Angle* that the firſt Ship bears from the ſecond, 56 deg. 15 min. or S. E. by E. The Complement whereof, 33 deg. 45 min. or N. W. by W. is the Bearing of the ſecond Ship from the firſt.

The Diſtance of the Ships from each other, is found by the VI. *Caſe*, by this Analogy.

As the Sine of the Bearing of the firſt Ship, 33 deg. 45 min. or N. W. by W.
Is to the Diſtance that the ſecond Ship ſailed;
So is the *Radius*, 90 deg.
To the Diſtance of the two Ships.

II. *A Ship at* A *diſcovers an Iſland at* C, *lying from her directly Eaſt; but ſhe ſails from* A *towards* B 33.26 *Leagues directly South; but her Compaſs coming to ſome Miſchance, that uſe cannot be made of it, ſhe again at* B *diſcovers the ſame Iſland, and ſails upon an unknown Point of the Compaſs directly upon the Iſland, and touches upon it, having ſailed* 40.00 *Leagues.—I demand upon what Point of the Compaſs the Ship ſailed from* B *to* C, *and alſo how far off the Iſland was from* A, *where it was firſt diſcovered.*

By *Protraction.*

DRaw a Line C A, repreſenting a Line of Eaſt and Weſt, and upon A erect a Perpendicular A B, and from A to B ſet off 40.00 Leagues, the Diſtance that the

the Ship sailed frhm A to B. Then take out of your Scale of Equal Parts 40.00 Leagues, the Distance that the Ship sailed from B to the Island; and setting one Foot of the Compasses in B, with the other describe an obscure Arch of a Circle *m m*, crossing the East and West Line in C; so is C the Place of the Island.

Now first, to find upon what Point of the Compas the Ship sailed from B to the Island, you must find the Quantity of the Angle at B, and you shall find it to contain 33 deg. 45 min. or the N. E. by N. and upon that Point did the Ship sail from B to the Island at C.—— Then, to know how far the Island C was from A, where it was first discovered, take in your Compasses the length of the Line A C, and measure it upon your Scale; so shall you find that to contain 24 *Cent.* or $\frac{4}{5}$ Leagues; and so far distant was the Island from A.

By *Calculation.*

The Point of the Compas that the Ship sailed upon from B to C, may be found by the V. *Case* of *Right-angled Plain Triangles*, by this Analogy.

As the Distance which the Ship sailed from B to C, 40.00 Leagues,
Is to the *Radius*;
So is the Distance sailed between A and B, 33.26 Leagues,
To the Co-sine of the Point that the Ship sailed upon from B to C, 56 d. 15 m.

The Distance that the Ship was from the Island, when first discovered, may be found by the VII. *Case* of *Right-angled Plain Triangles*, by the following Analogy.

(1) As the Distance that the Ship sailed from B to C, 40.00 Leagues,
Is to the *Radius*;
So is the Distance that the Ship sailed from A to B, 33.26 Leagues,
To the Bearing of the Island from B, 33 d. 45 m.

(2.) As the *Radius*,
Is to the Distance that the Ship sailed from C to B, 40.00 Leagues;
So is the Sine of the *Rumb* that the Ship sailed upon from B to C, 33 d. 45 m.
To the Distance of the Island from A, 29.41 Leagues.

III. *There are two Ports at* A *and* B, *which are distant* 33.26 *Leagues, and lie directly North and South of each other; from whence two Ships set sail, both for the Port* C: *The Ship at* B *sails away upon a South-West by South Point, and the Ship at* A *sails directly West.—— I demand, how many Leagues either of the Ships had sailed when they met at the Port* C, *and also how the Port* C *did bear from that at* B.

By *Protraction.*

DRaw a Right Line, A B, and upon it set off 33.26 Leagues. Now because the Ship at B steered a South-West by South Course, which is three Points from the South Westerly, therefore upon the Point B protract an Angle of 33 deg. 45 min. and draw the Line B C.—— Then, because the Ship at A steered a Westerly Course, which is a Quarter from the North, upon the Point A protract an Angle of 90 deg. and draw the Line A C, cutting the former Line B C in C.—— Now to know how many Leagues each Ship sailed, take in your Compasses the Length of the Line B C, and measuring it upon your Scale, you shall find it to contain 40.00 Leagues; and so many did the Ship that came from B sail. Also take the Length of the Line A C in your Compasses, and measuring that upon your Scale, it will be found to contain 22.22 Leagues; and so much did the Ship that came from A sail. Now to know how the Port at C did bear from that at B, find the Quantity of the Angle at C, which you shall find to be 56 deg. 15 min. that is, Five Points from the East Northerly, namely, N. E. by N. and so did the Port C bear from B.

By *Calculation.*

The finding of the Distance that each Ship sailed, may be done by the I. and II. *Cases* of *Right-angled Plain Triangles*, by this Analogy:

As the Distance of the two Ports, A and B, 33.26 Leagues,
Is to the Bearing of the Port C from B, 56 deg. 15 min.
So is the Sine of the *Rumb* that the Ship sailed upon from B to C, 33 deg. 45 min.
To the Distance that the Ship sailed from A to C, 22.22 Leagues.
And so is the *Radius*,
To the Number of Leagues that the Ship sailed from B to C, 40.00 Leagues.

IV. *A Ship at* C *discovers a Point of Land at* A, *bearing from her* S. S. E. *but she shapes a Course* E. *by* S. *and sails away* 40.00 *Leagues to* B, *and at* B *she discovers the same Point of Land bearing from her* W. S. W. —— *I demand how far the Ship was from Land, being at* C *and* B.

By *Protraction.*

DRaw a Line C B, containing 40 Leagues, and upon C protract an Angle of 56 deg. 15 min. or Five Points, which is the Difference the Point of Land did bear from the Ship being at C, and the Point upon which she sailed from C to B; and draw a Right Line C A.—— Then upon the Point B protract an Angle of 33 deg. 45 min. which is the Difference of the Ship's Bearing from C and A, she being at B, namely, W. S. W. and draw the Line B A, cutting the Line C A, before drawn, in A.

Now to find how far the Ship was from Land being at C, measure the Line C A upon your Scale of Equal Parts, and you shall find it to contain 22.22 Leagues; and so far was the Ship from the Land when she was at C. Also measure the Length of the Line B A, and you shall find that to contain 33.26 Leagues; and so far from Land was the Ship being at B.

By *Calculation.*

To find these Distances, you may do it by the III. *Case* of *Right-angled Triangles*, by this Analogy.

As the *Radius*,
Is to the Distance that the Ship sailed from C to B, 40.00 Leagues;
So is the Bearing of the Ship, being at C, 56 d. 15 m.
To her Distance from Land, being at B, 33.26 Leagues. Or,
The Bearing of the Ship, she being at B, 33 d. 45 m.
To her Distance from Land at C, 22.22 Leagues.

V. *A Ship being at* A, *discovers two other Ships at* C *and* B; *the Ship at* C *bears from her directly East, and the other Ship at* B *bears from her directly South: The Ship at* A *sails directly South,* 22.22 *Leagues to* B, *and being at* B, *steers away upon an unknown Course to* C, 40 *Leagues.*—— *I demand upon what Point the Ship sailed from* B *to* C, *and also how far* C *is distant from* A.

By *Protraction.*

DRaw a Right Line, A B, for the Bearing of the Ship B from the Ship A, which was direct South: Also from A draw another Line, A C, for the Bearing of the Ship C from the Ship A, which was directly East. Now because between the South and the East is 90 deg. or one Quarter of the Compass, therefore upon the Point A protract an Angle of 90 deg. drawing the Lines A C and A B at Right Angles. This done, take 33.26 Leagues out of your Scale of Equal Parts, which is the Distance that the Ship sailed South from A to B. Then take from the same Scale 40 Leagues, which is the Distance that the Ship sailed from B to C upon an unknown Point; and with this Distance, setting one Foot of the Compasses in B, with the other describe an obscure Arch of a Circle, *m m*, cutting the Line A C in the Point C, and draw the Line C B.—— Now to find upon what Point of the Compass the Ship sailed from B to C, find the Quantity of the Angle at B, which you shall find to contain 33 deg. 45 min. that is three Points from the North Easterly, namely, N. E. by N. and upon that Point did the Ship sail from B to C.—— Then to find how far C is distant from A, take the Line C A in your Compasses, and measuring it upon your Scale, you shall find it to contain 22.22 Leagues; and so far is C distant from A.

By *Calculation.*

The Point upon which the Ship sailed from B to C, may be found by the III. *Case* of *Right-angled Triangles*, by this Analogy.

As the Distance that the Ship sailed from B to C, 40.00 Leagues,
Is to the *Radius*;
So is the Distance that the Ship sailed from A to B, 22.22 Leagues,
To the Co-sine of the *Rumb* from the *Meridian*, 33 deg. 26 min.

Then

Then for the Distance of C *from* A.

As the *Radius*,
Is to the Distance that the Ship sailed from B to C, 40.00 Leagues;
So is the *Rumb* from the *Meridian* that the Ship sailed upon from B to C, 33 d. 45 m.
To the Distance of C A, 22.22 Leagues.

VI. *Two Islands at* A *and* C *are discovered by a Ship at* B; *the Island* A *bears from the Ship at* B, *N. N. W. and the Island at* C *bears N. by E. from* B: *The Ship being at* B *sails away N. N. W. to the Island* A, *and having sailed* 33.26 *Leagues, touches upon the Island, and being there, finds that the Island* C *bears from the Island* A, *E. N. E. I demand how far the Ship at* B *was from the Island* C, *and also how far the two Islands were asunder.*

By *Protraction.*

DRaw a Line A B, and upon it set 33.26 Leagues, which is the Distance that the Ship sailed from B to the Island at A. And because the Island A did bear from B, N, N W, and the Island at C, N by E, which are three Points, or 33 deg. 45 min. asunder, upon the Point B protract an Angle of 33 deg. 45 min. and draw the Line B C. Then because the Island at C bears from the Island at A, E N E, which is eight Points, or 90 deg. from N N W, upon the Point A protract an Angle of 90 deg. and draw the Line A C, cutting the Line B C in C.

Now to find the Distance of the Ship being at B from the Island C, take the Line C B in your Compasses, and applying it to your Scale, you shall find it to contain 40.00 Leagues; and so far was the Ship at B from the Island at C. And to find the Distance of the Islands one from the other, take C A in your Compasses, and measure it upon your Scale, you shall find it to contain 22.22 Leagues; and so far distant were the Islands one from the other.

By *Calculation.*

The Distance from A to C may be found by the I. *Case* of *Right-angled Plain Triangles*, by this Analogy.

As the Co-sine of the *Rumb* that the Ship sailed upon from B to A, 56 d. 15 m.
Is to the Distance that the Ship sailed from B to A, 33.26 Leagues;
So is the *Radius*,
To the Distance of the Ship at B, from the Island at C, 40.00 Leagues.

Then for the Distance of the two Islands, by the IV. *Case*, say,

As the *Radius*,
Is to the Distance C B, 40.00 Leagues;
So is the Sine of the Difference between the Bearing of the two Islands from B, 33 d. 45 m.
To the Distance of the two Islands C and A, 22.22 Leagues.

VII. *Two Ships set out from one and the same Port*, A; *the Ship* C *sails* 33.26 *Leagues directly East, and the Ship* B *sails away* 22.22 *Leagues directly South. When they have thus sailed, I demand how far the two Ships are from each other.*

By *Protraction.*

DRaw a Right Line A B, and set off upon it 33.26 Leagues, the Distance that the Ship sailed from A to B South. Then, because the other Ship sailed directly East, which is 90 deg from the South, upon the Point A erect the Perpendicular A C, and upon it set off 22.22 Leagues, from A to C, which was the Distance the other Ship sailed East. Then draw the Line C B, which being taken in your Compasses, and measured upon your Scale, will be found to contain 40.00 Leagues; and so far are the two Ships from each other.

By *Calculation.*

This Distance may be found by the VI. *Case* of *Right-angled Plain Triangles*, by this Analogy.

(1.) As the Distance that the Ship sailed from A to B, 33.26 Leagues,
Is to the Distance that the Ship sailed from A to C, 22.22 Leagues;

So

So is the *Radius*,
To the *Tangent* of the *Angle* at B, 33 deg. 45 min.
(2.) As the Sine of the *Angle* at B, 33 deg. 45 min.
Is to the Distance C A, 22.22 Leagues,
So is the *Radius*,
To the Distance C B, 40.00 Leagues.

VIII. *Two Ships set sail from the Port at* K; *the one sails* 3.77 *Leagues upon the* S. W. *Point towards* M, *the other sails* 8.00 *Leagues upon the* West *Point towards* L.— *I demand how many Leagues the Ships at* M *and* L *are asunder: and also how the Ship at* M *bears from the Port* K, *and the other Ship at* L.

By *Protraction.*

Fig. IV. DRaw a Right-Line K L, and by help of your *Scale* set off upon it 8 Leagues, the Distance that the Ship sailed from K to L upon the *West*-Point: Then because the other Ship sailed 3.77 Leagues from K towards M, upon the *S.W.* Point, which is 45 deg. or 4 Points from the *West*: therefore upon the Point K, Protract an *Angle* of 45 deg. and draw the Line K M, setting off upon it from K to M 3.77 Leagues, the Distance that the Ship sailed from K to M, and draw the Line M L.

Now to know, first, how far distant the Ships at M and L are from each other, take in your *Compasses* the length of the Line M L; which apply to your *Scale*, and you shall find it to contain 5.96 Leagues.— And, Secondly, to find how the Ship at M bears from the Port K, and the other Ship at L, you must find the quantity of the *Angle* at M, which you will find to be 108 deg. 28 min. that is Nine Points and a half and 1 deg. 35 min. Now, because the Course from K to M was *S. W.* therefore the Ship at M bears from the Port K North-east. And seeing that the *Angle* at M, is 108 deg. 28 min. or 9 Points and a half, and 1 deg. 35 min. therefore so many Points counted from the N E Point, is the Bearing of the Ship at M, from that at L; which will be W N W, half a Point and 1 deg. 35 min. *Westerly.*

By *Calculation.*

As the Summ of Leagues that both Ships have sailed, 11.77
Is to the Difference of their Leagues sailed, 4.23
So is the *Tangent* of half the Summ of the *Angles* at M and L, 67 deg. 30 min.
To the *Tangent* of 40 deg. 30 min.

Which added to 67 deg. 30 min. gives 108 deg. 28 min. for the *Angle* at M; and subtracted from 67 deg. 30 min. leaves 26 deg. 32 min. for the *Angle* at L.

And to find how far the two Ships are Distant.

As the Sine of the *Angle* at L, 26 deg. 32 min.
Is to the Leagues the first Ship sailed K M, 3.77
So is the Sine of the Course she steered 45 deg.
To 5.96 Leagues.

Which is the Distance of the Ship M from the Ship L.

IX. *There are three Ships,* K, L, *and* M: *the Ship* K *is distant from the Ship* L 8.00 *Leagues; the Ship at* L *is distant from that at* M 5.96 *Leagues; and the Ship at* M *is distant from that at* K 3.77 *Leagues; and they lie directly* North *and* South.— *I demand how the Ship at* M *bears to that at* L, *and how that at* L *bears to that at* K.

By *Protraction.*

DRaw a Right Line, and out of your Scale take 8.00 Leagues, and set them thereon from K to L, for the distance of the Ships at K and L. Then take 3.77 Leagues, the distance of the Ship's K and M, out of your Scale; and setting one Foot of the *Compasses* in K, with the other describe the obscure Arch of a Circle *o o*. Again, take 5.96 Leagues from your Scale, which is the distance that the Ship L was from the Ship M; and setting one Foot of the *Compasses* in L, with the other describe the obscure Arch of a Circle *n n*, crossing the former Arch in the Point M: Then draw the Lines M K and M L; so have you their true Positions.

Now

Now to find their Bearing one from another; forasmuch as the Ships M and K did lie *North* and *South* of each other, find the quantity of the Angle at M, which is 108 deg. 28 min. that is nine Points and a half, and 1 deg. 35 min. from the *South Eastward*, which will be the *N. W.* Point: and so doth the Ship M bear from that at K. And for the Bearing of that at K from that at L, find the quantity of the Angle at L, it will be found as before W. N W, half a Point, and 1 deg. 35 min. *Westerly*.

The Bearings of the Ships from each other may be found by the Fifth Case of Oblique-angled plain Triangles, by the Analogie in that Case set down.

Problems of Sailing by the Plain Sea-Chart, *both by* Trigonometrical Calculation, *and wrought upon the* Chart *it self.*

Among Sea-men *there are* Three *principal ways of* Sailing *most in* Use and Practice: *Two whereof are* Rectilineal, *performed by* Right-Lines; *the Third is* Spherical *or* Circular, *performed by* Arches *of* great Circles *of the* Sphere.

Of the Two first, the one is called Plain Sailing, *or* Sailing *by the* Plain Sea-Chart.

The other is called Mercator's Sailing, *or* Sailing *by* Mercator's Chart.

These two Charts *are both of them composed of* Right-Lines, *yet differ both in their* Construction *and* Use, *though not so much in their* Use, *as in their* Making *or* Construction.

The Plain Sea-Chart *consisteth of* Meridians *and* Parellels, *which are drawn in all parts equal from the* Æquinoctial *towards either of the* Poles; *which is erroneous, as hereafter shall be discovered.*

Mercator's Chart, (*or rather Mr.* Wright's Chart) *hath the Degrees of* Longitude *in every* Parallel of Latitude *equal to those in the* Æquinoctial, *as the* Plain-Chart *hath: But the Degrees of* Latitude *do increase more and more* (*as they grow nearer the* Poles) *in such a Proportion as every* Parallel of Longitude *doth decrease.*

The way of Sailing by the Plain Sea-Chart *is much in use, nay too much, considering the* Errors *that it leads* Sea-men *into*; *though they are not so easily discovered in* short *as* long Voyages, *nor in* Places near *the* Æquinoctial *as those* nearer *the* Poles. *But, I suppose, it is more used for the ease there is in* Projecting *of this* Chart, *than in that of* Mercator's: *otherwise I know not why that should be so as it is embraced, and the other* (*I mean that of* Mercator's) *so much neglected*; *which comes so near to the* Spherical *way of* Sailing, *that there is an insensible difference between them. But I shall compare them together, so that the* ingenious Sea-man *may see their Difference, and thereby abandon* Error, *and embrace the* Truth. *For in the following* Problems *I shall perform the* same *thing by* both Charts, *by which the* Errors *may more palpably be discovered. And I shall shew how the Doctrine of* Plain Triangles *may be made applicable to the Solution of the following* Nautical Problems.

The Making of the Plain Sea-Chart.

A *Sea-Chart* may be made either *general*, or *particular*. A *general Sea-Chart* is that whose Degrees of *Latitude* proceed from the *Æquinoctial* to either *Pole*, which in the *common Sea-Chart* may be done; but it will be egregiously false, as the Degrees of *Latitude* grow nearer the *Pole*, as I have already declared.— A *Particular Sea-Chart* is such a one as is made properly for one *particular Navigation*: as if your whole *Navigation* were not to exceed the *Latitudes* of 48 and 60 deg. of *Latitude*, and not to differ in *Longitude* above 8 degrees. *Fig.* V.

Now to *project* or *make* such a *Chart*; First, Draw a Right-Line A B, representing the *Meridian*, and cross it at right Angles in the Point A with another Right-line A D, representing the Parallel of your least *Latitude*, namely, of 48 deg.— Secondly, Consider what distance you will have your Parallels of *Longitude* and *Latitude* to be, (for in this *Chart* they are both equal,) whether 1, 2, 3, or 4 Inches, (for the larger the better.)

better.) But in this *Example* I have made them onely half an Inch. I take therefore half an Inch out of an exact *Scale*, and run it upon the *Meridian-Line* A B, from A to 49, from 49 to 50, from 50 to 51, *&c.* till I come to my greateſt *Latitude*, which is here ſuppoſed to be 60 deg. — Thirdly, Run the ſame Diſtance of half an Inch from A towards D, upon the Line A D, eight times, becauſe the Difference of *Longitude* in your whole *Navigation* will not exceed 8 deg. — Fourthly, Draw the Line C D, parallel to A B, and B C, parallel to A D, and run the ſame Diſtances upon the Line B C as are upon the Line A D, and the ſame upon C D as are upon the Line A B. — Fifthly, From each Degree of *Latitude* in the Line A B, draw to the like Degree of *Latitude* in the Line C D a Right Line, as 49, 49; 50, 50; 51, 51; 52, 52; *&c.* till you have drawn all your *Parallels* of *Latitude*. — Sixthly, For your *Meridians*, they are to be drawn in like manner as were the *Parallels* of *Latitude*, all of them equidiſtant, and parallel to your firſt *Meridian* A B, as the Lines 1, 1; 2, 2; 3, 3; *&c.* And by this means have you the *Meridians* and *Parallels* drawn.

The *grand Diviſions*, or *whole Degrees*, being thus ſet upon your *Chart*, we now come to ſub-divide them. And for the dividing of the Degrees of the *Æquinoctial* at the top and bottom of your *Chart*, let each of them be divided into 5 or 10 parts, and each of thoſe parts ſub-divided into 5 or 10 more leſs parts, according as quantity will permit; for every one of them is ſuppoſed to be divided into 100 or 1000 parts.

For the dividing of the Degrees of *Latitude*; they may be divided as thoſe of *Longitude* were, into 100 parts. But ſometimes each Degree is ſubdivided into 60 Minutes, or *Engliſh* Miles, or into 20 Leagues. — Now I have divided the Degrees of *Latitude* in this *Chart* each of them into 5 parts, by which means it is capable of the *Numeration*, either by Miles, Leagues, Centeſms, or 100 parts. —— For if you count by 60 Minutes, or Miles, then every of thoſe Diviſions will be 12 Minutes, or Miles; if by 20 Leagues, then every Diviſion will contain 4 Leagues; and if by Centeſms or 100 parts, then every of them is 20 Centeſms. And thus much concerning the Making or Projecting of this *Chart*. I now come to ſhew

Some Uſes of the Plain Sea-Chart.

PROB. I. *How to ſet any Place upon your* Chart *according to its* Longitude *and* Latitude.

IF the two Places lie under one and the ſame Parallel, differing not at all in *Latitude*, but onely in *Longitude*, then the Courſe leading from the one to the other is directly *Eaſt* or *Weſt*. As E and F are two Places lying under the Parallel of 50 deg. of *Latitude*, and differ in *Longitude* 5.5 deg. lay a Ruler to 5.5 deg. both at the top and bottom of the Chord, and where the Ruler croſſeth the Parallel of 50 deg. as at F, there is your other Place upon the *Chart*. So E and F lie in 50 deg. of *Latitude*, and differ in *Longitude* 5.5 deg.

But if the two Places to be ſet upon the *Chart* differ onely in *Latitude*, and lie under the ſame *Meridian* as G F, then the Courſe leading from the one to the other is directly *North* or *South*, and the difference of *Latitude* of F and G is 2 deg. G lying in the *Latitude* of 48 deg. and F in the *Latitude* of 50 deg.

But if the Places to be ſet upon the *Chart* differ both in *Longitude* and *Latitude*, as A and F, then the Courſe leading from the one to the other is upon ſome other Point of the *Compaſs*, ſo far diſtant from the *Meridian* as is the quantity of the Angle E A F; which here is 70 deg. 1 min. that is, upon the E N E Point 2 deg. 31 min. *Eaſterly*.

Fig. VI. This Angle may be found either by Protraction by your Line of Chords, or it may be protracted by a *Protracting Quadrant*, ſuch as is in *Fig.* VI. which in all theſe Operations upon the *Chart* is beſt, for that it avoids the drawing of Arches of Circles upon your *Chart* or *Blank*. So then if you were to Protract the Angle E A F by your *Protracting Quadrant*, lay the Centre A of your *Quadrant* upon the Point A in your *Chart*, and the *Meridian-Line* of the *Quadrant* A B upon the *Meridian-Line* of your *Chart*; then will the Line A C of the *Quadrant* lie upon the Parallel A D of your *Chart*; and the Angle that you are to Protract being 70 deg. 1 min. by the

edge

edge of your *Quadrant* make a ſmall Mark or Prick with your Needle, and from A through that Point draw a Right-Line, which will be the Line A F.

And in the ſame manner as you ſet any Place upon your *Chart*, you may find in what *Latitudes* and Difference of *Longitude* any Places already ſet upon your *Chart* are in.

II. *Any Places being ſet upon the* Chart, *to find in what* Latitudes *they are, and alſo how they differ in* Longitude.

LET the Points Q and R upon the *Chart* be two Places, and I would know in what *Latitudes* they lie. Firſt, Through the Point Q draw a Line parallel to the Line B C of your *Chart*; and alſo through the Point R draw another Line parallel to A C. The Line that is drawn through Q ſhoots upon the *Latitude* of 58 deg. 36 min. and the Line paſſing through R cuts the *Meridian* of the *Chart* on either ſide at the *Latitude* of 57 deg. 16 min. And under thoſe two *Latitudes* are the two Places Q and R.

Then to find their Difference of *Longitude*, take in your *Compaſſes* the Diſtance between R and S, and meaſuring it upon the bottom of the *Chart*, it will reach from A to 4 deg. 24 min. and ſuch is the Difference of *Longitude* of the two Places Q and R.

III. *Having the Rumb, and the Diſtance that the Ship hath run upon that Rumb, to find the Difference of* Longitude *and* Latitude.

The Analogie or Proportion.

As the *Radius*,
Is to the Diſtance run,
So is the Sine of the *Rumb*,
To the Difference of *Longitude*:
And
So is the Co-Sine of the *Rumb*,
To the Difference of *Latitude*.

So the *Rumb* being 70 deg. 1 min. that is E N E, 2 deg. 31 min. *Eaſterly*, and the Diſtance run 117 Leagues, the Difference of *Longitude* will be found to be 5.5 deg. and the Difference of *Latitude* 2 deg.

Upon the Chart.

UPon the Point A Protract an *Angle* of 70 deg. 1 min. as the *Angle* E A F, and and draw the Line A F, which is the *Rumb* upon which the Ship ſailed. Upon this Line ſet 117, the Number of Leagues that the Ship ſailed from A to F. Then through the Point F draw the Line F E parallel to A D. So ſhall E F be the Difference of *Longitude*, 5 deg. and an half, and A E, the Difference of *Latitude*, 2 deg.

IV. *The Difference of* Latitude, *and the* Rumb *being given, to find the Diſtance run and the Difference of* Longitude.

The Analogie or Proportion.

As the Co-Sine of the *Rumb*,
Is to the Difference of *Latitude*,
So is the *Radius*,
To the Diſtance run:
And
So is the Sine of the *Rumb*,
To the Difference of *Longitude*.

So the one *Latitude* being 48 deg. and the other 50 deg. the Difference is 2 deg. and the *Rumb* being E N E 2 deg. 31 min. *Eaſterly*, the Diſtance run will be found to be 117 Leagues, and the Difference of *Longitude* 5.5 deg.

Upon the Chart.

SET the difference of *Latitude* 2 deg. from A to E, and draw the Line E F parallel to A D. Then upon the Point A protract the *Angle* of the *Rumb* 70 deg. 1 min. E N E 2 deg. 31 min. *Easterly*, and draw the Line A F, cutting the other Line E F in F. Then taking in your *Compasses* the length of the Line A F, and measuring it upon the side of the *Chart*, you shall find it to contain 117; which is the number of Leagues the Ship sailed: and the Line E F, being so measured, will contain 5.5 deg. the Difference of *Longitude*.

V. *Having the Difference of* Longitude, *and the* Rumb *given, to find the Distance run, and Difference of* Latitude.

The Analogy or Proportion.

As the Sine of the *Rumb*,
Is to the Difference of *Longitude*,
So is the *Radius*,
To the Distance run:
And
So is the Co-Sine of the *Rumb*,
To the Difference of *Latitude*.

So the *Rumb* being E N E 2 deg. 31 min. *Easterly*, and the Difference of *Longitude* 5.5 deg. the Distance run will be found to be 117 Leagues, and the Difference of *Latitude* 2 deg.

Upon the Chart.

UPon the Point Point A Protract an *Angle* of the *Rumb* 70 deg. 1 min. and draw the Line A F. Then the Difference of *Longitude* being 5.5 deg. count 5.5 deg. upon the bottom of your *Chart* from A to G, and upon the Point G raise a Perpendicular G F, cutting the Line A F before drawn in F. Then the Line A F, being measured upon the side of your *Chart*, will be found to contain 117 Leagues, the Distance run: And F G, there also measured, will be found to be 2 deg. the difference of *Latitude*.

VI. *The Distance that the Ship hath run, and the Difference of* Latitude *given, to find the* Rumb, *and Difference of* Longitude.

The Analogy or Proportion.

As the Distance run,
Is to the *Radius*,
So is the Difference of *Latitude*,
To the Co-Sine of the *Rumb*:
And
So is the Sine of the *Rumb*,
To the Difference of *Longitude*.

So the Distance run being 117 Leagues, and the Difference of *Latitude* being 2 deg. the *Rumb* will be found to be E N E 2 deg. 31 min. *Easterly*, and the Difference of *Longitude* 5.5 degrees.

Upon the Chart.

SET the difference of *Latitude* 2 deg. upon your *Chart* from A to E, and draw the Line E F parallel to A B. Then out of the side of your *Chart* take the distance run 117 Leagues; and setting one Foot of the *Compasses* in A, turn the other about till it cross the Line E F, which it will do in F. Then F E, being measured upon the bottom of your *Chart*, will contain 5.5 deg. the difference of *Longitude*. And by your Line of *Chords*, or *Protracting Quadrant*, find the quantity of the *Angle* E A F, which will be 70 deg. 1 min. the E N E Point 2 deg. 31 min. *Easterly*.

VII. *The*

VII. *The Distance that the Ship hath run, and the Difference of* Longitude *being given, to find the* Rumb, *and Difference of* Latitude.

The Analogy or Proportion.

As the Distance run
Is to the *Radius*,
So is the Difference of *Longitude*
To the *Rumb*:
And
So is the Co-Sine of the *Rumb*
To the Difference of *Latitude*.

So the Difference of *Longitude* being 5.5 deg. and the Distance that the Ship hath run 117 Leagues; the *Rumb* will be found to be E N E 2 deg. 31 min. *Easterly*, and the Difference of *Latitude* 2 deg.

Upon the Chart.

COunt the Difference of *Longitude* upon the bottom of the *Chart* from A to G, and upon the Point G raise the Perpendicular G F; Then take out of the side of your *Chart* the Distance run, 117 Leagues, and setting one foot of the *Compasses* in A, with the other cross the Perpendicular F G in the Point F. Now if you take F G in your *Compasses*, and measure it on the side of your *Chart*, you shall find it to contain 2 deg. for the Difference of *Latitude*; and the *Angle* E A F, being measured by your *Chord* or *Quadrant*, will be 70 deg. 1 min. that is the E N E Point 2 deg. 31 min. *Easterly* for the *Rumb*.

VIII. *The Difference of* Longitude, *and Difference of* Latitude *being given, to find the* Rumb, *and the Distance run.*

The Analogy or Proportion.

As the Difference of *Latitude*
Is to the *Radius*,
So is the Difference of *Longitude*
To the *Tangent* of the *Rumb*:
And
As the Sine of the *Rumb*
Is to the Difference of *Longitude*,
So is the *Radius*
To the Distance run.

So the Difference of *Longitude* being 5.5. deg. and the Difference of *Latitude* 2 deg. the *Rumb* will be found to be E N E 2 deg. 31 min. *Easterly*, and the Distance upon the *Rumb* 117 Leagues.

Upon the Chart.

COunt the Difference of *Latitude* from A to E, and draw the Line E F parallel to A D: also count the Difference of *Longitude* from A to G, and upon the Point G raise the Perpendicular G F, cutting the Line E F in the Point F: Then take in your *Compasses* the length of the Line A F, and measuring it upon the side of the *Chart*, you shall find it to contain 117 Leagues, the Distance that the Ship hath run. And if by your Line of *Chords*, or *Quadrant*, you find the quantity of the *Angle* E A F, it will be the *Rumb*; which you may find to be E N E 2 deg. 31 min. *Easterly*, or 70 deg. 1 min.

IX. *The* Rumb *that a Ship hath sailed upon, and the number of Leagues she hath sailed upon that* Rumb, *being given, to know how much she hath raised or depressed the Pole.*

The Analogy or Proportion.

As the *Radius*
Is to the Distance run,
So is the Co-Sine of the *Rumb* from the *Meridian*
To the Difference of both *Latitudes*.

So the *Rumb* being E N E 2 deg. 31 min. Eaſterly, that is, 70 deg. 1 min. and the Diſtance that the Ship hath ſailed upon that *Rumb* 117 Leagues, the Pole will be found to be raiſed 2 deg.

Upon the Chart.

UPon the Point A, the leſſer Latitude, protract an *Angle* of 70 deg. 1 min. and draw the Line of the *Rumb* A F, and out of the Side of your *Chart* take 117 Leagues, (the Diſtance the Ship ſailed,) and ſet them upon the *Rumb* from A to F. Then through the Point F, draw the Line E F, parallel to A D, cutting the *Meridian* of your *Chart* in E, which is 2 deg. from A; ſo that the Ship hath raiſed the *Pole* 2 degrees.

X. *The* Longitude *and* Latitude *of the Place from whence you came, with the* Rumb *and* Diſtance *ſailed, being given, to find the* Longitude *and* Latitude *of the Place to which you are come.*

The Analogy or Proportion.

As the *Radius*,
Is to the Diſtance run;
So is the *Sine* of the *Rumb* from the *Meridian*,
To the Difference of *Longitude*: And,
So is the *Co-ſine* of the *Rumb*,
To the Difference of *Latitude*.

So the *Latitude* of the Place from whence you came being 52 deg. and the *Longitude* 35 deg. the *Rumb* upon which you have ſailed N E by N, 33 deg. 45 min. and the Diſtance which you have ſailed upon that *Rumb* 96.2 Leagues; you ſhall find the Difference of *Longitude* to be 2 deg. 40 min. and the Difference of *Latitude* 4 deg. So that the Place to which you are come is in the *Latitude* of 56 deg. and in the *Longitude* of 37 deg. 40 min.

Upon the Chart.

THe Place from whence you came being in the *Latitude* of 52 deg. and in the *Longitude* of 35 deg. is repreſented by H. The *Rumb* you have ſailed upon being N E by N, 33 deg. 45 min. upon the Point H protract an *Angle* of 33 deg. 45 min. and draw the Line H K for the *Rumb*. Then out of the Side of your *Chart* take 96.2 Leagues, which is ſo much as the Ship ſailed, and ſet that upon the *Rumb-line* from H to K, and through the Point K draw the Line K L, parallel to B C, (or perpendicular to A B,) and it will cut the Line A B in L: So K L being meaſured on the bottom of your *Chart*, will be found to contain 2 deg. 40 min. the Difference of *Longitude*, which added to 35 deg. the *Longitude* you came from, gives 37 deg. 40 min. for the *Latitude* you are in. Alſo the Line H L being meaſured on the Side of your *Chart*, will be found to contain 4 deg. and ſuch is the Difference of *Latitude*; which added to 52 deg. the *Latitude* from whence you came, gives 56 d. the Latitude in which you are.

XI. *The* Longitude *and* Latitude *of the Place from whence you came, the* Rumb *upon which you ſailed, and the* Latitude *of the Place to which you are come, being given, to find the Diſtance and Difference of* Longitude.

The Analogy or Proportion.

As the Difference of *Latitude*,
Is to the *Radius*;
So is the *Tangent* of the *Rumb* from the *Meridian*,
To the Difference of *Longitude*:
And,
As the *Sine* of the *Rumb*,
Is to the Difference of *Longitude*;
So is the *Radius*,
To the Diſtance run.

So the *Latitude* of the Place from whence you came being 52 deg. and the *Longitude* 35 deg. and the *Rumb* upon which you ſailed the third from the *Meridian*, N E by N,

N E by N, 33 deg. 45 m. you ſhall find the Diſtance run to be 96.2 Leagues, and the Difference of *Longitude* 2 deg. 40 min.

Upon the Chart.

UPon your Chart aſſign H for your Place from whence you came; in the *Latitude* of 52 deg. and *Longitude* 35 deg. Upon this Point H protract the *Angle* of the *Rumb*, 33 deg. 45 min. N E by N, and draw the *Rumb-line* H K. Then the *Latitude* of the Place where you are being found, by Obſervation, (or being otherwiſe given,) to be 56 deg. draw a Line quite croſs your *Chart* at the 56th Degree of *Latitude*, as the Line 56 56, in the *Chart* croſſing the *Rumb-line* in the Point K: So K L being meaſured at the bottom of your *Chart*, will be found to contain 2 deg. 45 min. which added to 35 deg. the *Longitude* you came from, makes 37 deg. 40 min. and that is the *Longitude* in which you are. In like manner, meaſure H K upon the Side of your *Chart*, and you ſhall find it to contain 96.2 Leagues; and ſo much hath the Ship run upon that Point N E by N.

XII. *The* Latitude *of two Places, and the Difference of* Longitude *between them, being known, to find what* Rumb *leadeth from one to the other; And how many Leagues diſtant they are aſunder.*

The Analogy or Proportion.

As the Difference of *Latitude*,
Is to the *Radius*;
So is the Difference of *Longitude*,
To the *Tangent* of the *Rumb*.
And,
As the *Sine* of the *Rumb*,
Is to the Difference of *Longitude*;
So is the *Radius*,
To the Diſtance of two Places.

So the *Latitude* of one of the Places being 50 deg. and the other 52 deg. 30 min. and the Difference of *Longitude* 6.5 deg. the *Rumb* will be found to be be 67 deg. 23 min. and the Diſtance upon the *Rumb* 6.5 deg. or 120 Leagues.

Upon the Chart.

UPon the Point of the greater *Latitude* at N, 52 deg. 30 min. draw a Line N M, parallel to A D, upon which Line ſet 6 deg. the Difference of *Longitude* of the two Places (being taken from the bottom of the *Chart*) from N to M. Then from the Point M draw the Line to E, the leſſer *Latitude*, 52 deg. which Line taken in the Compaſſes, and meaſured upon the Side of the *Chart*, will be found to contain 6 5 deg. or 130 League. Alſo the Angle N E M being meaſured by your *Chord*, or *Protracting Quadrant*, will be found to contain 67 deg. 23 min. which is the *Rumb* leading from one to the other, namely, ſhort of the E N E Point 7 deg. or N E by E 11 deg. 8 min. Eaſterly.

The End of the Second Part.

OF

NAVIGATION.

PART III.

OF

Mercator's-Sailing.

WHEREIN

The DOCTRINE of RIGHT-LINED TRIANGLES (with the supply of *Meridional Parts*) is applied to PRACTICE, in the Solution of sundry *Nautical Problems* in *Sailing* by the true *Sea-Chart:* All performed, (1.) By *Protraction.* (2.) By *Calculation.* And (3.) Upon the *Chart* it self.

Of the Table *of* Meridional Parts.

THE following *Table* of *Meridional Parts* is onely an Abridgment of Mr. *Edward Wright*'s large *Table* of *Latitudes*, which he long since publiſhed in his *Book* of the *Correction of Errors in Navigation*, it being to every Six Minutes of a Degree of *Latitude:* So that this *Table* ſheweth how many *Meridional Parts* every Degree, and Tenth part of a Degree of *Latitude* is from the *Equinoctial:* And the *Table* begins at 00. deg. 00 min. but proceedeth by every Sixth Minute to 90 deg. or a quarter of the *Compaſs:* And becauſe in this kind of Projection of the *Sea-Chart*, (as you may perceive by the working upon *Mercator's Chart* as hereafter,) the Degrees of *Longitude* and *Latitude* are not equal; for the Degrees of *Latitude* at every Parallel exceed thoſe of *Longitude* in ſuch proportion, as the Diametre of the *Equinoctial* exceeds the Diametre of the *Parallel:* And therefore theſe differences of *Longitude* and *Latitude* muſt firſt be expreſſed by ſome common Meaſure; for which purpoſe this *Table* ſerveth: For that it ſheweth how many equal Parts are contained between the *Equinoctial*, and every Degree of *Latitude:* namely, of ſuch equal Parts, as one Degree of *Longitude* contains 60. The Uſe of this *Table* will appear farther, by reſolving the *Problems* following.

A TABLE

A TABLE of Meridional Parts.

Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.
0	0	0	5	0	300	10	0	603	15	0	910	20	0	1225	25	0	1550
	6	6		6	306		6	609		6	917		6	1232		6	1557
	12	12		12	312		12	615		12	923		12	1238		12	1563
	18	18		18	318		18	621		18	929		18	1244		18	1570
	24	24		24	324		24	627		24	935		24	1251		24	1577
	30	30		30	330		30	634		30	942		30	1257		30	1583
	36	36		36	337		36	640		36	948		36	1264		36	1590
	42	42		42	343		42	646		42	954		42	1270		42	1596
	48	48		48	349		48	652		48	960		48	1276		48	1603
	54	54		54	355		54	658		54	966		54	1283		54	1610
1	0	60	6	0	361	11	0	664	16	0	973	21	0	1289	26	0	1616
	6	66		6	367		6	670		6	979		6	1296		6	1623
	12	72		12	373		12	676		12	985		12	1302		12	1630
	18	78		18	379		18	682		18	991		18	1308		18	1637
	24	84		24	385		24	689		24	998		24	1315		24	1643
	30	90		30	391		30	695		30	1004		30	1321		30	1650
	36	96		36	397		36	701		36	1010		36	1328		36	1657
	42	102		42	407		42	707		42	1016		42	1324		42	1663
	48	108		48	409		48	713		48	1023		48	1341		48	1670
	54	114		54	410		54	719		54	1029		54	1347		54	1677
2	0	120	7	0	421	12	0	725	17	0	1035	22	0	1360	27	0	1684
	6	126		6	427		6	731		6	1042		6	1367		6	1690
	12	132		12	433		12	738		12	1048		12	1373		12	1697
	18	138		18	439		18	744		18	1054		18	1380		18	1704
	24	143		24	445		24	750		24	1060		24	1386		24	1710
	30	150		30	451		30	756		30	1066		40	1393		30	1717
	36	156		36	457		36	752		36	1072		36	1399		36	1724
	42	162		42	463		42	768		42	1079		42	1406		42	1731
	48	168		48	469		48	774		48	1086		48	1412		48	1738
	54	174		54	475		54	781		54	1092		54			54	1744
3	0	180	8	0	482	13	0	787	18	0	1098	23	0	1419	28	0	1751
	6	186		6	488		6	793		6	1104		6	1425		6	1758
	12	192		12	494		12	799		12	1111		12	1432		12	1765
	18	198		18	500		18	805		18	1117		18	1438		18	1772
	24	204		24	506		24	811		24	1123		24	1445		24	1778
	30	210		30	512		30	818		30	1130		30	1451		30	1785
	36	216		36	518		36	824		36	1136		36	1458		36	1792
	42	222		42	524		42	830		42	1142		42	1464		42	1799
	48	228		48	530		48	836		48	1149		48	1471		48	1806
	54	234		54	536		54	842		54	1155		54	1477		54	1813
4	0	240	9	0	542	14	0	848	19	0	1161	24	0	1484	29	0	1820
	6	246		6	548		6	855		6	1168		6	1491		6	1826
	12	252		12	554		12	861		12	1174		12	1497		12	1833
	18	258		18	560		18	867		18	1181		18	1504		18	1840
	24	264		24	567		24	873		24	1187		24	1510		24	1847
	30	270		30	573		30	879		30	1193		30	1517		30	1854
	36	276		36	579		36	886		36	1200		36	1524		36	1861
	42	282		42	585		42	892		42	1206		42	1530		42	1868
	48	288		48	591		48	898		48	1212		48	1537		48	1875
	54	294		54	597		54	904		54	1219		54	1543		54	1881

A TABLE of Meridional Parts.

Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.
30	0	1888	35	0	2244	40	0	2623	45	0	3030	50	0	3475	55	0	3968
	6	1895		6	2252		6	2631		6	3039		6	3484		6	3979
	12	1902		12	2259		12	2638		12	3047		12	3493		12	3989
	18	1909		18	2266		18	2646		18	3056		18	3503		18	4000
	24	1916		24	2274		24	2654		24	3064		24	3512		24	4010
	30	1923		30	2281		30	2662		30	3073		30	3522		30	4021
	36	1930		36	2288		36	2670		36	3081		36	3531		36	4031
	42	1937		42	2296		42	2678		42	3090		42	3540		42	4042
	48	1944		48	2303		48	2686		48	3098		48	3550		48	4053
	54	1951		54	2311		54	2694		54	3107		54	3559		54	4063
31	0	1958	36	0	2318	41	0	2702	46	0	3116	51	0	3569	56	0	4074
	6	1962		6	2325		6	2710		6	3124		6	3578		6	4085
	12	1972		12	2333		12	2718		12	3132		12	3588		12	4096
	18	1979		18	2340		18	2726		18	3143		18	3598		18	4106
	24	1986		24	2348		24	2734		24	3150		24	3607		24	4116
	30	1993		30	2355		30	2742		30	3159		30	3617		30	4128
	36	2000		36	2363		36	2750		36	3168		36	3627		36	4139
	42	2007		42	2370		42	2758		42	3176		42	3636		42	4150
	48	2014		48	2378		48	2766		48	3185		48	3646		48	4161
	54	2021		54	2385		54	2774		54	3194		54	3656		54	4172
32	0	2028	37	0	2393	42	0	2782	47	0	3203	52	0	3665	57	0	4183
	6	2035		6	2400		6	2790		6	3212		6	3675		6	4194
	12	2043		12	2408		12	2798		12	3219		12	3685		12	4205
	18	2050		18	2415		18	2806		18	3229		18	3695		18	4216
	24	2057		24	2423		24	2814		24	3238		24	3705		24	4227
	30	2064		30	2430		30	2822		30	3247		40	3714		30	4238
	36	2071		36	2438		36	2830		36	3256		36	3724		36	4250
	42	2078		42	2446		42	2839		42	3265		42	3734		42	4261
	48	2085		48	2455		48	2847		48	3274		48	3744		48	4272
	54	2092		54	2461		54	2855		54	3283		54	3754		54	4283
33	0	2100	38	0	2468	43	0	2863	48	0	3294	53	0	3764	58	0	4295
	6	2107		6	2476		6	2871		6	3301		6	3774		6	4306
	12	2114		12	2484		12	2880		12	3310		12	3784		12	4317
	18	2121		18	2491		18	2888		18	3319		18	3794		18	4329
	24	2128		24	2499		24	2896		24	3328		24	3804		24	4340
	30	2135		30	2507		30	2904		30	3337		30	3814		30	4352
	36	2143		36	2514		36	2913		36	3346		36	3824		36	4363
	42	2150		42	2522		42	2921		42	3355		42	3834		42	4375
	48	2157		48	2530		48	2929		48	3364		48	3844		48	4386
	54	2164		54	2537		54	2938		54	3373		54	3855		54	4398
34	0	2171	39	0	2545	44	0	2946	49	0	3382	54	0	3865	59	0	4409
	6	2179		6	2553		6	2954		6	3391		6	3875		6	4421
	12	2186		12	2560		12	2963		12	3401		12	3885		12	4433
	18	2193		18	2568		18	2971		18	3410		18	3896		18	4445
	24	2201		24	2570		24	2979		24	3419		24	3906		24	4456
	30	2208		30	2584		30	2988		30	3428		30	3916		30	4468
	36	2215		36	2592		36	2996		36	3437		36	3927		36	4480
	42	2222		42	2599		42	3005		42	3447		42	3937		42	4494
	48	2230		48	2607		48	3013		48	3456		48	3947		48	4504
	54	2237		54	2615		54	3022		54	3465		54	3958		54	4516

A TABLE of Meridional Parts.

Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.	Latitude. D.	M.	Meridional Parts.
60	0	4528	65	0	5179	70	0	5967	75	0	6972	80	0	8277
	6	4540		6	5194		6	5984		6	6095		6	8312
	12	4552		12	5208		12	6002		12	7018		12	8347
	18	4564		18	5222		18	6020		18	7042		18	8383
	24	4566		24	5237		24	6038		24	7066		24	8418
	30	4588		30	5251		30	6056		30	7089		30	8455
	36	4600		36	5265		36	6074		36	7114		36	8491
	42	4613		42	5280		42	6092		42	7138		42	8528
	48	4625		48	5295		48	6110		48	7162		48	8566
	54	4637		54	5309		54	6128		54	7187		54	8603
61	0	4650	66	0	5324	71	0	6147	76	0	7211	81	0	8742
	6	4662		6	5339		6	6165		6	7236		6	8780
	12	4674		12	5354		12	6184		12	7261		12	8819
	18	4687		18	5369		18	6202		18	7287		18	8859
	24	4699		24	5384		24	6221		24	7312		24	8899
	30	4712		30	5399		30	6240		30	7338		30	8939
	36	4725		36	5414		36	6259		36	7364		36	8980
	42	4737		42	5429		42	6278		42	7390		42	9021
	48	4750		48	5444		48	6297		48	7418		48	9063
	54	4763		54	5459		54	6316		54	7442		54	9105
62	0	4775	67	0	5475	72	0	6336	77	0	7469	82	0	9148
	6	4788		6	5490		6	6355		6	7495		6	9192
	12	4801		12	5505		12	6375		12	7522		12	9236
	18	4814		18	5521		18	6394		18	7550		18	9280
	24	4827		24	5537		24	6414		24	7577		24	9325
	30	4840		30	5552		30	6434		30	7605		40	9371
	36	4853		36	5568		36	6454		36	7633		36	9417
	42	4866		42	5584		42	6474		42	7661		42	9464
	48	4879		48	5600		48	6495		48	7889		48	9512
	54	4892		54	5615		54	6515		54	7717		54	9560
63	0	4905	68	0	5631	73	0	6535	78	0	7746	83	0	9609
	6	4919		6	5648		6	6556		6	7775		6	9659
	12	4932		12	5664		12	6577		12	7804		12	9709
	18	4945		18	5680		18	6598		18	7834		18	9760
	24	4959		24	5696		24	6618		24	7864		24	9812
	30	4972		30	5712		30	6640		30	7894		30	9865
	36	4986		36	5729		36	6661		36	7924		36	9918
	42	4999		42	5745		42	6682		42	7954		42	9973
	48	5013		48	5762		48	6704		48	7985			
	54	5026		54	5779		54	6725		54	7916			
64	0	5040	69	0	5795	74	0	6747	79	0	8048			
	6	5054		6	5812		6	6769		6	8079			
	12	5067		12	5829		12	6791		12	8111			
	18	5081		18	5846		18	6813		18	8148			
	24	5095		24	5863		24	6835		24	8176			
	30	5109		30	5880		30	6857		30	8209			
	36	5123		36	5897		36	6880		36	8242			
	42	5137		42	5914		42	6903		42	8275			
	48	5151		48	5932		48	6925		48	8309			
	54	5165		54	5949		54	6948		54	8343			

Probl. I. *To find what* Meridional Parts *are contained in any* Difference *of* Latitude.

Let there be two Places, one in the *Latitude* of 32 deg. 25 min. North, and the other in the *Latitude* of 50 deg. North.

The *Meridional Parts* belonging to 50 deg. are	3475
The *Meridional Parts* belonging to 32 deg. 25 min. are	2058
Meridional Parts in 17 d. 35 m. the *Difference* of *Latitude*	1417

Which *Difference* are the *Meridional Parts* contained in the *Difference* of the two *Latitudes*.

This holds when the two given *Latitudes* are both North, or both South.

But if one *Latitude* be North, and the other South, then the Summ of the *Meridional Parts* of both *Latitudes* is the *Meridional Parts* answering to those *Differences* of *Latitude*.

So if one Place be in 50 deg. of North *Latitude*, and the other in South *Latitude* 26 deg. the *Meridional Parts* answering to the *Difference* of these two *Latitudes* will be 5091.

The *Meridional Parts* answering to 50 d. of *Latitude* are	3475
The *Meridional Parts* answering to 26 d. of *Latitude* are	1616
Their Summ	5091

Which are the *Meridional Parts* contained between those two *Latitudes*.

II. *To turn* Degrees *and* Minutes *of* Difference *of* Longitude *into* Meridional Parts.

This is performed by common *Multiplication*, in this manner: Multiply the *Degrees* of *Difference* of *Longitude* by 60, the Product is the *Meridional Parts* answering thereunto.

Let the *Degrees* of *Difference* of *Longitude* be 12; this multiplied by 60, produceth 720 for the *Meridional Parts* answering to 12 deg. of *Difference* of *Longitude*.

But if there be any odd *Minutes* joined with your *Degrees*, they must be added to the *Meridional Parts*: As,

Let the *Difference* of *Longitude* be 7 deg. 42 min. 7 multiplied by 60 produceth 420, to which add the 42 min. and it is 462, the *Meridional Parts* answering to 7 deg. 42 min. of *Difference* of *Longitude*.

Thus having shewed how to reduce *Degrees* of *Difference* of *Longitude* and *Latitude* into *Meridional Parts*, I will now proceed to the Solution of some *Problems* in *Sailing* by the True *Sea-Chart*, and that by the Doctrine of *Plain Triangles*.

III. *The* Latitudes *of two Places, and their* Difference *of* Longitude *given, to find the* Rumb *and* Distance.

In these following *Problems* I will make use of Mr. *Norwood*'s Example of two Places, *viz.* the *Lizard*, lying in 50 deg. of North *Latitude*, and the *Somer Island*, (commonly called *Bermudas*) lying in the *Latitude* of 32 deg. 25 min. North. Then,

Fig. VI. In the *Triangle* A D B, let A represent the *Lizard*, and A B the *Parallel* of the *Latitude* thereof, *viz.* 50 deg.—Let D represent the *Bermudas*, and D B the *Meridian* thereof. Then,

In the *Triangle* A D B there is given, (1.) D B, the *Difference* of *Latitude*, 17 deg. 35 min. which turned into *Meridional Parts*, are 1417 Parts. (2.) A B, the *Difference* of *Longitude*, 70 deg. which (turned into *Minutes* or *Miles*) makes 4200: Which two Sides (and the Right Angle) being given, we may find,

I. The *Rumb* A D B,

By the IV. *Case* of *Right-angled Plain Triangles*,

As the *Difference* of *Latitude* D B, (17 d. 35 m.) in Parts, 1417.	Co. Ar. 6.84863
Is to the *Difference* of *Longitude* A B, (70 deg.) in Miles, 4200	3.62325
So is the *Radius*, 90 deg.	10.
To the *Tangent* of A D B the *Rumb*, 71 deg. 21 min.	10.47188

So

So that the *Course* from the *Bermudas* to the *Lizard* is E N E, 3 deg. 51 m. Easterly; and from the *Lizard* to *Bermudas* W S W, 3 deg. 51 min. Westerly,

II. For the *Distance* in the *Rumb.*

By the II. *Case.* But first,

Reduce the *Difference* of *Latitude*, 17 deg. 35 min. into Miles, by multiplying them by 60, which makes 1055 Miles. Then,

As the *Co-sine* of the *Rumb* D A B, 18 deg. 39 min.	Co. Ar. 0.49514
Is to the *Difference* of *Latitude*, D B, in Miles, 1055.	3.02325
So is the *Radius*, 90 deg.	10.
To the *Distance* A D, 3299 Miles	*x*3.51839

Which is almost 1100 Leagues.

Note, In this or the like Cases, whensoever we speak of the *Distance* of two Places, is meant their *Distance* measured in the *Rumb.*

IV. *The* Latitudes *of two Places, and their* Distance *given, to find the* Rumb, *and* Difference *of* Longitude.

Suppose I sail from the *Lizard* at A, in the *Latitude* of 50 deg. upon some Point to the Westward, 3299 Miles, and then find my self in the *Latitude* of 32 d. 25 m. I would know upon what *Course* I have made my Way good, and how much I have altered my *Longitude.*

In the *Triangle* A D B there is given, (1.) A D, the *Distance* sailed, 3299 Miles. (2.) D B, the *Difference* of *Latitudes* in Miles 1055, by which I may find the *Rumb* and *Difference* of *Longitude.*

I. For the *Rumb.*

By the V. *Case* of *Right-angled Plain Triangles.*

As the *Distance* sailed, A D, 3299.	Co. Ar. 6.48161
To the *Radius*, 90 deg.	10.
So is the *Difference* of *Latitude*, D B, 1055 Miles	3.02325
To the *Co-sine* of the *Rumb* D A B, 18 d. 39 m.	*x*9.50486

That is, W S W, 3 deg. 51 min. Westerly.

II. For the *Difference* of *Longitude.*

Find (by *Prob.* I.) what *Meridional Parts* are contained in the *Difference* of *Latitude* 17 deg. 35 min. and they will be 1417 *Parts.* Then,

As the *Radius*, 90 deg.	10.
To the *Difference* of *Latitude* in Parts, D B, 1417	3.15137
So is the *Tangent* of the *Rumb* A D B, 71 d. 21 m.	10.47188
To the *Difference* of *Longitude* A B, in Parts 4200	*x*3.62325

Which Parts reduced into Degrees, (by dividing them by 60,) the *Quotient* is 70 deg. for the *Difference* of *Longitude* required.

V. *By the* Rumb, *and* Latitude *of two Places given, to find their* Distance *and* Difference *of* Longitude.

Admit I sail from the *Lizard* at D, (being in the *Latitude* of 50 deg.) W S W, 3 deg. 51 min. Westerly, till I find my self at A, in the *Latitude* of 32 deg. 25 min. I demand how far I have sailed, and how much I have altered my *Longitude.*

The *Distance* is found as in the Latter Part of *Probl.* III. The 17 deg. 35 min. converted into Miles is 1055 Miles. Say then,

As the *Co-sine* of the *Rumb* D A B, 13 d. 39 m.	Co. Ar. 0.48514
To the *Difference* of *Latitude* D B in Miles, 1055	3.02325
So is the *Radius*, 90 deg.	10.
To the *Distance* A D in Miles, *viz.* 3299	*x*3.51839

Thus

Thus for the *Distance*: And for the *Difference* of *Longitude*, that may be found as in the Latter Part of *Probl.* II. thus:

As the *Radius*,
Is to the *Difference* of *Latitude* in *Meridional Parts*;
So is the *Tangent* of the *Rumb*,
To the *Difference* of *Longitude* in Minutes.
Which divided by 60, gives the *Difference* of *Longitude* in *Degrees*, *viz.* 70 deg.

VI. *By the* Difference *of* Longitude, *the* Rumb, *and one* Latitude, *to find the other* Latitude *and the* Distance.

Admit I sail from the *Lizard* at D, (being in the *Latitude* of 50 deg.) W S W, 3 deg. 51 min. Westerly, till I have altered my *Longitude* 70 deg. how much have I depressed the *Pole*, and how far am I from the *Lizard*?

First, Reduce the *Difference* of *Longitude*, A B, 70 deg. into *Minutes*, by multiplying them by 60, it makes 4200: Then say,

As the *Tangent* of the *Rumb* A D B, 71 deg. 21 min.	Co. Ar.	9.52829
To the *Difference* of *Longitude* A B in Parts, 4200		3.62325
So is the *Radius*, 90 deg.		10.
To the *Difference* of *Latitude* of *Latitude* D B in Parts, 1417		*r* 3.15154

Now the *Meridional Parts* answering to 50 deg. of *Latitude* are 3475, from which subtracting 1417, (the Parts here found,) there remains 2058; which sought in the Table, against it you will find 32 deg. 25 min. which is the *Latitude* of the Place to which I am come; so that the *Difference* of *Latitude* is 17 deg. 35 min.

Secondly, For the *Distance*,

Having already the *Rumb* and *Difference* of *Latitude*, the *Distance* may be found as in the III. and V. *Problems*, in this manner.

As the *Co-sine* of the *Rumb* D A B, 18 d. 39 m.	Co. Ar.	0.49514
To the *Difference* of *Latitude* D B, 1055 Miles		3.02325
So is the *Radius*, 90 deg.		10.
To the *Distance* A D, 3299 Miles		*r* 3.51839

VII. *The* Rumb, *the* Distance, *and one* Latitude *given*, *to find the other* Latitude, *and the* Difference *of* Longitude.

Suppose I sail W S S, 3 deg. 1 min. Westerly, 3299 Miles, and then find my self in the *Latitude* of 32 deg. 25 min. I demand the *Latitude* of the Place from which I came, and the *Difference* of *Longitude* between That and This.

First, For the *Difference* of *Latitude*,

As the *Radius*, 90 deg.		10.
Is to the *Distance* sailed, A D, 3299 Miles		3.51838
So is the *Co-sine* of the *Rumb* D A B, 18 d. 39 m.		9.50486
To the *Difference* of *Latitude* D B, 1055 Miles		*r* 3.02324

Which 1055 Miles converted into Degrees, is 17 deg. 35 min. the *Difference* of *Latitude* required, which added to 32 deg. 25 min. makes 50 deg. for the *Latitude* of the First Place.

The *Difference* of *Longitude* is found as before in the IV. *Problem*, thus:

As the *Radius*,
To the *Difference* of *Latitude* in *Meridional Parts*;
So is the *Tangent* of the *Rumb*,
To the *Difference* of *Longitude* in Minutes.
And thus the *Difference* of *Longitude* will be found, as in this Example, to be 70 deg.

VIII. *To*

VIII. *To convert the* Difference *of* Longitude *found in any* Parallel, *in* Degrees *and* Minutes, *into* Miles.

Admit there be two Places, A and B, both in the *Parallel* of 50 deg. of *Latitude*, which differ in *Longitude* 70 deg. I demand the *Distance* of those two Places.

It is to be understood, that the *Minutes* of *Longitude* in any *Parallel* are in proportion to the *Distance* in *Miles*, as the *Equinoctial* is to the *Parallel*; or, as the *Semidiametre* of the *One* is to the *Semidiametre* of the *Other*. That is,

As the *Radius*, 90 deg.	10.
Is to the *Co-sine* of the *Latitude*, 40 deg.	9.80807
So is the *Difference* of *Longitude* in Miles, 4200	3.62325
To the *Distance* in that *Parallel* 2700 Miles	*3.43132

By *Protraction*.

Let it be required to protract the two Places before mentioned, one in the *Latitude* of 50 deg. the other in the *Latitude* of 32 deg. 25 min. and differing in *Longitude* 70 deg. both by the *Plain*, and by the *True Sea Charts*.

First, Draw a Right-line, E Æ, representing the *Parallel* of the *Latitude* of the *Lizard*, 32 deg. 25 min. and upon it set 70 deg. the *Difference* of *Longitude* (in Miles 4200) from Æ to E.

Secondly, From Æ let fall a Perpendicular Æ C, representing the *Meridian* of *Bermudas*: And because the *Difference* of *Latitudes* is 17 deg. 35 min. and the *Meridional Parts* answering thereunto are 1417, set them from Æ to C, and draw the Line E C; so shall you have constituted a Right-angled Triangle Æ E C.

Again, Reduce the 17 deg. 35 min. the *Difference* of *Latitude*, into Miles, by multiplying of them by 60, and they make 1055, which set from C to A, and through A draw the Line A B Parallel, and of the same length with E Æ, (*viz.* 4200 Miles) and draw the Line B C, constituting another Right-angled Triangle A B C.

Now is C *Bermudas*, and B the *Lizard*, distant 4329 Miles, according to the *Plain Chart*; but C D the *Distance* 3299 Miles, according to the *True Chart*:—The *Difference* 1030 Miles.

And now is B C the *Rumb-line* between the two Places, according to the *Plain Chart*; but C E the *Rumb-line*, according to the *True Chart*; and the Angle B C A, 75 deg. 53 min. is the *Course*, according to the *Plain Chart*; but the Angle E C Æ, 71 deg. 21 min. is the *Course* according to the *True Chart*, differing 4 deg. 32 min.

Also, E Æ is the true *Difference* of *Longitude*, and A D the true *Meridian Distance* or *Departure*.

In this manner may all the other *Problems* before Calculated be Protracted; but this one may suffice, the *Protraction* differing nothing from that of the *Plain Chart*, only you must use the *Proper Difference* of *Longitude* and *Latitude*, in stead of the other.

PROBLEMS of SAILING

According to

MERCATOR's PROJECTION,

Trigonometrically Performed, and also wrought upon the *Chart* it self.

With a *Parallel* between the PLAIN and TRUE SEA-CHART.

PROBL. I. *How to make a* Sea-Chart (*both* General *and* Particular) *according to* Mercator's *Projection.*

A *Sea-Chart*, according to this *Projection*, may be made *General*, from the *Equinoctial* towards either of the *Poles*; or *Particular*, for any one designed *Voyage* between any two *Parallels* of *Latitude*; both which are to be made by help of the following *Table*, which is fitted for dividing of a *Meridian Line* (or *Scale*) upon a *Ruler*, as Mr. *Gunter* hath taught how to doe: And such a *Meridian Scale* may be conveniently set upon such a *Ruler*, as is before described, *Lib.* 3. *Part* 2. of *Practical Astronomy*. But let it suffice here to shew how to Project a *Sea-Chart* by the following Table.

A

TABLE

For the Dividing the

MERIDIANS

IN A

True Sea-Chart,

According to Mr. *Wright's* (vulgarly called *Mercator's*) PROJECTION.

A TABLE

A TABLE *for Dividing the* Meridians *in a True* Sea-Chart.

D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.
0 0	.00	6 0	6.01	12 0	12.09	18 0	18.30	24 0	24.73	30 0	31.47	36 0	38.63	42 0	48.36
6	.10	6	6.11	6	12.19	6	18.41	6	24.84	6	31.58	6	38.76	6	46.50
12	.20	12	6.21	12	12.29	12	18.51	12	24.95	12	31.70	12	38.88	12	46.63
18	.30	18	6.31	18	12.39	18	18.62	18	25.06	18	31.82	18	39.00	18	46.83
24	.40	24	6.41	24	12.49	24	18.72	24	25.17	24	31.93	24	39.12	24	46.90
30	.50	30	6.51	30	12.60	30	18.83	30	25.28	30	32.05	30	39.25	30	47.04
36	.60	36	6.60	36	12.70	36	18.93	36	25.39	36	32.17	36	39.37	36	47.17
42	.70	42	6.71	42	12.81	42	19.04	42	25.50	42	32.28	42	39.50	42	47.31
48	.80	48	6.82	48	12.91	48	19.15	48	25.61	48	32.41	48	39.63	48	47.44
54	.90	54	6.92	54	13.01	54	19.25	54	25.72	54	32.52	54	39.75	54	47.58
1 0	1.00	7 0	7.02	13 0	13.11	19 0	19.36	25 0	25.83	31 0	32.63	37 0	39.88	43 0	47.72
6	1.10	6	7.12	6	13.21	6	19.46	6	25.94	6	32.75	6	40.00	6	47.86
12	1.20	12	7.22	12	13.32	12	19.56	12	26.05	12	32.87	12	40.13	12	47.99
18	1.30	18	7.32	18	13.42	18	19.67	18	26.16	18	32.98	18	40.25	18	48.13
24	1.40	24	7.42	24	13.52	24	19.78	24	26.27	24	33.10	24	40.38	24	48 27
30	1.50	30	7.52	30	13.62	30	19.89	30	26.38	30	33.21	30	40.50	30	48.40
36	1.60	36	7.62	36	13.73	36	19.99	36	26.50	36	33.34	36	40.63	36	48.54
42	1.70	42	7.72	42	13.83	42	20.10	42	26.60	42	33.45	42	40.76	42	48.68
48	1.80	48	7.82	48	13.93	48	20.21	48	26.72	48	33.57	48	40.88	48	48.82
54	1.90	54	7.92	54	14.04	54	20.31	54	26.83	54	33.69	54	41.01	54	48 96
2 0	2.00	8 0	8 03	14 0	14.14	20 0	20.42	26 0	26.94	32 0	33.80	38 0	41.14	44 0	49 08
6	2.10	6	8.13	6	14.24	6	20.52	6	27.05	6	33.92	6	41.26	6	49.23
12	2.20	12	8.23	12	14.35	12	20.63	12	27.16	12	34 04	12	41.42	12	49.37
18	2.30	18	8.33	18	14.45	18	20.74	18	27.27	18	34.16	18	41.52	18	49.51
24	2.40	24	8.43	24	14.55	24	20 84	24	27.39	24	34.28	24	41.65	24	49.65
30	2.50	30	8.53	30	14.66	30	20.95	30	27.50	30	34 40	30	41.77	32	49.79
36	2.60	36	8.63	36	14.76	36	21.05	36	27.61	36	34.52	36	41.20	36	49.93
42	2.70	42	8.73	42	14 86	42	21.16	42	27.72	42	34.63	42	42.03	42	50 07
48	2.80	48	8.83	48	14 96	48	21.29	41	27.83	48	34 75	48	42.16	48	50.22
54	2.90	54	8.94	54	15.07	54	21.37	54	27.95	54	34.87	54	42.29	54	50 36
3 0	3.00	9 0	9.04	15 0	15.17	21 0	21 49	27 0	28.06	33 0	34.99	39 0	42 41	45 0	50.50
6	3.10	6	9.14	6	15.27	6	21.59	6	28.17	6	35.11	6	42.54	6	50.64
12	3.20	12	9.24	12	15.38	12	21.70	12	28 28	12	35.23	12	42 67	12	50.78
18	3.30	18	9.34	18	15.49	18	21.81	18	28.40	18	35.35	18	42.80	18	50.92
24	3.40	24	9.44	24	15.59	24	21.91	24	28.51	24	35.47	24	42.93	24	51.06
30	3.50	30	9 54	30	15.69	30	22.02	30	28.62	30	35.59	30	43.06	30	51.21
36	3.60	36	9.64	36	15.79	36	22.13	36	28.73	36	35.71	36	43.19	36	51.35
42	3.70	42	9.75	42	15.90	42	22.24	42	28.85	42	35.83	42	43.32	42	51.50
48	3.80	48	9.85	48	16.00	48	22.34	48	28.96	48	35 95	48	43.45	48	51 64
54	3.90	54	9.95	54	16.10	54	22.45	54	29.07	54	36.07	54	43.58	54	51.78
4 0	4.00	10 0	10.05	16 0	16.21	22 0	22.56	28 0	29.18	34 0	36.19	40 0	43.71	46 0	51.93
6	4.10	6	10.15	6	16.32	6	22.67	6	29.30	6	36.3	6	43.84	6	52.07
12	4.20	12	10.25	12	16.42	12	22.77	12	29.41	12	36.43	12	43.97	12	52.21
18	4.30	18	10.35	18	16.52	18	22.88	11	29.52	18	36.55	18	44.10	18	52.36
24	4.40	24	10.46	24	16.63	24	22.99	24	29.64	24	36.67	24	44.23	24	52.50
30	4.50	30	10.56	30	16.73	30	23.10	30	29.75	30	36.79	30	44.37	30	52.65
36	4.60	36	10.66	36	16.84	36	23.21	36	29.87	36	36.91	36	44 50	36	52.79
42	4.70	42	10.76	42	16 94	42	23.32	42	29.98	42	37.04	42	44.65	42	53.94
48	4.80	48	10.86	48	17.05	48	23.43	48	30.09	48	37.16	48	44.76	48	53.08
54	4.91	54	10.96	54	17.15	54	23.53	54	30.20	54	37.28	54	44.89	54	53.23
5 0	5.01	11 0	11.07	17 0	17.25	23 0	23.64	29 0	30.32	35 0	37.40	41 0	45.02	47 0	53.38
6	5.11	6	11.17	6	17.36	6	23.75	6	30.44	6	37.52	6	45.16	6	53 52
12	5.21	12	11.27	12	17.46	12	23.86	12	30.55	12	37.64	12	45.29	12	53.67
18	5.31	18	11.37	18	17.56	18	23.97	18	30.67	18	37.77	18	45.42	18	53.82
24	5.41	24	11.47	24	17.67	24	24.07	24	30.78	24	37.89	24	45.56	24	53.97
30	5.51	30	11.57	30	17 77	30	24.18	30	30.90	30	38 02	30	45.69	30	54 11
36	5.61	36	11 68	36	17.88	36	24 30	36	31.01	36	38.14	36	45 82	36	54.26
42	5.71	42	11.78	42	17.99	42	24.40	42	31.13	42	38.26	42	45.99	42	54.41
48	5.81	48	11.88	48	18.09	48	24 51	48	31.24	48	38.39	48	46.06	48	54.56
54	5.90	54	11.98	54	18.20	54	24.62	54	31.36	52	38.51	54	46.23	51	54 71

A TABLE for Dividing the Meridians *in a True* Sea-Chart.

D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.	D. M.	D. par.
48 0	54.86	54 0	64.41	60 0	75.45	66 0	88.72	72 0	105.58	78 0	129.07	84 0	168.94
6	55.01	6	64.58	6	75.65	6	88.97	6	105.90	6	119.56	6	169.90
12	55.16	12	64.75	12	75.85	12	89.22	12	106.13	12	130.06	12	170.90
18	55.31	18	64.92	18	76.05	18	89.47	18	106.55	18	130.54	18	171.90
24	55.46	24	65.09	24	76.26	24	89.71	24	106.89	24	131.03	24	172.90
30	55.61	30	65.27	30	76.46	30	89.97	30	107.22	30	131.53	30	173.94
36	55.76	36	65.44	36	76.67	36	90.22	36	107.51	36	132.04	36	174.99
42	55.91	42	65.61	42	75.87	42	90.47	42	107.69	42	132.54	42	176.06
48	56.06	48	65.78	48	77.07	48	90.72	48	108.23	48	133.05	48	177.16
54	56.21	54	65.96	54	77.28	54	90.98	54	108.66	54	133.57	54	178.27
49 0	56.37	55 0	66.13	61 0	77.49	67 0	91.23	73 0	108.91	79 0	134.09	85 0	179.41
6	56.52	6	66.30	6	77.69	6	91.49	6	109.25	6	134.60	6	180.57
12	56.68	12	66.48	12	77.99	12	91.75	12	109.59	12	135.19	12	181.75
18	56.83	18	66.66	18	78.11	18	92.00	18	109.94	18	135.68	18	182.96
24	56.98	24	66.83	24	78.37	24	92.26	24	110.29	24	136.23	24	184.19
30	57.13	30	67.01	30	78.52	30	92.52	30	110.64	30	136.77	30	185.45
36	57 29	36	67.19	36	78.74	36	92.78	36	110.96	36	137.32	36	186.74
42	57.44	42	67.36	42	78.95	42	93.05	42	111.35	42	137.88	42	188.06
48	57.60	48	67.54	48	79.16	48	93.31	48	111.70	48	138.44	48	189.41
54	57.74	54	67.72	54	79.37	54	93.58	54	112.06	54	139.01	54	190.79
50 0	57.91	56 0	67.90	62 0	79.58	68 0	93.84	74 0	112.43	80 0	139.58	86 0	192.21
6	58.06	6	68.07	6	79.79	6	94.21	6	112.79	6	140.16	6	193.66
12	58.22	12	68.26	12	80.01	12	94.38	12	113.16	12	140.75	12	195.15
18	58.38	18	68.44	18	80.22	18	94.65	18	113.52	18	141.34	18	196.68
24	58.53	24	68.62	24	80.41	24	94.92	24	113.89	24	141.93	24	198.25
30	58.69	30	68.80	30	80.66	30	95.19	30	114.27	30	142.54	30	199.86
36	58.85	36	68.98	36	80.87	36	95.47	36	114.64	36	143.14	36	201.53
42	59.00	42	69.16	42	81.09	42	95.74	42	115.02	42	143.76	42	203.24
48	59.16	48	69.34	48	81.51	48	96.02	41	115.40	48	144.38	48	205.00
54	59.32	54	69.53	54	81.33	54	96.29	54	115.79	54	145.01	54	206.82
51 0	59.48	57 0	69.71	63 0	81.75	69 0	96.57	75 0	116.17	81 0	145.65	87 0	208.70
6	59.64	6	69.89	6	81.97	6	96.85	6	116.56	6	146.29	6	210.65
12	59.80	12	70.08	12	82.19	12	97.11	12	116.95	12	146.94	12	212.67
18	59.96	18	70.26	18	82.42	18	97.42	18	117.34	18	147.60	18	214.74
24	60.12	24	70.45	24	82.63	24	97.70	24	117.73	24	148.26	24	216.90
30	60.28	30	70.64	30	82.86	30	97.98	30	118.13	30	148.93	30	219.16
36	60.44	36	70.82	36	83.08	36	98.27	36	118.53	36	149.61	36	221.50
42	60.60	42	71.01	42	83.31	42	98.56	42	118.94	42	150.30	42	223.94
48	60.76	48	71.19	48	83.53	48	98.85	48	119.34	48	151.00	48	226.48
54	60.92	54	71.48	54	83.76	54	99.14	54	119.75	54	151.71	54	229.15
52 0	61.09	58 0	71.57	64 0	83.99	70 0	99.43	76 0	120.17	82 0	152.42	88 0	231.95
6	61.25	6	71.76	6	84.22	6	99.72	6	121.58	6	153.14	6	234.89
12	61.41	12	71.95	12	84.45	12	100.01	12	121.00	12	153.88	12	237.99
18	61.57	18	72.14	18	84.68	18	100.31	11	121.40	18	154.62	18	241.20
24	61.74	24	72.33	24	84.91	24	100.61	24	121.84	24	155.37	24	244.74
30	61.90	30	72.52	30	85.14	30	100.91	30	122.27	30	156.13	30	248.44
36	62.07	36	72.71	36	85.37	36	101.21	36	122.70	36	156.90	36	252.40
42	62.23	42	72.90	42	85.60	42	101.51	42	123.13	42	157.58	42	256.65
48	62.40	48	73.10	48	85.14	48	101.81	48	123.57	48	158.48	48	261.24
54	62.56	54	73.29	54	86.07	54	102.12	54	124.01	54	159.28	54	266.23
53 0	62.73	59 0	73.49	65 0	86.31	71 0	102.43	77 0	124.45	83 0	160.09	89 0	271.70
6	62.90	6	73.68	6	86.55	6	102.73	6	124.90	6	160.92	6	277.75
12	63 06	12	73.87	12	86.79	12	103.04	12	125.35	12	161.76	12	284.51
18	63.23	18	74.07	18	87.03	18	103 35	18	125.80	18	162.61	18	292.19
24	63.40	24	74.27	24	87.27	24	103 67	24	126.26	24	163.47	24	301.06
30	63.57	30	74.46	30	87.51	30	103.98	30	126.72	30	164.35	30	311.56
36	63.73	36	74.66	36	87.75	36	104.30	36	127.18	36	165.24	36	324.45
42	63.90	42	74.86	42	87.99	42	104.62	42	127.65	42	166.14	42	341.16
48	64.07	48	75.06	48	88.23	48	104.94	48	128.12	48	167.06	48	365.04
54	64.24	54	75.26	54	88.48	54	105.26	54	128.60	54	167.99	54	408.01

I. *The*

I. *The Use of the Table in making a* General Sea-Chart *according to* Mercator's *Projection.*

FOR the manner of Division of the *Parallels* of *Longitude* upon the *Æquator*, let the *Æquator* Æ æ be drawn, and divided and crossed with *Parallel Meridians*, in all respects as in the *Plain Sea-Chart*: But for the dividing the *Meridians* you must have recourse to the following *Table*, and work in this manner. *Fig.* VIII.

Example 1. To draw the *Parallel* of 10 deg of *Latitude* from the *Equinoctial.*

Look in the Table for 10 deg. against which you shall find 10.05; this Number being taken out of the *Diagonal Scale* from which your *Equinoctial* Æ æ was divided, (or from the *Equinoctial* it self, it being so divided,) and set them upon the *Meridians* Æ P and æ S, from Æ to 10, and from æ to 10, and draw the Line 10 10 for the *Parallel* of 10 deg. of *Latitude* from the *Equinoctial*, either Northward or Southward.

Example 2. Let it be required to draw the *Parallel* of 40 deg. of *Latitude* from the *Equinoctial.*

Look into the Table for 40 deg. and against it you shall find 43.71 Parts, which take from the same *Diagonal Scale*, and set them upon the *Meridians* Æ P and æ S, from Æ to 40, and from æ to 40, and draw the Line 40 40 for the *Parallel* of 40 d. of *Latitude.*

Example 3. To describe the *Parallel* of 55 deg. of *Latitude.*

Look in the Table for 55 deg. against which you find 66.13, which taken out of the *Scale*, (or divided *Equinoctially*) it will reach from Æ to F, and from æ to E; so the Line F E being drawn, it shall be the *Parallel* of 55 deg of *Latitude*: And so for any other.

II. *The Use of the foregoing Table in making a* Particular Sea-Chart, *according to* Mercator's *Projection.*

TO make a *Particular Sea-Chart* according to this *Projection*, first, draw the Line E F, serving for the First *Meridian*, and cross it with two *Parallels*, F G and E H, the one at the upper, the other at the lower end of the *Chart*, which may serve for the extreme *Parallels* of *Latitude* between which your *Navigation* will extend. *Fig.* IX.

Then consider at what *Latitude* the *Chart* is to begin and end; so this *Chart* here intended is between the *Latitudes* of 49 deg. and 57 deg. I therefore look into the Table, and find that 49 deg. of *Latitude* must be drawn at 56.37 Parts, and 57 deg. of *Latitude* at 69.71 Parts of the *Equinoctial*: Subtract 56.37 from 69.71, the Remainder will be 13.34, which is the *Meridional Distance* between the *Latitude* of 49 deg. and 57 deg. This done, take the Line E F in your Compasses, and measuring it upon a *Diagonal Scale*, you will find it to contain 12.75. Then say by the Rule of Proportion;

As 13.34, the *Meridional Distance* between the two *Latitudes*,
Is to 1000;
So is 12.75, the *Meridional Distance* between the two *extreme Latitudes*,
To 0.92, the *Parallel Distance* of the *Meridians.*

Wherefore, take 0.92 out of the same *Diagonal Scale* upon which you measured the Line E F, and with that Extent set off the other *Meridians* at both ends of the *Chart*, at 1, 2, 3, *&c.* and through those Points draw the *Meridians* 1, 1,—2, 2,—3, 3, parallel to the first *Meridian* E F, and number them with 1, 2, 3, *&c.* and divide each Space between 1 and 2, *&c.* into 10 or 100 Equal Parts; for it is supposed every of them to be divided into 1000 Equal Parts.

The *Meridians* being drawn, and the *Parallels* of the two *extreme Latitudes* divided, as is before said, proceed we now to prick down the other intermediate *Parallels* of *Latitude*, as followeth.

Look in the Table for 49 deg. against which is 56.37: Also look for 50 deg. against which is 57.91, their Difference is 1.54; which taken out of the divided *Parallel* of 49 deg. E H, will reach upon the *Meridians* E F and H G, from 49 to 50: so the Line 50 50 shall be the *Parallel* of 50 deg. of *Latitude* from the *Equinoctial.*

Then for the *Parallel* of 51 deg.

The Number against 51 deg. is	59.48
From which subtract the Merid. Distance of 49 deg.	56.37
The Remainder is	3.11

Which taken out of the divided Parallel of 49 deg. will reach upon the *Meridians* E F and H G to 51 and 51; ſo the Line 51 51 being drawn, ſhall be the Parallel of 51 deg. of *Latitude* from the *Equinoctial*: And ſo, by continual ſubtracting of 56.37 (the *Meridional Diſtance* of the leſſer given *Latitude* 49 deg.) from the *Meridional Diſtance*, againſt every Degree of *Latitude* in the *Table*, they ſhall give Points whereby to prick down all the Parallels beyond 49 deg. towards either *Pole*: As in this little *Table* for this *Chart*.

D.				
From the *Meridional Diſtance* of	50	57.909	Subtract 56.369, the *Meridional diſtance* of 49 deg. of *Latitude*, the remainder	1.540 Shall be the *Diſtance* of the reſpective Parallels of *Latitude* upon the Firſt *Meridian* from E and H, towards F and G.
	51	59.481		3.112
	52	61.088		4.719
	53	62.730		5.361
	54	64.412		8.043
	55	66.134		9.765
	56	67.900		11.531
	57	69.712		13.343

Viz.

For the Sub-diviſions of theſe Degrees of the *Meridian* they may be divided each of them into equal Parts, as the Diviſions at the top and bottom of the *Chart* ought to be; but the Degrees of the *Meridian*, as they grow higher, they ought ſtill to grow greater. But the difference is ſo ſmall, that it cannot produce any conſiderable Error, though the Sub-diviſions be all made equal between Degree and Degree. You may therefore divide them either into 60 Minutes or *Engliſh* Miles, or into 20 Leagues, or into 100 parts of Degrees, as you ſhall beſt like of.

Your *Chart* being thus prepared, I will now come to ſhew you how to reſolve ſeveral *Problems* upon it.

II. *To find how many Leagues do anſwer to one Degree of* Longitude *in every ſeveral* Latitude.

UPon the two edges of your *Protracting Quadrant* there are two Lines, the one divided into 20, the other into 60 equal parts.

Take therefore the leaſt diſtance from the Complement of the Parallel's diſtance from the *Æquator*, (or the Complement of the given *Latitude*:) this diſtance being meaſured upon the edge that is divided into 20, ſhall ſhew you what number of Leagues make one Degree of *Longitude* in that Parallel of *Latitude*. And the ſame Diſtance, being meaſured upon the other edge that is divided into 60, will give ſo many of our Miles, or ſo many Minutes of the *Æquinoctial*, or any other great Circle, as are anſwerable to one Degree of *Longitude* in that *Latitude*.

Example. Let it be required to find how many Leagues do anſwer to one Degree of *Longitude* in the *Latitude* of 18 deg. 12 min.

Set one Foot of your *Compaſſes* in 71 deg. 48 min. the Complement of the given *Latitude*, and with the other take the neareſt Diſtance to the ſide of the *Quadrant* which is divided into 20: that Diſtance, meaſured upon the Line 20, will reach from the beginning thereof to 19: and ſo many Leagues do anſwer to one Degree of *Longitude* in the *Latitude* of 18 deg. 12 min.

Or, if you take the leaſt Diſtance from 18 deg. 12 min. the *Latitude* it ſelf, in the Limb of the *Quadrant*, to that edge which is divided into 60, that Diſtance will alſo reach to 19 upon the Line 20, as before.

And the ſame Diſtance, being meaſured upon the Line 60 of the *Quadrant*, will give you 57 parts: and ſo many Minutes of the *Æquator* are anſwerable to one Degree of *Longitude* in the Parallel of 18 deg. 12 min. of *Latitude*.

So likewiſe in the *Latitude* of 25 deg. 15 min. if you take the leaſt Diſtance from the Complement thereof, or from the *Latitude* it ſelf, to the edges of the *Quadrant*, you ſhall find that Diſtance to reach 18 in the Line of 20: and ſo many Leagues do anſwer to one Degree of *Longitude* in the *Latitude* of 25 deg. 15 min. or unto 54 in the Line of 60: and ſo many Minutes of the *Æquator* do anſwer to one Degree of *Longitude* in that Parallel of *Latitude*.

The Analogie or Proportion is,

As the *Radius*,	10.
Is to the Co-Sine of the *Latitude* 64 deg. 45 min.	9.95638
So is 20 Leagues,	1.30103
To 18 Leagues,	11.25741
And	
So is 60 Minutes or Miles,	
To 54 Minutes or Miles.	

III. *By the* Latitude *of two Places and their Distance, to find the* Rumb.

The Analogie or Proportion.

As the *Distance* upon the *Rumb*, 6 deg. *Mer. Parts* 361,	2.55751
Is to the *Radius*, 90 deg.	10.
So is the *Difference* of *Latitude*, 5 deg. *Mer. Parts* 300,	12.47712
To the Co-Sine of the Rumb, 56 deg. 15 min.	9.91961

Thus if the Places given were one in the *Latitude* of 50 deg. and the other in the *Latitude* of 55 deg. and the *Distance* upon the *Rumb* 6 deg. or 120 Leagues; the *Rumb* leading from one to the other will be found to be the third from the *Meridian*, namely N E by N. 33 deg. 45 min.

Upon the Chart.

LET A represent the Place in the *Latitude* of 50 deg. and C that in 55 deg. whose *Distance* from A to C is 6 deg. Take 6 deg. out of the *Meridian-line*, by setting one Foot as much below the lesser *Latitude* as above the greater, which will be from K in the *Latitude* of 49½ deg. to L in the *Latitude* of 55½ either of which are half a Degree above and under the two given *Latitudes*. Take this *Distance* K L in your *Compasses*, and setting one foot in A, (the lesser *Latitude*) with the other cross the Parallel of the greater *Latitude* 55 deg. in the Point C, and draw a Right-Line from A to C. So shall the quantity of the *Angle* B A C, being found, (either by your *Chord* or *Quadrant*,) shew you the Inclination of the *Rumb* to the *Meridian* to be 33 deg. 45 min. the N E by N. Point.

Note, *That in the Propositions following, the Difference of* Longitude *must always be taken out of the* Æquator, *and measured thereupon also. But the Difference of* Latitude *and Distance upon the* Rumb *must always be measured upon, and taken out of the* Meridian-Line *of your* Chart. *And hereafter I shall call them the proper Difference, and proper Distance.*

IV. *The* Longitude *and* Latitude *of two Places being given to find the* Rumb.

The Analogie or Proportion.

As the *Proper Difference* of *Latitude* 5 deg. *Mer. Parts* 493,	2.69285
Is to the *Radius*, 90 deg.	10.
So is the *Difference* of *Longitude* 5 deg. 30. *Mer. Parts* 330,	12.51851
To the *Tangent* of 33 deg. 45 the *Rumb*.	9.82566

Thus if the Places should lie one in the *Latitude* of 50 deg. and the other in the *Latitude* of 55 deg. and the *Difference* of *Longitude* between them were 5 deg. 30 min. the *Rumb* leading from one Place to the other will be found to be the third from the *Meridian* N E by N. 33 deg. 45 min.

Upon the Chart.

THE *Meridians* and *Parallels* being drawn through the two Places at A and C, and a streight Line from A to C, for the *Rumb*, by your *Chord* or *Quadrant* find the quantity of the *Angle* B A C, which you will find to be 33 deg. 45 min. or the third *Lumb* from the *Meridian* N E *by* N.

But if this *Rumb* were to be found by the Common *Sea-Chart*, it would be found to be above 47 deg. that is, N E 2 deg. Easterly; that is, one whole Point and 2 deg. more Easterly than it should be.

V. *The*

V. *The* Latitude *of two Places, and the* Rumb *being given, to find the Difference of* Longitude.

The Analogy or Proportion.

As the *Radius*	10.
Is to the *Tangent* of the *Rumb*, 33 deg. 45 min.	9.82489
So is the *Proper Difference* of *Latitudes*, *Mer. Parts* 493,	2.69285
To 330, the *Differenee* of *Longitude* 6 deg. 30 min.	*x* 2.51774

Thus the *Latitude* of one Place being 50 deg. and the other 55 deg. and the *Rumb* leading from one to the other being the third from the *Meridian*, the *Difference* of *Longitude* will be found to be $5\frac{1}{2}$ deg.

Upon the Chart.

LET a *Meridian* be drawn through A, and a Parallel of *Latitude* through C. Then upon the *Angle* A Protract the *Angle* of the *Rumb* 33 deg. 45 min. So the *Distance* B C upon the Parallel, being measured upon the bottom of the *Chart*, will be found to contain 6 deg. 30 min.

But if this *Difference* of *Longitude* were to be found by the *Plain Sea-Chart*, the *Difference* of *Longitude* would be found to be but 3 deg. 20 min. which is more than 3 deg. less than the truth: And yet this Error would be yet greater, if either the *Latitude* be greater, or the *Rumb* farther from the *Meridian*.

VI. *The Difference of* Longitude *of two Places, the* Latitude *of one of them, and the* Rumb *leading from one to the other given, to find the* Latitude *of the other Place.*

The Analogy or Proportion.

As the *Radius*,	10.
To the *Co-Tangent* of the *Rumb*, 56 deg. 15 min.	10.17511
So is the *Difference* of *Longitude* 6 deg. 30 min. (or 330 *Parts*,)	2.51851
To 493, the *Proper Difference* of *Latitude* in *Parts*.	*x* 2.69362

Thus if the *Latitude* of one of the Places were 50 deg. the *Rumb* leading from that to the other N E *by* N. 33 deg. 45 min. and the *Difference* of *Longitude* between the two Places were 5 deg. 30 min. the *Latitude* of the other Place will be found to be in 55 deg.

Upon the Chart.

LEt A B and D C be two *Meridians* drawn through A and C, at $5\frac{1}{2}$ deg. the *Difference* of *Longitude*, and a Parallel of *Latitude* through A, crossing the *Meridian* C D in D: Then upon the Point A Protract an *Angle* equal to the *Rumb* from the *Meridian* given 33 deg. 45 min. So the Line C D, being measured upon the *Meridian* from A, the given *Latitude*, 50 deg. will reach to 55 deg. the proper *Difference* of *Latitude*. So that the other Place lies in the *Latitude* of 55 deg.

But if this *Difference* of *Latitude* were to be found by the *Plain Sea-Chart*, this *Difference* of *Latitude* would be found to be 8 deg. 13 min. and the *Latitude* sought would be found to be 58 deg. 13 min. above 3 deg. more than the truth; as by the *Triangle* for that purpose drawn upon the *Plain Sea-Chart*, marked with T V E may appear.

VII. *Having the* Latitude *of one Place, the* Rumb *leading from that Place to another unknown, and the* Distance *upon the* Rumb *from the first to the second Place, to find the* Difference *of* Longitude *of the two Places.*

The Analogy or Proportion.

As the *Radius*,	10.
Is to the Sine of the *Rumb* from the *Meridian*, 33 deg. 45 min.	9.74474
So is the *Proper Distance* upon the *Rumb* 6 deg. *Parts* 593,	2.77305
To 330 min. (or 5 deg. 30 min.) the *Difference* of *Longitude*.	*x* 2.51779

Thus

Thus if the two Places were one in the *Latitude* of 50 deg. and the other in a greater *Latitude*, but unknown; the *Proper Distance* upon the *Rumb* leading from one Place to the other being 6 deg. and the *Rumb* N E by N. 33 deg. 45 min. the *Difference* of *Longitude* will be found to be $5\frac{1}{2}$ deg.

Upon the Chart.

THrough the Point A in the *Latitude* of 50 deg. let be drawn a *Meridian* A B, and a Parallel A D; and upon the Point A Protract an *Angle* equal to the *Rumb* from the *Meridian* 33 deg. 45 min. Then take with the *Compasses* 6 Degrees, the *Proper Distance* upon the *Rumb*, out of the *Meridian-line*, (having respect to the *Latitude* of the Places,) as from K to L, and set that *Distance* upon the *Rumb* from A to C. Then through C draw another *Meridian* C D, crossing the Parallel drawn through A in the Point D. So the Line A D, being measured at the bottom of the *Chart*, will be found to contain $5\frac{1}{2}$ deg. the *Difference* of *Longitude* sought.

But if this *Difference* of *Longitude* had been to be found by the common *Sea-Chart*, it would be found to have been onely 3 deg. 20 min. which is 2 deg. 10 min. less than the truth; as in the *Plain Chart* may be seen, where the Third *Rumb* from the *Meridian* cuts the Parallel of 55 deg. of *Latitude* in 3 deg. 20 min. of *Longitude* at the Point X.

VIII. *The* Difference *of* Longitude *between two Places, the* Rumb *leading from one Place to the other, and the* Latitude *of one of the Places being given, to find their* Distance.

The Analogy or Proportion.

As the Sine of the *Rumb* 33 deg. 45 min.	9.74474
Is to the *Radius*,	10.
So is the *Difference* of *Longitude* 5 deg. 30 min. (in Parts 330.)	12.51851
To 6 deg. the proper *Distance* upon the *Rumb*, (or 593.)	2.77377

Thus, if the *Latitude* of one Place were in 50 deg. the other in a greater *Latitude* unknown, the *Difference* of *Longitude* between the two Places $5\frac{1}{2}$ deg. and the *Rumb* N E by N. 33 deg. 45 min. from the *Meridian*; the proper *Distance* upon the *Rumb* will be found to be 6 degrees.

Upon the Chart.

LET two *Meridians*, A B and C D, be drawn through A and C, according to the *Difference* of *Longitude*, and a Parallel of *Latitude* through A, crossing the *Meridian* C D in the Point D. Then upon the Point A Protract an *Angle* of 33 deg. 45 min. the quantity of the *Rumb* from the *Meridian*, and draw the Line A C crossing the *Meridian* C D in C. So the *Distance* C D, being taken in the *Compasses*, and measured upon the *Meridian-line* of the *Chart*, (respect being had to the *Latitude* of Places) that is, so much above the greater *Latitude* as below the lesser *Latitude*, you will find it to contain 6 deg.

But if this setting of the *Compasses* so much above one *Latitude* as below another seem difficult, it may be thus otherwise done.—— For, the *Rumb-line* being drawn, it will cut the *Meridian* C D in C: so a Parallel drawn through C will cut the *Meridian* A B in B: so is B the *Latitude* of the second Place, *viz.* 55 deg. Then divide the *Distance* between the two *Latitudes* A and B in two equal Parts in the Point M; also divide the *Rumb-line* A C in two equal Parts in N: Then take the *Distance* N C or N A, and setting one foot of the *Compasses* in M, the other will reach to L above the greater *Latitude*, and from M to K as much below the lesser *Latitude*, namely, 30 min. or half a Degree on either side; so that between K and L are contained 6 deg. and that is the proper *Distance* upon the *Rumb*.

But if this *Distance* were to be found by the *Plain Chart*, it would be almost 10 deg. or 197 Leagues, which is 77 Leagues more than in truth it should be: As may appear, if you measure the Line A L in the *Plain Chart*, upon the side thereof.

IX. *The Difference of* Longitude, *and Distance of two Places, with the* Latitude *of one of the Places, being given, to find the* Rumb *that leads from one to the other.*

The Analogy or Proportion.

As the proper *Distance* upon the *Rumb* 6 deg. (or 593,)	2.77305
Is to the *Difference* of *Longitude* 5 deg. 30 min. (or 330,)	12.51851
So is *Radius*,	10.
To the Sine of 33 deg. 45 min. the *Rumb* from the *Meridian*.	9.74546

Thus, if one of the Places lay in the *Latitude* of 50 deg. and the other in a greater *Latitude*, but unknown; the *Difference* of *Longitude* between them $5\frac{1}{2}$ deg. and their proper *Distance* upon the *Rumb* 6 deg. the Inclination of the *Rumb* to the *Meridian*, which leadeth from one Place to the other, will be found to be 33 deg. 45 min. that is the N E by N Point.

Upon the Chart.

LEt the *Meridians* A B and D C be drawn through A and C, and through A a Parallel of *Latitude* A D: Then open the *Compasses* (having respect to the *Latitudes*) from K to L, the quantity of 6 deg. in the *Meridian*; and setting one foot of that Extent in A, with the other foot cross the *Meridian* C D in C, and draw the Right-Line A C for the *Rumb*. Lastly, By your *Chord* or *Quadrant* find the quantity of the *Angle* B A C, 33 deg. 45 min. and that is the *Rumb* required N E by N.

But if you were to find this *Rumb* by the *Plain Sea-Chart*, it would be found almost the E N E Point within 1 deg. 30 min. differing from truth near 3 whole Points to the Eastward.

X. *The* Longitude *and* Latitude *of two Places being given, to find the Distance upon the* Rumb.

The Analogy or Proportion.

As the proper *Difference* of *Latitude* 493,	2.69285
Is to the *Difference* of *Longitude* 330,	12.51851
So is *Radius*,	10.
To the *Tangent* of 33 deg. 45 min. the *Rumb*.	9.82566
And	
As the Sine of the *Rumb*, 33 deg. 45 min.	9.74474
Is to the *Radius*,	10.
So is the *Difference* of *Longitude* 5 deg. 30 min. (or 330,)	12.51851
To 593, (or 6 deg.) the proper *Distance* upon the *Rumb*.	2.77377

Thus, the two Places being one in the *Latitude* of 50 deg. the other in the *Latitude* of 55 deg. and the *Difference* of *Longitude* between them being $5\frac{1}{2}$ deg. the proper *Distance* upon the *Rumb* will be found to be 6 deg.

Upon the Chart.

DRaw the *Meridians* A B and C D, the *Difference* of *Longitude* between them being $5\frac{1}{2}$ deg. and through A and B draw two Parallels B C and A D, and then the Line for the *Rumb* leading from the one to the other A C. So A C, being taken in the *Compasses*, and measured upon the *Meridian-line* of the *Chart*, with this condition, that at the resting of the *Compasses* upon the *Meridian-line*, one foot be so many Degrees above the greater *Latitude* as the other foot is below the lesser *Latitude*; so will the feet of the *Compasses* rest in the Points K and L, one being 30 min. below the lesser *Latitude*, and the other 30 min. above the greater.

But if this *Distance* upon the *Rumb* were to be found by the *Plain Chart*, it would be found to be almost 7 deg. 15 min. or 245 Leagues, which is 25 Leagues more than it should be.

XI. *The*

XI. *The* Latitude *of two Places, and their Distance upon the* Rumb *being given, to find their Difference of* Longitude.

The Analogy or Proportion.

As the proper *Distance* upon the *Rumb* 593,	2.77305
Is to the proper *Difference* of *Latitude* 493,	12.69285
So is *Radius*,	10.
To the Sine of 56 deg. 15 min. the Comp. of the *Rumb*:	9.91980

And

So is the Sine of the *Rumb* from the *Meridian*, 33 deg. 45 min.
To the *Difference* of *Longitude*, 330.

Thus, if one of the Places be in the *Latitude* of 50 deg. and the other in 55 deg. and their proper *Distance* upon the *Rumb* 6 deg. or 120 Leagues; their *Difference* of *Longitude* will be found to be 5 deg. 30.

Upon the Chart.

DRaw A D and B C, two Parallels of *Latitude*, through 50 deg. and 55 deg. which were the two given *Latitudes*: Then out of the *Meridian-line* take the proper *Distance* upon the *Rumb* (having respect to both *Latitudes*) from K to L: the *Compasses* being opened to this *Distance*, one foot being set in A, the lesser *Latitude*, the other will cross the Parallel of the greater *Latitude* in C. So the *Distance* B C, being measured at the bottom of the *Chart* from E, will reach to 5 deg. 30 min. and such is the *Difference* of *Longitude* between the two Places.

But if this *Difference* of *Longitude* were to be found by the *Plain Chart*, it would be but 3 deg. 20 min. which is no less than 2 deg. 10 min. less than the truth; as by the *Triangle* T V E, drawn upon the *Plain Chart*, may appear.

XII. *The Difference of* Longitude *of two Places, their Distance upon the* Rumb, *and the* Latitude *of one of the Places being given, to find the Difference of* Latitudes

The Analogy or Proportion.

As the proper *Distance* of the two Places upon the *Rumb*, 593	2.77305
Is to *Radius*,	10.
So is the *Difference* of *Longitude*, 330,	12.51851
To the Sine of 33 deg. 45 min. the *Rumb*:	9.74546

And

So is the Co-sine of the *Rumb*, 56 deg. 15 min.
To 493, the proper *Difference* of *Latitudes*.

Thus, the *Difference* of *Longitudes* being 5½ deg. their proper *Distance* upon the *Rumb* 6 deg. and the *Latitude* of one of the Places 50 deg. the *Difference* of *Latitudes* will be found to be 5 deg.

Upon the Chart.

THrough the given *Latitude* A draw a *Meridian* A B, and a Parallel A D, and upon the Parallel set the *Difference* of *Longitude* 5½ deg. taken from the bottom of the *Chart*, from A to D, and through D draw the *Meridian* D C: Then out of the *Meridian-line* take the proper *Distance* upon the *Rumb*, 6 deg. from K to L, and setting one foot of the *Compasses* in A, with the other cross the *Meridian* C D in C: so a Parallel of *Latitude* drawn through C will be the Parallel of 55 deg. So is 55 deg. the *Latitude* of the other Place, and 50 being taken from 55, leaves 5 deg. for the *Difference* of *Latitudes* required.

Which *Difference*, had it been to be found by the *Plain Chart*, would have been but 2 deg. 25 min. that is, 2 deg. 35 min. less than the truth; as by the *Triangle* T V E upon the *Plain Chart* may appear.

XIII. *The*

XIII. *The Latitude of two Places, and their Difference of Longitudes being given, to find the Rumb leading from one to the other, and alſo how many Degrees diſtant they are aſunder.*

THis Propoſition is already performed in the Example of the two Places A and B; but for variety I will take two other Places, and only ſhew the manner of working upon the Chart.

Suppoſe then two Places, one (as before) in the Latitude of 50 deg. the other in the Latitude of 52 deg. 30 min. whoſe Difference of Longitudes is 6 deg.

Upon the Chart.

Through the two given Latitudes, 50 deg. and 52 ½, at A and O draw two Parallels, O P and A D, upon which ſet the Difference of Longitudes from O to P, and from A to Q, 6 deg. Then draw the Line A P, which ſhall be the Line of the Rumb leading from one Place to the other: Wherefore, by your Chord or Protracting Quadrant find the Quantity of the Angle O A P, which ſhall be the Inclination of the Rumb to the Meridian, and will be found to be 56 deg. 15 min. that is, the N E by E Point; which was the firſt thing that was required.

Then to find the proper Diſtance upon the Rumb, take the Line A P in your Compaſſes, and meaſure it upon the Meridian-line, ſo that one Foot may be above the greater Latitude ſo much as the other is below the leſſer; and you will find the Compaſs-points to reſt in E and S, E being one whole Degree below the leſſer Latitude, and S one Degree above the greater: So that there is intercepted between E and S 4 ½ deg. and that is the proper Diſtance upon the Rumb, which was the ſecond thing required.

But if this Problem had been wrought upon the Plain Chart, the Rumb from the Meridian would be found to be 67 deg. 23 min. that is, within 7 min. of the ſixth Rumb; which is more than the truth by 11 deg. 8 min. almoſt a whole Point of the Compaſs.

XIV. *A Ship ſet ſail from the Latitude of 50 deg. upon the Fifth Rumb, N E by E; after that ſhe had made 36 Leagues of Way upon that Rumb, the Wind changing, ſhe was conſtrained to ſail 50 Leagues upon the Seventh Rumb E by N; I would know in what Longitude and Latitude the Ship is.*

Upon the Chart.

THe Rumb-line A P being drawn, ſet off thereupon 36 Leagues, (which was the Way that the Ship made upon the fifth Rumb before the Wind changed,) from A to T, (which Diſtance muſt be taken out of the Meridian-line by opening the Compaſſes from 50 deg. to 51.48, or better, to as much below 50 deg. as above 51 deg.) ſo ſhall the Point T be the Place that the Ship was in when the Wind altered: So a Parallel drawn through T upon the Chart, will cut the Meridian at V in 51 deg. and in that Latitude the Ship was. Now to find in what Longitude ſhe was, take in your Compaſſes the Line T V, and meaſure it at the bottom of the Chart, you ſhall find it will reach from E to 2 deg. 21 min. and in that Longitude the Ship then was.

This done, upon the Point T (where the Wind changed, and drove the Ship two Points more Eaſtwardly, namely, upon the E by N Point) protract an Angle of 22 deg. 30 min. namely, the Angle P T X, which is the Rumb upon which the Ship ſailed 50 Leagues after the Wind changed: Therefore take 50 Leagues out of the Meridian-line, and ſet them from T to X; ſo ſhall X be the Place that the Ship was in after ſhe had ſailed 50 Leagues upon the E by N Point; which, by drawing a Parallel through K, will be found in the Latitude of 51 deg. 30 min. and by drawing of a Meridian through K alſo, it will be found to be in the Longitude of 6 deg. 16 min.

But if theſe Courſes had been protracted according to the Plain Sea-Chart, the Point T would fall in the Latitude of 51 deg. and the Point X in the Latitude of 51 deg. 30 min. But the Longitude of T would be only 1 deg. 30 min. and the Longitude of X is 3 deg. 37 min. Both theſe Longitudes being added, make but 5 deg. 27 min.

27 min. for the Difference of Longitude between X and the first Meridian; whereas by the other Chart it is 6 deg. 16 min. So that the Ship at X is 33 min. Westward of the Place to which she was bound.

These Differences, which I have observed to be between the Plain and *Mercator's* Chart, may be seen by comparing the Scheme of the two Charts together.

Concerning a Sea-Voyage, and how to keep a Journal *for that purpose.*

LET the Book in which you keep your Account be in *Folio*, every Right-hand Page being ruled and divided as is done in the Journal following; and let the Left-hand Page against the same be blank, to insert therein the Occurrences which you meet with every day, or at any time, as Occasion offers; as of *Winds*, *Currents*, *Variation* of the *Compass*, *Storms*, and the *Death* of any *Person*, *&c.*

Then write the *Title* of your *Journal*, as from whence you came, in what *Latitude*, whither bound, and in what *Latitude*, *&c.* as you see in the Title of the *Journal* following.

In the *Journal* following there are Eleven Columns, which contain as followeth, *viz.*

The First contains the *Days* of the *Month*.

The Second, the *Month's Name*, and the *Latitude* that *Day*, as it was found by *Observation*.

The Third, the *Course*, as it is corrected by Allowance for *Lee-way*, or *Variation* of the *Compass*, if there be any.

The Fourth, the *Distance* sailed.

The Fifth, Sixth, Seventh, and Eighth, the *Easting*, *Westing*, *Northing*, and *Southing*; which are the *Differences* of *Latitude* and *Departure* upon the several *Courses* and *Distances*.

The Ninth is the *Latitude* found by the *Dead Reckoning*.

The Tenth and Eleventh, the *East* or *West Longitude*.

Every one that intends to keeps a true and just Account of the Ship's Way, whensoever demanded, ought (in a Private Book) to take off from the *Log-board*, every day at Noon, (1.) The *Time*. (2.) The *Course* the Ship made her Way upon. (3.) The *Way* the Ship hath made in that time in Knots and parts of a Knot of the *Log-line*. (4.) The *Depths* and *Soundings*. (5.) The *Wind*. And when Opportunity will permit, make Observation of your *Latitude*, and compare the *Observed Latitude* by your *Dead Latitude*, and reconcile them, and then accordingly make your *Entry* in your *Journal*: And the Manner of proceeding in the keepiug of this *Journal*, may be as followeth.

THE

Form of the Journal.

A Journal *of our Voyage intended, by God's Assistance, from the* Lizard, *in* 50 *deg. of North Latitude, to the Island of* Barbadoes, *in North Latitude* 13 *deg.* 12 *min. The Difference of Longitude between the* Lizard *and* Barbadoes *is* 52 *deg.* 58 *min. the Course S W ½ W,* March 20. 1684.

Days.	Month and Latitude by Observat.	The Courses Corrected.	Dist. in miles	Northing. Miles.	Southing. Miles.	Easting. Miles.	Westing. Miles.	Latitude by Reckoning.	Longitude. East. Miles.	Longitude. West. Miles.
	D. M.							D. M.		
22	March.	*Lizard* South	30		30.0			49 30		
23		South West	112		79.2		79.2	48 11		120.0
24	47 00	S W *by* S ½ W	72		55.6		45.7	47 15		68.1
	Correct by Observation.				15.0		11.0	47 00		16.0
25		S W *by* S	96		79.8		53.3	45 40		75.2
26		S W *by* S	79		65.7		43.9	44 34		62.8
27	42 37	S W *by* S	138		114.6		76.7	42 39		106.2
28		S S W ¼ W	120		110.7		66.3	40 48		88.8
29		S S W	72		66.5		27.6	39 41		36.4
30	37 44	S S W ½ W	102		90.0		48.0	38 11		62.0
	Correct by Observation.				27.0		16.0	37 44		20.0
1	*April.*	South West	85		60.1		60.1	36 44		75.0
2		W S W	20		7.6		18.5	36 36		24.1
3		S *by* W ½ W	32		30.6		9.3	36 05		11.9
4	34 10	S *by* W	120		117.7		23.4	34 07		27.9
	Correct by Observation.			3.00		1.00		34 10	1.00	
5		S *by* W ½ W	164		156.9		47.6	31 33		56.7
6	29 37	S S W	125		115.5		47.9	29 37		55.9
7		S W *by* S	110		91.4		61.2	28 06		69.5
8		S W *by* S	122		101.4		67.8	26 25		76.2
9	24 43	South West	116		82.0		82.0	25 03		91.0
	Correct by Observation.				20.0		11.0	24 43		12.0
10		South West	97		68.6		68.6	23 34		75.0
11		South West	96		67.9		67.0	22 26		74.0
12		South West	114		80.6		80.6	21 05		88.0
13		South West	118		83.5		83.5	19 41		89.0
14		S W *by* W	110		61.2		91.4	18 40		96.2
15		W S W	91		34.8		84.1	18 05		89.4
16		W S W	91		34.8		84.1	17 30		87.0
17		W S W	84		32.1		77.6	16 58		82.0
18		W S W	107		41.0		98.9	16 17		103.7
19		W S W	103		39.5		95.2	15 37		99.0
20	14 43	W S W	116		44.4		107.2	14 53		111.0
	Correct by Observation.				10.0		15.9	14 43		16.0
21		W *by* S	110		21.5		107.9	14 21		114.1
22		W *by* S	120		23.4		117.7	13 58		119.1
23		W *by* S	106		20.7		104.0	13 37		108.7
24		W *by* S	100		19.5		98.1	13 17		103.8
25		W *by* S	120		23.4		117.7	12 54		118.7
26	13 12	W *by* N	100	19.50			98.1	13 14		103.1
27		West	130				130.0	13 12		133.5
28		West	136				136.0	13 12		139.0
29	13 12	West	69				69.0	13 12		70.8
The whole Difference of Longitude is 3178 Miles.									1.00	3176.8

Example.

EXAMPLE.

On the 22d of *March*, at Noon, I find the *Lizard* to bear North, and to be distant 30 miles; therefore I am to the Southward of the *Lizard* 30 Miles, which I place in the South Column of my *Journal*, and that makes my *Latitude* to be 49 deg. 30 min. which I place in the Ninth Column of my *Journal*.

The 23d day the Course is South-West, and the *Distance* sailed 112 Miles: Having these two, you may by the *Difference* of *Latitude* and *Departure*, found by the *Traverse-Table* in *Section* I. before-going, find the Difference of *Latitude* to be 79.2, and the Departure the same; and (because the Course is South-Easterly) I place the Difference of *Latitude* 72.2 in the South Column, and the Departure 79.2 in the West Column, which in Degrees is 1 deg. 19 min. which subtracted from 49 deg. 30 min. (the *Latitude* I was in the day before) there remains 48 deg. 11 min. for the *Latitude* I am now in, which enter in the Ninth Column of the *Journal*. And now to find the Difference of *Longitude*,

In the preceding Column, the Ninth, you have both *Latitudes*, *viz.* 49 deg. 30 m. for the 22d day, and 48 deg. 11 m. for the 23d day, and the Course South-West; by which you may find the Difference of *Longitude*, as hath been shewed before, by this Proportion.

As the *Radius*,
To the Meridianal Difference of *Latitude*;
So is the *Tangent* of the Course,
To the Difference of *Longitude*.

Which Difference of *Longitude* will be found to be 120 Miles, which must be placed in the Column of *Longitude*, in the West part thereof, because the Course is Westerly.

The 24th day is to be wrought after the same manner as the 23 day; for having the *Course* and *Distance* given, you may find the *Difference* of *Latitude*, *Departure*, and *Difference* of *Longitude*, as in the day before; which must be entred in your *Journal* in their respective Columns.

How to correct your Dead Reckoning *by Observation of the* Latitude.

On the 24th of *March* by Observation I find the *Latitude* that I am in to be 47 d. whereas by my *Dead Reckoning* I should be in *Latitude* 47 deg. 15 min. so that the Difference is 15 min. Southerly: To correct which, I set 15 min. in the South Column, which subtracted from 47 deg. 15 min. makes my *Latitude* by *Dead Reckoning* to agree with the *Observation*. To correct your *Departure*, you must consider whether the Fault may be imputed to your *Course*, or to your *Distance*. If your *Course* be well steered, and you find no Current, nor any Variation of the Compass, then your Distance is faulty: But if you cannot trust to your Course steered, then the best way is to correct your *Latitude* only, not medling with your Departure. If there be a Current, and you know which way the Current sets, and how fast, then find the Difference of *Latitude*, and Difference of the Current, and add or subtract that *Latitude* and *Departure* to or from the Ship's Difference of *Latitude* and *Departure*, according as the Current doth further or hinder the Ship in her Course. But if by some probable Reason you only conjecture there is a Current, then give what Allowance you think meet in Difference of *Latitude* and Departure, and see if that will reform your Reckoning in your *Latitude*; which if it do, you have well guessed; but if it will not, it is to be supposed that you are mistaken in your Conjecture, or that there is some other Cause of this Error in your Reckoning.

If the Compass vary, (as it often doth) then finding what that Variation is, and which way, you must allow it in the Ship's Course. Now if you cannot impute the Error to any of these, then the *Distance* is faulty; and this is that which usually makes the Difference between the *Observed Latitude* and the *Latitude* by *Dead Reckoning*. And I here take this to be the Cause of the Error this 24th of *March*, and generally in this whole Reckoning.

Now to correct your *Departure* and *Difference* of *Latitude*, you must add up your *East*, *West*, *North*, and *South* Columns, from the day that you correct, to the beginning of your *Journal-Table*, if it be the First Correction you have made; or from the day of Correction to the last Correction, if it be the Second; and so between the Second and Third, the Third and Fourth, *&c.* Then subtract the Summs of the

North and *South* Columns from each other; and likewise the *East* and *West* Columns: And say by Proportion;

As the *Difference* of the *North* and *South* Columns,
Is to the *Difference* of the *East* and *West* Columns,
So is the *Difference* between the *Latitude* by Reckoning, and *Latitude* by Observation,
To the *Difference* in the *Difference*.

And for the *Difference* of *Longitude*; say,
As the *Difference* between the observed *Latitude*, and that by reckoning,
Is to the *Meridianal Difference* of those two *Latitudes*;
So is the *Difference* in the *Departure*,
To the *Difference* in the *Longitude*.

Example in this *Journal*.

This 24th. of *March*, you will find the Summ of the *North* Column to be 00.0. The Summ of the *South* Column (leaving out the 15 min.) to be 164.8; and therefore, their *Difference* is the same 164.8.—— The Summ of the *East* Column is 00.0. The Summ of the *West* Colum is 124.9, and therefore the *Difference* is the same: And then the Operation by the *Logarithms* will be

As the *Difference* of the *North* and *South* Columns 164.8	Co. Ar.	7.78304
To the *Difference* of the *East* and *West* Columns, 124.9		2.09656
So is the *Difference* between the two *Latitudes*, 15 min.		1.17609
To the *Difference* in the *Departure*, 11 Miles.		11.05559

Place this 11 Miles or Minutes in the *West* Column, because the Summ of the *West* Column exceeds that of the *East* Column.

The Tenths of the *Departure* are here neglected.

The Operation for the *Difference* in *Longitude*.

The two *Latitudes* are 47 deg. 15 min. and 47 deg. by which you will find in the *Table* of *Meridional Parts*, the *Meridional Difference* of *Latitude* to be 22 min. Therefore,

As the *Difference* of the two *Latitudes*, 15 min.	Co. Ar.	8.82391
To the *Meridional Difference* of those *Latitudes*, 22		1.34242
So is the *Difference* in the *Departure*, 11		1.04139
To the *Difference* in the *Longitude*, 16		11.20772

This 16 min. is to be placed in the *West* Column, because the *Departure* is *Westward*.

And after the same manner are the Corrections made in this *Journal* on the 30th of *March*, the 4th the 9th and the 20th of *April*; the Errour being supposed to arise from the computation of the *Distance* of the Ships Way.

If your Ship shape several Courses in 24 hours, you are to find your *Difference* of *Latitude* and *Departure* by working a *Traverse*; as hath been taught before: And so, your *Difference* of *Latitude* will give you the *Latitude* that your Ship is in. Then have you two *Latitudes*, namely, the *Latitude* the Ship was in the Day before at Noon, and the *Latitude* it is in at Noon this Day; by which you may find the *Meridianal Difference* of *Latitude* by the *Table* of *Meridianal Parts*, as before hath been taught.

Then for the *Difference* of *Longitude*; say,
As the *Difference* of *Latitude* found by the *Traverse*,
Is to the *Difference* of *Latitude* in *Meridianal Parts*;
So is the *Departure* found by the *Traverse*,
To the *Difference* of *Longitude* for that *Traverse*.

To find the whole *Difference* of *Longitude* of the two Places between which you make your Voyage.

Add up the Columns of *East* and *West Longitude*, and subtract the one from the other, the Remainder is the *Difference* of *Longitude* in Miles.

In this *Journal*, the *Difference* of the *East* and *West* Columns of *Longitude* is 3177.6, or 3178 Miles: which reduced into Degrees and Minutes is 52 deg. 58 min. for the *Difference* of *Longitude* between the *Lizard* and the *Barbadoes*.

The End of the Third Part.

OF NAVIGATION.

PART IV.

OF Circular-Sailing.

WHEREIN

Divers PROBLEMS of SAILING by the ARCH of a *GREAT CIRCLE* are performed, by *Spherical Projection*, and *Trigonometrical Calculation.*

IN the Two foregoing Ways of *Saling*, the one exceedeth the other: For, as Sailing by *Mercator's Chart* exceedeth that of the *Plain Chart*; ſo this, by the *Arch* of a *Great Circle*, exceedeth them both. For, foraſmuch as the Earth and Sea are a Spherical Body, therefore the moſt abſolute way of Sailing upon the Sea muſt needs be by the *Arch* of a *Great Circle*, drawn upon the Surface of the Sea, from Place to Place. And here I ſhall ſhew, both by *Spherical Projection*, and *Trigonometrical Calculation*, how from any two Places, howſoever ſituate upon the *Terreſtrial Globe*, to find,

I. Their *Neareſt Diſtance* in the *Arch* of a *Great Circle*.
II. The *Angle* of *Poſition* from one Place to the other.
III. By what *Longitudes* and *Latitudes* the *Arch* ſo drawn doth paſs between the two Places.

SECT. I. *For the* Neareſt Diſtance.

BEfore I come to ſhew how the *Neareſt Diſtance* between two Places is to be found upon the *Terreſtrial Globe*, it will be neceſſary, firſt, to deſcribe unto you the Manner how any two Places, whoſe *Longitudes* and *Latitudes* are given, may be laid down upon the *Projection*. Wherefore in the Scheme, the outward Circle thereof, N Æ S æ, repreſents the Firſt or General *Meridian*: The Line Æ æ is the *Equinoctial*, upon which the *Longitude* is counted from the *firſt Meridian*. The Circles N E S, N R S, N C S, and N D S, are Circles of *Longitude*, paſſing over ſeveral Places. The leſſer Circles C D F and L B M are Circles or Parallels of *Latitude*. The Points *, *, *, *&c.* are ſeveral Places whoſe Diſtance we are to find by the following Propoſitions. And the Points ⊙, ⊙, ⊙, *&c.* are the *Poles* of the *Arches* of *Great Circles*, which paſs through the reſpective Places whoſe Diſtance is to be found. *Fig.* X.

And here note, That the *Circles* of *Longitude* in this *Projection* are the ſame as the *Azimuth Circles* in the former *Meridional Projection*; and the *Centres* and *Poles* of them are found in the ſame manner. Likewiſe, the *Parallels* of *Latitude* in this *Projection* are the ſame with the *Parallels* or *Circles* of *Altitude* in the former *Meridional Projection*, and their *Centres* are found in the ſame manner as in the Deſcription of that *Projection* preſcribed; and therefore it ſhall here need no more Precepts for its Delineation, but we will proceed to the *Propoſitions* which ſhew how to find the Diſtance of Places.

PROP. I. *Two Places which differ only in* Latitude, *to find their* Distance.

IN this Proposition there are two Varieties.

1. If both the Places lie under one and the same *Meridian*, and on one and the same side of the *Equinoctial*, either on the North or South side thereof, then subtract the lesser *Latitude* from the greater, and the Difference converted into Miles, (by allowing 60 Miles to one Degree) shall give you the Distance.

Example. *London* and *Ribadio* lie both under one *Meridian*, but they differ in *Latitude*; for *London* hath 51 deg. 30 min. and *Ribadio* hath *Latitude* 43 deg. both North; the Difference of *Latitude* is 8 deg. 30 min. which being turned into Miles, makes 510 Miles.

2. If the two Places lie under one and the same *Meridian*, but one on the North, and the other on the South-side of the *Equinoctial*, add both the *Latitudes* together, the Summ is the Distance.

Example. *London* and the Island *Tristan Dacunhu* lie both under one *Meridian*; but *London* hath 51 deg. 30 min. North *Latitude*, and the Island hath 34 deg. South *Latitude*; their Summ is 85 deg. 30 min. which converted into Miles, (by dividing the Degrees by 60, and allowing for every Minute one Mile,) makes 5130 Miles: And such is the Distanc of *London* and the Island *Tristan Dacunhu*.

PROP. II. *Two Places which differ only in* Longitude, *to find their* Distance.

IN this Proposition there are two Varieties also. For,

1. The two Places may lie both under the *Equinoctial*, and have no *Latitude*: In this Case the Difference of their *Longitudes* (if it be less than 180 deg.) reduced into Miles, is their Distance; but if their Difference exceed 180 deg. take it out of 360 deg. the remaining Degrees turned into Miles will be the Distance of the two Places.

Example. The Island *Sumatra* and the Island of *S. Thoma* lie both under the *Equinoctial*, the Island of *S. Thoma* having 33 deg. 10 min. of *Longitude*, and the Island *Sumatra* 137 deg. 10 min. The lesser *Longitude* taken from the greater, leaves 104 d. 00 min. which converted into Miles, is 6240; and that is the Distance of the two Islands.

2. But if the two Places differ only in *Longitude*, and lie not under the *Equinoctial* but under some other intermediate *Parallel* of *Latitude*, between the *Equinoctial* and one of the *Poles*; then,

Let the two Places lie both in the *Latitude* of 50 deg. and let their Difference of *Longitude* be 135 deg.

I. By *Projection*.

Having drawn the first *Meridian Circle* N Æ S *æ*, and set the *Common Latitude* upon it, 50 deg. from Æ to C; and from *æ* to F, count 135 deg. upon the *Equinoctial* from Æ to E, and draw the *Meridian* N E S, passing through the other Place at D, in the same *Parallel* of *Latitude*, 50 deg.

Then through the Points C and D describe an *Arch* of a *Great Circle* C D G, passing through the two Places C and D, the *Pole* of which *Circle* is at A. Now to find the Distance between C and D in the *Arch* of the *Great Circle*, lay a Ruler to A and D, it will cut the first *Meridian* in *a*; so the Distance C *a* measured upon the *Scale* of *Chords*, will be 72 deg. 52 min. which converted into Miles, is 4372 Miles, for the Distance of the two Places.

II. By *Trigonometry*.

By the Intersection of the three *Great Circles* in the *Projection*, you have constituted the *Oblique-angled Spherical Triangle* C N D, in which is given,

(1.) The two Sides N C and N D, the *Complements* of the *Common Latitude*, 40 d.
(2.) The *Angle* C N D, the *Difference* of *Longitude*, 135 deg.

By which you may find the Side C D,
By *Case* VI. of *Oblique-angled Spherical Triangles*;
Or rather the Half of it, by the following *Analogy*.

As

As the *Radius*, 90 deg.	10.
To the Co-sine of the Common Latitude, C N or N D, 40 deg.	9.80806
So is the Sine of Half the Difference of Longitude, 67 deg. 30 min.	9.96561
To the Sine of Half the Distance, 36 deg. 26 min.	9.77367

Which doubled is 72 deg. 52 min. and that multiplied by 60, produceth 4372 Miles, as before; and that divided by 3, gives 1457 Leagues and one Mile, for the Distance in the *Arch* of a *Great Circle*.

PROP. III. *Two Places differing both in* Longitude *and* Latitude, *to find their* Distance.

THere are three Varieties contained in this Proposition. For,

1. One of the Places may lie under the *Equinoctial*, and have no Latitude; and the other under some Parallel of Latitude, between the *Equinoctial* and one of the *Poles*.

I. By *Projection*.

Thus, suppose the two Places to be, one at H, lying under the *Equinoctial*, and the other at C, under the Parallel of 50 deg. and differing in Longitude 158 deg.

The two Places upon the Projection are represented by C and H, the Point H lying under the *Equinoctial*, and the Point C under the Parallel of 30 deg. North, and differing in Longitude 158 deg. Through the two Places C and H (according to former Directions) draw an *Arch* of a *Great Circle*, C H G, and find the *Pole* thereof, which will be at the Point A. Then, a Ruler laid to A, and the Point H, will cut the first *Meridian* in *n*; the Distance *n* C being measured upon the Line of *Chords*, will be found to contain 126 deg. 23 min. which in Miles is 7583. or in Leagues 2527 and 2 Miles.

II. By *Trigonometry*.

By the Intersection of the three *Great Circles* in the Projection, there is constituted, the *Quadrantal Spherical Triangle* C N H, or rather its Opposite H S G, whose Side H S is a Quadrant, in which there is given, besides the Quadrant's Side H S,

(1.) The *Angle* H S G, 22 deg. the Complement of the Difference of Longitude to 180 deg.

(2.) The Side S G, equal to Æ C, the Latitude 50 deg. By which you may find the Side H G,

By the XVI. *Case* of *Right-angled Spherical Triangles*.

As the *Radius*, 90 deg.	10.
Is to the *Co-sine* of H S G, 68 deg. the Difference of Longitude	9.96716
So is the *Sine* of S G, the *Co-Latitude* 40 deg.	9.80807
To the *Sine* of 36 deg. 35 min.	9.77523

Whose Complement 53 deg. 25 min. is the Side H G, and that subtracted from 180 deg. leaves 126 deg. 23 min. for the Side C D H, which is the Distance between the two Places, and will be found as before to be 7583 Miles.

2. If both the Places proposed shall be without the *Equinoctial*, but on one Side thereof, either both towards the North, or both towards the South.

So the two Places propounded lying both in North Latitude, one in 50 deg. and the other in 15 deg. and differing in Longitude 60 deg. you may find their Distance as followeth.

I. By *Projection*.

The two Places in the Projection are represented by the Letters C and B, through which Points draw the *Arch* of a *Great Circle* C B G, and find its *Pole* at K. Wherefore lay a Ruler from K to B, it will cut the first *Meridian* in *g*; and the Distance *g* C measured upon the Scale of *Chords*, will be found to contain 59 deg. 25 min. and that is their Distance, which in Miles is 3565.

II. By *Trigonometry*.

By the Intersection of the three *Great Circles* in the Projection, there is constituted the *Oblique Spherical Triangle* N C B, in which is given,

(1.) The

(1.) The Side C N (40 deg.) the Complement of the Latitude of the Place at C.
(2.) The Side N B (75 deg.) the Complement of the Latitude of the Place at B.
(3.) The Angle C N B (60 deg.) the Difference of Longitude between the two Places.

By which may be found the third Side B C,

By *Case* VI. of *Oblique-angled Spherical Triangles*,

(1.) As the *Radius*, 90 deg.	10.
To the *Co-sine* of C N B, 30 deg. the Difference of Longitude	9.69897
So is the *Tangent* of C N, 40 deg.	9.92381
To a fourth *Tangent*, 22 deg. 46 min.	*x*9.62278

Subtract this fourth *Tangent*, 22 deg. 46 min. from the Side B N 75 deg. the Remainder will be 52 deg. 14 min. for the Segments of the Base (or Side) N O and O B. Then say,

(2.) As the *Co-sine* N O, 22 deg. 46 min.	Co. Ar. 0.03523
To the *Co-sine* O B, 52 deg. 14 min.	9.78707
So is the *Co-sine* N C, 40 deg.	9.88425
To the *Co-sine* C B, 30 deg. 35 min.	*x*9.70655

Whose Complement 59 deg. 25 min. is the Side C B; and that converted into Miles, makes 3565 for the Distance of the two Places in the *Arch* of a *Great Circle*.

3. The two Places may be so situate, that one of them may lie on the North, the other on the South-side of the *Equinoctial*: And to find their Distance,

I. By *Projection*.

The two Places in the Projection are represented by the Letters L and P, L being in 15 deg. of North-Latitude, and P in 62 deg. 30 min. of South Latitude. The *Meridian* passing through L is the *Primitive Circle*; the *Meridian* passing through P is N B P S; and the *Great Circle* passing through both the Places is L P; and the *Pole* thereof R. Now to find the Distance, lay a Ruler from R to P, it will cut the first *Meridian* in *k*; the Distance between L and *k* measured upon the Scale of *Chords*, will be found to contain 89 deg. 38 min. for the Distance of the two Places L and P, which make in Miles 5378.

II. By *Trigonometry*.

In the Projection, by the Intersections of the three *Great Circles*, is constituted the *Oblique-angled Spherical Triangle* L P S, in which is given,

(1.) The Side P S, 27 deg. 30 min. (the Complement of the greater Latitude.)
(2.) The Side L S, 105 deg. (the lesser Latitude) with 90 deg. added thereto.
(3.) The Angle P S L, 60 deg. (the Difference of Longitude between the two Places.

By which you may find the Side L P the Distance.

By *Case* VI. of *Oblique-angled Spherical Triangles*.

(1.) As the *Radius*, 90 deg.		10.
To the *Co-sine* of Difference of Longitude, P S L		9.69897
So is the *Co-tangent* of the greater Latitude P S, 27 deg. 30 min.		9.71647
To the *Tangent* of 14 deg. 36 min.		*x*9.41544
	deg. min.	
From the Side L S	105 00	
Subtract the Fourth Arch	14 36	
The Remainder is	90 24	

Then say,

(2.) As the *Co-sine* of the Fourth Arch	Co. Ar. 0.01425
Is to the *Co-sine* of the lesser Latitude, 15 deg.	9.94793
So is the *Co-sine* of the *Arch* remain. 90 deg. 24 min.	7.84092
To the *Co-sine* of the Distance, 89 deg. 38 min.	*x*7.80310

Which turned into Miles, is 5378, as before.

These are all the Varieties of Positions of Places upon the *Terrestrial Globe*: for no two Places (whose Distance can be required) but they must fall under one or other of

of the Varieties contained in some of these three Propositions. And note that this way of finding the *Distance* of Places is the most absolute and exact of any other. — And what is here said concerning finding these *Distances* I shall apply to *Circular Sailing*, of all other ways the most perfect.

SECT. II. *For the Angle of* Direct Position *from one Place to another.*

AS the *Distance* of Places were measured by the Sides, so the *Positions* of Places are measured by the *Angles* of *Spherical Triangles:* And in the resolving of some *Problems* of this nature, I shall retain the same method as in finding of *Distances, viz.* First by the *Projection*, and then by *Trigonometry*. *Fig.* XI.

PROB. I. *Two Places, one lying under the* Equinoctial, *the other in any* Latitude, *and the Difference of* Longitude *between them being given to find the direct Position of the First Place from the Second, and of the Second from the First.*

I. By *Projection.*

Having drawn a Primitive *Meridian*, as the Circle A B Q G P, let one Place be D, under the *Equinoctial*; the other A, in 50 deg. of *North Latitude:* and let the *Difference* of *Longitude* between them be 51 Degrees, as B D. The *Triangle* thus projected, the things required to be found are, (1.) The *Angle* B A D. And (2.) The *Angle* A D B.

1. For the *Angle* B A D, a Ruler laid from A to C will cut the *Primitive Circle* in *a*, and the *Distance a* E, measured upon the *Scale* of *Chords*, will be found to contain 58 deg. 11 min. for the *Angle* of *direct Position* from the Place at A, to the Place at C.

2. For the *Angle* A D B; a Ruler laid from A to C will cut the *Meridian* in *a*, (as before,) and a Ruler laid from H (the *Pole* of the *Circle* A C F) to *a*, will cut the *Circle* A C F in *b*: Then a Ruler laid from D to *b*, will cut the *Meridian* in *c*: The *Distance c* G, measured, will be found to be 56 deg. 53 min. for the *Angle* G D F; to which A D B is equal, and is the *Angle* of *direct Position* between D and A.

II. By *Trigonometry.*

By the Intersection of the *Equinoctial*, and the great Circle A D F, with the *Meridian*, is constituted the *Triangle* A B D, Right-angled at B; in which is given,

(1.) The side A B, the *Latitude*. 50 deg.
(2.) The side B D 51 deg. the *Difference* of *Longitude* of the two Places; by which you may find the *Angle* B A D.

By *Case* XV. of Right-angled Spherical *Triangles.*

	10.
As the *Radius*, 90 deg.	
Is to the *Sine* of A B 50 deg.	9.88425
So is the *Co-Tangent* of B D. 51 deg.	9.90837
To the *Tangent* of 30 deg. 49 min.	*x*9.79262

Whose Complement 58 deg. 11 min. is the *Angle* B A D.

Then for the *Angle* A B D, by the VI. *Case.*

	10.
As the *Radius*, 90 deg.	
To the *Co-Sine* of A B, 50 deg.	9.80806
So is the *Sine* of D A B, 58 deg. 11 min.	6.92928
To the *Sine* of 33 deg. 7 min.	*x*9.73734

Whose Complement 56 deg. 53 min. is the quantity of the *Angle* of *Position* A D B.

II. *Two*

II. *Two Places Differing in* Latitude, *and their Difference of* Longitude *being given, to find the* Angle *of* direct Position *from the First to the Second, and from the Second to the First.*

Let the two Places be the *Lizard*, lying in the *Latitude* of 50 deg. and the other the *Summer Islands*, lying in 32 deg. 25 min. of *North Latitude*, and their *Difference* of *Longitude* 70 deg. the several Positions of these Places one from the other is required.

I. By *Projection.*

Set the *Latitude* of the *Lizard* 50 deg. from G to L, (P being the *North Pole:*) also set the *Difference* of *Longitude* 70 deg. from G to T, and draw the *Meridian* P T Q, and from T to S set the *Latitude* of the *Summer Islands* 32 deg. 25 min. then through L and S, draw the great *Circle* M S L, passing through both Places, and by their Intersections is made the Oblique-angled *Triangle* P S L; in which,

P L is the Complement of the *Latitude* of the *Lizard*, 40 deg.
P S the Complement of the *Latitude* of the *Summer Islands*, 57 deg. 35 min.
S L the neareſt *Diſtance* between the two Places in the Arch of a great *Circle*; which may be found by the directions of the laſt *Section* to be 53 deg. 24 min.

Now the things we are to seek are, (1.) The *direct Position* from *Summer Islands* to the *Lizard*, P S L. And (2.) The *direct Position* from the *Lizard* to the *Summer Islands*, P L S.

1. For the *Angle* P S L.

A Ruler laid from N, the *Pole* of the *Meridian* P T Q, to S, will cut the *Primitive Circle* in *d*; and 90 deg. set upon the same *Circle* from *d*, will reach to *c*; a Ruler laid from N to *c* will cut the *Meridian* of the *Summer Islands* P T Q in *e*.

Also, a Ruler laid from K (the *Pole* of the great *Circle*, which passeth through both the Places, (*viz.* L S M) to S, will cut the *Primitive Circle* in *f*, and 90 deg. set from *f*, will reach to *g*; a Ruler laid from K to *g* will cut the great *Circle* M S L in *h*.

Lastly, A Ruler laid from S to *e* and *h*, will cut the *Primitive Circle* in *k* and *l*: and the *Diſtance k l* measured upon the *Scale* of *Chords*, will be found to contain 48 deg. 47 min. for the *Angle* M S Q, to which P S L is equal, and is the *direct Position* from the *Summer Islands* to the *Lizard*.

2. For the *Angle* P L S.

A Ruler laid from L to *o*, will cut the *Primitive Circle* in *q*, the *Diſtance* from *q* to V, measured upon the *Scale* of *Chords*, will be found to be 81 deg. 8 min. which is the *Angle* of the *direct Position* from the *Lizard* toward the *Summer Islands.*

II. By *Trigonometry.*

By the Intersections of the two great *Circles* passing through both the Places, with the general *Meridian*, there is constituted the *Oblique-angled Spherical Triangle* P S L; in which is given,

(1.) The side P S 57 deg. 35 min. the Complement of the *Latitude* of the *Summer Islands.*

(2.) The side P L 70 deg. the *Difference* of *Longitude* between the Places.

(3.) The side P L, 40 deg. the Complement of the *Latitude* of the *Lizard*: whereby you may find the *Angles* at S and L.

I. For the *Angle* P S L.

By *Case* V. of *Oblique-angled Sherical Triangles.*

(1.) As the *Radius*, 90 deg.	10.
To the *Co-sine* P, 70 deg.	9.53405
So is *Tangent* P L 40 deg.	9.92381
To a Fourth *Tangent*, *viz.* 16 deg. 1 min.	9.45786

	deg.	min.
From P S	57	35
Subtract	16	01
There Remains	41	34

(2.) As

(2.) As the *Sine* of 16 deg. 1 min. Co. Ar. 0.55966
Is to the *Sine* of the Remainder 41 deg. 34 min. 9.82183
So is the *Co-Tangent* P, 70 deg. 9.56106
To the *Tangent* of 41 deg. 13 min. *r*9.94255

Whose Complement, 48 deg. 47 min. is the quantity of the *Angle* P S L, and is the *Angle* of *direct Position* from *Summer Islands* to the *Lizard*.

II. For the *Angle* P L S, having attained the *Angle* P S L, say,

By *Case* I. of *Oblique-angled Spherical Triangles*,

As the *Sine* of P L, 40 deg. Co. Ar. 0.19193
Is to the *Sine* of PS L, 48 deg. 47 min. 9.87634
So is the *Sine* of P S, 57 deg. 35 min. 9.92643
To the *Sine* of P L S 81 deg. 5 min. *r*9.99470

Which is the *Angle* of *direct Position* from the *Lizard* to the *Summer Islands*.

SECT. III. *Of finding by what* Longitudes *and* Latitudes *the Arch of any great Circle, (drawn as before,) shall pass between any two Places.*

HAving in the two foregoing *Sections* shewed how to find the *Distance* of any two Places, howsoever situate in the Arch of a great Circle; as also, to find the *Angles* of direct *Position* from one to the other: The work of this Third *Section* shall be to find by what *Longitudes* and *Latitudes* the Arch of such a great Circle doth pass; both by *Projection* and *Trigonometrical Calculation*: And our first *Example* shall be between the same two Places as were mentioned in the Second *Problem* of the Second *Section* foregoing; *viz.* the *Lizard* and the *Summer Islands*.

For the performance of the Work of this Third *Section*, it will be necessary to have a perfect Projection of the whole *Sphere* or *Globe*, with all its *Meridians* and *Parallels*; the Projection whereof, (with some Uses of the same) is taught where we treat of Spherical Projection, but for our present occasion I have here again added another *Scheme* of the same Projection; referring the Reader for its Construction to the fore-cited Place; and shall here immediately proceed to the Use of the same Projection in *Navigation*.

PROB. I. *The* Latitudes *of two Places, and their Difference of* Longitude *being given, to find by what* Longitudes *and* Latitudes, *the great Circle leading to those Places doth pass.*

I. By *Projection*.

Let the two Places be the *Summer Islands*, lying in the *Latitude* of 32 deg. 25 min. and the *Lizard*, lying in the *Latitude* of 50 deg. both *North*, and differ in *Longitude* 70 deg.

1. Count upon your Projection the *Latitude* of one of the Places, as *Summer Islands*, 32 deg. 25 min. in the outermost Circle at S: [And herein regard must be had to the Bearing of the two Places: For if the Second Place lie *West* from the First, then set down the First Place on the *East* (or right) side of the Projection: But if the Second Place lie *Eastward* of the First, (as in this *Example*,) then you must set down the First Place on the *West* side of the Projection, as it is here at S.]

2. From the Point S, through the Centre E, draw the Diametre S E K, and cross it at Right-Angles, with the Line M E N.

3. The *Difference* of *Longitude* being 70 deg. count 70 deg. upon the *Equinoctial*, from C to 70, and upon that *Meridian*, count 50 deg. the *Latitude* of the *Lizard*, and there set L. And thus have you your two Places set down according to their true *Latitudes* and *Difference* of *Longitude* in the Projection.

4. Through the two Places S and L, you must draw the Arch of the great *Circle* S L K, as is taught in the 63*d*. *Geometrical Problem*, and also in *Spherical Projection*, several times.

5. The

5. The great Circle being drawn, you may find the *Pole* thereof at ⊙. The Projection thus prepared, if you would know,

1. By what *Longitudes* and *Latitudes* the Arch of the great Circle passeth: This may be done by inspection; for if you trace with your Eye, or with a Needle's Point, you shall see that the 10*th Meridian* from S, crosseth the great Circle at 38 deg. 51 min. of *Latitude*, at 20 deg. *Difference* of *Longitude*, it crosseth the great Circle at 43 deg. 34 min. of *Latitude*; and so of the rest, as in this *Table*.

At	Degrees Diff. of *Longit.*	D.	Degrees of *Latitude*	M.
00		32		25
10		38		51
20		43		34
30		46		54
40		49		04
50		50		15
60		50		33
70		50		00

2. The *Angle* of *Position* from S to L. To find this, take in your *Compasses* the *Distance* from T, (the Point where the great Circle crosseth the Diametre thereof M N) to N, that *Distance* measured upon the *Equinoctial* from C, will reach to about 49 deg. and that is the *Angle* of *Position North-easterly*.

3. The *Distance* between S and L. Lay a Ruler from ⊙, the *Pole* of the great Circle, to S, and note where it cuts the outermost *Meridian*, and from that Point take the *Distance* to S; that *Distance* measured upon the Primitive *Meridian* from C, shall give you the *Distance* 53 deg. 24 min.—— And in this manner you may measure any Part of the Arch of the great Circle, as between every 10*th* or 5*th Meridian*, or *Difference* of *Longitude*.

And in this manner may any two Places, howsoever situate in respect of *Longitude* or *Latitude*, be set upon the Projection; an Arch of a great Circle passing through them be drawn: their *Distance* measured: the *Angle* of their *Position* found: And by what *Longitudes* and *Latitudes* the Arch of the great Circle passeth. As by two or three *Examples* which follow will plainly appear.

II. By *Trigonometry*.

For the better Illustration of the *Trigonometrical Operations* following I have here inserted a particular Part of the former *Sphere*, it being an *Oblique-angled Spherical Triangle*, comprehending all such Parts of the several *Meridians*, as lie between the two Places, (or every 10*th* of them.)

In which *Triangle*,

Fig. XIII.

P represents the *North-Pole*.
S *Summer Islands*.
L the *Lizard*.

And in it there is given,

S P, the Complement of the *Latitude* of *Summer Islands*, 57 deg. 35 min.
P L, the Complement of the *Latitude* of the *Lizard*, 40 deg. 00 min.
S P L, the *Difference* of *Longitude*, 70 deg. 00 min.

These things known, it is required to find the *Longitudes* and *Latitudes* by which this Arch S L doth pass.

In this *Case* it will be requisite to let fall a Perpendicular from the *Pole* P to the Arch of the great Circle S L; which let be P B.

And then let it be required to find,

(1.) The Perpendicular it self.

(2.) The Parts of the Vertical *Angle*, *viz.* the *Angles* S P B, and L P B.

And these being known, all the others will be *Right-angled Triangles*, and so be resolved at One operation.

I. For the *Perpendicular* P B,

There is given, (1.) The *Hypotenuse* S P, the Complement of the *Latitude* of *Summer Islands*, 57 deg. 35 min. (2.) The *Angle* of *Position* at S, (as before it was found to be) 48 deg. 47 min. To find P B.

By the IV. *Case* of *Right-angled Spherical Triangles*.

	10.
As the *Radius*, 90 deg.	
To the *Sine* of P S, 57 deg. 35 min.	9.92643
So is the *Sine* of the *Angle* P S B, 48 deg. 47 min.	9.87635
To the *Sine* of the Perpendicular P B, 39 deg. 25 min.	9.80278

Whose Complement 50 deg. 35 min. is the greatest *Latitude* by which the great Circle S B L doth pass.

II. For

II. For the *Angles* S P B and L P B,

By the III. *Case* of *Right-angled Spherical Triangles*,

As the *Radius*, 90 deg.	10.
To the *Co-sine* of P S, 32 deg. 25 min.	9.72922
So is the *Tangent* of P S L, 48 deg. 47 min.	10.05752
To the *Tangent* of S P B, 31 deg. 29 min.	*x*9.78674

Whose Complement 58 deg. 31 min. is the Angle S P B.

And being the whole Angle S P L is 70 deg. 58 deg. 31 min. being taken therefrom, there remains 11 deg. 29 min. for the Angle B P L.

So that for the greateſt *Latitude* of this *Great Circle*, which is B, we have found the *Differenee* of *Longitude* from L to L P B, 11 deg. 29 min. and from the Angle S P B, 58 deg. 31 min.

Now the *Difference* of *Longitude* from S to L being 70 deg. let it be required to find by what *Latitudes* the *Arch* S L doth paſs, for every Tenth Degree of *Difference* of *Longitude* from S.

Suppoſe the Point K to differ in *Longitude* from S 10 deg. and I would know the *Latitude* of the ſame Point K.

Seeing we have before found the Angle S P B to be 58 deg. 31 min. and the Angle S P K being (by Suppoſition) 10 deg. therefore the Angle K P B is 48 deg. 31 min. And the Perpendicular P B we found before to be 39 d. 26 min. by which may be found the Complement of the *Latitude* P K,

By *Caſe* VII. of *Right-angled Spherical Triangles*,

As the *Radius*, 90 deg.	10.
Is to the *Co-ſine* of K P B, 48 deg. 31 min.	9.82112
So is the *Co-tangent* of P B, 39 deg. 26 min.	10.08492
To the *Tangent* of 38 deg. 51 min.	*x*9.90604

Whoſe Complement is 51 deg. 9 min. the *Latitude* at K.

In like manner, ſuppoſing,

	deg.		deg.	m.
The Point O / V / M / N	20 / 30 / 40 / 50	to differ in *Longitude* from S, we ſhall find the *Latitude* of the Point	O / V / M / N	43 44 / 46 54 / 49 04 / 50 15

And thus much for finding what *Longitudes* and *Latitudes* the *Arch* of a *Great Circle* between any two Places do paſs, by *Trigonometrical Calculation*.

But before I leave this Way of *Great Circle Sailing*, I will add two *Problems* more, neceſſary therein, to be performed upon the *Projection*; and thoſe are, Firſt, To find the *Diſtance*, *Angle of Poſition*, and the *Longitudes* and *Latitudes* by which the *Great Circle* ſhall paſs in ſeveral Poſitions, one Place being in the *Equinoctial*, and the other either in North or South *Latitude*: The other, one in North, and one in South *Latitude*.

II. *Two Places, one lying under the* Equinoctial Circle, *and the other in* South Latitude, *to find their* Diſtance, *their* Angle of Poſition, *and by what* Longitudes *and* Latitudes *the ſame* Great Circle *muſt paſs.*

Let one Place be St. *Thomas* Iſland, lying under the *Equinoctial*, and ſo hath no *Latitude*; let the other be the *Streights of Magelan*, lying in 53 deg. of South *Latitude*, and differing in *Longitude* from St. *Thomas* 90 deg. Weſtward.

I. *To place the two Places upon the* Projection, *and to deſcribe the* Great Circle.

Firſt, Set down the Iſland of St. *Thomas* at one end of the *Equinoctial* at D; then accounting thereon 90 deg. of *Longitude* from D to E, there is the *Meridian* of the *Streights of Magelan*; whereupon you muſt mark out the *Latitude* thereof, which is 53 deg. Southward, at the Point W: And thus have you three Points, D, W, and C, through which the *Arch* of the *Great Circle* muſt be deſcribed, as hath been taught before. Now this part of the *Great Circle* which lies between D and W, is the *Arch* of the *Great Circle*; by which may be found all the other things required: As,

II. *To find the* Longitudes *and* Latitudes *by which this* Great Circle *passeth.*

This is done by carrying your Eye along, or tracing with a Needle's Point the *Arch* it self, obſerving where it croſſeth the *Meridians* and *Parallels* of *Latitude* in the *Projection*, which you ſhall find for every Tenth Degree to be as is exhibited in this Table.

Degrees of *Longitude.*	Deg.	Degrees of *Latitude.*	Min.
10	12		58
20	24		25
30	33		34
40	40		28
50	45		31
60	43		58
70	51		16
80	52		35
90	53		00

III. *To find the* Angle *of* Poſition *between the two Places.*

This is ſhewed by the *Arch* D W croſſing the *Semidiametre* E B: So that if you take the *Diſtance* B W, and meaſure it in the *Equinoctial* C D, it will reach from C to 37 deg. which is the *Angle* of *Poſition* required, and ſhews it to be South-weſterly.

IV. *For the* Diſtance *of the two Places.*

If you take the Diſtance D W, and meaſure it upon the 37 *Meridian Circle*, (which anſwers to the *Angle* of *Poſition*,) it will reach from A (the *Pole*) to C, in the *Equinoctial* upon the 37th *Meridian*; which ſhews the *Diſtance* to be 90 deg.

III. *Two Places, one lying on the North, the other on the South-ſide of the* Equinoctial, *to find their* Diſtance, *the* Angle of Poſition, *and the* Longitudes *and* Latitudes *by which the* Arch *of the* Great Circle *paſſeth.*

Let the two Places be the *Summer Iſlands*, in 32 deg. 25 min. of North *Latitude*; and the other, the *Cape of Good Hope*, in 35 deg. of South *Latitude*; and their Difference in *Longitude* 90 deg.

I. *To place the two Places upon the* Projection, *and to draw the* Great Circle.

Count the *Latitude* of the firſt Place, *Summer Iſlands*, 32 deg. 25 min. from C to S, upon the *Primitive Circle*, and draw the Diametre S K, croſſing it at Right-Angles with another Diametre, N M: Then count the Difference of *Longitude*, 90 deg. upon the *Equinoctial* from C, and the *Meridian* of the *Cape of Good Hope* will fall to be the Diametre A B, upon which from E you muſt count the *Latitude* of the *Cape*, 35 deg. downwards towards B, at X; and by the Points S, X, and K, deſcribe the *Great Circle Arch* S X K, as hath been often taught before.

II. *To find the* Longitudes *and* Latitudes *by which the* Great Circle *paſſeth.*

Degrees of *Longitude.*	Deg.	Degrees of *Latitude.*	Min.
0	32		25
10	26		57
20	19		40
30	11		18
40	2		5
50	7		19
60	16		7
70	23		48
80	30		5
90	35		00

Trace with a Needle (or only caſt an Eye along the *Great Circle Arch*) obſerving where the *Arch* croſſeth the *Meridians* and *Parallels* of *Latitude* upon the *Projection*, which for every Tenth Degree of *Longitude* will be as in this Table.

III. *To find the* Angle of Poſition *between the two Places.*

Obſerve where the *Arch* S X K doth croſs the Line N M, which is at Z; then with your Compaſſes take the Diſtance Z M, and meaſure it in the *Equinoctial*, and you ſhall find it to reach from C to about 60 deg. *viz.* to 59 deg. 25 min. South-Eaſterly, which is the *Angle of Poſition.*

IV. *For the* Diſtance *of the two Places.*

If you take the *Diſtance* of the two Places in the *Arch* S X, and meaſure it in the *Meridian* of 60, (the Meaſure of the *Angle of Poſition*,) you ſhall find it to reach from A to Y, which is 107 deg. 34 min. and that is the Diſtance of the two Places.

The End of the Fourth Part.

Here followeth a Connex of ſuch TABLES, *and the Uſes of them, as are of continual Uſe in* Navigation.

KALENDARIUM NAUTICUM:

A

DIARY

OF THE

SUN, MOON, and FIXED STARS,

Fitted for the Daily Use of

MARINERS.

A TABLE *of the Moveable Feasts, from the Year* 1688, *to the Year* 1713. *In the* Julian *and* Gregorian *Accounts.*

	The *Julian* Account.							*Gregorian* Account.				
Years of Christ.	*Prime or Gold. Num.*	*Epact.*	*Domin. Letter*	*Shrove-Sunday.*	*Easter-Day.*	*Whit-Sunday.*		*Prime.*	*Epact.*	*Domin. Letter.*	*Easter-Day.*	*Whit-Sunday.*
1688	17	7	A G	*Febr.* 26	*April* 15	*June* 3		17	27	D C	*March* 18	*June* 6
1689	18	18	F	10	*March* 31	*May* 19		18	8	B	*April* 10	*May* 29
1690	19	29	E	*March* 2	*April* 20	*June* 8		19	19	A	*March* 26	14
1691	1	11	D	*Febr.* 22	12	*May* 21		1	1	G	*April* 19	*June* 7
1692	2	12	C B	7	*March* 27	15		2	12	F E	6	*May* 25
1693	3	3	A	27	*April* 17	*June* 5		3	23	D	*March* 22	10
1994	4	14	G	18	8	*May* 27		4	4	C	*April* 11	30
1695	5	25	F	*March* 3	*March* 24	12		5	15	B	3	22
1696	6	6	E D	*Febr.* 23	*April* 12	31		6	26	A G	22	*June* 10
1697	7	17	C	14	4	23		7	7	F	7	*May* 26
1698	8	28	B	*March* 6	*April* 24	*June* 12		8	18	E	*March* 23	11
1699	9	9	A	*Febr.* 19	9	*May* 28		9	29	D	*April* 19	*June* 7
1700	10	20	G F	11	*March* 31	19		10	10	C B	10	*May* 29
1701	11	1	E	*March* 2	*April* 20	25		11	21	A	13	*June* 11
1702	12	12	D	*Febr.* 15	5	24		12	2	G	15	3
1703	13	23	C	7	*March* 28	16		13	13	F	7	*May* 26
1704	14	4	B A	27	*April* 16	*June* 5		14	24	E D	*March* 22	10
1705	15	15	G	18	8	*May* 17		15	5	C	*April* 11	30
1706	16	26	F	3	*March* 24	12		16	16	B	3	22
1707	17	7	E	23	*April* 13	*June* 1		17	27	A	23	*June* 11
1708	18	18	D C	15	4	*May* 23		18	8	G F	7	*May* 26
1709	19	29	B	*March* 6	24	*June* 12		19	19	E	*March* 23	11
1710	1	11	A	*Febr.* 19	9	*May* 28		1	1	D	*April* 19	*June* 7
1711	2	22	G	11	1	21		2	12	C	4	*May* 23
1712	3	3	F E	24	14	*June* 2		3	23	B A	23	*June* 11

The SOLAR KALENDAR.

JANUARY.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	M.	I. After. 1681 1685 1689 1693 1697 D.	M.	M.	II. Aft. 1682 1686 1690 1694 1698 D.	M.	M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	M.
1	a	*Circumcis.*	21	50	10	21	43	10	21	45	10	21	47	9
2	b	22 ♑	56	21	40	10	21	33	11	21	35	10	21	38	10
3	c	23	57	21	30	10	21	22	11	21	25	11	21	28	11
4	d	24	59	21	20	11	21	11	11	21	14	11	21	17	11
5	e	26	0	21	9	12	21	00	12	21	3	12	21	6	12
6	f	*Epiphany.*		20	57	12	20	48	12	20	51	12	20	54	12
7	g	28	2	20	45	12	20	36	12	20	39	12	20	42	12
8	a	29	3	20	33	12	20	24	13	20	27	13	20	30	13
9	b	0 ♒	4	20	21	13	20	11	13	20	14	13	20	17	13
10	c	1	6	20	9	13	19	58	14	20	1	14	20	4	13
11	d	2	7	19	55	14	19	44	14	19	49	14	19	51	14
12	e	3	8	19	41	14	19	30	14	19	35	14	19	37	14
13	f	*Hilary.*		19	27	15	19	16	15	19	19	14	19	23	15
14	g	5	10	19	12	15	19	1	15	19	4	15	19	8	14
15	a	6	11	18	57	15	18	46	15	18	50	14	18	54	15
16	b	7	12	18	42	15	18	31	16	18	35	15	18	39	16
17	c	8	13	18	27	16	18	15	16	18	19	16	18	23	16
18	d	9	14	18	11	16	17	59	17	18	3	16	18	7	16
19	e	10	15	17	55	17	17	42	16	17	47	16	17	51	17
20	f	11	16	17	38	16	17	26	17	17	30	17	17	34	17
21	g	*Agnes.*		17	22	17	17	9	18	17	13	17	17	17	17
22	a	*Vincent.*		17	5	18	16	51	17	16	56	17	17	0	18
23	b	14	19	16	47	17	16	34	18	16	38	18	16	41	17
24	c	15	19	16	30	18	16	16	18	16	20	18	16	25	18
25	d	*Conv.Paul.*		16	12	18	15	58	18	16	2	18	16	7	18
26	e	17	21	15	54	19	15	40	19	15	44	18	15	49	19
27	f	18	22	15	35	19	15	21	19	15	25	19	15	30	19
28	g	19	23	15	16	19	15	2	19	15	7	18	15	11	19
29	a	20	23	14	57	19	14	43	11	14	48	19	14	52	19
30	b	21	24	14	38	20	14	22	18	14	28	20	14	33	20
31	c	22	25	14	18		14	4		14	8		14	13	

South Declination Decreasing.

Tables of the Sun's Right Ascenſion.

Month Days.	The *Sun's Right Aſcenſion,* Time. H.	M.	Compl. H.	M.
1	7	35	4	25
2	7	38	4	21
3	7	43	4	17
4	7	47	4	13
5	7	51	4	9
6	7	56	4	4
7	8	0	4	0
8	8	5	3	56
9	8	9	3	51
10	8	13	3	47
11	8	17	3	43
12	8	21	3	48
13	8	25	3	34
14	8	30	3	30
15	8	34	3	26
16	8	38	3	22
17	8	42	3	18
18	8	46	3	14
19	8	50	3	10
20	8	54	3	6
21	8	59	3	2
22	9	3	2	57
23	9	7	2	53
24	9	10	2	50
25	9	15	2	45
26	9	19	2	41
27	9	23	2	37
28	9	27	2	33
29	9	31	2	29
30	9	35	2	25
31	9	38	2	22

The LUNAR KALENDAR.

Years of Chriſt.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ Firſt Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Laſt Quarter D.	H.	M.	
1688	22	10	12	Nig.	29	1	0	Aft.	7	1	12	Aft.	15	4	50	Aft.
1689	11	10	10	Mor.	18	8	26	Mor.	25	10	14	Mor.	3	1	2	Aft.
1690	30	8	17	Mor.	8	4	53	Mor.	15	3	38	Mor.	22	6	8	Mor.
1691	19	10	8	Mor.	27	7	10	Mor.	4	11	40	Mor.	11	6	55	Mor.
1692	8	9	52	Mor.	16	12	46	Noo.	23	2	30	Aft.	30	5	34	Mor.
1693	26	5	38	Mor.	4	12	1	Noo.	12	4	0	Mor.	18	9	55	Nig.
1694	15	1	14	Aft.	23	7	39	Mor.	31	3	34	Mor.	8	7	52	Nig.
1695	5	3	5	Mor.	12	3	0	Mor.	20	5	41	Mor.	28	1	23	Mor.
1696	24	5	36	Mor.	30	11	55	Nig.	9	6	36	Mor.	17	10	0	Mor.
1697	12	9	0	Nig.	19	12	10	Noo.	27	1	53	Mor.	5	10	28	Mor.
1698	2	8	48	Mor.	9	9	21	Mor.	16	7	7	Mor.	24	3	26	Mor.
1699	21	4	0	Mor.	28	2	25	Aft.	5	7	0	Nig.	12	12	10	Nig.

The SOLAR KALENDAR.
FEBRUARY.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	Difference. M.	I. After. 1681 1685 1689 1693 1697 D.	M.	Difference. M.	South Declination Decreasing.	II. Aft. 1682 1686 1690 1694 1698 D.	M.	Difference. M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	Difference M.
1	d	23 ♒ 25	13	56	20	13	44	20		13	48	20	13	53	20
2	e	*Purif. Mar.*	13	39	20	13	23	21		13	28	20	13	33	20
3	f	25 27	13	10	20	13	3	20		13	8	20	13	13	21
4	g	26 27	12	58	22	12	43	21		12	48	20	12	52	20
5	a	27 28	12	38	20	12	22	21		12	27	21	12	32	21
6	b	28 28	12	17	21	12	1	21		12	6	21	12	11	21
7	c	29 29	11	56	21	11	40	21		11	45	21	11	50	21
8	d	0 ♓ 29	11	35	21	11	19	21		11	24	21	11	29	21
9	e	1 30	11	14	21	10	57	22		11	2	22	11	8	21
10	f	2 30	10	52	22	10	36	21		10	40	22	10	47	21
11	g	3 30	10	30	22	10	14	22		10	19	21	10	24	23
12	a	4 31	10	8	22	9	52	22		9	57	22	10	2	22
13	b	5 31	9	46	22	9	30	22		9	35	22	9	40	22
14	c	*Valent.*	9	24	22	9	7	23		9	18	22	9	18	22
15	d	7 32	9	2	22	8	45	22		8	50	23	8	55	23
16	e	8 32	8	39	22	8	22	23		8	27	23	8	33	22
17	f	9 32	8	17	23	8	0	22		8	5	22	8	11	22
18	g	10 32	7	54	23	7	37	23		7	42	23	7	44	23
19	a	11 32	7	31	23	7	14	23		7	20	22	7	26	22
20	b	12 32	7	8	23	6	51	23		6	57	23	7	3	23
21	c	13 32	6	45	23	6	28	24		6	34	23	6	39	24
22	d	14 32	6	22	23	6	4	24		6	10	24	6	16	23
23	e	15 32	5	59	23	5	41	23		5	47	23	5	53	23
24	f	S. *Matth.*	5	36	23	5	18	22		5	24	23	5	30	23
25	g	17 32	5	12	24	4	55	23		5	0	24	5	5	25
26	a	18 32	4	49	23	4	31	24		4	37	23	4	43	22
27	b	19 32	4	25	23	4	8	23		4	13	24	4	19	24
28	c	20 32	4	2	23	3	44	24		3	50	23	3	56	23
29			3	39											

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension.* Time. H.	M.	Compl. H.	M.
1	9	42	2	18
2	9	46	2	14
3	9	50	2	10
4	9	54	2	6
5	9	58	2	2
6	10	2	1	58
7	10	6	1	54
8	10	10	1	50
9	10	14	1	46
10	10	17	1	43
11	10	21	1	39
12	10	25	1	35
13	10	29	1	31
14	10	33	1	27
15	10	36	1	24
16	10	40	1	20
17	10	44	1	16
18	10	48	1	12
19	10	52	1	8
20	10	55	1	5
21	10	59	1	1
22	11	5	0	55
23	11	8	0	52
24	11	12	0	48
25	11	15	0	45
26	11	19	0	41
27	11	23	0	37
28	11	26	0	34
29				

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ First Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Last Quart. D.	H.	M.	
1688	21	8	24	Mor.	27	12	36	Nig.	6	7	4	Mor.	14	10	30	Mor.
1689	9	11	21	Nig.	16	3	41	Aft.	24	2	12	Mor.	1	11	56	Nig.
1690	28	11	4	Nig.	6	1	49	Aft.	13	8	39	Mor.	21	2	24	Mor.
1691	18	4	28	Mor.	25	5	39	Aft.	2	9	51	Nig.	9	12	9	Nig.
1692	7	4	31	Mor.	15	6	26	Mor.	21	11	48	Nig.	28	7	37	Nig.
1693	24	10	50	Nig.	3	7	53	Mor.	10	3	52	Aft.	17	8	30	Mor.
1694	14	2	50	Mor.	22	1	51	Mor.	No Full Moon.				7	3	7	Mor.
1695	3	1	52	Aft.	10	9	40	Nig.	18	12	13	Nig.	26	8	1	Mor.
1696	21	3	20	Aft.	28	3	43	Aft.	8	2	16	Mor.	15	12	21	Nig.
1697	11	9	34	Mor.	17	10	36	Nig.	25	8	10	Nig.	4	5	43	Mor.
1698	1	1	26	Aft.	7	4	22	Aft.	14	10	2	Nig.	22	11	33	Nig.
1699	19	7	52	Nig.	26	9	42	Nig.	4	2	49	Mor.	11	8	0	Nig.

The SOLAR KALENDAR.

MARCH.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.		II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.		D. M.	M.	D. M.	M.
1	d	*David* B.	3 15	24	3 20	23	South Decl. decreasing. *	3 26	23	3 32	23
2	e	♓ 22	2 51	23	2 57	24		3 3	24	3 9	24
3	f	23	2 28	24	2 33	24		2 29	23	3 45	24
4	g	24	2 4	24	2 9	23		2 16	24	2 21	23
5	a	25	1 40	24	1 46	24		2 22	24	2 58	24
6	b	26	1 16	23	1 22	23		1 28	23	1 34	23
7	c	27	0 53	24	0 59	24		1 5	24	1 11	24
8	d	28	0 29	24	0 35	24		0 41	24	0 46	23
9	e	29	0 5	24	0 11	24		0 17	11	0 23	22
10	f	♈ 0 28	0* 18	24	0* 13	23		0* 6	24	0* 1	24
11	g	1 27	0 42	23	0 37	24		0 30	24	0 25	24
12	a	2 27	1 6	24	0 0	23		0 54	24	0 49	23
13	b	3 26	1 29	23	1 24	24	North Decl. Increasing.	1 18	24	1 12	24
14	c	4 25	1 53	24	1 47	24		1 42	23	1 36	23
15	d	5 25	2 16	24	2 11	23		2 5	23	2 0	23
16	e	6 24	2 40	24	2 35	23		2 29	24	2 23	23
17	f	7 23	3 4	23	2 58	23		2 52	23	2 46	23
18	g	*Edward.*	3 27	23	3 21	24		3 15	23	3 10	24
19	a	9 21	3 50	24	3 45	23		3 36	24	3 33	23
20	b	10 20	4 14	23	4 8	23		4 2	23	3 56	23
21	c	11 19	4 37	23	4 31	23		4 25	23	4 19	23
22	d	12 18	5 0	23	4 54	23		4 49	24	4 43	24
23	e	13 17	5 23	23	5 17	23		5 12	23	5 6	23
24	f	14 16	5 46	23	5 40	23		5 35	23	5 29	23
25	g	*Annun. M.*	6 9	23	6 3	22		5 57	22	5 52	23
26	a	16 14	6 31	22	6 25	23		6 20	23	6 15	23
27	b	17 13	6 54	23	6 48	23		6 42	22	6 37	22
28	c	18 12	7 16	22	7 11	22		7 5	23	7 0	23
29	d	19 11	7 38	22	7 33	22		7 28	23	7 22	22
30	e	20 9	8 1	22	7 55	22		7 50	22	7 45	23
31	f	21 8	8 23	22	8 17	22		8 12	22	8 7	22

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension*, Time. H. M.	Compl. H. M.
1	11 29	0 31
2	11 33	0 27
3	11 35	0 25
4	11 40	0 20
5	11 43	0 17
6	11 46	0 14
7	11 50	0 10
8	11 53	0 7
9	11 57	0 3
10	0 1	11 59
11	0 5	11 55
12	0 8	11 52
13	0 12	11 48
14	0 15	11 45
15	0 19	11 41
16	0 23	11 37
17	0 26	11 34
18	0 30	11 30
19	0 33	11 27
20	0 37	11 23
21	0 41	11 19
22	0 44	11 16
23	0 48	11 12
24	0 51	11 9
25	0 55	11 5
26	0 59	11 1
27	1 3	10 57
28	1 6	10 54
29	1 10	10 50
30	1 14	10 46
31	1 17	10 43

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D. H. M.		☐ ☉ ☾ First Quart. D. H. M.		☍ ☉ ☾ Full Moon. D. H. M.		☐ ☉ ☾ Last Quarter D. H. M.	
1688	21 5 15	Aft.	28 2 29	Aft.	7 2 0	Mor.	15 1 14	Mor.
1689	11 10 0	Nig.	17 12 17	Nig.	25 6 4	Nig.	4 5 8	Mor.
1690	30 10 8	Mor.	7 8 6	Nig.	14 10 14	Nig.	22 9 46	Nig.
1691	19 7 52	Nig.	27 1 30	Mor.	4 8 25	Mor.	11 6 30	Nig.
1692	7 10 30	Nig.	15 1 42	Aft.	22 8 53	Mor.	29 11 36	Mor.
1693	26 4 0	Aft.	5 2 17	Mor.	12 1 30	Mor.	18 5 19	Nig.
1694	15 5 41	Aft.	23 7 52	Nig.	{ 1 4 13 N. / 31 2 3 M. }		8 9 40	Mor.
1695	5 1 21	Mor.	12 5 3	Aft.	20 2 50	Aft.	27 1 46	Aft.
1696	23 1 0	Mor.	30 9 22	Mor.	8 7 51	Nig.	16 10 55	Mor.
1697	12 5 14	Aft.	19 11 24	Mor.	27 1 23	Aft.	5 10 20	Nig.
1698	{ 1 10 34 N. / 31 4 15 M. }		8 12 4	Nig.	16 4 18	Aft.	24 5 20	Nig.
1699	21 8 54	Mor.	28 3 30	Mor.	5 7 14	Nig.	12 3 45	Aft.

The SOLAR KALENDAR.

APRIL.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.	II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.	D. M.	M.	D. M.	M.
1	g	22 ♈ 7	8 45	22	8 39	22	8 34	22	8 29	22
2	a	23 5	9 7	22	9 1	22	8 56	22	8 51	22
3	b	24 4	9 29	22	9 23	21	9 18	22	9 13	21
4	c	25 2	9 50	21	9 44	22	9 39	21	9 34	22
5	d	26 1	10 11	21	10 6	21	10 1	21	9 56	21
6	e	26 59	10 32	21	10 27	21	10 22	21	10 17	21
7	f	27 58	10 53	21	10 48	21	10 44	21	10 38	21
8	g	28 56	11 14	21	11 9	21	11 3	21	10 59	21
9	a	29 55	11 35	20	11 30	20	11 25	21	11 20	20
10	b	0 ♉ 53	11 55	20	11 50	20	11 46	20	11 40	21
11	c	1 51	12 15	20	12 10	20	12 6	20	12 1	20
12	d	2 50	12 35	20	12 30	20	12 26	20	12 21	20
13	e	3 48	12 55	20	12 50	20	12 46	20	12 41	20
14	f	4 46	13 15	19	13 10	19	13 6	19	13 1	19
15	g	5 44	13 34	19	13 29	19	13 25	19	13 20	20
16	a	6 42	13 53	19	13 48	19	13 44	19	13 40	19
17	b	7 40	14 12	19	14 7	19	14 3	19	13 59	19
18	c	8 39	14 31	19	14 26	19	14 22	19	14 18	18
19	d	9 37	14 50	18	14 45	18	14 41	18	14 36	19
20	e	10 35	15 08	18	15 3	18	14 59	18	14 55	18
21	f	11 33	15 26	18	15 21	17	15 17	18	15 13	18
22	g	12 31	15 44	17	15 39	18	15 35	18	15 31	17
23	a	S. *George.* 16 1		17	15 56	17	15 53	17	15 48	18
24	b	14 26	16 18	17	16 14	17	16 10	17	16 6	17
25	c	*Mar. Eva.* 16 35		17	16 31	16	16 27	17	16 23	17
26	d	16 22	16 52	16	16 47	18	16 44	17	16 40	17
27	e	17 20	17 9	16	17 5	16	17 1	16	16 57	16
28	f	18 18	17 25	15	17 21	16	17 17	16	17 13	16
29	g	19 16	17 41	15	17 37	15	17 43	16	17 29	15
30	a	20 13	17 56		17 52		17 59		17 44	

North Declination Increasing.

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension.* Time. H.	M.	Compl. H.	M.
1	1	21	10	39
2	1	25	10	35
3	1	29	10	31
4	1	33	10	27
5	1	36	10	24
6	1	40	10	20
7	1	44	10	16
8	1	47	10	13
9	1	51	10	9
10	1	54	10	6
11	1	58	10	2
12	2	2	9	58
13	2	6	9	54
14	2	10	9	50
15	2	13	9	47
16	2	17	9	43
17	2	21	9	39
18	2	25	9	35
19	2	29	9	31
20	2	32	9	28
21	2	36	9	24
22	2	40	9	20
23	2	44	9	16
24	2	48	9	12
25	2	51	9	9
26	2	55	9	5
27	2	59	9	1
28	3	3	8	57
29	3	7	8	53
30	3	10	8	50

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ First Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Last Quart. D.	H.	M.	
1688	20	3	27	Mor.	27	6	14	Mor.	5	6	4	Nig.	13	12	0	Noo.
1689	9	6	32	Nig.	16	10	42	Mor.	24	11	0	Mor.	2	9	31	Nig.
1690	28	7	0	Nig.	6	2	2	Mor.	14	9	34	Mor.	21	3	7	Aft.
1691	18	9	40	Mor.	25	7	4	Mor.	2	7	22	Nig.	10	1	5	Aft.
1692	6	3	6	Aft.	14	3	33	Mor.	20	5	49	Aft.	28	4	6	Mor.
1693	25	8	20	Mor.	3	4	52	Aft.	10	9	44	Mor.	17	6	0	Mor.
1694	14	9	6	Mor.	22	12	5	Noo.	29	10	13	Mor.	6	4	50	Aft.
1695	3	1	41	Aft.	11	11	46	Nig.	19	2	0	Mor.	25	7	11	Nig.
1696	21	11	0	Mor.	29	2	41	Mor.	7	11	10	Mor.	14	5	42	Aft.
1697	11	1	32	Mor.	18	1	52	Mor.	26	5	27	Mor.	4	11	0	Mor.
1698	30	2	14	Mor.	7	10	8	Mor.	15	6	19	Mor.	23	7	55	Mor.
1699	19	6	46	Nig.	25	1	4	Aft.	4	8	50	Mor.	12	9	44	Mor.

The SOLAR KALENDAR.

MAY.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	Difference. M.	I. After. 1681 1685 1689 1693 1697 D.	M.	Difference. M.	II. Aft. 1682 1686 1690 1694 1698 D.	M.	Difference. M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	Difference. M.
1	b	*Phil. & Ja.*	18	11	15	18	8	15	18	4	15	18	0	15
2	c	22 ♉ 9	18	26	15	18	23	14	18	19	15	18	15	15
3	d	23 6	18	41	14	18	37	15	18	34	14	18	30	15
4	e	24 4	18	55	14	18	52	14	18	48	14	18	45	14
5	f	25 2	19	9	14	19	6	13	19	2	14	18	59	14
6	g	25 59	19	25	14	19	19	14	19	16	14	19	13	14
7	a	26 57	19	36	13	19	33	13	19	30	13	19	27	13
8	b	27 54	19	49	13	19	46	13	19	43	13	19	40	13
9	c	28 52	20	2	12	19	59	12	19	56	12	19	53	12
10	d	29 49	20	14	12	20	11	13	20	8	12	20	5	13
11	e	0 ♊ 47	20	26	12	20	24	12	20	20	12	20	18	12
12	f	1 44	20	38	11	20	36	11	20	32	11	20	30	11
13	g	2 41	20	49	11	20	47	10	20	43	12	20	41	11
14	a	3 39	21	0	10	20	57	11	20	55	11	20	52	11
15	b	4 37	21	11	10	21	9	10	21	6	10	21	3	11
16	c	5 34	21	21	10	21	19	10	21	16	10	21	14	10
17	d	6 31	21	31	9	21	29	9	21	26	10	21	24	10
18	e	7 29	21	40	10	21	38	9	21	36	9	21	34	9
19	f	8 26	21	50	9	21	47	9	21	45	9	21	43	9
20	g	9 23	21	59	9	21	56	9	21	54	8	21	52	9
21	a	10 21	22	7	8	22	5	8	22	2	9	22	1	8
22	b	11 18	22	15	8	22	13	8	22	11	8	22	9	8
23	c	12 15	22	22	7	22	21	7	22	19	9	22	17	7
24	d	13 12	22	29	7	22	28	7	22	24	9	22	24	7
25	e	14 10	22	36	6	22	35	7	22	33	7	22	31	7
26	f	15 7	22	43	6	22	41	6	22	40	6	22	48	6
27	g	16 4	22	49	5	22	47	6	22	46	6	22	44	6
28	a	17 1	22	55	5	22	53	5	22	52	5	22	50	6
29	b	17 58	23	0	5	22	58	5	22	57	5	22	56	5
30	c	18 56	23	5	4	23	3	5	23	2	5	23	1	5
31	d	19 53	23	9	4	23	8	5	23	7	5	23	6	5

North Declination Increasing.

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension*, Time. H.	M.	Compl. H.	M.
1	3	14	8	46
2	3	18	8	42
3	3	22	8	38
4	3	26	8	34
5	3	30	8	30
6	3	34	8	26
7	3	38	8	22
8	3	52	8	18
9	3	56	8	14
10	3	50	8	10
11	3	54	8	6
12	3	58	8	2
13	4	2	7	58
14	4	6	7	54
15	4	10	7	50
16	4	14	7	46
17	4	18	7	42
18	4	22	7	38
19	4	26	7	34
20	4	30	7	30
21	4	34	7	26
22	4	38	7	22
23	4	42	7	18
24	4	46	7	14
25	4	50	7	10
26	4	54	7	6
27	4	58	7	2
28	5	2	6	58
29	5	6	6	54
30	5	11	6	49
31	5	15	6	45

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ Firſt Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Laſt Quarter D.	H.	M.	
1688	19	4	10	Mor.	26	10	41	Nig.	5	9	36	Mor.	12	6	56	Nig.
1689	9	1	34	Mor.	15	11	21	Nig.	24	2	10	Mor.	{ 2 / 31	10 / 7	26 M. / 40 N.	}
1690	28	2	31	Aft.	5	9	0	Mor.	13	3	19	Mor.	21	5	53	Mor.
1691	17	6	29	Nig.	26	9	50	Mor.	{ 2 / 31	8 / 7	12 M. / 54 N.	}	10	5	37	Mor.
1692	6	4	48	Mor.	12	10	48	Nig.	20	3	40	Mor.	27	9	9	Nig.
1693	24	11	7	Nig.	{ 2 / 31	5 / 3	47 Af. / 6 Af.	}	9	5	37	Nig.	16 / 6	8 / 2	0 Nig. / 2 Mor.	
1694	13	12	41	Nig.	22	1	11	Mor.	28	5	56	Nig.				
1695	3	3	0	Mor.	11	4	54	Mor.	18	11	8	Mor.	24	12	53	Mor.
1696	20	9	52	Nig.	28	8	12	Nig.	6	11	45	Nig.	13	10	33	Nig.
1697	10	9	39	Mor.	17	5	37	Nig.	25	7	13	Nig.	3	7	46	Nig.
1698	29	9	10	Mor.	6	8	24	Nig.	14	9	55	Nig.	22	7	11	Nig.
1699	19	2	43	Mor.	25	5	7	Nig.	3	11	12	Nig.	12	2	10	Mor.

The SOLAR KALENDAR.

JUNE.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.		II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.		D. M.	M.	D. M.	M.
1	e	20 ♊ 50	23 13		23 12		North Declin. Increasing.	23 11		23 10	
2	f	21 47	23 17	4	23 16	4		23 15	4	23 14	4
3	g	22 44	23 20	3	23 19	3		23 19	4	23 18	4
4	a	23 41	23 23	3	23 22	3		23 22	3	23 21	3
5	b	24 38	23 25	2	23 25	3		23 24	2	23 24	3
6	c	25 36	23 27	2	23 27	2		23 26	2	23 26	2
7	d	26 33	23 29	2	23 28	1		23 28	2	23 28	2
8	e	27 30	23 30	1	23 29	1		23 29	1	23 29	1
9	f	28 27	23 31	1	23 30	1		23 30	1	23 30	1
10	g	*Corpus Chr.*	23 *31	0	23 31	1		23 31	1	23 31	1
11	a	S. *Barnab.*	23 31	0	23 *31	0	*	23 *31	0	23 *31	0
12	b	1 ♋ 18	23 30	1	23 31	1		23 31	0	23 31	0
13	c	2 15	23 29	1	23 30	1		23 30	1	23 30	1
14	d	3 12	23 29	0	23 29	1		23 29	1	23 29	1
15	e	4 9	23 27	2	23 27	2		23 28	1	23 28	1
16	f	5 6	23 25	2	23 25	2		23 26	2	23 26	2
17	g	6 3	23 22	3	23 23	2		23 23	3	23 24	2
18	a	7 0	23 19	3	23 20	3		23 20	3	23 21	3
19	b	7 57	23 16	3	23 17	3		23 17	3	23 19	2
20	c	8 55	23 12	4	23 13	4		23 13	4	23 15	4
21	d	9 52	23 8	4	23 9	4		23 9	4	23 11	4
22	e	10 48	23 3	5	23 4	5		23 5	4	23 6	5
23	f	11 46	22 58	5	23 0	5		23 1	4	23 2	4
24	g	S. *Jo. Bapt.*	22 53	5	22 55	5	North Declination Decreasing.	22 56	5	22 57	5
25	a	13 40	22 47	6	22 49	6		22 51	5	22 51	6
26	b	14 27	22 41	6	22 43	6		22 45	6	22 45	6
27	c	15 34	22 34	7	22 36	7		22 38	7	22 39	6
28	d	16 31	22 27	7	22 28	8		22 31	7	22 33	7
29	e	S. *Pet.* & *P.*	22 20	7	22 22	8		22 34	8	22 26	8
30	f	18 25	22 12		22 14			22 16		22 18	

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension.* Time. H.	M.	Compl. H.	M.
1	5	19	6	41
2	5	23	6	37
3	5	27	6	33
4	5	31	6	29
5	5	36	6	24
6	5	40	6	20
7	5	44	6	16
8	5	48	6	12
9	5	52	6	8
10	5	56	6	4
11	6	0	6	0
12	6	4	5	56
13	6	8	5	52
14	6	12	5	48
15	6	17	5	43
16	6	21	5	39
17	6	25	5	35
18	6	29	5	31
19	6	33	5	27
20	6	38	5	22
21	6	42	5	18
22	6	46	5	14
23	6	50	5	10
24	6	54	5	6
25	6	58	5	2
26	7	2	4	58
27	7	6	4	54
28	7	10	4	50
29	7	14	4	46
30	7	19	4	41

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ First Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Last Quart. D.	H.	M.	
1688	17	8	5	Nig.	25	3	34	Aft.	3	9	56	Nig.	10	5	46	Aft.
1689	7	10	8	Mor.	14	1	51	Aft.	22	4	18	Aft.	30	1	56	Mor.
1690	25	9	21	Mor.	3	6	30	Nig.	11	4	13	Aft.	19	5	43	Aft.
1691	16	2	31	Mor.	22	4	24	Aft.	30	10	26	Mor.	8	10	46	Nig.
1692	4	4	28	Aft.	11	12	37	Noo.	18	2	22	Aft.	26	2	4	Aft.
1693	23	11	56	Mor.	30	3	44	Aft.	8	1	36	Mor.	15	11	37	Mor.
1694	12	3	22	Aft.	20	11	22	Mor.	26	11	51	Nig.	4	1	28	Aft.
1695	1	4	56	Aft.	9	7	56	Nig.	16	6	17	Nig.	23	9	26	Mor.
1696	19	10	32	Mor.	27	12	8	Noo.	5	9	48	Mor.	12	2	42	Mor.
1697	8	6	36	Nig.	16	10	16	Mor.	24	7	0	Mor.	1	11	10	Nig.
1698	27	5	0	Aft.	5	5	32	Mor.	13	12	40	Noo.	27	3	0	Mor.
1699	17	9	32	Mor.	23	11	45	Nig.	2	12	12	Noo.	10	3	16	Aft.

The SOLAR KALENDAR.

JULY.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	Diff. M.	I. After. 1681 1685 1689 1693 1697 D.	M.	Diff. M.	II. Aft. 1682 1686 1690 1694 1698 D.	M.	Diff. M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	Diff. M.
1	g	*Visit. Mar.*	22	4	8	22	6	8	22	8	8	22	10	8
2	a	20 ♋ 19	21	56	9	21	58	9	22	0	9	22	2	9
3	b	21 17	21	47	9	21	49	9	21	51	9	21	54	9
4	c	22 14	21	38	10	21	40	10	21	42	9	21	45	10
5	d	23 11	21	28	10	21	31	10	21	33	10	21	35	9
6	e	24 8	21	18	10	21	21	10	21	23	10	21	26	10
7	f	25 5	21	8	11	21	11	11	21	13	10	21	16	11
8	g	26 2	20	57	11	21	0	12	21	3	11	21	5	11
9	a	26 59	20	46	12	20	49	11	20	52	12	20	54	11
10	b	7 *Brethr.*	20	34	11	20	37	12	20	40	11	20	43	12
11	c	28 54	20	23	12	20	26	12	20	29	12	20	31	11
12	d	29 51	20	11	12	20	14	13	20	17	12	20	20	12
13	e	0 ♌ 48	19	59	13	20	2	13	20	5	13	20	8	13
14	f	1 46	19	46	13	19	49	13	19	52	13	19	55	13
15	g	*Swythin.*	19	33	13	19	36	13	19	39	13	19	42	13
16	a	3 40	19	20	13	19	23	13	19	26	13	19	24	13
17	b	4 37	19	6	14	19	10	14	19	13	13	19	16	13
18	c	5 35	18	52	14	18	56	14	18	59	14	19	2	14
19	d	6 32	18	30	14	18	42	15	18	45	14	18	48	14
20	e	*Margaret.*	18	23	15	18	27	15	18	30	15	18	34	14
21	f	8 27	18	8	15	18	12	15	18	15	15	18	19	15
22	g	9 23	17	53	15	17	57	16	18	0	15	18	4	15
23	a	10 21	17	37	16	17	41	16	17	45	16	17	49	16
24	b	11 19	17	21	16	17	25	16	17	29	16	17	23	16
25	c	*James* A.	17	5	16	17	9	16	17	13	16	17	17	16
26	d	13 14	16	49	16	16	53	17	16	57	16	17	1	16
27	e	14 11	16	33	17	16	37	17	16	41	17	16	45	17
28	f	15 9	16	16	17	16	20	18	16	24	17	16	28	17
29	g	16 6	15	59	17	16	3	18	16	7	17	16	11	17
30	a	17 4	15	41	18	15	45	18	15	50	18	15	54	18
31	b	18 1	15	23	18	15	27	18	15	32	18	15	36	18

North Declination Decreasing.

Tables of the Sun's Right Ascension.

Month Days.	Week Days.	The *Sun's Right Ascension.* Time. H.	M.	Compl. H.	M.
1	c	7	23	4	57
2	d	7	27	4	33
3	e	7	31	4	29
4	f	7	35	4	25
5	g	7	39	4	21
6	a	7	43	4	17
7	b	7	47	4	13
8	c	7	51	4	9
9	d	7	55	4	5
10	e	7	59	4	1
11	f	8	3	3	57
12	g	8	7	3	53
13	a	8	11	3	49
14	b	8	15	3	45
15	c	8	19	3	41
16	d	8	23	3	37
17	e	8	27	3	33
18	f	8	31	3	29
19	g	8	35	3	25
20	a	8	39	3	21
21	b	8	43	3	17
22	c	8	47	3	13
23	d	8	51	3	9
24	e	8	55	3	5
25	f	8	58	3	2
26	g	9	2	2	58
27	a	9	6	2	54
28	b	9	10	2	50
29	c	9	14	2	46
30	d	9	18	2	42
31	e	9	21	2	39

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ Firſt Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Laſt Quart. D.	H.	M.	
1688	17	7	47	Mor.	25	8	34	Mor.	3	8	38	Mor.	10	3	10	Mor.
1689	6	5	28	Aft.	18	1	54	Mor.	22	5	0	Mor.	29	6	7	Mor.
1690	25	4	42	Nig.	3	6	27	Mor.	11	9	28	Mor.	19	2	45	Mor.
1691	15	9	49	Mor.	22	1	43	Mor.	30	1	29	Mor.	8	1	2	Aft.
1692	3	4	25	Mor.	10	5	14	Aft.	18	3	9	Mor.	26	5	51	Mor.
1693	22	11	7	Nig.	29	7	33	Nig.	7	10	40	Mor.	15	4	22	Mor.
1694	12	6	6	Mor.	16	6	1	Nig.	26	8	50	Mor.	4	3	9	Mor.
1695	1	7	41	M.	9	8	20	Mor.	16	1	10	Mor.	22	8	42	Nig.
	30	10	50	N.												
1696	18	11	52	Nig.	27	2	56	Mor.	4	6	56	Nig.	11	7	54	Mor.
1697	8	5	0	Mor.	16	3	2	Mor.	23	5	1	Aft.	1	11	10	N.
													30	9	10	M.
1698	27	1	51	Mor.	4	12	55	Nig.	13	2	20	Mor.	20	8	36	Mor.
1699	16	4	38	Aft.	23	4	53	Aft.	2	5	22	M.	10	2	27	Mor.
									31	7	52	N.				

The SOLAR KALENDAR.
AUGUST.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.	II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.	D. M.	M.	D. M.	M.
1	c	*Lammas.*	15 5	18	15 10	18	15 14	18	15 19	18
2	d	19 ♌ 56	14 17	18	14 52	18	14 56	18	15 1	19
3	e	20 54	14 29	19	14 34	19	14 38	19	14 42	18
4	f	21 52	14 10	19	14 15	19	14 19	19	14 24	19
5	g	22 49	13 51	19	13 56	19	14 0	19	14 5	19
6	a	23 47	13 32	19	13 37	19	13 41	19	13 46	19
7	b	24 45	13 13	19	13 18	19	13 52	19	13 27	19
8	c	25 43	12 54	20	12 59	20	13 3	20	13 8	20
9	d	26 40	12 34	19	12 39	20	12 43	20	12 48	20
10	e	*Laurence.*	12 15	20	12 19	20	12 23	20	12 28	20
11	f	28 36	11 54	21	11 59	21	12 3	20	12 8	20
12	g	29 36	11 33	20	11 38	20	11 43	20	11 48	20
13	a	0 ♍ 32	11 13	21	11 18	21	11 23	20	11 28	21
14	b	1 30	10 52	20	10 57	21	11 3	21	11 7	20
15	c	*J. Bapt. N.*	10 32	21	10 36	21	10 42	21	10 47	21
16	d	3 26	10 11	21	10 15	21	10 21	21	10 26	21
17	e	4 24	9 50	22	9 54	21	10 0	22	10 5	21
18	f	5 22	9 28	22	9 33	21	9 38	21	9 44	22
19	g	6 20	9 6	22	9 12	22	9 17	22	9 22	21
20	a	7 18	8 44	22	8 50	21	8 55	22	9 1	22
21	b	8 16	8 23	21	8 29	22	8 33	21	8 39	22
22	c	9 14	8 1	22	8 7	22	8 12	22	8 17	22
23	d	10 12	7 39	22	7 45	22	7 50	22	7 55	22
24	e	*Barthol.* A.	7 17	23	7 23	23	7 28	23	7 33	22
25	f	12 9	6 56	22	7 0	22	7 5	22	7 11	23
26	g	13 7	6 32	22	6 38	23	6 43	22	6 45	22
27	a	14 5	6 10	23	6 15	22	6 21	23	6 26	22
28	b	15 4	5 47	22	5 53	23	5 58	22	6 4	23
29	c	16 2	5 25	23	5 30	22	5 35	23	5 41	23
30	d	17 1	5 2	23	5 8	24	5 13	23	5 13	23
31	e	17 59	4 39		4 44		4 52		4 55	

North Declination Decreasing.

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension*, Time. H.	M.	Compl. H.	M.
1	9	25	2	35
2	9	29	2	31
3	9	33	2	27
4	9	37	2	23
5	9	40	2	20
6	9	44	2	16
7	9	48	2	12
8	9	51	2	9
9	9	55	2	5
10	9	58	2	2
11	10	2	1	58
12	10	6	1	54
13	10	10	1	50
14	10	14	1	46
15	10	17	1	43
16	10	21	1	39
17	10	25	1	35
18	10	28	1	32
19	10	32	1	28
20	10	35	1	25
21	10	39	1	21
22	10	43	1	17
23	10	46	1	14
24	10	50	1	10
25	10	53	1	7
26	10	57	1	3
27	0	2	0	58
28	0	4	0	56
29	0	8	0	52
30	0	11	0	49
31	0	15	0	45

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ First Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Last Quarter D.	H.	M.	
1688	15	9	10	Nig.	23	12	34	Nig.	1	5	49	Af.	8	8	45	Mor.
									31	2	26	M.				
1689	25	3	21	Mor.	12	11	13	Nig.	20	4	17	Aft.	27	10	38	Mor.
1690	24	1	21	Mor.	1	9	18	Ni.	9	11	49	Nig.	17	9	32	Mor.
					31	2	23	Af.								
1691	13	4	50	Aft.	20	2	22	Aft.	28	5	16	Aft.	7	1	6	Mor.
1692	2	9	48	M.	8	11	44	Nig.	16	5	55	Aft.	24	9	5	Nig.
	31	5	49	Af.												
1693	21	9	26	Mor.	27	12	39	Nig.	5	9	50	Nig.	13	10	0	Nig.
1694	10	7	27	Nig.	17	11	12	Nig.	24	6	30	Nig.	2	7	4	Nig.
1695	29	2	13	Aft.	7	6	20	Nig.	14	8	33	Mor.	21	11	10	Mor.
1696	17	3	10	Aft.	25	4	15	Aft.	2	1	35	Mor.	9	2	36	Aft.
1697	6	5	27	Aft.	14	8	1	Nig.	22	1	54	Mor.	28	3	48	Aft.
1698	25	12	52	Noo.	3	5	40	Aft.	11	2	25	Aft.	18	12	25	Mor.
1699	14	12	32	Nig.	22	8	49	Mor.	30	10	22	Mor.	8	10	10	Mor.

The SOLAR KALENDAR.

SEPTEMBER.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	M.	I. After. 1681 1685 1689 1693 1697 D.	M.	M.		II. Aft. 1682 1686 1690 1694 1698 D.	M.	M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	M.
1	f	18 ♍ 58	4	16	23	4	22	23		4	27	23	4	32	23
2	g	19 56	3	53	23	3	59	24		4	4	23	4	9	23
3	a	20 55	3	30	23	3	35	23		3	41	24	3	46	23
4	b	21 53	3	7	24	3	12	23		3	17	23	3	23	23
5	c	22 52	2	43	23	2	49	23		3	54	23	3	0	23
6	d	23 51	2	20	23	2	26	23		2	31	23	2	37	24
7	e	24 49	1	57	23	2	3	24		2	8	23	2	13	23
8	f	*Mary Nat.*	1	33	24	1	39	23		1	45	23	1	50	23
9	g	26 47	1	10	23	1	16	24		1	21	24	1	27	24
10	a	27 46	0	46	24	0	52	23		1	0	23	0	3	24
11	b	28 44	0	23	23	0	29	24		0	51	24	0	40	24
12	c	29 43	0	1	22	0	5	24		0	30	24	0	16	24
13	d	0 ♎ 42	0	*24	23	0	* 8	23	*	0	* 10	7	0	* 7	9
14	e	*Holy Croſs.*	0	48	24	0	42	23		0	36	23	0	31	24
15	f	2 40	1	11	23	1	5	24		1	0	26	1	54	23
16	g	3 39	1	35	24	1	29	24		1	23	23	1	18	24
17	a	4 38	1	58	23	1	52	23		1	47	24	1	41	23
18	b	5 37	2	22	24	2	16	24		2	10	23	2	5	24
19	c	6 36	2	45	23	2	40	24		2	34	24	2	28	23
20	d	7 36	3	9	24	3	3	23		2	57	23	2	52	24
21	e	*Matth.* Ap.	3	32	23	3	27	24		3	21	24	3	15	23
22	f	9 34	3	56	24	3	50	23		3	44	23	3	38	23
23	g	10 33	4	19	23	4	13	23		4	7	23	4	2	24
24	a	11 33	4	42	24	4	36	23		4	30	23	4	25	23
25	b	12 32	5	6	23	4	59	23		4	54	24	4	48	24
26	c	13 31	5	29	23	5	23	23		5	17	23	5	12	23
27	d	14 31	5	52	23	5	36	23		5	41	24	5	35	23
28	e	15 30	6	15	23	6	9	23		6	4	23	5	58	23
29	f	*Mich.* Arc.	6	38	23	6	32	23		6	26	22	6	21	23
30	g	17 29	7	1		6	55			6	49	23	6	44	

North Declin. Decreaſing. / South Declination Increaſing.

Tables of the Sun's Right Aſcenſion.

Month Days.	Week Days.	The *Sun's Right Aſcenſion.* Time. H.	M.	Compl. H.	M.
1	a	11	19	0	41
2	b	11	23	0	37
3	c	11	26	0	34
4	d	11	30	0	30
5	e	11	33	0	27
6	f	11	37	0	23
7	g	11	41	0	19
8	a	11	44	0	16
9	b	11	46	0	12
10	c	11	51	0	9
11	d	11	55	0	5
12	e	11	59	0	1
13	f	0	2	11	58
14	g	0	6	11	54
15	a	0	9	11	51
16	b	0	13	11	47
17	c	0	17	11	43
18	d	0	20	11	40
19	e	0	24	11	36
20	f	0	27	11	33
21	g	0	31	11	29
22	a	0	35	11	25
23	b	0	38	11	22
24	c	0	42	11	18
25	d	0	45	11	15
26	e	0	48	11	12
27	f	0	53	11	7
28	g	0	57	11	3
29	a	1	1	10	59
30	b	1	4	10	56

The LUNAR KALENDAR.

Years of Chriſt.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ Firſt Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.		□ ☉ ☾ Laſt Quart. D.	H.	M.	
1688	14	12	49	Noo.	22	3	43	Aft.	29	11	0	Noo.	6	4	28	Aft.
1689	3	3	39	Aft.	11	5	17	Aft.	19	5	46	Mor.	25	8	13	Nig.
1690	22	12	32	Noo.	30	9	12	Mor.	8	1	42	Aft.	15	3	8	Aft.
1691	12	1	13	Mor.	19	6	13	Mor.	27	9	8	Mor.	5	11	0	Noo.
1692	30	2	24	Mor.	7	10	4	Mor.	5	10	26	Mor.	23	9	13	Mor.
1693	19	7	0	Nig.	25	1	16	Aft.	4	11	34	Noo.	12	1	0	Aft.
1694	9	8	10	Mor.	16	5	8	Mor.	23	7	8	Mor.	1	12	55	Noo.
1695	28	4	35	Mor.	6	2	5	Mor.	12	5	30	Aft.	2	4	30	Mor.
1696	16	9	46	Mor.	24	3	29	Mor.	1 30	9 8	21 Ni. 52 Af.		8	5	17	Mor.
1697	5	8	17	Mor.	13	12	0	Noo.	20	10	10	Mor.	27	2	6	Mor.
1698	24	2	29	Mor.	2	12	06	Noo.	10	1	30	Mor.	16	5	1	Aft.
1699	13	10	29	Mor.	21	3	4	Mor.	26	2	10	Mor.	6	4	15	Aft.

The SOLAR KALENDAR.
OCTOBER.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.		II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.		D. M.	M.	D. M.	M.
1	a	18 ♎ 28	7 24	22	7 17	23	South Declination Increasing.	7 12	22	7 6	23
2	b	19 29	7 46	23	7 40	22		7 35	23	7 29	23
3	c	20 28	8 9	22	8 2	23		7 58	22	7 52	23
4	d	21 28	8 31	22	8 25	22		8 20	22	8 15	22
5	e	22 28	8 53	22	8 47	22		8 42	22	8 37	22
6	f	23 27	9 15	22	9 9	22		9 4	23	8 59	21
7	g	24 27	9 37	22	9 30	23		9 27	22	9 21	22
8	a	25 27	9 59	22	9 54	22		9 49	21	9 43	22
9	b	26 27	10 21	22	10 16	22		10 10	22	10 5	22
10	c	27 27	10 43	21	10 37	21		10 32	21	10 27	21
11	d	28 27	11 4	21	10 59	22		10 53	22	10 48	22
12	e	29 27	11 25	21	11 20	21		11 15	21	11 18	21
13	f	0 ♏ 27	11 46	21	11 41	21		11 36	21	11 31	21
14	g	1 27	12 7	21	12 2	21		11 57	21	11 52	23
15	a	2 27	12 28	21	12 23	21		12 18	21	12 15	21
16	b	3 27	12 48	20	12 44	20		12 39	20	12 34	20
17	c	4 27	13 9	21	13 4	20		12 59	21	12 54	20
18	d	*Luke* Eva.	13 29	20	13 24	20		13 20	20	13 14	20
19	e	6 27	13 49	20	13 44	20		13 40	19	13 34	20
20	f	7 27	14 9	20	14 4	20		13 59	20	13 54	29
21	g	8 28	14 28	19	14 24	19		14 19	19	14 14	20
22	a	9 28	14 48	20	14 43	19		14 38	19	14 34	19
23	b	10 28	15 7	19	15 2	19		14 57	19	14 53	19
24	c	11 29	15 26	19	15 21	19		15 16	19	15 12	19
25	d	*Crispine.*	15 44	18	15 40	19		15 35	18	15 31	19
26	e	13 29	16 2	18	15 58	18		15 53	18	15 49	18
27	f	14 30	16 20	18	16 16	18		16 11	18	16 7	18
28	g	*Sim.& Ju.*	16 38	17	16 33	17		16 29	18	16 25	17
29	a	16 31	16 55	17	16 51	18		16 45	17	16 32	18
30	b	17 31	17 12	17	17 8	17		17 4	17	17 0	17
31	c	18 32	17 29	17	17 25	17		17 21	17	17 17	17

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension*, Time.		Compl.	
	H.	M.	H.	M.
1	1	8	10	52
2	1	12	10	48
3	1	15	10	45
4	1	19	10	41
5	1	22	10	38
6	1	26	10	34
7	1	30	10	30
8	1	34	10	26
9	1	38	10	22
10	1	41	10	19
11	1	55	10	15
12	1	49	10	11
13	1	53	10	7
14	1	57	10	3
15	2	0	10	0
16	2	4	9	56
17	2	8	9	52
18	2	12	9	48
19	2	16	9	44
20	2	20	9	40
21	2	24	9	36
22	2	28	9	32
23	2	32	9	28
24	2	36	9	24
25	2	39	9	21
26	2	43	9	17
27	2	47	9	13
28	2	51	9	9
29	2	55	9	5
30	2	59	9	1
31	3	3	8	57

The LUNAR KALENDAR.

Years of Christ.	♂ ☉ ☾ New Moon.				□ ☉ ☾ First Quart.				♃ ☉ ☾ Full Moon.				□ ☉ ☾ Last Quarter			
	D.	H.	M.		D.	H.	M.		D.	H.	M.		D.	H.	M.	
1688	14	6	40	Mor.	22	5	0	Mor.	28	8	12	Nig.	6	4	56	Mor.
1689	3	6	45	Mor.	11	11	0	Noo.	18	12	40	Noo.	25	2	21	Mor.
1690	22	1	10	Mor.	30	3	19	Mor.	8	2	26	Mor.	14	10	35	Nig.
1691	11	1	4	Aft.	19	1	9	Aft.	26	12	29	Nig.	4	6	42	Nig.
1692	29	12	36	Noo.	6	12	30	Nig.	15	3	52	Mor.	22	9	32	Nig.
1693	19	4	17	Mor.	25	9	10	Nig.	4	3	41	Mor.	12	7	25	Mor.
1694	8	7	32	Nig.	15	10	48	Mor.	22	10	34	Nig.	31	2	36	Mor.
1695	26	7	0	Nig.	4	9	26	Nig.	12	4	45	Mor.	19	12	0	Nig.
1696	15	12	0	Nig.	23	12	52	Noo.	30	4	30	Mor.	7	9	23	Nig.
1697	5	1	7	Mor.	13	2	40	Mor.	19	7	4	Nig.	27	2	28	Mor.
1698	23	7	24	Nig.	2 31	5 11	53 M. 20 Ni.		9	12	2	Noo.	16	2	3	Mor.
1699	12	10	41	Mor.	20	10	43	Nig.	28	12	26	Noo.	5	9	47	Nig.

The SOLAR KALENDAR.

NOVEMBER.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696 D.	M.	Diff. M.	I. After. 1681 1685 1689 1693 1697 D.	M.	Diff. M.		II. Aft. 1682 1686 1690 1694 1698 D.	M.	Diff. M.	III. Aft. 1683 1687 1691 1695 1699 D.	M.	Diff. M.
1	d	*All Saints.*	17	46	16	17	41	17		17	38	16	17	32	17
2	e	20 m	33	18	2	16	17	58	16		17	54	16	17	50	16
3	f	21	34	18	18	16	18	1	16		18	10	16	18	6	16
4	g	23	35	18	33	15	18	30	15		18	26	16	18	22	16
5	a	*Powd. Tr.*	18	48	15	18	45	15		18	41	15	18	37	15
6	b	24	36	19	3	15	19	0	15		18	56	15	19	53	16
7	c	25	37	19	18	15	19	14	14		19	11	15	19	8	15
8	d	26	37	19	32	14	19	29	14		19	25	14	19	22	14
9	e	27	38	19	46	14	19	43	13		19	39	14	19	36	14
10	f	*Martin.*		19	59	13	19	56	14		19	53	14	19	50	14
11	g	29	40	20	13	14	20	10	13		20	6	13	20	3	13
12	a	0 ♐	41	20	26	13	20	23	13		20	19	13	20	16	13
13	b	1	42	20	38	12	20	35	12		20	31	12	20	29	13
14	c	2	43	20	50	12	20	47	12		20	43	12	20	41	12
15	d	3	43	21	2	12	20	59	12		20	55	12	20	53	12
16	e	*Hugh.*		21	13	11	21	10	11		21	7	12	21	5	12
17	f	5	45	21	23	10	21	21	10		21	18	11	21	16	11
18	g	6	46	21	34	9	21	31	10		21	29	11	21	26	10
19	a	7	47	21	44	10	21	41	10		21	39	10	21	36	10
20	b	8	48	21	54	10	21	51	9		21	49	10	21	46	10
21	c	9	49	22	3	9	22	0	9		21	58	9	22	56	9
22	d	10	50	22	11	8	22	9	9		22	7	9	22	5	8
23	e	11	52	22	20	8	22	18	8		22	10	8	22	13	9
24	f	*Katherine.*		22	28	7	22	26	8		22	24	7	22	22	8
25	g	13	54	22	35	7	22	34	7		22	31	8	22	30	7
26	a	14	55	22	42	7	22	41	7		22	39	6	22	37	7
27	b	15	56	22	48	6	22	48	6		22	45	7	22	44	6
28	c	16	57	22	54	6	22	54	6		22	52	6	22	50	6
29	d	17	58	23	0	6	22	59	5		22	58	6	22	56	6
30	e	*Andr.* Ap.		23	5	5	23	4	5		23	3	5	23	2	

South Declination Increasing.

Tables of the Sun's Right Aſcenſion.

Month Days.	The *Sun's Right Aſcenſion.* Time. H.	M.	Compl. H.	M.
1	3	7	8	53
2	3	11	8	49
3	3	15	8	45
4	3	19	8	41
5	3	23	8	37
6	3	27	8	33
7	3	31	8	29
8	3	36	8	24
9	3	40	8	20
10	3	45	8	15
11	3	49	8	11
12	3	53	8	7
13	3	58	8	2
14	4	2	7	58
15	4	7	7	53
16	4	11	7	49
17	4	15	7	45
18	4	19	7	41
19	4	23	7	37
20	4	28	7	32
21	4	32	7	28
22	4	36	7	24
23	4	40	7	20
24	4	45	7	15
25	4	49	7	11
26	4	53	7	7
27	4	57	7	3
28	5	2	6	58
29	5	6	6	54
30	5	11	6	49

The LUNAR KALENDAR.

Years of Chriſt.	☌ ☉ ☾ New Moon. D.	H.	M.		□ ☉ ☾ Firſt Quart. D.	H.	M.		□ ☉ ☾ Laſt Quart. D.	H.	M.		☍ ☉ ☾ Full Moon. D.	H.	M.	
1688	12	12	52	Nig.	20	4	8	Aft.	27	7	5	Mor.	4	2	51	Aft.
1689	1	12	35	Nig.	10	3	35	Mor.	16	10	30	Nig.	23	5	22	Aft.
1690	20	7	34	Nig.	28	9	51	Nig.	6	12	25	Noo.	13	4	52	Mor.
1691	9	12	15	Nig.	17	9	44	Nig.	25	2	30	Aft.	3	1	42	Mor.
1692	27	11	48	Nig.	5	6	46	Nig.	13	9	46	Nig.	21	6	43	Mor.
1693	17	2	9	Aft.	24	12	42	Noo.	2	9	16	Nig.	10	10	2	Nig.
1694	7	6	20	Mor.	13	8	46	Nig.	21	4	42	Aft.	{ 1	7	44	Ni. }
													{ 29	8	14	Ni. }
1695	26	4	7	Mor.	3	3	10	Aft.	10	7	0	Nig.	18	10	37	Nig.
1696	14	5	12	Aft.	21	8	16	Nig.	28	5	22	Aft.	6	4	31	Aft.
1697	3	7	46	Nig.	11	3	13	Aft.	18	5	51	Mor.	25	10	14	Mor.
1698	22	2	10	Aft.	30	3	16	Aft.	7	10	0	Nig.	14	2	3	Aft.
1699	11	2	43	Aft.	19	6	37	Nig.	26	12	3	Nig.	3	8	25	Nig.

The SOLAR KALENDAR.

DECEMBER.

Tables of the Sun's Declination, *with their* Differences, *for the Years of our Lord.*

Month Days.	Week Days.	Fixed Feasts, and The Sun's Place.	Leap-y. 1680 1684 1688 1692 1696	Difference.	I. After. 1681 1685 1689 1693 1697	Difference.		II. Aft. 1682 1686 1690 1694 1698	Difference.	III. Aft. 1683 1687 1691 1695 1699	Difference.
			D. M.	M.	D. M.	M.		D. M.	M.	D. M.	M.
1	f	20 ♐ 1	23 10	4	23 9	4	South Decl. Increasing.	23 8	4	23 6	5
2	g	21 2	23 14	4	23 13	4		23 12	4	23 11	4
3	a	22 3	23 18	4	23 17	3		23 16	4	23 15	4
4	b	23 4	23 21	3	23 20	3		23 20	4	23 18	3
5	c	24 6	23 24	3	23 23	3		23 23	3	23 22	3
6	d	*Nicholas.*	23 26	2	23 26	2		23 25	3	23 25	2
7	e	26 8	23 28	2	23 28	2		23 27	2	23 27	2
8	f	27 9	23 30	1	23 29	1		23 29	2	23 28	1
9	g	28 11	23 31	0	23 30	1		23 30	1	23 30	2
10	a	29 12	23 *31	0	23 *31	0		23 *31	1	23 *31	1
11	b	0 ♑ 13	23 31	1	23 31	0		23 31	0	23 31	0
12	c	1 14	23 30	1	23 31	1		23 31	0	23 31	0
13	d	*Lucy.*	23 29	1	23 30	1		23 30	1	23 30	1
14	e	3 17	23 28	1	23 29	2		23 29	1	23 29	1
15	f	4 18	23 26	2	23 27	2		23 27	2	23 27	2
16	g	5 20	23 24	2	23 25	2		23 25	2	23 25	2
17	a	6 21	23 21	3	23 22	3		23 22	3	23 23	2
18	b	7 22	23 18	3	23 19	3		23 19	3	23 20	3
19	c	8 23	23 14	4	23 15	4		23 16	4	23 16	4
20	d	9 25	23 9	5	23 10	5		23 12	4	23 12	4
21	e	S. *Tho.* Ap.	23 5	4	23 6	6		23 7	5	23 8	4
22	f	11 27	23 0	5	23 1	6		23 2	5	23 3	5
23	g	12 28	22 54	6	22 55	7		22 57	5	22 54	6
24	a	13 29	22 48	6	22 49	8		22 51	6	22 52	6
25	b	*Christ. N.*	22 41	6	22 43	8		22 45	6	22 46	7
26	c	S. *Stephen.*	22 34	7	22 36	8		22 38	7	22 39	7
27	d	S. *Joh.* Ev.	22 27	7	22 28	8		22 31	7	22 32	7
28	e	*Innocents.*	22 19	8	22 20	8		22 23	8	22 25	8
29	f	18 35	22 11	8	22 12	9		22 15	8	22 17	8
30	g	19 37	22 9	8	22 4	9		22 6	9	22 8	9
31	a	20 38	21 53		21 55			21 57	9	31 59	9

Tables of the Sun's Right Ascension.

Month Days.	The *Sun's Right Ascension,* Time. H. M.	Compl. H. M.
1	5 15	6 45
2	5 21	6 39
3	5 25	6 35
4	5 29	6 31
5	5 34	6 26
6	5 38	6 22
7	5 42	6 18
8	5 47	6 13
9	5 51	6 9
10	5 56	6 4
11	6 0	6 0
12	6 5	5 55
13	6 9	5 51
14	6 14	5 46
15	6 19	5 41
16	6 24	5 36
17	6 28	5 32
18	6 33	5 27
19	6 37	5 23
20	6 41	5 19
21	6 45	5 15
22	6 48	5 12
23	6 54	5 6
24	6 58	5 2
25	7 3	4 57
26	7 7	4 53
27	7 11	4 49
28	7 16	4 44
29	7 20	4 40
30	7 25	4 35
31	7 30	4 30

The LUNAR KALENDAR.

Years of Christ.	☌ ☉ ☾ New Moon. D. H. M.	□ ☉ ☾ First Quart. D. H. M.	☍ ☉ ☾ Full Moon. D. H. M.	□ ☉ ☾ Last Quarter D. H. M.
1688	12 7 0 Nig.	20 1 2 Mor.	26 7 42 Nig.	4 6 15 Nig.
1689	1 1 4 Af. / 31 3 2 Af.	9 6 0 Nig.	16 10 2 Mor.	23 10 42 Mor.
1690	20 2 42 Aft.	28 5 5 Nig.	6 1 12 Mor.	12 4 25 Aft.
1691	9 1 51 Aft.	17 6 33 Nig.	25 3 9 Mor.	2 10 54 M. / 31 4 29 Af.
1692	27 1 35 Aft.	5 2 40 Aft.	13 1 50 Aft.	20 12 46 Noo.
1693	16 11 17 Nig.	24 8 3 Mor.	2 5 16 Aft.	10 10 28 Mor.
1694	6 4 43 Aft.	13 10 20 Mor.	21 12 8 Noo.	29 11 31 Noo.
1695	25 6 11 Nig.	3 11 12 Nig.	10 12 0 Noo.	18 4 25 Aft.
1696	14 8 4 Mor.	21 3 49 Mor.	28 8 49 Mor.	6 12 42 Noo.
1697	3 2 0 Aft.	11 1 25 Mor.	17 5 31 Nig.	25 6 8 Mor.
1698	22 9 37 Mor.	30 4 0 Mor.	7 7 54 Mor.	14 5 48 Mor.
1699	11 8 51 Mor.	19 12 56 Noo.	26 11 00 Mor.	3 8 11 Mor.

A TABLE whereby to appropriate the Tables of the Sun's Declination *to any Hour of the Day, or to any other* Meridian, *either Eastward or Westward of* London, *for which they were Calculated.*

Hours and Minutes after Noon, till Midnight.

The Difference of the Sun's Declination in Min. in 24 Hours.	1		2		3		4		5		6		7		8		9		10		11		Midn.	
	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.
1	0	2	0	5	0	5	0	10	0	12	0	15	0	17	0	20	0	20	0	25	0	27	0	30
2	0	5	0	10	0	15	0	20	0	25	0	30	0	55	0	40	0	45	0	50	0	55	1	0
3	0	7	0	15	0	22	0	30	0	37	0	45	0	52	1	0	1	7	1	15	1	22	1	30
4	0	10	0	20	0	30	0	40	0	50	1	0	1	10	1	20	1	30	1	40	1	50	2	0
5	0	12	0	25	0	37	0	50	1	2	1	15	1	27	1	40	1	52	2	5	2	17	2	30
6	0	15	0	30	0	45	1	0	1	15	1	30	1	45	2	0	2	15	2	30	2	45	3	0
7	0	17	0	35	0	52	1	10	1	27	1	45	2	2	2	20	2	37	2	55	3	12	3	30
8	0	20	0	40	1	0	1	20	1	40	2	0	2	20	2	40	3	0	3	20	3	40	4	0
9	0	22	0	45	1	7	1	30	1	52	2	15	2	37	3	0	3	22	3	45	4	7	4	30
10	0	25	0	50	1	15	1	40	2	5	2	30	2	55	3	20	3	45	4	10	4	35	5	0
11	0	27	0	55	1	22	1	50	2	17	2	45	3	12	3	40	4	7	4	35	5	2	5	30
12	0	30	1	0	1	30	2	0	2	30	3	0	3	30	4	0	4	30	5	0	5	30	6	0
13	0	32	1	5	1	37	2	10	2	42	3	15	3	47	4	20	4	52	5	25	5	57	6	30
14	0	35	1	10	1	45	2	20	2	55	3	30	4	5	4	40	5	15	5	50	6	25	7	0
15	0	37	1	15	1	52	2	30	3	7	3	45	4	22	5	0	5	37	6	15	5	52	7	30
16	0	40	1	20	2	0	2	40	3	20	4	0	4	40	5	20	6	0	6	40	7	20	8	0
17	0	42	1	25	2	7	2	50	3	32	4	15	4	57	5	40	6	22	7	5	7	47	8	30
18	0	45	1	30	2	15	3	0	3	45	4	30	5	15	6	0	6	45	7	30	8	15	9	0
19	0	47	1	35	2	22	3	10	3	57	4	45	5	32	6	20	7	7	7	55	8	42	9	30
20	0	50	1	40	2	30	3	20	4	10	5	0	5	50	6	40	7	30	8	20	9	10	10	0
21	0	52	1	45	2	37	3	30	4	22	5	15	6	7	7	0	7	52	8	45	9	37	10	30
22	0	55	1	50	2	45	3	40	4	35	5	30	6	25	7	20	8	15	9	10	10	5	11	0
23	0	57	1	55	2	52	3	50	4	47	5	45	6	42	7	40	8	37	9	35	10	32	11	30
24	1	0	2	0	3	0	4	0	5	0	6	0	7	0	8	0	9	0	10	0	11	0	12	0

Hours and Minutes from Midnight, till the next Morning.

The Difference of the Sun's Declination in 24 Hours.	1		2		3		4		5		6		7		8		9		10		11	
	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.	M.	Sec.
1	0	32	0	35	0	37	0	40	0	42	0	45	0	47	0	50	0	52	0	55	0	57
2	1	5	1	10	1	15	1	20	1	25	1	30	1	35	1	40	1	45	1	50	1	55
3	1	37	1	45	0	52	2	0	2	7	2	15	2	22	2	30	2	37	2	45	2	52
4	2	10	2	20	2	30	2	40	2	50	3	0	3	10	3	20	3	30	3	40	3	50
5	2	42	2	30	3	4	3	20	3	32	3	45	3	57	4	10	4	22	4	35	5	47
6	3	15	3	30	3	45	4	0	4	15	4	30	4	45	5	0	5	15	5	30	5	45
7	3	47	4	5	4	22	4	40	4	57	5	15	5	32	5	50	6	7	6	25	6	42
8	4	20	4	40	5	0	5	20	5	40	6	0	6	20	6	40	7	0	7	20	7	40
9	4	52	5	15	5	37	6	0	6	22	6	45	7	7	6	30	7	52	8	15	8	37
10	5	25	5	50	6	15	6	40	7	5	7	30	7	55	8	20	8	45	9	10	9	35
11	5	57	6	25	6	52	7	20	7	47	8	15	8	42	9	10	9	37	10	5	10	32
12	6	30	7	0	7	30	8	0	8	30	9	0	9	30	10	0	10	30	11	0	11	30
13	7	2	7	35	8	7	8	40	9	12	9	45	10	17	10	50	11	22	11	55	12	27
14	7	35	8	10	8	45	9	20	9	55	10	30	11	5	11	40	12	15	12	50	13	25
15	8	7	8	45	9	22	10	0	10	37	11	15	11	52	12	30	12	67	13	45	14	22
16	8	40	9	20	10	0	10	40	11	20	12	0	12	40	13	20	14	0	14	40	15	20
17	9	12	9	55	10	37	11	20	12	2	12	45	13	27	14	10	14	52	15	35	16	17
18	9	45	10	30	11	15	12	0	12	45	13	30	14	15	15	0	15	45	16	30	17	15
19	10	17	11	5	11	52	12	40	13	27	14	15	15	2	15	50	16	37	17	25	18	12
20	10	50	11	40	12	30	13	20	14	10	15	0	15	50	16	40	17	30	18	20	19	10
21	11	22	12	15	13	7	14	0	14	52	15	45	16	37	17	30	18	22	19	15	20	7
22	11	55	12	50	13	45	14	40	15	35	16	30	17	25	18	20	19	15	20	10	21	5
23	12	27	13	25	14	22	15	20	16	17	17	15	18	12	19	10	20	7	21	5	22	2
24	13	0	14	0	15	0	16	0	17	0	18	0	19	0	20	0	21	0	22	0	23	0

The

The Use of the Table of Referring the Sun's Declination *to any* Hour *or* Meridian.

THE Tables of the *Sun's Declination* in the foregoing *Kalendar*, are calculated properly for the Sun's true Place at Noon in the *Meridian* of *London*; but may be referred to any other *Meridian*, by help of this Table.

Alſo take notice, That the Degrees and Minutes of *Declination* found againſt every Day of the Year, are the Degrees and Minutes which the *Sun* hath every Day at Noon, for the *Aſtronomical Account* of *Days* is from *Noon* to *Noon*; ſo that One in the Afternoon is called the Firſt Hour after Noon, Two the Second, Three the Third, *&c.* till Twelve at Night, or Midnight. Then One in the Morning is called the Thirteenth Hour, or the Firſt Hour from midnight; Two in the Morning the Fourteenth Hour, or the Second Hour from Midnight, *&c.* So that Eleven the next Forenoon is called the Twenty third Hour, or the Eleventh Hour from Midnight; and then at Twelve begins the next Day.

Alſo obſerve, That when you make uſe of the Table of *Declination*, you make uſe of the Degrees and Minutes found againſt the Tenth day, till the Eleventh day at Noon; and of thoſe againſt the Eleventh day, till the Twelfth day at Noon, *&c.* Theſe things being premiſed, the Uſe of the Table follows.

How to Appropriate the Sun's Declination *to any Hour after Noon.*

You muſt firſt find the *Sun's Declination* for the Day propoſed, (in its proper Year,) in the Table, and note it down, with the *Difference* between that and the next day; then knowing at what Hour after Noon you would make uſe of the *Declination*, find that Hour in the Head of this Table, and the Minutes of the *Difference* of the *Sun's Declination* in the Firſt Column towards the Left-hand, and in the common *Angle* or Meeting of thoſe two, you ſhall find the Number of *Minutes* and *Seconds* which are to be added to, or ſubtracted from the *Degrees* and *Minutes* found in the Table: To be added, if the *Declination* increaſe; but to be ſubtracted, if the *Declination* do decreaſe.

Example. *What* Declination *ſhall the* Sun *have upon the Fifth of* April, *at Seven Hours after Noon, in the* Leap-year?

Look in the Table for the Fifth of *April*, and againſt it, in the Column (*Leap-year*) you ſhall find 10 deg. 6 min for the *Declination*, and 21 min. for the *Difference*: Now the *Declination* found, 10 deg. 6 min. is the *Declination* at Twelve at Noon; but to find it at Seven after Noon, go to this Table, and find 7 Hours at the Head thereof, and 21 (the *Difference*) in the firſt Column towards the Right-hand, and againſt 21, and under 7 Hours, you ſhall find 6 *Minutes* 7 *Seconds*; the *Seconds*, being under 30, may be omitted; but the 6 *Minutes* muſt be added (becauſe the *Declination* increaſeth) to the *Declination* at Noon, 10 deg. 6 min. making it 10 deg. 12 min. and ſuch *Declination* ſhall the *Sun* have on the Fifth of *April* at Seven in the Afternoon, in the *Leap-year*.

In like manner, if it were required to find what *Declination* the *Sun* ſhould have the ſame day at 19 Hours after Noon, that is, the Seventh Hour after Midnight, or Seven the next Morning: The Hour 7 being found in the Head of the Table, and 21 min. (the *Difference* of *Declination*) in the firſt Column, againſt 21, and under 7 Hours, you ſhall find 16 min. 37 ſec. which *Seconds* being above 30, add 1 min. to 16, making it 17 min. which added to 10 deg. 6 min. the *Declination* found in the Table, maketh 10 deg. 23 min. for the *Declination* at 7 the next Morning.

THE

Aſtral Kalendar.

Containing all the

FIXED STARS

OF THE

FIRST, SECOND, and THIRD,

MAGNITUDES.

THEIR

Longitudes corrected, and reduced from the Pole of the *Ecliptick* to the Pole of the *World*; That is to ſay, Brought into Right Aſcenſions and Declinations.

To which are added,

Their Right Aſcenſions in Time, and Semidiurnal Arks; whereby their Riſing, Southing, and Setting may (at at time) be eaſily obtained: Alſo upon what Day of the Year each of them will be upon the *Meridian* at Midnight, and conſequently the Stars known one from another.

The Northern

STARS NAMES.	Upon the Meridian at Midnight.
Ursa Minor. Cynosura.	
In the Extremity of the Tail, or *Pole-Star*	*Decemb.* 5
The Upper of the Two preceding in the Square	*Januar.* 17
The Lower of the same	24
Ursa Major. Helice.	
In the former Left Knee	*Januar.* 11
The Northerly of the two in the Right Foot	7
The Southerly	8
The Upper of the foregoing in the greater Square	19
The Lower of the same Square	23
The Upper of the following in the Square	*February* 4
The Lower of the same	4
The last save Two of the Tail	12
The last save one	18
The last of the Tail	*March* 2
That between this *Bear*'s Tail and the *Lion*	*Februar.* 26
That between the Fore-foot of the *Bear* and the First of the *Lion*'s *Head*	*Januar.* 16
The following Southern	30
The foregoing of the Two in the Base of the *Oxigon*	*February* 2
The following one	5
Draco.	
The foregoing of the Two Bright ones in the Head	*May* 18
The Bright one in the Head	*June* 4
The Northern of the Square in the second Bending	*Septem.* 26
The Southern of the same Side	*October* 11
That in the Flexure (or Bending) of the third Joint	*February* 8
The meanly Bright one near the *Pole*	*March* 8
The last but Two foregoing the Flexure	23
The last but one to the Flexure	10
That which next follows the Flexure	*Februar.* 11
The last but one of the Tail	*Januar.* 20
The last of the Tail	15
Cepheus.	
In the Girdle	*October* 14
The Bright one in the Right Shoulder	*Septem.* 21
In the Left Foot	*Novemb.* 7
Bootes. Artophylax.	
In the Left Shoulder	*March* 24
In the Head	30
In the Right Shoulder, above the Crown	*April* 8
In the Hip, below the Right Arm	3
That in the Right Leg	8
The Upper one of the Leg	*March* 24
In the Skirt. *Arcturus*	30

Corona

Constellations.

Magnitude	Longitude. Sin.	Deg.	Min.	Latitude. Deg.	Min.	Right Ascension. Deg.	Min.	Declination. Deg.	Min.	Right Ascension in Time. Ho.	Min.	Semidiurnal Ark. Ho.	M.
						The Lesser Bear.							
2	♊	24	2	66	2	7	53	87	33	9	31	Sets not.	
2	♌	8	16	72	51	42	46	75	36	2	51	Sets not.	
3	♌	15	41	75	23	51	14	73	15	3	25	Sets not.	
						The Greater Bear.							
3	♌	1	32	34	34	135	56	53	10	9	3	Sets not.	
3	♋	26	56	29	15	127	20	49	20	8	29	Sets not.	
3	♋	28	10	28	38	128	34	48	21	8	34	Sets not.	
2	♌	10	34	49	40	160	48	63	32	10	43	Sets not.	
2	♌	14	43	45	3	160	18	58	8	10	41	Sets not.	
2	♌	26	25	51	37	179	48	58	51	11	59	Sets not.	
2	♌	25	45	47	6	173	39	55	33	11	56	Sets not.	
2	♍	4	10	54	18	9	53	57	47	0	39	Sets not.	
2	♍	10	56	56	22	17	37	56	41	1	30	Sets not.	
2	♍	22	12	54	25	23	37	51	00	1	34	Sets not.	
2	♍	18	43	40	6	9	4	40	34	0	36	Sets not.	
3	♌	7	17	17	55	135	15	35	45	9	1	10	16
3	♌	20	57	24	50	153	8	37	49	10	12	11	16
3	♌	24	22	21	28	155	6	33	30	10	20	9	46
3	♌	27	9	20	44	157	56	31	48	10	30	9	30
						The Dragon.							
3	♐	7	19	75	21	80	46	52	34	5	23	Sets not.	
3	♐	23	24	75	3	87	15	51	36	5	49	Sets not.	
3	♈	13	26	82	49	108	14	67	7	7	13	Sets not.	
3	♈	28	47	79	25	117	27	69	30	7	49	Sets not.	
3	♍	0	44	81	4	167	22	69	24	4	29	Sets not.	
3	♍	27	51	84	46	256	59	66	8	5	7	Sets not.	
3	♑	13	28	74	11	239	4	58	58	3	56	Sets not.	
3	♑	0	22	71	4	229	27	60	3	3	18	Sets not.	
2	♍	3	10	66	36	209	40	65	56	1	59	Sets not.	
3	♌	11	26	61	33	184	13	71	34	0	17	Sets not.	
3	♌	5	37	57	7	167	15	71	5	11	9	Sets not.	
						Cepheus.							
3	♉	1	13	71	7	321	3	69	9	9	24	Sets not.	
3	♈	8	13	68	54	317	38	61	10	9	10	Sets not.	
3	♉	25	23	64	28	351	59	75	41	11	27	Sets not.	
						The Bear-keeper, or Bear-ward.							
3	♑	14	5	49	33	215	29	39	24	2	22	Sets not.	
3	♑	19	43	54	15	222	32	41	44	2	50	Sets not.	
3	♑	28	29	49	1	225	34	34	34	3	2	8	1
3	♑	23	29	40	40	217	40	28	29	2	31	8	53
3	♑	28	26	27	57	216	24	15	11	2	26	7	19
3	♑	14	42	28	9	204	45	20	5	1	39	7	48
1	♒	19	39	31	2	210	13	20	58	2	1	7	55

The

STARS NAMES.	Upon the Meridian at Midnight.	
Corona Borea.		
The Bright one of the Crown	*April*	17
Engonasi Hercules.		
In the Head	*May*	22
In the Right Shoulder		7
The laſt ſave one of the Right Arm	♊	5
In the Left Shoulder	♋	21
That in the Left Hip	♋	8
The more Eaſterly than this in the Left Thigh		15
In the Left Knee	*June*	4
That in the Left Leg Calf, nigh the *Dragon's Head*	*May*	27
In the Upper Right Thigh	♌	5
Lyra, Vultur cadens.		
The Bright one of the Harp	*June*	22
The Northern of the two foremoſt in the Neck	♌	26
The Northern of the two following in the Neck	♌	29
Olor. Cygnus.		
In the Bill	*July*	9
In the Breaſt	*Auguſt*	2
In the Tail	*Novemb.*	3
The Firſt and Brighteſt in the Pinnion of the Upper Wing	*July*	25
That in the Pinnion of the Lower Wing	*Auguſt*	6
The uttermoſt of the Lower Wing		11
Below the Wing, towards *Pegaſus* Foot		18
Caſſiopeia.		
In the Breaſt. *Schedir*	*October*	16
In the Flexure (or Bending) at the Hips		22
In the Leg		26
At the Knee	*Novemb.*	2
The Bright one of the Chair	*October*	13
Perſeus.		
In the Right Shoulder	*Novemb.*	7
The very Bright one in the Right Side	♋	9
That at the Flexure of the ſame Side		2
The Head of *Meduſa.* *Algol*		3
That in the Left Knee	♌	13
The following one of the Left Foot		10
Auriga, Heniochus, Erichthonius.		
The Bright one in the Left Shoulder. *Capella*	*Novem.*	28
The Bright one in the Right Shoulder	*Decemb.*	7
Ophyuchus, Serpentarius.		
In the Head	*May*	29
In the Right Shoulder	*June*	1
The Lower and Follower in the Right Shoulder		3

The

Magnitude	Longitude. Sin.	Deg.	Min.	Latitude. Deg.	Min.	Right Ascension. Deg.	Min.	Declination. Deg.	Min.	Right Ascension in Time. Ho.	Min.	Simidiurnal Ark. Ho.	M.
The Northern Crown.													
2	♏	7	38	44	23	230	12	27	51	3	21	Sets not.	
Engonasi Hercules.													
3	♐	11	31	37	23	254	12	14	50	4	57	7	18
3	♏	26	27	42	48	245	12	22	16	4	21	8	4
3	♏	24	36	40	5	241	52	19	59	4	7	7	47
3	♐	10	10	47	47	255	24	25	16	5	2	8	26
3	♏	27	2	53	10	247	19	32	14	5	29	8	4
3	♐	3	45	53	21	251	58	31	28	4	47	10	56
3	♐	23	56	60	47	266	17	37	21	5	41	Sets not.	
3	♐	15	17	66	22	262	34	46	14	5	30	Sets not.	
3	♏	24	8	60	22	247	56	39	35	4	31		
The Harp: Or, Falling Vultur.													
1	♑	10	43	61	47	276	27	38	30	6	26	Sets not.	
3	♑	14	16	56	5	279	27	33	2	6	38	9	39
3	♑	17	11	55	6	281	32	32	16	6	46	9	31
The Swan.													
3	♑	26	44	49	2	289	23	27	18	7	18	8	43
3	♒	20	25	57	9	302	39	39	13	8	10	Sets not.	
2	♏	20	53	59	56	307	32	44	5	8	30	Sets not.	
3	♒	11	53	64	28	393	44	44	22	7	35	Sets not.	
3	♒	23	9	49	26	308	14	32	45	8	33	9	36
3	♒	28	43	43	44	314	54	28	57	8	59	8	57
3	♓	5	33	38	39	322	20	26	20	9	29	8	35
Cassiope.													
3	♉	3	17	46	35	5	32	54	45	0	22	Sets not.	
3	♉	9	27	48	46	9	22	58	57	0	38	Sets not.	
3	♉	13	21	46	22	16	10	58	30	1	5	Sets not.	
3	♉	20	13	47	29	22	51	61	1	1	31	Sets not.	
3	♉	0	35	51	14	357	59	57	22	11	51	Sets not.	
Perseus.													
3	♉	25	26	34	30	40	17	52	12	2	41	Sets not.	
2	♉	27	17	30	5	44	16	48	36	2	57	Sets not.	
3	♉	20	15	27	14	38	0	43	49	2	32	Sets not.	
3	♉	21	37	22	22	41	46	39	39	2	47	Sets not.	
3	♊	1	8	19	4	54	1	39	2	3	36	Sets not.	
3	♉	28	36	11	17	53	27	30	54	2	3	9	15
The Carter, Wagoner, or Wain-man.													
1	♊	77	16	22	50	73	7	45	37	4	52	Sets not.	
2	♊	26	52	21	27	85	53	44	56	5	43	Sets not.	
The Serpent Bearer.													
3	♐	17	50	35	57	259	55	12	52	5	19	7	7
3	♐	20	45	28	1	261	49	4	46	5	27	6	20
3	♐	22	5	26	11	262	53	2	51	5	31	6	9

STARS NAMES.	Upon the Meridian at Midnight.	
The more Northern in the Left Hand	*May*	9
The more Southern following		10
In the Right Knee	*May*	25
In the Left Knee		15
That in the Right Shin		26
The foremoſt of the Four in the Right Foot.		26

Serpens Ophiuchi.

That in the Mouth	*April*	25
That in the Temples, or Side of the Head		28
In the ſtretching out of the Neck		25
The Second in the Neck, below the Head		24
In the Middle of the Knot in the Neck		27
The laſt ſave two of the Tail	*June*	6
The laſt ſave one		12
The laſt		23

Sagitta, ſive Telum.

There is no Eminent Star

Aquila, ſeu Vultur volans.

In the Neck	*July*	11
The Bright one in the Shoulder-blade		10
That in the Left Shoulder		8
The Tail of the *Vultur*	*June*	27
That which next precedes the Tail		25

Antinous.

In the Left Hand	*July*	13
In the Right Side		3
In the Knee		2
In the Right Arm		1
In the Breaſt		7
In the Right Foot	*June*	24

Delphinus.

The Bright one of the Tail	*July*	22
The more Southerly of the foremoſt of the *Rhomboid*		24
The more Northerly of the ſame Side		25
The more Southerly of the following Side		27
That in the Head.		28

Equuſeus, Equiſectio.

No Eminent Star

Pegaſus, Equus Alatus.

The Mouth of *Pegaſus*	*July*	27
The Bright one of the Neck	*Aug.*	24
The Right Knee	*Sept.*	4
The Firſt of the Wing *Marchab*	*Aug.*	31
The Drawing forth of the Leg *Scheat*	*Sept.*	8
The Uttermoſt of the Wing.		17

Magnitude	Longitude. Sin.	Deg.	Min.	Latitude. Deg.	Min.	Right Ascension. Deg.	Min.	Declination. Deg.	Min.		Right Ascension in Time. Ho.	Min.	Simidiurnal Ark. Ho.	M.
3	♏	27	44	17	19	239	19	2	49	S.	3	57	5	41
3	♏	28	57	16	30	240	17	3	51	S.	4	1	5	36
3	♐	13	24	7	18	252	55	15	14	S.	4	51	4	36
3	♐	4	39	11	30	244	48	9	49	S.	4	19	5	3
3	♐	15	23	2	12	254	23	20	31	S.	4	58	4	7
3	♐	15	1	2	16	254	0	20	25	S.	4	56	4	7

The Serpent of Ophiucus.

3	♏	15	24	39	6	234	19	20	58		3	37	7	55
3	♏	18	6	35	25	235	21	16	49		3	41	7	30
3	♏	15	21	34	27	232	49	16	32		3	31	7	28
3	♏	13	46	28	58	229	50	11	41		3	19	7	0
2	♏	17	30	25	35	232	4	7	30		3	28	6	32
3	♐	25	34	19	57	265	50	3	30	S.	5	43	5	27
3	♑	1	12	20	37	271	8	2	53	S.	6	4	5	41
3	♑	11	10	26	59	279	7	3	51		6	39	6	10

The Arrow, or Dart.

in this Conſtellation.

The Eagle, or Flying Vultur.

3	♑	27	53	26	49	294	48	5	41		7	39	6	24
2	♑	27	9	29	21	293	41	8	3		7	35	6	35
3	♑	26	26	31	18	292	43	9	54		7	31	6	48
3	♑	15	15	36	16	282	36	13	25		6	50	7	8
3	♑	13	44	37	40	281	12	14	41		6	45	7	16

Antinous.

3	♒	0	21	18	48	298	36	1	45	S	7	54	5	52
3	♑	21	17	20	14	289	56	1	47	S	7	19	5	52
3	♑	20	17	14	28	289	48	7	40	S	7	19	5	21
3	♑	19	1	24	56	287	12	2	33		7	9	6	8
3	♑	25	50	21	88	293	54	0	14		7	35	6	1
3	♑	12	46	17	41	282	12	5	17	S	6	49	5	26

The Dolphin.

3	♒	9	32	29	8	304	24	10	14		8	18	6	49
3	♒	11	56	31	57	305	40	13	30		8	23	7	8
3	♒	12	50	33	5	306	6	14	48		8	24	7	17
3	♒	14	36	32	0	307	54	14	11		8	31	7	13
3	♒	14	52	32	41	307	53	15	00		8	31	7	17

The Horſe-Colt, or Little Horſe.

in this Conſtellation.

Pegaſus. The Winged Horſe.

3	♒	27	22	22	7	322	3	8	24		9	28	6	31
3	♓	11	39	17	41	336	21	9	8		10	25	6	43
3	♓	21	10	35	7	336	54	28	31		10	27	8	53
2	♓	18	56	19	26	342	7	13	28		10	48	7	8
2	♓	24	49	31	7	341	59	26	18		10	48	8	35
2	♈	4	38	12	35	359	8	13	22		11	56	7	8

STARS NAMES.	Upon the Meridian at Midnight.
Andromeda.	
The Head	*Septem.* 22
The more Southern in the Girdle	*October* 9
The Bright one in the Southern Foot	22
The Clearer and Upper in the Left Shoulder	*Sept.* 30
Triangulus, Deltoton.	
No Eminent Star	
Coma Berenices.	
In the top of the Firſt and Northern Triangle	*June* 10

The 12 Zodiacal

ARIES.

The Bright one in the top of the Head	*October* 16
The Eaſtern in the Baſe of the Triangle	26

TAURUS.

In the Face, the Firſt of the *Suculæ*, in the Noſtrils.	*Novem.* 13
Between this and the Northern Eye.	14
In the Southern Eye. *ALDEBARON. Palilicium.*	17
In the Northern Eye.	16
In the extremity of the South Horn	*Decemb.* 1
In the extremity, common with the Right Foot of *Heniochus*	*Novem.* 30
The Middle and Bright one of the Seven Stars	7

GEMINI.

In the upper Head, *Caſtor. APOLLO.*	*Decemb.* 26
In the lower Head. *Pollux. HERCULES.*	29
In the Northern and upper Knee	16
In the Left Knee	21
That in the Belly of the Southern Twin	24
That in the Foot, called the Heel	11
The Bright one of the Foot.	15

CANCER.

In the Southern Claw	*Januar.* 18

LEO.

The more Southerly in the Head	*Jan.* 25
The three Northern in the Neck	*Feb.* 1
The Middle and Bright one in the Neck	2
The Southern one	1
The Lyons Heart. *Regulus, Baſiliſcus*	3
The Bright one that follows	14

Magnitude	Longitude. Sin.	Deg.	Min.	Latitude. Deg.	Min.	Right Ascension. Deg.	Min.	Declination. Deg.	Min.	Right Ascension in Time. Ho.	Min.	Semidiurnal Ark. Ho.	M.
						Andromeda.							
2	♈	9	47	25	42	357	54	27	18	11	51	8	23
2	♈	25	49	25	59	12	49	33	55	0	51	9	52
2	♉	9	39	27	46	25	57	40	44	1	43	Sets not.	
3	♈	17	19	24	20	5	33	29	5	0	22	8	58
						The Triangle.							
	in this Constellation.												
						Berenice's Hair.							
3	♌	29	17	28	25	182	39	30	6	0	10	9	8

Constellations.

						The Ram.							
3	♉	3	6	9	57	27	12	21	54	1	59	8	4
3	♉	13	40	10	24	34	44	25	53	2	19	8	31
						The Bull.							
3	♊	1	12	5	46 S	60	17	14	59	4	1	7	17
3	♊	2	16	4	2 S	61	1	16	44	4	4	7	29
1	♊	5	12	5	31 S	64	17	15	48	4	17	7	22
3	♊	3	53	2	36 S	62	23	18	26	4	10	7	39
3	♊	20	12	2	14 S	79	30	20	56	5	18	7	55
2	♊	17	59	3	0	76	23	28	21	5	3	8	51
3	♉	25	24	4	0	52	00	23	3	3	28	8	10
						The Twins.							
2	♋	15	41	10	2	108	24	32	33	7	14	9	34
2	♋	18	43	6	38	111	19	28	46	7	25	8	35
3	♋	5	22	2	11	95	57	25	35	6	23	8	29
3	♋	10	26	2	6 S	101	11	21	00	6	44	7	56
3	♋	13	56	0	13 S	105	7	22	34	7	0	8	8
3	♋	0	44	0	53 S	88	48	22	38	5	55	8	8
2	♋	4	31	6	48 S	94	1	16	38	6	19	7	28
						The Crab.							
3	♌	9	3	5	8 S	130	7	16	6	8	40	7	24
						The Lion.							
3	♌	16	5	9	40	141	44	25	16	9	27	8	26
3	♌	22	57	11	50	149	34	25	2	9	58	8	24
2	♌	24	59	8	47	150	26	21	29	10	2	7	59
3	♌	23	20	4	52	147	22	18	22	9	50	7	39
1	♌	25	17	0	26	147	43	13	33	9	51	7	7
2	♍	6	41	14	20	164	8	22	20	10	56	8	7

STARS NAMES.	Upon the Meridian at Midnight.	
The Foremoſt and Northern of the two in the Buttock	*Feb.*	17
In the Thigh		21
The Bright one in the End or Tip of the Tail.		25
VIRGO.		
In the Tip of the Southern and Left Wing.	*March*	2
In the Right Side, under the Girdle		6
The more Northerly, called *Vindemiatrix*		15
In the Left-Hand, called the *Virgins Spike*		30
Under the —— in the Right Buttock.		26
LIBRA.		
The Southern Balance	*April*	20
The Northern Balance		24
The Third from the ſame Balance to the Eaſt		30
Under the Northern Balance, in the Left Claw of the *Scorpion*		26
That which follows.		26
SCORPIO.		
The Uppermoſt in the Forehead	*May*	9
The Middle one in the Forehead		9
The Southern of the Three Bright ones in the Forehead		19
The Red ſhining one in the Middle, *Antares, Scorpio's Heart*		16
SAGITTARIUS.		
In the Point of the Arrow or Shaft	*June*	7
In the Haft or Handle in the Left Hand		7
CAPRICORNUS.		
The Northern of the Three in the former Horn	*July*	12
The Southern one		12
The foregoing of the Two Bright ones in the Tail	*July*	30
That which follows	*Aug.*	1
AQUARIUS.		
The Brighter in the Right Shoulder	*Aug.*	4
In the Left Shoulder	*July*	30
In the Elbow of the Right Arm	*Aug.*	15
The Southern one in the Right-Leg Calf, *Scheat*		17
The laſt in the Water-pouring, *Fomahant.*	*Aug.*	12
PISCES.		
The Bright one in the Faſtning of the *Purſuers.*	*Oct.*	7

The

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3	♍	8	50	9	41	164	15	17	14	10	1	7	33
3	♍	12	58	6	7	166	43	12	21	11	7	7	3
2	♍	17	3	12	18	173	4	16	25	11	32	7	27
						The Virgin.							
3	♎	22	32	0	43	273	26	3	38	6	14	6	12
3	♎	6	55	8	41	189	48	5	13	0	39	6	21
3	♎	5	23	16	15	191	30	12	45	0	46	7	5
1	♎	19	16	1	59 S	196	56	9	31 S	1	7	5	3
3	♎	16	22	8	10	198	13	1	5	1	13	6	2
						The Ballance.							
2	♏	10	31	0	26	218	13	14	37 S	2	33	4	41
2	♏	14	48	8	35	224	52	8	7 S	2	59	5	9
3	♏	20	33	4	28	229	19	13	38 S	3	17	4	43
3	♏	16	27	7	37 S	221	35	24	5 S	2	46	3	43
3	♏	16	17	1	48 S	223	15	18	29 S	2	53	4	17
						The Scorpion.							
2	♏	28	36	1	5	236	36	18	51 S	3	46	4	16
3	♏	27	59	1	54 S	235	15	21	38 S	3	41	4	2
3	♏	28	25	5	22 S	234	50	25	6 S	3	39	3	36
1	♐	5	13	4	27 S	242	23	25	37 S	4	9	3	31
						The Archer, or Bow-man.							
3	♐	26	30	6	54 S	265	58	30	22 S	5	43	2	49
3	♐	26	51	6	50 S	269	50	30	21 S	5	59	2	49
						Capricorn.							
3	♑	29	18	7	2	299	58	13	29 S	7	52	4	44
3	♑	29	31	4	41	300	40	15	44 S	8	3	4	34
3	♒	17	14	2	26 S	320	29	18	2 S	9	22	4	21
3	♒	19	00	2	29 S	322	15	17	32 S	9	29	4	24
						The Water-keeper.							
3	♒	28	49	10	42	337	16	1	52 S	9	49	5	46
3	♒	18	51	8	42	318	7	6	56 S	9	12	5	16
3	♓	2	10	8	17	332	30	2	59 S	10	10	5	41
3	♓	4	22	8	10 S	339	22	17	31 S	10	38	4	24
1	♒	29	11	21	00 S	339	46	31	17 S	10	39	2	40
						The Fishes.							
3	♈	24	47	9	4 S	26	16	1	11	1	45	6	3

The Southern

STARS NAMES.	Upon the Meridian at Midnight.
Cete.	
The Bright one of the *Whale's* Jaw	October 23
The Middle one in the Mouth	18
The foremoſt of the Three at the Cheek	15
The Northern one of the Belly	Septem. 30
The more Eaſterly of the two Bright ones in the Back	24
The more Weſterly of the ſame	20
The Northern one of the Tail	9
The Southern and Brighter of the Tail	11
Orion.	
The following, or Bright one in the Right Shoulder	Decemb. 5
The Left, or foremoſt	Novem. 28
The Firſt of the Belt	30
The Middle one	Decemb. 1
The Laſt	2
In the Hilt or Handle of the Sword	Novem. 27
The middle one of the Sword	29
The Southern one	30
The Bright one in the Left Foot. *Regel*	23
In the Right Knee	Decemb. 3
Eridanus.	
Above the Foot of *Orion*, in the River	Novem. 22
The Firſt of thoſe which touch the *Whale*	October 17
The Third that follows	22
Lepus.	
That in the Back or Middle of the Body	Novem. 28
In the Left Shoulder	27
The more Southerly of the two in the hinder Feet	Decemb. 2
The more Northerly of them	4
Canis Major.	
The Bright ſhining one in the Mouth. *Sirius*	Decemb. 20
That under the Left Ear	Januar. 24
That in the End or Top of the Fore-foot	12
That in the Belly	27
The foremoſt of the Right Foot	Decemb. 13
That in the Tail	January 5
Canis Minor. Procyon.	
In the Neck	Decemb. 28
In the Thigh. *Procyon*	January 1

Constellations.

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						The Whale.							
2	♈	9	47	12 37 S		41	21	2	48	2	45	6	9
3	♈	4	53	12 2 S		36	37	1	50	2	26	6	5
3	♈	3	2	14 32 S		35	44	1	7 S	2	23	5	49
3	♈	17	25	20 19 S		23	52	11	54 S	1	35	4	51
3	♈	11	42	15 46 S		16	58	9	52 S	1	7	5	3
3	♈	7	11	16 55 S		13	23	12	39 S	0	55	4	48
3	♓	26	23	10 1 S		0	43	10	37 S	0	3	4	58
2	♓	27	56	20 47 S		6	45	19	48 S	0	27	4	11
						Orion.							
2	♊	24	12	16 6 S		84	23	7	18	5	38	6	31
2	♊	16	23	16 53 S		76	54	6	1	5	7	6	24
2	♊	17	50	23 38 S		78	52	0	35 S	5	15	5	55
2	♊	18	54	24 33 S		79	55	1	26 S	5	19	5	49
2	♊	20	6	25 21 S		81	4	2	9 S	5	24	5	44
3	♊	15	37	25 36 S.		77	3	2	44 S.	5	8	5	42
3	♊	18	24	28 45 S.		79	48	5	39 S.	5	16	5	42
3	♊	18	27	29 17 S.		79	52	6	10 S.	5	19	5	20
1	♊	12	17	31 11 S.		74	44	8	37 S	4	59	5	6
3	♊	21	49	33 8 S.		83	3	9	49 S.	5	32	5	3
						The River.							
3	♊	10	40	27 54 S.		72	56	5	32 S.	4	51	5	23
3	♈	4	00	24 34 S.		40	8	11	1 S.	2	40	4	56
3	♈	9	16	25 59 S.		45	11	10	4 S.	3	0	5	2
						The Hare.							
3	♊	16	49	41 5 S.		79	35	18	03 S.	5	18	4	21
3	♊	15	6	43 57 S.		78	35	21	2 S.	5	14	4	4
3	♊	20	21	45 49 S.		82	32	22	33 S.	5	30	3	56
3	♊	22	36	44 18 S.		48	20	20	56 S.	3	13	4	5
						The Great Dog.							
1	♋	9	35	39 30 S.		97	42	16	14 S.	6	31	4	34
3	♋	15	6	38 2 S.		102	16	15	10 S.	6	49	4	40
2	♋	2	42	41 18 S.		92	8	17	49 S.	6	8	4	25
3	♋	18	55	48 30 S.		103	48	25	53 S.	6	55	3	29
3	♋	2	7	51 46 S.		91	29	28	16 S.	6	6	3	9
3	♋	25	11	51 24 S.		107	45	29	26 S.	7	11	2	59
						The Lesser Dog.							
2	♋	17	39	13 33 S.		107	22	8	54	7	9	6	42
2	♋	21	18	15 57 S.		110	34	6	03	7	22	6	25

STARS NAMES.	Upon the Meridian at Midnight.
Argo. Navis.	
That in the uppermoſt Part of the Caſtle	*Januar.* 16
The Uppermoſt of the	11
The former of	9
The following one	*Februar.* 13
That between the Sail and the Milke Way	*March* 5
Hydra.	
The Bright one of *Hydra*, or the Heart	*Januar.* 31
Under the Tail of the *Raven*	*April* 2
That before *Hydra*'s Head	*January* 9
Crater.	
	No Eminent Star
Corvus.	
The former of the two Upper ones in the Square	*March* 16
The following one hf the ſame Square	18
The following one of the Lower in the Square	22
Centaurus. Chiron.	
	No Eminent Star

Finis' Catalogus.

Magnitude	Longitude. Sin.	Deg.	Min.	Latitude. Deg.	Min.	Right Ascension. Deg.	Min.	Declination. Deg.	Min.		Right Ascension in Time. Ho.	Min.	Simidiurnal Ark. Ho.	Min
						The Ship, Argo.								
3	♌	6	53	43	18 S.	118	25	14	48 S.		7	54	4	42
3	♌	1	35	44	58 S.	113	57	12	22 S.		7	35	4	55
3	♋	29	0	47	28 S.	111	24	6	6 S.		7	26	5	30
3	♍	5	20	22	29 S.	148	55	11	22 S.		9	55	5	1
3	♍	24	44	30	30 S.	162	14	25	43 S.		10	49	3	30
						The Water Serpent.								
1	♌	22	45	22	24 S.	137	54	7	15 S.		9	11	5	22
3	♎	22	24	13	43 S.	195	15	21	25 S.		1	1	4	1
3	♋	29	44	10	19 S.	119	43	10	11 S.		7	59	6	53
						The Cup.								
	in this Constellation.													
						The Raven.								
3	♎	6	13	14	25 S.	179	49	15	39 S.		11	35	4	35
3	♎	8	55	12	7 S.	183	16	14	39 S.		0	13	4	39
3	♎	12	49	17	59 S.	184	20	21	33 S.		0	17	4	0
						The Centaure.								
	in this Constellation.													

The End of the Catalogue.

Example 1. *In* 17 *deg.* 15 *min of the* Equinoctial, *how many Hours and Minutes of Time.*

Seek 17 deg. in the top of the *Table*, (which you ſhall find in the Second *Line*, at the end whereof ſtands I. or one Hour.) Alſo find 15 min. in the Firſt Column of the lower part of the *Table*, and againſt 15 min. and under 17 deg. you ſhall find 9 min. which with the one Hour at the top of the *Table*, makes one Hour 9 Minutes. And ſo much in *Time* is contained in 17 deg. 15 min. of the *Equinoctial.*

Example 2. *In* 68 *deg.* 25 *min. of the* Equinoctial, *how many Hours and Minutes of Time?*

Look for 68 deg. in the Head of the *Table*, which you ſhall find in the Fifth *Line*, at the end whereof ſtandeth IV. or Four Hours: Likewiſe find 25 min. in the Firſt Column of the under part of the *Table*; and againſt 25, and under 68, you ſhall find 33 min. 40 ſeconds; which, with the Four Hours above, makes 4 Hours, 33 min. 40 ſeconds of *Time*; and ſo much is contained in 68 deg. 25 min. of the *Equinoctial.*

In like manner you may find, that

	D.	M.			H.	M.	S.	
In	27	0	of the *Equinoctial* is contained		1	48	0	of *Time.*
	84	10			3	12	40	
	72	25			4	49	40	

II. *To find how many Degrees and Minutes of the* Equinoctial, *are contained in any Number of Hours, Minutes, and Seconds of* Time.

Seek the Minutes, (or Minutes and Seconds) in ſome of the Columns in the lower part of the *Table*, and note what number of Minutes of the *Equinoctial* ſtands againſt the ſame in the Firſt Column. Alſo ſeek the Hours in the Head of the *Table*, and carry your Eye along that *Line*, till you come over the Column where you found the Minutes and Seconds; for thoſe are the Degrees; and the Number in the Firſt Column the Minutes.

Example 1. *In* 3 *Hours*, 48 *min.* 20 *Seconds of* Time, *how many Degrees and Minutes of the* Equinoctial?

Firſt ſeek 48 min. 20 Seconds among the Columns in the lower part of the *Table*, (which you will find in the 14th Column,) and over it (in the *Line* at the top of the *Table*, which hath three Hours at the end of it,) you ſhall find 57 deg. and againſt 48 min. 20 ſec. in the firſt Column you ſhall find 5 min. So that in 3 hour. 48 min. 20 ſec. of *Time*, are contained 48 deg. 5 min. of the *Equinoctial.*

In like manner,

	H.	M.	S.		D.	M.	
In	1	7	0	of *Time* are contained	16	45	of the *Equinoctial.*
	3	28	20		51	50	
	5	18	40		79	40	

And ſo of any other number of Hours, Minutes, *&c.*

Note here, That this *Table* extendeth but to the Degrees of one *Quadrant* of the *Equinoctial*; wherefore, if your number of Degrees of the *Equinoctial* to be reduced into *Time*, be above 90, add 6 Hours; if above 180, add 12 Hours; if above 270, add 18 Hours to what you find in the *Table.*

Some

Some Uſes of the foregoing Table *of* FIXED STARS.

THE Uſes of this *Table* are many, ſome of which follow.

USE I. *To find at what Hour any of the* Stars *in the* Table, *will be upon the South part of the* Meridian *any time of the Year.*

The RULE.

From the Right Aſcenſion *of the* Star *in Degrees and Minutes (as found in this* Table*) ſubtract the* Right Aſcenſion *of the* Sun, *in Degrees and Minutes, (as it is in the Kalendar for the Time propoſed,) the Degrees and Minutes remaining (being reduced into Time, either by the little* Table *at the end of the Catalogue of* Stars, *or by allowing* 15 *deg. for one Hour, and* 4 *deg. for one Minute of Time,) ſhall give you the Hour and Minute that that* Star *ſhall be upon the South part of the* Meridian *on the day propoſed.*

Note. If it ſo fall out (as often times it will) that the *Right Aſcenſion* of the Sun be greater than that of the Stars, (ſo that ſubtraction cannot be made,) you muſt add 180 deg. to the Star's *Right Aſcenſion*, and then the remaining Degrees (reduced into *Time* as before) will be the time that the Star will be upon the South part of the *Meridian* the next Morning.

This *Rule* ſhall be illuſtrated by *Examples.*

Example 1. *Let it be required to know at what Hour the Head of* Meduſa (*called* Algol) *will be upon the South part of the* Meridian *upon the firſt of* February.

The *Right Aſcenſion* of *Algol*, (or *Head of Meduſa*,) according to this *Table*, is 41 deg. 46 min. And the *Right Aſcenſion* of the Sun for the firſt of *February*, (by the *Table*) is 145 deg. 30 min. which being greater than 41 deg. 46 min. the Star's *Right Aſcenſion*, I do add (according to the Note, or Caution before given) 180 deg. to the Star's *Right Aſcenſion*, and the Summ will be 221 deg. 46 min. from which if you ſubtract 145 deg. 30 min. (the Sun's *Right Aſcenſion*,) the Remainder will be 76 deg. 16 min. which reduced into *Time* (as is ſhewed before) is 5 hour. and 5 min. and at ſuch time the next Morning, (*viz. February* 2.) will the Star *Algol* be upon the South part of the *Meridian.*

A *Synopſis* of this Firſt *Example.*

	D.	M.
The *Right Aſcenſion* of the *Head of Meduſa, Algol*,	41	46
Add 180 deg.	180	00
The Summ is	221	46
The *Right Aſcenſion* of the Sun *February* 1. ſubtract	145	30
The Remainder is	76	16

Which 76 deg. 16 min. converted into *Time*, is 5 hou. and 5 min. at which time in the Morning, (*February* 2.) the Star *Algol* will be upon the South part of the *Meridian*, becauſe the Sun's *Right Aſcenſion* was greater than that of the Star's.

I have been ſomewhat large in the Explanation of this *Rule*, and this firſt *Example*, becauſe I intend the more brevity in thoſe which follow: — And alſo, for that there is in it all the varieties that can poſſibly ariſe in any other *Example* of the like kind.

Example 2. *Let it be required to know at what time the great Star in the Skirt of* Bœotes *Garment, called* Arcturus, *doth come to the* Meridian *on the* 31*ſt of* May.

The *Right Aſcenſion* of *Arcturus*, by the *Table*, is 30 deg. 13 min. The *Right Aſcenſion* of the Sun, for the 31 of *May*, is 78 deg. 45 min. (which ſubtracted from the *Right Aſcenſion* of the Star 180 deg. being added, the Summ will be 210 deg. 13 min. from which ſubtract 78 deg. 45 min. the *Right Aſcenſion* of the Sun, and there reſts

131 deg. 48 min. which corrected into *Time* by the former *Table*, giveth 8 hours 46 min. So that *Arcturus* will be upon the *Meridian* at 46 min. after 8 the next Morning (*June* 1.)

See the following *Synopsis*.

	D.	M.
The *Right Aſcenſion* of *Arcturus*	30	13
180 deg. add	180	
The Summ	210	13
The Sun's *Right Aſcenſion*, *May* 31 ſubtract	78	45
Arcturus on the *Meridian*	131	28

That is in *Time* 8 hou. 46 min. the next Morning, *June* 1.

Example 3. *Let it be required to find at what Hour the* Great Dog, *or* Syrius, *will be upon the* Meridian *on the* 18*th of* October.

I ſhall uſe no more words to Explain this Example, but the following *Synopſis*.

	D.	M.
The *Right Aſcenſion* of *Syrius*, the *Great Dog*	97	42
The Sun's *Right Aſcenſion*, *October* 18. ſubtract	32	57
The *Great Dog* South	64	45

Which 64 deg. 45 min. being converted into *Time*, gives 4 hou. 19 min. for the time of the *Great Dog's* coming to the South the 18th of *October* in the Evening; becauſe the Sun's *Right Aſcenſion* was leſs than the Star's; ſo that 19 min. after 4 in the Afternoon the *Great Dog* on the 18th of *October* will be upon the *Meridian*.

The Three foregoing *Examples* have been all wrought in Degrees and Minutes of *Right Aſcenſion*; and ſo at the concluſion did require a Reduction of Degrees and Minutes of *Right Aſcenſion* into Degrees and Minutes of *Time*: Wherefore, I will work the ſame Three *Examples* again in Hours and Minutes of *Right Aſcenſion* in *Time*, as they were before in Degrees and Minutes of *Right Aſcenſion*.

Example 1. *At what time will* Algol, *or* the Head of Meduſa, *be upon the* Meridian *on the Firſt of* February?

The *Right Aſcenſion* of *Meduſa's Head*, in the *Table* of Stars, is 2 hou. 47 min. And the *Right Aſcenſion* of the Sun on the Firſt of *February*, in *Time*, is 9 hou. 42 min. which ſhould be ſubtracted from 2 hou. 47 min. the *Right Aſcenſion* of *Algol*; but being it is greater, add 12 Hours to the *Right Aſcenſion* of *Algol*, and then from the Summ ſubtract the Sun's *Right Aſcenſion*, the Remainder will be 5 hou. 5 min. for *Time* of *Algol's* coming to the *Meridian* the next Morning, *February* 2. becauſe the Sun's *Right Aſcenſion* was greater than the Star's *Right Aſcenſion*.

	H.	M.
The *Right Aſcenſion* of *Algol* in *Time*	2	47
12 Hours added makes	12	00
The Summ	14	47
The Sun's *Right Aſcenſion*, *February* 1. ſubtract	9	42
The time of *Algol's* coming to the *Meridian*	5	05

Example 2. *At what Hour upon the* 31*ſt of* May, *will* Arcturus *be upon the* Meridian?

	H.	M.
The *Right Aſcenſion* of *Arcturus*	2	1
12 Hours add	12	0
Summ	14	1
Sun's *Right Aſcenſion*, *May* 31 ſubtract	5	15
Time of the Star's being upon the *Meridian* in the Morning, *June* 1.	8	46

Example

Example 3. *At what Hour will the* Great Dog, Syrius, *be upon the* Meridian *on the 18th of* October?

	H.	M.
The *Right Aſcenſion* of *Syrius* in *Time*	6	31
Sun's *Right Aſcenſion*, *October* 18	2	12
The *Great Dog*, *Syrius*, South at	4	19

In the Evening, becauſe the Sun's *Right Aſcenſion* was leſs than the Stars.

This way of finding the time of a Stars coming to the *Meridian* is ſomewhat eaſier than the former, becauſe the Reduction of Degrees and Minutes into Hours and Minutes is avoided —— There is yet a more expeditious way by Addition, and not Subtraction, and that is this.

The RULE.

To the Complement of the Sun's Right Aſcenſion (*found in the Sun's Kalendar*) *add the Stars* Right Aſcenſion (*found in the* Table,) *the Summ of them is the Hour and Minute of the Star's coming to the* Meridian.

Note, If the Summ of the Addition do exceed 12 Hours, take 12 therefrom, and the Remainder will be the time that the Star will be upon the *Meridian* in the Evening.— But if the Summ be leſs than 12 Hours, it will give the time of the Star's coming to the *Meridian* the next Morning.

I will make this *Rule*, and the *Note* upon it, plain, by the Three foregoing *Examples*, wrought by this way of Addition only.

Example 1. *At what time*, February 1. *will* Algol *be upon the* Meridian?

The *Synopſis*.

	H.	M.
The Complement of the Sun's *Right Aſcenſion*, *Feb.* 1.	2	18
The *Right Aſcenſion* of *Meduſa*'s Head, *Algol*	2	47
Algol comes to the *Meridian* at	5	05

The next Morning (*Feb.* 2.) becauſe the Summ is leſs than 12 Hours.

Example 2. *At what time upon the* 31*ſt of* May, *will* Arcturus *be upon the* Meridian?

	H.	M.
Complement of the Sun's *Right Aſcenſion*, *May* 31.	6	45
The *Right Aſcenſion* of *Arcturus*	2	01
Arcturus comes to the *Meridian* at	8	46

The next Morning, (*June* 1.) becauſe the Sun is leſs than 12 Hours.

Example 3. *At what time upon the* 18*th of* October, *will* Syrius, the Great Dog, *be upon the* Meridian?

	H.	M.
The Complement of the Sun's *Right Aſcenſion*, *Oct.* 18.	9	48
The *Right Aſcenſion* of the *Great Dog*, *Syrius*	6	31
Syrius comes to the *Meridian* at	16	19

From which take 12 Hours, and there remains 4 hou. and 19 min. at which time in the Evening, *Oct.* 18. (becauſe the Summ exceeded 12 Hours) *Syrius* will be upon the *Meridian*.

Thus have you Three ways to find the *Time* that any Star in the *Table* will be upon the South part of the *Meridian*, and you ſee (by obſerving the *General Rules*) that they all agree, ſo that every Man may uſe that way which he beſt liketh: and ſo this ſhall ſuffice for the firſt Uſe of this *Table*.

USE II. *To find any Day of the Year, at what Hour any Star in the Table doth riſe and ſet.*

For this purpoſe in the Table of Stars you have a Column whoſe Title is *Semidiurnal Ark*, by help whereof (having the Time of the Star's coming to the *Meridian*) you may find the Time of the Star's Riſing and Setting, by the Directions following.

The RULE.

From the Time that any Star comes to the Meridian, *ſubtract the* Semidiurnal Ark *of that Star, and the Remainder ſhall be the Time of the Star's Riſing.* Alſo, *If to the Time of any Star's coming to the* Meridian, *you add the* Semidiurnal Ark *of that Star, the Sum ſhall be the Hour and Minute of that Star's ſetting. But (if Subtraction cannot be made) add Twelve Hours where Occaſion ſerves.*

Example 1. *At what Hour, upon the 2d. of* February, *will the Star* Algol *riſe and ſet?*

By the foregoing Examples you found, that *Algol* came to the *Meridian* on the 2d. of *February*, at 5 Hours and 5 min. in the Morning; which known, look for the Star *Algol* (the *Head of Meduſa*) in the Table of Stars, and againſt it (under the Title of *Semidiurnal Ark*) you ſhall find theſe Words, [*Sets not*]; which ſhews, that *Algol* (in this Latitude of *London*, 51 deg. 30 min. for which the Tables in this Book are calculated) never ſets, nor never riſes, but is always above the *Horizon*: So that this Example is impertinent in this Latitude, though this Star riſes and ſets in other Latitudes. And to be aſcertained of what Stars do riſe and ſet in any Latitude, obſerve this

GENERAL RULE.

If the Declination of any Star (found in the Table) having North Declination in North Latitude, do exceed the Complement of the Latitude of the Place you would know the Time of his or Setting Riſing in, you may conclude, that that Star never riſes nor ſets in that Latitude.

As in this Example, the Declination of *Algol* is 39 deg. 39 min. North; which being greater than 38 deg. 30 min. the Complement of the Latitude of *London*, you may conclude, that it never riſes nor ſets in that Latitude.

And, *If the Declination of the Star be equal to the Complement of the Latitude*, (as the Bright Star in the *Harp*, called ſometimes *Lucida Lyra*, whoſe Declination is 38 deg. 30 min.) *that Star doth only touch the North part of the* Meridian, *upon the very Verge of the* Horizon, *but never riſes nor ſets.*

Alſo, *Thoſe Stars whoſe Declinations are South, and greater than the Complement of the North Latitude you are in, do never to us riſe or ſet, but are always hid from our ſight, under the* Horizon.

Example 2. *At what Hour upon the 18th. of* October *doth the great Dog* Sirius *riſe and ſet?*

By the foregoing Examples you found, that the *Great Dog* was upon the *Meridian* on the 18th. of *October*, at 19 min paſt Four in the Evening; and (by the Table of Stars) you find that his *Semidiurnal Ark* is 4 ho. 34 min. which ſubtracted from 4 ho. 19 min. the Time of the Star's Southing, the Remainder is 11 ho. 45 min. for the Time of the Star's Riſing; and being added thereto, the Summ is 8 ho. 53 min. the Time of the Star's Setting.

The

	H.	M.	
The *Great Dog* South, *October* 18	4	19	Evening.
His *Semidiurnal Arch*, subtract	4	34	
Leaves the Time of the Star's Rising	11	45	Before Noon.
And the *Semidiurnal Ark* added, gives the Time of the Star's Setting——	8	53	Evening.

Example 3. *At what time upon the First of* June *will* Arcturus *rise and set?*

	H.	M.	
Arcturus South upon the First of *June* at	8	46	Morning.
His *Semidiurnal Ark* subtracted	7	55	
Leaves the Time of the Star's Rising	0	51	Morning.
And the *Semidiurnal Ark* added makes	16	41	
From which subtract	12	00	
Leaves the Time of the Star's Setting	4	41	After Noon.

USE III. *To find what* Latitude *you are in, at any time when the Stars appear, by taking of the Star's* Meridian Altitude, *and finding of his* Declination *in this Table.*

In finding of the *Latitude* by the *Meridian Altitude* of a Star, and his *Declination*, there are several various Workings to be observed.

(1.) According to the Situation of the Star, he having North or South *Declination.*

(2.) In respect of your observing his *Altitude*, whether he be upon the North or South part of the *Meridian.*

(3.) And in respect of the Part of the World in which you are.

Examples of all which Varieties I shall give in these few brief Rules.

1. If the Star have no *Declination*, and you observe his *Meridian Altitude* on the
- *NORTH* Side of the *Meridian*, Subtract the *Meridian Altitude* from 90 deg. the Remainder is the *Latitude South.*
- *SOUTH* Side of the *Meridian*, Subtract the *Meridian Altitude* from 90 deg. the Remainder is the *Latitude North.*

2. If the Star's *Declination* be
- *SOUTH*, And the *Meridian Altitude* less than 90 deg. and the Star upon the South-side of the *Meridian*, add the *Meridian Altitude* and *Declination* together, which taken from 90 deg. the Remainder is the *Latitude North.* But if the Summ exceed 90 deg. take 90 from it, the Remainder is the *Latitude South.*
- *NORTH*, And the *Meridian Altitude* less than 90 deg. and the Star upon the South-side of the *Meridian*, subtract the Star's *Declination* from his *Meridian Altitude*, and the Remainder taken from 90 deg. gives the *Latitude North.*

3. If the Star's *Declination* be
- *SOUTH*, { And the *Meridian Altitude* be leſs than 90 deg. and the Star upon the North-ſide of the *Meridian*, ſubtract the *Declination* from the *Meridian Altitude*, and the Remainder taken from 90 deg. leaves the *Latitude South*.
- *NORTH*, { And the *Meridian Altitude* leſs than 90 deg. and the Star upon the North ſide of the *Meridian*; the Summ of the *Altitude* and *Declination* taken from 90 deg. leaves the *Latitude South*. But if the Sum be more than 90 deg. the Remainder above 90 is the *Latitude North*.

If the Star's *Declination* be {*South*, / *North*,} and the *Meridian Altitude* be juſt 90 deg. the Star's *Declination* is the *Latitude* {*South*. / *North*.}

If the *Meridian Altitude* be obſerved within the Bounds of the *Polar Circles*, in this Caſe the Star's *Declination* muſt be taken from 90 deg. and the Remainder is the Star's *Diſtance* from the *Pole*; which added to the *Meridian Altitude*, gives the *Latitude*.

¶ Note, *That theſe are all the Varieties that can poſſibly happen at any time, in any Part of the World: And what is here ſaid of the* Stars, *the ſame is to be underſtood of the* Sun *alſo. And therefore theſe* Rules *will ſerve for the* Sun's Meridian Altitude *alſo, by changing the Word* Star's *for* Sun's, *&c. And I place thoſe* Rules *here the rather, for that they will be more ſerviceable to Seamen, than to obſerve by the* Sun, *for theſe Reaſons.*

1. Becauſe you can have the *Sun* upon your *Meridian* but once in 24 Hours, there being but one *Sun* in the Firmament. And,
2. Becauſe there are infinite of *Stars*, and at any time that the *Stars* may be ſeen, ſome or other of them will be always upon the *Meridian*; ſo that you may make ſeveral Obſervations of them in a ſhort time.

Rules to find the Latitude *by the* Sun's Meridian Altitude *and* Declination.

CASE I. If the *Sun* be on the *Meridian* to the *Southwards*, and have *South Declination*;

Add the *Sun's Meridian Altitude* and *Declination* together, the total taken from 90 deg. leaves the *Latitude North*.

	D.	M.
The *Sun' Declination* Southerly	20	00
The *Sun's Meridian Altitude* obſerved	17	55
Their Summ	37	55
Which ſubtracted from 90 deg. or	89	60
There remains the *Latitude North*	52	05

But if the Summ of the *Meridian Altitude* and *Declination* do exceed 90 deg. ſubtract 90 deg. from it, and the Remainder is the *Latitude South*.

	D.	M.
The *Sun's Declination South*	20	11
The *Meridian Altitude* by Obſervation	70	35
Their Summ	90	46
From which ſubtract 90	90	00
There remains the *Latitude South*	00	46

CASE

CASE II. If the Sun be on the *Meridian* to the Southwards, and have North Declination.

Subtract the Sun's Declination from the *Meridian Altitude*, and that which Remains subtract from 90, the last Remainder is your *Latitude* North.

	D.	M.
The *Sun's Declination* Northerly	15	10
The *Meridian Altitude* observed	64	22
The *Declination* subtracted, the Remainder is	49	21
Which subtracted from 90	89	60
There Remains the *Latitude* North	40	39

CASE III. If the Sun be on the *Meridian* to the Northwards, and have North *Declination.*

Add the *Sun's Declination* and *Meridian Altitude* together, the Total taken from 90, the Remainder is the *Latitude* South.

But if the Total exceed 90 deg. subtract 90 deg. from it, and the Remainder is the *Latitude* North.

CASE IV. If the *Sun* be Northward at Noon, and have South *Declination.*

Subtract the *Declination* from the *Meridian Altitude*, and that which Remains subtract from 90 deg. the last remainder is the *Latitude* South.

CASE V. If you observe when the *Sun* hath no *Declination*;

The *Meridian Altitude* observed when the *Sun* hath no *Declination*, the *Meridian Altitude* taken from 90 deg. is the *Latitude.*

CASE VI. If you observe when the *Sun* is in the *Zenith.*

Look in the *Table* for the *Sun's Declination* that day, for that is your *Latitude.*

CASE VII. If the *Sun* come to the *Meridian* beneath the *Pole.*

If you be within the *Arctick* or *Antarctick* Circles, and observe the *Sun* upon the *Meridian* under the *Pole*, subtract the *Sun's Declination* from 90, the Remainder is the *Sun's* Distance from the *Pole*; which added to his *Meridian Altitude*, the *Summ* is the *Latitude.*

What is said here concerning the *Sun*, the like is to be understood of the *Stars* also.

Of Magnetical Inclination, *and how to know the* Latitude, *by the* Dipping Needle, *without the help of* Sun *or* Stars.

THE *Inclinatory Needle*, was the Useful invention of Dr. *Butler*, and hath been since approved of by divers learned Men of this, as well as of former Ages; It is now contrived so as to hang in a broad Brass Ring, within two Glasses to keep the Needle from the Air; in the Brass Ring are two *Quadrants* divided into 90 Degrees, and the Needle hangs upon an *Axis*, playing upon two Pevets: This Instrument is placed in a Box, and so ordered to hang therein, that the Ship moving up and down, the Instrument hangs Perpendicular and Horizontal also, as the common Compass doth. The Needle in this Brass Frame between the Glasses (before it be touched with the Load Stone) is made to hang upon its *Axis* Horizontal, but when it is touched it hath an inclination, respecting a certain Point of Elevation or Depression, according to the *Latitude* wherein it is used: And the Depressions are such, in every Degree of *Latitude*, as in this *Table* is expressed.

A TA-

A TABLE of Magnetical Inclination.

Latitude.	*Magnetical Inclination.*	*Latitude.*	*Magnetical Inclination.*	*Latitude.*	*Magnetical Inclination.*	*Latitude.*	*Magnetical Inclination.*
D.	D. M.	D.	D. M.	D.	D. M.	D.	D. M.
0	00 00	23	41 39	46	68 24	69	83 40
1	02 11	24	43 6	47	69 17	70	84 7
2	04 20	25	44 30	48	70 9	71	84 32
3	06 27	26	45 54	49	70 59	72	84 57
4	08 31	27	47 15	50	71 48	73	85 21
5	10 34	28	48 36	51	72 36	74	85 44
6	12 34	29	49 54	52	73 23	75	86 7
7	14 32	30	51 11	53	74 8	76	86 28
8	16 28	31	52 27	54	74 52	77	86 48
9	18 22	32	53 41	55	75 35	78	87 8
10	20 14	33	54 53	56	76 17	79	87 26
11	22 4	34	56 4	57	76 57	80	87 44
12	24 52	35	57 13	58	77 37	81	88 1
13	25 38	36	58 21	59	78 15	82	88 17
14	27 21	37	59 28	60	78 52	83	88 33
15	29 4	38	60 33	61	79 29	84	88 47
16	30 45	39	61 37	62	80 4	85	89 1
17	32 24	40	62 39	63	80 38	86	89 14
18	34 0	41	63 40	64	81 11	87	89 27
19	35 36	42	64 39	65	81 43	88	89 39
20	37 9	43	65 38	66	82 13	89	89 50
21	38 41	44	66 15	67	82 43	90	90 0
22	40 11	45	67 30	68	83 12	—	—

Of

Of the North-Star, *and how to find the* Latitude *thereby.*

	The True Points of the Compass.	*A Table of the* North-Star *in these several* Latitudes.												Of Declinat.	The old way of Reckoning the Points of the Compass.
		0		20		30		40		50		60		70			
		D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.	D.	M.		
If the former of the *Guards* be ascending from North, or lower part of the *Meridian.*	North	2	10	2	10	2	10	2	9	2	9	2	8	2	7	Above the *Pole.*	South
	N by E	1	53	1	53	1	53	1	52	1	52	1	51	1	49		S by E
	NNE	1	31	1	31	1	30	1	30	1	29	1	28	1	25		SSE
	NE by N	1	6	1	5	1	4	1	3	1	2	1	1	0	58		SE by S
	NE	0	39	0	38	0	37	0	36	0	35	0	33	0	30		SE
	NE by E	0	10	0	9	0	0	0	7	0	6	0	4	0	1		SE by E
	ENE	0	18	0	19	0	20	0	22	0	22	0	23	0	26	Under the *Pole.*	ESE
	E by N	0	49	0	50	0	50	0	51	0	52	0	53	0	56		E by S
	East	1	15	1	15	1	16	1	17	1	18	1	19	1	21		East
	E by S	1	38	1	39	1	39	1	40	1	41	1	42	1	44		E by N
	ESE	2	0	2	0	2	0	2	1	2	1	2	2	2	2		ENE
	SE by E	2	15	2	15	2	15	2	16	2	16	2	16	2	16		NE by E
	SE	2	25	2	25	2	25	2	25	2	25	2	25	2	25		NE
	SE by S	2	30	2	30	2	30	2	30	2	30	2	30	2	30		NE by N
	SSE	2	29	2	29	2	29	2	29	2	29	2	29	2	29		NNE
	S by E	2	22	2	21	2	22	2	22	2	22	2	22	2	22		N by E
If the former of the *Guards* be ascending from South, or upper part of the *Meridian.*	South	2	10	2	17	2	10	2	11	2	11	2	11	2	12	Under the *Pole.*	North
	S by W	1	58	1	53	1	54	1	53	1	55	1	55	1	57		N by W
	SSW	1	31	1	32	1	32	1	33	1	34	1	35	1	38		NNW
	SW by S	1	7	1	7	1	8	1	10	1	10	1	11	1	13		NW by N
	SW	0	39	0	40	0	41	0	40	0	43	0	44	0	47		NW
	SW by W	0	10	0	11	0	12	0	13	0	14	0	16	0	19		NW by W
	WSW	0	19	0	18	0	17	0	16	0	15	0	13	0	10	Above the *Pole.*	WNW
	W by S	0	48	0	47	0	46	0	45	0	44	0	43	0	42		W by N
	West	1	14	1	14	1	13	1	11	1	11	1	10	1	8		West
	W by N	1	39	1	39	1	38	1	37	1	36	1	35	1	33		W by S
	WNW	1	59	1	59	1	59	1	58	1	58	1	57	1	56		WSW
	NW by W	2	15	2	15	2	14	2	14	2	14	2	13	2	12		SW by W
	NW	2	25	2	25	2	25	2	25	2	25	2	24	2	24		SW
	NW by N	2	30	2	30	2	30	2	30	2	30	2	30	2	30		SW by S
	NNW	2	29	2	29	2	29	2	29	2	29	2	29	2	29		SSW
	N by W	2	22	2	22	2	22	2	22	2	22	2	21	2	21		S by W

The Use of this Table.

WHen you would observe the *Altitude* of the *North-Star*, mark as near as you may upon what Point of the Compass the former *Guard* of the *Little Bear* is: But if the Star be not upon a just full Point of the Compass, stay a while, till it come to be upon one; and then take the Height of the *North-Star* as exactly as you can. Then knowing within a Degree or two what *Latitude* you are in, see which *Latitude* in this Table is neareſt thereunto, and use that Column. Then find the Point of the Compass that the *Guard-Star* is upon in the first or last Column of the Table, and in the Column under your neareſt *Latitude* you shall find how many Degrees and Minutes the *Pole-Star* is either above or under the *Pole.* If the *Star* be above the *Pole*, subtract the Number in the Table from the *Altitude* Observed, and the Remainder is the *Latitude*: But if the *Star* be under the *Pole*, add the Number in the Table to the Height Observed, and the Summ thereof is the *Latitude.*

A Table

A TABLE of the Procession, *or* Retrogression *of the* Fixed Stars, *whereby their true Places in* Longitude *may be known for any time past or to come.*

Years.	Progression of the Fixed Stars in Longitude. D. M. S.	Years.	Progression of the Fixed Stars in Longitude. D. M. S.	Years.	Progression of the Fixed Stars in Longitude. D. M. S.	Years.	Progression of the Fixed Stars in Longitude. D. M. S.
0	0 0 0	30	0 25 0	60	0 50 0	90	1 15 0
1	0 0 50	31	0 25 50	61	0 50 50	91	1 15 50
2	0 1 40	32	0 26 40	62	0 51 40	92	1 16 40
3	0 2 30	33	0 27 30	63	0 52 30	93	1 17 30
4	0 3 20	34	0 28 20	64	0 53 20	94	1 18 20
5	0 4 10	35	0 29 10	65	0 54 10	95	1 19 10
6	0 5 0	36	0 30 0	66	0 55 0	96	1 20 0
7	0 5 50	37	0 30 50	67	0 55 50	97	1 20 50
8	0 6 40	38	0 31 40	68	0 56 40	98	1 21 40
9	3 7 30	39	0 32 30	69	0 57 30	99	1 22 30
10	0 8 20	40	0 33 20	70	0 58 20	100	1 23 20
11	0 9 10	41	0 34 10	71	0 59 10	200	2 46 41
12	0 10 0	42	0 35 0	72	1 0 0	300	4 10 1
13	0 10 50	43	0 35 50	73	01 0 50	400	5 33 22
14	0 11 40	44	0 36 40	74	01 1 40	500	6 56 42
15	0 12 30	45	0 37 33	75	1 2 30	600	8 20 3
16	0 13 20	46	0 38 20	76	1 3 20	700	9 43 23
17	0 14 10	47	0 39 10	77	01 4 10	800	11 6 44
18	0 15 0	48	0 40 0	78	01 5 0	900	12 30 4
19	0 15 50	49	0 40 50	79	01 5 50	1000	13 53 25
20	0 16 40	50	0 41 40	80	01 6 40	2000	27 46 50
21	0 17 30	51	0 42 30	81	01 7 30	3000	41 40 15
22	0 18 20	52	0 43 20	82	01 8 20	4000	55 33 40
23	0 19 10	53	0 44 10	83	01 9 10	5000	69 27 5
24	0 20 0	54	0 45 0	84	01 10 0	6000	83 20 30
25	0 20 50	55	0 45 50	85	01 10 50	7000	97 13 55
26	0 21 40	56	0 46 40	86	1 11 40	8000	111 7 20
27	0 22 30	57	0 47 30	87	1 12 30	9000	125 00 45
28	0 23 20	58	0 48 20	88	1 13 20	10000	138 54 10
29	0 24 10	59	0 49 10	89	1 14 10		

The Use of the foregoing Table *of the* Retrogression *and* Progression *of the* Fixed Stars *in* Longitude.

THE *Catalogue* of *Fixed Stars*, before given, has the Places in *Longitude* exactly calculated for the Year of Christ 1671: And although the *Table* will serve (without any sensible difference) for many Years before and after 1671, because their *Progression* in *Longitude* is but small, namely but one Degree in 72 Years (as this *Table* apparently shews,) yet, being I have framed *Tables* for the finding of the true

true Places of the *Sun* and *Moon* in *Longitude*, for any time past or to come, I held it convenient to do the like for the *Fixed Stars*: And how to effect it by this *Table*, this is the

RULE.

The Radix *of the* Table *being for the Year* 1671. *If you would find the true Place of any* Fixed Star *in* Longitude *for any Years past; subtract the Years from the* Radix 1671, *and you shall find the Difference.— But if the* Star's *Place be required for any Years to come, subtract the* Radix 1671, *from the Year proposed, and the Difference shall give you the Interval or Space between the time proposed and the* Radix.— *With which Interval (or Difference) enter this* Table, *and find the* Retrocession *or* Progression *of the* Star *in that Interval; which being added to, or subtracted from, the* Longitude *found in the* Catalogue, *shall give you the true Place of the* Star *in* Longitude *for the Year proposed.*

Example 1. *Let it be required to find what was the true Place of* Aldebaron *(or the* Bull's Eye*) in* Longitude *in the Year of our Lord* 515.

From the *Radix* 1671 subtract 515, the residue 1156 is the Interval of Years between the time proposed and the *Radix*; the *Retrocession* whereof I collect from this *Table* in this manner,

Year.	D.	M.	S.		D.	M.	S.
1000	13	53	25	Place of *Aldebaron* 1671.	65	12	30
100	0	23	20	*Retrocession* in 1156 Years, subtract	16	03	25
56	1	46	40				
1156	16	03	25	Place of *Aldebaron, Anno Christi* 515.	49	09	05

That is in 19 deg. 9 min. 5 seconds of *Taurus*.

Example 2. *Let the Place of* Aldebaron *be required for the Year* 1910.

From the Year given 1910, subtract the *Radix* 1671, the Difference 239 is the Interval; which collect from this *Table*, and add it to the *Radix* in this manner,

Year.	D.	M.	S.		D.	M.	S.
200	2	46	41	Place of *Aldebaron* 1671	65	12	30
39	0	32	30	*Progression* in 239 Years, add	3	19	11
239	3	19	11	True Place of *Aldebaron, Anno Christi* 1910.	68	31	41

That is, in 8 deg. 31 min. 41 sec. of *Gemini*.

CURSUS

CURSUS MATHEMATICUS.

BOOK VIII.

OF

DIALLING:

ARITHMETICAL,
GEOMETRICAL,
INSTRUMENTAL,

WHEREIN

The Doctrine of *PLAIN* and *SPHERICAL TRIANGLES* is applied to Practice in that *ART*.

ALSO OF

PROJECTIVE and REFLECTIVE

DIALS:

And Variety of other

SHADOWS.

By *WILLIAM LEYBOURN*, Philomathemat.

LONDON,

Printed *Anno Domini* MDCXC.

OF DIALING.

Præem.

CConcerning *this* Sciographical *Art, who was the first Inventor thereof is hard to determine: But the first* Gnomonical Organ *which stands upon Record* 2 Reg. 20. 11. *is the Dial of King* Ahaz; *whereby the Almighty was pleas'd to express a Miracle for the Recovery of King* Hezekiah, Esay. 38. 8. *and moved the* Babylonish Ambassadors, 2 Chron. 32. 31. *to enquire of the Wonder which was done in the Land. What kind of* Dial *this was, is also uncertain, but Authors generally suppose it to be either a* Convex *or (rather)* Concave Hemisphere, *those being afterwards* Dials *familiar among the* Chaldeans. *Some there are who crown* Anaximenes, *the* Lacedemonian, *with giving the first breath to this Science. It crept into* Rome *later, (but after a rude manner;) for* Accensus *the* Consul *determined only the* Noon-tide, *and that only in serene Days: And no farther Light had they, till the* Consul M. Val. Messala *beautified a* Column *with a* Dial *near the* Rostra, *as* Varro *relates. The Invention of the* excavated Hemisphere *is attributed to* Berosus, *the* Chaldean, *after the taking of* Catana *in* Sicily: *But others attribute it to the witty* Samian Aristarchus, *as first shadowing out the* Hour-Lines *on a* Plain. *Yet sure, the* Scaphe *or* Concave Hemisphere *was in use before his time, for* Eratosthenes, *before him, determined by* Shadows *some* Celestial *and* Terrestrial Distances; *but upon what kind of* Superficies *is not said. And now, since both the first Invention and the Inventor of this most* Celestial Science *are buried in oblivion, I shall in this following* Tractate *expose to publick view, in what lustre it is in this our Time arrived unto.*

Arithmetical DIALLING.

PART I.

Shewing how to describe the Hour-Lines, Stile, Substile, Meridian, Horizon, *&c. upon all sorts of* Dial Plains, *by* Trigonometrical Calculation; *deduced from the* Sphere *it self, it being* Projected in Plano, *suitable to any* Horizon; *In which* Projection, *all such* Spherical Triangles *as appertain to the finding of the above mentioned* Requisites, *are discovered and resolved.*

IN the *Third Book* of *Practical Astronomy*, in the first Part thereof, I have largely treated of the *Doctrine of the Sphere*, and of the *Circles* thereof, and to what uses each of them is appropriate: And in the *Second Part* of the said *Third Book*, I have treated of the *Doctrine* of *Spherical Projection*, shewing how to *Project* the *Sphere in Plano* upon any of the *Great Circles* thereof. Also in the *Second Part* of the *Fourth Book* of *Trigonometry* I have shewed how the Doctrine of *Spherical Triangles* is to be performed by *Spherical Projection*, in both which the Reader ought (at least in some measure) to be acquainted, before he enter upon the Practice of this *Treatise*; for so shall he find what is herein delivered very familiar and easie: And therefore I shall say nothing of either of them in this Place, but proceed to the matter in hand, *viz. Dialling*; and that by *Trigonometrical Calculation.*

CHAP. I.

Of the several sorts of Dial Plains.

A *Sun Dial* may be drawn upon any *Plain Superficies* in any Latitude, and the *Hour Lines* so drawn are *Great Circles* of the *Sphere*, but being projected upon a *Plain Superficies* become streight Lines.

Now all *Plains* are situate in one or other of these Three Positions, in respect of the *Horizon* of the *Place* wherein the *Dial* is made; for all *Plains* are either

Parallel
Perpendicular } To the *Horizon* of the Place.
Oblique

1. *Parallel* to the *Horizon*: (or Level thereunto) and such are called *Horizontal Plains* or *Dials*.

2. *Perpendicular* to the *Horizon* (or *erect* thereunto) and such are all *Plains* (or *Dials*) standing against any upright *Wall* or *Building*; and these are of two sorts, either *Direct* or *Declining*. For, (1.) If the *Wall* or *Plain* doth directly behold the true *East*, *West*, *North*, or *South Points* of the *Horizon*, the *Dial* made upon such a *Plain* is called an *Erect Direct*, *East*, *West*, *North* or *South Plain*. But (2.) If the *Wall* or *Plain* do not directly respect one of these four *Cardinal Points* of the *Horizon*, then it lieth open to Two of them, for it must necessarily lie open to the *South* and the *East*, or the *South* and the *West*; and accordingly it is denominated, and called an *Erect South Plain*, declining *East* or *West*; but if it lie open to the *North*, and *East* or *West*, it is called an *Erect North Plain* declining *East* or *West*.

3. *Ob-*

3. *Oblique*, (or *Reclining* from the *Zenith*, or *Inclining* to the *Horizon*;) and these also are of two sorts, namely, *direct Reclining*, or *Declining* and *Reclining*: For if the *Reclining Plain* do lie open to the true *East*, *West*, *North*, or *South* points of the *Horizon*, then it is call'd a *direct East*, *West*, *North* or *South Recliner*; but if it lie open to (or do behold) the *South-East* or *South-West*, the *North-East* or *North-West*, it is then called a *Reclining Plain*, *Declining* from the *North* towards the *East* or *West*: Or a *Reclining Plain*, *Declining* from the *North* towards the *East* or *West*.

These *Reclining Plains* are best represented by the *Roof* of an ordinary *House*, as the Tiling; and of these I shall hereafter call those *Recliners*, whose upper faces behold the *Zenith* of the *place*, as the Tiles of all *Houses* do; and the under faces of them, which respect or look down to the *Horizon*, I call *Incliners*.

According to this description, the number of *Dial Plains* are 25, which I shall reduce to 17, by supplying of the *Inclining Plains* with the hours of the *Reclining Plains* opposite to them, for they are indeed the same *Examples*, of all which 17 *Varieties* shall follow in due place.

CHAP. II.

How to find the Declination *and* Reclination *of a* Plain.

HAving in the foregoing *Chapter*, given you an account of such *Plains* and their *Positions*, upon which Dials are usually made; it will be necessary, before you can describe *Hour Lines*, &c. upon any *Plain* proposed, first, to know the quantity of its Situation; that is, (1) whether it be truly *Horizontal*; (2) or whether it be truly *Perpendicular* to the *Horizon*; (3) or whether it do *Recline* from the *Zenith*, and *Incline* to the *Horizon*; (4) or whether any such *Plain* do directly behold the true *North*, *South*, *East*, or *West* points of the *Horizon*; and if they do not, (5) to find how much they *deviate*, or *decline* from either of those *Four Cardinal points*, and *which way*: And to find all these, shall be the work of this *Chapter*. But before the *Declination* of a *Plain* can be attained, there are required two *Observations* to be made at one instant of time, when the Sun shineth upon the *Plain*; and those never after the hour of 10 in the Forenoon, nor before 2 in the Afternoon, but the *earlier* in the *Morning*, or *later* in the *Evening* are the best times: The first is to find the *Horizontal Distance* of the *Sun*, from the *Pole* of the *Plain*; the second is of the *Sun's Altitude*, thereby to get the *Azimuth*; these two Observations, as I said before, ought to be made at one instant of time, as near as may be, that the parts of the work may agree together the better, and may best be done by two persons, one to take the *Sun's Altitude*, and the other the *Horizontal Distance*.

§ I. *For the* Horizontal Distance *of the* Sun *from the* Pole *of the* Plain.

APply one edge of a *Quadrant* to the *Plain*, so that the other may be Perpendicular to it, and the *Limb* of the *Quadrant* towards the *Sun*; and hold the whole *Quadrant*, parallel to the *Horizon*, as near as you can conjecture: Then holding a *Thred* and *Plummet* at full liberty, so that the shadow of the *Thred* may pass both through the *Centre* and *Limb* of the *Quadrant*; and carefully then observe what degrees of the *Limb* are cut off by the shadow of the *Thred*, numbring them (always) from that side of the *Quadrant* that standeth *Perpendicular* from the *Plain*; for those degrees so cut by the Shadow of the *Thred*, are the degrees of the *Sun's Horizontal Distance* from the *Pole* of the *Plain*.

§ II. *How to observe the* Sun's Altitude.

TAke your *Quadrant* in both your hands, laying your *Right Hand* somewhat near that side which hath the *Sights*, and your *Left Hand* towards the other side; by which means you may let it slip lower, or raise it higher, as occasion requires. Then turning the left side of your *Body* to the *Sun*, move the *Quadrant* up and down, till the Sun, shining through that *Sight* which is next the *Centre* of the *Quadrant*, do

do cast its *Ray*, or *Beam of Light*, upon the *Hole* of the other *Sight*; the *Thred* and *Plummet*, all this time, playing at free liberty; and then look in the *Limb* of the *Quadrant*, what *degrees* and part of a *Degree*, are cut by the *Thred*, for those *degrees* are the *degrees* of the *Sun's Altitude* at that time.

By help of these two Observations, *the* Declination *of any* Plain *may be attained at any time of the* day, *and any time of the* year: *But first you must find the Sun's* Azimuth *from the* North *or* South *for the same time; and to find it shall be taught in the following Section.*

§ III. *To find the Sun's* Azimuth *at any time of the day.*

THE Sun's *Azimuth* may be found several ways taught already is this Book, as by the *Celestial Globe*, Book 3. Part 1. Prob. 18. by several *Spherical Projections*, and by *projecting* of the *Sphere in Plano*, as in Book 3. Part 2. and in divers other places of Book 4 and 5. But the way which I will here deliver, shall be by *Trigonometrical Calculation*, because my intent in this VIII Book, is to apply the *Doctrine* of *Spherical Triangles* to practice in *Dialling*, and resolving of *Problems* to that Art belonging.

The Sun's *Azimuth* is form'd by the XI of *Olique angled Spherical Triangles.*

Example. Upon *Monday* the 14th of *April*, 1690. in the Morning, I was required to take the *Declination* of *a Plain*, whereon to make a *Dial*, in *London*, whose Latitude is 31 Degrees, 32 Minutes.

First, I applied the side of my *Quadrant* to the *Plain*, horizontally, and found the *Horizontal Distance* of the *Sun* from the *Pole* of the *Plain* to be 70 degrees *Eastward* of the *South*; and at the same time, the Sun's *Altitude* I observed to be 27 deg. from whence I compute the Sun's *Azimuth* as followeth.

First, By the Intersection of the Arches of three *Great Circles* of the *Sphere* (as is at large declared, both in the III. and IV. Books foregoing) is constituted an *Oblique Angled Spherical Triangle* Z ⊙ P, such as you see in the following Figure, in which there is given the three sides, *viz.* (1) Z P, the *Complement of the Latitude* 38 deg. 28 min. (2) Z ⊙, the *Complement of the Sun's Altitude* observed, 63 deg. (3) ⊙ P, the *Complement of the Sun's Declination*, (or the Sun's Distance from the *Elevated Pole*) 76 deg. 56 min. And it is required to find the *Angle* at Z, which is the Sun's *Azimuth* from the *North* part of the *Meridian*: Which to perform, *add* the three *sides* together, and find the *summ* of them. Then, from half that *summ* take the *Complement of the Sun's Declination*, and note the difference, as is here done.

			Deg.	Min.			Deg.	Min.
April 14, 1690. Morning,	Latitude,		51	32	its Comp.		38	28
	Sun's Altitude,		27	00			63	00
	Sun's Declination,	*No.*	13	04			76	56
					Their Summ,		178	24
					The half Summ,		89	12
The difference between the Complement of the Sun's Declination and the half Summ.							12	16

Being thus prepared, the Sun's *Azimuth* may be found by these two *Analogies* following.

(1)	As *Radius*, *s.* 90 deg.	10.00000
	Is to the Sign of Z ⊙, the Co-Altitude 63 d. 00'.	9.94988
	So is the Sign of Z P, the Co-Latitude, 38 d. 28 m.	9.79383
	To the Sign of 33 d. 39 min.	9.74371
(2)	As the Sign of 33 d. 39 min. Co-Ar.	0.25629
	Is to the Sign of the half Summ, 89 d. 12 m.	9.99995
	So is the Sign of the Difference 12 d. 16 m.	9.82728
	To this Sign,	19.58352
	The half whereof is	9.79176

Which

Which is the Sign of 38 deg. 15 min. whose Complement 51 deg. 45 min. being doubled, is 103 deg. 30 min. which is the Sun's *Azimuth* from the *North* part of the *Meridian*: Wherefore its Complement to 180 deg. being 76 deg. 30 min. is the Sun's *Azimuth* from the *South*: And the Complement thereof 90 deg. *viz.* 12 deg. 30 min. is the Sun's *Azimuth* from the *East*, it being in the Forenoon that the *Observation* was made.

Another way to Calculate the Sun's Azimuth.

Take half the difference between the Complement of the *Latitude*, and the Complement of the *Altitude*, and add it to half the Complement of the Sun's *Declination*; and also subtract it from the same, noting the *summ* and *difference*. Which done, Sun's observe this

RULE.

To the Complement Arithmetical, of the Signs of the Co-Latitude and Co-Altitude; Add the Signs of the Summ and Difference before found: Half that Summ shall be the Sign of half the Azimuth *from the* North.

As in the foregoing Examples.

	D. M.
Co-Lat.	38.28
Co-Alt.	63.00
Differenee,	24.32
half Differ.	12.16
½ Co-⊙ d.	38.28
Summ	50.44
Difference,	26.12

Z P, Co-Latitude 38 d. 28 m. Co-Ar. ——	0.20616
Z ⊙, Co-Altitude 63 d. 00. Co-Ar. ——	0.05012
Sign of the Summ 50.44 ——	9.88885
Sign of the Differ. 26.12 ——	9.64493
The Summ of all, ——	19.79006
The half, ——	9.89503

Which is the Sign of 51 deg. 45 min. and is the Sun's *Azimuth* from the *North* part of the *Meridian*, as in the former *Example*.

§ IV. *To find the* Plain's Declination.

HAving found the Sun's *Azimuth* from the *South* part of the Meridian (for that is the most Convenient) then by comparing the *Azimuth* and the *Horizontal Distance* together, you may find the *Plain's Declination* by these following *Rules*.

Rule 1. When you make your Observation of the Sun's *Horizontal Distance*, mark whether the Shadow of the *Thred* do fall between the *South*, and that side of the *Quadrant* which is Perpendicular to the *Plain*: For,

1. If the *Shadow* fall between them, then the *Horizontal Distance* and the Sun's *Azimuth* added together, do make the *Declination* of the *Plain*; and in this case, the *Declination* is upon the same *Coast*, whereon the Sun is; *i. e.* if the Sun be *Eastward* of the *South*, the *Plain* declines *Eastward*: If *Westward*, the *Plain* declines *Westward*.

2. If the Shadow fall not between the *South*, and that side of the *Quadrant* which is Perpendicular to the *Plain*; then the *Difference* between the *Horizontal Distance* and the Sun's *Azimuth* from the *South*, is the *Declination* of the *Plain*: And in this case, if the *Azimuth* be the *greater* of the two, then the *Plain* declineth to the *same Coast* whereon the Sun is: Otherwise, if the *Horizontal Distance* be the *greater* of the two, then the *Plain* declineth to the *Coast*, contrary to that on which the Sun was, at the time of Observation.

Rule 2. Note here farther, that a *Plain Declination* thus found, is always accounted from the *South*; and that all *Declinations* are numbred from either the *South* or *North*, towards either *East* or *West*, and must never exceed 90 degrees: Wherefore,

1. If the number of Degrees of *Declination* do exceed 90 deg. take the residue of that number to 180 deg. and that shall be the *Declination* of the *Plain* from the *North*, towards that *Coast* whereon the Sun was: But,

2. If the number of degrees of *Declination* do exceed 180 deg. then the excess above 180 deg. are the degrees of the *Plain's Declination* from the *North*, towards that *Coast* which is *contrary* to that whereon the Sun was at the time of Observation.

Example

Example in the formentioned Observation.

Upon a piece of Paper, or Paistboard, draw a Line, as A C B, representing the *Dial Plain*, and upon C, with 60 deg. of your *Scale* of *Chords*, describe the Semicircle A D B; then when the side of your *Quadrant* C B was applied to the *Plain*, the side C D was Perpendicular thereto, and holding up your *Thred* and *Plummet*, you found the *shadow* of the *Thred* to pass through 70 deg. of your *Quadrant*, wherefore take 70 deg. out of your *Chords*, and set them from D to *a*, and draw the line C *a* ⊙, for the line of the Sun's *Azimuth* from the Perpendicular line of the *Plain*, at the time of Observation. Then (at the same time taking the Sun's *Altitude*) you hereby, by the III. Sect. hereof, find the Sun's *Azimuth* to be 76 deg. 30 min. from the *South*; therefore take 76 deg. 30 min. and set them from *a* to *b*, and draw the Line C *b* S; so shall the Line C *b* S be the *Meridian line*, and *b* the *South*

Point of the *Horizon*: Now, if you take 90 deg. and set them from *b* to *c*, and draw the Line C *c* E, that point *c* shall be the *East* point of the *Horizon*, and the Line C *c* E, the Line of *East* and *West*: The *Scheme* being thus drawn, you may see that the Line of *shadow* does not fall between the Line C *b* S the *South*, and the Perpendicular C D; wherefore, (by the second part of the first *Rule* before going) the *difference* between the *Azimuth* and *Distance* (which is 6 deg. 30 min.) is the *Declination* of the *Plain*: And also, because (by the same second part of the same *Rule*) because the *Azimuth* is the *greater* of the two, the *Plain declines* to the same *Coast* to that whereon the Sun was at the time of the *Observation*, which is *Eastward*, because the *Observation* was in the Forenoon: So that this *Plain* must be called, *An upright Plain, declining from the* South, *towards the East*, 6 *deg.* 30 *min.* And so it appears by the *Scheme*, where you see, that the Line A C B, representing the *Dial Plain*,

Plain beholds S the *South*, and E the *East* Points of the *Horizon*; the *North* and *West* Points being on the back part thereof, as appears by the foregoing Figure.

§ V. *To find the* Reclination *of a* Plain.

IN the foregoing Figure, let A B C D, being the Roof of the Building P L R X, represent a *Reclining Plain*. First, draw thereon an *Horizontal Line*, which may be done by an ordinary *Level*; or by applying a *Ruler* to the *Plain*, letting one hold the *Ruler* so to the *Plain*, till you find by the edge of the *Quadrant* applied to the *Ruler*, the Line and Plummet to fall just upon the edge of the *Quadrant*; then draw a Line by the side of the *Ruler*, as E F; and that shall be the *Horizontal Line* of the *Reclining Plain* A B C D.

Secondly, To this *Horizontal Line* E F, draw another Line at Right Angles thereto, as the Line G H; to which Line, apply the side of a *Ruler* M N, and towards that end of the *Ruler* which hangeth over the *Plain*, as N, apply the side of a *Quadrant*, letting the *Thred* and *Plummet* have free liberty to play by the side of the *Quadrant*; and when it resteth, note what degrees of the *Quadrant* the *Thred* resteth upon; for those degrees counted from the side of the *Ruler*, are the quantity of the *Plain's Reclination*. The manner of the work of this *Chapter*, will best appear by the foregoing *Figure*.

CHAP. III.

How to know which Pole, *whether the* North *or the* South, *is to be elevated over any* Dial Plain, *in any Position.*

THE *Stile* of every *Dial*, represents the *Axis* of the *World*, and therefore the two ends thereof must directly respect the two *Poles* of the *World*: And therefore, if the *South Pole* be elevated above any *Dial Plain*, a *Dial* made on the *Back-side* thereof, must have the *North Pole* elevated above it: And to know which *Pole* is to be elevated over any *Plain*, observe these few

General RULES.

1. Upon the *Horizontal Plain*, in *North Latitude*, the *North Pole*; in *South Latitude*, the *South Pole*.

2. Upon all *Erect Plains*, whether *direct* or *declining*; if the *Plain* lie open to the *South*, the *South Pole* is elevated; but if it behold the *North*, the *North Pole* is elevated.

3. Upon direct *East* or *West Plains*, *Reclining*, (how far soever,) the *North Pole* is elevated: And upon the *East* and *West Incliners* opposite to them, the *South Pole*.

4. Over all *North Reclining Plains*, (whether *direct* or *declining*) the *North Pole* is elevated: And over the *Inclining Plains* opposite to them, the *South Pole* is elevated.

5. Over all *South Reclining Plains*, whether *direct* or *declining* (if the *Plain* pass between the *Zenith* and the *Pole*) the *Axis* of the *Stile* must have respect to the *South Pole*: And on the *Inclining Plains* opposite to them, the *North Pole*. But if the *Plain* pass between the *Horizon* and the *Pole*, the *North Pole*; and on the *Incliners* opposite to them, the *South Pole* must be elevated.

CHAP. IV.

Of an Horizontal Projection, *upon which any* Dial Plain *may be described; and all the* Requisites *belonging to any such* Plain, *found out Geometrically, and also by Trigonometrical Calculation.*

FOR the better understanding of what I have hitherto delivered concerning the several sorts of *Plains*, upon which *Dials* may be described: And also how to find in what *position any Plain* is situated, in respect of *Declination* or *Reclination*, or both; I have here inserted a Figure or Scheme, which I shall hereafter call

The Fundamental Diagram.

Which is no other then a Projection of the Sphere upon the *Plain* of the *Horizon* of *London*, whose Latitude is 51 deg. 32 min. upon which I have described every several *Plain* before mentioned; and shall, (when I come to the construction of such or such a *Dial*) shew what Line or Circle upon the *Fundamental Diagram* do represent that *Plain*; and also what *Spherical Triangle*, (or Triangle) is produced by the intersections of other *Great Circles* of the *Sphere*, with the Line or Circle representing the *Plain*, from whence all the *Requisites*, relating to every *Plain*, may be found by *Trigonometrical Calculation*.

CHAP V.

How to find the distances of the Hour-Lines upon an Horizontal Plain.

THis *Plain* is represented in the *Fundamental Diagram*, by the outward (or *Primitive*) *Circle* thereof, N W S E, wherein Z is the *Zenith*, and *Pole* of the *Plain*; therefore, if a *Ruler* be laid to Z, and to the several Points, 3, 2, 1, 12, 11, 10, *&c.* where the Hour-Circles do cut the *Plain*, right Lines be drawn, those shall be the true Hour-Lines of an *Horizontal Dial*, for the *Latitude* for which the *Fundamental Diagram* was made; *viz.* for *London*, whose *Latitude* is 51 deg. 32 min. represented by the Arch of the *Meridian* N P.

The Arithmetrical or Trigonometrical Calculation.

For the making of an *Horizontal Dial*, there is nothing required to be given, but the *Latitude* of the place for which the *Dial* is made, which is here represented by N P 51 deg. 32 min. Now the *Hour Lines* in the *Fundamental Diagram* meeting altogether in P the *Pole*, do cut the *Primitive Circle* N W S E in unequal parts in the point 1, 2, 3, 11, 10, 9, *&c.* but make equal *Angles* at the *Pole*; and do constitute several *Right Angled, Spherical Triangles*, as N P 11, in which is given (1) the side P N, 51 deg. 32 min. the *Latitude*: (2) the *Angle* N P 11, 15 deg. the *Æquinoctial* distance for one hour: (3) the *Right Angle* at N, to find the side N 11.

Thus,

As the *Radius*, 90 deg. —— —— —— —— ——	10.000000
Is to the Sign of the *Latitude* N P 51 deg. 32′ —— ——	9.893544
So is the Tangent of N P 11, 15 deg. for 1 hour, —— ——	9.428052
To the Tangent of the side N 11, 11 deg. 51′. —— ——	9.321596

This 11 deg. 51 min. is the true *Hour-distance* of the hours of 11 or 1 a Clock, from the *Meridian* upon the *Dial Plain*:

Where-

Wherefore, make a *Table*, as this following; In the first Column whereof set the Hours in order from *Noon*, as 11, 10, 9, 8, 7, and 1, 2, 3, 4, 5, 6; and against them in the second Column, set the *Equinoctial distances*, allowing 15 deg. for one hour, 30 for two hours, 45 for three hours, *&c.* Then by the former *proportion*, the first hours distance being found 11 d. 51 m. set that in the third Column against 11 and 1. Then

As *Radius*, —— ——	10.000000
Is to the Sign of the *Latitude* N P, 51 d. 32 m.	9.893544
So is the Tangent of 30 deg. N P 10,	9.761439
To the Tangent of 24 deg. 19 min.	9.654983

Which is the second hours distance upon the *Plain*, namely of 10 and 2 a clock; which set in the third Column of the Table.

Latitude 51 *deg.* 32 *min.*

Hours from Noon.	*Equinoctial distances.*	*True H. distance on the Plain.*
	De. Mi.	De. Mi.
12	00 00	00 00
11 1	15 00	11 51
10 2	30 00	24 19
9 and 3	45 00	38 3
8 4	60 00	53 36
7 5	75 00	71 06
6	90 00	90 00

The Geometrical Projection of this Dial.

Fig 1.

First, Draw two streight Lines, A B for the *Meridian*, and *Hour Line* of 12, and D F for the hour of 6; crossing each other at right Angles in C, for the Centre of your *Dial*. Upon the Centre C, with 60 deg. of the *Scale of Chords*, describe the Semicircle D E F; then lay your Table before you, and out of your *Scale of Chords* take 11 deg. 51 min, and set them upon the Semicircle both ways from E, for the hours of 11 and 1 a clock: Likewise take 24 deg. 19 min. and set them on both sides from E, for the hours of 10 and 2: Also 38 deg. 3 min. for the hours of 9 and 3; and so of the rest. Through these Points, and the Centre C, draw streight Lines; and they shall be the true *Hour-Lines* upon your *Plain*, from 6 in the *Morning* till 6 at *Night*, which number as you see done in the *Figure*. And for the other four hours, *viz.* 4 and 5 in the *Morning*, and 7 and 8 at *Night*, they are drawn by extending the hours of 4 and 5, and of 7 and 8, through the Centre.

Then for the *Stile* of your *Dial*, take 51 deg. 32 min. from your *Scale*, and set it from E to G, and draw the Line C G for the *Stile* of your *Dial*, which must be placed upright upon the *Hour-Line* of 12; and so is your *Dial* finished, which you may draw into any Form, Square, Round, Oval, or what other Figure you please.

CHAP. VI.

How to find the Distances of the Hour-Lines upon an Erect Direct South *or* North Plain.

THese *Plains* in the *Fundamental Diagram* are represented by the Right Line W Z E, the upper part respecting the *North Pole*, and the lower part respecting the *South*; and the *Hour-Circles* in the *Diagram*, which all meet in P, the *Pole*, do cross the *Plain* W Z E in the Points 6, 5, 4, 3, 2, 1, and 11, 10, 9, 8, 7, 6. The *Poles* of these *Plains* are N or S, to either of which Points if you lay a Ruler, and to the several Points where the *Hour Circles* cross the *Plain*, and make marks upon the outer Peripherie of the *Fundamental Diagram*; those Distances shall be the true Hour-Distances to be set upon the *Dial Plain*; and the Arch of the *Meridian*, P Z, between the *Pole* and the *Plain*, equal to the complement of the *Latitude*, is always the height of the *Stile* or *Pole* above the *Plain*.

The Trigonometrical Calculation.

For the making of these *Dials*, there is nothing requir'd to be known, but the *Latitude* of the place in which the *Dial* is to stand; and that in the *Fundamental Diagram* is represented by the Arch of the *Meridian* N P, 51 deg. 32 min. and the Complement thereof by the Arch P Z, 38 deg. 28 min. which is the *height* of the *Pole* or *Stile* above these *Plains*.

Now the hour Circles in the *Fundamental Diagram* meeting in P, the *Pole*, and intersecting the *Plain* W Z E in the points 5, 4, 3, 2 and 1, do constitute several Right Angled *Spherical Triangles*, as Z P 1, 15 deg. Z P 2, 30 deg. *&c.* in which Triangle Z P 1, there is given (1) the Right Angle at Z, (2) the Side Z P, the *Complement of the Latitude* of the place, 38 d. 28 m. (3) the Angle 1 P Z, 15 deg. or one hour's distance from the *Meridian*: To find the Side 1 Z, or Z 11, (which are both equi-distant from the *Meridian*,) by this Analogy:

As *Radius* 90 deg.	10.00000
Is to P Z the *Co-Latitude* 38 deg. 28 min.	9.79383
So is the Tangent of 1 P Z, 15 de.	9.42805
To the Tangent of the side 1 Z, 9 deg. 28 min.	9.22188

Which 9 deg. 28 min. is the distance of the hours of 1 and 11 from the *Meridian* N. S. upon the *Plain*, and will be found by the foregoing Proportion to be such as this Table expresseth.

Co-Latitude 38 *deg.* 28 *min.*			
Hours.	*Equinoctial dist.*	*True Ho. distance.*	
	De. Mi.	De. Mi.	
12	00 00	00 00	
1 11	15 00	9 28	
2 10	30 00	19 45	
3 and 9	45 00	31 53	
4 8	60 00	47 8	
5 7	75 00	66 42	
6	90 00	90 00	

The Geometrical Projection of these two Dials.

I. *For the* South.

Fig. II. First, Draw a Perpendicular Line O M, representing the *Meridian* and Hour-Line of 12, and another Perpendicular thereunto for the Hour-Line of 6, as N P, intersecting each other at O, the Centre of the Dial.

Secondly, Upon O, describe the Semicircle N Q P, with the Scale of *Chords*, and laying your Table before you, take 9 deg. 28 min. out of your Scale, and set them from Q upon the Semicircle, both ways; for the points of the hours of 11 and 1. Also, take 19 deg. 45 min. and set them always from Q, for the points of the hours of 10 and 2, and so for all the rest, as you find, by your Table, their distances to be from the *Meridian*.

Then Lastly, Take 38 deg. 32 min. the Complement of the *Latitude* from the Scale of *Chords*, and set them from Q to R, and draw the Line O R for the Stile of the Dial, which must stand upon the Hour-Line of 12, and must point *downwards* to the *South Pole*, and so is your Dial finished.

II. *For the* North Dial.

Fig. III. The *North* Dial is the same with the *South*, only the Stile must point upwards towards the *North* Pole, and the Hours about *Midnight*; as 9, 10, 11, at Night, and 1, 2 and 3 in the Morning must be left out; and 4 and 5 in the Morning, and 7 and 8 at Night, must be drawn through the Centre, as in the *Horizontal* Dial; and so is your Dial finished.

CHAP.

CHAP. VII.

How to describe Hour-Lines *upon a Direct* East *or* West *Plain.*

THese Plains are represented in the Fundamental *Diagram* by the Line N Z S, which passeth directly through P, the Pole of the World, and therefore the Pole hath no Elevation above these Plains; and consequently the Dial will have no Centre, but all the Hour Lines will be parallel one to the other: But Dials may be drawn upon such Plains by the following Directions: Figure IV and V.

The Geometrical Construction of these Dials.

Let A B C D be a Dial Plain, upon which you would draw an *East* or *West* Direct Dial.

1. Upon the point C, at the lowermost corner, if it be an *East* Dial, or upon the point D, at the other lowermost Corner, if it be a *West* Dial, with 60 d. of your Line of Chords describe an obscure Arch of a Circle E F; then from the same Line of Chords take 38 deg. 28 min. the Complement of the Latitude of the Place (which is also the Elevation of the Equinoctial above the Horizon) and set that distance upon the Arch from E to F, and draw the Line C F quite through the Plain, which Line shall represent the Equinoctial Circle.

2. That you may the better proportion your Stile to your Plain, and that all the Hours may come on, and be at a convenient distance one from another, assume two points in the Equinoctial Line, one towards the end C, for the hour of 11 in the *East* Dial (or of 1 in the *West* Dial) as the point G, and another towards the other end thereof, for the hour of 6, as the point H; and through these two points, G and H, draw two Lines at right Angles to the Equinoctial Line, for the Hour Lines of XI and VI a Clock.

3. Upon the point G with 60 d. of the Line of Chords describe an obscure Arch of a Circle (below the Equinoctial Line) as I K, setting thereon 15 d. of your Line of Chords, from I to K, and draw the obscure Line G K, extending it till it cut the Hour Line of VI in the point L; so shall the distance L H be the height of the perpendicular Stile proportioned to this Plain.

4. Open your Compasses to 60 d. of your Line of Chords, and setting one foot in the point L; with the other describe an obscure Arch of a Circle M N; between the Hour Line of VI and the Line G L.

5. Divide the Arch M N into five equal parts (which 15 d. of your Line of Chords will do) at the points ⊙⊙⊙⊙⊙, and lay a Ruler from L, to each of these points, ⊙⊙⊙⊙⊙, and the Ruler shall cut the Equinoctial Line C H in the points ∗∗∗∗, through which points draw right Lines parallel to the Hour Line of VI, as the Lines VII ∗ VII, VIII ∗ VIII, IX ∗ IX, X ∗ X, and they shall be the true Hour-Lines of an *East* Plain, from Six in the Morning to Eleven before Noon.

6. For the Hour-Lines before VI, namely of IV and V in the Morning, you may put them on by transferring the same Distances upon the Equinoctial Line before VI, as there is between VI, and the Hour-Lines of VII and VIII after VI, and through those Points draw Lines parallel to the Hour Line of VI, and they shall be the Hour Lines of IV and V in the Morning.

7. For the Stile of these *East* or *West* Dials, it may be either a streight Pin or Wire pointed, of the just length of the Line H L, fixed in the point H, or some other part of the Line of VI, perpendicularly to the Plain; which will shew the true hour only by the shadow of the very top thereof, as in the *West* Dial Figure V. —— Or, (which is better) it may be a Plate of Brass or Iron, of the same breadth as is the distance between the Hour Lines of VI and IX upon the Equinoctial, as in the *East* Dial, Figure IV; which Plate must be set perpendicularly upon the Hour Line of VI, which shall shew the hour by the shadow of the upper edge thereof; and so is your Dial finished.

8. If you would insert the halfs and quarters of hours into these Dials, you may easily effect it, by dividing each space between ⊙ and ⊙ on the Arch M N, into four

four equal parts, and so transferring them to the Equinoctial Circle, as you did the whole hours. All which may be plainly seen in Figure V.

In the making of this Dial you have made two Dials, namely, a West Dial as well as an East, for it is the same in all respects as to the Hour-Distances, and Height of the Stile. Only whereas the Arch E F in the East Dials (through which the Equinoctial passeth) was described on the right hand of the Plain, upon the Centre C; in the West Dial it must be described on the left hand, upon the Centre D; and the Hour-Lines of IV, V, VI, VII, VIII, IX, X, and XI in the Forenoon on the East Dial, must be VIII, VII, VI, V, IV, III, II, and I in the Afternoon, on the West Dial; as the Figure IV and V do evidence.

CHAP. VIII.

How to find the Hour-Distances, and the other Requisites *belonging to an Erect* South *or* North *Plain, declining* East *or* West.

LET our *Example* be of an upright *South* Plain, in the Latitude of 51 deg. 32 min. declining *Eastward* 30 deg: And such a Plain is represented in the Fundamental Diagram by the Line C D, the Pole whereof is B. Now the Hour-Circles in the Fundamental Diagram, meeting in P, the Pole, do cut the Plain C D, dividing it into several unequal parts; if a Ruler be laid to B, the Pole of the Plain, and to these several Intersections, the points where the Ruler, so laid, shall cut the outer Circle of the Fundamental Diagram, shall be the Hour-distances, and Lines drawn from Z through those Points, shall be the true Hour-distances upon the Plain: And if you imagine an Arch of a great Circle to pass through P, the Pole of the *World*, and B, the Pole of the Plain, that Circle shall cut the Plain at Right Angles in the point Y; and so there shall be constituted a Right Angled Spherical Triangle, Z P Y, Right Angled at Y.

The Trigonometrical Calculation.

Before you can calculate the Hour-distances for these Plains, there are three *Requisites* to be first enquired; and all of them are to be found in the Right Angled Spherical Triangle P Y Z, right Angled at Y, in which there is given:

1. The side P Z, 38 deg. 28 min. the Complement of the *Latitude*.
2. The Angle P Z Y, the Complement of the Plain's Declination, 60 deg.
3. The Right Angle at Y.

And by help of these the other three Requisites *may be attained; which are,*

1. The Height of the Pole or Stile above the Plain, represented by the side P Y.
2. The *Deflexion*, (or *Substile's* distance from the *Meridian*,) represented by Z Y.
3. The Plain's Difference of *Longitude*, represented by the Angle Y P Z.

I. *For the Height of the* Pole *or* Stile *above the* Plain P Y.

As the *Radius*, 90 deg.	10.00000
Is to the Co-Sine of the Latitude P Z, 38 deg. 28 min.	9.79383
So is the Co-Sine of the Plain's Declination P Z Y, 60 deg.	9.93753
To the Sign of P Y, 32 deg. 36 min.	9.73136

Which is the height of the Pole or Stile above the Plain.

II. *For the* Deflexion, *or Substile's Distance from the Meridian* Z Y.

As the *Radius*, 90 deg.	10.00000
Is to the Sign of the Plain's Declination, 30 deg.	9.69897
So is the Co-Tangent of the Latitude Z P, 38 deg. 28 min.	9.90008
To the Tangent of Y Z, 21 deg. 40 min.	9.59905

Which is the *Substile's* Distance from the *Meridian*.

III. *For*

III. *For the Plain's Difference of Longitude,* Y P Z.

As the Co-Sign of the Latitude P Z, 38 deg. 28 min.	9.79383
Is to the *Radius*, ——— ——— ———	10.00000
So is the Sign of the Deflexion Y Z, 21 deg. 40 min.	19.56736
To the Sign of Y P Z, 36 deg 25 min.	9.77353

Which is the Plain's Difference of Longitude.

From the Plain's Difference of Longitude thus found, by allowing 15 deg. of the Equinoctial for one hour, and 1 deg. for 4 min. of time; it will follow, that the Substile will fall between the Second and Third hours from the *Meridian*; that is, between the hours of 2 and 3 in the Afternoon, if the Plain decline *Westward*; or between 9 and 10 in the Forenoon, if the Plain decline *Eastward*.

For becauſe the Plain's Difference of Longitude falls between 30 d. and 45 d. (which is the Second and Third Hour's Equinoctial Diſtance) there will be two Compleat Hours, and 6 deg. 25 min. more: Wherefore prepare a Table of the Hours fit for the Plain, as is here done in the done in the firſt Column of the Table.

Then againſt XII, ſet the Plain's Difference of Longitude 36 deg. 25 min. from which ſubtract 15 deg. and there will remain 21 deg. 25 min. which ſet againſt XI and I; and from 21 deg. 25 min. ſubtract 15 deg. and there will remain 6 deg. 25 min. which ſet againſt X and II, and (becauſe it is leſs than 15 deg.) write the word Subſtile over it, and ſubtract it from 15 deg. and and the remainder 8 deg. 35 min. ſet over Subſtile, and againſt the Hours of IX and III; to this 8 deg 35 min. by the continual addition of 15 deg. you ſhall have ſuch Equinoctial Hour-diſtances, as in the Table: Which Table thus prepared, the next thing will be to find the true Hour diſtances upon the Plain from the Subſtiles.

A Table of the Hour-Diſtances for a South Dial, *declining* 30 *deg.* East *or* Weſt.

	D. M.
Latitude of the place,	51.32
Deflexion,	21.40
Stile's Height,	32.36
Difference of Longitude,	36.25

Hours for the East	*Hours for the* Weſt	*Equinoctial Ho. Diſtances.* De. Mi.	*The true Ho. Diſtances on the Plain from the Subſtile.* Deg. Min.
IV	VIII	83 35	78 12
V	VII	68 35	53 57
	VI	53 35	36 08
VII	V	38 35	23 16
VIII	IV	23 35	13 14
IX	III	8 35	4 36
		Subſtile	
X	II	6 25	3 28
XI	I	21 25	11 56
	XII	36 25	21 40
I	XI	51 25	34 03
II	X	66 25	51 00
III	IX	81 25	74 21

IV. *To find the true Hour-diſtance upon the Plain from the Subſtile. For which this is the Proportion.*

As the *Radius* 90 deg. ——— ——— ———	10.00000
Is to the Sign of the Stile's Height, 32 deg. 36 min.	9.73140
So is the Tangent of the ſeveral Equinoctial diſtances from the Subſtile, as 6 deg. 25 min.	9.05101
To the Tangent of 3 deg 28 min.	18.78241

Which is the diſtance of the Hour-Lines of 10 or 2 a Clock from the Subſtile.

So again,

As the *Radius* 90 deg. ——— ——— ———	10.00000
Is to the Sign of the Stile's Height 32 deg. 36 min.	9.73140
So is the Tangent of the next Equinoctial diſtance, 21 deg. 25 min.	9.59394
To the Tangent of 11 deg. 56 min.	19.32534

Which

Which 11 deg. 56 min. is the distance of the hour-lines of 11 and 1 of the Clock from the Substile.

And according to the Method here prescribed, you must deal with all the *Equinoctial Distances* in the Second Column of the Table; so shall you find the true Hour-distances to be such as are exhibited in the third Column from the Substile.

The Geometrical Projection of these Dials.

Fig. VI. Draw a Right Line C D, for the *Meridian* or Hour-line XII, and towards the top thereof cross it with another Line A B, at Right Angles in C; so shall C be the Centre of the Dial, upon which, with 60 deg. of your *Chords*, describe the Semicircle A H B; then laying your Table before you, take 21 deg. 40 min. (the Deflection, or Substile's Distance from the *Meridian*,) and set it upon the Semicircle from H to E, if the Plain decline *Westward*, as in this Example it doth; or towards K, if the Plain had declined *Eastward*, and draw the Line C E, for the Substilar Line of your Dial: Also take 32 deg. 36 min. from your Scale of Chords, and set them from E to F, and draw the Line C F for the *Axis*, or Stile of your Dial.

This done, Out of your Table take the respective Hour-distances from a Scale of Chords, and set them off upon the Semicircle from E; so Lines drawn from C, the Centre of the Dial through those Points, shall be the true Hour-Lines proper for the Declining Plain.

Now in the making of this *South* Dial, declining *Westward* 30 deg. you have made four Dials: Namely,

A *South* declining *West*	30 deg.
A *South* declining *East*	30 deg.
A *North* declining *East*	30 deg.
A *North* declining *West*	30 deg.

Only by changing the names of the Hours, and placing the Substile on the contrary side of the Line of XII, for the *South* declining *East* Dial. Then if you turn these two Dials upside down, you have two *North* declining Dials, one *East*, the other *West* 30 deg. the Stile, Substile, and Hour-distances being the same in all four *Plains*; but in the two *North* decliners, the Stile must point upwards, towards the *North* Pole, as the *South* decliners do downwards, towords the *South* Pole; and to remove all doubts concerning this matter (which is very material for a Diallist to be perfect in) I have in Figure VI. drawn the Hour-Lines of the two *South* Decliners through the Centres, and they produce the two *North* decliners, only the hours before 4 in the *Morning*, and after 8 at *Night* are omitted, the Sun being then under our *Horizon*; so that XII, being drawn through the Centre, will be XII at *Midnight*; but no more of this, for the sight of the Figure will work more upon the Fancy, than a whole Chapter of Information.

CHAP. IX.

How to draw Hour Lines upon a South *or* North Plain, *which declines* many Degrees *towards the* East *or* West.

Fig. VII. IF a Plain shall be found to decline many Degrees from the *North* or *South* towards either *East* or *West*; as above 60 deg. although the *Requisites* may be found, and the Dial made in all respects as the former Dial was, in the last *Chapter*, yet by reason that the Pole of the World will have but small Elevation above such a Plain, the Hour-Lines will fall so close one to another, that there will be no competent distance between them, till they be extended very far from the Centre: And therefore, it was the way of the Ancients, to draw the Dial upon a large Floor, and extend the Hour-Lines, Stile, and Substile, to a great length, that so the Hour-Lines might be of a convenient distance; and then to cut the Dial off, Stile and all, and so transfer it to the Plain. But this being too *Mechanical*, I will here shew a more Artificial way how to draw such a Dial *Geometrically*, by help of a Line of Chords only, having no regard to the Centre of the Dial. But, I. *By*

I. *By* Trigonometrical *Calculation.*

You must find the *Requisites*, Namely, (1.) *The Height of the* Pole, *or Stile, above the* Plain. (2.) *The Distance of the Substile from the* Meridian. (3.) *The* Plain's *Difference of* Longitude. All which may be found by the foregoing *Analogies* or *Proportions.*

Suppose therefore an upright Plain, in the Latitude of 51 deg. 32 min. should decline from the *South Eastward* 85 deg.

I. *For the* Stile's *Height.*

As the *Radius* 90 deg. —— 10—00000

Is to the Co-Sine of the Latitude 38 deg. 28 min. —— 9—79384
So is the Co-Sine of the Plain's Declination 5 deg. —— 8—94029

To the Sine of 3 deg. 6 min. the Height of the Pole or Stile —— *x*8—73413

II. *For the* Substile's *Distance from the* Meridian.

As the *Radius* 90 deg. —— 10—00000

To the Sine of the Declination 85 deg. —— 9—99834
So is the Co-Tangent of the Latitude 38 deg. 28 min. —— 9—90008

To the Tangent of 38 deg. 22 min. —— *x*9—89842
For the *Deflexion*, or the *Substile's* Distance from the *Meridian.*

III. *For the* Plain's *Difference of Longitude*

As the Sine of the Latitude 51 deg. 32 min. —— 9—89374

Is to the *Radius* 90 deg. —— 10—00000
So is the Tangent of the Declination 85 deg. —— 11—05804

To the Tangent of 86 deg. 5 min. —— 11—16430
Which is the *Plain's* Difference of Longitude.

These *Requisites* being thus found, you may proceed to the making of a Table for the Hour Distances, in all respects as in the last Chapter: By, first, setting down the Hours proper for the Plain in Order, as in this Table: And against XII set the Plain's Difference of Longitude 86 deg. 5 min. from which subtract 15 deg. and there will remain 71 deg. 5 min. which set against XI and I. Also from 71 deg. 5 min. subtract 15 deg. the remainer will be 56 deg. 5 min. which set against X and II; and so by the continual subtraction of 15 deg. you shall find 11 deg. 9 min. to stand against VII and V, under which write *The Substile's Place*, and then 11 deg. 5 min. being subtracted from 15 deg. there will remain 3 deg. 55 min. which set under *The Substile's Place*, against VI; and then by the continual addition of 15 deg. thereto, you shall have such Equinoctial Distances, as the second Column of the Table affordeth: And then for the true Hour-distances upon the Plain, say,

As the *Radius* 90 deg. —— 10—00000

To the Sine of the Stile's Height 3 deg. 6 min. —— 8—73302
So is the Tangent of 86 deg. 5 min. the Equinoctial Distance of XII. —— 11—16453

To the Tangent of 33 deg. 23 min. —— *x*9—89755
Which is the Distance of XII a Clock from the Substile upon the Plain: And so proceeding with the rest of the Equinoctial Distances, you shall exhibit such Numbers as the Third Column affordeth.

	D.	M.
South Declin. West	85	00
Distance Substile and Meridian	39	22
Height of the Stile	3	06
Plain's difference of Longitude	86	05

Hours. East.	Hours. West.	Equinoctial Distances. D.	Equinoctial Distances. M.	Hour-Dist. on the Plain D.	Hour-Dist. on the Plain M.
XII		86	5	38	23
XI	I	71	5	8	58
X	II	56	5	4	36
IX	III	41	5	2	42
VIII	IV	26	5	1	31
VII	V	11	5	0	36
The Substil'es Place.					
VI		3	55	0	13
V	VII	18	55	1	4
IV	VIII	33	55	2	6

By this Table you may see that the Hour-distances about the Substile (and indeed all the rest, except the extream Hour of XII) do fall so near together, that without an infinite extention of them, there will be no competent Distance between Hour-line and Hour-line; wherefore, laying aside your Table, proceed to make your Dial Geometrically, according to these following Directions.

The Geometrical Projection of this (or the like) Dial.

Fig. VIII. 1. Draw a right Line A B, perpendicular to one side of your Plain, and towards the right hand, because the Plain declineth *Eastward*; and with 60 deg. of your Line of Chords, describe an obscure Arch of a Circle C D E, and upon it (from C to D) set off 38 deg. 23 min. the Substile's Distance from the *Meridian*, and draw the Line A D for the Substile, quite through the Plain.

2. Out of your Line of Chords take 3 deg. 6 min. the Height of the Stile, and set them upon the former Arch from D to E, and draw the Line A E for the Stile.

3. Now (because the Stile is but of small Elevation, *viz.* but 3 deg. 6 min.) draw another Line (as G H) parallel to the Line of the Stile A E, at such convenient distance as you shall think fit; which shall be your Augmented Stile.

4. Assume any two Points in the Substilar-Line A D, at some convenient distance each from other, as R and S, and through those two Points draw two infinite right Lines, both of them at right Angles to the Substilar-Line A D, as the Lines Z Z, and X X.

5. From the Point R, with your Compasses, take the nearest distance to the new Augmented Stile G H, and set that distance upon the Substilar-Line from R to K. Also, from the Point S, take the nearest distance to the new Augmented Stile G H, and set that distance also upon the Substilar-Line from S to L.

6. Upon these two Points K and L, with 60 deg. of the Line of Chords, describe two Semicircles, and in either of them set off 86 deg. 5 min. the Plain's Difference of Longitude; as from R to M, and also from S to M; both of them on the same side of the Substilar-Line, on which the first perpendicular Line A B was drawn.

7. Divide either of the Semicircles last drawn into 12 equal parts, beginning at the Point M, as the Points ⊙⊙⊙, &c. which 15 deg. of the Line of Chords will effect.

8. Lay a Ruler to the Point L, and the respective Points ⊙⊙⊙, &c. in the Semicircle, and the Ruler will cut the Line XX in the Points ∗∗∗, &c. Also lay a Ruler to K, and the several Points ⊙⊙⊙, &c. and the Ruler will cut the Line Z Z in the several Points ∗∗∗, &c.

Lastly, Lines drawn from the first Point ∗ in the Line Z Z, to the first Point ∗ in the other Line X X, (which the Substilar-Line will direct you how to doe,) those Lines so drawn shall be the true Hour-Lines proper for the Plain, and will appear as in the Figure, and be at a competent distance one from another, without having any relation at all to the Centre.

Now in the making of this Dial you have made four Dials, *viz.*

A { South declining West, South declining East, North declining West, North declining East } 85 degrees,

only by changing of the names of the Hours, and placing the Stile on the contrary side

side of the Line A B, for the South declining East. And by turning the Dial upside downwards, for the two North Decliners; so that the Stiles may point upwards to the North-Pole, and the Hours about Midnight omitted, as in the former North Dial, Figure VI.

CHAP. X.

How to find the Hour-distances, *and the other* Requisites *belonging to a* Direct East *or* West Plain, *which* Reclineth *from the* Zenith *towards the* Horizon.

SUppose a direct *West* Plain, should recline from the *Zenith* towards the *Horizon* 35 deg. Such a Plain is represented in the Fund. Diagram by the Circle N X S, whose Pole is at *a*. Now if a Ruler be laid to *a*, and to the several Points where the Hour-Circles of the Projection do cross the Plain in the Fundamental Diagram, it will cut the Primitive Circle N W S E in several Points, through which if Lines be drawn from Z, they will be the true Hour-Lines of the *East* or *West* Reclining Plain. And if you imagine an Arch of a Great Circle to pass through P the Pole of the World, and *a* the Pole of the Plain, as the Arch P R, you shall have constituted a Right Angled Spherical Triangle N P R, right angled at R.

Now before the Hour-Distances can be found upon these *East* or *West* reclining Plains, there must be three things found, as there were in upright declining Plains, *viz.*

1. The Height of the Stile or Pole above the Plain.
2. The Deflexion, or Substile's Distance from the Meridian.
3. The Plain's Difference of Longitude.

All which may be found by restoring of the foresaid Spherical Triangle N P R, right Angled at R, in which is given,

1. The Side N P, the *Latitude* of the Place 51 d. 32'.
2. The Angle P N R, the Plain's Reclination 35 deg.
3. The Right Angle at R.

And by these things given, the three former *Requisites* may be found.

I. *For the Height of the* Stile *or* Pole *above the Plain* P R.

As the *Radius* 90 deg.	10.
Is to the Sine of the Latitude 51 deg. 32 min. N P	9.89374
So is the Sine of the Plain's Reclination P N R	9.75859
To the Sine of the Stile's Height P R 26 deg. 41 min.	9.65233

II. *For the* Substile's Distance *from the Meridian* R N.

As *Radius* 90 deg.	10.
Is to the Tangent of the Latitude N P, 51 deg. 32 min.	10.09991
So is the Co-Sine of P N R, the Reclination, 55 deg.	9.91336
To the Tangent of N R, 45 deg. 52 min.	10.01327

Which is the Deflexion, or Distance of the Substile from the Meridian.

III. *For the Plain's Difference of Longitude* N P R.

As the Sine of the Latitude 51 deg. 32 min.	9.89374
Is to the *Radius* 90 deg.	10.
So is the Sine of the Deflexion N R 45 deg. 52 min.	9.85595
To the Sine of N P R 66 deg. 27 min.	9.96221

Which 66 deg. 27 min. is the Plain's Difference of Longitude, N P R.

The three Requisites belonging to this Plain being found, you are to prepare a Table, such as followeth, wherein set the Hours proper for the Plain, in the first Column; namely, from 4 in the Morning till 2 in the Afternoon for the *East*, and from 10 in the Forenoon till 8 at Night for the *West*.

Now the Plain's Difference of Longitude being 66 deg. 27 min. there are contained therein four compleat Equinoctial Hours, and 6 deg. 27 min. over; wherefore, the Substile must stand between the Hours of 4 and 5 in the *East* Dial, and between 7 and 8 in the *West*; as is visible in the Fundamental Diagram by the Arch P R. And therefore between those Hours in your Table write the word Substile, and the 6 deg. 27 m. remaining above four Hours, set over the word Substile in the second Column of the Table against the Hours of 4 and 8, and subtracting 6 deg. 27 min. from 15 deg. the remainder, 8 deg. 33 min. set under the word Substile, against the Hours of 5 and 7. Then by the continual Addition of 15 deg. to these two numbers, you shall find the Equinoctial Distances belonging to each Hour, to be such as are exhibited in this Table following, in the second Column thereof: As 21 deg. 27 min. for 3 and 9; and 23 deg. 33 min. for 6 and 6; and so of the rest.

	D.	M.
West Reclining	35	00
Latitude	51	32
Stile's Height	26	41
Deflexion	45	52
Differ. of Long.	66	27

Hours.		Equinoctial Distances. D.	M.	True Hour Distances. D.	M.
11	1	81	27	71	29
12		66	27	45	52
1	11	51	27	29	24
2	10	36	27	18	20
3	9	21	27	10	0
4	8	6	27	2	54
The		Sub-		stile.	
5	7	8	33	3	52
6		23	33	11	4
7	5	38	33	19	41
8	4	53	33	31	18
9	3	68	33	48	49
10	2	83	33	75	50

Your Table thus prepared, your next work will be to find the true Hour-Distances upon the Dial-Plain, from the Substilar-Line, all which may be found by this following *Analogy*:

As the *Radius*, 90 deg. 10.00000
Is to the Tangent of 6 deg. 27′ (for 4 and 8) 9.05328
So is the Sine of the Stile's Height P R 26 d. 41′. 9.65233
To the Tangent of 2 deg. 45 min. 8.70561

Which 2 deg. 45 min. is the Distance of the Hour-Lines of 4 or 8 upon the Plain, from the Substile. And by continual working of this Proportion you shall find Numbers answering to every Hour's Distance from the Substile upon the Plain, such as in the last Column of the Table.

The Geometrical Construction of these Dials.

Fig VIII. First draw a Right Line S L, for the Horizontal Line of the Plain, which in all these Plains the Meridian and hour-line of 12, must lie towards the end thereof, as at S, (on the Right hand for the *East*, but on the Left hand for the *West* Recliner,) make the Centre, upon which, with 60 deg of your Scale of Chords, describe the Arch A D C N B, upon which, set 45 deg. 52 min. the Deflexion from N to C, and draw S C for the Substilar Line of the Dial: Also, set 26 deg. 41 min. from C to D, and draw S D for the Stile or *Axis*.

This done, have recourse to your Table of Hour-distances, from thence take out the several Degrees and Minutes as you find them in the Third Column, and set them upon the Arch A D C B from D the Substilar Line of your Dial; so shall right Lines drawn from the Centre S, through those Points, be the true Hour-Lines belonging to your Reclining or Inclining *East* or *West* Plains.

For in the making of one of these Dials you have made Four, as in upright declining Plains you did, by only changing the Names of the Hours, and leaving out such as the Sun can never shine upon in the Inclliners; all which is visible in the Figure, and needs no words for farther Explanation.

CHAP.

CHAP. XI.

Of South Reclining Plains, and how to draw Hour-Lines *upon them.*

I. *The First Variety, of a* South-Plain *Reclining Equal to the Complement of the Latitude.*

THere are three Varieties of *South Reclining Plains*; for the Plain may so recline, that it may fall just into the Pole, and that is when the Reclination of the Plain is equal to the Complement of the Latitude of the Place; and such a Reclining Plain in the Fundamental Diagram is represented by the Circle W P E, passing through the Pole at P, so that the Pole hath no Elevation over the Plain; and therefore the Dial will have no Centre but all the Hour-lines will be parallel one to the other: And the making of such a Dial is the same with the *East* and *West* erect Dials; only, as the Stile in those Dials stood upon the Hour-line of Six, in these it must stand upon the Hour-line of Twelve: and the Equinoctial Line in those, is the Horizontal Line in this: An *Example* of such a Plain you have in Figure IX. Fig IX.

II. *The Second Variety, of a* South-Plain *Reclining less than the Complement of the Latitude.*

Such Plains as recline from the Zenith less than the Complement of the Latitude, do always fall between the Zenith and the Pole. Suppose therefore in the Latitude of 51 deg. 32 min. a *South* Plain should recline from the Zenith 25 deg. Such a Plain is represented in the Fundamental Diagram by the Circle W F E, whose Pole is at *b*; so that if you lay a Rulet to *b*, and to the several Points where the Hour-Circles in the Fundamental Diagram do cross the Reclining Plain W F E, it shall divide the Periphery of the Fundamental Diagram into 12 unequal Parts, through which, if Lines be drawn from the Centre Z, they shall be the true Hour-lines belonging to the Reclining Plain. Fig. X.

The Trigonometrical Calculation.

In this Plain there is nothing required to be found by Trigonometrical Calculation, but the Hour-distances from the Meridian; and before they can be calculated the Height of the Pole, or Stile, above the Plain, must be known; and that is represented by the Arch of the Meridian P F. Wherefore the Plain falling between the Pole and the Zenith, if you subtract 25 deg. the Plain's Reclination, Z F from Z P, 38 deg. 28 min. there will remain F P, 13 deg. 28 min. for the Height of the Pole or Stile above the Plain; which being known, the Hour-distances may be found by this *Analogy*:

As the *Radius*, 90 deg.	10.00000
Is to the Sine of P F (the Stile's Height) 13 deg. 28 min.	9.42805
So is the Tangent of 15 deg. the Equ. Distance for 11 and 1,	9.36713
To the Tangent of 3 deg. 34 min	8.79518

The Distance of the Hours of 11 and 1 a Clock from the Meridian, of 12 a Clock Hour-line: And so for all the rest of the Hour-distances as they are exhibited in the third Column of this Table.

Stile's Height 13 deg. 28 min.					
Hours.		Equinoct. Distance.		Hour-Distance.	
		D.	M.	D.	M.
	12	00	00	00	00
1	11	15	00	3	34
2	10	30	00	7	39
3 and	9	45	00	13	07
4	8	60	00	21	58
5	7	75	00	41	00
	6	90	00	90	00

The

The Geometrical Conſtruction of theſe Dials.

Fig. X. There is no other Geometrical Conſtruction of this and the other following direct Recliners, than in a common Horizontal Dial; for having found the Height of the Stile, and the Hour-diſtances as in the Table, you have no more to doe, than to deſcribe an Arch of a Circle upon the Centre of the Dial, and ſet off the Hour-Arches, as you find them in the Table. The Stile is to be elevated upon the 12 a Clock-line, equal to an Angle of 13 deg. 28 min. for this Dial will be an Horizontal Dial in that Latitude. And ſuch a Dial you have in Figure X.

This Dial ſerves alſo for a North *Inclining Plain, by drawing the Hour-lines through the Centre, and changing their Names.*

III. *The Third Variety, of a* South-Plain *Reclining more than the Complement of the Latitude.*

Fig. XI. Such Plains as recline from the *Zenith* more than the Complement of the Latitude, do always fall between the Pole and the Horizon.

Suppoſe therefore, that in the Latitude of 51 deg. 32 min. a *South* Plain ſhould recline from the Zenith 70 deg. and ſuch a Plain is repreſented in the Fundamental Diagram, by the Circle W G E, whoſe Pole is at C; ſo that if you lay a Ruler to c, and to the reſpective Points where the Hour Circles of the Fundamental Diagram do cut the Dial-Plain W G E, the Ruler ſo laid will divide the Primitive Circle of the Fundamental Diagram into 12 unequal Parts, through which, if Lines be drawn from the Centre Z, they ſhall be the true Hour-lines for the Reclining Plain.

The Trigonometrical Calculation.

In this Plain (as in the laſt beforegoing) there is nothing required to be found by Calculation, but the Hour-Diſtances upon the Plain: And before they can be found, the Height of the Pole or Stile above the Plain muſt be known; and that is repreſented in the Fundamental Diagram, by the Arch of the Meridian P G: wherefore Z G 70 deg. being equal to the Reclination, P Z, the Complement of the Latitude 38 deg. 28 min. taken therefrom, leaveth 31 deg. 32 min. for the Arch P G, equal to the Height of the Pole or Stile above the Plain; which being known, the true Hour-diſtances from the Meridian may be found by this general *Analogy*.

As the *Radius*, 90 deg.	10.00000
Is to G P (the Height of the Stile) 31 deg. 32 min.	9.71850
So is the Tanget of 15 deg. the Equinoct. Diſtance for 11 and 1	9.42805
To the Tangent of 7 deg. 59 min.	9.14655

The Diſtance of the Hour-lines of 11 and 1 a Clock from the Meridian, or 12 a Clock Hour-line. And ſo you may find all the reſt to be ſuch as are in the Third Column of this Table.

Stile's Height 31 *deg.* 32 *min.*

Hours.	*Equinoct. Diſtances.*		*Hour-Diſtances.*	
	De.	*Mi.*	*De.*	*Mi.*
12	00	00	00	00
1 11	15	00	7	59
2 10	30	00	16	48
3 and 9	45	00	27	36
4 8	60	00	42	10
5 7	75	00	62	52
6	90	00	90	00

For the Geometrical Conſtruction of this Dial, it is no other than to make an Horizontal Dial for the Latitude of 31 deg. 32 min. which is equal to the Stile's Height of this Dial, and the Work is done; and ſuch a Dial you have in Figure XI.

This Dial alſo ſerves for a *North* Inclining Plain oppoſite thereto, by only changing the Names, and leaving out of the Hours above Midnight.

CHAP.

CHAP. XII.

Of North Reclining Plains, and how to draw Hour-Lines upon them.

I. *The first Variety of a* North *Plain, Reclining equal to the Latitude of the Place.*

AS of *South*, so of *North* Reclining Plains there are three Varieties, for the Plain may so recline, that it may fall just in the Equinoctial Circle, and that is, when the Reclination of the Plain is equal to the *Latitude* of the place: And such a Plain is represented in the Fundamental Diagram, by the Circle W Æ E, whose Pole is at P, the Pole of the *World*; unto which Point P, a Ruler laid, and to the several Intersections of the Hour-Circles in the Fundamental Diagram, with the Plain W Æ E, it will divide the Primitive Circle N W S E into 24 equal Parts, and Lines drawn from Z, (the *Zenith*) shall be the true Hour-Lines of a *Plain*, whose *Reclination* is equal to the Latitude of the place: So that to make such a Dial, is no more but to describe a Circle, and divide it into 24 equal Parts, and through those draw right Lines, so many as the longest day in the Latitude wherein the Dial is made doth contain hours. Fig. XII.

For the *Stile* of this Dial, it is represented in the Fundamental Diagram by the Arch of the Meridian P Æ; the Arch Z Æ being equal to the Latitude 51 deg. 32 min. and Z P equal to the Complement thereof, 38 deg. 28 min. which added together, make 90 deg. so that the Stile is no other, than a streight Pin or Wyre, set perpendicularly up in the Centre: And such a Dial you have described in Figure XII.

And in making this Dial, you have made two Dials *viz.* a *South* Inclining Dial also; the *North* (or upper) face whereof being serviceable only from the 10th or 11th of *March*, till the 12 or 13th of *September*, during which time, the Sun hath *North* Declination; and the inclining (or under) face all the time that the Sun hath *South* Declination, namely, from *September* till *March*.

II. *The second Variety of a* North *Plain, Reclining less than the Latitude.*

If a *North* Plain do recline from the *Zenith* less than the Latitude of the place, it falleth between the *Zenith* and the Equinoctial Circle. Fig. XIII.

Suppose therefore that a *North* Plain should recline from the *Zenith* 25 deg. and such a Plain is represented in the Fundamental Diagram, by the Circle W H E, whose Pole is at *d*: Wherefore, if you lay a Ruler to *d*, and to every of the Intersections of the *Hour-Circles* of the Fundamental Diagram, with the Plain W H E, Lines drawn from Z, through those Points, shall be the true Hour-Lines for such a Reclining Plain.

The Trigonometrical Calculation.

In this Plain there is nothing required to be found by Calculation, but the Hour-distances from the Meridian; and before they can be calculated, the Height of the Pole (or Stile) above the Plain must be known, and that is represented by the *Arch* of the Meridian H P, which is compounded of P Z, the Complement of the Latitude, 38 d. 28 m. and Z H, the Reclination, 25 deg. which two added together, make P H 63 deg. 28 min. for the Height of the Stile or Pole above the Plain; and this being obtained, the Hour-distances may be found by the general Analogy, or Proportion following:

As the *Radius*, 90 deg.	10.00000
Is to P H, the Stile's Height, 63 deg. 28 min.	9.95166
So is the Tangent of 15 deg. the distance of 11 and 1.	9.42805
To 13 deg. 29 min.	9.37971

The distance of 11 and 1 a Clock from the Meridian or 12 a Clock Hour-Line, and so of all the rest, as in the third Column of this Table.

Stile's

Stile's Height 63 deg. 28 min.				
Hours.		*Equinoctial Distance.*		*Hour-distances.*
		Deg. Min.		*De. Mi.*
12		00	00	00 00
1	11	15	00	13 29
2	10	30	00	27 19
3 and	9	45	00	41 49
4	8	60	00	57 10
5	7	75	00	73 20
6		90	00	90 00

The Geometrical Construction of these Dials.

The Hour-distances being calculated as in the Table, you have no more to doe but to make an Horizontal Dial for the Latitude of 63 deg. 28 min. and such is the Dial. Fig. XIII.

This Dial serves also for a *South* Inclining Plain opposite thereto, by changing the Names, and leaving out the Superfluous Hours.

III. *The third Variety of a* North *Plain Reclining more than the Latitude.*

Such Plains as do *recline* more than the Latitude of the place, do always fall between the Equinoctial and the Horizon.

Suppose therefore, that in the Latitude of 51 deg. 32 min. a *North* Plain should recline from the *Zenith* 70 deg. and such a Plain is represented to the eye in the Fundamental Diagram, by the *Circle* W K E, whose Pole is at *e*; so that if you lay a Ruler to *e*, and to the respective Points, where the Hour-Circles in the Fundamental Diagram do cross the Plain W K E, the Ruler so laid will divide the Primitive Circle of the Fundamental Diagram into 12 unequal parts; through which Lines being drawn from the Centre Z, shall be the Hour-Lines proper for the Reclining Plain.

The Trigonometrical Calculation.

In this Plain (as in the rest foregoing) there is nothing to be found by Calculation, but the Hour-distances upon the Plain: And before they can be found, the Height of the Pole or Stile above the Plain must be known, and that is represented in the Fundamental Diagram by the Arch of the Meridian P K. Now from P to Z is 38 deg. 28 min. equal to the Complement of the Latitude, and from Z to K is 70 deg. equal to the Plain's Reclination; which two being added together, do make 108 deg. 28 min. for the Height of the Pole above the Plain, which should be the Height of the Stile; but that cannot be the true Height of the Pole above the Plain, for that can never exceed 90 deg. But this Plain falling between the Equinoctial and the Horizon, must needs have so much less Altitude than 90 deg. as the Reclination is more than the Latitude: Therefore if you add 38 deg. 28 min. P Z, to Z K, the Summ will be 108 deg. 38 min. as before: whose Complement to 180 deg. is 71 deg. 32 min. and that is the true Height of the Pole (or Stile) above the Plain.—Or, if you add Z Æ (the Latitude) 51 deg. 32 min. to Z *e*, the Complement of the Plain's Reclination 20 deg. the Summ of them will be 71 deg. 32 min. for the Height of the Pole (or Stile) above the Plain also: And now the Height of the Stile being by this means attained, the Hour-distances upon the Plain may be calculated by the common *Analogie*:

As the *Radius*, 90 deg.	10.00000
Is to the Sine of the Stile's Height, 71 deg. 32 min. Æ *e*,	9.97704
So is the Tangent of 15 deg.	9.42805
To the Tangent of 14 deg. 16 min. for the Distance of 11 and 1 a clock.	19.40509

And

And so repeating this Work for all the hours, you shall have such distances as are expressed in the third Column of the Table.

The Stile must be erected Perpendicularly upon the Hour-Line of 12, (as in all the other direct *North* or *South* Reclining and Inclining Dials,) making an Angle therewith equal to the Height of the Pole above the Plain; which in this Example is 71 deg. 32 min.

The Geometrical Construction of this Dial is no more than to make a common Horizontal Dial for the Latitude of 71 deg. 32 min. And such a Dial you have in Figure XIV.

And in making of this Dial you have made a *South* Inclining Dial also, by drawing the Hour-Lines through the Centre, changing their Names, and leaving out such as the Sun cannot shine upon.

Stile's Height, 71 d. 32 m.

Hours.		*Equinoctial Dist.* De.	Mi.	*Hour-Distances.* De.	Mi.
	12	00	00	00	00
1	11	15	00	14	16
2	10	30	00	28	43
3	9	45	00	43	30
4	8	60	00	58	41
5	7	75	00	74	14
	6	90	00	90	00

CHAP. XIII.

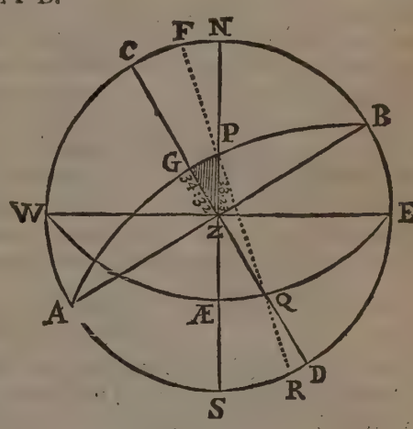
Of South *Declining Reclining Plains, how to find the several Requisites to them belonging, and the Hour-Distances proper to each Plain.*

AS there were three Varieties of direct *South* Reclining Plains, so many also are there of *South* Reclinings which do Decline also: For to any Declination, the Plain may be made so to recline; that (1.) it may pass through the Pole of the *World*; or (2.) it may recline so, that it shall fall between the Pole and the Zenith; or (3.) it may recline so, that it shall pass between the Pole and the Horizon.

I. *The First Variety of a* South *Plain, Declining* Eastward 30 *deg. and Reclining from the Zenith* 34 *deg.* 32 *min.*

This Plain in the F. Diagram, is represented by the Circle C P D, whose Pole is at *q*; it passeth directly through P the Pole of the *World*, and therefore hath no Elevation about it; wherefore the Hour-Lines will be all parallel to one another: But before you can draw them, two things must be found.

1. The Distance of the Meridian from the Horizon P B.
2. The Plain's Difference of Longitude Z P Q.



All which may be found by the F. Diagram; but that comes now to be so incumbred with Lines and Letters, that there can conveniently be no more inserted; wherefore for these six Varieties of *South* and *North* Declining Reclining Plains, I will insert in their due places six particular Schemes, wherein shall be described such Circles of the Sphere, as shall represent such Triangles as are necessary for the finding out of the several *Requisites* belonging to each Dial Plain.

In this Diagram, and so in all the other five which follow, the Primitive Circle, W N E S, represents the Horizon of *London*, whose Lat. is 51 deg. 32 min. N Z S the Meridian, W Æ E the Equinoctial, W E the Prime Vertical, P the Pole of the *World*, Z the Zenith: And in this Scheme the Circle A P B represents the Declining Reclining Plain, passing through P the Pole of the World, Q the Pole of the Plain, and P Q R, the Arch of a great Circle, passing through both the Poles, C G D, a Line cutting the Declining Reclining Plain at right Angles in G; by the Intersection of those Circles of the Sphere, there is constituted a right Angled Spherical Triangle Z G P, right Angled at G; by the solution whereof, the two forementioned Requisites may be found.

I. *By Trigonometrical Calculation.*

In the Triangle P G Z, is given (1.) The side G Z, 34 d. 32 m. the Plain's Reclination, (2) Z P, 38 d. 28 m. the Complement of the Latitude. (3.) The Angle G Z P 30 d. the Plain's Declination, to find G P.

As the *Radius*, Sine 90. Z G P	10.00000
Is to the Sine of P Z, the Co-Latitude 38 deg. 28 min.	9.79383
So is the Sine of P Z G, the Declination 30 deg.	9.69897
To the Sine of P G 18 deg. 7 min.	*x*9.49280

Whose Compl. 71 d. 53 m. is equal to P B, the Dist. of the *Meridian* from the *Horizon*.

2. For the Plain's *Difference of Longitude* Z P Q. An Arch of a great Circle being drawn through P the Pole of the World, and Q the Pole of the Plain; constitutes the Oblique △ P Z Q; the Angle Z P Q being the Plain's Difference of Longitude, and may be found by resolving the Triangle Z P G, in which is given (1.) Z P 38 d. 28 m. (2.) Z G the Reclination 34 d. 32 m. To find the Angle Z P G.

As the Sine of Z P, 38 deg. 28 min. the Co-Latitude	9.79383
Is to *Radius* 90 deg. Z G P	10.
So is the Sine of Z G the Reclination 34 deg. 32 min.	19.75349
To the Sine of Z P G, 65 deg. 41 min.	9.95966

Whose Compl. to 90 d. is 24 d. 19 m. for the Angle Q P Z, the Plain's *Diff. of Longitude.*

II. By *Spherical Projection.*

The Distance of the Meridian from the Horizon is represented in the Scheme by the Arch of the Plain A P B, and may thus be found;

Lay a Ruler from Q, the Pole of the Plain, to P, the Pole of the World, and it will cut the Primitive Circle near O, at 71 deg. 53 min. distant from B, and that is the *Distance* of the *Meridian* from the *Horizon.*

The Plain's *Difference of Longitude* is represented in the Scheme by the Angle Q P Z, and may be thus found: A Ruler laid from P, the Pole of the World, to Q the Pole of the Plain, will cut the Primitive Circle at R 24 deg. 19 min. from S, and that is the Plain's Difference of Longitude.

III. *The Geometrical Protraction of this Dial.*

Fig. XV. 1. Draw a Line at pleasure, as A Z B, representing the Horizontal Line of the Plain; then considering of what length you would have the Stile to be, answerable to the bigness of your Plain; suppose Z D, take that distance in your Compasses, and setting one foot in Z, with the other describe a Semicircle A C D B, upon which (by help of a Scale of Chords) set 71 deg. 53 min. the distance of the Meridian from the Horizon, from A to C, and draw the Line A C E from the Substile.

2. Take 24 deg. 19 min. the Plain's Difference of Longitude, and set it from C to D.

3. Through the point C, draw a Line F G, Perpendicular to the Substile Z E, and another Line H I, at any distance parallel to F G.

4. Lay a Ruler from Z to D, it will cut the Tangent Line F G, in 12, through which point draw the Line 12, 12, parallel to the Substile Z E, for the Hour-Line of 12.

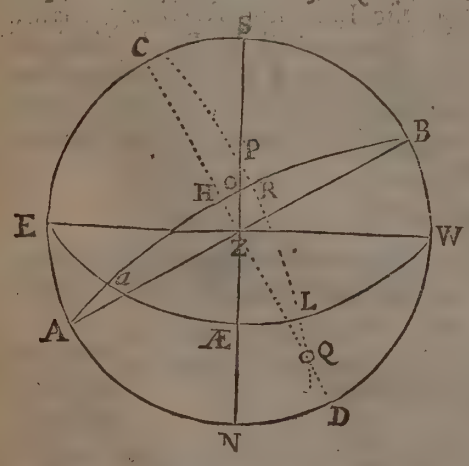
5. Divide the Semicircle A D B into 12 equal parts, at the points * * *, &c. beginning at D.

6. Lay a Ruler unto Z, and upon every of the marks * * *, &c. and it will cut the Line F G, in the points 7, 8, 9, 10, 11, 12, 1, 2, 3; through which points draw Lines parallel to the Substile Z E, so is your Dial finished.

As for the Stile, it may be a streight Pin or Wyre, of the just length of the Line C Z, as K L, set Perpendicular to the Plain upon any part of the Substile, whose point will give the Shadow.— Or it may be a Plate of Brass or Iron, of the same breadth with the Line Z C, which will shew the Hour by the shadow of the side thereof.

II. *The second Variety shall be of a* South *Plain, Declining from the* South Eastward *30 deg. and Reclining from the Zenith 20 deg.*

This Plain in the F. Diagram, is represented by the Circle C M D, and passeth between the Pole and the Zenith, whole Pole is at ♈: And before the Hour-Lines can be described, four Requisites must be found, *viz.*



1. The Dist. of the Merid. from the Horizon O B.
2. The Height of the Pole, (or Stile above the Plain P R.)
3. The Deflexion, or Substile's Dist. from the Merid.
4. The Plain's Difference of Longitude O P R.

In this Scheme, the Circle A H B represents the Declining Reclining Plain, Q the Pole thereof: and P Q R, the Arch of a great Circle passing through P the Pole of the World, and Q the Pole of the Plain, whose Pole is at *a*; and by the Intersection of these Circles, and those of the Sphere, there are constituted several Spherical Triangles, whereby the four forementioned Requisites may be found.

I. *By Trigonometrical Calculation.*

The first is the right Angled △, Z H O, right Angled at H, in which there is given, (1) the Angle H Z O, the Plain's Declination 30 deg. (2) the side H Z, the Plain's Reclination 20 deg. to find the side H O. As

As the *Radius* Z C 90 deg. —— —— —— 10.00000
Is to the Sine of Z H, 20 deg. the Reclination, —— —— 9.53405
So is the Tangent of C S, 30 deg. the Declination, —— —— 9.76144
To the Tangent of H O, 11 deg. 10 min. —— —— —— *r*9.29549

Whose Complement 78 deg. 50 min. is O B, the distance of the Meridian from the Horizon.

In the same Triangle H Z O, you have given, (1) the Angle H Z O 30 deg. the Declination; (2) the side H O, 11 deg. 10 min. to find the side Z O.

As the Sine of H Z O, the Declination 30 deg. —— —— 9.69897
Is to the Sine of the side H O, 11 deg. 10 min. —— —— 9.28705
So is the Sine of Z H O 90 deg. —— —— 10.
To the Sine of Z O, 22 deg. 37 min. —— —— 9.58808

Which taken out of Z P, 38 deg. 28 min. there remains P O, 15 deg. 41 min.

And now we have another vertical and oppositе right Angled Spherical Triangle, O R P; the Sines of whose *Hypotenuses* and Perpendiculars, are proportional, use may be made of both the Triangles together, to find P R: For,

As the Sine of the *Hypotenuse* Z O, Co-Ar. —— —— 0.41200
Is to the Sine of the Perpendicular Z H, 20 deg. —— 9.53405
So is the Sine of the *Hypotenuse* P O, 15 deg. 41 min. —— 9.43188
To the Sine of the Perpendicular P R, 13 deg. 49 min. —— —— *r*9.37793

Which 13 deg. 49 min. is P R, the Height of the Pole, or Stile above the Plain.

Again, In the same Triangle P R O, there is now given, (1) the side P R, 13 deg. 49 min. (2) and P O, 15 deg. 41 min. to find the side O R.

As the Co-Sine of P R, 76 deg. 11 min. —— —— 9.98725
Is to the *Radius*, P R O, 90 deg. —— —— 10.
So is the Co-Sine of P O, 74 deg. 19 min. —— —— 19.98752
To the Co-Sine of O R, 82 deg. 30 min. —— —— 9.99627

Whose Complement 7 deg. 30 min. is the distance of the Deflexion, or Substile from the Meridian.

Lastly, In the same Triangle P R O, there are all the three sides given, and the right Angle at R, to find the Angle O P R, the Plain's Difference of Longitude.

As the Sine of the side P O, 15 deg. 41 min. the Stile's Height, 9.43188
Is to the Sine of the Angle P R O, 90 deg. —— —— 10.
So is the Sine of the side R O, 7 deg. 30 min. —— —— 19.11569
To the Sine of the Angle O P R, 28 deg. 52 min. —— —— 9.68381

Which is the Plain's Difference of Longitude.

The next thing you are to seek, is the Hour-Distances upon the Plain.

The Plain's Difference of Longitude being 28 deg. 52 min. which is but one hour, and 13 deg. 52 min. of the Equinoctial over; wherefore, the Plain declining *Eastward*, the Stile must stand between the Hours of 10 and 11 in the Forenoon: Therefore prepare a Table, and write down the Hours that the Plain is capable to receive; namely, from 5 in the Morning, till 4 in the Afternoon, and between the Hours of 10 and 11, where the Substile must stand, write the word Substile, and under it write 13 deg. 52 min. which subtract from 15 deg. the remainder will be 1 deg. 8 min. which write over Substile. Then to these two Numbers, by the continual addition of 15 deg. (one Equinoctial Hour's-distance) you shall produce such Numbers as are set down in the second Column of the Table. And now to find the true Hour-distances upon the Plain, make use of the accustomed Analogy.

Latitude		51 32
Declination		30 00
Reclination		20 00
Diff. Mer. & Hor.		78 50
Stile's Height		13 49
Deflexion		7 30
Differ. of Longit.		28 52

Hours from Noon.		Equinoctial Distances.		True H. Distances.	
		De.	Mi.	De.	Mi.
5	7	76	8	44	3
	6	61	8	23	25
7	5	46	8	13	57
8	4	31	8	8	12
9	3	16	8	3	57
10	2	1	8	0	16
The		Sub-		stile.	
11	1	13	52	3	22
	12	28	52	7	30
1	11	43	52	12	55
2	10	58	52	21	43
3	9	73	52	39	32
4	8	88	52	85	16

As the *Radius* 90 deg. —— —— ——	10.00000
Is to the Sine of the Stile's Height 13 deg. 49 min.	9.37806
So is the Tangent of 76 d. 8 m. the Equinoctial Dist. of 5 or 7 a Clock,	10.60755
To the Tangent of 44 deg. 3 min. the true Distance of the Hours of 5 or 7 upon the Plain from the Substile.	9.98561

And thus working for every Hour, you shall produce such numbers as are in the third Column of the Table, which with the Meridian, Substile, Stile, or Deflexion, being set upon your Plain, your Dial is finished.

II. *By Spherical Projection.*

1. For O B, a Ruler laid from Q to O, will cut the Primitive Circle at B 7 deg 50 min. from B, and that is the distance of the Meridian from the Horizon.

2. For P R, You must first find the Pole of the great Circle P R Q, which will be at *a*, a Ruler laid from *a*, to P and R, will cut the Primitive Circle in two points, the distance between which will be 13 deg. 49 min. for P R the Stile's Height.

3. For O R, A Ruler laid from Q to O and R, will cut the Primitive Circle in two points, the distance between which will be found to be 7 deg. 30 min. for O R, the Deflexion, or Substile's Distance from the Meridian.

4. For O P R, A Ruler laid from P to L, where the great Circle P R Q crosses the Equinoctial, will cut the Primitive Circle at 28 deg. 52 min. from N, for the Angle O P R, the Plain's Difference of Longitude.

5. For the Hour Distances upon the Plain, a Ruler laid upon *r*, which is the Pole of this Plain C M D, in the Fundamental Diagram, the Intersections of the several Hour Circles in that Diagram with the Plain, will give the true Hour Distances upon the Plain; and will be found such, (either from the Meridian of the Place, or the Meridian of the Plain,) as in the foregoing Table are exhibited.

III. *The Geometrical Protraction of this Dial.*

Fig. XVI. 1. Draw a right Line A C B, and upon it, assume C for the Centre of the Dial, and thereon with 60 deg. of the Chords, describe the Semicircle A F B; and (because the Plain declines *Eastward*,) take 78 deg. 50 min. the distance of the Meridian from the Horizon, from your Scale of Chords, and set them from A to E, and draw the Line C E for the Hour-Line of 12: Also take 7 deg. 30 min. of your Chords, and set them from E to D, and draw the pricked Line C D for the Substile. Again take 13 deg. 49 min. the Stile's Height from your Scale of Chords, and set them (either way) from D, as to F, and draw the Line C F for the Stile or *Axis* of your Dial. And lastly, Upon the Semicircle A F B, set off all the Hour-distances from D the Substile, as you find them in the foregoing Table; so Lines being drawn from C the Centre, through those points, shall be the true Hours proper for the declining Plain, and so is your Dial finished, as in Figure XVI.

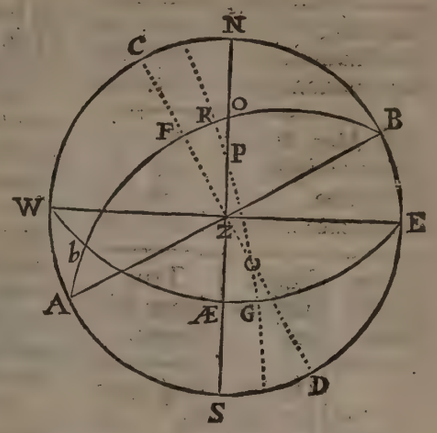
III. *The Third Variety shall be of a* South *Plain, declining* Eastward 30 *deg. and Reclining* 55 *deg.*

Such a Plain is represented in the Fundamental Diagram, by the Circle C R D, which passeth between the Pole and the Horizon, whose Pole is at *s*, and before the Hour-Lines can be drawn, these four Requisites must be obtained. *viz.*

1. The Distance of the Meridian and Horizon O B.
2. The Height of the Pole or Stile above the Plain P R.
3. The Distance of the Substile, and Meridian R O.
4. The Plain's Difference of Longitude R P O.

In

In this Scheme, the Circle A F R O B represents the Declining Reclining Plain, Q the Pole thereof, R P Q an Arch of a great Circle, passing through P the Pole of the World, and Q the Pole of the Plain, whose Pole is at *b*; now by the Interfections of these Circle, with the other great Circles of the Sphere, are constituted several *Spherical Triangles*; by the solution of which, the four forementioned *Requisites* may be found.



I. *By Trigonometrical Calculation.*

1. In the Triangle Z F O, right angled at F, there is given, (1) the side Z F, the Reclination, 55 deg. (2) the Angle Z F O, the Declination 30 deg. to find the side F O.

As the Sine of 90 Z C,	1.000000
Is to the Sine of F Z, the Reclination 55 deg.	9.91336
So is the Tangent of F Z O, the Declination 30 deg.	9.76144
To the Tangent of F O, 25 deg. 9 min.	19.67480

Whose Complement 64 deg. 41 min. is the Ark O B, the distance of the Substile from the Meridian.

2. In the right Angled Triangle B N O, there is given, (1) the side B O, 64 deg. 41 min. the Distance of the Meridian from the Horizon; (2) the Angle N B O 35 deg. the Complement of the Reclination, to find the side N O.

As the Sine of B N O, 90 deg. or *Radius*,	10.00000
Is to the Sine of B O, the dist. of the Meridian and Horizon, 64 d. 41 m.	9.95615
So is the Co-Sine of the Reclination N B O, 35 deg.	9.75859
To the Sine of the side N O, 31 deg. 14 min.	19.71474

Take N O, 31 deg. 41 min. out of N P, 51 deg. 32 min. the remainder will be 20 deg. 18 min. for P O.

3. The Sines of the *Hypotenuses*, and Perpendiculars of the Triangles R O P, and N O B, being proportional, you may deal with both the Triangles together, for the finding of R P: For,

As the Sine of the *Hypotenuse* B O, 64 deg. 41 min. *Co-Ar.*	0.04384
Is to the Sine of the *Hypetenuse* P O, 20 deg. 18 min.	0.54025
So is the Sine of the Perpendicular N B 60 deg.	9.93753
To the Sine of the Perpendicular R P, 19 deg. 25 min.	19.52162

Which 19 deg. 25 min. R P, is the Height of the Pole or Stile above the Plain.

4. In the Triangle R P O, you have given, (1) R P, 19 deg. 25 min. the Distance of the Substile from the Meridian; (2) P O, 20 deg. 18 min. to find R O.

As the Co-Sine of R P, 70 deg. 35 min.	9.97457
Is to the *Radius* P R O, 90 deg.	10.
So is the Co-Sine of P O, 69 deg. 42 min.	19.97215
To the Co-Sine of R O, 83 deg. 58 min.	9.99758

Whose Complement 6 deg. 2 min. is equal to R O, the Distance of the Substile from the Meridian.

5. In the same Triangle R P O, whose three sides are all given, as also the right Angle at R, by which you may find the Angle R P O, equal to Z P Q, the Plain's Difference of Longitude: For,

As the Sine of P O, 20 deg. 18 min.	9.54025
Is to *Radius*, the Angle O R P, 90 deg.	10.
So is the Sine of the side R O, 6 deg. 2 min.	19.02163
To the Sine of the Angle R P O, 17 deg. 38 min.	9.48138

Which 17 deg. 38 min. is the Plain's Difference of Longitude.

This

Latitude,	51	32
Declination,	30	00
Reclination,	55	00
Dist. Mer. & Hor.	64	41
Stile's Height,	19	25
Deflexion,	6	02
Difference of Long.	17	38

Hours from Noon.	Equin. Distances.	True Hour Distances.
	De. Mi.	Deg. Min.
5 7	87 22	82 7
6	72 22	46 17
7 5	57 22	27 26
8 4	42 22	16 52
9 3	27 22	9 46
10 2	12 22	4 10
The	Sub-	stile.
11 1	2 38	0 53
12	17 38	6 2
1 11	32 38	12 1
2 10	47 38	20 1
3 9	62 38	32 42
4 8	77 38	56 35

This 17 deg. 38 min. is but 1 Hour, and 2 deg. 38 min. of the Equinoctial over, it denotes (the Plain declining *Eastward*) that the Stile must stand between the Hours of 10 and 11, a small distance beyond 10: Wherefore prepare your Table, and set down all the Hours the Plain is capable to receive, as from 5 in the Morning, till 4 in the Afternoon; and between the Hours of 10 and 11, write Substile, and under it, the 2 deg. 38 min. remaining, which subtract from 15 deg. and the remainder 12 deg. 22 min. set over Substile, and by the continual addition of 15 deg. to these numbers above and below the word Substile, you shall produce such numbers as those in the Second Column of this Table are.

Now for the Hour-distances upon the Plain, they are found by the usual Canon, *viz.*

As the *Radius* 90 deg.	10.00000
To the Sine of the Stile's Height 19. deg. 25 min.	9.52171.
So is the Tang. of 42 d. 22 m. Equi. dist. for 8 and 4 a clock.	9.96002
To the Tangent of 16 d. 52 m. the true dist. of those Hours upon the Plain from the Subst.	9.48173

And so for all the rest of the numbers in the Table: And having found all the Requisites, you may by a Scale of Chords transfer them, and the Hour-distances also from this Table to your Plain.

II. *By the Spherical Projection.*

1. For O B, A Ruler laid from Q, the Pole of the Plain, to O, will cut the Primitive Circle at 64 deg. 41 min. from B, and that is the distance of the Meridian from the Horizon.

2. For P R, A Ruler laid from *b*, the Pole of the great Circle R P Q, to the Points R and P, it will cut the Primitive Circle in two Points, distant one from the other 19 deg. 25 min. for P R which is the Height of the Pole above the Plain.

3. For R O, A Ruler laid from Q to R and O, will cut the Primitive Circle in two Points, distant from each other 6 deg. 2 min. for R O, the Deflexion, or Substile's Distance from the Meridian.

4. For the Angle R P O, (or rather the Angle Æ P Q,) a Ruler laid from P to G, where the Arch of the great Circle R P Q crosses the Equinoctial, will cut the Primitive Circle, at 17 deg. 38 min. from S, which is the Plain's Difference of Longitude.

5. For the Hour-distances upon the Plain, A Ruler laid upon *s*, (which is the Pole of this Plain C R D in the Fundamental Diagram,) to the Intersections of the several Hour Circles with that Plain in the Fundamental Diagram, will give upon the Primitive Circle the true Hour-distances upon the Plain, and will be the same as in the foregoing Table.

III. *The Geometrical Protraction of this Dial.*

Towards the bottom of the Plain, (because the Centre must be downwards, and Stile and Hour-lines point upwards,) draw a right Line A C B, making choice of the Point C for the Centre of your Dial; upon which, with 60 deg. of your Chords, describe the Semicircle A D E B: Then from B, the *East* end of the Horizontal Line, set off 64 deg. 41 min. the distance of the Meridian and Horizon from B to I, and draw the Line C I for the Hour line of 12; then take 6 deg. 2 min. and set them from I, Westward (because the Plain declines Eastward) to E, and draw the Line C E for the Substile: Also take 19 deg. 25 min and set them from E, on either side, to D, and draw C D for the Stile of the Dial. And lastly, Upon the Semi-

circle

circle A D E B, set off all the Hour-distances, as you find them in the foregoing Table, from the Substile at D, so Lines drawn from C the Centre of the Dial, through those Points, shall be the Hour-lines proper to that Plain; and so is your Dial finished, as in Figure XVII.

CHAP. XIV.

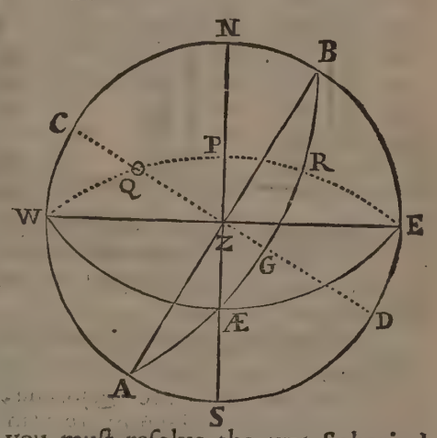
Of North *Declining Reclining Plains, how to find the several Requisites to them belonging, and the Hour-Distances proper to each Plain.*

OF *North* Declining Reclining Plains, there are the same Varieties as in South Recliners declining, namely, three; for to any Declination, a Reclination may be fitted; so that the Plain may pass by the Intersection of the *Meridian* with the Equinoctial — Or the Reclination may be such, that the Plain may pass between the *Zenith* and the Equinoctial,— Or it may recline so, that the Plain shall pass between the *Æquator* and the Horizon, of all which Examples follow.

I. *The first Variety: Of a* North *Plain Declining Westward 60 deg. and Reclining from the Zenith 32 deg. 11 min.*

In the Fundamental Diagram such a Plain is represented by the Circle A Æ B, the Pole whereof is at *t*; and the Plain it self passeth through Æ, at the Intersection of the Meridian with the Equinoctial Circle W Æ E.

In this Scheme the Circle B G A, represents the Declining Reclining Plain; Q the Pole thereof, W P E an Arch of a great Circle, passing through P the Pole of the World, and Q the Pole of the Plain; which in this Example comes to be the same with the Hour-Circle of 6, W P E, in the Fundamental Diagram, the Pole whereof is at Æ: So that in this Dial, the Hour-line of six, and the Substilar Line will be the same: And before the Hour-distances can be found, two other Requisites must be had, *viz.*



1. The Distance of the Meridian from the Horizon A Æ.

2. The Height of the Pole or Stile above the Plain P R: And for the finding of them, you must resolve the two Spherical Triangles Z G Æ, right Angled at G, and the Quadrantal Triangle P Æ R, made by the intersection of the several great Circles of the Sphere, and those relating to the Declining Plain.

I. *By Trigonometrical Calculation.*

In the right Angled Triangle Z G Æ, there is given (1) Z G, the Plain's Reclination 32 deg. 11 min. (2) the Angle Æ Z G, the Plain's Declination 60 deg. by which you may find Æ G.

As *Radius* 60 deg.	——	10.00000
To the Tangent of Æ Z G 60 deg. the Declination	——	10.23856
So is the Sine of Z G 32 deg. 11 min. the Reclination	——	9.72642
To the Tangent Æ G 42 deg. 42 min.	——	19.96498

Whose Complement 47 deg. 18 min. is the Distance of the Meridian from the Horizon.

II. *For*

II. *For* P R, *the Height of the Pole above the Plain.*

In the Quadrantal Triangle P Æ R.

As the Sine of Æ G 42 deg. 42 min. —— —— ——	9.83133
To *Radius* Æ R 90 deg. —— —— ——	10.00000
So is the Tangent of Z G 32 deg. 11 min. —— ——	19.79887
To the Tangent of R P 42 deg. 52 min. —— ——	9.96754

Which is the Height of the Pole, or Stile above the Plain.

3. and 4. For the other two *Requisites*, the Distance of the Substile from the Meridian R Æ, and the Plain's Difference of Longitude Q P Æ, or Æ P R, they are either of them 90 deg. as is evident in the Projection.

Latitude	15	32
Declination N.	60	00
Reclination	32	11
Dist. Mer. & Hor.	47	18
Stile's Height	42	52
Deflexion	90	00
Difference of Longit.	90	00

Hours from Noon.	Equinoctial Distances. Deg.	Min.	True Hours Distances. Deg.	Min.
6	0	0	0	00
7 5	15	00	10	20
8 4	30	00	21	27
9 3	45	00	34	14
10 2	60	00	49	41
11 1	75	00	68	30
12	90	00	90	0

And for the Hour-distances, they are calculated as the Hour-distances for an Horizontal Dial; wherefore prepare a Table as is here done, setting the Equinoctial Distances against their proper Hours; and then by the general Analogy or Proportion, say,

As *Radius* 90 deg. —— ——	10.00000
To the Sine of 42 d. 52 m. the Stile's Height.	9.83270
So is the Tangent of 15 deg. *&c.*	9.42805
To the Tangent of 10 d. 20 m.	19.26075

For the two first Hours from fix the Substile, of 5 and 7 on either side; and so for all the rest, as in the Table.

II. *By Spherical Projection.*

1. For A Æ, A Ruler laid from Q, the Pole of the Plain A G B, will cut the Primitive Circle at 47 deg. 18 min. from A, and that is the distance of the Meridian from the Horizon.

2. For P R, A Ruler laid from Æ, the Pole of the great Circle W P E, to R *v*, will cut the Primitive Circle at 42 deg. 52 min. from N; and that is the Height of the Pole, (or Stile) above the Plain.

3. For Æ R: A Ruler laid from Q to R and Æ, will cut the Primitive Circle in two Points, whose distance will be 90 deg. for the Substile's Distance from the Meridian.

4. For the Angle Q P Z, or Æ P R, either of them are 90 deg. for the Plain's Difference of Longitude: For it is evident from the Scheme, that all the sides of the Triangle Q Æ R are Quadrants, and consequently all the Angles right Angles.

5. For the Hour-distances, they may be found by applying a Ruler to *t*, the Pole of the Plain A Æ B in the Fundamental Diagram, and to the several Intersections of the Hour Circles therewith, and it will cut the Primitive Circle at such distances as are set down in the foregoing Table.

In this Dial, the Hour-line of 6 is the Substile; and the Meridian (or Line of 12) is distant from it 90 deg. so that they cut each other at right Angles, as in all direct Dials they do: And indeed this Dial is no other than an Horizontal Dial, for the Latitude of 42 deg. 52 min. if you call the Hour of 6, the Hour of 12, and so the rest in order.

III. *The Geometrical Protraction of this Dial.*

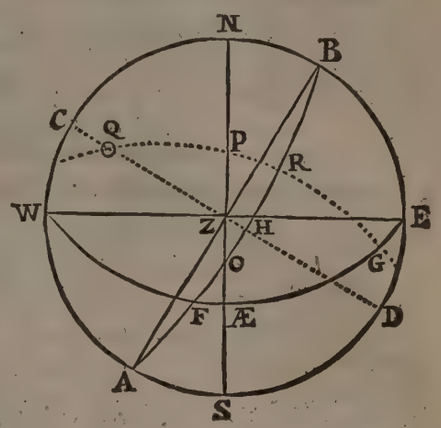
Fig. XVIII. About the middle of the Plain, draw an Horizontal Line A B, and upon any part thereof as C, with 60 deg. of your Chords, describe a Circle A E B D, upon which from A set off 47 deg. 16 min. the Distance of the Meridian from the Horizon, to D, and draw C D for the Hour-line of 12: Also take 90 deg. and set them from D to G, and draw the Line C G for the Substilar and Hour-line of six: Also take 42 deg. 52 min. and set them from G to E, and draw C E for the Stile or *Axis*: Lastly, by help of your Scale of Chords, set upon the Circle E G A D, the

the Hour-distances, as you find them in the Table, both ways from G; through which points, draw Lines from C; and so is your Dial finished, as in Fig. XVIII.

II. *The Second Variety, Of a North Plain, declining Westward 60 deg. and reclining from the Zenith 16 deg.*

This Plain in the Fundamental Diagram, is represented by the Circle B X A, and passeth between the Zenith and the Equinoctial, whose Pole is at ν. But,

In this Scheme it is represented by A O R B, intersecting the Meridian at O, between the Zenith and the Equinoctial, the Pole whereof is at Q, and Q P R is an Arch of a great Circle, passing through P the Pole of the World, and Q the Pole of the Plain, and cutteth the Plain at right Angles in R, the Pole of which Circle is at F: These Circles, by their Intersections with the Circles of the Sphere, do constitute several Spherical Triangles, whereby the *Requisites* belonging to this Plain may be found, and Four things must be known before the Hours can be drawn, *viz.*



1. The Distance of the Meridian from the Horizon A O.
2. The Height of the Pole or Stile above the Plain P R.
3. The Deflexion, or Distance of the Substile from the Meridian R O.
4. The Plain's Difference of Longitude O P R.

All which may be found as followeth:

I. *By Trigonometrical Calculation.*

1. For A O: In the right Angled Spherical Triangle Z H O, right Angled at H, there is given, (1) the side Z H the Reclination, 16 deg. (2) the Angle Z H O the Declination 60 deg. to find the side A O.

As the *Radius* 90 deg.	10.00000
Is to the Sine of Z H, the Reclination 16 deg.	9.44034
So is the Tangent of H Z O, the Declination 60 deg.	10.23856
To the Tangent of H O, 25 deg. 31 min.	9.67890

Whose Complement 64 deg. 29 min. is A O, the Distance of the Meridian and Horizon.

2. For P R: In the Triangle P R O, in which there is not yet enough given; wherefore the side P O, in the Triangle H O Z, must first be found, thus:

As the Sine of H Z O, the Declination 60 deg.	9.93753
Is to the *Radius*	10.
So is the Sine of H O, the Arch before found 25 deg. 31 min.	19.63425
To the Sine Z O, 29 deg. 50 min.	9.69672

Unto this Z O, 29 deg. 50 min. add Z P, 38 deg. 28 min. the Summ will be 68 deg. 18 min for the side P O.

And now the two Triangles O Z H, and R O P, being proportional; the side P R may be thus found.

As the Sine of the *Hypotenuse* O Z, 29 deg. 50 min. Co-Ar.	0.30322
Is to the Sine of the *Hypotenuse* O P 68 deg. 18 min.	9.96807
So is the Sine of the Perpendicular H Z, 16 deg.	9.44034
To the Sine of the Perpendicular P R, 30 deg. 59 min.	9.71163

Which 30 deg. 59 min. is the side P R, the Height of the Pole or Stile above the Plain.

3. For O R: In the two former Triangles; the Sines of their Bases, and Tangents of their Perpendiculars being proportional, OR may be found, thus:

As the Tangent of the Perpendicular Z H, 16 deg. Co-Ar. 10.54251
Is to the Tangent of the Perpendicular P R, 30 deg. 59 min. 9.77849
So is the Sine of the Base H O, 25 deg. 31 min. —— 9.63425
To the Sine of the Base R O, 64 deg. 26 min. —— 29.95525

Which 64 deg. 26 min. is the Deflexion, or Substile's Distance from the Meridian O R.

4. For the Angle O P R, that may be found in the same Triangle P R O: Thus,

As the Sine of P R, 30 deg. 59 min. —— —— 9.71163
Is to the *Radius* P R O, 90 deg. —— —— —— 10.
So is the Tangent of R O, 64 deg. 26 min. —— —— 20.32020
To the Tangent of O P R, 76 deg. 10 min. —— —— 10.60857

Which 76 deg. 10 min. is the Angle O P R, for the Plain's Difference of Longitude

5. For the Hour-distances: In this distance is contained, &c.

Latitude	51	32
Declination	60	00
Reclination	16	00
Dist. Mer. & Hor.	64	29
Stile's Height	30	59
Deflexion	64	26
Difference of Longitude	76	10

Hours from the Substile.		Equinoctial Distances.		Hour Distances from the Substile.	
		Deg.	Min.	Deg.	Min.
1	10	88	50	87	44
2	11	73	50	60	37
3	9	58	50	40	24
4	8	43	50	26	18
5	7	28	50	15	49
6	6	13	50	7	13
The		Sub-		stile.	
7	5	1	10	0	36
8	4	16	10	8	29
9	3	31	10	17	18
10	2	46	10	28	12
11	1	61	10	43	5
12	12	76	10	64	26

In this distance is contained five compleat Hours, and 1 deg. 10 min. more, wherefore the Stile must stand between the fifth and sixth Hours from the Meridian. Prepare therefore a Table as is here done, and write down all the Hours, and between the Hours of 5 and 6, or 6 and 7 (which are all one, according as the Plain declines Eastward or Westward) write Substile, and under it, the remainder 1 deg. 10 min. against 7 and 5, which subtract from 15 deg. and there will remain 13 deg. 50 min. which write over Substile against the Hours of 6 and 6. And so by the continual addition of 15 deg. to the numbers next above and below Substile, you shall produce such numbers as this Table affords in the second Column thereof; from which numbers, which are Equinoctial Distances, the true Hour-distances upon the Plain, may be collected by this Proportion.

As *Radius* 90 deg. —— —— —— 10.00000
To the Sine of 30 deg. 59 min. the Stile's Height —— 9.71163
So is the Tangent of 88 deg. 50 min. the first Equinoctial distance, 11.69112
To the Tangent of 87 deg. 44 min. —— —— 11.40275

Which is the true Hour-distance of the Hours of 1 and 11, upon the Plain from the Substile; and so doing for all the rest, you shall produce such numbers as the third Column of the Table affordeth, which may be transferred from this Table, by help of a Scale of Chords; and so is your Dial finished.

II. *By Spherical Projection.*

1. For A O; A Ruler laid from Q, the Pole of the Plain to O, the Intersection of the reclining Plain with the Meridian; it will cut the Primitive Circle at 64 deg. 29 min. from A, and that is the distance of the Meridian and Horizon.

2. For P R: A Ruler laid upon F, the Pole of the great Circle Q P R, to P and R, will cut the Primitive Circle in two Points distant from each other 30 deg. 59 min. and that is the Height of the Pole or Stile above the Plain.

3. For

3. For R O: A Ruler laid to Q, and the Points R and O will cut the Primitive Circle in two points distant from each other 64 deg. 26 min. and such is the distance of the Substile from the Meridian.

4. For the Angle O P R: A Ruler laid from P the Pole of the World, to G, where the Arch of the great Circle Q P R crosseth the Equinoctial Circle W Æ E; it will cut the Primitive Circle at 76 deg. 10 min. distant from S; and that gives the Plain's Difference of Longitude.

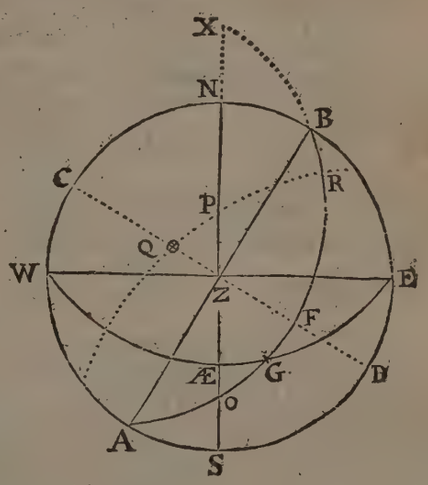
5. For the Hour-distances upon the Plain: A Ruler laid upon *v*; (which is the Pole of this Plain B X A, in the Fundamental Diagram) to the several Intersections; where the Hour Circles there do cut the Reclining Plain, it will upon the Primitive Circle give such Distances as are exhibited in the third Column of the foregoing Table.

III. *The Geometrical Protraction of this Dial.*

About the middle of the Dial Plain, draw an Horizontal Line A B, and towards the end thereof as at C, assume a point for the Centre of the Dial, and upon C describe part of a Circle D E F, upon which, from A, set 64 deg. 29 min. the distance of the Meridian and Horizon; and draw C D for the Hour-line of 12, through the Centre, to D above: Also take 64 deg. 26 min. the distance of the *Substile* from the Meridian, and set them from D above to E, and draw C E for the Substile. Then take 30 deg. 59 min. the Height of the Pole or Stile above the Plain, and set them from E to F, and draw C F for the *Axis*. Lastly; Take the Hour-distances as you find them in the third Column of the Table, and set them off upon the Circle from E the Substile, and through the several Points draw Lines from C the Centre: So is your Dial finished, as in Figure XIX. Fig. XIX.

III. *The Third Variety of a North Plain, declining Eastward 60 deg. and reclining from the Zenith 54 deg.*

A Plain according to this Declination and Reclination, is represented in the Fundamental Diagram, by the Circle B T A, whose Pole is at *x*. But in this Scheme it is represented by the Arch of a great Circle A O F R B, whose Pole is at Q, and intersects the Meridian at O; the Plain passing between the Horizon and the Equinoctial; and Q P R is the Arch of another great Circle, passing through P the Pole of the World, and Q the Pole of the Plain; by the Intersections of his great Circular Arch, and the Circles of the Sphere are constituted several Spherical Triangles; by the resolution of which, the four *Requisites* belonging to this Plain, before the Hour-lines can be drawn, may be found; and they are



1. The Distance of the Meridian and Horizon A O.
2. The Height of the Pole or Stile above the Plain P R.
3. The Deflexion, or Substile's Distance from the Meridian O R, or R X.
4. The Plain's Difference of Longitude O P R, or X P R.

And these *Requisites* may be found as followeth,

I. *By Trigonometrical Calculation.*

1. For A O: In the Triangle Z F O, there is given Z F, the Reclination Z F, 54 deg. (2) O Z F, the Plain's Declination, with the right Angle at F, to find F O.

As the *Radius* Z F O, 90 deg. —— —— ——	10.00000
Is to the Sine of Z F, the Reclination 54 deg. —— ——	9.90795
So is the Tangent of O Z F, the Declination 60 deg. —— ——	10.23856
To the Tangent of F O, 54 deg. 29 min. —— ——	10.14651

Whose Complement, A O, 35 d. 31 m. is the Distance of the Mer. from the Horizon.

2. For R O: In the Triangle O P R, there is not yet enough given; therefore find the side Z O in the Triangle O Z F. Thus,

As the Sine of O Z F, the Declination, 60 deg. — — 9.93753
Is to the *Radius* 90 deg. — — — — 10.
So is the Sine of O F, the distance of the Merid. and Horiz. 54 d. 29 m. 19.91059
To the Sine of Z O, 70 deg. 2 min. — — — 9.97306

To this 70 deg. 2 min. add Z P, 38 deg. 28 min. the Summ is 108 deg. 30 min. whose Complement to 180 deg. is P X, 71 deg. 30 min.

Then in the two Triangles, O Z F and O P R, whose *Hypotenuses* and Perpendiculars are proportional.

As the Sine of the *Hypotenuse* O Z, 70 deg. 2 min. Co-Ar. — 0.02694
Is to the Sine of the *Hypotenuse* P X, 71 deg. 30 min. — — 9.97695
So is the Sine of the Perpendicular Z F, 54 deg. — — 9.90796
To the Sine of the Perpendicular P R, 54 deg. 43 min. — 19.91185

Which 54 deg. 43 min. is the Height of the Pole, or Stile above the Plain.

3. For R O, or its Complement to 180 deg. R X, if you continue the sides O P, and O R to X, you will constitute another Spherical Triangle X B P, in which you are to find the Base X R. Thus:

As the Tangent of the Perpendicular Z F, 54 deg. Co-Ar. 9.86126
Is to the Tangent of the Perpendicular P R, 54 deg. 43 min. 10.15021
So is the Sine of the Base F O, 54 deg. 29 min. — — 9.91059
To the Sine of the Base R X, 56 deg. 42 min. — — 9.92206

Which 56 deg. 42 min. is the Deflexion, or Substile's Distance from the North part of the Meridian; whose Complement to 180 deg. is 123 deg. 18 min. its distance from the South part thereof.

4. For the Angle O P R, or its Complement to 180 deg. X P R.

As the Sine of P R the Stile's Height, 54 deg. 43 min. — — 9.91184
Is to the Sine of P R X, *Radius* 90 deg. — — — 10.
So is the Tangent of R X, the Substile's Distance 56 deg. 42 min. 20.18251
To the Tangent of R P X, 61 deg. 48 min. — — — — 10.27067

Latitude	51	32
Declination	60	00
Reclination	54	00
Dist. Mer. & Hor.	35	31
Stile's Height	54	49
Deflexion	123	19
	56	41
Differ. of Long.	118	13
	61	74

Hours from the Substile.	Equinoctial Distances.		Ho. Dist. from the Substile.	
	Deg.	Min.	Deg.	Min.
11 1	76	47	73	57
12	61	47	56	41
1 11	46	47	40	59
2 10	32	47	26	50
3 9	16	47	13	50
4 8	1	47	1	27
The	Sub-		stile.	
5 7	13	13	10	51
6 6	28	13	23	39
7 5	43	13	37	29
8 4	58	13	52	48
9 3	73	13	69	43
10 2	88	13	87	49

Which is the Plain's Difference of Longitude, R P X being 61 deg. 48 min. is the Angle counted from the North, therefore the Angle O P R, the Complement to 180 deg. is 118 deg. 13 min. the Plain's Difference of Longitude reckon'd from the South.

These things prepared, I proceed to make the Table for the Hour Distances from the Substile, wherein considering that the Angle P, the Plain's Difference of Longitude, is 118 deg. 12 min. reckon'd from the South, whereof 90 deg. is answerable to 6 Hours, and 105 to 7 Hours, and there is yet remaining 13 d. 13 m. it is evident, for the Substile must be drawn between the 7th and 8th Hour from the South part of the Meridian, or between the 4th or 5th Hour, reckoned from the North part.

Wherefore set the Hours down as in the Table, and between the Hours of 4 or 8, and 5 or 7, write Substile, setting the remainder 31 deg. 13 min. over Substile, and by the continual addition of 15 deg. to both these numbers, you shall produce the Equinoctial Distances, as in the Second Column of the Table. And those by the general Canon, will help you to the true Hour-distances upon your Plain. For,

As

As *Radius* 90 deg. —— —— —— ——	10.00000
To the Sine of 54 deg. 43 min. the Stile's Height.	9.91185
So is the Tangent of 28 d. 13 m. the Equinoctial Distance for 6 a Clock,	9.72963
To the Tangent of 23 deg. 39 min. —— ——	9.64148

Which is the true distance of the 6 a Clock Hour from the Substile of the Dial.

And thus continuing this proportion through all the Equinoctial Distances, you will in the end have such numbers to every Hour as the third Column of the Table sheweth, which are the true Hour-distances counted from the Substile upon your Plain.

The Meridian, Horizon, Substile, and Hour-distances, may be transferred from this Table to the Dial Plain at pleasure, by help of a large Cord and Beam Compasses.

II. *By Spherical Projection.*

1. For A O: A Ruler laid from Q, the Pole of the Plain to O; it will cut the Primitive Circle at 35 deg. 31 min. from S, which is the distance of the Meridian from the Horizon.

2. For P R: A Ruler laid from G, the Pole of the Arch of the great Circle Q P R, to the points P and R; it will cut the Primitive Circle in two Points, the distance between which will be 54 deg. 43 min. for the Height of the Pole, or Stile above the Substile.

3. For O R: A Ruler laid from Q, to O and R, will cut the Primitive Circle in two Points, whose distance will contain 123 deg. 18 min. the Substile's Distance from the *South* part of the Meridian; whose Complement to 180 deg. is 56 deg. 42 min. its Distance from the *North* part thereof.

4. For the Angle O P R: A Ruler laid from P the Pole of the World, to R the Intersection of the reclining Plain, with the Equinoctial Circle, will cut the Primitive Circle at 118 deg. 13 min. from S, which is the Plain's Difference of Longitude.

For the Hour-distances upon the Plain: A Ruler laid x, (which is the Pole of this Plain B T A, in the Fundamental Diagram,) to the several Intersections of the Hour Circles with the reclining Plain; it will give such Distances upon the Primitive Circle, as are in the third Column of the foregoing Table.

III. *The Geometrical Protraction of this Dial.*

First, Draw the Horizontal Line B C A, about the middle of the Plain: In any part thereof, as at C, place the Centre, and with 60 deg. of the Chords, describe the Circle A D F B, and from the West point thereof, set 35 deg. 31 min. the Distance of the Meridian from the Horizon, from A to O, and draw the line O C G, for the Hour-line of 12. Fig. XX.

Secondly, From G, the North part of the Meridian, set the Distance of the Substile from the Meridian from G to D, and draw the Line C D for the Substilar Line of the Dial.

Thirdly, From D to F, set 54 deg. 43 min the height of the Stile above the Substile, and draw the Line C F for the Stile.

Lastly, For the Hour-distances, they must be taken out of the Table, and by the Chords set upon the Circle from D the Substilar; upon which the Stile or Axis C F being erected perpendicularly above the Substile, your Dial is finished, as in Fig. XX.

And thus have you in this one (and in all the six Varieties of North and South Reclining Decliners) Dial, (as hath been often before intimated) made Four several Dials; *viz.* This for one; and his opposite South declining Easterly 60 deg. and inclining to the Horizon 36 deg. (which is the Complement of this Reclination.) Also, a North Declining Easterly 60 deg. and Reclining 54 deg. And his opposite, South Declining Westerly 60 deg. and Inclining to the Horizon 36 deg. only by drawing the Hour-Lines, Stile and Substile, quite through the Centre; and turning the Dial up and down, Stile and all; and changing the Figures of the Hours for Forenoon or Afternoon; according to the Nature of the Plain; and as Reason it self will sufficiently direct; and as is already done in an upright Declining Dial in Figure VI. and in an East and West Reclining Dial Figure VIII. Figure VI. Fig. VIII.

DIAL-

DIALLING

Geometrical and Instrumental.

PARTS II, and III.

THESE Two Parts I shall unite in one, and that for this Reason; The Geometrical Way may be performed by the Circular Scales of Natural Sines, Tangents, and Chords, set upon the Ruler, whose Construction is shewed in Book 3. Part 2. Page 396. of Spherical Projection, and therefore I shall say nothing of their Construction here; for that the same Scales are inscribed upon the Edges of the Quadrant following, by which I intend to shew how, Instrumentally, to make Dials upon all sorts of Plains. As for the finding of the Situation of any Plain, in respect of Declination, Reclination, or both; I refer you to Book 6. Part I. Chap. 1. and 2. hereof: And therefore the Work of this double Part shall be to shew how to describe Hour-lines, and to find all other Requisites belonging to any Dial-Plain in any Latitude: And that,

I. *Geometrically*, By the Scales inscribed upon the Sides of the Quadrant: And,

II. *Instrumentally*, By the Scales and Circles delineated upon the Superficies (or Plain) of the Quadrant.

CHAP. I.

A Description of the Quadrant.

1. THE Limb of the Quadrant is divided into 90 equal Parts or Degrees, according to the usual manner, and sub-divided into as many equal Parts, as the bigness thereof will permit; and numbred by 10, 20, 30, *&c.* to 90 degrees.

2. On that Edge of the Quadrant which is next to the Left hand, where the degrees of the Limb begin to be numbred, is a Scale of Natural Tangents, the whole Scale containing 63 deg. 27 min. *fere*. Numbred by 10, 20, 30, *&c.* to 63 deg. 27′.

3. On the other Edge of the Quadrant, towards the Right hand, is a Scale of Natural Sines, Numbred from the Centre downwards by 10, 20, 30, *&c.* to 90 deg. equal to 45 deg. of the Tangents. At the end of the Scale of Sines, there is a Line of Tangents, of 45 degrees, numbred by hours, halves, and Quarters; and is called the Scale of Three Hours.

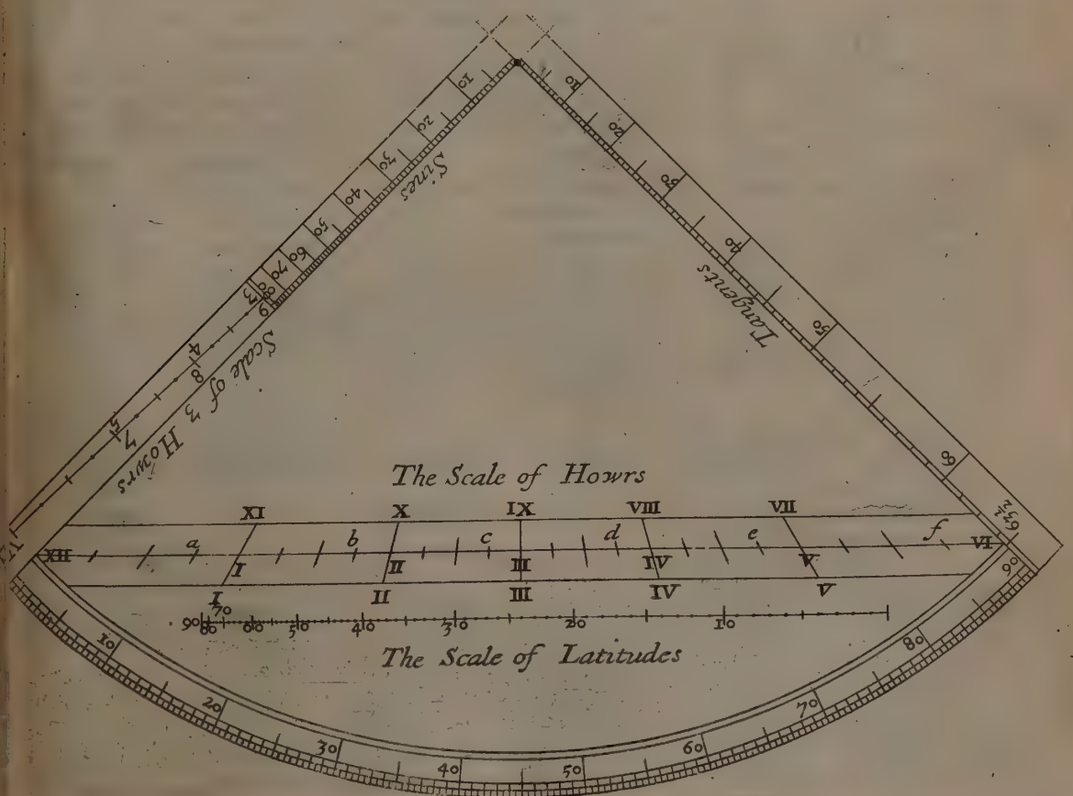
These are the Scales which are inscribed upon the two Edges or Sides of the Quadrant; upon the Superficies thereof is,

1. A Line

1. A Line of Hours divided into six unequal parts, and numbered with numerical Letters, by XII—I. XI—II. X—III. IX—IV. VIII—V. VI and VI at the other end.
2. A Line or Scale of Latitudes, divided into 90 unequal parts or degrees; by which the Latitude of the Place, or the height of the Pole (or Stile) above any Plain, is to be taken; which Line is to be divided according to the Numbers in this Table.

A Table for the dividing of the Scale of Latitudes.

1	247	17	3969	33	6764	49	8519	65	9496
2	493	18	4176	34	6900	50	8600	66	9539
3	739	19	4378	35	7036	51	8678	67	9578
4	984	20	4577	36	7186	52	8753	68	9615
5	1228	21	4772	37	7292	53	8825	69	9651
6	1470	22	4961	39	7414	54	8895	70	9685
7	1711	23	5146	38	7532	55	8962	72	9745
8	1949	24	5328	40	7647	56	9016	74	9801
9	2186	25	5505	41	7758	57	9088	75	9825
10	2419	26	5678	42	7865	58	9147	76	9846
11	2650	27	5846	43	7968	59	9203	78	9888
12	2879	28	6010	44	8068	60	9258	80	9924
13	3104	29	6169	45	8165	61	9311	85	9982
14	3325	30	6325	46	8259	12	9360	90	10000
15	3543	31	6475	47	8348	63	9408		
16	3758	32	6622	48	8436	64	9454		



CHAP. II.

To make an Horizontal Dial in any Latitude.

I. *Geometrically, By the Lines on the Sides of the Quadrant.*

Fig. I. LET our Example be of an Horizontal Dial in the Latitude of 41 degrees. First draw a right Line A B C, for the Hour-line of Six; and in that Line assure C for the Centre of your Dial. Also draw the Line C D perpendicular thereunto, for the Hour-line of 12. Then, out of the Line of Sines, take 41 deg. the Latitude, and set it from C, both ways, to A and B. Then take the length of the whole Line of Sines (*i. e.*) from 0 deg. to 90 deg.) and set it upon the Meridian (or Hour-line of 12) from C to D; and draw the Lines A D and B D.—Then, take in your Compasses the length of the Line A D or B D; and setting one foot of the Compasses in the Point VI. of the Line of Three Hours, turn the other foot of the Compasses about, till it do only touch the Thred, (which is fixed in the Centre of the Quadrant) and there keep the Thred: (which may be done by a bit of Wax, a Screw, or a Spring fitted for that purpose. Then, set one foot of the Compasses in the Point V and VII of the Line of Three Hours, and open the other till it do but only touch the Thred; and set that distance from D to 7, from D to 5, from A to 1, and from A to 11.—Also, set one foot of the Compasses in the Point IV and VIII of the Line of Three Hours; and open the other to the nearest distance to the Thred; and set that distance from D to 4, from D to 8; from A to 2, and from B to 10.—Again, set one foot of the Compasses in the Point IX and III of the Line of Three Hours, and open the other till it do only touch the Thred, and set that distance from D to 3, and from D to 9; and (if you have committed no former Error in your Operation) this distance shall divide the Lines A D and B D into two equal parts in the Points 3 and 9.—Lastly, If from the Centre C, you draw right Lines through the Points 1, 2, 3, *&c.* 11, 10, 9, *&c.* they shall be the true Hour-distances of an Horizontal Dial for the Latitude of 41 deg. and will be such, if taken out of a Scale of Chords, and set upon a Semicircle from the Hour-line of 12 both ways; such as are exhibited in this little Table in the Margin. And this manner of working is general for all direct Plains beholding directly the North or South Points, whether they be Erect, Reclining or Inclining; and therefore by it may be made all the Dials whose Construction are taught in the 5, 6, 11, and 12 Chapters of the First Part hereof: And therefore I shall not give Examples of any of them in this Place. Only remember this, that the Stile must always be elevated above the Substile, (which in all these Plains is C D, the Hour-line of 12) so much as the Latitude is, as in this Example 41 deg. And, as the whole Hours are pricked on, in the same manner may the Halves and Quarters, by taking the nearest distance to the Thred, from the several half and quarter Points in the Line of Three Hours.

Hours.		*Distances upon the Plain.*	
		De.	Mi.
12		00	00
11	1	9	57
10	2	20	41
9	3	33	16
8	4	48	39
7	5	67	47
6		90	00

II. *Instrumentally, By the Lines and Arches delineated upon the Plain of the Quadrant.*

First (as before) draw the Line A B for the Line of VI, and, at right Angles thereto, C D, for the Hour line of 12.—Then, out of the Line of Latitudes, (from the beginning of it) take the Latitude 41 deg. and set it both ways from C, to A and B.—Also out of the Quadrant take the whole Line of Hours, and setting one foot of the compasses in A, with the other describe the obscure Arch *b b*; and in B, and describe the Arch *a a*, crossing each other in D.—Then take in your Compasses (from the Line of Hours the Distance from XII) to V and VII, and set it upon the Lines D A and D B, from D, to 5 and 7.— Also take the Distance from XII to VIII and IV, and set it from D to 4, and from D to 8.— Also take the Distance from XII,

to

to III and IX, and set it from D to 3 and 9; then take the Distance from XII to II and X, and set it from D to 2 and 10.— Also take the Distance from XII to I and XI, and set it from D, to 1 and 11.— Lastly, Lines drawn from the Centre C, through these Points shall be the true Hour Lines for an Horizontal Plain, in the Latitude of 41 deg. And by this one general way of working, you may make any direct South or North reclining inclining Dial; but in South reclining Plains, observe (1,) if the Reclination of the Plain be equal to the Complement of the Latitude, the Plain is a Polar Plain, lying parallel to the *Axis* of the World, and must be made by the directions in Chap. 11. Sect. 9. Part 1 hereof, Fig. IX. (2,) if the Plain recline more than the Complement of the Latitude; then the North Pole is elevated above the Plain, so much as the Reclination is more than the Complement of the Latitude, and must be made, as in Chap. 11. Sect. 3. Part 1. hereof, Fig. XI. (3,) if the Plain recline less than the Complement of the Latitude, the South Pole is elevated so much above the Plain, as is the Difference between the Complement of the Latitude, and the Plain's Declination; and must be made as in Chap. 11. Sect. 2. Part 1. hereof, Fig. X. Also in North recliners; (1,) if the Plain recline equal to the Latitude, then the North Pole is elevated 90 deg. above the Plain; and the Dial must be made as in Chap. 12. Sect. 1. Part 1. hereof, Fig. XII. (2,) if the Plain recline less than the Latitude, the Reclination added to the Complement of the Latitude, gives the Height of the North Pole above the Plain, and must be made as in Chap. 12. Sect. 2. Part 1. hereof, Fig. XIII, (3,) if the Plain recline more than the Latitude, then the Reclination added to the Complement of the Latitude, gives the Height of the Pole above the Plain; but if the Sun exceed 90 deg. subtract it from 180 deg. and the remainder is the Height of the North Pole, and may be made by Chap. 12. Sect. 3. Part 1. hereof, Fig. XIV.

CHAP. III.

How to make an upright South or North Dial, declining East or West in any Latitude.

I. *Geometrically, By the Lines on the sides of the Quadrant.*

LET it be required to make an upright Dial, declining from the South, Eastward, 20 deg. in the Latitude of 48 deg. Fig. II.

First, Draw a right Line A B, for the Horizontal Line of the Plain, and a Line C D, Perpendicular thereunto for the Meridian, and Hour-line of 12.

Secondly, Out of the Scale of Tangents, take the Tangent of the Latitude 48 deg. and set it from C to G, and the Tangent Complement of the Latitude 42 deg. and set it from C to E, and make the parallellogram C E F G, on the left hand of the Hour-line of 12, (because the Plain declineth Eastward.)

Thirdly, Out of the Scale of Sines, take 20 deg. the Declination of the Plain, and set it from G to M; and from E to P, drawing the Line C M for the Substile, and C P for the Hour-line of 6.

Fourthly, Take 70 deg. the Complement of the Plain's Declination, and set it from C to K, and from M to N, setting M N Perpendicular to M C.

Fifthly, Take the Sine of the Complement of the Latitude 42 deg. and set it from C to H, and draw H O, parallel to E F or C G, till it cut the Hour-line of 6 in O; then make C L equal in C O, and draw the Lines O K and L K, constituting the Triangle O K L.

Sixthly, Take in your Compasses, the Line K O, and setting one foot in the Point 6, of the Line of three Hours, turn the other foot about, till it do only touch the Thred which is in the Centre, and there keep the Thred.— Then setting one foot in the Point V and VII, take the neareſt Distance to the Thred, and set that Distance upon your Dial Plain; from K to 7, and from O to 11.— Also take the neareſt distance from the Point VIII and IV, to the thred; and set it upon the Plain, from K to 8,

and from O to 10.— Then take the Distance from the Point III and IX, to the thred, and set it from K or O to 9; for this Distance will divide the Line O K into two equal parts in 9.— Then take in your Compasses the Line K L, and setting one foot in the Point VI of the Line of three hours, bring the Thred to the neareſt Distance, and there keep it; and take the neareſt Distance from the Point V and VII, to the thred, which set from K to 5, and from L to 1.— Also take the neareſt Distance from the Point IV and VIII to the thred, and set it from K to 4, and from L to 2. Also take the neareſt Distance from the Point III and IX, to the thred, and set it from K or L to 9, for this Distance will divide the Line K L into two equal parts, also in the point 3.

Lastly, If you lay a Ruler to C, and draw Lines through the several Points 7, 8, 9, 10, 11, and 1, 2, 3, 4, 5, they shall be the Hour-lines proper for an upright Plain in the Latitude of 48 deg. declining from the South Eastward 20 deg.

II. *Instumentally, by the Lines and Arches described upon the Quadrant.*

Before this, (or any other) declining Dial can be made, three things must be first found.

1. The Deflexion, or Substile's distance from the Hour-line of 12.
2. The Height of the Stile above the Substile.
3. The Plain's Difference of Longitude.

And all these may be found by help of the Limb, and Scale of Tangents on the side of the Quadrant, and the thred, Thus:

I. *For the Deflexion, or Substile's Distance from 12.*

Set one foot of the Compasses upon the Sine of the Plain's Declination, (in this Example 20 deg.) and extend the other to the Centre of the Quadrant: The Compasses being open to this Distance; set one foot in the Tangent of 45 deg. and bring the thred to the neareſt Distance, and there keep it; then set one foot in the Tangent of the Complement of the Latitude, (in this Example 42 deg.) and take the neareſt Distance to the thred; this Distance measured upon the Tangent Scale from the Centre of the Quadrant, will reach to 17 deg. 7 min. and that is the Deflexion, or Substile's Distance from the Meridian.

II. *For the Height of the Stile above the Substile.*

Set one foot of the Compasses in the Sine Complement of the Latitude (in this Example 42 deg.) and extend the other to the Centre of the Quadrant; and with this Distance, set one foot in the Sine of 90 deg. bringing the thred to the neareſt Distance, and there keep it; then set one foot of the Compasses in the Sine of the Complement of the Plain's Declination, (in this Example 70 deg.) and with the other take the neareſt Distance to the Thred; this Distance measured upon the Scale of Sines, will reach from the Centre to 38 deg. 58 min. And that is the Height of the Stile above the Substile.

III. *For the Plain's Difference of Longitude.*

Out of the Scale of Sines, take the Sine of the Complement of the Latitude (in this Example 42 deg.) and setting one foot in the Sine of 90 deg. bring the thred to the neareſt Distance, and there keep it: Then take the Sine of the Deflexion (in this Example 17 deg. 7 min.) out of the Scale of Sines from the Centre, and setting one foot of the Compasses upon the Scale of Sines, move it along, till the other foot being turned about, will only touch the thred; and so doing, you shall find the Compass Point to rest upon the Sine of 26 deg. 6 min. and that is the Plain's Difference of Longitude.

IV. *For the Hour-distances.*

Fig. IV. First, draw a right Line A B, for the Meridian or Hour-line of 12, and assuming the Point A for the Centre, upon it with 60 deg. of your Scale of Chords describe an Arch B C, upon which, set the Deflexion, or Substile's Distance from the Meridian, 17 deg. 7 min. from B to C, and draw the Line A C for the Substile: Also take 38 deg. 58 min. and set them from C to D, and draw A D for the Stile: Then, Through the Centre A draw the Line R A T, perpendicular to the Substilar A B.

Secondly, Out of the Line of Latitudes, take 38 deg. 58 min. the Height of the Stile, and set them from A both ways upon the Line R A T, from A, to R and T;

Also

Also take the whole length of the Line of Hours in your Compasses, and setting one foot in R, with the other describe the Arch *c c*, and removing the Compass Point to T, with the other; describe the obscure Arch *d d*, crossing the other in the Point V then draw the two Lines R V and T V: so shall you have constituted the Equicrural Triangle R V T, upon the two equal sides of which, R V and T V, the Hour Points are to be pricked down, as followeth.

Thirdly, Having recourse to your Quadrant, and having found before that the Plain's Difference of Longitude is 26 deg. 6 min. lay the Thred to 26 deg. 6 min. counted in the Limb of the Quadrant, and it will cross the Line of Hours in the point *b*, near three quarters after XI; take with your Compasses the Distance from XII, to the poin twhere the Thred crosseth the Line of Hours, and that Distance shall reach from V to 12, and from R to 6, through which points draw the Lines A 12, and A 6 for the Hour-lines of XII and VI. Then for the rest of the Hour-distances, do as followeth.

The Plain's Difference of Longitude being 26 deg. 6 min from it subtract 15 deg. as often as you can, (which in this Example is but once) and there will remain 11 deg. 6 min. which being less than 15 deg. set down in the second Column of a little Table, as here, with the word Substile over it, and then, continually add 15 deg. thereto, and it will be 26 deg. 06 min. (equal to the Difference of Longitude,) against which set XII; then add 15 deg. thereto, and it makes 41 deg. 06 min. and so adding 15 continually, till you come as near 90 deg. as you can, you shall find them to be as in the Table, against which, set the Hours proper for the Plain, as you see here done.

Hours.	*Substile.*
XI V	11 06
XII	26 06
I VII	41 06
II VIII	56 06
III IX	71 06
IV X	86 06

The Table being thus prepared.

	Deg.	Min.									
Lay the Thred to	11	06	and it will cross the Line of Hours in	*a*	And the Distance from XII to	*a*	will reach upon the Plain from V, to	11	And from R, to	5
	26	06		*b*		*b*		12		6
	41	06		*c*		*c*		1		7
	56	06		*d*		*d*		2		8
	71	06		*e*		*e*		3		9
	86	06		*f*		*f*		4		10

Through which Points from A, Right Lines being drawn, as A 5, A 6, A 7, A 4, A 3, *&c.* they shall be the true Hour-lines proper for a South Plain, declining Eastward 20 deg. in the Latitude of 48 deg.

CHAP. IV.

How to make an East or West Reclining Dial.

THese Dials are no other than upright Declining Dials referred to a New Latitude: which New Latitude is no other than the Complement of that Latitude wherein your Plain is a reclining Plain: And the Declination in that New Latitude, is only the Complement of the Reclination.

Example. *Suppose that in the Latitude of 42 deg. an East or West Plain should Recline from the Zenith towards the Horizon 70 deg. In what Latitude will that be an upright Declining Plain?*

The Old	Latitude is 42 deg. 00 min.	Compl.	48 deg 00 min.	New Latitude.
	Reclination 70 deg. 00 min		20 deg. 00 min.	New Declin.

Wherefore such an East or West Plain, as in the Latitude of 42 deg. reclines 70 deg. shall be an Erect Plain, Declining East or West 20 d. in the Latitude of 48 d.

So that the Declining Dial before made, will serve for an East or West Dial, Reclining 70 deg. in the Latitude of 42 deg. Observing the Cautions given for placing of the Meridian Line or Hour-line of 12, which in all such Plains must lie parallel to the Horizon, whereas in Erect Plains it is perpendicular thereunto.

CHAP V.

Of Declining, Reclining, or Inclining Dials; how to describe Hour-Lines upon them.

THE best way whereby to find the Requisites, and to describe the Hour-lines upon these Plains, is (as in East and West Recliners) to refer them to a New Latitude, and a New Declination, wherein they will stand as upright Declining Plains; and so may be made by the Rules beforegoing, both Geometrically by the Scales on the Sides, and by the Lines and Arches on the Superficies of the Quadrant.

I. *Geometrically, by the Scales on the sides of the Quadrant.*

Let our Example be of a South Plain, Reclining from the Zenith Northwards 60 deg. and Declining towards the East 60 deg. in the Latitude of 52 deg. 30 min.

1. *To find the New Latitude.*

Fig. III. Upon Paper or Paistboard describe a Quadrant A B C, and upon B erect the Tangent-line B D; Then from your Scale of Sines, take 30 deg. the Complement of the Plains Declination, and with that distance describe the Quadrant H E G; also take the Tangent of the Complement of the Reclination, and set it from B to D and draw the Line A D, cutting the Quadrant A H G in E: The least distance taken from E, to the Line B A, being measured upon the Tangent Scale, will be found to be 16 deg. 6 min. equal to B K.—This Tangent thus found, 16 deg. 6 min. in South Recliners, (as this is) must be compared with the Old Latitude 52 deg. 30 min. and their difference 36 deg. 24 min. is the New Latitude.

2. *For the New Declination.*

Out of the Scale of Chords take 60 deg. the Plain's Reclination, and set it from B to O, and draw the Line A O. Also take the Sine of 60 deg. the Plain's Old Declination, and set it from A to N; the least distance taken from N, to the Line C A measured upon the Scale of Sines, will be found to be 25 deg. 39 min. for the New Declination.

And thus the New Declination, and New Latitude being found, if you make a Dial for that New Declination, and in that New Latitude, it will be a South Plain declining West 60 deg. in the Latitude of 52 deg. 30 min.

New { Latitude 36 deg. 24 min.
New { Declination 25 deg. 40 min South-West.

So that if according to the Rules delivered in the foregoing Sections hereof, if you make an upright Dial to decline 25 deg. 40 min. from the South, in the Latitude of 36 deg. 24 min. It shall serve for a South Plain, Declining 60 deg. and Reclining Northward 60 deg. in the Latitude of 52 deg. 30 min.

II. *Instrumentally, by the Scales and Arches described on the Superficies of the Quadrant.*

Our Example shall be of a South Plain, Declining Eastward 30 deg. and Reclining from the Zenith Northward 55 deg. in the Latitude of 51 deg. 32 min.

Before the Hours can be drawn, four Requisites must be found.

1. The Distance of the Meridian from the Horizon.
2. The Height of the Pole or Stile above the Plain.
3. The Deflexion or Substile's Distance from the Meridian.
4. The Plain's Difference of Longitude.

All which may be found by the Scales on the Sides of the Quadrant.

1. *For the Distance of the Substile from the Meridian.*

First, Out of the Scale of Sines take 55 deg. the Reclination, and setting one foot in the Tangent of 45 deg. bring the Thred to the neareſt distance, and there keep it. Then set one Foot in the Tangent of the Plain's Declination 30, and take the neareſt distance to the Thred; that distance measured upon the Tangents from the Centre, will reach to 25 deg. 19 min. the Complement whereof 64 deg. 41 min. is the Distance of the Meridian from the Horizon.

2. *For*

2. *For the Height of the Stile above the Substile.*

Take 64 deg. 41 min. the distance of the Meridian from the Horizon, out of the Line of Sines; and setting one foot in the Sine of 90 deg. bring the Thred to the nearest distance, and there keep it: Then set one foot in the Sine of 35 deg. (the Complement of the Plain's Reclination,) and take the nearest distance to the Thred; that distance will reach from the Centre to the Sine of 31 d. 14 m. which subtracted out of the Latitude 52 deg. 32 min. the remainder is 21 deg. 18 min. Then again, Take the Sine of 21 deg. 18 min. from the Scale of Sines, and setting one foot in the Sine of 64 deg. 41 min. the distance of the Merid. and Horizon, bring the Thred to the nearest distance, and there keep it. And setting one foot in 60 deg. take the nearest distance to the Thred, and that distance measured upon the Sines will reach from the Centre to 19 deg. 25 min. and that is the height of the Pole above the Plain.

3. *For the Deflexion or Substile's Distance from the Meridian.*

Out of the Scale of Sines, take 19 deg. 25 min. the Stile's height; and setting one foot in the Sine of 90, bring the Thred to the nearest distance, and there keep it; Then set one foot in 20 deg. 8 min. and take the nearest distance, which measured will reach from the Centre to 6 deg. 2 min. and that is the Deflexion.

4. *For the Difference of Longitude.*

Take out the Sine of 20 deg. 18 min. and setting one foot in the Sine of 90, bring the Thred to the nearest distance, and there keep it; then take out the Sine of 6 deg. 2 min. the Deflexion, and setting one foot upon the Scale of Sines, move it along the Line, and turn the other gently about, till it do only touch the Thred; so shall you find the Compass point to rest at 17 deg. 38 min. which is the Plain's Difference of Longitude.

5. *To find the Hour Distances.*

First draw a right Line A B, for the Horizontal Line of the Plain, and making choice for a Centre at C, upon C with 60 degrees of your Chords describe the Semi-circle A F B. Then the Distance of the Meridian from the Horizon being 64 deg. 41 min. set them from B to D, and draw C D for the Hour-line of 12.—Then the Distance of the Substile from the Meridian being 6 deg. 2 min. set them from D to E, and draw C E for the Substile. And the Stile's height being 19 deg. 25 min. set them from E F, and draw C F for the Stile or *Axis*.—Then through C draw a Line perpendicular to the Substile C E, as G H. These things done, Fig. V.

Out of the Line of Latitudes take 19 deg. 25 min. the height of the Stile, and set that distance from C to G, and from C to H: also take the whole Line of Hours out of the Quadrant; set one foot in G, and with the other describe the occult Arch o o; likewise set one foot in H, and with the other describe the Arch S S, crossing one another in the Substile in the Point H. And now the Plain's Difference of Longitude being 17 deg. 38 min. subtract from it 15 deg. as often as you can, which you may in this case doe only once, and then there will remain 2 deg. 38 min. Then make a Table, by adding of 15 deg. continually to 2 deg. 38 min. the Numbers will be such as this Table exhibits.—Your Table thus finished, repair to your Quadrant; And

Hours.		*Substile.*	
V	XI	2	38
VI	XII	17	38
VI	I	32	38
VIII	II	47	38
IX	III	62	38
X	IV	77	38

	De.	*Mi.*					
Laying the Thred upon	2	38	Note where the Thred cuts the Line of Hours, for the distance from XII, taken to the Thred, will give the Distances from H to	11	and from G, to	5
	17	38		12		6
	32	38		1		7
	47	38		2		8
	62	38		3		9
	77	38		4		10

Through which Point Lines drawn from the Centre C, shall be true Hour-lines for the Declining Reclining Plain.

DIAL-

DIALLING

Projective and Reflective.

PARTS IV, and V.

CHAP. I.

Of Projective Dials.

FOR to perform the Work of this and the following Chapters, it will be neceſſary to have a Semicircle divided into an 180 deg. by two Quadrants beginning the Numbers at the Semidiametre, by 00, 10, 20, 30, &c. degrees, both ways, to 90 deg. at the Diametre, into which Diametre, a Ruler muſt be ſo fitted by a Grove, Screw, or the like; that the Semicircle may move along the Ruler, and be ſtayed in any part thereof: And of theſe Rulers, it were neceſſary to have 2 or 3 of them of ſeveral lengths, or (inſtead of a Semicircle) a Quadrant, if it be divided on both ſides, and a Ruler fitted to it; it will ſupply the uſe of the Semicircle.

The uſe of this Semicircle (or Quadrant) in the general, is,

How, upon a Line drawn any where, to project any Altitude or Depreſſion, above or below the Horizon, from a fixed Point that ſtands at a diſtance from that Line.

The manner how this is to be done, is very eaſie: For, if you hold the Edge of the Ruler to the fixed Point; and alſo apply the Point of that Edge of the Ruler to the Line given, removing it higher or lower, (as occaſion requires) till the Thred hanging down by the ſide of the Semicircle at full liberty, do fall upon the degrees of Altitude (or Depreſſion) intended; for then doth the Ruler lie at the Altitude, or Depth, and ſo doth Project it from the fixed Point into the Line, as was required.

It will be neceſſary in many Caſes to ſlip the Semicircle higher or lower upon the Ruler, as alſo to change the ſides of the Ruler; but in this Diſcretion muſt be your Director.

CHAP. II.

A general and eaſie way to project Hour-lines upon all kinds of Superficies without any regard had to their ſtanding, either in reſpect of Declination or Inclination.

1. LET a Gnomon, being firſt ſharpned into a Point, be ſhaped, and faſtned in ſuch wiſe, that it no way hinder either the draught of the Horizontal Line, or the Point of the ſhadow, from having free acceſs to the Dial at all times of the Year.

2. Draw

2. Draw an Horizontal Line, by help of your Semicircle in a true Level both in regard of it self, and also to the Point of the Gnomon, through the whole Superficies on which the Dial is to be described. Or having two Points in the same Level with the Point of the Gnomon, project it upon your Superficies, if it be a rugged one. And if the Superficies be more than one, or if any of them be very much inclined toward the Horizon, or else be very rugged, or far remote from the Gnomon, so that it will not at all, or not so well, receive an Horizontal Line upon it, you may either set up some Board, or such like Object, upon which for a time you are to inscribe the Horizontal-line, and by help of which the Hours are to be projected upon the Superficies; or else (which perhaps will be better) you may extend a Thred in the Air, (it matters not which may, nor whether from the Gnomon towards the Sun, or from the Sun: whether stretcht out in one length, or with returns, so long as it lieth justly parallel, in every Point of it, to the Horizon, and in the same Level with the Point of the Gnomon:) which being fixed in this manner, will very well supply the use of the Horizontal-line: or the Horizontal-line may be partly Thred, and partly drown upon the Superficies, as occasion shall be. And upon it may any Point be transferred, and signed out by slipping knots of Thred tied upon it.

3. Upon the Superficies of the Dial, observe the Point of the Shadow of the Gnomon (making a mark at it) and the Sun's Altitude, both of them at the same instant of time.

4. By the Altitude observed, compute the Azimuth of the Sun from the Meridian.

5. The same Azimuth must be transferred unto, or projected upon, the Horizontal Line by help of a Perpendicular Thred, covering to your sight (as it hangeth down) the Points of the Gnomon and Shadow both together; and at the same view cutting through the Horizontal-line: observe then punctually where it cuts through the same Line, for that same Section being signed thereon, shall be the Azimuth projected into the Horizontal-Line.

6. Let any kind of board or past-board be now applied to the Point of the Gnomon; so, as that it may be staid, either upon the Horizontal-line, (where it may so be conveniently) or at least so placed toward the Horizontal-line, that it may have a just respect unto it, and in that posture may have some stay for the edge of it to rest upon, that after it is furnished with such necessary Lines as must be drawn upon it, it may be placed in its former just posture without any Impeachment. Upon this Plain so placed, let the Point of the Gnomon be signed, which may be called the Centre; and from this Centre, to the Sign of the Azimuth, before projected into the Horizontal Line, draw a right Line: this right Line so drawn, shall represent upon the board or past-board, the same Azimuth which was before computed.

7. Then taking away the same Plain, draw upon it the Meridian or Line of 12; extending it from the Centre before noted, as the true Angle that it hath from the Azimuth before computed and described, and also toward the true Coast of the World. And let it be extended on both sides the Centre, if need be.

8. To the Meridian so pitched upon the past-board, draw (from the Centre) the Lines of an Horizontal-dial made to that Latitude wherein you are.

9. Then again, let the plain board or past-board be applied to its former situation, the Centre of the Horizontal-dial resting upon the Point of the Gnomon, and every thing else answering to the same just posture that it had at the first. Which done, let a Thred be fixed in the Centre of the Horizontal-dial, by help whereof you may transfer every hour from the past-board into the Horizontal-line. Let every hour be therein noted (by fixing marks upon the Horizontal-line where it is drawn, or by slipping knots set upon the Thred, where a Thred Horizontal-line is used) especially mark out the hour of 12: For which (if it chance to run besides the Superficies) some kind of Object, (whereon the Horizontal-line is also to be drawn,) or an Horizontal Thred must be fastned, that may receive it, till such time as your Dial be finished.

10. After all this, take your Plain away, (for there will now be no more need of it,) and conjecture whereabout the Axis of the World would pass from the Point of the Gnomon to the Poles of the World, for into that place is the Mridian to be projected. Which that it may be done more commodiously, if no object stand in the way that will receive it, you must place one there, it matters not whether above or below the Gnomon, chuse that which is most convenient: Or, a Thred laid

aslope

aſlope in the Meridian juſtly as it ought, will ſerve as well as may be. If then you hold up a Perpendicular Thred, ſo that by your eye you may ſee the Point of the Gnomon, and alſo the Point of 12 in the Horizontal-line, both together, the ſame Thred ſo hanging, ſhall ſhew where the Meridian is to be drawn. Or, you may extend a Thred from the Point of the Gnomon to the Point of 12 in the Horizontal-line, which Thred ſhall repreſent the Line of 12: And ſtaying your Thred there, cloſe to it, hang up two Perpendicular Threds at a good diſtance; ſo ſhall the ſame two Threds give you the track of the Meridian-line.

11. The next Work will be to project one of the Poles of the World (that namely, which lies the ſame way that this projected Meridian doth from the Point of the Gnomon) into this Meridian. And this is done by elevating or depreſſing your Semicircle, from the Point of the Gnomon towards the Meridian-line, according to the Latitude of your Place; for ſo will the Ruler of the Semicircle, or a Thred extended along by it, ſign out the very Pole Point. If now you extend a Thred from this Pole Point, to the Point of the Gnomon, the ſame ſhall repreſent the Axis of the World.

12. Laſt of all; by theſe helps, all the hours may eaſily be projected. For if the Eye do lay, or project, this Thred or Axis upon each Point of thoſe hours that were inſerted before into the Horizontal-line, the Axis upon an hour Point, or a Point upon the Axis, each one of thoſe Projections ſhall repreſent upon your Dial each of the Hours required, and will ſhew upon every Object that ſtands in the way, where the Hours are to be drawn. Or, where convenient room is wanting to place the Eye, ſo as it may make this Projection, there may two Threds be uſed for the ſame purpoſe, one whereof muſt be faſtned to the Point of the Gnomon, the other to the Pole deſigned in the Meridian-line. Then ſtretching one of the Threds to any of the Points noted in the Horizontal-line, and holding it there, you may take the other, and extend it to the Superficies, ſo as it may cloſely paſs by the firſt Thred, by which Work you may make as many Points upon your Superficies as you pleaſe, through which each Hour is to be drawn. Having thus traced the way before hand, you may afterward draw the hours without any difficulty, be the Superficies never ſo irregular. Among which Lines, the Shadow of the Point of the Gnomon, as it creepeth along, will ſhew the Time of the Day.

The foregoing Problem may be propounded more generally than before, in manner following;

If a Point be aſſigned upon any Superficies, Flat or Curved, one or more, wherein the Hour-lines and Axis *ſhall concur, how to project the Hours to that Point, and to ſet up an* Axis *after the ordinary manner to give Shadow to them without any knowledge how the Dial ſtandeth, in reſpect either of Declination or Inclination.*

1. To the Point aſſigned (upon any ſide of it) by direction of your Semicircle, or other Level, ſtretch out an Horizontal Thred, ſerving for the Horizontal-line; this Horizontal-line need not be one direct Line, but may be turned at one or more Angles, provided that it lie totally in the Superficies of the Horizon.

2. With a Perpendicular Thred held up, project the Sun into the aſſigned Point, and into the Horizontal Thred, and tie a little mark of Thred upon the ſame Horizontal, through which the Shadow cutteth. At the ſame inſtant alſo, take the Sun's Altitude.

3. By the Altitude taken, find out the Azimuth; This Azimuth, what ever it be, is repreſented by the knot.

4. Apply a paſte-board to the aſſigned Point, and hold it flat that it may anſwer to the Horizontal Thred alſo, and upon this paſte-board protract your Azimuth by a Thred extended from the Point aſſigned for the Centre, to the mark upon the Horizontal Thred. This done,

5. By help of that Azimuth upon your paſte-board protact the Meridian-line, obſerving the true Coaſt, and quantity of the Angle from the Azimuth; and to the Meridian deſcribe an Horizontal Dial.

6. Applying the paſte-board to its place again, all things ſtanding right as before, project all the hours into the Horizontal Thred from off the paſte-board, and ſet marks upon the ſame for the Points of each ſeveral hour, which marks may be little moveable knots to ſlip to and fro upon the ſame Thred.

7. Pro-

7. Project the Meridian Point by a Perpendicular Thred upon some object into that place whereabouts you imagine the Axis of the World would pass, above or below from the Point assigned for the Centre.

8. With your Semicircle elevated or depressed (as it shall be required) from the Point assigned for the Centre, according to your Latitude project the Pole of the World.

9. Extend a Thred from the Point assigned for the Centre to the Poles of the World, which shall represent the Axis.

10. By the Point upon the Horizontal Thred, and this Axis, (either by your eye, laying the Axis to the hour-points, or laying the hour-knots to the Axis,) you may project all the hours, and draw them; Or else you may let the Axis alone, and content your self with the Pole-point projected into the Meridian, for if from the Point assigned to be the Centre or meeting of the hours and Axis, you extend a Thred to each hour-point in the Horizontal-line, and do repose (with your eye) the same Thred upon the Pole-point, then shall the Shadow of the Thred give you that hour-line, and do so in all the rest.

11. Your Thred or Axis lying in its true Situation, you may easily fit an Axis to the same Posture. If your Dial be described upon a plain Superficies, you may then (by one side of a Nominal Square, applied to a Thred or Axis, and the other side lying upon the Plain) find out the Substile, and measure from it the Elevation of the Axis above the Plain: But if the Dial be described upon a curved Superficies, you must be content to set up your Axis by the direction of the Thred only.

12. This Point assigned for the Centre being a Point of the Axis, is as it were the Apex of the Gnomon, unto which all the Work is projected. But if it be required to set up an Axis to such a Superficies, upon which the Axis and Hours will not meet in any tolerable manner, because perhaps the Axis may be but of very small Elevation above the Superficies, and yet an Axis is required: In this case, set up any point (of Wire, or such like) of such distance from the Superficies, as that the Axis and Hours may be distinct: And through that Point let it be required to make the Axis pass, you have no more to do but only to project to this Point, as before, by letting the Shadow of a Perpendicular Thred pass through that Point, and noting the same upon your Horizontal Thred, and counting that end of the Wire as your Centre, proceed as before, for the Thred that lies to project the hours is a Pattern for the Axis.

This way is as general as the former, serving to project the hours upon many Superficies, be they plain or curved, and however situate whether contiguous, or seperate; and that without any laborious Inquisition of any of their Situations, in respect of Inclination or Declination. If you will put in that Furniture which is usual, you must make some mark (notch, or button) upon your Axis, unto which (as representing the Centre of the World) by help of your Semicircle you are to project the Altitudes of such great or lesser Circles as you intend to insert; as hereafter shall be taught.

The twelve Propositions in the first way were to project to an Apex.

These twelve Propositions answerable in the second way are to project to an Axis.

CHAP. III.

From a Hole made in any Pane of a Glass Window, how to draw Hour-lines within any Room upon the Window-board, Window-Jambs, and Floor: Nay, under the Intertise where the Sun can never shine: By which, the spot of Light passing through the Hole, shall shew the time of the day, upon any of the Plains upon which the Spot of Light shall fall; or to which it may be transferred.

FIrst, if the Glass-window of any Room, one Pain or Quarry of Glass be darkned, and a hole about half a quarter of an Inch Diametre made about the middle thereof, the Sun shining upon the Window will, through that hole, cast a bright Spot of Light into the Room, which as the Sun in his motion passeth by the Window, the Spot of Light will be also removed from place to place, sometimes upon the Window-board, sometimes upon the Jambs, sometimes upon the Sides, and sometimes upon the Floor of the Room. If such a Hole should be supposed to be the *Gnodus* or Point of the top of the Perpendicular Stile of any Dial, I say from it the Hour-lines of a Sun-dial or of several Dials (for every side or part of the Room is a different Plain) may be made about the Room, If,

1. Horizontally you apply an Horizontal Dial to the Hole in the Glass-window, and extend a third Horizontally also from the Hole over every Hour-line (or half and quarter Hour-line) till it touch the Sides, Doors, Windows, Jambs, or other Objects or Impediments (standing in the way) about the Room: Then,

2. The Twelve a Clock Hour-line being both an Hour-line and an Azimuth also, you may (by a Perpendicular Thred, or Threds, transfer the same to the Cieling or Floor of your Room, or to which of them will best serve your turn, and sometime there may be occasion for both: Then,

3. In this Meridian Line find another Point, from which, a Line or Thred extended to the Hole in the Window may represent either the Direct (or Reverted) Axis of the World, and unto that Point or Points all the Hour-lines which you draw in that Room will have respect unto (or be in the same Plain with) this Axis: And therefore,

4. If you fix a Thred in one or both of these Points, (or Poles rather,) and extend that String by the Side of another String extended from the Hole over any Hour-line or Point found on the side of the Room as before, that moveable String being gently moved by the Side of the Horizontal String shall trace out (upon all Objects that it meets withal) the Hour-line which the extended Horizontal String doth represent.

CHAP. IV.

Of Reflected Dials.

REflexion is best made by a piece of Looking-glass, which is so much the better, by how much it is Thinner. for the thickness of it causeth a double Ray of Light to be Reflected; the breadth may be about half an Inch: And to prevent this double Ray of Light, colour the lower Superficies thereof with some thick Oyl Colour, or Rub it upon a rough Brick or Grind-Stone rather, which will prevent the double Ray.

CHAP.

CHAP V.

How from a Glass Horizontally placed (or from Water or other Liquors) to Reflect Hours upon any Superficies, Flat or Curved, one or more.

THis is for Reflected Dials within a Room, and is not yet limited to a Plain Superficies, but is general to all, Flat or Curved, lying Horizontal or standing Upright, or else Leaning. It is general, in respect of the Superficies, whereto the Description is to be made, but particular in respect of the posture of the Glass, for that is here required that it should be Horizontal. Now to place it Horizontal, do thus: Lay it into a little Board, flat and even with the Board, and either make it fast, by forcing it into the Board (the place of it being streight and just fit) or else overlay it with a Plate of Lead beaten very thin, but be sure it lie true to the flat of the Board; then with a small Level lay the Board truly Flat or Horizontal, which will be done without any great difficulty, which is to lie for the most part in a Window.

1. The Glass being laid, observe the Spot of Light that the Sun casts, and make a mark at it.

2. And immediately observe the Sun's Altitude, and find the Azimuth.

3. Then extend an Horizontal Thred in the same Level with the Glass, but within the Room.

4. Afterwards project the Azimuth into the Horizontal Thred, by holding up a Perpendicular Thred in such a place, that though it hang at liberty, you may at once discern both the mark of the spot of Light, and the Glass likewise; and then observe where the Perpendicular Thred seems to cut the Horizontal Thred; at that apparent intersection tie a short Thred for a mark of the Azimuth.

5. Apply a Past-board to the Glass, and so that it may be staid upon some rest, that after it is taken away it may be restored to the first posture without impeachment: Let it be also held Horizontally, so as that it may have full relation to the Horizontal Thred.

6. At the Glass's Centre make a Point for a Centre upon the Paste-board, and extending a Thred from the Centre of the Paste-board to the mark of the Azimuth upon the Horizontal Thred, draw upon the Paste-board that Line which the Thred so extended figures out thereon. Afterwards, unto the same Azimuth, upon the Paste-board, draw a Meridian, and to it an Horizontal Dial, and applying the Paste-board to its first Situation, project the Hours thereon, unto the Horizontal Thred, and the Knots; all which is done in the Room.

7. Then project the Meridian (by a Perpendicular Thred, covering, in appearance, both the Knot of 12 and the Glass) unto the contrary Coast to that wherein the Pole is elevated (above the Horizon) that is to say, in our Northern Climates you must project the Meridian Southwards from the Glass, because the North Pole is elevated. And in the Meridian, elevate your Semicircle from the Glass Southwards, till it rise up to your Latitude; so shall the Ruler of the Semi-Circle point out (upon some Object set to receive it) the North Pole Reflected. Or else, if that be not convenient, (because in Windows, or such like Places that stand towards the South, the North Pole will be without the Room, and so the *Axis* above the Glass, extended towards that Pole will be without also) you may in such cases find out the opposite Pole to it; that is to say, that Pole which the former reflected *Axis* being extended through the Glass, and below it, would sign out, and they may be effected in this manner:

Project the Meridian Line towards the Pole that is elevated; that is, with us towards the North Pole, and then (because the North Pole is elevated by Reflection towards the South, so, by the same reason, the South Pole must be depressed towards the North) with the Ruler of your Semi-Circle, directed even with the Centre of the Glass, expres or project your Latitude downwards, (but towards the North,) so shall the Ruler of the Semi-Circle point out the reflected South Pole in the Meridian.

Now whether you will or can (moſt conveniently) uſe the reflected North Pole above the Glaſs, or the reflected South Pole below it, you are to take your choice, for both the one and the other of them do repreſent the reflected *Axis* of the World.

8. By this reflected *Axis*, and the Hour Points ſigned out upon the Horizontal Thred, you may eaſily project the reflected Hours, in the ſame manner that hath been heretofore declared upon any kind of Superficies, one or more, whatever they be that ſtand in the way.

How ſuch Circles of the Sphere, as have relation to the Courſe of the Sun, may be inſcribed upon all Dial Plains, whether plain, projected, or reflected.

PART V.

AS Hour-lines may be deſcribed upon all ſorts of Plains, to ſhew the Hour of the the Day, by the ſhadow of a Stile or *Axis*; ſo may other Lines alſo be deſcribed upon the ſame Plains, which ſhall have relation to the Sun's Courſe; which ſhall trace out upon the Plain (by the ſhadow of an *Apex*, or Point in the *Axis*) the Sun's courſe, whereby at the ſame time, not only the Hour of the day may be known, but (2) the time of the year, (3) the riſing and ſetting of the Sun, (4) the length of the Day and Night, (5) the Sun's Azimuth, or on what Point of the Mariner's Compaſs the Sun is at any time of the day, (6) the *Almicanthar*, or Circle of the Sun's Altitude, whereby the proportion that any object bears to its ſhadow is diſcovered, (7) the *Babyloniſh*, *Jewiſh*, and *Italian* Hours, (8) the Sine of the *Zodiack*, in which the Sun is; with (9) thoſe Aſcending, and Deſcending; and (10) the Circles of Poſition, ſhewing in which of the twelve Cœleſtial Houſes the Sun is at any time of the day, and dives other Gnomonical Concluſions; and before any of theſe can be inſcribed upon any Plain, a convenient Point or *Apex* in the Stile muſt be found, for it is that which gives the ſhadow: Therefore,

§ I. *Of the Perpendicular Stile.*

THis muſt be Perpendicular to the Subſtile, and the top thereof determined in the Stile or *Axis*.

If the Plain be ſmall, conſider whether it be direct or declining, and much declining.

If direct, the Subſtile may be placed in the middeſt, if declining, then on the part oppoſite to the Declination.

The Subſtile well placed, (and room left for the figures,) divide it into two parts, ſo as that part next the Centre of the Dial, may be the Tangent Complement of the Height of the Pole above the Plane; and the other part, the Tangent Complement of the Sun's Meridional Height in the beginning of that Tropick, which is to be more remote from the Centre of the Dial.

And the *Radius* proper to theſe Tangents ſhall be the Perpendicular Stile, to be placed in the point of Diviſion in the Subſtile perpendicular thereunto.

§ II. *Of the Sines, or Parallels.*

A Sine is a twelfth part of the Ecliptick, and contains therefore 30 deg.

A Parallel, according to the vulgar ſenſe, is the Sun's diurnal Motion day by day: And becauſe there are 47 deg. from Tropick to Tropick, there may be ſo many Parallels; that is, Circles which the Sun deſcribes every 24 Hours, ſuppoſed Parallel to the *Æquator*, though not exactly ſo; and although there are 47 of theſe, yet in our Latitude of 51. 32′. we accompt but 9. *viz.* thoſe which are the day from Sun to Sun, when it is 8, 9, 10, 11, 12, 13, 14, 15, or 16 Hours long. The Deſcription of theſe parallels, and of the Signs is made the ſame way, only due reſpect muſt be had to the quantity of the Sun's Declination; for in all direct Horizontals, the Perpendicular Stile being *Radius*, the Tangent Complement of the Sun's Height in any

any Sign or Parallel, at any hour of the day; set off from the foot of the said Stile, and extended to the Hour-line, gives a mark, by which the Parallel of that day shall pass: So that this work repeated so often as the number of Parallels to be inscribed, and the Hour-lines require, shall give respective points enough in each hour, to draw each Parallel by.

Example.

In the Latitude 51. 32. the Sun being in *Pisces*, (the beginning thereof,) the degrees of the Sun's Height above the Horizon at every hour being as followeth; that is, 25. 37'. at one of the clock, 21, 49'. at two; 15, 57'. at three; 8. 32'. at four, and the same for eight, nine, ten, and eleven respectively, if the Perpendicular Stile being *Radius*, the Tangents of the Complements of 25. 37'. 21. 49'. 15. 57'. 8. 32: be applied from the foot of the Stile, to the respective Hour, that is, the Co-Tangent of 25. 37'. from the foot of the Stile, to the Hours of 1 and 11, and so the others, they shall give points, in every Hourline one; by which a Line being drawn with an even hand, shall be the Parallel at the beginning of *Pisces*: And the like of all the rest.

And therefore generally in Verticals; as also in all Recliners; that is to say, upon all Planes whatsoever, draw a Horizontal Dial proper to the Plane, and inscribe the Signs or Parallels upon it, by setting off from the foot of the Perpendicular Stile, the Tangents Complements of the Sun's Height at every Hour in the beginning of every such Sign above that Plane, taken as an Horizon, the Perpendicular Stile being ever *Radius*; and at the end of these Tangents so set off, upon every respective Hour-line will be a Points; by which Point, Lines drawn with an even hand, shall give the Parallels desired. This Horizontal Dial being drawn in obscure Lines, the Dial for the Plane may be drawn afterwards: The Parallels serving, which were drawn before.

Example.

Suppose a Plane declining 30 deg. and reclining 55 deg. the Height of the Pole above the Plane 19 deg. 25 min. the Sun's Height at the beginning.

	12 H.	82 D.	5'.
Of *Taurus* to be at the Hours of	1	73	30
	2	60	3
	3	46	1
	4	31	53
	5	17	47

The Tangents of the Complements of 82, 5'. and 73, 3'. and 60, 3'. *&c.* set off from the foot of the Perpendicular Stile, (the said Stile being the *Radius* to those Tangents) to the obscure Horizontal Hours of 12, 1, 2, *&c.* give the true Distances between the foot of the Stile, and those Auxiliary Hours for the Parallel of *Taurus*, and so the other Parallels may be found.

It is true, the Height of the Sun at every Hour of the day, at the beginning of every Sign in any Latitude, is not easily found out without Trigonometrical Calculation, or by the Globes, as in Lib. 3. Part 1. Prob. 16. Page 371. where there is a Table calculated for the Sun's Altitude at every Hour of the day, when he enters into any of the 12 Signs in the Latitude of 51 deg. 30 min. And the like may be done by the *Analemma*, for any Latitude, as in the use thereof.

§ III. *Of the Vertical Circles.*

THese are vulgarly called *Azimuths*; and are great Circles, whose Poles lie in the Horizon, and intersecting one another in the Zenith and Nadir of the Place.

The whole Horizon being divided into 32 equal parts, these Circles shewing those divisions are called Points of the Compass, and marked S. S *b* E. S S E. *&c.* every one distant from other by 11 ¼ degrees.

But the better way of accompting them is 10, 20, 30, 40, 50, 60, 70, *&c.* degrees from the Meridian.

1. In all Horizontal Dial, the Perpendicular Stile being chosen, making the foot thereof the Centre, at any convenient Distance describe a Circle, and accompt from the Meridian both ways, Arches equal to 10, 20, 30, *&c.* degrees, from which divisions right Lines drawn to the foot of the Stile aforesaid, shall represent those *Azimuths* upon that Dial.

2. Upon

2. Upon a prime Vertical (or South) Dial, through the foot of the Perpendicular Stile, draw a right Line parallel to the Horizon, and making the said Stile *Radius*, upon the Parallel Line set off both ways from the Meridian Tangents of 10, 20, 30, 40, &c. degrees, through which divisions right Lines drawn, all at right Angles, with the parallel Line, shall be the *Azimuths*.

3. Upon any declining Vertical, the same being done, shall give the *Azimuths* of 10, 20, 30, &c. from the Meridian of the Plane, or from the Meridian of the place, just allowance being made for the Distance of Meridians.

4. In South declining reclining Planes, the Perpendicular Stile being chosen, and made the *Radius*, the Tangent Complement of the Reclination applied from the foot of the said Stile, to the Meridian of the place, shall determine the Zenith of the place; through which, and the foot of the Stile, that is the Zenith of the Plane, a right Line drawn, shall be a Perpendicular to the Horizontal Line, which shall concurr with the *Æquator* in the Hour of 6; and therefore if from the foot of the Stile upon the said Perpendicular towards the North, (for the former application is made towards the South,) be set off the Tangent of the Reclination, a Line drawn from the end thereof at right Angles with it, shall be the Horizontal Line; upon which, the Tangents of 10, 20, 30, &c. (the secant of the Reclination being now made *Radius*) set from the said right Angle, Lines drawn from them to the Zenith of the place, shall be the Azimuths.

5. The Distance betwixt the Meridians being known upon the Horizontal Line, the Azimuths which were accompted from the Meridian of the Plane, may be fitted for accompt from the Meridian of the place with ease.

For example, Let that Distance be the Tangent of 20 deg. then that Azimuth which is 10 from the one, is 10 from the other also, and that which is 30 on the same side of the Substile, is 10 on the other side of the Meridian of the place, the like Method serves for any Distance.

§ IV. *Almicanters*

ARE lesser Circles of the Syhere, and may be called the Parallels of Declination from the Horizon; having in all respects the same relation and habitude to the Azimuths, that the Signs have to the Meridians, although these are accompted by 15 d. and those usually by 10 d. and therefore as in the description of the Signs, an Horizontal Dial proper to the Plane, being first (obscurely) delineated, it was shewed, that the Points thro' which the Signs or Parallels must pass upon every hour, might be had by applying the Tangents of the Complements of the Sun's Height at those Hours in those Parallels, from the foot of the Perpendicular Stile, to the respective Hours, so here making use of that Azimuth which is Perpendicular to the Plane, (which in all Planes is that which passeth through the foot of the Perpendicular Stile,) the rest of the Azimuths being also inscribed, the Tangents Complements of the Sun's Height above the Plane, when he is in any Azimuth, applied from the foot of the Stile to the said Azimuth gives a point, through which, that Height or Almicanter upon that Azimuth must pass.

And these Altitudes must be found by Trigonometrical Calculation, or by the Globes, as in Lib. 3. Part 1. Prob. 17. Page 372. where there is a Table, shewing what Altitude the Sun hath, he beingupon every tenth Azimuth from the Meridian, in the beginning of every Sign in the Latitude of 51 deg. 30 min. and the like may be made by the *Analemma* for any Latitude, as in the uses thereof, or by any other Projection made for any particular Latitude.

So χe and $s e$ being severally found, the Difference between them, namely $se - \chi e$, is the Complement of the Sun's Height above the Horizon.

Then find how high the Sun is above the Plane of the Dial at the same time, the Tangent Complement of that Height applied from the Stiles foot to the Azimuth, representing the Angle $p \chi s$, gives upon it the Almicanter's point, or passage.

Or because s, $p s'$. s, $p \chi s''$. s, $\chi s'$. s, $\chi p s''$. the Hour from Noon, that is the Angle $\chi p s$ is found, which will cross the Azimuth aforesaid in the same point also.

Which Hour, if it be uneven, and unfit to remain with the rest, may be drawn obscurely.

SECT.

§ V. *Of the Jewish, Babylonish, and Italian Hours.*

THE Babylonish are accounted equal Hours from Sun rising, and may be inscribed upon any Plane by help of those two Parallels, which shew the longest, and shortest day consisting of intire hours, as here 16 and 8 hours, and of the *Æquator*; for a Line drawn through the hour of 5 in the first, 7 in the *Æquator*, and 9 in the other, is the hour of 1 from Sun rising.

Likewise in the same order, through 6, 8, and 10, shall pass the hour of 2, the like order in all.

In Winter when the Parallel of 8 hours shall fail, the other two points will serve; because the hours to be drawn are right Lines.

But after the first six hours are inscribed, the *Æquator* failing also, some other Diurnal Arch, as of 9 or 10 hours, must be described to supply that want.

The Italian hours are accounted 1, 2, 3, &c. from Sun-setting; to inscribe these the same Diurnal Arches will serve, and a Line drawn through them in the hours 9, 7, and 5, Afternoon, (observing the same order as before) shall be the hour of 1; the like through 7, 5, and 3, shall be the hour 23; the Night hours of 9, 10, &c. are the Morning hours produced.

The Jewish hours are reckoned like the Babylonish, from Sun-rising, but unequally, their sixth hour being Noon; and every hour a twelfth part of the Artificial day, of what length soever that be.

The vulgar hours proper to the Plane being first drawn, and the Diurnal Arches of 15, 12, and 9, (if it may be) divide the degrees in each by 12, and the Quotients by 15; or else (which is all one) divide the said Arches by 180, the three Quotients shall give the just times in hours or usual parts of hours from 12 of Clock upon the two Parallels and the *Æquator*; through which lines drawn by a Ruler shall be the Jewish hours desired.

Example, In Latitude 51. 32', the Diurnal Arch of 15 hours, is in degrees 225, which divided by 180 the Quotient is 1¼. and so much the Jewish hours of 5 and 7 are distant from Noon, an hour and quarter being a twelfth part of the Diurnal Arch of 15 hours, which hour and quarter being doubled, gives the place of 4 and 8, tripled the place of 3 and 9, &c. from Noon, upon that Parallel of 15 hours.

In like manner the Diurnal Arch of 9 hours, is 135 d. which divided by 180, Quotient is $\frac{135}{180}$ that is ¾ of an hour, which shews the place of the Jewish 7 and 5, to be three quarters after 10 before Noon, and doubled is 1½. which gives the place of 8 and 4, all one with our 1½. and 10½. and so tripling and quadrupling and quintupling of ¾ gives the places of the other hours on this Parallel of 9 degrees.

And these parts doubled and tripled as is said, will always (in this Parallel and the former) fall upon even hours, halves, or quarters of our hours, which is the only reason why these two Parallels of 15 and 9 are preferred; there being no necessity of using them more than the Tropicks or other Parrallels, only this convenieney of even parts.

Lastly, In the Diurnal Arch of 12, that is, the *Æquator*, the equal and unequal hours concur, that is, the Jewish hours of 5 and 7, with our hours of 11 and 1, so their 4 and 8, with our 2 and 10, &c. so that a line drawn from 1¼. in the Arch of 15 to 1, in the *Æquator*, and from thence to ¾ in the Arch of 9 is their 7, &c.

CONCLUSION.

I Might (in this Place) have illustrated the foregoing *Precepts* by *Figures* and *Examples*; but the *Book* swelling to a bulk beyond my Expectation, I pass that over; And the rather, because I have (in my late Book of *Dialling*, Printed *Anno* 1680.) at large insisted on these Matters, and of many other *Gnomonical* Conclusions: And so shall here satisfie the Reader, by inserting the following *Table* of *Natural Sines*, *Tangents* and *Secants*; which will be of great use in the putting in of the *Furniture* upon any *Dial*, whether *Plain*, *Projected*, or *Reflected*.

A Table

A Table of Natural Sines, Tangents and Secants, to every degree of the Quadrant to a Radius of 1.00000. of great use, for the ready inserting of the Furniture into all manner of Sun Dials.

D.	Sine.	Tangent.	Secant.	D.	Sine.	Tangent.	Secant.
0	0.00000	0 00000	1.00000	45	0.70710	1.00000	1.41421
1	0.01745	0.01745	1.00015	46	0.71933	1 03553	1.43955
2	0.03489	0 03492	1.00060	47	0.73135	1 07236	1.46627
3	0.05233	0.05240	1.00137	48	0.74314	1.11061	1.49447
4	0.06975	0.06817	1.00232	49	0.75470	1 1 036	1.52425
5	0.08715	0.08748	1.00000	50	0.76604	1.19175	1 55572
6	0.10452	0 10510	1 00015	51	0.77714	1.23489	1.58901
7	0.12186	0.12278	1.00060	52	0 78801	1.27994	1.62426
8	0.13917	0.14054	1.00137	53	0.79863	1.32704	1 66164
9	0.15643	0.15838	1.00232	54	0.80901	1.37638	1.70130
10	0.17364	0.17632	1.01542	55	0.81915	1 42814	1.74344
11	0.19080	0.19438	1.01871	56	0 82903	1.47659	1 78367
12	0.20791	0.21255	1.02234	57	0.83867	1.53986	1.83607
13	0.22495	0.23086	1.02620	58	0 84804	1 60023	1 88707
14	0.24192	0.24932	1.03061	59	0 85716	1.66427	1 94 60
15	0.25881	0.26794	1.03527	60	0.86602	1.73205	2 00000
16	0.27563	0.28674	1.04229	61	0.87461	1.80404	2.06266
17	0.29237	0.30573	1.04569	62	0.88294	1 88072	2.12308
18	0.30901	0.32491	1.05146	63	0.89100	1.96261	2.20268
19	0.32556	0.34432	1.05762	64	0.89879	2 05030	2.28017
20	0.34202	0.36397	1.06417	65	0.90630	2.14450	2.36620
21	0.35836	0.38386	1.07114	66	0.91354	2.24603	2.45859
22	0.37460	0.40402	1.07852	67	0 92050	2 35585	2.55930
23	0.39073	0 42447	1.08636	68	0.92718	2.47508	2 66946
24	0.40673	0.44522	1.09463	69	0 93358	2.60508	2.79042
25	0.42261	0.46630	1.10337	70	0 93969	2.74747	2.92380
26	0 43837	0.48773	1.11260	71	0.94551	2 90421	3.07155
27	0.45399	0.50952	1.12232	72	0.95106	3 07768	3.23606
28	0 46947	0.53170	1.13257	73	0.95630	3 27085	3.42030
29	0.48480	0.55430	1.14335	74	0.96126	3 48741	3.62795
30	0.50000	0.57735	1.15470	75	0.96592	3.73205	3.86370
31	0.51503	0.60323	1.16705	76	0.97029	4.01078	4.13356
32	0.52991	0 62486	1.17917	77	0.97437	4.33147	4.44541
33	0 54463	0.64940	1.19371	78	0.97824	4 70463	4.80973
34	0 55919	0.67450	1.20621	79	0.98162	5.14415	5.24085
35	0.57357	0.70020	1.22077	80	0.98480	5.67128	5.75877
36	0.58778	0.72654	1.23606	81	0.98768	6 31375	6.39245
37	0.60181	0.75355	1 25213	82	0 99026	7.11536	7 18529
38	0.61566	0.78128	1.26901	83	0 99254	8.14434	8.20550
39	0.62932	0.80970	1.28675	84	0.99452	9.51436	9.56677
40	0.64278	0.83909	1.30540	85	0.99619	11.43005	11.47371
41	0 65605	0.86928	1 32501	86	0 99756	14.30036	14.33558
42	0 66913	0.90040	1 84563	87	0.99862	19.08113	19.10732
43	0.68199	0.93251	1.36732	88	0 99939	28.63625	28.65370
44	0 69465	0.86568	1.39056	89	0 99984	57.28996	57.29868

The End of the Eighth Book.

CURSUS MATHEMATICUS.

BOOK IX.

OF THE

THEORY

OF THE

PLANETS:

OR,

Of the *HEAVENLY BODIES*, that have a Second Motion of their own.

In Five Parts, or Sections.

I. Of the *Phænomena.*

II. Of the *Hypotheſis*, by which the *Phænomena* is Explained.

III. How all Lines and Angles may be determined from *Obſervations.*

IV. Of Computing the *Motions* of the *Planets* by *Tables.*

V. A New *Hypotheſis*; How to *Calculate* according to it: And thoſe *Calculations* compared with *Obſervations.*

By *WILLIAM LEYBOURN*, Philomathemat.

LONDON,

Printed *Anno Domini* MDCXC.

OF THE THEORY OF THE PLANETS: OR, Of the *Heavenly Bodies*, that have a Second Motion of their own.

Procem.

THIS Ninth Book of the Theory *of the* Planets, *is a Tranſlation of Mr.* Nicholas Mercator *of* Halſatia, *his Second Book of* Theorical Aſtronomy, *by him publiſhed in Latin*, Anno 1676. *And the Method which he obſerves in it is This:* (1.) *I will lay down the* Phænomena, (i. e.) *thoſe Things that appear in, or relating to the* Heavenly Bodies; *upon the Obſervation of which men firſt began to think of other Motions beſide the firſt.* (2.) *I will deliver the chief* Hypotheſis, *by the help of which they have endeavoured to demonſtrate, or give an Account of what they have obſerv'd.* (3.) *I will ſhew the manner, how all the parts of the* Hypotheſis *might be determin'd by the Obſervations that have been made.* (4.) *I ſhall bring the Tables, that have been made according to the determin'd Lines and Angles; and alſo explain the Uſe of them by* Rules *and* Examples *too, as far as may be.*

SECTION I.

Of the Phænomena.

OF the *Phænomena*, which ariſe from other Cauſes than the daily Motion that has been explained in the former Part; ſome are common to all the Stars, others are peculiar to theſe or thoſe particularly.

1. In the firſt place 'tis obſerv'd, as to all the Planets, That they do not always riſe or ſet in the ſame places of the *Horizon*; but that they riſe ſometimes from that place where the Sun riſes in *Summer*, and ſometimes from that where it riſes in *Winter*; and in like manner, that they go down ſometimes in the place of the *Winter-Sun-ſetting*, and ſometimes in the place of the *Summer-Solſtice*. Again, That under the *Meridian* they go ſometimes higher towards the *North*, and ſometimes lower towards the *South*. Beſides, That there are certain Bounds or Limits, beyond which they neither run *North* or *Southward* in their riſe or ſetting; nor are they elevated or depreſs'd beyond them, when they are culminant.

2. 'Tis found, That ſometimes all the *Planets* move faſter, ſometimes ſlower.

3. That ſometimes they appear greater, and ſometimes leſs, though at both times the Refractions be the ſame, as being in the ſame Altitude, or in the ſame Temperature of Air.

4. That they have divers Situations, both with relation to one another, and alſo to the fixed Stars: And that ſometimes in their Conjunctions or Approaches to one another, the fixed Stars are cover'd by the Planets, and ſometimes the Planets by one another; and yet not all alike with reſpect to all the Inhabitants of the Earth.

5. But particularly as to the Sun, 'tis obſerv'd, That after it is ſet, it has certain Stars viſibly apparent, that will ſet after it, which ſome days after appear no more; and that when 'tis about to riſe, it has ſome before it that appear not, which yet after ſome days appear, and riſe before it.

6. That the Sun ſpends 187 Days in paſſing from the *Vernal* to the *Autumnal Æquinox*, and but 178 in paſſing on from the *Antumnal* again; ſo that it tarries nine whole days longer in the *Northern* than in the *Southern Signs*.

7. That the Sun ſometimes ſuffers a total Eclipſe, but for the moſt part only a partial one; and both theſe only in the New of the Moon, and yet not in every New Moon neither.

8. That the greateſt Declination or Departure of the Sun from the *Æquator*, as well *Northward* as *Southward*, ſeems to be grown leſs; ſince it is not now accounted ſo great as it has been formerly.

9. As to the Moon, 'tis obſerv'd, That from the New, or Change of it, it does day by day ſo remove from the Sun, that it does always go more and more towards the more *Eaſtern* Stars, till it hath perform'd its Circuit.

10. That in every Lunation (or Time betwixt Change and Change) it has divers *Phaſes*, or appearances: In its Increaſe it appears horned, or ſomething reſembling the form of a Sickle; by and by it appears halfed, or, as it were, cut in two; and afterwards gibbous, or bunching out on both ſides: and then after the Full, when it decreaſes, it again becomes gibbous, halfed, and horned.

11. That it does likewiſe ſometimes ſuffer a total Eclipſe, but oftner a partial one; and both only when 'tis in the Full, and yet not in every Full neither; but for the moſt part once in ſix Months, and ſometimes in five.

12. Laſtly, That it departs from the *Æquator* as well *Northward* as *Southward*, ſometimes more, and ſometimes leſs than the Sun does.

13. As to *Mercury* and *Venus*, 'tis obſerved, That they conſtantly accompany the Sun; and as for *Venus*, ſhe ſcarce ever goes farther from him, than about half a Sign, and *Mercury* not a whole one.

14. And at the ſame time, That they ſometimes go before the Sun, or are ſeen early before its riſing; and ſometimes follow the Sun, riſing in the evening after the Sun ſets.

15. That they ſometimes are direct, that is, move into the Signs following, or according to the Order and Succeſſion of the Signs, as from ♈ into ♉, from ♉ into ♊; but ſometimes contrary to the Order of the Signs, as from ♈ into ♓, from ♓ into ♒; and ſometimes that they are Stationary, that is, for ſome times they ſeem to move neither into the antecedent nor conſequent Signs.

16. That their Digreſſions alſo, or Departures *North* and *South*, are ſometimes greater, ſometimes leſs than that of the Sun.

17. As to *Mars*, *Jupiter*, and *Saturn*, 'tis noted, That they are not ſo tied to the Sun as are the other two, but do ſo depart from it, that ſometimes they are in oppoſition to it, or remov'd from it ſix whole Signs.

18. That every year they proceed more and more towards the *Eaſtern* Stars; but yet that ſometimes they are Direct, ſometimes Retrograde, and ſometimes Stationary.

19. That they are always Retrograde, and are both ſwifteſt in their Retroceſſion, and greateſt in Aſpect or Appearance, when being in oppoſition to the Sun they ſhine all night above the *Horizon*: And that a larger ſpace of Retrogradation belongs to *Mars* than to *Jupiter*, and to *Jupiter* than to *Saturn*; but then again, that a longer time of Retrogradation belongs to *Saturn* than to *Jupiter*, and to *Jupiter* than to *Mars*.

20. And

20. And lastly, That they also depart sometimes more, and sometimes less from the *Æquator* than does the Sun.

21. In the last place of all, as to the fix'd Stars 'tis observ'd, That they do not always keep the same distance from the Æquinoctial Points; and that they do likewise move, but that very slowly, towards the consequent Signs; and (as it appears to some) unequally, that is, sometimes more swiftly, and sometimes more slowly: For, as for instance, the Star call'd *Spica Virginis*, which (not long after the Death of *Alexander*) *Timocharis* observ'd to go before the Point of the Autumnal *Æquinox* eight degrees, was observ'd, 200 years after, by *Hipparchus*, to go before it only six; and 260 years after that, *Ptolomy* observ'd not much more than three: That I may not add, that at this time, or 1500 years after that last Observation, 'tis observ'd, that the forementiond Star follows the same Point almost eighteen Degrees: In like manner the first Star in Aries, which, in the time of *Timocharis*, was distant from the Vernal *Æquinox* only two Degrees, is now found to be distant more than twenty eight: So also 'tis worthy to be mention'd, that the Star at the end of the Tail of the Lesser Bear, which is call'd the Pole-star, is now only two Degrees distant from the Pole, with a little above half a Degree more; whereas, in the Time of *Hipparchus*, it was distant more than twelve: And so of the rest.

SECTION II.

Of the Hypotheses, *by which the* Phænomena *now premis'd may be explain'd.*

THE *Hypotheses* that have been invented to explain and solve these *Phænomena* are many; but they may all be reduc'd to these principal ones, namely, that which supposes the Earth to be Immovable, and that which supposes it to move. Whether of these is the more ancient does scarce appear, for the *Pythagoreans* did long ago reckon the Earth among the Plannets; yet because before *Copernicus*, there was scarce extant any exact Demonstration of the *Phænomena* by the Motion of the Earth, and so since *Ptolomy's Hypothesis* is not a little older, therefore I shall first of all briefly lay down this and explain it, and then afterwards discourse of the other.

CHAP. I.

Of the Motion of the Sun.

1. HOW the various rise and setting of Stars, sometimes in one, and sometimes in another part of the *Horizon*; as also how their Ascent under the *Meridian*; and lastly, how the Approaching of the Planets to the fixed Stars, and their departing from the same, may be demonstrated, is evident from the Spherical part of *Astronomy*, which gave us the Oblique Circle, call'd the *Zodiack*, for the salving of these Motions.

2. The Sun's sometimes appearing greater than it self, and this in the same Altitude above the *Horizon*, (unless the Refraction may seem to be the cause,) is a plain Argument, That at some seasons 'tis farther from the Earth, and at others nearer again. If we consider its Approachment to the fixed Stars, and its Departure from them, we are convinc'd that its Motion is Circular, or Elliptical, and so running back into it self again. If therefore the Sun run round, and be sometimes nearer, and sometimes farther off, that Circle which it runs must needs be Eccentrical: For though in an Elliptical Circle, that is concentrical, the movable Body may be at sometimes farther from the Centre than at others; yet 'tis otherwise in the Sun, which approahces nearer the Earth but once

once a year, and once a year departs farther from it; whereas in concentrical Elliptick, the movable Body is twice in its Apogée, and twice in its Perigée; therefore the Sun moves in an Eccentrick.

Fig. I. 3. The Motion of the Sun in the Eccentrick from the Earth, seems to be unequal; that is, in Figure I. the Angle A T ☉ in equal spaces of time is not equally increas'd; for the Sun being about the Apogée A, does increase every day less than 59′, but being about the Perigée P, every day more than 59′. This is plain from ☉'s continuing longer in the *Northern*, than in the *Southern Signs*: For supposing ♈ ♋ ♎ ♑ to be the Ecliptick, whose Centre let T be; now though the Semicircle ♈ ♋ ♎ be equal to the Semicircle ♎ ♑ ♈, yet does the Sun scarce finish the former in 187 days, but the latter it runs through in 178.

4. Yet does not this hinder, but the Motion of the Sun from a certain other Point out of the Earth T, as for instance from C, the Centre of the Circle A ☉ P, may appear equal; that is, the Angle A C ☉ may be seen to be equally increased in equal spaces of time, *viz.* every day 59′, 8″. for so it will come, that while the motion about C appears equal, the same about T may seem unequal: And this also follows from thence, that seeing C is the Centre of the Circle A ☉ P, about which the Motion is conceiv'd to be equal; therefore the same Motion in its very Circle becomes equable and uniform.

5. Now in the Scheme I have given, the Distance of the Points T C, that is, of the Earth, from the Centre of the Eccentrick, is call'd commonly the Eccentricity: And the same T C, being continued on both sides to the Circumference of the Eccentrick unto A and P, it makes the Line of the *Apses* A P. For A, the farthest Point from the Earth, is call'd the Apogée, and *Aux*, and the *Highest Apse*; as the Point P, that is next to the Earth, is call'd the Perigée, and also the opposite to the *Aux*, and the *Lowest Apse*. And farther, if to the Line C ☉, drawn from the Centre of the Eccentrick C to the Sun in its Circle, be drawn another Line parallel to it, *viz.* T M, or T Q, this Line is call'd the Line of the Middle Motion of the Sun, and is to be continued even to the Ecliptick unto *m*, where it denotes the Sun's middle place; as the Line of its true Motion, T ☉, being continued from the Earth through the Centre of the Sun, even unto *v*, does denote there the true place of the Sun in the Ecliptick: So that the difference between the middle and true place of the Sun, is that Arch of the Ecliptick *v m*, or the Angle *v* T *m*, and that which is equal to it, *viz.* T ☉ C, which they call the Equation, and also the *Prosthaphæresis*, a word compounded of the name of Addition and Subtraction: For in the former Semicircle A M P, which they also call the Descendent (because the Sun does in it descend from the Apogée into the Perigée,) the Arch *m v*, or the *Prosthaphæresis*, is substracted from the Middle Motion of the Sun ♈ ♋ *m*; so that ♈ ♋ *v* remains the true Motion of it. But in the latter Semicircle P Q A, (which is also call'd the Ascendent,) the Arch of the Ecliptick *m v*, or the *Prosthaphæresis*, is added to the Middle Motion of the Sun ♈ *m*; so that ♈ *m v* becomes the true. Now to find out the *Prosthaphæresis m* T *v*, or T ☉ C, is requir'd the Angle A C ☉, (to which is equal A T M,) call'd the middle Anomaly of the Sun or the Argument, as A T ☉ is call'd the true Anomaly, or the Co-equate. Lastly, If upon the Line of the *Apses* A P, be from T erected a perpendicular R T S; this is call'd the Line of the Middle Longitudes, because the Sun moving in the Eccentrick A R P S, as in A it is the farthest from the Earth, and in P the neareſt; so in the middle Longitudes R and S, it is at its middle, or in different distance from the Earth.

6. Nor is it to be conceal'd, that the Line of the *Apses* A P, has indeed the point T fixed; but yet the Line itself, together with its whole Eccentrick, is carried by little and little towards the following signs; so that as time goes on, the Angle ♈ T A increases more and more: At present the Point A is under the sixth Degree of ♋, in the following Age it will be under the seventh, and so on.

7. And this also you may take notice of, if you please, That the Eccentricity C T to some has seem'd to be changeable; for that which is now near upon the 28th part of the Eccentrick Ray C A, was formerly greater, or else less: Others say, that Opinion arose from the Uncertainty of ancient Observations, and that nothing is to be determin'd concerning the Change of the Eccentri-

city,

city, before it be confirm'd by more certain Tokens, which cannot be done without the more diligent Observations of more Ages still.

8. But though the change of the Obliquity of the Ecliptick may seem to be owing to the like uncertainty of Observations; yet if you desire an Account of it, it may be salved by conceiving a certain sort of wavering Motion of the Poles of the Ecliptick, by which they do very gently nod, as it were, to and fro, under the Colour of the Solstices, towards the Poles of the *Æquator*; whence it comes to pass, that the Ecliptick it self must needs be sometimes parted more from the *Æquator*, and again sometimes less.

CHAP. II.

Of the Equipollency of the Homocentre-Epicycle, and of Eccentrick, in Salving the Motion of the Sun, and of other Planets.

1. I Call that Circle an Homocentre, which has the same Centre that the Earth has; and an Epicycle is a little Circle that has its Centre fix'd in the Circumference of a greater, together with which Circumference it is carried round: Therefore an Homocentre-Epicycle, is a greater Circle, that has the same Centre with the Earth, and that in its Circumference carries about with it the Centre of the Epicycle, and so the Epicycle it self. Thus in Scheme II. if A be the Centre of the Earth, and also the Centre of the Circle A C E F, this will be the Homocentrick Circle, and its Circumference is fixt C, the Centre of the Epicycle C D G. Therefore the greater Homocentrick Circle A C E F, together with the Epicycle C D G, is, by a compounded name, call'd the Homocentre-Epicycle; just as in like manner the Circle B D E F, having its Centre B different from the Centre of the Earth A, is an Eccentrick, in this place, without an Epicycle: but if to this Eccentrick Circle you should add an Epicycle, whose Centre should be carried about in the Circumference of the Eccentrick, it would be call'd, by a compounded word, an Eccentre-Epicycle. Fig. II.

2. Now if the Ray A C of the Concentrick A C E F be suppos'd to be equal to the Ray B D of the Eccentrick B D E F, and the Ray C D of the Epicycle be suppos'd equal to the Eccentricity A B; besides let two Motions be imagin'd, the first of the Centre of the Epicycle C, in the Circumference C E F, and of the Planet it self, the Sun for example, in the Circumference of the Epicycle from G to D, and so on, till it return to G again; so that the Motion of the Planet in the Epicycle, in the Semicircle farthest from the Earth A, be contrary to the Motion of the Centre of the Epicycle carried from C through E and F; but in the nearer Semicircle let them tend the same way, and let the Revolutions, as well of the Planet in the Epicycle, as of the Centre of the Epicycle in the Circumference of the Homocentre, be not only equable and uniform, that is, going on equally in an equal space of time, but let both the Revolutions also be perform'd in the same Compass of time, so that the right Line C D, drawn from the Centre of the Epicycle C, to the Body of the Planet D, may always remain parallel to itself, as may be learnt from the Scheme. And then let the Sun be conceiv'd to move separately in the Eccentrick B D E F, with an uniform Motion, that is to be finish'd in the same time that the Homocentre A C E F is rolled about. These things being suppos'd, I say, there is a perfect Equipollency, or 'tis the same thing, whether according to the first way the Sun be mov'd in the Circumference of the Epicycle that is carried about in the Homocentre, or whether after the other manner it go about in the Eccentrick alone; for in both Cases 'tis plain, it will have the same place. And in this *Hypothesis* of Homocentre-Epicycle the Line of the Middle Motion A C, is drawn from the Centre of the Earth to the Centre of the Epicycle; but the Line of the true Motion is A D, drawn from the Earth to the Body of the Planet in the Epicycle.

The

The Demonstration.

In the Quadrilateral or fourſided Figure A B C D, take A for the Earth, B is the Centre of the Eccentrick, C the Centre of the Epicycle, D the Sun. In the *Hypotheſis* of the Homocentre-Epicycle, the Line of the Middle Motion A G, together with the Line of the Apogée G A B C, makes the Angle C A G, (the ſpace of which the Line of the Middle Motion is departed from the Apogée,) to which, according to ſuppoſition, the Angle G C D is equal, or the Arch G D, (the ſpace of which the Sun is departed from the Apogée of the Epicycle;) ſo that it is in the Point D. If therefore from the Centre of the Eccentrick B, you draw a ſtreight Line B D, to the Sun D, it will be equal and parallel to A C, ſeeing they are joyned by Lines that are parallels and equals, and ſo the Point D will fall into the Circumference of the Eccentrick; and alſo the Angle C B D, being equal to the Angle C A G, proves, that the Sun in the Eccentrick came in the ſame time to the ſame Point D, that it had come to it in the Epicycle, which is the thing, that was to be Demonſtrated.

CHAP. III.

Of the Æquations of Time.

SInce all Motion is in time, therefore the very Motions of the Stars are circumſcrib'd in certain Periods of time; but yet again 'tis the daily and yearly Motion of the Sun, that deſcribes and divides Time: So that the one cannot be underſtood without the other; that is, neither Time without Motion, nor Motion without Time. But farthermore in all Heavenly Motions, there is a certain Inequality or Irregularity, as 'tis plain, concerning the Sun, from what has been ſaid. Now Time, which as a meaſure is applied to ſuch various Motions, and which are every moment alterable, ought to be as uniform and equable, as poſſible, that a confuſion that cannot be rectified may not ariſe. But now ſince ſuch a moveable Body, that by its Motion does meaſure out an uniform and equable Time, can no where be found; therefore Aſtronomers are of different opinions, whilſt every one frames to himſelf, what to him ſeems moſt agreeable. But howſoever they differ, we muſt ſuppoſe ſome ſuch moveable Body, as is continually rolled about to the Meridian in the very ſame ſpaces of time, and ſo makes an equable day; and alſo that it moves uniformly to the Equinoctial Points, and ſo meaſures out a Tropical Year of an indifferent Quantity. Now to me there ſeems nothing more fit for this, than a certain fictitious or ſuppoſed Sun, that in a moſt regular Motion, and that equal to the middle Motion of the Sun, may move, not in the Ecliptick, but in the *Æquator* itſelf; whence it would come to paſs, that while the ſuppoſed Sun is moved from the Meridian to the Meridian again, all the Degrees of the *Æquator*, in number 360, would paſs one after another; and beſides, that part of the *Æquator*, which the ſuppoſed Sun had in the mean while paſs'd through by its middle Motion, *viz.* 59'. 8''. and ſo one day would perpetually be equal to another, which therefore I call the Aſtronomical day; becauſe 'tis very fit for computing the Motions of the Stars by, and Aſtronomers in their Tables can uſe none other than this. Different from this is the apparent or Natural day, which the true Sun does make by his apparent Motion, being moved from the Meridian to the Meridian again.

Which apparent Motion of the Sun, 'tis certain, is unequal, as is evident from what goes before; for the Sun does continually move faſter and faſter or elſe abate of its Motion: Whence it comes, that that portion which is daily to be added to the perfect Revolution of the *Æquator*, is not always of the ſame quantity; and if it ſhould be ſo, yet there are not equal portions of the *Æquator* (between the Circles of Declinations, that go through the Sun's yeſterday's and to day's Place) to anſwer thoſe portions, which by the Sun are daily gone through in the Ecliptick.

tick. Hence the apparent days must needs be unequal. Now the difference of Time, which is between the true and supposed Sun's coming to the same Meridian is that Equation of Time, which is here treated of: For sometimes the supposed Sun comes sooner to the Meridian than the true one, and then the Noon-day of the Astronomical Time comes before the apparent Noon-day; that is, the Astronomical Time is reckon'd twelve hours, when the apparent Time is not reckon'd many; though in reality it be altogether at the same moment of Time, which is call'd by divers Names, according to the first it being reckon'd more of the Clock than according to the last. But again, on the other hand it sometimes happens, that the true Sun comes sooner to the Meridian than the supposed one; and then, according to the apparent, 'tis reckon'd twelve a Clock, when according to the Astronomical Time 'tis not reckon'd so much. Now 'tis plain, that the difference that is between the coming of the true Sun and of the supposed Sun to the Meridian, is the Arch of the *Æquator*, that is between the place of the supposed Sun and the right Ascension of the True, which Arch, if it be turned into Time, by reckoning 4′ of an hour for every Degree, we shall then have the Equation of Time. In Scheme III. let ♈ be the true *Æquinox*, ♈ M V part of the Vernal Quadrant of the Ecliptick, ♈ *r m v* the *Æquator*, M the middle place of the Sun in the Ecliptick, V the true place: Let the Arch ♈ *m* be suppos'd equal to the Arch ♈ M, and the Arch ♈ *v* to the Arch ♈ V; and so let *m* be the place of the Sun suppos'd in the *Æquator*, as far distant from ♈ as M is distant from it: Let V *r* be the Circle of Declination, passing through V the true place of the Sun, and in the *Æquator* making its right Ascension in the Point *r*. Now therefore the Equation of time is the Arch *r m*, that lies between the supposed Sun *m*, and the right Ascension of the true Sun *r*. And in this example the true Sun V with its right Ascension *r*, comes sooner to the Meridian, than the supposed Sun M: wherefore the apparent Noon-day comes sooner than the Astronomical. You may also observe, that the Arch *r m*, which makes the Equation of time, is alway either the Summ or the difference of two Arches, of which one is the difference of the true Motion of the Sun, and of its right Ascension, and the other the difference of the true and middle place of the Sun. Thus in this Example the difference of the true Motion of the Sun ♈ V or ♈ *v*, and of its right Ascension ♈ *r* is *r v*; and the difference of the true and middle place of the Sun M V, or *m v*. Now therefore if from *r v* you take away *m v*, there remains the Arch of Equation *r m*, to be turn'd into time. The like Examples in the other Quadrants of the Ecliptick, every Student in Astronomy may draw to exercise himself by. Fig. III.

CHAP. IV.

Of the Motion of the Moon.

1. BEcause the Moon, as well as the Sun, and the rest of the Planets, moves sometimes faster, and sometimes slower; hence it comes, that its Longitude in the Zodiack is variously alter'd. But besides, because both the Moon, and the lesser Planets do go wide of the *Æquator*, sometimes more, and sometimes less than the Sun: This occasion'd the Consideration of their Latitude or Departure from the Ecliptick: Wherefore in the Sun indeed its Longitude was only to be consider'd, but in the Moon, and the other Planets, we are to consider both Longitude and Latitude.

2. Now as in the Sun we find but only one Inequality of Motion, that is to be made up in a perfect Revolution; so we find, that the Moon, and the other Planets, are liable to a twofold Inequality of Motion; both of which do indeed come into a kind of roundness of Motion (or are made equable) in a certain space of Time: But since there is not a Revolution of both in the same time, therefore from the various compounding or Conjunction of this twofold Inequality, there arises a various Complication of Motions, which

that we may distinctly understand, it will be best to take a View of both Inequalities separately from one another.

For the one, which is called the first, is absolute of it self, because it happens to the Moon and Planets, without any Relation to their Configurations with the Sun: But the other, in the Moon, is called the Monthly; and in the Rest, the Yearly; and also the second Inequality, as being tied to, and depending upon the Moons monthly Congress, or coming together with the Sun; or of the Sun with the Superior Planets: Or, lastly, Upon the Revolution of the Inferior on this side, or beyond the Sun.

3. The first Inequality of the Moon *Ptolomy* explains by an Epicycle, the latter by an Eccentrick: But since the Eccentrick does not only carry an Epicycle, but does also influence its Motion, and in some sort govern it; therefore the second, or the monthly Inequality of the Moon, is first of all to be understood.

Fig. IV. Now therefore in Scheme IV. Suppose T to be the Earth, A T E the Line of the middle Motion of the Sun, going on together with the middle place of the Sun every day 59′, 8″, and governing this second Inequality of the Moon; which monthly Inequality, if it should be made up only once in every Lunation, (*i. e.*) betwixt every Change and Change, the Motion of the Moon's Eccentrick would differ in nothing from the Sun's; and, as in the *Syzygies*, (by which common Name the Full, as well as the New of the Moon, is meant,) the Centre of the Eccentrick is in the Line of the middle Motion of the Sun T A, namely, in the Point 1, so it would perpetually continue in the same Line of the middle Motion of the Sun; and the Moon in every time of its going from, or returning to the Sun, would come once to the Perigée of the Eccentrick, and once to the Apogée. But now, since in every Lunation the Motion of the Moon is twice quickned and abated, by reason of this second Inequality; therefore it must needs be twice in the Apogée, and twice in the Perigée of the Eccentrick: In the Apogée at every Change and Full, in the Perigée in every Quadrature.

This then may be done, if we conceive these three Lines, First, Of the middle Motion of the Sun. Secondly, Of the Moon's monthly Apogée. Thirdly, Of the middle Motion of the Moon, which in the midst of every New and Full Moon do fall into one Line A T E; but after every New, or Full Moon, they are so parted, that the Line of the middle Motion of the Sun lies in the midst between the other two, which lie on each side at an equal Distance: For let the Line of the monthly Apogée A F, which has the point A farthest from the Earth, and the point F nearest, be conceiv'd to be fixt indeed at the Point T, but in the whole length besides to be moveable; so that from its first Situation A 1 F, it may be carried to a second B 2 G, and thence to a third, C 3 c, from thence to a fourth D 4 H, and last of all to a fifth E 5 I. And so while the Apogée of the Eccentrick passes through the Points A B C D E, on the other side the Perigée passes the Points F G c H I. And the Point 1, (which is the Centre of the Eccentrick,) that lies between the Apogée A, and the Perigée F, does in the same time pass the Points 1, 2, 3, 4, 5, and so does by its motion make the little Circle 1, 2, 3, 4, 5: But now if from the Centres 1, 2, 3, 4, 5, be made Circles, whose Semidiametre shall be the space between 1 A, or 1 F; they will give the Position or Situation of the Eccentrick at the several times between the New and the Full of the Moon: For as for the other half, which is from the Full to the New of the Moon, it was not convenient to express it in the Scheme, lest there should be too much Confusion. Now suppose this Motion about the Point T, to be equable, proceeding day by day towards the Antecedent Signs, (that is from A towards B,) eleven degrees and two minutes; so that its return to the Line of the middle Motion of the Sun A T E, may be in a Synodical Month, that is in that space of Time in which the Moon departs from the Sun, and returns to it again, *viz.* in twenty nine days, twelve hours, and forty four minutes. But the Line of the monthly Apogée is something longer in running through the whole Zodiack, *viz.* thirty two days, three hours, and almost five minutes. For since the Line of the middle Motion of the Sun, while the monthly Apogée goes back into the Antecedent Signs, does for its part go forward into the consequent Signs,

and

and so moves towards the other; thence it comes to pass, that they meet one another sooner than the Line of the Apogée can have gone through the whole Zodiack. And the Line of the middle Motion of the Moon departing likewise from the same A T E, it is also equally moved about T into the consequent Signs from the position T A, to the position T *b*, thence to T *c*, T *d*, T E; so that the Angles, which the Lines of the monthly Apogée, and of the middle Motion of the Moon, do make on both sides to A T E, the Line of the middle Motion of the Sun, are constantly equal, A T B to A T *b*, and A T C to A T *c*, *&c.* Now this Line of the middle Motion of the Moon does every day proceed thirteen degrees and eleven minutes; so that its Revolution to the same place of the Zodiack is performed in twenty seven days, seven hours, and forty three minutes; which makes a periodical Month: For the Sun being in the mean while gone a whole Sign from the former place of Conjunction, the Moon does not overtake it till two days after. Hence it comes, that the Moon changing, and the Line of the middle Motion of the Sun being almost in the midst, when the Apogée of the Eccentrick is in B, the Centre of the Epicycle (though the Epicycle be not yet express'd in this Scheme) is in *b*: When the Apogee is in C, the Centre of the Epicycle is in *c*; when in D, the Centre of the Epicycle in *d*; till such times as they meet one another in E. And farther, hence it comes to pass, That when at the New Moon the Centre of the Epicycle is in the Apogée of the Eccentrick in A; then, when it has past a Sign and an half, or is in the midst of the first Quarter, it is in T *b*, the middle distance from the Earth; in the Quadrature or first Quarter, 'tis in the Perigée of the Eccentrick, and in the other half quarter it is again in T *d*, the middle distance from the Earth, till at the Full Moon it return to the Apogée in E. Likewise after the Full Moon, the Centre of the Epicycle comes to the Perigée of the Eccentrick, and about the New Moon it returns to the Apogée; so that in the whole Lunation it makes not a Circle, but a kind of Elliptick, or an Oval Line, such as is shown in this and the following Scheme, by the Points A *b c d* E *f g h* A. And so at length it hence follows, that the distance of the Moon from the Apogée, for instance, B T *b*, is always double to its Elongation, or farthest departure from the Sun A T *b*. And whereas in the Sun, the distance of the Line of the middle Motion from the Line of the Apogée, is call'd the Anomaly or the Argument; here, in the Moon, it is called, not by the same, but a different Name, to wit, The Centre of the Moon. Now all that this second, or monthly Inequality of the Moon does, is this; It makes the Centre of the Epicycle, twice in a Lunation, draw near the Earth, and makes it twice be removed farther from the same, as it plainly appears in Scheme V. Whence it happens, that the second Inequality alone, without the Consideration of the Epicycle, does nothing alter the Motion of the Moon's Longitude: For though the Centre of the Epicycle does, by such a Motion, come sometimes nearer to the Earth, and does again depart farther from it; yet the Angles A T *b*, A T *c*, (the space of which it recedes from the Line of the middle Motion of the Sun,) are always increased equally; and so to our sight, who are plac'd in T, the Centre of the Epicycle seems to move equally. Therefore hitherto we have been laying the Foundation for the second Inequality, and have shown from whence it arises; though we cannot, as yet, understand the Effect of it, before we make use of an Epicycle; which, besides receiving and drawing into it self the Effects of the monthly Inequality, does also in its Gyration, or wheeling about, add another Inequality of its own, called the Absolute, which I am now about to explain. Fig. V.

4. Now then in Scheme V. where there are the same Letters over again, that there were in the former, and they signifie the same; *viz.* A T E, the Line of the middle Motion of the Sun, *&c.* Now the Epicycle by the Motion of the Eccentrick above explain'd, obtains a various distance from the Earth T, and moreover it goes round by its own proper Motion, carrying about with it the Body of the Moon, sometimes from *i*, towards K and *y*, sometimes again from *y*, toward Z and *i*; whereby it happens, that the Moon does by this Inequality (which is call'd the first) depart from the Line of the middle Motion T A, sometimes towards the side K, sometimes toward Z; so that running through Fig. V. the upper part of the Epicycle Z *i* K, (which is the part far- Fig. V.

theſt from the Earth,) it is carried contrary to the order of the Signs, and ſo abates of its Motion: But on the other hand, being in the lower Semicircle of the Epicycle K y Z, which is neareſt the Earth, it is carried according to the Order of the Signs, and ſo its Motion is made ſwifter. And this Revolution of the Moon in the Epicycle, and the firſt Inequality which riſes from it, is perform'd neither in a Periodical month, nor a Synodical, but in a ſpace of time leſs than a Synodical one, but greater than a Periodical.

For whereas a Periodical Month is made up of twenty ſeven days, ſeven hours and 43′, and a Synodical one of twenty nine days, twelve hours and 44′; this Revolution of the Moon in the Epicycle is finiſh'd in twenty ſeven days, thirteen hours and almoſt 9′; and for this reaſon this Inequality is called the Abſolute, as being tied to no certain Configuration of the Moon with the Sun. For when, for inſtance at any new Moon, the Moon is in *i* the Apogée of the Epicycle, in the half Quarter it will be in *b*, diſtant from the Epicycle's middle Apogée K the Arch K L, but from the Epicycle's true Apogée *r* the Arch *r* L; K L indeed is call'd the middle Anomaly, or the middle Argument, but *r* L is the true; but the true Apogée of the Epicycle, is a Point in it very remote from the Earth, which ſhews the Right Line, drawn from the Earth T, thro' the Centre of the Epicycle: But now the middle Apogée of the Epicycle K, as ſhown by the right Line 6 *b* K, drawn through the Centre of the Epicycle from 6 the Point, that is oppoſite to the Centre of the Eccentrick when 'tis in ♑. For in the *Syzygies* and Quarters of the Moon, the true Apogée of the Epicycle falls in with the middle Apogée; but in the times betwixt thoſe they vary from one another. Then the Centre of the Epicycle being mov'd on into *c*, the Moon will be in M in a Quarter, that is, ninety degrees remov'd from the Sun; and its middle Anomaly as well as its true will be I M. So the Centre of the Epicycle going on into *d*, the Moon will be in N, diſtant from the Epicycle's middle Apogée *m*, the Arch *m s* N, but from the true Apogée *s* the Arch *s* N. In oppoſition the Moon will be in O, diſtant from the middle, as well as true Apogée of the Epicycle *n*, the Arch *n* ♑ O, which will be ſomething greater than a Semicircle: For the whole Epicycle runs round ſooner than the Lunation is compleated; and therefore in half a Lunation, the Epicycle makes more than a Semicircle: And ſo farther on you may obſerve the Courſe of the Moon in P Q R, till it be in X, at the end of its Lunation; namely, when in a Synodical Month it has run through the whole Epicycle, and the Arch *i x* beſides: And thence beginning another Month, *viz.* from X, it will paſs through the Points S T V X Y Z, 9, 10. Hence it is, that the Inequality that riſes from the Motion of the Epicycle is call'd Abſolute, becauſe when the Lunation is ended, the Moon does not return to the ſame Point of the Epicycle, from whence it did depart: But by the Elliptical Motion of the Epicycle, which is conſtantly finiſh'd in every Lunation, the Epicycle it ſelf does approach the Earth twice in every Month, and is twice remov'd farther from it: Wherefore it muſt needs be, that not only the whole Epicycle, but alſo its parts, and its Arches of Anomalies K L, L M, M N, muſt according to the different diſtance from the Earth, afford different Appearances of magnitude to thoſe that behold them from T, and ſo that departure of the Moon from the Apogée of the Epicycle, or from the Line of middle Motion, though it be in it ſelf the ſame, yet it muſt ſeem greater or leſs, according as the Epicycle is nearer to, or farther from the Eye. And thus the Motion of the Eccentrick, making ſuch an Approachment, and again ſuch a Retreat of the Epicycle, is the cauſe of the ſecond Inequality of the Moon, which is call'd the monthly: And the effect of the ſame we perceive in the Epicycle it ſelf, which for that reaſon appears ſometimes leſs, and and ſometimes greater, and ſo ſhews the Arch of the Moon's Digreſſion in like manner, ſometimes greater, and ſometimes leſs. But however the very Eye does without much difficulty diſcern, that the Epicycle, when in *c*, is ſeen under a greater Angle M T S, than if the ſame being plac'd in A, ſhould be ſeen under the Angle Z T K; and that the utmoſt Digreſſion of the Moon *l* T M ſeems greater there, than even the greateſt *i* T K does ſeem here. Now what is ſaid of variouſneſs of the greateſt Digreſſion, the ſame is likewiſe underſtood of any other Digreſſion whatever. But farther, the declining of the true Apogée of the Epicycle from the middle one, does alſo help the ſecond Inequality; by which it comes to paſs, that the Epicycle, about the time of Conjunction or oppoſition, com-

coming from *b* by A to *b*, the middle Apogée *q* comes nearer and nearer to the true one *u*; untill it be joyn'd to it in *i*, and thence it runs beyond the true Apogée *r* into K, the Motion tending towards the Antecedent Signs, and so the same way that the Moon is carried; and therefore for this reason the Motion of the Moon in the Epicycle must needs be quicken'd: For seeing the Moon departs from the middle Apogée, as from a certain Goal, always with an equal swiftness, if the Goal it self, from whence the Motion is computed, do together with it go on towards the same place, the progress of the Moon must certainly be so much the more advanc'd, by how much the Goal helps it by its Motion. But it is true, that the contrary happens about the Quarters for the contrary reason, that is, before and after them the Motion of the Moon in the Epicycle is made slower. And now the Arch *r* K, or the Angle *r b* K, by which the middle Apogée differs from the true, is call'd the *Prosthaphæresis*, or the Equation of the Centre, and is equal to the Angle 6 *b* T. But the Angle *r* T L, that lies between the Lines of the middle Motion of the Moon *r* T, and of the true Motion of the same T L, (drawn from the Earth through the Body of the Moon,) is called the Equation of the Argument; and whereas there the *Prosthaphæresis* of the Centre is added to the middle Argument, that the true may be found; if the Centre of the Moon be less than six Signs; and is substracted, if it be greater: Here on the contrary, the Equation of the Argument is substracted, if the Centre of the Moon be lesser than six Signs; and is added, if it be greater.

5. But seeing 'tis usual in the old Astronomy to compute the Inequality of the Moon, and so of the other Planets, by excess and proportional Scruples, that the Reader may not be quite ignorant of this doctrine, let him take briefly thus: The Epicyclical *Prosthaphæreses*, as for instance *r* T L, by reason of the Epicycle's Approachment to the Earth, are increas'd, and so do admit of as many varieties as there are divers distances of the Epicycle it self from the Earth: But to go about to compute the *Prosthaphæreses* of the Moon as in every degree of the Epicycle, for so many diverse distances would be too troublesome and mad a piece of work: Therefore the Ancients computed the said *Prosthaphæreses* only for two distances of the Epicycle, to wit, the greatest and the least; whence were made the least of the Epicyclical *Prosthaphæreses*, belonging to the Moon, when in the Apogée of the Eccentrick, which *Prosthaphæreses* they laid down in Tables; but the greatest that were found by the same Computation, which belong to the Situation of the Epicycle in the Perigée, they did not draw up in Tables, but they put down the differences, whereby these exceeded the least, by the name of Excess, divided into sixty Scruples. Then to any distance of the Epicycle from the Earth they added proportional Scruples, which shew how much of the whole Excess, or rather Increase of the *Prosthaphæreses* was owing to such or such a Situation of the Epicycle between the farthest and the neareſt. For the proportional Scruples being brought into the whole Excess, out of them came a portion to be added to every *Prosthaphæresis* of the Epicycle in the Apogée, that so may come a just *Prosthaphæresis* for such a distance of the Epicycle from the Earth. And thus have I laid down what seem'd requisite, of *Ptolomy*'s Theory of the Moon.

6. The other *Phænomenon*, wherein the Moon appears to depart from the *Æquator* sometimes more, and sometimes less than the Sun does, is explain'd by the Motion of Latitude: For as the way of the Sun, or the Ecliptick, does cut the *Æquator* obliquely in the two opposite Equinoctial Points, so the way of the Moon or the Eccentrick which it moves in, does not directly lie under the Ecliptick, but cuts it obliquely in two opposite Points, call'd Nodes or Knots; and that is nam'd the Ascendent, by which it passes from South and North; and that the Descendent, by which it passes from North to South: And therefore it comes, that the Moon, in moving in its Orbit or way, is found twice in the Ecliptick in every Period or Revolution of it; namely, when it is in the Nodes. At all other times, it is distant from it more or less: For its greatest Digression happens about ninety degrees from the Nodes as may be understood from this Figure VI; where ☊ is the Ascendent Node, or the Dragon's Head, ☋ the Descendent Node, or the Dragon's Tail; ☊ B, ☋ A, the Orbit of the Moon, ☊ E, ☋ the Ecliptick, B the Northern Bound, A the Southern. Now the distance of the Moon from the Ecliptick, taken in the Circle of Latitude, that is Fig. VI.

drawn

drawn perpendicular through the body of the Moon to the Ecliptick, is call'd the Latitude of the Moon; which, when greateſt, is of five degrees. But here 'tis to be noted by the way, That the Nodes of the Moon are not fixt in any certain Points of the Ecliptick, but do go on by little and little towards the Antecedent Signs, every day ſome little more than three Minutes, ſo within almoſt nineteen years they go their round.

Now this Theory of the Moon, which I have now explained, is ſtill but lame and defective, upon ſeveral Accounts. Later Ages have indeed, in a great meaſure, mended its Imperfections; but yet one thing above all the reſt ſeems to be intolerable in it, That in the Quarters, it makes the Moon be too near the Earth; whereby the apparent Magnitude of the Moons Diametre would, about thoſe times, become almoſt as big again as in the *Syzygies*; which even the Vulgar themſelves know to be falſe.

CHAP. V.

Of the Phaſes, *or the Appearances of the Moon.*

THE Moon is an Opaque, or thick and dark Body, and does in no wiſe let the Rays, which it receives from the Sun, paſs through it, but reflects them back again, as the Earth does: And farther, ſeeing it is of a Spherical or round Figure, no more than about the half, or little more, is enlightned by the Sun; and the other part, which is turn'd from the Sun, is in Darkneſs, and does not caſt any Light to our Eyes, except perhaps ſome faint kind of Light, about the New Moon eſpecially. But as the Rays that come from the Sun, and fall upon the Moon, do diſtinguiſh the enlightened Hemiſphere from the Opaque; ſo again, The Rays that go from our Eyes to the ſame Globe of the Moon, do diſtinguiſh the viſible Hemiſphere, or half of the Moon, from that which is not ſeen. If therefore it ſo happen, that the Rays that go both from the Sun, and from our Eye, do fall upon the Moon on the ſame ſide, then the enlighten'd Hemiſphere muſt needs be the ſame with that which is viſible; ſince the Sun and the Eye do look upon the Moon from the ſame parts, or on the ſame ſide, as I ſaid, and then the Full Moon appears to us. But on the contrary, when the Moon is plac'd in the midſt, between the Sun and our Eye, it happens, That the Hemiſphere that is enlighten'd by the Sun, is to us inviſible; but the other Opaque Hemiſphere that is turn'd from the Sun, is then towards us; and then the Moon is not taken out of the Heavens, but to our Eyes diſ-appears; at which time is that which we call the New Moon, or the ſpace between one Moon and another, or the thirſty Moon. It may alſo happen, that not the whole enlighten'd Hemiſphere may be ſeen by us, as 'tis in the Full Moon; nor the whole Opaque Hemiſphere be turn'd towards us, as 'tis in the New Moon; but part of the enlighten'd, with part of the Opaque, may be turn'd towards us, as 'tis at the times between the Change and Full, and then the Moon ſhines not with an whole Hemiſphere, but with a greater or leſs part of it, according as leſs or more of that which is enlighten'd does offer it ſelf to our Sight: And hence are all theſe different *Phaſes* of the Moon, which appears horned, then an Half Moon, then bunching out on each ſide, and then a Full Moon, as may be ſeen
VII. in Scheme VII. where the Moon running through its Orb, from the time of its Conjunction with the Sun, does ſucceſſively appear ſuch as it is here repreſented by thoſe parts of the Circle that are left white. And the Cauſe of theſe
VIII. *Phaſes* may in ſome meaſure be underſtood from Scheme VIII. while the Moon goes about the Earth, the Sun does continually enlighten one half of it; but of that half ſometimes more, and ſometimes leſs, and ſometimes none at all, is turn'd towards us. And the Semicircles that are turn'd toward the Earth, or toward the Eye, are to be conceiv'd as if they were half Globes; and the bending which cannot be repreſented in a Plain, is to be ſupplyed by Imagination. Now here three things eſpecially are to be obſerv'd; Firſt, That the

ſame

ſame ſide of the Moon is always towards our Eyes, which is prov'd by the Spots of the Moon, which always appear to be the ſame. For if ſometimes one, and ſometimes another ſide of the Moon ſhould be towards us, different Spots would ſurely appear. Some indeed object, That this can in no wiſe conſiſt with the Motion of the Moon in the Epicycle; For in Scheme XXX. while the Centre of the Epicycle comes from A, through *b*, *c*, *d*, to E; if the Epicycle ſhould be depriv'd of its proper Motion about its own Centre, and the Moon that is fixt in it be in *i*, the Moon would be carried, together with the Epicycle, into *n*; and the Parts, as well of the Epicycle, as of the Moon, which in A were turn'd to or from the Earth T, the ſame would alſo be turn'd to or from it in E: But if the Epicycle ſhould moreover be conceiv'd to roll about in the mean time, with the Body of the Moon fixt in it, from *n* through 12 to O; then truly that part of the Moon, which being in *n*, was turn'd from the Eye plac'd in T, the ſame being in O would be turn'd from it, and ſo contrariwiſe. But the Anſwer to this is eaſie; For 'tis but ſuppoſing the Body of the Moon to be mov'd about its own Centre, that it may be as much turn'd towards the Earth by its own Motion, as by the Motion of the Epicycle it is turn'd away from it. Fig. XXX.

Fig. VIII. The ſecond thing to be obſerved is, That the Moon does firſt of all appear with very ſmall Horns, ſometimes one day, ſometimes two days, and ſometimes not till three days from the time of its Conjunction or Change: And the ſame may be ſaid of its hiding itſelf before its Conjunction; of which three Cauſes may be aſſign'd. The firſt is the ſlowneſs or ſwiftneſs of the Moon's Motion: For with that part of the Epicycle, which is towards the Earth, the Body of the Moon tends towards the Conſequent Signs; and ſo the ſame way that the Centre of the Epicycle does; whereby its Motion is made more ſwift. The contrary happens, when the Moon, being in that part of the Epicycle which is fartheſt from the Earth, moves againſt the order of the Signs, and ſo takes off from the Motion, by which the Centre of the Epicycle does continually move into the Signs Conſequent. Now, by how much the Moon moves the faſter, ſo much the ſooner does it depart from the Sun, and ſhew its Horns. Another Cauſe is the Moons being plac'd in Signs of long or ſhort Aſcenſions or Deſcenſions: For when the New Moon is in Signs of long Aſcenſions, namely, ♍, ♎, ♐, ♑, ♉, ♊; then the degrees of the Ecliptick, that are between the Sun and the Moon, are longer in going down, and make the Moon ſtay longer above the Horizon after Sun-ſet; and ſo it gets out of the Twilight the ſooner, from the Time of its Change, ſo as that it may be ſeen. In like manner, the Old Moon being in the Signs of long Aſcenſions, ♋, ♌, ♍, ♎, ♏, ♐, ariſes the earlier before the Sun; and ſo, though it be near the Conjunction, yet however it is ſeen. The contrary happens to the New Moon, when in Signs of ſhort Aſcenſions; and to the Old Moon, when in Signs of ſhort ones. The third Cauſe is the Northern or Southern Latitude of the Moon: For in the Northern Hemiſphere of the Earth, by how much the Moon is more North, ſo much the ſooner does it ariſe before the Sun, and ſets the later after it; whereby again, it is the ſooner ſeen, when 'tis New; and the longer when 'tis Old. But it happens to the contrary, when it is more South. Fig. VIII.

The third Thing worth Obſervation, is that faint kind of Light, which beſides its Silver Horns, appears both in the New and the Old Moon, and makes the reſt of its Hemiſphere viſible: 'Tis call'd its ſecondary Light, and is commonly thought to be the Native Light of the Moon. But 'tis to be obſerv'd, That it is owing to the Earth, which reflects towards the Sun, and the neighbouring Region, the Rays that it receives from it. When therefore the Moon, about the time of its Conjunction, is within that Region, it partakes of that Reflected Light: But does not ſo, when it is remov'd from the Sun more than a Quarter, or 90 Degrees: And ſo it being too far out of the Region of Reflexion, that faint Light now ſpoken of vaniſhes.

CHAP.

CHAP. VI.

Of the Eclipſe of the Moon.

1. THE Greek word *Eclipſis* denotes a defect or want of Light, which in the Moon is real, for as much as it has no Light of its own; wherefore the Earth coming between the Sun and it, the Rays muſt needs be intercepted, whereby otherwiſe it is wont to be enlighten'd: For the Earth is an opaque Body, that caſts a ſhadow from it, which when the Moon falls into, it is darken'd. Now the ſhadow of the Earth does not extend it ſelf beyond the Region of the Moon, becauſe no other Star or Planet is found to fall into it, to ſuffer ſuch a defect; whence it manifeſtly follows, that the Sun muſt be greater than the Earth; for if it was equal, the ſhadow of the Earth would be cylindrical, as in Scheme IX. or if the Sun was leſs than the Earth, its ſhadow would be found to be Calathoide, or of the Form of a Basket, growing wider and wider from the Top downwards, as in Scheme X. And either way the ſhadow would reach *in infinitum*, whereby other Planets alſo would ſometimes be involv'd in it. But it is Conical or like a Top, ſuch as it appears in Scheme XI.

Fig. IX.

Fig. X.

Fig. XI.

2. But farther hence it may be underſtood, that ſeeing the Sun moves perpetually under the Ecliptick; therefore its ſhadow muſt always fall upon the oppoſite Point of the Ecliptick: Wherefore if the Moon ſhould conſtantly move, as the Sun does, under the Ecliptick, it would neceſſarily ſuffer an Eclipſe every full Moon, when it came into that Point of the Ecliptick, that is oppoſite to the Sun, and is within the ſhadow of the Earth. But it was ſaid in the Theory of the Moon, that it walks in a Circle, that inclines obliquely to the Sun's way, and paſſes the Ecliptick onely twice every Month; when therefore the Sun is far from the Nodes of the Moon, then alſo the Moon, that is oppoſite to it, is as far remov'd from them, and ſo has a Latitude or diſtance from the Ecliptick, greater than that it can come under the ſhadow that lies there. Yet becauſe the Sun paſſes through the whole Zodiack by its yearly Motion, it muſt needs in that time come to both the Nodes of the Moon, little leſs than ſix Months always being ſpent between its coming to the Aſcendent and the Deſcendent Node. And though the full Moon does not continually happen at the very ſame moment of the Sun's coming to the Node, yet the Moon can ſcarce eſcape the ſhadow, which is ſo broad, that though the Full Moon be more than ten days diſtant from the meeting of the Sun and of the Node; yet the Moon is not then gotten ſo far from the Ecliptick, as to eſcape untouch'd. But yet if the meeting of the Sun and of the Node, happen to be at the New-Moon, or a day either before or after; then both the foregoing and following Full Moon will be ſo far from thence, that it will clearly avoid an Eclipſe. Now by how much the farther the Full Moon is from the Nodes, ſo much leſs of the ſhadow will it fall under; whence it comes, that ſome Eclipſes of the Moon are Total, which happen either in the very Node, or ſomething near it; others are only partial, and ſo much the leſs, by how much they happen the farther from the Nodes. Beſides, the Moon's diſtance from the Earth, which is ſometimes greater and ſometimes leſs, is the cauſe why Eclipſes appear ſometimes leſs, and greater than they do at other times; for the Cone of the ſhadow growing ſmaller and ſmaller, till it come into a Point, therefore the farther it is extended from the Earth, the ſlenderer it grows: Wherefore the Moon in the Apogée, has a ſhadow to paſs thorow, that is not ſo thick as that it has in its Perigée, and ſo it is leſs obſcur'd. Moreover the Motion of the Moon being ſometimes ſwifter and ſometimes ſlower, is the reaſon why it paſſes thorow the ſhadow the ſooner, or ſtays the longer in it. Again, a total Eclipſe is Central, when the Full Moon happens to be in the very Node, as in Figure XII. where A B is the Ecliptick, C D the Orbit of the Moon, E the Moon firſt entring into the ſhadow, F the Motion going out again, G the Moon in the very Centre of the ſhadow. But now a Non-central, though a total one, may be underſtood by Figure XIII. where the Centre of the Shadow, at the time of the greateſt Obſcuration, is not of the Node H, but out of it.

Fig. XII.

Fig. XIII.

But

But before I come to that, you may perceive by the XII Figure, that Central Eclipſes, that are of the greateſt duration, can ſcarce laſt a few Minutes above four Hours: For ſeeing the Moon, in the time from the beginning of its Obſcuration to the end, does with its Centre go through the Line E F, which is equal almoſt to four Moons, (now the Moon takes up in the Heavens about half a degree,) therefore E F will be about two degrees. Now the Motion of the Moon for every hour is almoſt half a degree above the Sun, (and ſo above the Shadow;) wherefore the Moon will go through thoſe two Degrees in about four hours time. And when the total Immerſion of the Moon into the Shadow happens in H, and the firſt Recovery of Light in I, ſo that H I is the ſpace the Moon has to paſs, while it ſtays in total Darkneſs; and ſeeing H I is almoſt the half of the whole Line E F; hence it comes to paſs, that half the Duration, in this Inſtance two hours, is the time of the Moon's ſtay in total Darkneſs: For the time of Incidence, while the Moon goes from E to H, from the beginning of the Eclipſe to the very Moment of its Immerſion, or total Obſcuration, is but about an Hour; and the time of Regreſs, while it goes from I to F, from the Moment of its firſt Emerſion, or Recovery of Light, to the very End of the Eclipſe, is about the ſame. And now as for a partial Eclipſe, it may be underſtood from one or two Schemes following. The Orbit or way of the Moon, is repreſented double in each of them, that it may be underſtood, why an Eclipſe happens ſometimes towards the South, and ſometimes towards the North. Now Fig. XIV. ſhews, how the Moon may ſometimes chance to be eclips'd, before it come to the Node; but Fig. XV. how it may be ſo, after it have paſs'd it; which Things are likewiſe to be ſupplyed about Fig. XIII. which is of Eclipſes that are total, but not central. And now the Diametre of the Moon being ſuppos'd to be divided into twelve equal parts, which they call Digits, the quantity of the Eclipſe, or want of Light, is wont to be repreſented by Digits, (and alſo by minutes of Digits, and is accounted greater, or leſs, according as the Obſcuration is of more or fewer Digits.

Fig. XII. Fig. XIV. Fig. XV. Fig. XIII.

But beſides all theſe, there are found in the Moon, when eclips'd, two *Phænomena* more; namely, a faint kind of Shadow, call'd a *Penumbra*, or the utmoſt Edge of the Shadow, ſomething leſs obſcure than the reſt; and a certain reddiſh colour'd Light, that the Moon ſhews out of the very Darkneſs, and which, in total Eclipſes eſpecially, is ſo much the redder and obſcurer, by how much the Moon comes the more towards the Axle or Centre of the Shadow. As for the *Penumbra*, the Cauſe of it is plain enough; becauſe the Earth going under the Sun by little and little, at firſt only part of the Sun is taken from the Moon, but that by degrees grows greater and greater. Now that part of the Moon from whom more parts of the Sun are hidden, muſt needs be involv'd in the thicker Darkneſs; wherefore the *Penumbra* towards the perfect Shadow, does, by little and little, grow more and more obſcure: But now, as for the reddiſh Light that appears out of perfect Darkneſs, which is by ſome thought to be the Moon's own native Light, ſeems rather to ariſe from the Refraction of the Sun's Rays, that paſs through the Earth's Atmoſphere, or Shpere of Vapours; and that decline towards the Axle of the Shadow, and that temper the Shadow with a kind of Twilight, more about the outmoſt Edges, leſs towards the Centre or Axle, whither few or no Rays are reflected; by which the Moon becomes ſcarce, or not ſcarce viſible.

CHAP. VII.

Of the Eclipſe of the Sun.

THAT which is called an Eclipſe of the Sun, might more fitly be called an Eclipſe of the Earth: For it is the Earth that is then depriv'd of the Light of the Sun by the Moon's coming between; juſt as the Moon when eclips'd, is depriv'd of it by the Earth's coming between it and the Sun: whereas

the Sun all the while keeps its Light untouch'd and unobscur'd; yet the Sun is said to suffer an Eclipse, in as much as with respect to us it does so. And here, that the Sun is to us Eclips'd, by reason of the Moon's putting between us and the Sun, is manifest even from this, That it is not eclips'd but at New-Moons, or when the Moon is in Conjunction with it. But that it is not eclips'd every New-Moon, the Reason is the Moon's Latitude; because of which it is, that the Moon passes by either higher towards the North, or else lower towards the South, and does not pass directly between us and the Sun: But that happens when the two Luminaries are about the same Node, and that not much above half a Sign from it. And let no one wonder, that there happen far more Eclipses of the Moon than of the Sun; for the Globe of the Moon being much less than the Globe of the Earth, it casts a less shadow: And again, The place of our Habitation is a great deal less than the whole Globe of the Moon; and so consequently the greater shadow (*viz.* of the Earth) does the more easily fall upon some part of the greater space, (*viz.* of the whole Moon,) than the lesser shadow (of the Moon) can fall upon the lesser space of our Habitation. But now this must be understood of some determinate place of the Earth, for instance, of this in which we now live; for otherwise, if we consider the whole Hemisphere of the Earth, (or the half of its Surface, taken for a Plain,) there are more frequent Eclipses of the Sun, than of the Moon: For every six, (and sometimes every five Months) there happen some, either here, or in some other parts of the Earth; nay, when there happen any Central Eclipses of the Moon, the New-Moon, both before and after, is for the most part made remarkable for some little Eclipse of the Sun: The reason of which is the great Bulk of the Earth, and the great nearness of the Moon; which, though it cast no great shadow, yet it threatens the Earth with a *Penumbra*, or faint shadow, that is so much the greater, by how much it is the nearer; which that it should fall upon some part or other of the Earth, is no wonder. But besides, the same Bulk of the Earth causes, That in some places there is a Total Eclipse of the Sun, while in others there is only a partial One, and in others none at all, as may be understood from Fig. XVI. Scheme XVI. where *a* is the Sun, *b* the Earth, *c* the Moon, *f b g* the Cone of the Moon's shadow, whose Base is the Hemisphere of the Moon *f g*; *h o l* is the other Cone of the Moon's *Penumbra*, whose Base is part of the Surface of the Earth *h l*. Now I say, That that little space of Earth *i k*, which falls under the perfect shadow of the Moon, will see a Total Eclipse of the Sun; since a right Line can be drawn from no point between *i* and *k* to any part of the Sun's Hemisphere, but it will be intercepted by the Body of the Moon *f c g*. Again, They that live between *i h* and *k l*, will have a partial Eclipse of the Sun, and that so much the greater, by how much they are nearer to *i* and *k*; but so much the less, by how much they are the farther off towards *h* and *l*. Lastly, They that dwell between *h m* and *l n*, will behold the whole Hemisphere of the Sun, eclips'd in no part of it. This Diversity also of Sight arises from the nearness of the Moon, which if farther off, and near the Body of the Sun, would cover it alike from all the Inhabitants of the Earth. And here by the way, we may in some measure take Notice of the nature of Parallaxes, (for so that Diversity of Sight is call'd;) namely, That they are owing, as to the Bulk of the Earth, so to the nearness of the Moon too: I design to explain them something more largely in the following Chapter. Now as for the Quantity of a partial Eclipse of the Sun, like that of the Moon, it is wont to be estimated or measured by Digits, or by twelve parts of the Sun's Diametre; and as for a Total Eclipse of the Sun, there is this difference between it and a Total Eclipse of the Moon, *viz.* That such an Eclipse of the Moon is generally of some remarkable Duration, since the Moon by its own Motion cannot in a little time free it self from the shadow of the Earth; but a Total Eclipse of the Sun cannot be of any long continuance, at least, not to be very sensible: And so neither can that Darkness be very long, that sometimes happens to be very thick at Noon-day, so that the Stars are seen, the Birds hide themselves, or else fall down; because the Moon by its own Motion running underneath the Sun, towards the East, as soon as by its Eastern Verge or Edge it is come to the Sun's Eastern Verge, and so covers the

the whole Sun, it presently begins with its Western Verge to quit the Sun's Western, and so to discover the Sun again. But farther, it sometimes happens, because the apparent Hemiphere of the Moon in the Apogée is less than when in the Perigée, and therefore is less than the Hemisphere of the Sun, that when the Moon in its Apogée runs underneath the Sun, and shews Centre joyn'd with Centre, there is over and above of the Sun and edge all round, like a Golden Bracelet, or a kind of Golden Circle, because the Moon does not cover it all. Lastly, If you ask, of how long Duration is the greatest Eclipse of the Sun? It seems clear, that it is of about two hours. For since the Diametre of the Sun, which first the Eastern Verge of the Moon, and then the Western must pass beyond, is of about half a Degree, which half degree of the Moon passes in one hour; it follows, that the Time of the Incidence from the Moon's touching the Western edge of the Sun with its Eastern, even to the Central Conjunction, when the Eastern Verge of the Moon has gone over all the Sun, will be about the space of one hour. Then the Time of Emersion, namely, while the Western edge of the Moon goes from the Sun's Western to its Eastern edge, will likewise amount to about an hour more, till the Eclipse be ended.

CHAP. VIII.

Of Parallax.

HAving thus far been explaining the *Phænomena* of the Sun and Moon, that may be salv'd by the *Hypotheses* of the Ancients: There remain still two other *Phænomena*, to wit, the Parallaxes and Refractions, the Efficient Cause of which is not to be ascrib'd to any *Hypotheses* or Circles, but they depend wholly upon optick Grounds and Reasons. Now therefore as to Parallaxes, I have said already, that they are most observable in Eclipses of the Sun, and are owing to the Bulk of the Earth, and the nearness of the Planet. For though the whole Globe of the Earth, with respect to that vast and almost unmeasurable and infinite Expansion, and compass of the highest Heavens, be but like a Point, so that the fix'd Stars are by us observ'd, and to us appear no otherwise from the Earth's Surface, than if our Eye was plac'd in the Centre of the Earth; yet we must conclude otherwise with respect to the Sun and the Moon, the latter especially, whose nearness begets some diversity of sight, which the Greeks have call'd a Parallax. For the Bulk and Thickness of the Earth do make the Sun and Moon seem to have another position in the Heavens, than they really have with respect to the Centre of the Earth: Wherefore the Parallax may be thus describ'd; that it is the difference between the true and the visible, or seeming place of any Star. Now the true place of a Star, is that Point of the Firmament, which a right Line, drawn from the Centre of the Earth through the Centre of the Star, shall terminate in: And the visible and seeming place is likewise that point of the Firmament, which a right Line, drawn from the Eye of the beholder, (plac'd on the Earth's Surface,) through the Centre of the Star, shall then be terminated in. Wherefore because those two Points of the Firmament do fall into one and the same vertical Circle, a Parallax is wont to be defin'd to be the Arch of the Vertical Circle, that lies between the true and the seeming place of the Star. As for example, in Figure XVII. let A be the Centre of the Earth, C B D its Surface, B the Eye of the Beholder, E F G the Vertical in the Firmament. Now let the Star be first of all suppos'd to be in the visible Horizon in the Point H; and then a right Line A H I, drawn from A, the Centre of the Earth, through this Point H, does in the Firmament find the Point I, which is therefore the true place of the Star. Again, A right Line drawn from the Eye B, through the Star H, does in the Firmament find the seen or seeming place K; therefore the Parallax is I K. Afterwards the Star being ascended to L, the Parallax will be M N; if it come to O, the Parallax will be P Q; lastly, if it come to the Zenith R, there will be no Parallax at all: For then the Lines drawn from the Centre A, and the Eye B through the Star R, Fig. XVII.

will fall both upon the same Vertical Point F. For whereas the Parallax is the greatest in the Horizon; as the Star ascends higher and higher, the Parallax is more and more lessen'd, till in the Zenith it come to be none at all. Besides, 'tis to be remembred, that the nearer a Star is to the Earth, so much the greater Parallax is has. For, for instance, a Star in S, that is in the same Line of Vision with H, has the Parallax T K, which is plainly greater than I K, the Parallax of the Star H. Likewise the very same Star, if it ascend to V, when it comes into the same Visual Line with L, it will have the Parallax N X, which is greater than M N the Parallax of the Star L. Lastly, A Star plac'd in Y, in the same Visual Line with O, has Q Z for its Parallax, which is still greater than is Q P, the Parallax of the Star O. Now the measure of the Arch of the Parallax, for instance of I K, is the Angle I H K, or the Angle that is oppos'd to it *Ad Verticum* A H B, which the Lines of the true and seeming place of the Planet do make in H, the Centre of it, and to which the Semidiametre of the Earth is the side of the Triangle, which is Subtended to the Angle aforesaid. For here the Angle A H B, is the difference of the two Angles F B H and F A H, of which the former gives the seeming distance of the Star from the Zenith, and the latter gives the true. But now besides these Parallaxes of Altitude, which are taken in the Vertical Circle, there are also two other kinds of Parallax, to wit, of Longitude and Latitude; the first of which is that Arch of the Ecliptick, which lies between two Circles of Latitude, that pass thorow the true and seeming place of the Star, the second is the Arch of any Circle of Latitude that lies between two Circles, parallel to the Ecliptick, that do likewise go through the seeming and true place of the Star. The chief use of these is in the Eclipses of the Sun. Now this is common to all sorts of Parallaxes, that by them the seeming place of the Star is always lower towards the Horizon, than the true place is: For 'tis a known Observation in opticks, that by how much the Eye is more elevated from the Centre of the Earth towards the Surface, the thing seen will appear so much the lower.

CHAP. IX.

Of Refractions.

HE that desires in some measure at least to understand the Nature of Refractions, let him put a Counter, or the like, in the bottom of a Bason, and then fix his Head in such a posture, that the edge of the Bason may hinder him from seeing the Counter. Then let him bid some body fill it up with water to the brim; and he shall find by Experience, that the Counter, which before could not be seen, will then appear to the Eye, though neither the Counter nor the Eye be mov'd out of their first places: Which is, because the Ray that comes from the Counter, which before light upon the Forehead, is so refracted by passing out of the water into the Air, that it falls upon the Eye, and makes the Counter visible. From this Experiment two things may be gather'd; the first is, That the Passage of the Ray through a twofold Medium, for example, out of Water into Air, or on the contrary, does make a Refraction; the other is, That the Rays that go out of a thicker Medium, as particularly Water, into a thinner, such as the Air is, being refracted, they decline from the perpendicular that falls upon the Surface which bounds both the Mediums: And again, the other way, when they come out of a thinner Medium into a thicker, they incline towards the said perpendicular. Therefore if we conceive the Atmosphere of the Earth, or the Vapours that surround the Earth, to be as they really are, a Body of a Globous Figure, whose Centre is the same with the Centre of the Earth, and suppose the Sun to be in the Horizon; 'tis clear, that the Rays will fall very obliquely, or very much aslope, upon that part of the Atmosphere, that hangs over our Heads; for they will fall upon it in the same manner, that they fall upon the Surface of the Earth, that is under our Feet. And when the Rays of the Sun, out the more pure and thin *Æther*, do enter into

into the Vaporous Air, that inclines something to the Nature of Water; then they will be very much refracted towards the perpendicular, that falls from our Zenith; whence it must needs be, that the Ray that is so refracted to our Eyes, must shew the Sun to be higher than it would be seen to be, if there was no Atmosphere.

Now, though the Quantity of the Horizontal Refraction be by *Tycho Brahe* defin'd to be some little greater in the Sun and Moon, than in the Stars; yet the same thickness of the Air seems to affect, after the same manner, the Rays that come from the same side of the Heavens, let them have pass'd never so great space of *Æther* before they reach the Atmosphere. And, as for that difference, which *Tycho* thought there is betwixt the Refractions of the Sun and of the Stars, it arose from the Parallax of the Sun, which by Him was determin'd to be of three Minutes, which yet scarce makes up the fourth part of a Minute. For by how much the more he suppos'd the Sun to be lower than it is, by giving to it a greater Parallax than it has, he was forc'd by Refractions to advance it so much the higher above the Stars that have no Parallax. Hence it is, that he makes the Horizontal Refraction of the Sun and Moon to be thirty three or thirty four Minutes, whereas He attributes to the Stars no greater than of thirty. But it seems more true, that an Horizontal Refraction of about thirty Minutes belongs to the Sun and Moon, and all the Stars; though indeed Experience plainly Testifies, That Refractions, even in the same Horizon, are greater or less, according to the Thickness or Thinness of the Vapours; and towards the Northern side of the World do generally increase, because the Air is thicker. Whence it came to pass, That the *Hollanders* Wintering in *Nova Zembla*, after a continual Night of some Weeks, the Sun appear'd to them above the Horizon, sooner than they expected, according to the Elevation of the Pole which they had taken. And for the same reason it is, That the Sun being in the Horizon, and being seen by a refracted Ray, seeming to be 30, that is, its whole Diametre, higher than really it is, when it seems to touch the Horizon with the lower Edge of it; it is at that time wholly beneath it.

Moreover, because of the continual Diminution of Refractions it is, that the lower Edge of the Sun being a little more refracted than the upper, it seems to be drawn nearer together to it; whence it comes, That the Sun at its Rising or Setting does not appear exactly round, but Ecliptical or Oval, having its perpendicular Diametre more contracted than the Cross-Diametre is. But though the Moon be as much elevated by Refraction as the Sun itself is; yet it is so, that when it touches the Rational Horizon with its lower Edge, it is as then wholly beneath the Visible: For indeed, if it was not as far from the Earth as the Sun, it would not only All appear, but by the help of Refractions it would leave the Horizon with all its Diametre: But the Parallax of the Moon, which is so near, depressing the same upper Edge again a whole Degree, it makes it all be hidden under the Horizon.

CHAP. X.

Of the Theory of the three higher Planets, Saturn, Jupiter, *and* Mars.

IT has been said above, That the Motion of Longitude of the three higher, as well as of the Moon, is affected with a double Inequality; the one sort of which is absolute, the other is tied to certain Approachments to the Sun. There is Difference between the Moon and the lesser Planets, that the first Inequality of the Moon depends upon the Revolution of the Epicycle, the second upon the Motion of the Eccentrick that carries the Centre of the Epicycle; but the first Inequality of the Planets depends upon the Eccentrick, and the second is owing to the Epicycle. And besides, the Epicycles of the lesser

Planets

Planets move a contrary way, to that of the Moon's Epicycle: For they about the Apogée of the Epicycle, do go towards the Conſequent Signs; but about the Perigée of the ſame towards the Antecedent. Now the Theories of theſe three upper Planets, do agree in having the like Circles and Lines; and are content with the ſame Laws, only they differ in the quantity of their Circles and Motions.

Fig. XVIII. 2. In Figure XVIII. let T be the Earth, C the Centre of the Eccentrick A D E, B the Centre of the Equant, as far diſtant from C, as this is from T; let A be the Apogée of the Eccentrick, P the Perigée, A B C T P the Line of the *Apſes*, T E the Line of the middle Motion of the Planet, to which is always Parallel the Line B D, drawn from the Centre of the Equant, to the Centre of the Epicycle, which being continued to M, ſhews the middle Apogée of the Epicycle, as the Right Line T D V, drawn from T through the Centre of the Epicycle D, ſhews in V the true Apogée of the Epicycle; but the Line C D Q, being drawn from C, the Centre of the Eccentrick, through the ſame D, finds in Q the Point of Contact. Let the Planet be in F, and then T F will be the Line of the true Motion of the Planet.

3. Now here a threefold Motion is to be conſider'd: The firſt of the Line of the Apogée A B C T P, which being affix'd to the Point T, with the reſt of its Length, is, according to *Ptolomy*'s opinion, mov'd with the ſame Motion that the Sphere of the fix'd Stars has. The ſecond is of the Eccentrick, that carries about the Centre of the Epicycle, from A through D to P, by an orderly Motion, not about its own Centre C, as in the Sun, nor about the Centre of the world T, as in the Moon, but about the Centre of the Equant B; namely, ſo that A B D in equal ſpaces of time is equally increas'd; as is alſo the Angle A T E between the Lines of the *Apſes* A T, and of the middle Motion T E. Thirdly, The Motion of the Epicycle carries the Body of the Planet equally from M, the middle Apogée of the Epicycle towards F and G.

4. Now theſe Motions perform'd their Revolution thus: That of the Apogée in all the leſſer Planets once in 36000 years, according to *Ptolomy*, to wit, in the ſpace of time that the Firmament it ſelf moves once round; but the Eccentrick of *Saturn* moves every day two Minutes, that of *Jupiter* five, of *Mars* 31 $\frac{1}{2}$; and ſo *Saturn* goes his Round once in 29 *Egyptian* years, *Jupiter* once in 11 *Egyptian* years and 316 days, *Mars* in one year and near upon 322 days. Laſtly, Theſe Planets are carried about in the Circumference of their Epicycles, the firſt of them in one year and thirteen days and $\frac{1}{4}$, the ſecond in one year and almoſt 34 days; the third in two years and almoſt 60 days.

5. Now the Angle A B D, the ſpace of which the Line of the middle Motion B D, (or T E) has receded from the Line of the *Apſes* A B T, is call'd here as in the Moon, the middle Centre of the Epicycle; but the Angle A T D, is call'd the true or equaliz'd Centre of the Epicycle; and therefore the Angle B D T, which the middle Centre differs from the true, is call'd the Equation of the Centre. And M F the diſtance of the Planet from the middle Apogée of the Epicycle, is nam'd the middle Argument; as V M F the diſtance from the true Apogée, is call'd the true. Now the Angle D T F, is the Equation of the Orb or Revolution; and is ſometimes greater, ſometimes leſs in the ſame diſtance of the Planet F from the Perigée G, according as the Centre of the Epicycle D is farther or nearer to the Earth T. And therefore here again, there is alſo the ſame Diverſity of the Diametre, that there was before in the Moon; and the ſame proportional Scruples too, and the ſame ſort of Exceſs, though of a different quantity in each of the Planets.

6. Now firſt of all, That the Concurrence or Agreement of the Superiour Planets with the Sun, to Followers of the old Aſtronomy, ſeem'd to be very wonderful; *viz.* That the Revolution of every one of them in their own Epicycles, ſhould be compleated juſt in ſo much time, as there is from one middle Conjunction or oppoſition with the Sun to another; ſo that in every middle Conjunction, the Planet is in the middle Apogée of the Epicycle, and in every oppoſition it is in the Perigée; and ſo the Planet is always as far diſtant from the middle Apogée of the Epicycle, as the Line of the middle place of the Sun, has departed from the Line of the Motion of the Planet, and conſequently by Subtracting the middle Motion of the Planet from the middle Motion of the Sun,

Sun, the middle Argument of the Planet is found remaining. Whence it appears, that by how much the more slowly the Centre of the Epicycle goes forward in the Eccentrick, as particularly in *Saturn*; so much the sooner does the Sun return around to it again, and likewise the Planet goes the sooner round in the Circumference of the Epicycle, and on the contrary.

7. The Motion of the Superiour Planets in Latitude has also a twofold variety, (according to *Ptolomy*'s opinion at least:) For besides that the Orbit of the Planet inclines towards the Ecliptick, the Plain also of the Epicycle declines so from the Orbit, that it is parallel or almost parallel to the Plain of the Ecliptick. In Figure XIX. let the Line *a* T *b* represent the Ecliptick; (for it is well known, that a Circle may be so turn'd to the Eye, that it may appear to nothing else but a right Line;) and let *l* T *m* be the Orbit of the Planet, or the Eccentrick, cutting the Ecliptick in T. Now I say the Plain of the Epicycle *f l e*, or *g m h*, is not continued on with the Plain of the Eccentrick *c* T *d*, but declines from it the Angle *c l e*, which is almost equal to the Angle *c* T *a*, the space of which the Orbit inclines to the Ecliptick; so that *f l e* the Epicycle, and *a* T *b* the Ecliptick, are as it were parallel one to another. And hence it comes to pass, that the Latitude of the Planet does not only run out from the Ecliptick in the Periodical Motion of the Eccentrick, sometimes Northward, and sometimes Southward; but in every Revolution of the Epicycle, the same is increas'd by the Planet's coming to the Perigée of the Epicycle *f* or *g*, and is diminish'd by the Planet's being about the Apogée of the Epicycle *e* or *h*. For if the Epicycle *c l i* was instead of *f l e*, and so should fall upon the same Plain with the Orbit *c l i* T; the Planet, being in the Apogée of the Epicyle *c*, would then be farther distant from the Ecliptick *a* T, than it is now found to be in *e*, the Apogée of the Inclined Epicycle: And, on the other side, *f* the Perigée of the Inclined Epicycle, is farther distant from the Ecliptick *a* T, than *i* the Perigée of the Epicycle, when it falls upon the same Plain with the Orbit *c l i* T. Now the greatest Latitude of *Saturn* can scarce come to two Degrees, and five or six Minutes; nor of *Jupiter* to above one degree, and also five or six Minutes; nor of *Mars* to above seven degrees in all. Fig. XIX.

CHAP. XI.

Of the Theory of Venus.

1. THE Motion of *Venus* in Longitude, according to *Ptolomy*, has the same Circles and Lines belonging to it, that the superiour Planets have, and is explain'd with the same terms: And moreover the Motions of the Eccentrick, and of the Epicycle do tend the same way, though the Motion of the Eccentrick is not in every place govern'd by the same Law; for that Line of the middle Motion, which in the superiour Planets was absolute, or not tied to any other, is here so tied to the Line of the middle Motion of the Sun, that it is the very same; I say the Line T E, of the Figure XVIII. does in *Venus* always agree with the Line of the middle Motion of the Sun, to which is always parallel the Line B D, (which therefore may be also call'd the Line of middle Motion,) drawn from the Centre of the Æquant B, through the Centre of the Epicycle D. Fig. XVIII.

2. Now though the Inequality, which happens to *Venus*, by reason of the Motion of the Eccentrick, that agrees with the Motion of the Sun, may seem to be tied, (as well as the other which is deriv'd from the Epicycle;) yet in a true and sound sense, it is to be reckon'd absolute; namely, because the Motion neither of the Sun, nor of the Eccentrick of *Venus*, is tied to the coming together of *Venus* and the Sun; but she may be in Conjunction with him, in what place soever of the Eccentrick the Centre of the Epicycle is found.

3. But now from this continual Connexion of the Lines of the middle Motion of the Sun, and of *Venus*, there follows another certain thing, through which it is, that her *Phænomena* are very much different from those of the Superior

Superior Planets; namely, That *Venus* can never go farther from the Sun, than her Epicycle will suffer her: And for this reason she is the Sun's constant Companion, scarce going from him at any time above half a Sign. And whereas the Superior Planets, in the Perigée of their Epicycles, are in Opposition to the Sun, and the Apogée in Conjunction with Him; She both in the Apogée and Perigée is in Conjunction with the Sun.

4. Although, by reason of the Epicycles of all the lesser Planets tending the same way, it happens, that they are all Retrograde in the Perigée of the Epicycle, because it is contrary to the Motion of its Centre, and move the fastest when in the Apogée of the Epicycle, because 'tis carried the same way with its Centre, and therefore quickens the Motion of the Planet; yet they are Stationary, when they are a little below the Contingent Line drawn from the Earth to the Circumference of the Epicycle, where the Planet goes as far back in the Circumference of the Epicycle, as it is mov'd forward by the Centre of the same.

5 Now *Venus* moves from the Apogée of the Epicycle round to it again, in almost nineteen Months; so she is about nine Months and an half in going from Apogée to Perigée, and is all that time the Morning Star: but the other nine Months and an half, while she is ascending from the Perigée to the Apogée of the Epicycle, she rises as the Morning-Star before the Sun.

CHAP. XII.

Of the Theory of Mercury.

1. THIS Planet in all things agrees with *Venus*, excepting the Motion of the Eccentrick, and the different Quantity of Lines and Motions. Now as for the Motion of the Eccentrick, in Figure XX, let T be suppos'd to be the Earth, B the Centre of the Equant; 1, 2, 3, 4, 5, 6, the Equant it self; C the Centre of the little Circle, in whose Circumference goes about the Centre of the Eccentrick, according to the order of the Points, *g*, *h*, *i*, B, *k*, *l*. The Circles made with Points represent the Eccentrick in its various Positions; and I have made G and *g*, as also H and *h*, and so on, that is, the great and little Letters of the same kind, to signifie for instance, that G is the Circumference of the Eccentrick, when its Centre is in *g*, and H when it is in *h*, and so on: Whence it follows, That it happens only once in every Revolution of the Eccentrick, (which is yearly, being equal to the middle Motion of the Sun,) that the Centre of the Eccentrick is joyn'd in B to the Centre of the Equant, and so that the Eccentrick is the same with the Equant 1, 2, 3, 4, 5, 6, but at all other times they are different. Now T B is equal to B C, and T B C drawn out on each side is the Line of the middle Apogée, which goes forward with the same Motion that the Firmament has, and is the same with the Line of the true Apogée, when the Eccentrick and the Equant are coincident, otherwise it differs from it: For when, for Example, the Centre of the Eccentrick is in *h*, then T B C *g* does indeed remain the Line of the middle Apogée, but the Line of the true Apogée is to be drawn from the Earth T, through *h* the Centre of the Eccentrick.

Fig. XX.

2. Now therefore the Motion of the Eccentrick is compounded of two Motions, *viz.* Of the Centre of the Eccentrick, which is carried about in the little Circle *g h i* B *k l* with a Motion equal to the middle Motion of the Sun; and then of the Line of the middle Motion of *Mercury*, passing through the Centre of the Equant B, and which is always parallel to the Line of the middle Motion of the Sun: So that these two Motions agree in the Quantity of the time of their Revolution, but do tend two contrary ways; the first from *g* through *h* and *i*, but the second from being in the Situation of B 1, to the Situation of B 2, B 3, *&c.* for in the place where the Lines B 1, B 2, *&c* do cut the Eccentrick, thither you shall find the Centre of the Epicycle carried by Motion of the Eccentrick. Let the Centre of the Eccentrick be in *g*, then the

the Line of the middle Motion of *Mercury* will be B 1, and the Centre of the Epicycle in G. Again, Let the Centre of the Eccentrick be in *h i* B *k l*, the Line of the middle Motion of *Mercury* will be B 2, B 3, B 4, B 5, B 6, and the Centre of the Epicycle in H I *b* K L.

3. And now hence 'tis evident, That the Centre of the Epicycle being carried about by such a compound Motion of the Eccentrick as this, does, in moving about the Earth T, make an Oval Figure G H I *b* K L. And farther, by reason of the same Motion it happens, that the Centre of the Epicycle is twice in every Revolution in its Perigée in the Points I and K, (which are nearer the Earth T, than is the Point *b*,) but is only once the farthest from the Earth in the Point G.

4. And so the Motion of the Centre of the Epicycle, which agrees with that compounded of the Motion of the Eccentrick, being thus conceiv'd, it will be afterwards easie to take a view of those other things, that farther happen to the Epicycle: For in Figure XXI, the Line drawn from the Centre of the Equant B, through the Centre of the Epicycle, shews the middle Apogée of the Epicycle in the Point M; but one drawn from the Earth T, through the Centre of the Epicycle, (being the Line T V,) it shews in V the true Apogée of the Epicycle. Lastly, The Line *g* G Q, or *h* H Q, or *i* I Q, being drawn from the Centre of the Eccentrick *g h i*, *&c.* through the same Centre of the Epicycle, it finds in Q the Point of Contact: And so *Mercury* is moved in the Circumference of the Epicycle, always by equal Motions, departing farther and farther from M, his middle Apogée, till he return to the same again, after the space of one hundred and fifty days: Whence it follows moreover, that he is in Conjunction with the Sun in the Apogée of the Epicycle, as well as in the Perigée, as was said before of *Venus*; Namely, because his as well as her Line of middle Motion T E is always joyn'd with the Line of the middle Motion of the Sun, and to it is continually parallel the Line B M, which may also be called the Line of the middle Motion. It also follows, that *Mercury* departs no farther from the Sun, than his Epicycle will suffer him; which since, in proportion to its Eccentrick, it is less than the Epicycle of *Venus*, it thence comes to pass, that his Departure from the Sun is the less, *viz.* scarce at any time a whole Sign. But his greatest Digression in the Epicycle, when his Centre, is nearest the Earth in I and K, appears to be near upon 23^d, $52'$; which yet, when the Centre of the Epicycle is in the Apogée of the Eccentrick G, does but come to little more than 19^d; which is to be imputed to the Diversity of the Diametre of the Epicycle, which is seen sometimes under the Appearance of a greater Angle, and sometimes of a less. Now *Mercury* appears in the Evening, while it is carried from the Apogée of the Epicycle V, through F, to the Perigée G; and in the Morning, while it is again ascending from G to V: And if there be any other things, that follow the Motion of the Epicycle just now explain'd, those, I say, may be understood from the Theory of *Venus*. It remains now, that I say something of the Latitude of both.

Figure XXI.

In Figure XXII, Let the Circle *a b c d e* represent the Plane of the Eccentrick of *Venus* and *Mercury*, and let the Face of the Plain, that you see with your Eyes, be suppos'd to be turn'd towards the Northern Pole of the Ecliptick: And let *b a d* be the Line of the *Apses*, *b* the Apogée, *d* the Perigée; and let the Right Line *c a e* be drawn through the Centre of the World *a*; so as to cut the Line of the *Apses* at right Angles, and also to represent the Ecliptick, and the Plain of the Eccentrick, mutually cutting one another: And so the Ascending Node is *e*, and the Descending *c*. Now the Plane of the Eccentrick comes to the Plane of the Ecliptick, and again goes from it by a sort of wavering Motion, that perfects its Revolution in the same space of time, in which the Centre of the Epicycle goes about through *e b c d e*. For the Centre of the Epicycle being in *e*, both the Planes, (*i. e.*) of the Eccentrick and the Ecliptick, are united; but the Centre of the Epicycle proceeding from *e* to the Apogée *b*, the Plane of the Eccentrick does deviate from that of the Ecliptick, departing in the upper part of it *e b c*, in *Venus* indeed towards the North, but in *Mercury* towards the South; but at the same time the lower part *c d e* does leave the Ecliptick the contrary way, (*i. e.*) in *Venus* Northward,

Figure XXII.

and in *Mercury* Southward. And so the Centre of the Epicycle proceeding from *b* to *c*, the Plane of the Eccentrick does again come to the Ecliptick by little and little, till it be united in the descending Node *c*. And again, the Epicycle proceeding from *c* to *d*, the lower part of the Plane of the Eccentrick *c d e*, departs from the Ecliptick, in *Venus* Northward, but in *Mercury* Southward, but in the mean time, the upper part *e b c*, declines from the Ecliptick the other way (*i. e.*) in *Venus* Southward, but in *Mercury* Northward. And by and by the Epicycle proceeding from *c* to *d*, the Plane of the Eccentrick comes again to the Ecliptick by little and little, till it be again united to it in the Ascending Node. Hence it may be understood, that the Centre of the Epicycle of *Venus*, has always its distance from the Ecliptick Northward, unless when in the Points *e* and *c*, it falls upon the Plane of the Ecliptick, and so that part of the Plane of the Eccentrick, which the Centre of the Epicycle of *Venus* does pass over, (whether that be the upper part, as *e b c*, or the lower, as *c d e*,) does always deviate from the Ecliptick towards the North; but we are to understand the quite contrary of *Mercury*. And now this Libration of the Eccentrick they commonly call the Deviation; but besides this, there is also an Inclination of the Epicycle to the Plane of the Eccentrick, which gives a Latitude to *Venus* and *Mercury*: For in the same Figure XXII. let *f g*, or *h i*, or *k l*, be taken for the Line of the mutual Section of the Epicycle and Eccentrick, which, while the Centre of the Epicycle is mov'd through *b c d e*, *d* is always situated parallel to the Line of the *Apses b a d*. Also let the left part of the Epicycle *f m g*, *h o i*, or *k q l* be understood perpetually to decline from the Plane of the Eccentrick, in *Venus* to the North, in *Mercury* to the South; but let the right side of the Epicycle *f n g*, or *h p i*, or *k r l* be understood perpetually to tend quite the other way, in *Venus* to the South, in *Mercury* to the North

CHAP. XIII.

Of the true System of the World.

1. THough *Ptolemy*'s *Hypothesis*, which I have been hitherto explaining, do salve most of the *Phænomena* of the Stars, that could be obsery'd by the bare Eye, if so be the quantities of Lines and Motions be limited by those Observations he had got of many Times; yet certain it is, that not only the same may be all accounted for, by an easier and far more probable *Hypothesis*, but that also there are certain *Phænomena*, found by new-invented Tubes or Telescopes, which can in no wise consist with his System of the World: For he places the Earth in the Centre, and then supposes, that the Orbs of the Planets do surround in this order, first the Moon, then *Mercury*, *Venus*, the Sun, *Mars*, *Jupiter*, *Saturn*, and, last of all, the Sphere of the fix'd Stars; as Figure XXIII. does present them to our view. But now our Telescopes do shew us those *Phases* of *Venus* and *Mercury*, that do manifestly prove, that those two Planets are distant from us sometimes beyond the Sun, and sometimes on this side: For, sometimes they are horned, other times half full, then gibbous, and last of all in the full, no otherwise than the Moon is. Whence 'tis thus argued, if they were always under the Sun, (as *Ptolemy* thought,) they would either appear halfed, or like a Sickle, or not at all, seeing they would either hold towards us their whole enlighten'd Hemisphere that is turn'd to the Sun, or else the greatest part of it would be turn'd away from us; but if they were continually above the Sun, they would always appear Full, or as if they were so, for the contrary reason: Therefore since they appear sometimes like half Moons, and sometimes in the Full, they must be sometimes beneath, and sometimes above the Sun; therefore I say, their Orbits do encompass the Sun: But they cannot surrond the Earth, because they never go far from the Sun, nor ever come into Opposition with him; which yet they must necessarily do, if the said Orbits did include not only the Sun, but the Earth too. And farther, Figure XXIII. the Orbit of *Mercury* is included within the Orb of *Venus*, because his greatest Digression from the Sun is less than that of *Venus*; and also because he being nearer the Sun,

Figure XXIII.

is

is of a more lively brightneſs, than either *Venus* or any other Planet. On the contrary, *Mars*, *Jupiter*, and *Saturn*, unleſs they mov'd about the Earth, they could not come into Oppoſition with the Sun: But they alſo go about the Sun too; for if they went under it, at the time of Conjunction they would appear horned, as the Moon does. 'Tis plain alſo, that the Earth, though it be included in the Orbits of the Superiour Planets, *Mars*, *Jupiter*, and *Saturn*; yet it is not placed in the Centre of them: For *Mars* is five times nearer the Earth when in Oppoſition, than he is when in Conjunction; and ſo appears five times greater there, than he does here: For the apparent Diametre of a Planet, obſerv'd by the Teleſcope, is leſſen'd in proportion to the Increaſe of its diſtance from the Earth. Now the Diametre of *Jupiter* is increaſed only the half from its Conjunction to its Oppoſition; whence 'tis therefore likewiſe inferr'd, that the Earth is plac'd beyond the Centre of the Orbit of *Jupiter*, but not in the ſame proportion to the whole Circumference that it is diſtant from the Centre of the Orbit of *Mars*: And beſides the *Phænomena* ſeem to intimate, that the Sun is not far from the Centre of the Orbit of every primary Planet; ſo that they have the common Centre of their Orbits (namely the Sun) at an equal diſtance from the Earth: And moreover 'tis manifeſt, that that Planet, to whoſe Orbit that common diſtance of the Centre has the leaſt proportion, that has the greater Circuit about the Sun, and ſo the Orbit of *Jupiter* encompaſſeth that of *Mars*, and for the ſame reaſon it ſelf is contain'd within the Orbit of *Saturn*. Laſtly, That the Moon moves about the Earth is evident from hence, *viz.* becauſe it may not only be in Conjunction with the Sun, but alſo in Oppoſition with it: And that it is the neareſt of all to us, is plain; becauſe no Planet has ſo great a Parallax, as it has; wherefore it muſt needs be the neareſt to the Earth.

2. Now therefore from theſe *Phænomena* now mention'd, it will be no difficult thing to frame a true Syſtem of the world: For in Figure XXIV. let us ſuppoſe the Earth to be in *x*, and let the little Circle that encompaſſes it next of all, be ſuppos'd to be the Orbit of the Moon. Then from thence let a ſpace be taken from the Earth *x*, even to *a*, beyond the Orbit of the Moon; and let *x a* be the neareſt diſtance of *Venus* from the Earth; and then from *a* to *b*, let *a b* be five times as much as *x a*, ſo that *x b* the greateſt diſtance of *Venus* from the Earth, may bear the ſame proportion to *x a* the leaſt diſtance, that 6 has to 1. Let *a b* be divided into two parts, and let *c* be the Sun it ſelf; ſo that *Venus*, with reſpect to the Earth *x*, may ſometimes be as well on this ſide the Sun in *a*, as beyond it in *b*. And now within the Orbit of *Venus a b*, let the Orbit of *Mercury d* be drawn; and then let the Orbit of *Mars e f*, encompaſs both the Earth *x* and the Sun *c*, ſo that *x e*, the neareſt diſtance of *Mars* from the Earth, may be a little greater than *x a*, the leaſt diſtance of *Venus*; ſince the Parallaxes of both theſe Planets, when they are neareſt to the Earth are almoſt the ſame, only that of *Mars* is ſomething the leſs. Then the Circle *e f*, made with the Semidiametre *c e*, will be the true Orbit of *Mars* for a twofold Reaſon; firſt, becauſe by theſe means the Sun *c* will be in the Centre of the Orbit of *Mars*, or very near it, as the *Phænomena* hereafter to be ſpoken of do require: And then ſecondly, becauſe ſo the greateſt diſtance of *Mars x f*, has a fivefold proportion to the leaſt diſtance *x e*, or is five times greater, as the proportion of the greateſt apparent Diametre to the leaſt does require. Laſtly, Let the Orbit of *Jupiter g h*, be drawn round them all, whoſe Semidiametre *c g* let be almoſt five times greater than the Semidiatre either of the Orbit of the Earth, or of the Sun *c x*; and let *i k* be the Orbit of *Saturn*, whoſe Semidiametre *c i*, let be almoſt ten times as big as the Semidiametre *c x*, the *Phænomena* hereafter to be explain'd requiring both to be ſo: And thus ſhall we have the true Syſtem of the World compoſed, as the black Circles do ſet it out. And if the fix'd Stars did not appear in the Heavens, that ſhew that the Sun does every year walk through the twelve Signs of the Zodiack; and if the Planets, beſides their firſt Inequality, had not alſo a yearly one; and laſtly, if we had not the Viciſſitudes of Summer and Winter, as alſo the lengthening and ſhortening of Days and Nights; there would be no need of Pointed Circles, the one of which, *c m*, does ſhew the Orbit of the Sun moving about the Earth, the other, *x l*, the Orbit of the Earth, moving about the Sun: For the Appearances of the firſt Inequality in the Planets might be as well explain'd, if neither the

Figure XXIV.

Sun was carried about the Earth, nor the Earth about the Sun, by an Annual Motion; ſuppoſing the Centres of the Sun and of the Earth, do every year come a little nearer to one, and again remove to a farther diſtance. But ſo many other *Phænomena*, as were juſt now nam'd, do neceſſarily require, that either the Sun muſt be ſuppos'd to move yearly about the Earth, or elſe the Earth about the Sun; ſo that if the Sun be carried about, the Earth reſts, or elſe 'tis the contrary way. If with *Tycho* we ſuppoſe the Sun to move, only the Earth ſtands ſtill in x, and the Sun moving from c, through n and m, all the Orbits of the Planets do go along with it, (excepting the Moon;) ſo that the Centre c being transferr'd out of its place, they are transferr'd likewiſe, and by the ſame Motion all the Points of every Orbit do deſcribe a Circle equal to the Circle of the Sun; as for inſtance, the Point h of the Orbit $h g$, does deſcribe the Circle $h o p$, equal to the Circle $c n m$, which is deſcribed by the Centre c. But now, if, following *Copernicus*, we ſuppoſe the Earth to move; then all the Orbits remain in their places, as the Sun their Centre does alſo remain immovable in c; and only the Earth muſt be mov'd from x, through q to l in the ſame time, that the Sun ſhould be mov'd from c, through n to m. And now the Arch of the Earth's Motion $x q$, being ſuppos'd to be equal to the Arch of the Motion of the Sun $c n$, to our Eye, that is plac'd in the Earth, the Sun will ſeem to move to the ſame part of the World, whether it be moved, or the Earth: For, firſt, the Sun being moved from c to n, and the Eye reſting in x, the Ray from the Eye to the Sun is $x n$: And again, the Earth being moved from x to q, (whither alſo the Eye will come, while the Sun reſts in c,) the Ray from the Eye to the Sun is $q c$. Now I ſay, this Ray $q c$ is directed towards the ſame part of the World, towards which the Ray $x n$ is alſo directed; and that the Ray $q c$ is parallel to the Ray $x n$; which parallel Rays, if they reach a vaſt length, even to the Sphere of the fix'd Stars, they will conſequently ſeem to fall in with one another into one and the ſame Line; for the vaſt length to which they are extended, does make their diſtance as it were vaniſh into nothing; and for this Reaſon the Sun ſeems to be in the ſame point of the Firmament, whether it be ſeen by the Ray $q c$, or by the other $x n$; and ſo there will be the ſame *Phænomena*, whether the Sun move about the Earth, or on the contrary: Neither is there any Reaſon for any one to except againſt ſo vaſt an Extenſion of the Firmament, for that it is really ſuch, and ſo great one Argument is, that for inſtance, the *Sirius*, or Dog-ſtar, which to the bare Eye ſeems to be almoſt as big as *Jupiter*, yet by the Teleſcope it appears to be but a mere Point; whereas *Jupiter* being by the ſame Inſtrument ſeen by one Eye, and compar'd with the Moon ſeen by the other Eye, being bare, does ſeem to have three times a greater Diametre than the Moon has: But if the Dog-ſtar was no farther off than *Jupiter*, it would ſcarce ſeem to be leſs than *Jupiter* by the Teleſcope: Whence we may gather, that there is in a manner no proportion between the diſtance of *Jupiter* and the Dog-ſtar. But beſides, conſidering the ſo various and manifold Motions of all the Planets, *viz.* how they move directly, are ſometimes Stationary, ſometimes Retrograde; are alſo ſlower and ſwifter, do approach, and again recede from the Earth, and have an Increaſe and Decreaſe of their Apparent Latitudes and Magnitudes, (all which *Phænomena* may be accounted for barely by the yearly Motion of the Earth about the Sun,) who cannot but judge it very probable, that the Earth does move about the Sun indeed? Eſpecially ſince the Orb of *Saturn* is almoſt ten times greater than the Orb of the Sun, than the Orb of *Jupiter* five times, and ſix times greater than that of *Mars*: Nor is it likely or credible, that the leſſer Circles, ſuch as is that of the Sun, ſhould carry the greater Orbits of the ſuperiour Planets, as *Tycho*'s *Hypotheſis* does require: To ſay nothing, that in this the Orbit of *Mars* cuts that of the Sun, and ſo at certain times the Body of *Mars* may come into that ſpace of the Airy Region of the World, in which the Sun is at other times, which as to the primary Planets is very incredible; *viz.* that the Regions they paſs through ſhould be ſo confounded. Moreover for the Analogy or near reſemblance there is between the Earth and the other Planets, as they are all opaque or groſs Bodies, and the Earth does ſhine, as the reſt do, by a reflected Light, as is plain from the brightneſs of reflecting Clouds, and from the Earth's illuſtrating the Moon with a dim Light; they are alſo all (as far as can be obſerv'd) of rough, uneven ſurfaces, and, like the Earth, are turn'd about their own Axles, and thoſe inclining to the Ecliptick in their daily Motion, and

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so have the same changes of night and day, Summer and Winter; nay, and farther are accompanied with Moons proper to them, *Jupiter* with four, and *Saturn* with three, which they Eclipse, and again are Eclipsed by them. For this Analogy, I say, the Earth seems to be of right to be reckon'd among the Planets; and the Sun, because he has not the same Analogy, seems to be plac'd in the only Centre of them all; especially seeing not only the Motions of every one of them would appear uniform enough, if they were look'd upon from the Sun; and seeing their distances from it are almost every where equable, which from the Earth appear to be very unequal; but also the Revolutions of the Stars about *Jupiter* with respect to the Sun, would be more regular, than they are seen to be from the Earth. And as their Revolutions have among themselves the proportion of half as much more as their distances are from the Centre of *Jupiter*, (that is, the Squares of the times, in which they finish their Revolutions, are as the Cubes of their distances from the Sun,) which is also true of the Companions of *Saturn*, so that this seems to be the general Rule of the Concentral celestial Motions; so likewise the Times of the primary Planets Revolutions, have the proportion to one another of half as much more as their indifferent distances are from the Sun, as *Kepler* observ'd; and therefore these ought to be accounted no other than Lunar Companions, that mov'd about the Sun.

And hence it is, that now most Astronomers receive *Copernicus*'s System of the World, as far more probable than any other, which accordingly I shall endeavour to illustrate in what follows, though in explaining the Motions of the Luminaries, *Tycho*'s Calculation be generally to be preferr'd before the rest. Wherefore I shall lay down, in the first place, *Tycho*'s Theory of the Luminaries as distinctly as I can, and shall retain it in computing the Motion of the Moon, and of the Eclipses of both Luminaries.

CHAP. XIV.

Of Tycho's *Theory of the Moon.*

1. BEcause *Tycho*'s Theory of the Sun does altogether agree with *Ptolomy*'s, differing only in the quantity of Motions and Lines; therefore I streightway enter upon his Theory of the Moon. And here first of all 'tis to be noted, that *Tycho* does explain the first Inequality of the Moon, and of the rest of the Planets, by a certain Circle that carries two Epicycles, *viz.* one bigger than another. As in Figure XXV. the Circle that carries the Epicycle is B C D E, in whose Circumference the Centre of the greater Epicycle is carried about from B towards C, D, and E; and again the greater Epicycle does carry about the less with it, whose Centre *n* it has fix'd in its Circumference. Figure XXV.

And now the quantity and proportion of these Circles is this: The Semidiametre of the Deferent, (or Circle that carries the Epicycles about,) namely, the Line A B, is equal to 56½ Semidiametres of the Earth. Now if in the said Semidiametre A B, there be suppos'd to be 100000 Parts; then of such parts 2174, are to be allow'd to the Semidiametre A H, and to the Semidiametre of the greater Epicycle B *n* or C *n* 5800; and lastly, to the Semidiametre of the lesser Epicycle 2900, which is half of the former.

2. Now these Lines are so mov'd, that the Line of the Apogée D A B, is first conceiv'd to be fix'd to the point A; but is movable in the rest of its length, so that in about nine years it passes through the Zodiack: Therefore from this Line of the Apogée, the Centre of the greater Epicycle being so many Months departed, it returns back to the same from B, through C D E, till it again overtake B, the Apogée of the Moon, a little beyond the Point of the Zodiack, in which it had last left it. Again, the right Line B *n*, C *n*, D *n*, E *n*, drawn from the Centre of the greater Epicycle to the Centre of the less, is always parallel to this Line of the Apogée D A B; whence it follows, that the Centre of the greater Epicycle being in C, the Angle *m* C *m*, or the Arch *m n*, the space of which the Centre of the lesser Epicycle is departed from the Apogée of the greater

greater *m*, is always equal to the Angle B A C, or to the Arch B C, the ſpace of which, the Centre of the greater Epicycle is departed from the Line of the Apogée D A B; and they both are call'd the middle Anomaly of the Moon: But the Motion of the leſſer Epicycle is always double to the Motion of the greater Epicycle, (as alſo the Ray of the greater C *n*, is double to the Ray of the leſſer *n i*,) and it always begins from *a*, the point of its Circumference, which the Line that joins the Centres B *a n*, C *a n*, &c. do paſs through: So the Centre of the greater Epicycle being in the Apogée B, the Motion of the greater Epicycle begins from *n*, and of the leſſer, and ſo conſequently of the Body of the Moon from *a*. Again, The Centre of the greater Epicycle being in C, if *m n* be the Quadrant of the Circle, the Semicircle will be *a e i*, and the Moon will be in *i*; alſo the Centre of the greater Epicycle being in D, if *m o n* be the Semicircle, the whole Circle will be *a e i o a*, and the Moon will return from *a* to *a*. Laſtly, The Centre of the greater Epicycle being in E, ſo that the Arch B C D E conſiſt, for inſtance, of ten Signs; then likewiſe the Motion of the greater Epicycle will be through ten Signs, from *m* to its Perigée in *n*: But the Motion of the leſſer Epicycle will be through twenty Signs, to wit, from *a* through *e i o*, its own Circle being wholly run through, and *a e i o*, an Arch of eight Signs, beſides. And all theſe things are exactly alike in the leſſer Planets. And in every one of them, the Line A B, or A C, drawn from the Centre of the Deferent to the Centre of the greater Epicycle, is the Line of middle Motion; and the Angle C A *i*, that lies between this and the other Line A *i*, drawn from the Centre of the Deferent through the Body of the Moon, is call'd the Epicyclical *Proſthaphæreſis*, and A *i* the Elongation of the Moon from the Centre of the Deferent: Laſtly, the Arch of the Deferent between A B and A *i*, or between A B and A *o*, (computed according to the order of the Signs B C D, to A *o*,) is call'd the Anomaly of the Moon, firſt equall'd.

3. The ſecond Inequality of the Moon is explain'd by a little Circle, whoſe Diametre A H F, either falls into the Line of the Apogée D A B, or elſe is parallel to it, and its Circumference paſſeth through the Centre of the Earth A. In this little Circle is carried about the Centre of the Deferent twice in every Lunation, ſo that in every true Syzygy, the Centre of the Deferent agrees with the Centre of the Earth A, as in Scheme XXV: But in the true Quadratures the Centre of the Deferent is in F, the point remoteſt from the Earth, as in Scheme XXVI. it may be ſeen; where, beſides the Lines of the foregoing Scheme, there appears alſo the Line of the true Motion of the Moon A *e*, drawn from the Earth A, to the Body of the Moon *e*; and the length of it, A *e* is call'd the diſtance of the Moon above the Earth. But the Angle to the Moon A *e* F, that lies between the Lines drawn from the Centre of the Earth A, and from the Centre of the Deferent F to the body of the Moon, is call'd the *Proſthaphæreſis* of the Eccentricity; and the Arch of the Eccentrick between F B and F *e*, or between F B and F *o*, (reckoning according to the order of the Signs B, C, D, to F *o*,) is call'd the Anomaly equall'd: For the ſingle or middle Anomaly here, as well as there, is B C, or the Arch B C D E, from which that other (firſt equall'd) differs always from the Epicyclical *Proſthaphæreſis* C F *e*, or E F *o*.

4. But out of the Syzygies or Quadratures, the Diametre of the little Circle A F, does not (as before) fall into the Line of the middle Apogée D B, but is parallel to it, as may be ſeen in Scheme XXVII. For let the Centre of the Deferent E, wander as it will through the little Circle A L F E, yet thoſe three Lines, *viz.* the Diametre of the little Circle A F, and the Line of the middle Apogée D B, and that which joyns the Centres of the Epicycles C *n*, will continue parallel. But in this third Caſe beſides the Lines of the foregoing Schemes, there is alſo found A E R, the Line of the true or monthly Apogée, drawn from the Centre of the Earth A, through the Centre of the Deferent E, ſo that R is the true Apogée, and S the true Perigée. And ſince the Motion of the Centre of the Deferent E, *viz.* the Arch of the little Circle A L F E, is always equal to the doubled Arch, that lies between the true place of the Sun and the place of the Moon, firſt equall'd, (which they call the doubled diſtance of the Luminaries;) thence 'tis manifeſt, that the Arch F E, (which in this place is the Exceſs of the whole A L F E, above the Semicircle A L F,) may be obtain'd, to which is equal the Angle to the Centre E H F; but the half of the ſame is the Angle to the Circumference F A E, by 20 III. *M. I.* And again, to this is equal B E R, which they call the Angle of

Figure XXVII.

of the second Equation, with respect to the single Anomaly, which is twice to be equall'd: So in this place, the single Anomaly is B C, or B E C the Angle; but the Anomaly first equalled, is B E *e*; and the Angle of the second Equation B E R, being subtracted from it, there remains R E *e*, the Anomaly equall'd in the second place, lying between the Line of the true Apogée E R, and the other which is drawn from the Centre of the Deferent E, to the Body of the Moon. But besides, there is farther to be consider'd the Line A E, that joyns together the Centres of the Earth A, and of the Deferent E, which they call the Eccentricity; and that is subtended, or lies under the Angle to the Moon A E *e*, which I have nam'd the *Prosthaphæresis* of the Eccentricity: For it shews the difference that is between the place of the Moon, that is seen from the Centre of the Deferent E, and from A the Centre of the Earth; and the Moon going from the true Apogée R, through D, to the Perigée S, it is taken from the Moon's place that is first equall'd; as in the other Semicircle S B R, it is added, that in both cases, the Moon's place that is secondly equalled, may be found.

5. But now besides the Inequalities of the Moon hitherto spoken of, *viz.* the first Epicyclical, and the other of Eccentricity, *Tycho* also found a certain third, but very small one, which never exceeds forty Minutes and ½; and 'tis call'd the Variation, and is nothing else but a certain Libration, or wavering of the Centre of the greater Epicycle, whose Motion is swifter about the time of the Syzygies, and slower about the Quadratures: For in every Syzygy and Quadrature, the Centre of the Epicycle is in C, (in Figure XXVIII.) the Centre of the little Circle; where it would also be, if there was no Libration at any time. But in an Octile Aspect, before the Syzygies, it is in *g*, and goes on from thence; so that in the very Moment of the Syzygy it comes to the Point C, and proceeds even to the Octile after the Syzygy into *f*. Now here when the Octile is before the Quadrature, it goes back; so at the very moment of the Quadrature, it returns into C, and goes backward still, even to the Octile after the Quadrature into *g*. Now the manner of this Motion of Libration is, as if we should suppose a certain Point to begin at the very moment to go forward from *s*, with an equal Motion towards *f*; whither it comes in the Octile after the Syzygy, and proceeds from thence, till at the time of the Quadrature it is in *y*, and so on. Now from any Point of the Circumference of the little Circle, in which the feigned movable is supposed to move, let a perpendicular be cast upon the Diametre of the little Circle *f g*: As for instance, the feigned Moveable being in *h*, a Perpendicular let fall, would find in the Diametre of the little Circle the Point *t*, where the Centre of the Epicycle will then be. In the mean time 'tis easily understood, that this libratory Motion of the Centre of the greater Epicycle, does the same way affect not it self only, but also the lesser Epicycle, and the Body of the Moon that is fix'd in it. And this is the Moon's Motion of Longitude according to *Tycho Brahe*. Figure XXVIII.

CHAP. XV.

Of the Latitude of the Moon according to Tycho.

BEsides those things that our Predecessors have delivered concerning the Latitude of the Moon, which were built upon the Deviation of the Moon's Orbit, five degrees from the Ecliptick, and upon the Revolution of the Nodes to the Antecedent Signs, which was suppos'd to be finish'd in about nineteen years time; *Tycho* has farther found out a double Irregularity, the one of which he ascribes to the trembling of the Nodes that sometimes move more swiftly than ordinary toward the Antecedent Signs, sometimes go something back into the Consequent; but the other he attributes to the Orbit that sometimes inclines, or nodds, as it were, towards the Ecliptick, and again sometimes declines from it. But both the said Irregularities happen through a wavering Motion, just like that attributed to the Variation of the Moon in the foregoing Chapter: For to speak first of the former of the two, in Figure XXIX. there is a little Circle to be describ'd from the Node A, which by its middle Motion, tends to the Antecedent Signs towards Figure XXIX.

wards D. In every Syzygy and Quadrature, the true Node agrees with the middle one A, thence it goes on to the conſequent Signs towards B, till the Octile after the Syzygy or Quadrature be completed; and ſo it goes back towards A and C, in the ſame manner, that the Semidiametres, or the inverted Signs of all the degrees of the little Circle, do increaſe or decreaſe, (as was ſaid before concerning the Variation of the Moon,) and the Ray of the little Circle A B, is of one degree and 46′. The other Inequality, *viz.* of the inclining and declining Orbit, is made under a Circle of Latitude, drawn through Bounds 90 degrees remov'd from the Nodes. In every Syzygy, the Angle of the Orbit of the Moon, and of the Ecliptick, or the Arch of the Circle of Latitude drawn through the ſaid Bounds, is at leaſt of 4 degrees 58′. 30″. but in every Quadrature, the ſame Angle or Arch is of 5 degrees, 17′. 30″. So that the Diametre of the Circle of Libration, is of 19′; and the manner and meaſure of the Libration (as before) follows the proportion of the inverted Signs, that anſwer to every degree of the Circle of Libration.

CHAP. XVI.

Of Copernicus's *Sphere.*

HAving now explain'd the chief *Hypotheſes* of thoſe that will have the Earth to ſtand ſtill, I now enter upon their *Hypotheſes*, that ſuppoſe it be mov'd about the Sun. But before the *Phænomena* of the ſecondary Moveables can be explain'd by this ſuppoſition, we muſt firſt underſtand, how according to this the firſt Motion may be demonſtrated, that is common to all the Stars, which is otherwiſe wont to be explain'd by the common Sphere. But here we have need of a Sphere, different from the common one; and we muſt form our Conception a little otherwiſe than before, perhaps in this manner: The Centre of the Sun is ſuppos'd to be the Centre of our planetary World, which is encompaſs'd on all ſides by the Lucid Bodies, which we call the fix'd Stars; which fix'd Stars, how far they are from the Sun, is out of Ken and reach of our Senſes and Imagination to comprehend: For who can ſet any Bounds to the Univerſe, no bounds of which he could ever yet perceive? Wherefore we make the Sun the Centre, not with reſpect to the whole Univerſe, which is unmeaſurable, (at leaſt to us,) and therefore has neither middle not extream; but with reſpect to our Earth and the reſt of the Planets, which are carried round the Sun, as about a Centre or certain middle Point. And the ſame Sun may be ſaid to be the Centre of the Sphere of the fix'd Stars, not as if this was diſtinguiſh'd from the Extramundane Void by certain Surface or outward Cruſt; but becauſe our Imagination, ſtretching it ſelf equally on every ſide from the place where we are, and being unable to comprehend the Immenſity, it is bounded on each ſide into a kind of Sphere. Or if any one had rather extend this Sphere of the World, as far as the Eye is able to go, when aſſiſted by the beſt perſpectives, the Univerſe will loſe nothing of its Immenſeneſs. And now if the Sphere of the fix'd Stars be unmeaſurable, it will alſo for that reaſon be unmovable. Nay, if you ſuppoſe a certain Extramundane Void, yet no man can imagine, how the outmoſt Surface of the World forſakes one ſpace after another of that Void, where there cannot be ſo much as any Sign given, by which that departure may be perceiv'd. If therefore the utmoſt Surface of the World, do not change its Situation, with reſpect to the void ſpace that is without it, (for we cannot ſo much as feign ſuch a thing by the ſtrength of Imagination,) but changes its Situation only with reſpect to the Bodies contain'd within it; What remains, but that we may think the Sphere of the fix'd Stars is immovable, but the encloſed Earth, with reſpect to the ſame Sphere, does change its Situation, that is, is really turn'd about its own Axle? Now therefore, whether the Sphere of the fix'd Stars be unmeaſurable, or bounded, yet certainly 'tis immovable, and may be ſaid to be of a Spherical Surface, whoſe Centre the Sun poſſeſſes, and never moves out of it, though 'tis found to roll about its own Axle, by the ſpots which in a certain ſpace

of

of time are removed from one edge of its Hemisphere to the other. Now as the Immense Sphere of the fix'd Stars, consider'd in it self, does want a Centre or middle; but, because of the Motion of the Planets, does obtain a Centre, which I have said to be the Sun, rather than the Earth: So also by reason of the same Motion of the Planets, which in a certain Track are mov'd about the Sun as their Centre, the same Immense Sphere of the fixed Stars has a certain Girdle, which we call the Zodiack *i. e.* the Rays passing from the Sun by the Planets that are carried about; all which Rays do fall upon those Asterisms of the Sphere of the Stars, which from the Resemblance of Animals, they call *Zodia*. In the middle of which Girdle, seeing the Earth always walks, as by a certain High-way between the Planets that deviate on each side; therefore that Circle is call'd the Circle that passes through the midst of the Animals, that is, the Circle that goes through the middle of the Signs, (the Plane of the Circle, which the Earth describes, being continued to the Sphere of the fix'd Stars:) And the Area, or plane space, extended on every side between the Circumference, delineated in the Firmament, and the Centre of the Sun, is call'd the Plane of the Ecliptick: And a perpendicular erected from the Centre upon this Plane, is the Axle of the Ecliptick, which, when continued to the fixed Stars, has in its Extremities or Ends, the Poles of the Ecliptick; and these Poles, as also the Ecliptick it self, has always the same Situation with respect to the fix'd Stars. Again, from the Poles of the Ecliptick to the Ecliptick it self, are drawn Circles of Latitude: But as in the Terrestrial Globe, no Meridian is in nature before any other, but only by Men's Institution and Agreement; so among the Circles of Latitude, there is no one the first, but that which men would have so. *Copernicus* draws the first Circle of Latitude through the first Star in *Aries*, and from this computes the Longitudes of the fix'd Stars; which was begun from thence, because about the Age those lived in, that have left us the most antient Observations, the first Star in *Aries* was near the Vernal *Æquinox*. But in the beginning of 1660. the Colure of Solstices was distant from the first Circle of Latitude, drawn through the first Star in *Aries* 61 degrees, 32′, 51″. towards the consequent Signs. Now the other Colure (of *Æquinoxes*) is distant from the Colure of Solstices 90°, to be computed on this side, and beyond in the Ecliptick. But in this the two Colures differ from one another, *viz.* that the Solstitial Colure passes both through the Poles of the Ecliptick, and the *Æquator* too; but that of the *Æquinoxes* goes not through the Poles of the Ecliptick, but only of the *Æquator*. Now the Poles of the *Æquator* are thus found out; from the Pole of the Ecliptick in the same Colure of Solstices, let 23°, 30′. be reckon'd towards the same part of the Starry Globe, where are the bright Heads of *Gemini*; and so where the Account is ended, there is the Arctick Pole of the *Æquator*, to which the Antarctick is diametrically oppos'd. But now as the yearly Motion of the Earth about the Sun, produc'd the Ecliptick; so the daily Motion of the Earth about its own Axle, produc'd the *Æquator*; which is properly a Terrestrial Circle, which any point in the Surface of the Earth, that is on each side equally distant from the Poles, does describe by its daily Motion. But if any one will conceive the *Æquator* to be in the Heavens; then for certain he must not conceive only Mathematical Points, Lines, and Surfaces, that want all profundity; but he must philophize something more grosly: For all that Sphere, whose Semidiametre is not less than the distance of the Earth from the Sun, is in proportion but a Point or a Centre: And by the flowing of this so gross a Point from the Centre to the Ecliptick, is made the Ray of the Ecliptick, which being carried round, it describes the Area of the Ecliptick, which is indeed of a vast depth, but with respect to the Firmament, it is unperceivable. So likewise, if that gross Point be suppos'd to flow from the Centre perpendicularly to the Plane of the Ecliptick, then the Axle of the Ecliptick is drawn: And so reckoning 23 degrees in the Colure of the Solstices, from the Pole of the Ecliptick, towards the Heads of *Gemini*, there will again appear from that gross Centre at the end of 23°, 30′. an Axle of the *Æquator*, as equally gross or thick, which yet with respect to the vast Firmament, is no more than a bare Line. And so at length, both the *Æquator* and the Tropicks will be conceiv'd to be delineated by gross Circumferences in the Sphere of the fix'd Stars. And now these things being thus suppos'd, in what part soever of its Orbit the Earth is, it will always be in the Centre of the Firmament: For that Centre is as big

as the very Orbit of the Earth. And yet nevertheleſs, if any Circle that is drawn in the ſurface of it (that is, of the Earth,) be divided, as the *Æquator*, for inſtance, into 360°; there may be drawn from the Centre of the Earth, through every Diviſion that is made in its Surface, to the very Cœleſtial *Æquator* right Lines, that will in like manner divide the whole Cœleſtial *Æquator* into juſt ſo many degrees: And to every degree of the Terreſtrial *Æquator* every degree of the Cœleſtial ones will anſwer, as the whole Terreſtrial *Æquator* lies under all the Cœleſtial, in what part ſoever of its Orbit the Earth is plac'd. And ſo alſo the Tropicks of the Earth will lie under the Tropicks of the Firmament, and the Arcticks under the Arcticks. Hence we muſt know, that the firſt Circle of Latitude (which goes through the firſt point of *Aries*) is immoveable; but the Colure of the Solſtices comes by little and little towards the firſt Circle of Latitude, moving with a ſlow Motion every year 51″ about the Poles of the Ecliptick, that is in 70 years about one degree. And by that Motion it carries about with it the Pole of the *Æquator*, (which is in the Colure of the Solſtices;) whence it comes, that alſo the other Colure (of *Æquinoxes*, which is diſtant from the Colure of the Solſtices always 90°,) does creep on together with it towards the Antecedent Signs; And ſo the Equinoctial Sections do recede from the fix'd Stars towards the Antecedents, the Ecliptick however remaining immoveable; and the *Æquator*, with all its Appurtenances, *viz.* its Parallels and Meridians, which are carried about the Axle of the Ecliptick. And this Motion is wont to be called the Proceſſion of the Æquinoxes. Now the daily Motion of the Earth is from Weſt to Eaſt, according to the Order of the Signs about its own Axle; which Axle, while the Centre of the Earth is by its yearly Motion carried around in its own Orbit, does always continue Parallel to it ſelf, excepting only that ſlow Motion of it about the Axle of the Ecliptick, which has been juſt now mentioned, which is only 51″ yearly. For whereſoever the Earth is in its Orbit, always the Right Line, drawn from its Centre to the Pole of the Cœleſtial *Æquator*, does repreſent the Axle of the Earth, about which it is moved by its daily Motion. Now how the *Phænomena* of the Seaſons of the year, and of the Changes of Days and Nights may be hence deduc'd, I ſhall explain in what follows. But now as concerning the Motions of the reſt of the Planets, Firſt, The Arch of the Planet in the Ecliptick, taken from the vernal Interſection, or the point of the Spring-*Æquinox*, to the Aſcending Node, is the Longitude of the Node, which is always equally increaſed in the other Planets, but in the Moon it decreaſes equally, becauſe the Nodes of the Moon go backward. Secondly, Moreover from the Aſcending Node are reckoned in the Ecliptick 90ᵈ, and from thence to the Pole of the Ecliptick is drawn the Circle of the greateſt Inclination. For as the diſtance of the Sun from the *Æquator* is the greateſt in the beginning of *Cancer* and *Capricorn*, which is 90ᵈ diſtant from either *Æquinox*, and is called the greateſt Declination: So the greateſt diſtance of every Planet from the Ecliptick, conſider'd as from the Sun, happens in the Northern or Southern Bounds of the Planet's Motion, which are both 90ᵈ diſtant from each Node, and it is called the greateſt Inclination. And now from the Aſcending Node through the Northern Bound, is drawn the Orbit of the Planet to be conceiv'd in the Firmament, in whoſe Plane is afterwards delineated the very Track of the Planet, as it goes about the Sun, which is alſo call'd more properly the Orbit of the Planet. Thirdly, The Arch that is reckoned from the Aſcendent Node to the *Aphelion*, (or place of the Planets fartheſt diſtance from the Sun,) when added to the Longitude of the Node, gives the Longitude of the *Aphelion* from *Aries*. And though theſe two Arches be taken and computed in diverſe Circles, the firſt in the Ecliptick from *Aries* to the Node, and the ſecond in the Orbit of the Planet, that inclines upon the Ecliptick, from the Node to the *Aphelion*; yet ſeeing both the Motions are regular, one Motion may be compounded of them, which may alſo be regular, to wit, of the *Aphelion* from the *Æquinox*: For a Regular added to a Regular produces a Regular. 4 Laſtly, from the *Aphelion* in the Orbit of the Planet (underſtand here always that Orbit which is drawn in the Sphere of the fixed Stars) to the Line of the Middle Motion, is the middle Anomaly, but to the Line of the true Motion is the equaliz'd Anomaly, which being added in the firſt Semicircle to the place of the *Aphelion*, and ſubſtracted in the latter, it gives the Heliocentrick place of the Planet, (or the place it is in with reſpect to the Sun, when taken for the Centre,) which is afterwards to be brought to the Ecliptick, and then the Heliocentrick Place of the Planet in the Ecliptick does appear, namely,

ly, That Point which is found by the Circle of Inclination, that is drawn through the Place in the Orbit.

CHAP. XVII.

Of the Motion of Reflexion of the Earth's Axle, and of the Declination of the Sun that arises from thence.

IN Figure XXX. let the great Circle ♈ ♋ ♎ ♑ be the Ecliptick of the Firmament, whose Southern Pole *a*, let it be conceiv'd to be turn'd to us, who are suppos'd to be without the Firmament at an infinite distance, while the other the Northern Pole is hid behind it in the other Hemisphere. Now this Ecliptick ought to be conceiv'd so large, that the whole Orbit of the Earth *b c d e*, with respect to it, may be consider'd as no more but a Point or Centre; which, since it cannot be expres'd in a Scheme, is to be supplyed by the Fancy. Now the Colure of the Solstices is here cast into a right Line ♋ *a* ♑. In the Plane of the Ecliptick is delineated the Orbit of the Earth, going forward in its yearly Motion from *b* to *c d e*, &c. and that so, that the Plane of the Earth's Ecliptick, or of the lesser Circle ♈ ♋ ♎ ♑ does perpetually remain in the Plane of the greater Circle, ♈ ♋ ♎ ♑; (not mentioning at present the Earth's daily Motion about its own Axle:) And all the Signs of the Earth's Ecliptick do each of them constantly remain under those particular Signs of the great Ecliptick, which bear the same Names, and are never separated from one another, though the Earth's Centre be carried about by a yearly Motion; but by a certain magnetical Power every Sign is kept in the same Situation by the Sign of the same Names: Therefore ♋ *b* ♑, ♋ *c* ♑, &c. will represent the Earth's Solstitial Colure, which passes through the Southern Pole of the Earth's Ecliptick, that is turn'd towards us, *b*, *c*, *d*, &c. and is also hid by the Northern behind *b*, *c*, *d*, &c. and likewise through both the Poles of the Earth's *Æquator*, *viz.* the Southern, which is turn'd to us in *p*, and the Northern, which is plac'd in *o* in the Hemisphere, that is turn'd from us. But if from the Poles of the Earth's Ecliptick, namely, from the Southern one *b*, that is turn'd to us, and from the Northern one, which is hid behind *b*, be taken on each side the Distance of 23° ½; And if there be two lesser Circles understood to be describ'd of that Interval about both the Poles of the Ecliptick, the one *m n o p*, and the other that which lies hid behind it in the Hemisphere that is turn'd from us: Then (if from the Centre of the Earth there go out on each side a Ray to the Circumference of the said lesser Circles, so that the two Rays be extended streight forward into one Diametre; and the middle Point of that Diametre being fixt in the Centre of the Earth *b*, if the ends of the Diametre be carried about in the Circumference of the lesser Circles *m n o p*, and that which lies behind it,) there will be describ'd by the Diametre the Surface of two Cones, whose tops do come into a Cone in the Centre of the Earth, but the Bases are the Planes of the lesser Circles last mentioned, in whose Circumference the Ends of the Diametre were carried about; moreover the Plane of the Circle of Illumination that is cast upon the Right Line *m n*, does divide this twofold Cone in the Middle. For wheresoever the Earth is in its Orbit, the Ray that comes from the Sun *a*, to the Centre of the Earth *b*, *c*, *d*, &c. will be perpendicular to the Plane of the Circle of Illumination *m n*, and this same Plane *m n* does divide equally both the twofold Cone and its Bases *m n o p*, and that which lies behind it. And now these being thus suppos'd, when the Earth is in *b* in the beginning of *Aries*, the Axle of the *Æquator o b p*, that extends it self from the Northern Pole *o*, (which is turn'd away from us,) through the Centre of the Earth *b*, to the Southern Pole, (which is turn'd to us,) does then fall upon the Plane of the Circle of Illumination, in which the Axle *o p* is plac'd for a time. Here therefore, seeing the Circle of Illumination passes through both the Poles of the Earth, it may be easily imagin'd, that both the *Æquator*, and all its Parallels that are to be describ'd by the daily Motion of the Earth, must be cut in two by that same Circle of Illumination; and so that every Point of the Earth does in its daily Motion make one Semicircle parallel

Figure XXX.

to the *Æquator* in the enlighten'd Hemiſphere, and another in the dark, that is, that there is an apparent *Æquinox* throughout the whole World. Farthermore, the Ray *a b* at that time does paſs over the Plain of the *Æquator*, when it is perpendicular to the *Æquator*'s Axle *o p*. And whatſoever Line is perpendicular to the Axle, and paſſes through the Centre of the Earth, the ſame is plac'd in the *Æquator*: For the *Æquator* it ſelf is made up, as I may ſay, of innumerable Lines, perpendicular to the Axle, that proceed on all ſides from the Centre of the Earth. And the ſame will alſo happen, when the Earth is under *Libra* in *f*. But under *Cancer* the Earth being in *d*, the Plain of Illumination *m n*, is bor'd through obliquely by the Axle *o p*, which touches the ſide of the twofold Cone; and therefore the Northern Pole *o*, for inſtance, does mount above the Plane of the Illumination *m n*, 23 degrees and $\frac{1}{2}$. You will learn this better from Scheme XXXI. which agrees ſo far with Scheme XXX. that that which this does expoſe to the Eye directly, that other exhibits to be ſeen on one ſide, and on the contrary. Thus the Ecliptick, which in Scheme XXX. appear'd in a whole Circle, in Scheme XXXI. is caſt into a right Line ♋ ♈ ♎, which is to be underſtood as well of the great Ecliptick, as of the Ecliptick of the Earth. And again, that which in Scheme XXXI. appears a whole Circle, namely, the Colure of the Solſtices ♋ *a* ♑; that in Scheme XXX. is caſt into a right Line. Therefore I now ſay in Scheme XXXI. that *o p* the Axle of the Earth, when under *Cancer*, do paſs obliquely through the Plane of the Circle of Illumination *q y*, and the Northern Pole *o* does mount above the Plane of Illumination *q y*, the quantity of the Angle *o* ♈ *q*, 23° $\frac{1}{2}$, which degrees are meaſur'd by the Arch *q o*, which is that portion of the Colure of the Solſtices *q o* ♋, that lies between the Poles of the Ecliptick *q*, and of the *Æquator o*. For it is worth noting, that the Circle of Illumination does always paſs through the Poles of the Earth's Ecliptick; which is evident from Scheme XXX. where I made ♈, ♋, ♎, ♑, the Earth's Ecliptick, and its Southern Pole *b c d*, *&c.* behind which, both the Northern lies hid, and alſo the Plane of Illumination *m n*, which always goes through both Poles. Now returning to Figure XXXI. we ſhall find that the Ray, which joins the Centres of Sun, and of the Earth ♈ ♋, does go through the Earth's *Æquator* 90 270, with the ſame obliquity of Angle ♑ ♈ 270, with which the Axle *o p*, did alſo make an hole through the Plane of Illumination *q y*; and that the Declination of the Sun, (which under *Aries* is none at all, the Ray that goes through the Centre, paſſing juſt over the Plane of the *Æquator*,) is now the greateſt, *viz.* 23° $\frac{1}{2}$. For as much as the Axle of the Earth *o p*, does riſe higher than the Plane of Illumination *q y*; juſt ſo much does alſo the Ray that joins the Centres ♈ ♑ ♈, riſe above the Plain of the *Æquator* 90 270, that is, the Sun declines juſt ſo much from the *Æquator*.

Figure XXXI.

For the Circle of Illumination, and the *Æquator*, are the greateſt Circles of the Terreſtrial Sphere; and to thelatter of them its own Axle is Perpendicular, and to the former, the Ray that joyns the Centres; and conſequently both theſe being Perpendicular, each of them will find the Poles of its own Circle in the Surface of the Earth. Why therefore may not the Pole of the *Æquator o*, be as far diſtant from the Circle of Illumination *q y*, as the Pole of the Circle of Illumination ♑, is diſtant from the *Æquator* 90, 270? For the part 4270, which is common to both, being taken from the Quadrants, (or Quarters of the Circle,) 270, and *q* ♑, then *o q*, and 27 *o* ♑ remain equal. It will be profitable to remember this hereafter. And there is alſo the ſame reaſon, when the Earth is under ♑: But the Earth being out of the Cardinal Points for inſtance in *c*, in Scheme XXX. the Pole *o* is then alſo above the Plane of Illumination *m n*, but not all out 23°. $\frac{1}{2}$. which that it may be the more readily underſtood, I will firſt of all explain the Motion of Reflexion, ſeeing we have now all things ready that belong to it. Imagine therefore the Plane of Illumination to cut both the Baſes of the double Cone; and call the common Section of the Plane of Illumination, and of the Baſis of the Cone, the Fiſſure of the Baſis. Now the Northern Pole of the Earth *o*, is always in the top of the Baſis that is turn'd from us, and the Southern *p* is always in the bottom of the Baſis that is turn'd to us: 'Tis plain then, that the Earth being in *b*, the Pole *o* falls upon the Fiſſure *m n*, of the Baſis that is turn'd away; the Earth being in *c*, the Pole *o* has left the Fiſſure *m n* of the averted Baſe, the ſpace of the Arch *n o*; the Earth being in *d*, the Pole *o* has now left the Fiſ-

ſure

ſure *m n*, a Quarter of a Circle *n o*; the Earth being in *e*, the Pole *o* has now left the Fiſſure *m n*, more than a Quarter, namely, the Arch *n o*; and ſo the Arch *n o*, (which is to be computed contrary to the order of the Signs ♈, ♉, ♎, ♑,) will increaſe on more and more, till by that Motion of Reflexion of the Pole *o*, moving all along more and more from *n*, contrary to the order of the Signs, does at laſt go its Round, the Earth being carried about through *f g h i*, into *b*, where the Pole *o* is again join'd with the end of the Fiſſure *n*. The ſame may be imagin'd of the Southern Pole *p*, the end of the Fiſſure of the Baſis, that is turn'd towards us, departing likewiſe more and more from *m*, contrary to the order of the Signs, till the Earth being return'd into *b*, the Pole *p*, and the end of the Fiſſure be join'd again. And this is the Motion of Reflexion, as *Copernicus* call'd it; by which it comes to paſs, that the Axle of the Earth always continues in the ſame Parallelism, with the Fibres or ſtreight Lines of the groſs or thick Axle of the Cœleſtial *Æquator*, deſcrib'd in the XVI. Chapter.

For neither is it to be conceal'd, That that ſlow Motion (by which in Scheme XXXI. the Axle of the Cœleſtial *Æquator* O P, does with the end O, for inſtance deſcribe the Circle O 7, about the Pole of the Ecliptick Z, of which alſo I made mention in the XVI Chapter, that that ſlow Motion, I ſay, does likewiſe affect the Earth's Axle *o p*, which, as I ſaid, does always keep the ſame Parallelism with that thick Axle O P.

For thence it is, That in Figure XXX. the Pole of the Earth *p* or *o*, do not in a Sidereal year, run all out through the whole Circle *n o m p*; but only reaches every year a Point 51″ on this ſide; namely, juſt ſo much as the Axle declines from its parallelism in the ſame ſpace of time. Now therefore ſeeing 'tis evident, that for example, the Pole *o* does ſometimes fall upon the Fiſſure *m n*, and that the Plane of Illumination is the cauſe of this, and ſometimes it is more elevated: It may therefore be deſir'd to be known, how great in every moment is that Elevation or Eminency of the Pole *o*, above the Plane of Illumination, and its Fiſſure *m n*. We muſt know therefore, that the Motion of Reflexion, or the Arch *n o*, when the Earth is for inſtance in *c*, is always equal to the Motion of the Centre of the Earth in the Orbit, or to the Arch *b c*. For ſeeing the Diametre of the great Ecliptick ♋ *a* ♑, is always parallel to the Diametre of the Earth's Ecliptick ♋ *c* ♑, and the Ray *a c* which joyns the Centres, does always fall upon theſe parallels, 'tis plain, the Alternate Angles *p c a*, and *c a d*, are equal, wherefore alſo their Complements *p c m*, (or *o c n*) and *c a b* will be equal. Therefore there being given the Arch *n o*, equal to the Arch *b c*, there may be found no Perpendicular *o s*, that falls from the Pole *o*, to the Fiſſure *m n*, by ſaying: As the Ray is to the Sign of the Angle *o c n*, ſo is *c o* the Sign of 23° ½, to *o s*, the Sign of the Elevation, or Eminency of the Pole above the Plane of Illumination, to which is equal the Eminency of the Ray *a c* above the *Æquator*, or the Declination of the Sun, which I will'd you to remember from what went before.

CHAP. XVIII.

Of the Seaſons of the year, and the Change of Days and Nights.

HOW it happens, that when the Earth is under *Aries* or *Libra*, there are equal Days all over the World, I have explain'd in the foregoing Chapter: Now in Figure XXXI. let the Earth be under *Cancer*, the Ray of the Sun ♈ ♑ ♈, falls directly upon the Tropick of *Capricorn* ♑ *u l*, one Point of which after another, as the Day goes on by little and little, will meet directly with the Sun, which conſequently will be Vertical to every Point in order. And ſeeing the Circle of Illumination is *q y*, dividing the Tropick of *Capricorn* into the greater ♑ *u*, and *u l* the leſs; 'tis plain, that all the Points of this Tropick are longer in deſcribing the greater part ♑ *u* by their daily Motion, than in the leſs *u l*, that is, they

they ſtay longer in the enlighten'd Hemiſphere, than in the dark one; and ſo in them, the Day is longer than the Night. And by how much any parallel is the more diſtant from the *Æquator* 90 270, towards the Southern Pole *p*; ſo much the more unequal will its Diviſion be, and the greater part will be enlighten'd, and the Day for that reaſon will be the longer; till you ſee the Antarctick *x y*, wholly withdraw it ſelf from the dark Hemiſphere, and in all its Revolution remain in the Light, with all the Southern Zone *x p y* included in it. And ſince 'tis clear from Scheme XXX. that the Southern Pole *p*, the Earth being under *Aries* in *b*, begins to decline from *m n*, the Plane of Illuminations, and to depart from the ſame, inclining towards the Sun *a*; while the Earth is in *c d e*, that is in the Northern Signs, where you ſhall always find *p* plac'd between *m n* and *a*: You may therefore eaſily underſtand, that all that time, the Southern Pole remains turn'd towards the Sun; as on the contrary, *o* the Northern Pole at the ſame time remains turn'd from *a* the Sun, beyond the Plane of Illumination *m n*; whence it muſt needs come, that the Pole *p* muſt be ſix whole Months in the Light, while the Pole *o* is hidden with darkneſs. But on the contrary, while the Earth goes through the Northern Signs from *f* through *g h i*, even to *b*; the Northern Pole *o* is turn'd to the Sun *a*, on this ſide, the Plane of Illumination *m n*, but the other *p* beyond *m n* is hidden; whence by way of alternate change, the contrary will happen, namely, a day of Six Months to the Pole *o*, and a night of as many Months to the Pole *p*. Let us return now to Scheme XXXI. that we may finiſh this whole Speculation. The Earth being under ♋, I ſaid, the Tropick of *Capricorn* ♑ *u l*, in its greater portion *Capricorn u*, and the whole Antarctick *x y*, is in the enlighten'd Hemiſphere *q* ♑ *y*. But on the other ſide, there you ſee the Tropick of *Cancer k t* ♋, in its leſſer portion, and the Arctick *q w*, to be in no meaſure enlighten'd by the Sun, that is plac'd in the Centre of the Ecliptick behind ♈, and ſo that every point of the Tropick *k* ♋, is longer in paſſing through the greater portion *t* ♋ in its daily Motion, than in paſſing through the leſs *k t*, that is, it is longer in Darkneſs than in Light. But again, the Earth being plac'd under ♑, the Ray of the Sun ♈, ♋, ♈, falls directly upon every Point of the Tropick of *Cancer* ♋ *t k*, as it meets with them one after another; and the greateſt part of it ♋ *t*, and ſo the whole Arctick *q w*, with the Northern Frigid Zone *q o w*, that are included in it, are in Light; while on the other ſide, the greater part of the Tropick of *Capricorn* ♑ *u*, and the whole Antarctick *x y*, with the Southern Frigid Zone *x p y*, that is included therein, are in Darkneſs.

CHAP. XIX.

Of Equation of Time in the Syſteme wherein the Earth is ſuppos'd to move.

Figure XXXII.

IN Figure XXXII. let there be a projection of the Copernican Sphere, the Eye being ſuppos'd to be plac'd in the Axle of the World, and looking at an infinite diſtance directly upon the Southern Pole of the *Æquator*, the *Æquator* will appear as a Circle, whoſe third Quadrant or fourth part is ♎ 270, and the Ecliptick will fall into an Oval Figure, whoſe Quadrant let ♎ ♑ be ſuppos'd to be. Let ☉ be in the Centre of the Sphere, and in the Ecliptick let the middle place of the Earth be mark'd in M, and the true place in V. For nothing hinders, but we may advance the Earth out of its true Orbit, which it has in the Region of the Planets, according to the Line of its middle and true Motion to the very Firmament, if ſo be it be but underſtood, to move there in its daily Motion about its own Axle, juſt after the ſame manner as before. And in this mean while 'tis plain, that the Earth's *Æquator*, as alſo that of the Heavens, is caſt into a Circle, and all the Meridians, as likewiſe the Circles of Declination, do appear as it were Right Lines. Therefore the Circle of Declination will be drawn through V, the true place of the Earth, and will be the Line ☉ V *r*, marking in *r* the Right Aſcenſion of the Earth. Farthermore, in the Colure of the

the Solſtices ☉ ♑ 270, the leſſer parallel Circles drawn through M and V, that is, through the middle and true place of the Sun, will reduce them both to the *Æquator*, to wit, to the Points *m* and *v*, which, are as far diſtant from ♎, as M and V are: And ſo *m* will be the Feigned Earth, which, in like manner as the Feigned Sun in the third Chapter, is to moderate the Equable or Aſtronomical Time, its Yearly as well as daily Motion, being ſuppoſ'd to be as regular as is poſſible. And the Regularity of the yearly Motion is perceiv'd, if the Centre of the feigned Earth *m* be ſuppoſ'd to move every day the ſame ſpace in the *Æquator* ♎ 270; but the Regularity of the daily Motion in this, that whereſoever the Earth is in its true Orbit, every Meridian of it always moves around in the ſame ſpace of time, to the ſame Paralleliſm with the Colure of the Equinoxes ♎ ☉; which Revolution would alſo reſtore again the ſame Meridian to the Sun, if it had no yearly Motion, and the Centre *m* would always be in the ſame Point of the *Æquator*. But now, while the Centre of the Earth is mov'd from ♎ to *m*, 'tis not enough, that the Meridian *o s*, being carried about by its daily Motion, ſhould return to the ſame Paralleliſm with the Line ♎ ☉: For the Meridian *t u*, which, when the Earth was in ♎, was directed towards the Sun, the Earth being mov'd on to *m*, though it return to its Paralleliſm *o s*; yet it is not as yet directed towards the Sun, till *o s* be carried a little farther, ſo that it falls into the ſame right Line with B *m* E ☉. For juſt ſo much, as the Centre of the Earth has gone on from ♎ to *m*, ought the Meridian *o s*, to go beyond the Paralleliſm that it obtain'd with *t u*, before it can fall in with B *m* E ☉, and be directed towards the Sun; ſince the Angles ♎ ☉ *m*, (or the Arches ♎ *m*,) and B *m o*, (or *s m* ☉,) are equal. Whence it follows, that the daily Motion of every Meridian from Sun to Sun, does amount to a perfect Revolution to the ſame Paralleliſm with ♎ ☉, and moreover to a portion of the ſame Revolution, that is equal to the Motion of the Earth's Centre, that is to be made from *Æquinox*, while that ſame Meridian rolls about again to the Sun. But farther, let us ſuppoſe the true Earth, being in the Ecliptick in the point V, to have its Revolution about its own Axle, govern'd by the ſame Law of Regularity, as the feigned Earth in *m*; to wit, that any one Meridian of the true Earth V, which is once parallel to any one Meridian of the feigned Earth *m*, will always remain parallel to the ſame; which is likewiſe to be underſtood of the Earth, when plac'd by Fiction or Imagination in any other Point, as for inſtance in M, *r*, *v*, &c. And now from hence 'tis eaſie to underſtand the Equation of time, which is nothing elſe but the Arch of the *Æquator r m*, that lies between *r*, the right Aſcenſion of the true Earth that is plac'd in V, and between *m*, the feigned Earth, and which is turn'd into Time: To which is equal the Angle *r* ☉ *m*, or that which is alternate to it, E *m d*, or B *m a*. For when the parallel Lines A D, *a d*, ♈ ♑, do through the Antecedent Signs, bring back again the Plane of the ſame Meridian of the Earth, that is feign'd to be in *r m v*; yet it appears, that only the Meridian A D is directed towards ☉ the Sun, from whom the other *a d* is as yet diſtant the Angle *d m* ☉, and the Third ♈ ♑, a greater ſtill, *viz.* ♑ *v* ☉; whence it is, that when A D the Meridian of the true Earth V has reach'd the Sun, the Correſpondent Meridian of the feigned Earth *a d*, has not as yet come to the Sun, and ſo the true Noon-day does come before the Equable or Aſtronomical one, the Angle *d m* ☉, or *m* ☉ *r*, or laſtly, the Arch *m p*, as I ſaid before. But here alſo, as in the third Chapter, the Arch *r m* is always either the ſumm or the difference of two Arches, the one of which is the difference of the true Motion of the Earth, and of its right Aſcenſion, and the other is the difference of the true and middle place of the Earth. So in this Example, the difference of the true Motion of the Earth ♎ V or ♎ *v*, and of its right Aſcenſion ♎ *r*, is *r v*; and the difference of the true and middle place of the Earth, is M V or *m v*: Therefore now if from *r v*, you take *m v*, there remains the Arch of Equation *r m*; or if from the Angle ♈ V C, you take the Angle *b* V C, there remains the Angle of Equation ♈ V *b*, or *a m* B, that is to be turn'd into Time; which will be the difference between the Apparent and middle Noon-day or any other hour.

CHAP.

CHAP. XX.

Of Kepler's *Hypotheſis.*

HAving thus explain'd the Spherical Doctrine of the Syſtem, wherein the Earth is ſuppos'd to move, the next thing to be done is, that I explain the *Phænomena* of the ſecond moveables by fit *Hypotheſes* in the ſame Syſtem. And here I ſhall wave *Copernicus*'s Circular *Hypotheſis*, as being too diſagreeable; as alſo the Circular *Hypotheſis* of *Lansbergius*: And ſhall follow only the Elliptical ones, ſince *Kepler* in his Commentary concerning the Planet *Mars*, has ſufficiently proved, That no other Orbits but Elliptical will ſerve for the accurate and exact Explication of the Motion of the Planets. Now the chief *Hypotheſis* among the Elliptical ones is *Kepler*'s, who ſuppoſes the Sun to be placed in the lower Focus of the Elliptical Orbit of every Planet, and that by its rolling about its own Axle in about 27 days, it carries about the Planets that are plac'd around it, by virtue or force of its Rays, as if it were by a kind of a Leaver; And thoſe that are nearer, it carries the more ſwiftly, and thoſe that are farther off, the more ſlowly; juſt as a Leaver lifts up a weight that is nearer to the moving power with the greater force. Whence it would have come, that, if every Planet always kept the ſame Diſtance from the Sun, it would move by a Motion that would be altogether regular: But ſeeing our Author does conclude, that the Bodies of the Planets are furniſh'd with magnetical Fibres, which with one end do tend towards the Sun, and with the other do flee from it; it follows that they ſometimes go farther from the Sun, and at other times come nearer, according as they turn their agreeing or diſagreeing Pole towards it. Whence it conſequently follows, that the Planet which goes farther off, is carried about the more ſlowly by the weaker power of the Sun's Rays, and on the contrary when it is otherwiſe: And all this in this manner and meaſure, that the Area, which the carrying Ray, that is extended from the Sun to the Planet, does bruſh or paſs over, do increaſe equally in equal Moments of Time. In Figure XXXIII. let A be the Sun, and alſo the lower Focus of the Elliptick, let L be the other Focus, B the Centre of the Elliptick, and the Elliptick it ſelf P E X I, which the Planet moves in, in its Motion about the Sun. Let there be a Circle drawn about the Sun P D X T. The Line of the Apſes is extended through the Focuſſes A and L, and is X A L P; The *Aphelium* is P, and the *Perihelium* X, and A B is the Eccentricity, and the length of it doubled is call'd the Diſtance of the Focuſſes. The greateſt Diſtance of the Planet from the Sun is A P, and the leaſt A X, and the middle diſtance A E, which is the Arithmetical middle between A P and A X, and alſo equal to the Semidiametre of the greateſt Elliptick B P; but the leaſt Semidiametre is B E. If the Planet be in C, the carrying Ray will be A C, extended from the Sun A to the Body of the Planet. 'Tis certain, that this Line A C, or the carrying Ray, does paſs over all the Area of the whole Elliptick, while the Planet returns from P through C E X I, to P again, after it has perform'd its whole Revolution. So while the Planet deſcends from P into C, the ſame carrying Ray A C does paſs over, as it were bruſhing or ſweeping the Area P A C, that lies between the Line of the Apſes A P, the carrying Ray A C, and the Arch of the Elliptick P C. Now I ſay, This is the Law or manner of the Motion, that as the Time of the whole Revolution, in which the Planet goes about from P to P, is to the Time in which it deſcends from P into C; ſo alſo is the Area of the whole Elliptick P E X I P to the Area P A C: So that the Times, in which the Planet C proceeds, and the Area which the carrying Ray does in the mean while paſs over, have each of them the ſame proportion to their ſeveral Integers or wholes. And for this reaſon the middle Anomaly is not by *Kepler* expreſſed by an Arch, or by any Angle, as it commonly is; but either by Times, as theſe are three hundred and ſixty parts of the whole Period or Revolution; or elſe by the *Area*, (for inſtance P A C,) which the carrying Ray does paſs over, as theſe are alſo three hundred and ſixty parts of the whole Area P E X I P. For why may we not divide the whole Periodical Time, or the whole Area of the Elliptick, as well as the Circumference, into 360 parts, which

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we may afterwards call degrees, and ſubdivide them into leſſer Parts? Why may we not alſo give the Name of Anomaly either to the Time, or to an Area, as well as to Arches and Angles, ſeeing Names are given to Things according to the pleaſure of their Inventors? And here, leſt it ſhould ſeem difficult to any one to find the Quantity of the Area P A C, as it is a portion of the Elliptick; *Kepler* has found out a Remedy for this, by putting for a portion of the Elliptick P A C the portion of the Circle P A K; to wit, by drawing a perpendicular through the Planet C to the Line of the Apſes C L, (which they call the ordinated Line) and by continuing it to the Circumference of the Circle that is drawn in K, and by making it A K. Now I ſay, That the part of the Circle P A K hath the ſame proportion to the whole, that the part of the Elliptick P A C has to the whole Elliptick; for the part of the Circle P A K being divided into two parts, P L K, and K L A, and alſo the part of the Elliptick P A C into two parts, P L C, and C L A; firſt the Area P K L has the ſame proportion to the Area P C L, that K L has to C L, or that D B has to E B; and ſo the very ſame, that each of the ordinated Lines of the Circle have to the ordinated Lines of the Elliptick: For the Area, as well of the Circle as of the Elliptick, is made up, as it were, of their innumerable ordinated Lines, which have been already ſaid to have the ſame proportion one to onother. Whence it is, that not only the Semicircle P D X has the ſame proportion to the Semi-Elliptick P E X, that D B has to E B; but alſo the whole Circle has the ſame to the whole Elliptick, and the portion of the Circle P K L to the portion of the Elliptick P C L. And alſo the Triangles K L A and C L A bear the ſame proportion to one onother, that their Altitudes K L and C L do: And therefore the compos'd Area P A K has the ſame proportion to the compoſed Area P A C, that K L has to C L; or that the whole Circle has to the whole Elliptick; And for the Surface P A C may be ſubſtituted the ſurface P A K, if ſo be at the ſame time for the whole Elliptick be ſubſtituted a whole Circle, which muſt now be underſtood to repreſent the middle Anomaly, and is likewiſe to be divided into 360 degrees, as the Elliptick has been divided above.

It remains now, that to every middle Anomaly, for inſtance, to the Surface P A C, we find out an agreeable Anomaly co-equall'd to it, to wit, the Angle P A C, that lies between the Line of the Apſes and the carrying Ray. Now this, neither *Kepler*, nor any other after him, could ever find out by a direct way; yet we want not a way whereby it may be come to indirectly. For the Arch P K, which is cut by the Line L C K, that is ordinated or directed through the Planet, he takes to be of as many Degrees as he pleaſes, and calls it the Anomaly of the Eccentre. Then he ſeeks an agreeable Area P A K, as the middle Anomaly, and alſo the Angle P A C, as the co-equall'd Anomaly. Whence he afterwards infers, that ſeeing the Anomalies ſo found, as well the middle as the co-equall'd, do agree with the ſame Anomaly of the Eccentre, it cannot be but they muſt mutually anſwer one another. Now the Method is this, Firſt, The Anomaly of the Eccentre P K, is taken to be of as many degrees as you pleaſe, the middle Anomaly, that agrees with it, or belongs to it, is the Surface P A K, that conſiſts of two parts, P B K, and K B A. And the Area of the Sectour P B K is the ſame part of the whole Circle, that the Arch P K is of the whole Circumference; but the Area of the Triangle K B A is equal to the Altitude K L, drawn into the half of B A.

Now L K is found, when firſt B D, the Ray of the Circle, is known, whoſe Area contains 360°, 'tis found by ſaying, As 22 to 7, (or as 314159 to 100000) ſo is the Area of the Circle to the Square of the Ray or Semidiametre. Then you muſt ſay, As the Tabular Ray to the Ray D B, lately found, ſo is the Sign P B K to K L. But alſo B A is to be found in like parts, by ſaying; As the greateſt Semidiametre of the Elliptick of the Planet D B, is (in parts of which the middle Diſtance from the Sun is 100000) to the ſame D B, as a Ray of the Circle that contains an Area of 360°; ſo is A B, the Eccentricity, to the ſame A B, which anſwers to the Area of the whole Circle 360°. And then again, for finding the co-equall'd Anomaly, *viz.* the Angle P A C; firſt in the Triangle K B A there are given K B, B A, and the Angle P B K, whence alſo will appear the Angle B A K. But farther, as D B, the greateſt Semidiametre of the Elliptick, is to E B, the leaſt Semidiametre; ſo is K L, the Tangent of the Angle K A L, newly found, to C L, the Tangent of the ſought-for Angle P A C. As for Example: In *Mars* let the Anomaly of the Eccentre P K be ſuppos'd to be of 60°: let the middle Anomaly

that anſwers it be ſought after, *viz.* the Surface P A K; and alſo the co-equall'd Anomaly, the Angle P A C. Out of the *Rudolphine* Tables, which are added at the latter end of this Book, there is given the Log. of the greateſt Diſtance of *Mars* from the Sun, 4.22131, and its abſolute Number 16646 is the very A P: So the Log. of the leaſt Diſt. is 4.14060, whoſe Abſolute 13823 is A X. A P − A X = A L = 2823; therefore A B = 1411.5: Therefore alſo A B + A X = B X = 15234.5. In the Triangle B A K are given B K = 15234.5, B A = 1411.5, and P B K the external Angle = 60°; there is found B A K = 55.615. The Rule is; As the greateſt Diſtance A P is to the leaſt A X; So is the Tangent of half the Anomaly of the Eccentre (*i. e.* the Tangent of ½ P B K) to the Tangent of ½ X < <. That is, Always add 991929 to the Tangent of ½ the Anomaly of the Eccentre; The Summ, the Semidiametre being ſubtracted, is the Tangent ½ X < <. Then the half of the Anomaly of the Eccentre + ½ X < < = B A K. For the finding of the leaſt Semidiametre B E in the Triangle A B E are given A B = 1411.5, A E = 15234.5, and A B E the right Angle. There is found, Firſt, the Angle A E B = 5.32. Secondly, B E = 15169. Then ſay, The Log. of the greateſt Semidiametre 4.18283 gives the Log. of the leaſt Semidiametre 4.18096; therefore the Artificial Tangent of the Angle B A K newly found out, gives the Artificial Tangent of the Angle B A C, which is the very co-equall'd Anomaly 55.50. Or, always add 999813 to the Tangent B A K, the Summ, the Semidiametre being ſubtracted, is the Tangent B A C. Then for the finding the middle Anomaly, or the Surface P A K, firſt, the Sector P B K is 60°, to wit, the ſame part of the Area of the Circle, that the Arch P K is of the Circumference. But the Ray D B of the Circle, whoſe Area is 360°, is found by ſaying,

Abſolute Numbers.	*Logarithms.*
As 314159	5.49715
is to 100000,	5.00000
So is the Area 360°	2.55633
to the Square Root,	2.05915
Therefore the Root, *&c.*	1.02958

Again:

As the Logarith. of the mid. diſt.	is to the Log. of Eccentricity		So is the Root D B from the Area 360°	to the Eccentricity A B from 360°
4.18283	3.14968	::	1.22958	$\bar{1}$.99643.

And by taking away the Log. of two 0.30103

There remains the Logarithm. ½ A B $\bar{1}$.69540.

Farther:

As the Tabular Ray is to the Ray from the Area of 360; ſo is the Sign P B K, to K L from 360° 10.00000. 1.02958 :: 993753. 0.96711.

Taking away the firſt Term from the ſecond, there always remains $\bar{9}$.02958 to be added. Laſtly, K L in ½ A B = to the Area K B A; and in the Logarithms 0.96711 + $\bar{1}$.69540 = 0.66251. And ſeeing the firſt of the Components, namely, 0.96711 equals $\bar{9}$.02958 + the Sign P B K (as was juſt now ſaid) the compounded Number will be 0.96711 = $\bar{9}$.02958 + the Sign P B K + $\bar{1}$.69540 = $\bar{10}$.72498 + the Sign P B K. Wherefore for finding the Area of the co-equalizing Triangle, the perpetual Rule is this: Always add $\bar{10}$.72498 to the Artificial Sign of the Anomaly of the Eccentre, the Summ is the Logarithm of the Area K B A; which you muſt add to P B K, an Area of as many Degrees, as the Anomaly of the Eccentre P K conſiſts of, and you will have the whole ſurface P A K, which we call the middle Anomaly.

Now to the Logarithm of the *Area* K B A 0.66251 does anſwer the abſolute Number 4.5974, that is to be added to the Area P B K of 60°, and there is made the Area P A K 64.5974.

And

And now from the middle Anomaly 64.5974 take the co-equall'd Anomaly found above ——— ——— 55.50.

There remains the *Prosthaphæresis* 9.0974.

Also the Table of Equation of the Centre of *Mars* does give the same to the same middle Anomaly; whence is manifest the way that all those Tables of the Equation of the Centre were made. In this *Hypothesis* of *Kepler* some disapprove only of this, That from the middle Anomaly, when given, he deduces the co-equall'd Anomaly only by an indirect way; commending at the same time both its Agreement with the Heaven's, and the handsome way of demonstrating Motions from their Causes; which indeed *Kepler* valued so much, (not without the Approbation too of all candid Judges,) that he chose rather to follow an indirect Method of Computation, than to devise another *Hypothesis* less agreeable to Nature: For they that object against Him an Ungeometricalness in the *Hypothesis*, have not yet solv'd his Problem; and they seem to be deceiv'd, as if they believe, that even an indifferent Geometrician might not find out a way to solve *Phænomena à priori*, from their Causes; except he had studied that which was to no one's damage, but was well approv'd by many, namely, to explain the natural Causes of the Motions.

But omitting these, I shall give you from *Kepler* the Dimensions of the Orbs of every Planet, taken in parts, of which the middle distance of the Earth from the Sun is 100000.

	The Interval.			
	Aphelion.	*Middle.*	*Perihelion.*	*Eccentricities.*
Of *Saturn*	1005207	951000	896793	5700
Of *Jupiter*	544701	519650	494592	4822
Of *Mars*	166465	152350	138235	9263
Of *The Earth*	101800	100000	98200	1800
Of *Venus*	72900	72400	71900	694
Of *Mercury*	46955	38806	30657	2100

Of such as the Semidiametre of the Orb of the Planet is 100000. Of such as Semidiam. of the yearly Orb is 100000.

Nor must we conceal here the most admirable Theorem of the same Author, wherein he shews, that the Periodical Times of the Planets, compar'd with one another, have the proportion of one and an half with respect to their middle Distances from the Sun; that is, a proportion of a single proportion, and half as much more: Or, which comes to the same, the Squares of the Periodical Times are as the Cubes of the middle Distances, which it may be worth our while to illustrate by an Example:

The Middle Distance of the Earth from the Sun, to the Middle Distance of *Saturn*, is as an 100 to 951.

Hence it is inferr'd, that the Periodical Time of *Saturn* is 29 years and 327 days. For as the Cube of the Number 100 is to the Cube of the Number 951; So the Square of the Number 1, (that of the Earth's Revolution, or of one Tropical year,) is to the Square of the Number 29 years, 327 days; that is,

As 1000000 is to 86008,351: So is 1 to 860.085351. For the Numbers 860.085351 is the Square Root of 29.327.

CHAP. XXI.

Of the Hypothesis *brought by the Learned* Seth Ward, D. D. *afterward Lord Bishop of* Salisbury, *and by* C. P.

Fig. XXXIV. IN Figure XXXIV. s is the Sun, plac'd in the lower *Focus* of the Elliptick; f is the upper *Focus* of the Elliptick; a the *Aphelium*, p the *Perihelium*, e the wandring Star or Planet; fe, or the Line parallel to it, $s\,m$, is the Line of the Middle Motion of the Planet; $s\,e$ is the Line of the true Motion of the Planet, (beheld from the Sun;) the Angle $a\,f\,t$ is the middle Anomaly, increasing equally in equal times; the Angle $a\,s\,e$ is the co-equall'd Anomaly; $f\,e\,t$ is a right Line, $e\,t = e\,s$.

'Tis now purpos'd to be shown, how from the Middle Anomaly, when given, or from the Angle $a\,f\,t$, the co-equall'd Anomaly may be drawn, or the Angle $a\,s\,e$.

The Demonstration.

Of the Triangle $s\,f\,e$ the three sides $s\,f + f\,e + e\,s = 2\,a\,s$, according to the Nature of an Elliptick Figure.

Now $2\,a\,s = a\,s + a\,f + f\,s$,
And by putting $p\,s$ for $a\,f$, is made
$2\,a\,s = a\,s + p\,s + f\,s$, β
Therefore by α, and β, and Ax. 1. $s\,f + f\,e + e\,s = a\,s + p\,s + f\,s$; And taking $s\,f$ from both sides, there remains $f\,e + e\,s = a\,s + p\,s$;
And putting $e\,t$ for $e\,s$, by way of supposition, there is made $f\,e + e\,t$ (i. e. $f\,t$) $= a\,s + p\,s$.
Again, $a\,s - a\,f = f\,s$.
And putting $p\,s$ for $a\,f$, there is made
$a\,s - p\,s = f\,s$. δ
Now therefore in the Triangle $s\,f\,t$ there are given two sides, $s\,f$ and $f\,t$, with the Angle comprehended in them, $s\,f\,t$.
For $f\,t = a\,s + p\,s$ by γ
And $f\,s = a\,s - p\,s$ by δ
And $a\,f\,t$ is the Middle Anomaly, which Angle being external to the Angle $s\,f\,t$, it is equal to the two internal and opposite Angles $f\,t\,s$ and $f\,s\,t$, that is, the Angle $a\,f\,t$ is Z $<<$ of the unknown ones $f\,t\,s$ and $f\,s\,t$.

Now by the *Trigonometrical* Rule:
Z II. XII :: $t\,\frac{1}{2}$ Z $<<$. $t\,\frac{1}{2}$ X $<<$, that is,
$f\,t + f\,s$. $f\,t - f\,s$ :: $t\,\frac{1}{2}$ of the Angle $a\,f\,t$. $t\,\frac{1}{2}$ X $<<$ $f\,t\,s$ and $f\,s\,t$,
That is by Interpretation,
$2\,a\,s$. $2\,p\,s$ :: $t\,\frac{1}{2}$ of the Angle $a\,f\,t$. $t\,\frac{1}{2}$ of the Angle $f\,s\,e$ ε :
For $f\,t = a\,s + p\,s$ by γ
And $f\,s = a\,s - p\,s$ by δ } Add first, and then subtract, and there is made
Z $= f\,t + f\,s = 2\,a\,s$
X $= f\,t - f\,s = 2\,p\,s$, therefore
$\frac{1}{2}$ Z $= a\,s$, and
$\frac{1}{2}$ X $= p\,s$.
Again, X $<<$ $f\,s\,t$ and $f\,t\,s = f\,s\,t - f\,t\,s$,
And by putting $e\,s\,t$ for $f\,t\,s$ is made
X $<<$ $= f\,s\,t - e\,s\,t = f\,s\,e$,
Therefore $t\,\frac{1}{2}$ X $<<$ $= t\,\frac{1}{2}$ of the Angle $f\,s\,e$, : :
And therefore also $\frac{1}{2}$ Z II. $\frac{1}{2}$ XII. :: $t\,\frac{1}{2}$ Z $<<$. $t\,\frac{1}{2}$ X $<<$ That is, by ε.
$a\,s$. $p\,s$ :: $t\,\frac{1}{2}$ of the Angle $a\,f\,t$. $t\,\frac{1}{2}$ of the Angle $f\,s\,e$, which is a perpetual Rule for finding out the first Inequality of the Planets; and it is expressed in words thus: As is the *Aphelian* Distance of the Planet to the *Perihelian* Distance; so is the Tangent

Tangent of half the middle Anomaly, to the Tangent of half the co-equate Anomaly. And becauſe the diſtance of the Planet, as well *Aphelian* as *Perihelian*, is conſtantly the ſame, 'tis plain, the two firſt Terms of the propos'd Analogy are always the ſame, to wit, in the Sun, for inſtance theſe:

The Logarithm of the Earth's greateſt diſtance 4.00775
The Logarithm of the leaſt diſtance 3.99216.

Whence by the Rule of Proportions; the ſecond being divided by the firſt, or in Logarithms, the firſt being ſubtracted from the ſecond, there remains a number, that is always to be added to the third; and that number which is to be added, will be always the ſame; becauſe the two firſt Terms of the Analogy, by the Subtraction of which it ariſes, are always the ſame.

As for Example in the Sun, the firſt Term is 4.00775
Let it be ſubtracted from the ſecond, 3.99216
And there always remains the number that is to be added to the third, 9.98441.

So for the reſt of the Planets, the Numbers that are always to be added are theſe:

☉	998441
♄	995049
♃	995808
♂	991922
♀	999400
☿	981477
☾	996209

The uſe of this Table is this:

Add the Numbers always to the Artificial Tangent of half the middle Anomaly, the ſumm will be the Artificial Tangent of half the Co-equate Anomaly.

But farther, to find out the Line *s e*, or the diſtance of the Planet from the Sun, the Operation is to be thus wrought:

a f t, — *a s e*, = *s e f*, then
s of the Angle *s e f* . *s f* :: *s* of the Angle *a f t*, (*s f e*:) *s e*

But now, though this method do, from the middle Anomaly given, gather the Co-equate Anomaly directly, or *à Priori*; and though C. P. believ'd, that if there was any Variance in this *Hypotheſis* from the *Hypotheſes* of others, it was to be attributed to the errour of Aſtronomers, which is very great about the Octants from the *Aphelion* and *Perihelion*; yet 'tis certain, that this *Hypotheſis* is almoſt as far wide from the Obſervations made in the Heavens, as he thought the Aſtronomers had err'd: Nor is that Diſagreement from the Heaven only in ſome few Minutes, but ſo in *Mars* it may ſometimes amount to almoſt half a degree, which no one can ever aſcribe to the Faultineſs of Obſervations.

Nor was *Kepler* ignorant, that this Inconvenience would follow the making an equable Motion about the upper *Focus*: For he ſaid in his Epitome, that it might be *almoſt* ſo taken; but that *almoſt* was of ſuch extent, that he thought it better to let it alone.

CHAP. XXII.

Of the Limitation, which Bullialdus *made to the foregoing* Hypotheſis.

IN Figure XXXV, let *s* again be the Sun, plac'd in the lower *Focus*, *f* the upper *Focus*, *c* the Centre of the Elliptick *a q p*, and of the Circle that is drawn about it, *a* L *p*, *a* the *Aphelion*, *p* the *Perihelion*. Let the Line of the middle Motion be carried uniformly about the upper *Focus f*, as in the foregoing, and let it be *f e*, (or *f* E,) but with this difference, that whereas the Planet was before in the Line of the middle Motion, where it cuts the Elliptick Orbit, to wit, in *e*, (or E:) Now it is in the right Line *f m*, (or *f* M,) drawn from the upper *Focus f*, to the end *m*, (or M,) which the Ordinated Line *n m*, (or N M,) that is extended through *e*, (or E) the common Section of the Line of the middle Motion and of the Elliptick, does reach the Circle: For where this Right Line *f m*, (or *f* M,) cuts the Elliptick in *e*, (or E,) there let the Centre of the Planet be ſuppos'd to be plac'd. Farther, from the Planet in *e*, (or E,) to each *Focus* let there be drawn the right Lines *e f*, *e s*, (alſo E *f*, *e s*,) and alſo from the ſame Point, let a Line be drawn parallel to the Line of the middle Motion, *viz.* *e o*, parallel to *e f*, (and *e o* to E *f*.) The Figure XXXV.

The thing now propos'd, is from the middle Anomaly when given, or from the Angle *a f e*, to find the Co-equate Anomaly, or the Angle *f s ε*: Therefore is first to be found the Angle *a f m*, which *Bullialdus* calls the middle true Anomaly. Now, as the least Semidiametre of the Elliptick *c q*, is to the greatest *c* L; so is *n e* to *n m*; so also is the Tangent of the middle Anomaly *a f e*, to the Tangent of the middle true Anomaly *a f m*.

And now the Angle *a f m*, or *a f ε* being found, which is External to the Triangle *f s ε*, *f s ε* is found by the same Analogy, as in the foregoing Chapter, by saying; *as. p s :: t ½* of the Angle *a f e. t ½* of the Angle *f s ε*.

The found Angle *f s ε* is taken from *a f ε*, and there remains *f ε s*. Then, as the Sign *f ε s* to *f s*; so is the Sign *a f ε*, (or *ε f s*,) to *s ε* the distance of the Planet from the Sun.

Now the difference of the middle Anomaly *a f e*, or *a o ε*, and of the co-equate *f s ε*, is *s ε o*, and is call'd the *Prosthaphæresis*. And now by this Limitation, *Bullialdus* has brought it to pass, that from the middle Anomaly might be gather'd the co-equate *à Priori*; and also that at the same time the Computation should answer the Observation, which in the Elliptical *Hypothesis* no one ever did before him.

CHAP. XXIII.

Of finding the Geocentrick place of the Planet.

HAving been hitherto in explaining the first Inequality of the Planets, I come now to the second: For by what goes before, is found the place of the Planet, as it would be seen from the Sun, if our Eye was there plac'd, and is call'd for that reason, the Heliocentrick place of the Sun. But now seeing the Stars or Planets that are wheel'd about the Sun, do appear to us on Earth in another place from what they would, if look'd upon from the Sun; there happens to the Planets a second Inequality, not that there is really any such thing in their notions; but because the yearly Motion of the Earth (*i e*,) the Motion of our Eye seems to be in them. Wherefore this second Inequality is properly call'd the *Parallaxis* of the Orb, (understand it of the Earth's yearly Orb,) because it solely depends upon the moving of the Eye, from the Centre of the universe, (the Sun) and the place of the Planet that is seen from the Earth, is call'd the Geocentrick place; and it seems to be an useful distinction between the Line of the true, and the Line of the apparent Motion; that the former is drawn from the Centre of the Sun, through the Centre of the Planet, but that the latter is drawn from the Centre of the Earth, through the Centre of the Planet too; yet that each of them is continued to the very Firmament. The Reason of the name may be, because the place that is seen from the Sun is come to, by the true and proper Motion of the Planet; but that which is seen from the Earth, is owing to no other than an apparent seeming Motion. But now before the Geocentrick place of the Planet can be found, we must first bring its Heliocentrick place in the Orbit to the Ecliptick, which is done by the help of a Spherical Triangle, that lies between the Node and the Planet in the Orbit, and the Point, which the Circle of Inclination, drawn through the Planet, does find in the Ecliptick: For the difference between the *Hypoteneuse*, or the subtended side of this Triangle, and the greater Leg or side, is the very Reduction to the Ecliptick; and the lesser Leg of the same Triangle is call'd the Inclination. Then the distance of the Planet from the Sun is to be abbridged, which is done by the help of a certain Plane Rectangular Triangle, that lies between the Sun, that is plac'd in the Centre of the Sphere, and the Planet in its Orbit, and the Point, which a Right Line, falling perpendicularly from the Planet upon the Plane of the Ecliptick, does at length reach to: For the greater Leg of this Triangle, (to wit, the Line that is drawn in the Plane of the Ecliptick, from the Sun to the very Point, which it finds when it falls perpendicular,) is the very abbridged distance of the Planet.

Now

Now therefore to find out the Geocentrick Place of the Planet, we ought to have ready, 1. The true Place of the Sun. 2. The Heliocentrick Place of the Planet. 3. The Distance of the Earth from the Sun. 4. The Abbridged Distance of the Planet from the Sun. And being furnish'd with these, we conceive a rectilinear Triangle in the Plane of the Ecliptick, between the Sun, the Earth, and the Heliocentrick Place of the Planet in the Ecliptick. As for Instance, in Figure XXXVI. let S be the Sun, T the Earth, E the wandring Star or Planet. And let there be given, First, The true Place of the Sun seen from the Earth in the Line T S V in 12° ♑. Secondly, The Heliocentrick Place of the Planet in the Ecliptick, seen from the Sun in the Line S E H in 22° ♉. Thirdly, The Distance of the Earth from the Sun S T, in Log. let it be 3.99255. Fourthly, The Abbridged Distance of *Jupiter* from the Sun S E, in Log. 4.72821. 'Tis now propos'd to us, from these things given to find out the Geocentrick Place of the Planet, that is seen from the Earth in the Line T E G.

1. From the true Place of the Sun	5ˢ	12°	
Take away the Heliocentrick Place of the Planet	1	22	
There remains the Angle to the Sun V S H	3	20	

Now therefore in the Triangle S T E there being given the two sides S T and S E, with the external Angle V S H, which is equal to the Summ ∠ ∠ of the unknown ones S T E and S E T; the Angle S T E is thence sought for, which is call'd the Angle to the Earth, lying between the Line of the true Motion of the Sun T S V, and the Line of the Apparent Motion of the Planet, T E G.

The working of it is thus:

S E 4.72821 }
S T 3.99255 } Subtract.

Prefixt to the remainder 1, there is made 10.73566, which is the Artificial Tangent 79° 35′, whence there being 40° taken away, there remains 34° 35′. The Artif. Tang. of which is 9.83850. Then the half of the Ang. V S H 55 00
The Tangent is 10.15477
The Summ, the Root being deducted, is 9.99327 The Tangent of the Arch 44 33

The Summ of the Arches 99 33

is the greater unknown Angle S T E, which is call'd the Angle to the Earth. Therefore the Apparent Place of *Jupiter* is on this side the true Place of the Sun in the Angle S T E, or degrees 99° 33′.

For take from the true place of the Sun	5ˢ	12°	00′
the Angle to the Earth	3	09	33
And there remains the Geocentrick place of ☿	2	02	27

Lastly, to find the Latitude of the Planet, or it's distance from the Ecliptick, as it is seen from the Earth, say,

As the Sine of the Angle to the Sun
is to the Sign of the Angle to the Earth,
So is the Arch of Inclination (computed in degrees and minutes)
to the Arch of Latitude.

And having been hitherto in laying down and explaining the *Hypothesis* of others, I have a mind to subjoyn one of my own, which I began to frame about twenty four years ago; And I did some time ago cast it into the form of a little Book, which is to be now inserted.

SECT.

SECTION III.

A New Astronomical Hypothesis, *and its Agreement with Observations.*

The Contents of the BOOK.

The Chapters of this Book are three:

The First lays down the New *Hypothesis.*
The Second teaches how to calculate according to it.
The Third compares the Calculations with the Observations.

CHAP. I.

Laying down the Hypothesis.

1. I Suppose the Motion of the Earth about it's own Axle to be equable, that is, that the Revolution of every *Meridian* to any fixt Star is always perform'd in equal Spaces of time, let the Earth be in what Place of its Orbit it will. Which Supposition, though it be not new; yet since several have thought otherwise, and yet nevertheless the Daily Revolution, together with the Difference of the True and middle yearly Motion, does constitute one Element or Principle of the Equation of Time, (for the Difference of Longitude, and of the Right Ascension of the Earth is another Thing;) therefore it was to be declar'd, what it is, that I judg'd fit to be follow'd here.

2. I shall now use no other Procession of the *Æquinoxes*, than what *Kepler* did, *viz.* 51″ for every year; that without any troublesome Reduction I may give you, as near as I can, the Observations deliver'd by him in his Commentary concerning the Planet *Mars*, which are affected by a Procession of the *Æquinoxes* as great as has been said; yet I am not at the same time ignorant, that possibly the Motion of the *Æquinoxes* may be something otherwise.

3. Again, let us suppose the *Aphelia* and the Nodes of the Planets to have perpetually the same Place under the fixt Stars, or, which comes to the same, not to be mov'd by any other Motion, than that of the Procession of the *Æquinoxes.*

Scheme XXXVII.

4. In Scheme XXXVII. let S be the Sun, *f* the Superior *Focus*, *c* the Centre of the Elliptick, *m* the Point of the divided Section, namely, in which the Distance of the *Focusses s f* is cut according to its extreme and middle Proportion; And let the same be also the Centre of a Circle made by the Ray *m d*, which is equal to *c a* or *c p*, the Semidiametre of the greatest Elliptick. Let *a* be the *Aphelion*, *p* the *Perihelion*, *a f d* the Middle Anomaly, *a s d* the co-equate Anomaly, *f d s* the *Prosthaphæresis*, *e* the Planet, *s e* the Distance of the Planet from the Sun.

5. The

5. The Determination of the Lines.

	In the *Sun.*	In *Mars.*
c a, or *m d*	10000000	15236900
s f	344477	2835800
s m	212898	1752621
m f	131578	1083179

CHAP. II.

Teaching how to Calculate according to the Hypothesis *now laid down.*

TO the Calculation belong these; 1*st*. The Equation of Time. 2. The Computation of the Middle Motions. 3. The Equation of the Centre. 4. The Parallax of the Orb. Every one of them will be made the more ready and easie by the Tables that are added at the End. Of which the first does present you with the perpetual Equation of Time in two Parts. The second contains the Middle Motions of the Sun and of *Mars*, and also their *Epochæ*. Thirdly, Then follow three *Laterculi*, with the Numbers written upon them that are constantly requisite for the finding out of the Equation of the Centre, and of the Distance of the Earth and of *Mars* from the Sun. Lastly, I have transcrib'd out of the *Rudolphine* Tables *Morinus*'s Tables of the Inclinations of *Mars*, to wit, for shortening the Distance, and of his Reduction to the Ecliptick.

1. The Equation of Time consists of two parts, The first is taken from the former part of the Table that is to be gone to with the true place of the Sun. And the little Particles that are taken away are added to the Apparent Time, (of which kind is that of the Observations,) or subtracted according to the Titles expres'd in the Table, that the Time may first of all become equall'd. But the latter part of Equation may be then found out, when to the Time first equall'd you have computed the Middle Anomaly of the Sun out of the Tables of middle Motions: For with that middle Anomaly out of the latter Table of the Equation of Time, you may take the Minutes and Seconds that are to be applied to the Time first equall'd, (as the Titles written upon them do teach,) that there may be had the truly Astronomical or middle Time. And to or from the same Minutes and Seconds add or take away a competent Portion of the Motion of the Sun, according as the same Tables direct to the Middle Motion, as well of Longitude as of the Anomaly of the Sun before found.

2. Now the middle Motions are by Computation brought to Astronomical Time out of the Second Table, by a common way known to all; so that I need say nothing of that.

3. The Equation of the Centre, from the middle Anomaly given *a f d*, makes the true *a s d*, and the very *Prosthaphæresis* too, *f d s*, consisting of two parts, *f d m*, and *m d s*. The first part is found in the Triangle *f d m*, whose sides *f m* and *m d* being given, they are of a constant length with *a f d*, the external Angle of the middle Anomaly, that is given; wherefore M *d*. M *f* :: to the Sign *a f d* : to the Sign *f d m*. For the Log. *m d* being subtracted from the Log. *m f*, there remains the constant Logarithmical Number, noted in the first *Laterculus* by the Letter A; and is always to be added to the Sign of the middle Anomaly that is given, *a f d*, that there may be made the Sign *f d m*. The latter part of the Equation of the Centre is found in the Triangle *m d s*, whose sides *m d* and *m s* being given, they are of a constant Length with the External Angle *f m d* = *a f d* − *f d m*; wherefore *m d* + *m s*. *m d* − *m s* :: to the Tang. ½ *f m d*. Tang. ½ X<<. For the Log. *m d* + *m s* being subtracted from the Log. *m d* − *m s*, there remains the constant Logarithmical Number, noted

in the first *Laterculus* by the Letter B, and is always to be added to the Tangent $\frac{1}{2} f m d$, That it may be made the Tang. $\frac{1}{2}$ X <<, &c.

4. The Parallax of the Orb requires the Distance of the Earth and of the Planet from the Sun, which I shall teach you to find two ways; to wit, by using the first way the Numbers of the second *Laterculus*; but the latter way the Numbers of the Third. The first way I use mere Artificial Figures: For to half of the Sign of the Complement of the Angle *a s d* I always add the Logarithmical Number of the Second *Laterculus*, noted by the Letter C. I enquire for the Summ between the Signs, when the Angle *a s d* is acute, or between the Tangents, when *a s d* is obtuse; and I double the Sign of the Complement that answers to that Sign or this Tangent; and when doubled I take it from the Number of the Second *Laterculus*, that is noted by the Letter D, if *a s d* be acute; or I add it, if it be obtuse, (borrowing there, and casting away here the doubled Ray;) so there comes from thence the Logarithm of the Distance of the Planet from the Sun. In the latter way I use partly natural Numbers, and partly Artificial As to the Natural 'tis to be observ'd, that I take the Ray = 1. Now to the Number of the third *Laterculus* noted with the Letter E, (which is natural,) I take away also the natural Sign of the Complement of the Angle *a s d*, when it is acute, or I add it, when that is obtuse; and the Logarithm of the Remainder or Aggregate I always subtract from the Number of the Third *Laterculus*, mark'd by the Letter F; and what remains is also the Logarithm of Distance of the Planet from the Sun, to wit: *a p* the Sign of Complement of

Root $\overline{fs}$

asd. $\overline{2fs}$ :: *ap* 2 — *fs* 2 . *se*.

The first way may be us'd, when only the Artificial Canon of Triangles is at hand: The later requires both the Natural Canon and the Logarithms of absolute Numbers, And it is something more easie, than the former. Moreover, to the middle Anomaly of *Mars* I always add three Signs, 12°, that there may be the Argument of Latitude, with which I take away the Inclination to shorten the Distance that's already found, as also the Reduction to the Ecliptick. This only I add, That when the Planet draws nigh to it's Acronychial Position, I then seek the Angle to the Earth by this Analogy:

As the Distance of the Planet from the Earth
is to the Distance of the same from the Sun,
So the Angle to the Sun
is to the Angle to the Earth,

which I add to, or subtract from ☌ of the true place of the Sun, according as the Planet is on this side or beyond ☌. Now the Distance of the Planet from the Earth in it's Acronychial Position, or thereabouts, is the Difference of the Distances of the Planet and of the Earth from the Sun, or much thereabouts.

Example.

In the Year 1580. *Novemb.* 18. h. 1. 31′, *Mars* was observ'd by *Tycho* at *Urenburgh*, to be near his Acronychial Position in ♊ 6°, 28′, 35″. The Sun being in ♐ 6½. There the first Equation of Time is 7′, 29″ subtracted. From whence we have the Time first equall'd hor. 1. 23′ 31″, to which I take out of the second Table the Middle Motions of the Sun and of his Apogée after this manner.

	Sun from the *Æquinoctial*.				*Sun*'s Apogée.			
	Si.	De.	Min.	Sec.	Si.	De.	Min.	Sec.
1560	09	20	35	19	03	05	23	00
19	11	29	25	10	00	00	16	09
October compleat	09	29	38	15	00	00	00	43
Day 18	00	17	44	31	00	00	00	03
Hour 1	00	00	02	28	00	00	00	00
Min. 23	00	00	00	57	00	00	00	00
Sec. 31	00	00	00	01	00	00	00	00
Sun's mean Motion	08	07	26	41	03	05	39	55
Sun's Apogée	03	05	39	55				
Sun's mean Anomaly	05	01	46	46				

With

With the middle Anomaly of the Sun 5ˢ, 1°, 46′, 46″, I take away the latter part of the Equation of Time 3′, 49″, by ſubtraction; wherefore the Time truly Aſtronomical is h. 1, 19′, 42″; And a Portion of the Motion of the Sun, agreeing with the latter part of the Equation of Time, *viz.* 9″ being ſubtracted from the middle Motion of the Sun, leaves it 8ˢ, 7°, 26′, 32″; but being ſubtracted too from the middle Anomaly, it leaves 5ˢ, 1°, 46′, 37″, or 151°, 78′, omitting the leſſer and more minute Particles, which are above an hundred parts of a Degree. For the Calculation will be much the eaſier by loſing now and then one or two firſt minutes; And yet it will come nearer the Obſervations, than the moſt accurate Calculation that has been yet found.

To the Artificial Sign 9.67473 does anſwer the middle Anomaly that is given ——— 151.78 } Subt.
Add always A 8.11918
The Sum. is the Art. Si. 𝔵7.79391 of the Angle *f m d* —— ——— 0.36 } α
There remains *f m d* ——— 151.42
Whoſe half is ½ Z << 75.71 }
of which the Artificial Tangent is 10.59395
Add always B ——— ——— 9.98151 } Subt.
The Summ is the Tangent 10.57546, to which anſw. ½ X<< 75.12

There remains the Angle *m d t* 0.59 β
a + β is the *Proſthaphæreſis f d s* 0.95 Subt.
or 57′, 0″, Subt.

	Si.	*Deg.*	*Min.*	*Sec.*	
Therefore now from the middle Motion of the Sun	08	07	26	32	
take the *Proſthaphæreſis*	00	00	57	00	Subt.
There reſts the true place of the Sun ——— ——	08	06	27	32	

Farther, from the middle Anomaly given *a f d* 151.78
take always the *Proſthaphæreſis f d s* 0.95
there remains the co-equate Anomaly *a s d* 150.83
Of this Sign of Complement the half is ——— ——— —— 4.97055
Add always C 4.11807
The Summ (becauſe *a s d* is obtuſe) is the Tangent 9.08862
to which anſwers the Sign of Complement doubled, 19.99352
the which (*a s d* being obtuſe) I add to D, 4.99987
The Summ (the doubled Ray being caſt away) 𝔵4.99339
is the Diſtance of the Earth from the Sun.

Again, at the time truly Aſtronomical above found, *viz.* 1580. *Novemb.* 18, h. 1, 19′, 42″, let the middle Anomaly of *Mars* be ſought.

	Mars from the Equin.				*Aphelion* of *Mars*.			
	Si.	De.	Min.	Sec.	Si.	De.	Min.	Sec.
1560	07	00	32	13	04	28	12	04
19	01	06	32	45	00	00	16	09
October compleat	05	09	19	07	00	00	00	43
Day 18	00	09	26	00	00	00	00	03
Hour 1	00	00	01	19	00	00	00	00
Min. 19	00	00	00	25	00	00	00	00
Sec. 42	00	00	00	01	00	00	00	00
Mean Motion of *Mars*	01	25	51	50	04	28	28	59
Aphelion of *Mars*	04	28	28	59				
Mean Anomaly of *Mars*	08	27	22	51				

Therefore to the middle Anomaly 8s, 27o, 22', 51'', or it's Complement to the whole Circle 3s, 2o, 37', 9'', that is, 92.62. I seek the *Prosthaphæresis*.

To the Artificial Sign 9.99955 answering the middle Anom. given	92.62	Subt.
Add always A 8.85180		
The Summ is the Sign Art. 8.85135 of the Angle *f d m*	4.07	α
There rests *f m d*	88.55	
whose half is ½ Z<<	44.27	
whose Artificial Tangent is 9.98893		Subt.
Add always B 9.89965		
The Summ is the Artif. Tan. 19.88858 to which answer ½ X<<	37.73	
There remains the Angle *m d s*	6.54	β
α + β is the *Prosthaphæresis f d s*	10.61	Add
Or 10°, 36', 36'', A.		

	Si. De. Min. Se.	
Now therefore to the middle Motion of *Mars*	01 25 51 50	
Add the *Prosthaphæresis*	00 10 36 36	A
The Summ is the place of the Eccentricity of ♂ in the Orbit,	02 06 28 26	

	Si. De. Min. Se.	
To the middle Anomaly of *Mars* given,	08 27 22 51	
Add always	03 12 00 00	
There is made the Argument of Latitude	00 09 22 51	
to which answers the Inclination, or there abouts,	00 00 18 00	
that is 0.30 to be kept; and the Reduction	00 00 00 17	Subt.
Now therefore from the place of Eccent. of *Mars* in the Orbit	02 06 28 26	
take away the Reduction	00 00 00 17	S
There remains the place of Eccent. of *Mars* in the Ecliptick	02 06 28 09	

But farther, from the middle Anom. given *a f d*	92.62	
take always the *Prosthaphæresis f d s*	10.61	
there remains	82.01	
Of this Sign of Complement the half is	4.57151	
Add always C	4.48442	
The Summ (because *a s d* is acute) is the Sign	9.05593	
to which answers the doubled Sign of Complement	19.99435	Subt.
which I subt. (*a s d* being acute) from D, the doubled Ray increas'd,	25.17912	
There remains the Distance of *Mars* from the Sun	5.18477	γ

The same Distance of *Mars* from the Sun is found also the latter way, Thus:

To the Co-equate Anomaly *a s d*	82.01	
answers the Natural Sign of Complement	15900	
to be subtracted (when *a s d* is acute) from E	10.74610	
There remains	10.60710	
whose Logarithm is	1.02559	
to be always taken from F	6.21037	
There remains the Distance of *Mars* from the Sun	5.18477	

The same that was above at γ.

The

The Inclinations of *Mars* reserv'd above 0°.30 the Sign of Complement is	999999
whose Difference from the Ray is	1
which is abbreviation, whereby the Distance of *Mars* becomes shorten'd	5.18476

	Si. De. Min. Se.
The true place of the Sun above found was	08 06 29 32
The Eccentrick place of *Mars* in the Ecliptick	02 06 28 09
The Arch that *Mars* is distant from ♂ ⊙, or the Angle to ⊙ is	00 00 01 23

The Distance of *Mars* from the Sun 5.18476, answered by the absolute Number 153023
Also the Distance of the Earth from the Sun 4.99339, answered by the absolute 98489 } Subt.

The Difference is the Distance of the Earth from *Mars*	54534
As therefore the Distance of the Earth from *Mars*	4.73667
to the Distance of the Sun from *Mars*	5.18476
So the Angle to the Sun (*viz.* the Logarithm 83″)	1.91908
to the Angle to the Earth	2.36717

Whose absolute is 3′, 53″, which *Mars* from on Earth seems to be distant from ♂ of the Sun.

	Si. De. Min. Se.
Therefore from ♂ of the Sun	02 06 29 32
(because *Mars* is on this side the ♂ ⊙) take	05 00 03 53
There remains the true Place of *Mars* computed,	02 06 25 39
But the Place of *Mars* observed was	02 06 28 35
The Difference is	00 00 02 56

CHAP. III.

Comparing the Calculation with Observations.

According to the Method laid down in the former Chapter, I have computed fourteen Acronychial Observations, and twenty eight out of his Acronychial Position, the Product of which Calculation take as follows:

Fourteen

Fourteen of the Extream Fulſions, or of the brighteſt ſhinings of Mars, *of which the twelve firſt are out of* Kepler's *Commentary concerning* Mars, *the two laſt are taken out of* Longomotanus's *two Books of Theoricks, and either taken or reduc'd to the Meridian of* Urenburgh.

					The *Sun*'s true Place.				The mean Anomaly of *Mars*.				The Place of *Mars* by Computation.				The Place of *Mars* by Obſervation.				Difference	
Years.	Months	Da.	Ho.	Mi.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	S.	D.	M.	S.	M.	S.
1580	Novem.	18	01	31	♐	06	29	32	08	27	22	51	♊	06	25	39	♊	06	28	35	02	56+
1582	Decem.	23	03	58	♑	16	53	06	10	10	56	52	♋	16	56	24	♋	16	55	30	00	54−
1585	January.	30	19	14	♒	21	34	23	11	21	38	48	♌	21	35	29	♌	21	36	10	00	41+
1587	March.	06	07	23	♓	25	38	14	01	02	16	19	♍	25	46	03	♍	25	43	00	03	03−
1589	April.	14	06	23	♉	04	20	41	02	15	45	09	♏	04	27	29	♏	04	23	00	04	29−−
1591	June.	08	07	43	♊	26	42	22	04	07	09	00	♐	26	40	41	♐	26	43	00	02	19+
1593	Auguſt.	25	17	27	♍	12	21	52	06	11	18	21	♓	12	16	37	♓	12	16	00	00	37−
1595	October.	31	00	39	♏	17	35	15	08	08	35	23	♉	17	30	10	♉	17	31	40	01	30+
1597	Decem.	13	15	54	♑	02	28	02	09	24	31	52	♋	02	24	04	♋	02	28	00	03	56+
1600	January.	18	14	02	♒	08	34	39	11	05	54	22	♌	08	35	41	♌	08	38	00	02	19+
1602	Febru.	20	14	13	♓	12	21	33	00	16	16	19	♍	12	24	24	♍	12	27	00	02	36+
1604	March.	28	16	23	♈	18	33	50	01	28	14	59	♎	18	33	42	♎	18	37	10	03	28+
1608	July.	24	02	00	♌	11	14	10	05	15	23	18	♒	11	13	18	♒	11	11	00	02	18−
1610	October.	08	16	50	♎	25	33	51	07	18	04	34	♈	25	28	14	♈	25	31	00	02	46+

As to the two laſt it is to be noted, that Longomontanus *delivers the laſt but one to be* ♒ 11°, 10′, 00″, *and the laſt to be* ♈ 25°, 30′, 00″. *But his Proceſſion of the* Æquinox *is leſs than* Kepler's *by* 1′, *which Minute I have therefore reſtor'd.*

The

The Places of *Mars* out of his Acronychial Position, observed at *Uranburgh*, and recorded by *Kepler* in his Commentary of *Mars*.

Years.	Months.	Da.	Ho.	Mi.	*Sun*'s true place. S.	D.	M.	Se.	Mean Anomaly ♂ S.	D.	M.	Se.	Place ♂ by Com. S.	D.	M.	Se.	Place ♂ by Ob. S.	D.	M.	Se.	Difference M.	Se.
1582	Novem.	23	16	00	♐	11	41	25	09	22	51	48	♋	26	40	13	♋	26	38	30	01	43—
	Decem.	26	08	30	♑	15	02	20	10	09	59	54	♋	17	38	57	♋	17	40	30	01	33+
	Decem.	30	08	10	♑	19	05	59	10	12	05	17	♋	16	03	15	♋	16	00	30	02	45—
1583	Janu.	26	06	15	♒	16	28	04	10	26	11	50	♋	08	22	40	♋	08	20	30	02	10—
1584	Decem.	21	14	00	♑	10	41	41	11	00	34	01	♍	01	15	17	♍	01	13	30	01	47—
1585	Janu.	24	09	00	♒	15	04	43	11	18	16	44	♌	24	09	43	♌	24	07	30	02	13—
	Febru.	04	06	40	♒	26	05	42	11	23	59	31	♌	19	47	30	♌	19	47	00	00	30—
	March.	12	10	30	♈	02	11	25	00	12	56	17	♌	11	45	37	♌	11	46	00	00	23+
1587	Janu.	25	17	00	♒	15	56	13	00	11	31	22	♎	04	42	25	♎	04	42	00	00	25—
	March.	04	13	24	♓	23	54	00	01	01	21	19	♍	26	26	03	♍	26	25	40	00	23—
1587	March.	10	11	30	♓	29	46	28	01	04	27	27	♍	24	05	56	♍	24	05	15	00	41—
	April.	21	09	30	♉	10	45	09	01	26	25	09	♍	15	51	45	♍	15	48	20	03	25—
1589	March.	08	16	24	♓	28	30	17	01	26	35	10	♏	12	15	17	♏	12	16	50	01	33+
	April.	13	11	15	♉	03	34	45	02	15	20	06	♏	04	43	11	♏	04	43	20	00	09+
	April.	15	12	05	♉	05	33	17	02	16	24	04	♏	03	59	04	♏	03	58	20	00	44—
1589	May.	06	11	20	♉	25	46	15	02	27	23	19	♎	27	10	15	♎	27	07	20	02	55—
1591	May.	13	14	00	♊	02	08	03	03	23	39	39	♑	02	18	51	♑	02	20	00	01	09+
	June.	06	12	20	♊	24	59	38	04	06	12	09	♐	27	11	45	♐	27	15	00	03	15+
	June.	10	11	50	♊	28	47	11	04	08	17	17	♐	25	58	40	♐	26	02	36	03	56+
	June.	28	10	24	♋	15	51	44	04	17	41	26	♐	21	07	20	♐	21	10	00	02	40+
1593	July.	21	14	00	♌	08	28	13	05	22	53	30	♓	17	45	01	♓	17	45	45	00	44+
	August.	22	12	20	♍	09	14	54	06	09	37	21	♓	13	08	26	♓	13	10	15	01	49+
	August.	29	10	20	♍	15	57	52	06	13	14	44	♓	11	15	05	♓	11	14	00	01	05—
	October.	03	08	00	♎	20	19	42	07	01	31	55	♓	07	46	06	♉	07	50	10	04	04+
1595	Septem.	17	16	45	♎	04	22	26	07	15	53	09	♉	26	08	38	♉	26	07	12	01	26—
1595	October.	27	12	20	♏	14	02	39	08	06	44	54	♉	18	47	49	♉	18	51	15	03	26+
	Novem.	03	12	00	♏	21	05	17	08	10	24	36	♉	16	16	28	♉	16	18	30	02	02+
	Decem.	18	08	00	♑	06	42	10	09	03	54	36	♉	11	40	22	♉	11	40	00	00	22—

The Close.

'TIS *plain therefore from what goes before, that our* Hypothesis *does give the Equations of the Centre* à Priori, *not only as accurately and as easily, but even more accurately and more easily than any* Hypothesis *that has been yet invented. And let no one object, that in Calculation I neglect the more minute Particles; for any one may take notice of them, if he please; and yet the Calculation will not be so full of trouble as any other, but rather the very great variance of Calculation will be something diminish'd; for it will never run out to the four first Minutes.*

TABLE I.

TABLE I.

Shewing the Equation of Time in two parts.

Subtract from the Apparent

The true place of the Sun.								*The middle Anomaly of the Sun.*																		
Degrees.	S. ♈ ♎		♉ ♍		♊ ♐		Degrees.	Degrees.	Sign. 0.		1		2		3		4		5		Degrees.					
	Min.	sec.	Min.	sec.	Min.	sec.			Min.	sec.	Min.	sec.	Min.	sec.	Min.	sec.	Min.	sec.	Min.	sec.						
0	00	00	08	25	08	47	30	0	00	00	03	54	06	48	07	56	06	57	04	03	30					
1	00	20	08	36	08	37	29	1	00	08	04	01	06	52	07	56	06	53	03	55	29					
2	00	40	08	45	08	26	28	2	00	16	04	08	06	56	07	56	06	49	03	48	28					
3	00	59	08	55	08	15	27	3	00	24	04	15	07	00	07	56	06	44	03	40	27					
4	01	19	09	04	08	03	26	4	00	33	04	22	07	04	07	56	06	40	03	33	26					
5	01	39	09	12	07	50	25	5	00	41	04	28	07	08	07	55	06	35	03	25	25					
6	01	59	09	19	07	36	24	6	00	49	04	35	07	11	07	55	06	30	03	18	24					
7	02	18	09	25	07	22	23	7	00	57	04	42	07	15	07	54	06	25	03	10	23					
8	02	38	09	32	07	07	22	8	01	05	04	48	07	18	07	53	06	20	03	02	22					
9	02	57	09	37	06	52	21	9	01	13	04	55	07	21	07	52	06	15	02	54	21					
10	03	16	09	42	06	36	20	10	01	21	05	01	07	24	07	51	06	10	02	46	20					
11	03	35	09	46	06	20	19	11	01	29	05	07	07	27	07	49	06	05	02	38	19					
12	03	53	09	50	06	03	18	12	01	37	05	14	07	30	07	48	05	59	02	30	18					
13	04	12	09	52	05	46	17	13	01	45	05	20	07	33	07	46	05	54	02	22	17					
14	04	30	09	54	05	28	16	14	01	53	05	26	07	35	07	45	05	48	02	14	16					
15	04	47	09	56	05	10	15	15	02	01	05	32	07	37	07	43	05	42	02	06	15					
16	05	05	09	56	04	51	14	16	02	09	05	38	07	40	07	41	05	36	01	58	14					
17	05	22	09	56	04	32	13	17	02	16	05	43	07	42	07	38	05	30	01	49	13					
18	05	38	09	55	04	12	12	18	02	24	05	49	07	44	07	36	05	24	01	41	12					
19	05	55	09	53	03	53	11	19	02	32	05	54	07	46	07	34	05	18	01	33	11					
20	06	11	09	51	03	33	10	20	02	40	06	00	07	47	07	31	05	11	01	25	10					
21	06	28	09	48	03	12	9	21	02	47	06	05	07	49	07	28	05	05	01	16	9					
22	06	41	09	44	02	51	8	22	02	55	06	10	07	50	07	25	04	58	01	08	8					
23	06	56	09	40	02	31	7	23	03	02	06	15	07	51	07	22	04	52	00	59	7					
24	07	10	09	34	02	09	6	24	03	10	06	20	07	53	07	19	04	45	00	51	6					
25	07	24	09	28	01	48	5	25	03	17	06	25	07	54	07	16	04	38	00	42	5					
26	07	37	09	21	01	27	4	26	03	25	06	30	07	54	07	12	04	31	00	34	4					
27	07	50	09	14	01	05	3	27	03	32	06	35	07	55	07	09	04	24	00	25	3					
28	08	02	09	06	00	43	2	28	03	39	06	39	07	56	07	05	04	17	00	17	2					
29	08	14	08	57	00	22	1	29	03	47	06	44	07	56	07	01	04	10	00	08	1					
30	08	25	08	47	00	00	0	30	03	54	06	48	07	56	06	57	04	03	00	00	0					
	S. ♓	♍	♎	♌	♍	♋			Sig. 11		10		9		8		7		6							

Add to the Apparent

TABLE II.

Shewing the middle Motions of the Sun *and* Mars.

The *Epocha* of the middle motions of the *Sun* and *Mars*, for the Meridian of *Uranburgh*, in the Kalends of *January*, the Mid-day of Aſtronomical Time, according to the *Julian* Account.

Years of Chriſt compl.	Sun *from the* Æquinox.				Sun's *Apogée.*				Mars *from the* Æquinox.				Aphelion *of* Mars.			
	Sig.	gr.	min.	ſec.	Sig.	gr.	min.	ſec.	Sig.	gr.	min.	ſec.	Sig.	gr.	min.	ſec.
1560	09	20	35	19	03	05	23	00	07	00	32	13	04	28	12	04
1580	09	20	45	20	03	05	40	00	02	18	53	37	04	28	29	04
1600	09	20	55	21	03	05	57	00	10	07	15	01	04	28	46	04

Years expanded.	Sun *from the* Æquinox.				Mars *from the* Æquin.				*Proceſſion of the* Æquin.			
	Sig.	grad.	min.	ſec.	Sig.	grad.	min.	ſec.	Sig.	grad.	min.	ſec.
1	11	29	45	43	06	11	17	13	00	00	00	51
2	11	29	31	26	00	22	34	25	00	00	01	42
3	11	29	17	09	07	03	51	38	00	00	02	33
4	00	00	02	00	01	15	40	17	00	00	03	24
5	11	29	47	43	07	26	57	29	00	00	04	15
6	11	29	33	26	02	08	14	42	00	00	05	06
7	11	29	19	09	08	19	31	55	00	00	05	57
8	00	00	04	00	03	01	20	34	00	00	06	48
9	11	29	49	43	09	12	37	46	00	00	07	39
10	11	29	35	26	03	23	54	59	00	00	08	30
11	11	29	21	09	10	05	12	11	00	00	09	21
12	00	00	06	01	04	17	00	50	00	00	10	12
13	11	29	51	44	10	28	18	03	00	00	11	03
14	11	29	37	27	05	09	35	16	00	00	11	54
15	11	29	23	10	11	20	52	28	00	00	12	45
16	00	00	08	01	06	02	41	07	00	00	13	36
17	11	29	53	44	00	13	58	20	00	00	14	27
18	11	29	39	27	06	25	15	32	00	00	15	18
19	11	29	25	10	01	06	32	45	00	00	16	09
20	00	00	10	01	07	18	21	24	00	00	17	00

Months completed.	Sun *from the* Æquin.				Mars *from the* Æquin.				*Proceſſion of the* Æquin.			
	Sig.	grad.	min.	ſec.	Sig.	grad.	min.	ſec.	Sig.	grad.	min.	ſec.
January	01	00	33	19	00	16	14	47	00	00	00	05
February	01	28	09	13	01	00	55	14	00	00	00	09
March	02	28	42	31	01	17	10	00	00	00	00	13
April	03	28	16	41	02	02	53	20	00	00	00	17
May	04	28	49	59	02	19	08	06	00	00	00	21
June	05	28	24	09	03	04	51	26	00	00	00	25
July	06	28	57	28	04	21	06	13	00	00	00	30
Auguſt	07	29	30	46	04	07	21	00	00	00	00	34
September	08	29	04	56	04	23	04	19	00	00	00	38
October	09	29	38	15	05	09	19	07	00	00	00	43
November	10	29	12	25	05	25	02	26	00	00	00	47
December	11	29	45	43	06	11	17	13	00	00	00	51

The

The Residue of TABLE II.

Shewing the middle Motions of the Sun *and* Mars.

Days.	☉ *from Æqu.*	♂ *from Æq.*	Æqu. Pr.	Hours and Minutes. ☉ *from Æ.*	Hours and Minutes. ♂ *from Æ.*	Minutes.	☉ *from Æq.*	♂ *from Æq*
	Deg. min. sec.	Deg. min. sec.		Deg. mi. sec.	Min. sec.		M. se. III.	M: se. III.
1	00 59 08	00 31 27	00	0 02 28	01 19	31	1 16 23	0 40 37
2	01 58 16	01 02 53	00	0 04 56	02 37	32	1 18 51	0 41 55
3	02 57 25	01 34 20	00	0 07 24	03 56	33	1 21 19	0 43 14
4	03 56 34	02 05 47	01	0 09 51	05 14	34	1 23 47	0 44 33
5	04 55 42	02 37 14	01	0 12 19	06 33	35	1 26 14	0 45 51
6	05 54 50	03 08 40	01	0 14 47	07 52	36	1 28 42	0 47 10
7	06 53 58	03 40 07	01	0 17 15	09 10	37	1 31 10	0 48 28
8	07 53 07	04 11 33	01	0 19 43	10 29	38	1 33 38	0 49 47
9	08 52 15	04 43 00	01	0 22 11	11 47	39	1 36 06	0 51 06
10	09 51 23	05 14 26	01	0 24 38	13 06	40	1 38 34	0 52 24
11	10 50 32	05 45 54	02	0 27 06	14 25	41	1 41 02	0 53 43
12	11 49 40	06 17 20	02	0 29 34	15 43	42	1 43 29	0 55 01
13	12 48 49	06 48 47	02	0 32 02	17 02	43	1 45 57	0 56 20
14	13 47 57	07 20 13	02	0 34 30	18 20	44	1 48 25	0 57 39
15	14 47 05	07 51 40	02	0 36 58	19 39	45	1 50 53	0 58 57
16	15 46 13	08 23 06	02	0 39 25	20 58	46	1 53 21	1 00 16
17	16 45 21	08 54 33	02	0 41 53	22 16	47	1 55 49	1 01 34
18	17 44 31	09 26 00	03	0 44 21	23 35	48	1 58 16	1 02 53
19	18 43 39	09 57 27	03	0 46 49	24 53	49	2 00 44	1 04 12
20	19 42 47	10 28 53	03	0 49 17	26 12	50	2 03 12	1 05 30
21	20 41 55	11 00 20	03	0 51 45	27 31	51	2 05 40	1 06 49
22	21 41 03	11 31 47	03	0 54 13	28 49	52	2 08 08	1 08 07
23	22 40 12	12 03 13	03	0 56 40	30 08	53	2 10 36	1 09 26
24	23 39 20	12 34 40	03	0 59 08	31 27	54	2 13 03	1 10 45
25	24 38 29	13 06 07	04	1 01 36	32 45	55	2 15 31	1 12 03
26	25 37 37	13 37 34	04	1 04 04	34 04	56	2 17 59	1 13 22
27	26 36 45	14 09 00	04	1 06 32	35 22	57	2 20 27	1 14 40
28	27 35 54	14 40 27	04	1 09 00	36 41	58	2 22 55	1 15 59
29	28 35 02	15 11 53	04	1 11 27	38 00	59	2 25 23	1 17 18
30	29 34 10	15 43 20	04	1 13 55	39 18	60	2 27 50	1 18 36

LATERCULUS I.

For the finding out the Equation of the Centre.

	Of the Sun.	*Of* Mars.
A	8.11918.37	8.85180.37
B	9.98150.51	9.89964.78

LATERCULUS II.

For the finding the Distance from the Sun *the former way.*

	Of the Sun.	*Of* Mars.
C	4.11806.52	4.48437.45
D	4.99987.11	5.17911.95

LATERCULUS III.

For the finding the Distance from the Sun *the latter way.*

	Of the Sun.	*Of* Mars.
E	58|5903	10|74610
F	6.76374.09	6.21037.05

A Table of the Inclination of Mars.

The Inclination.

Degrees.	Sign 0 / Sign 6	Sign 1 / Sign 7	Sign 2 / Sign 8	Degrees.
	Deg. m. s.	Deg. m. s.	Deg. m. s.	
0	00 00 00	00 55 15	01 35 42	30
1	00 01 56	00 56 54	01 36 39	29
2	00 03 52	00 58 32	01 37 34	28
3	00 05 47	01 00 09	01 38 28	27
4	00 07 42	01 01 45	01 39 20	26
5	00 09 38	01 03 21	01 40 10	25
6	00 11 33	01 04 55	01 40 58	24
7	00 13 28	01 06 28	01 41 44	23
8	00 15 23	01 08 00	01 42 28	22
9	00 17 17	01 09 31	01 43 10	21
10	00 19 11	01 11 01	01 43 50	20
11	00 21 05	01 12 30	01 44 28	19
12	00 22 58	01 13 57	01 45 05	18
13	00 24 51	01 15 23	01 45 40	17
14	00 26 43	01 16 47	01 46 13	16
15	00 28 35	01 18 10	01 46 44	15
16	00 30 26	01 19 31	01 47 13	14
17	00 32 17	01 20 50	01 47 40	13
18	00 34 07	01 22 08	01 48 05	12
19	00 35 57	01 23 25	01 48 28	11
20	00 37 46	01 24 40	01 48 49	10
21	00 39 34	01 25 54	01 49 08	9
22	00 41 22	01 27 06	01 49 25	8
23	00 43 09	01 28 16	01 49 40	7
24	00 44 55	01 29 25	01 49 53	6
25	00 46 40	01 30 32	01 50 04	5
26	00 48 25	01 31 37	01 50 13	4
27	00 50 09	01 32 41	01 50 20	3
28	00 51 52	01 33 43	01 50 25	2
29	00 53 34	01 34 43	01 50 28	1
30	00 55 15	01 35 42	01 50 30	0
	Sign 11 / Sign 5	Sign 10 / Sign 4	Sign 9 / Sign 3	

A Table of the Reduction of Mars to the Ecliptick.

The Reduction:

Subtract.

Degrees.	Sign 0 / Sign 6	Sign 1 / Sign 7	Sign 2 / Sign 8	Degrees.
	M. sec.	M. sec.	M. sec.	
0	00 00	00 47	00 47	30
1	00 01	00 48	00 46	29
2	00 03	00 48	00 45	28
3	00 05	00 49	00 44	27
4	00 07	00 49	00 43	26
5	00 09	00 49	00 42	25
6	00 11	00 50	00 41	24
7	00 13	00 50	00 39	23
8	00 15	00 51	00 38	22
9	00 16	00 51	00 37	21
10	00 18	00 52	00 36	20
11	00 20	00 52	00 34	19
12	00 22	00 52	00 33	18
13	00 24	00 52	00 32	17
14	00 26	00 53	00 30	16
15	00 28	00 53	00 28	15
16	00 30	00 53	00 26	14
17	00 32	00 53	00 24	13
18	00 33	00 52	00 22	12
19	00 34	00 52	00 20	11
20	00 36	00 52	00 18	10
21	00 37	00 51	00 16	9
22	00 38	00 51	00 15	8
23	00 39	00 50	00 13	7
24	00 41	00 50	00 11	6
25	00 42	00 49	00 09	5
26	00 43	00 49	00 07	4
27	00 44	00 49	00 05	3
28	00 45	00 48	00 03	2
29	00 46	00 48	00 01	1
30	00 47	00 47	00 00	0
	Sig. 11 / Sign 5	Sig. 10 / Sign 4	Sign 9 / Sign 3	

Add

SECTION

SECTION IV.

Shewing how all Lines and Angles may be determin'd from Observations.

PROBLEM. I.

To find the Meridian *Line.*

IN a Plane that is Poliſh'd, and that is brought to an *Equilibrium* with the Horizon, and firmly fixt, let a great many Circles be drawn from the ſame Centre, and let there be fixt perpendicularly in the Centre a piece of Wood about three Fingers high, having in the top of it a Copper-plate, through the little Hole of which let a Ray of the Sun paſs: And then in the Fore-noon is to be accurately obſerv'd, and noted with a point, the Incidence of the Sun's Ray upon as many of the Circles that are drawn, as the Ray can come to; and in the After-noon is to be obſerv'd the Return of the Sun's Rays to the ſame Circles one after another. Which being done, if all the Arches that lie between the two Correſpondent Points be Geometrically cut in two, the Line that cuts them in two is the *Meridian*.

PROBLEM. II.

To find the Elevation of the Pole.

IF by a Quadrant (that is large enough, and exquiſitely made, and that is fixt preciſely upon the *Meridian* Line,) the greateſt as well as leaſt Altitude of any Star, that is near to the Elevated Pole be obſerv'd; the Half of the Difference of the Altitudes being added to the leſs, or Subſtracted from the greater, (or Half of the Summ of the Altitudes,) ſhews the Elevation of the Pole which is ſought for.

PROBLEM. III.

To find the Horizontal Parallax of the Sun.

IF you obſerve for two days together the Planet *Mars*, when Acronychial, in 90° of the Ecliptick, very diligently marking both the time and his Longitude, (by making Computation of it with reference to the fix'd Stars, in the way hereafter deliver'd,) you may from thence conclude the quantity of the Motion for the time that then paſſes; whence farther may be computed the Longitude of the ſame at any intermediate moment of Time. But if at the ſame intermediate moment his Longitude be again obſerv'd, by comparing it to the fix'd Stars, while he is in his Corner-Altitude, you ſhall find the Longitude that is found by Calculation to differ a very little from the Longitude obſerv'd, which difference will go to the Parallax of Longitude. And ſeeing at the ſame moment may be found the Angle, which the Vertical, that paſſes through *Mars*, does make with the Parallel of the

the Ecliptick: There are in the Rectangular Angle that is made by the Vertical, the Parallel of the Ecliptick, and the Circle of Latitude that passes through *Mars*, all the Angles and one Leg; whereby will be made known the *Hypotenusa*, which will be the Parallax of Altitude to such an Almucantarath, whence will likewise be found out his Horizontal Parallax. Then you may say: As the Distance of the Sun from the Earth is to the distance of *Mars* from the same; so is the Horizontal Parallax of *Mars* to the Horizontal Parallax of the *Sun*. Now, they that have search'd this thing very narrowly, do affirm, That the Parallax of *Mars*, when neareſt the Earth, is found to be so very small, that it scarce amounts to any at all, as never coming fully to 1′. Hence therefore I consequently infer, That if the Horizontal Parallax of *Mars*, when Acronychial, which is when He is almost four times nearer the Earth than the *Sun* is, do not come to 1′; then certainly the Parallax of the Sun, which is almost four times farther off, will scarce come to $\frac{1}{4}$ of a Minute. And hence it is, that the Horizontal Parallax of the *Sun* is by Modern Astronomers concluded not to be above 15″. at the most.

PROBLEM. IV.

To find the Obliquity of the Ecliptick.

IF on the Day of the Summer Solstice the Meridian Altitude of the *Sun* be taken, which may be limited by the true Parallax, as if it had been seen from the Centre of the Earth; And if from this Solstitial *Meridian* Altitude, and which is limitted by the Parallax, be taken the Altitude of the *Æquator*, or the Complement of the Elevation of the Pole; there remains the greatest Declination of the *Sun*, which they call the Obliquity of the Ecliptick; as for Example: The Solstitial *Meridian* Altitude of the *Sun*, may be concluded to be as great as it was by *Tycho* really observ'd to be, from the Obliquity of the Ecliptick, which he determines, and from the Horizontal Parallax. For he makes the Obliquity of the Ecliptick to be 23°. 31′. 30″. and the Horizontal Parallax 3′. and determines the Elevation of the Pole at *Uranburgh* to be 55°. 54½′. therefore the Elevation of the *Æquator* was 34°. 5½′. And that we may conclude the *Meridian* Parallax of the *Sun* from the Horizontal Parallax of three Minutes suppos'd, it is also necessary to know pretty near the Altitude of the same, which was compounded of the Altitude of the *Æquator*, and the greatest Obliquity of the Ecliptick, which supposing for a while with Him to be 23°. 31½′. we gather the *Meridian* Altitude to be 57°. 37′. which is as great as *Tycho* believ'd it would be, if it was seen from the Centre of the Earth. And to this Altitude agrees the Parallax, 1′. 36½″. if so be the Horizontal Parallax be suppos'd to 3′. That Parallax therefore 1′. 36½″. is contain'd together in the supposed Altitude 58°. 37′. Therefore 1′. 36½″. being subtracted, there remains the Altitude, which is found by Instruments to be 57°. 35′. 23½″. And then farther adding to this the true Parallax 7½″. that arises from the supposed Horizontal Parallax 15″. we gather the Solstitial *Meridian* Altitude, limited by the true Parallax to be 57°. 35′. 31″. from whence being taken the Altitude of the *Æquator* 34°. 5′. 30″. there remains the Obliquity of the Ecliptick 23°. 30′. 1″.

PROBLEM. V.

To take the Longitude of the Sun.

AT Noon-day let the Altitude of the Sun be taken, and limited by the true Parallax; then the Difference of the limited Altitude, and of the Elevation of the *Æquator*, will be the Declination of the *Sun*. Whence by the first Inverted Problem of the Spherical part will appear the Longitude of the *Sun*, that

is sought for. But farther 'tis to be obſerv'd, if the Obſervation be made in the Winter-Months, that then the Refractions are to be conſider'd: And ſo to avoid theſe, the Summer time is us'd rather to be choſen, when the Noon-day Sun has almoſt got quite out of the Labarinth of Refractions; and if beſides, a calm and clear ſeaſon be choſen, the leſs Inconvenience will be to be fear'd. Now theſe things may be the more certainly practiſed about the *Æquinoxes*, when the Declination is very much changed in a ſhort ſpace: So the *Æquinoxes* themſelves are found, the Meridian Altitude of the Sun being ſought for, on the very day of the *Æquinox*, or on the next to it; which Meridian Altitude, if when limited by the true Parallax, it be equal to the Elevation of the *Æquator*; 'tis an Argument, that the time of the true *Æquinox* does fall upon the very Noon-day that the obſervation is made; but if it be leſs or greater, the *Æquinox* will almoſt ſo many hours go before the time of the making of the Obſervation, or follow it as many Minutes, as there are between the limited Altitude of the Sun, and the Elevation of the *Æquator*; for the greater limited Altitude argues the Vernal *Æquinox* to have gone before the making of the obſervation, but that the Autumnal *Æquinox* will follow. And the leſs Altitude denotes the contrary.

PROBLEM VI.

To find the Apogee and Eccentricity of the Sun.

THE three true Longitudes of the Sun are to be found out by the foregoing Problem, avoiding the too great nearneſs of the Solſtices, and the Winter-Signs. Then in Figure XXXVIII. let the Orbit of the Earth be *a b c e*, *s* the Sun, *t* the other *Focus*, let *a b c* be the true places of the Earth, diametrically oppos'd to the obſerved places of the Sun; now therefore there are given true diſtances of theſe places, to wit, the Angles *a s b*, *b s c*, *a s c*; and there are alſo given the middle Motions of the Sun, that agree with the Temporary Intervals of the Obſervations *a b*, *b c*, *a c*; which Intervals are known by marking the moments of making the obſervations, and they have the ſame proportion to the time of the whole Revolution of the Sun, *viz.* to 365 *d.* 5 *h.* 49 *m.* That the Angles *a f b*, *b f c*, *a f c*, have to the whole Circumference. Nor does it ſignifie any thing, that there is a difference among the Aſtronomers about the Seconds and Thirds, above the Hours and Minutes of the Sun's Revolution; for they are here almoſt of no conſideration. Now the Angles at *s* and *f*, being thus given from the Centre *f*, at the compaſs of *f* A, equal to *d e*, the whole Line of the *Apſes*, let there be drawn the Circumference A B C G, which let the right Lines, drawn from the *Focus f* through *a b c*, cut in A B C; and let *s* A, *s* B, *s* C, be joyn'd, the Triangles *s a* A, *s b* B, *s c* C, will be of equal Legs; ſeeing according to the nature of an Ellipſis to *d e*, (*i. e.* to *f* A,) is equal *s a* + *a f*; wherefore taking away on both ſides *f a*, there remains *s a* = *a* A. And therefore the Angle *f a s*, (which is external to the Triangle *a* A *s*,) will be double to the Angle *b* B *s*. Laſtly, *f c s* will be double to *c* C *s*. And now take the Angle *a s b* from *a f b*, and there will remain the Summ of the Angles *s a f*, and *s b f*; for the Angle *s a f* is the difference of the parts *a f d*, and *a s d*, and the other Angle *s b f*, is the difference of the reſt of the parts *b f d*, and *b s d*. Wherefore when the whole Angles *a s b*, and *a f b* are given, the Summ of the differences *s a f* and *s b f*, will be alſo given, the ſumm of whoſe half *s* A *f*, and *s* B *f*, or *a s* A, and *b s* B, being added to the Angle *a s b*, it gives the Angle A *s* B; and by the ſame way may be found the Angles B *s* C, and A *s* C. Now there being given after this manner the Angles that are made by the right Lines that are drawn at both *Focuſſes* to A B C; let A *s* be drawn out to G, and let B G and C G be joyn'd. Firſt in the Triangle *s* B G of the given Angles, (ſince B *s* G is the complement of the given Angle A *s* B to 180 degrees, and *s* G B is the half of the given Angle A *f* B,) let *s* B be taken and ſuppos'd to be of as many parts as you pleaſe, ſuppoſe 100000, *s* G will be found by Trigonometry in the ſame parts. Again, in the Triangle *s* C G are given, firſt, *s* G juſt now found; ſecondly, G *s* C the complement of the given Angle A *s* C to 180 degrees; thirdly, *s* G C the half of the given Angle A *f* C;

Figure XXXVIII

A *f* C; Therefore *s* C will so be found. Farther, in the Triangle B *s* C are given, first, *s* B, which was first taken; secondly, *s* C, just now found; thirdly, B *s* C, from observations; and therefore *s* B C and B C will likewise be found. Again, in the Æquicrural Triangle B *f* C (because *f* B and *f* C are Rays of the same Circle) are given all the Angles; for B *f* C is known from what goes before, and the rest *f* B C, *f* C B, together, are its complement to 180 degrees, and the same are also equal to one another; and there is also given the side B C, lately found: so that *f* B will be found out. Lastly, in the Triangle *f s* B are given, first, *s* B, that was taken first of all; secondly, *f* B, newly found; thirdly, the Angle *f* B *s*, *s* B C, being subtracted from *f* B C; wherefore both the unknown Angles may be found, and the side *f s*, the distance of the *Focusses*, in parts, such as *s* B was taken to be 100000. Whereby may be found the Eccentricity in parts, of which the middle distance of the Earth from the Sun is suppos'd to be 100000, by saying; *f* B newly found, (and the same is equal to *d e*, the Diametre of the greater *Ellipsis*,) gives *f s*, that is also newly found; therefore the ½ of *d e*, or the middle distance of the Earth from the Sun, suppos'd to be 100000 parts, will give the ½ of *f s*, or the Eccentricity in like parts. Last of all, you must subtract *f* B *s*, that is just now found, or what is equal to it, the Angle *b s* B, from the given Angle *d s* B, that there may remain the Angle *d s b*, which the place of the middle observation *b*, is distant from the *Aphelion d*. Wherefore the place *b*, being given by observation, the place of the *Aphelion* of the Earth *d* is also given. This way of finding the *Aphelion* and Eccentricity, first propos'd by *Herigon*, and afterwards applied by Mr. *Tho. Street*, to the Elliptical *Hypothesis*, (though yet he draws the Circle from the Sun, which I have chose rather to draw from the upper *Focus*, that there might be the greater Agreement between the Lines of Scheme XXXIV. and so the Reader might be wearied with the less variety of Conceptions.) This way, I say, does Geometrically excell both those in the *Hypothesis* of Dr. *Ward*, and of *Count de Pago*; nor is there any thing else wanting in it, but that the *Hypothesis* should agree more with the Heavens. Wherefore that Respect may be had to *Bullialdus*'s Limitation, a repeated operation is necessary, by which through the seat of the Apogée next found, and through the Eccentricity, may be found the middle true Anomaly at three Observations given, as I have taught in the XXII. Chapter; and by those middle true Anomalies, let the Angles to *f* be corrected, suppose *a f b*, *b f c*, *a f c*, which now must be the differences of the middle true Anomalies, as they were before of the middle ones only: And so following the foregoing Calculation step by step the same way, you may find the *Aphelium* of the Earth, and the Eccentricity very exactly.

PROBLEM VII.

To find the Apogee and Eccentricity of the Sun, by the method us'd by the Ancients.

THey were wont to observe the time of both *Æquinoxes*, *viz.* the Vernal and Autumnal, and also of the Summer Solstice; then they computed the Interval of the time that pass'd between. Now *Hipparchus* found 94 days and ½ from the Vernal *Æquinox* to the Solstice, from the Solstice to the Autumnal *Æquinox* 92 days and ½, which being join'd, do make 187 day: Therefore seeing thus it is found, that the Sun tarries two days longer in the Vernal Quadrant, or Spring Quarter of the Circle, than in the Summer Quarter; it follows, that the Apogée of the Sun, (in which it stays the longest,) is plac'd in the Spring Quadrant.

In Figure XXXIX. let then A be the Earth, from which, as from the Centre of the Universe, let there be describ'd the Ecliptick ♈, ♋, ♎, ♑, and let ♈, ♋, ♎, ♑, be the Cardinal Points; there will be plac'd, as I said, in the Vernal Quadrant, ♈ A ♋, both the place of the Apogée C, and the Centre of the Sun's Eccentrick B; wherefore from the Centre B, let there be describ'd the Eccentrick of the Sun C K M N, and through B, to ♈ ♎, and ♋ ♑, let there be drawn parallel the Lines E

Figure XXXIX.

B M, H B N, dividing the Eccentrick into Quadrants. And now the time being given, in which ☉ goes from the begining of ♈, to the beginning of ♋, *viz.* 94 days and ½; there is also given the Arch of the Eccentrick D K, which it goes through in the mean time by its middle Motion, *viz.* 93 degrees, 8′. 37″. So from the beginning of ♋, to the beginning of ♎, in 92 days and ½, the Sun passes through the Arch of the Eccentrick K O, *viz.* 91°. 10′. 21″. Wherefore there is given the whole Arch D K O, of 184°. 18′. 58″. from whence being taken, the Semicircle E K M, there remain the Arches D E and M O, which together are equal to 4°. 18′. 58″. Therefore each of them by it self, for instance D E, will be 2°. 9′. 29″. The Sign of which Arch is D F. 3766 = B G = L A. Again, from the Arch D K of 93°. 8′. 37″. take away D E of 2°. 9′, 29″. there remains E K of 90°. 59′. 8″. from whence again, the Quadrant E H being taken, there remains H K of 59′. 8″. whose Sign is K I, 1720 = L B = A G. Now therefore in the rectangular Triangle A B G, there being given the Legs B G and G A, the Eccentricity A B may be found in parts, of which the Ray of the Eccentrick B C is suppos'd to be 100000: As also the Angle G B A = B A L, which the Line of the *Apses* A C, was formerly distant from the beginning of ♋, *viz.* 24°. 33′. 6″. or according to *Ptolomy*, 24°. 30′. Wherefore the Apogée of the Sun in the time of *Hipparchus* was in 5°. 30′ of ♊. Now in this operation some (*Copernicus* and *Tycho* particularly) do think, a sufficient Certainty of things assum'd is wanting: For the Solstices can scarce be found by an Instrument, because the Declination of the Sun about that time is scarce so alter'd as to be perceiv'd by sense. Yet since the Antients are both said to have observ'd the Solstice, and have made known the time, they found it to happen, though they have not discover'd the manner of observing it; yet it cannot be doubted, but that they were both furnished with, and did really use such means as they could perform it by: For that there is still a way of observing the Solstices, *Gassendus* witnesses in his II. Epistle to *Wendelin*, where he says, he found in the year 1626, on the 19th day of *June*, St. *N.* at *Marseilles*, that the length of the shadow, cast from a *Gnomon* 89321 particles high, was 31766 particles; again, on the 20th day, that it was 31753 part. and on the 21st day, that it was 31751; lastly, on the 22d day, that it was 31759 part. from whence we may thus infer; If on the 20th and 21st days the length of the shadow had been the same, it would have also decreas'd as much on the 19th and 20th days, as it had again increas'd from the 21st to the 22d, and the Solstice would have happen'd at mid-night, between the 20th and 21st days. But now seeing from the 20th to the 21st, it decreases two particles, and from the 21st to the 22d it increases less than it had decreas'd from the 19th to the 20th, 'tis certain, the Solstice had been before the Noon-day of the 21st, but yet was nearer to this, than to the foregoing Meridian; and so that it happen'd the 21st day in the Fore-noon: And, if I be not deceiv'd, *Nicolaus Tagautius* says, the *Chinese* do at this day observe the Solstice, by a way not much unlike this. Whence I am apt to believe, the Ancients might do the same; since 'tis evident, the use of the *Gnomon* was very ancient: And that the Ancients did not err much, (I do not say *Ptolomy*, who did distrust himself too much, but *Hipparchus*, and they that were before him,) in determining the Time of the Solstice, the Ancient Lunar Eclipses, that were observ'd at *Babylon*, do evince, whose Intervals do not admit of any other Seat of the Sun's Apogée, than that which was lately concluded by the foregoing ways, as I have long ago demonstrated, and shall sometime or other, publish to the World from what lies by me, if the matter require it. Hence therefore we argue, that the Apogée of the Sun, from *Hipparchus* to our times, has gone forward a whole Sign.

PRO-

PROBLEM VIII.

To define the middle Motion of the Sun, and the Tropical Year.

THere is need of two Obſervations, made at very far diſtant Intervals of time, at which the Sun was in the point of the *Æquinox*: And ſeeing from what goes before, both the Eccentricity and the Apogée of the Sun may be known at the time of making both the Ancient and Modern Obſervations; the *Proſthaphæreſis* of the Sun, which it had at the time of the *Æquinox*, cannot lie hid, whereby the true Place of the Sun may be reduc'd to the middle in both Obſervations. Then are to be computed both the whole Tract of Time that comes between both Obſervations, and the whole Revolutions, with the degrees that are over and above, if there be any, which the Sun has finiſh'd in the mean time. Whence, by the Rule of Proportions, you may infer: If in ſo much time (*viz.* in that which has paſt between both obſervations) the Sun has finiſh'd ſo many Revolutions, and moreover ſo many degrees and minutes; in how much time will it go one Circuit by its middle Motion from *Æquinox* to *Æquinox*? For what the Product is, is the Quantity of the middle Tropical Year. And tranſpoſing the Terms of the ſame Proportion, you may ſay; If ſo many Revolutions, and Degrees, and Minutes, have been paſt in ſo great ſpace of Time, *viz.* as has paſt between the two Obſervations; How many Degrees and Minutes muſt be allowed for one Day or one Hour.

PROBLEM IX.

To obſerve the viſible Diametre of the Sun and Moon.

IF to a Lath that is well plain'd and pretty long, be perpendicularly fixed two Plates, the one to the one end, and the other to the other; and the one have a little hole, through which the Rays of the Sun's Diametre may paſs into a dark Chamber, ſo that they may be receiv'd by the oppoſite Plate; there will be painted in this the Image of the Sun, whoſe Diametre is to be taken by a ſharp pair of Compaſſes, and to be meaſured by a certain accurate Scale, deducting the Particles, which the hole takes up: Then by the ſame Scale let the Diſtance of the Plates be meaſur'd; ſo ſhall you find the Proportion, that the diſtance of the Sun from the Earth has to its Diametre. Whence you may farther thus infer; As is the Diſtance of the Sun to its Semidiametre; ſo is the Ray to the Tangent of the Apparent Semidiametre, or the half-Angle, under which the face of the Sun is ſeen. The Semidiametre of the Sun being this way found out, when an Eclipſe of it happens; let the Images of both the Luminaries be taken by a Teleſcope in a dark Chamber, and in that part wherein the Margin or Edge of the Moon does cut the Face of the Sun, let there be taken by a three-pointed Compaſſes three points, here one, and there one in the Circumference of the Moon; and take alſo three points in the Circumference of the Sun: Then when you have found the Centres of theſe three Points, you may compare the Diametre of both Luminaries, and from the Semidiametre of the Sun before known may alſo conclude the Apparent Semidiametre of the Moon.

PROBLEM X.

To find the Eccentricity and Apogee of the Moon.

LET there be taken three Eclipses of the Moon, observ'd within the space of one year, marking accurately both the Moments of Time, and the Places of the Ecliptick, in which they happen'd: Now the last of these will be known by Calculation by the Place of the Sun, (which is then diametrically opposite to the Moon.) Then to the Intervals of Time between the first and second, as also between the second and third Eclipse, compute from any Tables the Motion both of the middle Anomaly, and of the Apogée of the Moon. For the Motion of the Apogée being deducted from the true Motion of the Moon, (which is known from the Places which it possess'd in every Eclipse,) there remains the Motion of the true Anomaly at the same Intervals. And so returning to the Method explain'd in Prob. VI. there are given there in Scheme XXXVII. the Angles at *f* from the Motion of the Middle Anomaly, and the Angles at *s* from the Motion of the true Anomaly. Wherefore by following the same way of calculating, you will find both the Eccentricity and the Apogée of the Moon.

Figure XXXVII.

PROBLEM XI.

To determine the Horizontal Parallax of the Moon.

LET the time be accurately observ'd, at which the beginning and end of an Eclipse of the Sun does happen, when the apparent Latitude of the Moon is also small or none: For at the same time the Longitude of the Sun, found by Calculation, will differ from the Apparent Latitude of the Moon, as much as the Semidiametres of both Luminaries, when joyned together, do both make. Now how to find out these Semidiametres has been taught Prob. IX. therefore the Apparent Longitude of the Moon will be known, and the true Longitude of the same may be computed to the same Moment, therefore the difference of the true Longitude and the Apparent will be the Parallax of the Moon's Longitude: Whereby may be gathered by Prob. III. the Parallax of Altitude, first at the Almucantarath given, and then the Horizontal Parallax.

PROBLEM XII.

To find the visible Diametre of the Earth's Shadow.

Figure XL.

IN Fig. XL. let *a* be the Sun, *b* the Earth, *c* the Centre of the Shadow, thro' which the Moon *e* does pass, the Apex or Point of which Shadow ends in *d*; let *h g f d* be the visible Horizon, which both Luminaries do touch with their upper Margin or Edge. Now 'tis certain, that right Lines, drawn from the Centre and Surface of the Earth *b* and *g* to the same point of the Sun *h*, will intercept the Angle *g h b*, equal to the Horizontal Parallax of the Sun; As the right Lines drawn from the same Points *b* and *g* to one and the same Point of the Moon *f*, do contain the Angle *b f g*, equal to the Horizontal Parallax of the Moon. And *f b* being drawn out to *i*, *i b h* the external Angle of the Triangle *h b f* will be equal to the Summ of the Horizontal Parallaxes *b h g* and *b f g*. From that Summ therefore *i b h* take away *a b h*, the visible Semidiametre of the Sun, there will remain *i b a* = *c b f*, the apparent Semidiametre of the Earth's Shadow.

PRO-

PROBLEM XIII.

To find out the Declination and Right Ascension of the fix'd Stars, and to infer from thence their Longitude and Latitude.

NOW the Declination of a fix'd Star is found by observing its Meridian Altitude, as is also the Declination of the Sun; but the Right Ascension of fix'd Stars has something more difficulty in it. *Tycho* for this purpose chose the Planet *Venus*, when in her greatest Elongation from the Sun, she shines by Day, and is seen after Sun-set. If therefore on the Day-time be taken the Altitude of the Sun by an Azimuthal Quadrant, and also the Altitude and Azimuth of *Venus*; there will in Figure XLI. presently be given the three sides in the Triangle P Z ☉, *First*, The Complement of the observed Altitude of the Sun Z ☉. *Secondly*, P Z the Complement of the Elevation of the Pole. *Thirdly*, P ☉ the Complement of the Declination of the Sun, greater or less than a Quadrant; Z P ☉ is found the Distance of the Sun from the Meridian. Again, in the Triangle P Z ♀ are given, *First*, P Z as before. *Secondly*, Z ♀ the Complement of the observed Altitude of *Venus*. *Thirdly*, P Z ♀ the observed Azimuth of *Venus*: Therefore Z P ♀ is found the Distance of *Venus* from the Meridian. From the Angle Z P ☉ take the Angle Z P ♀, there remains the Angle ☉ P ♀, or the Difference of the Right Ascensions of the Sun and *Venus*. Afterwards when the Sun is set, let the Altitude of the same fix'd Star be taken by the same Quadrant; as also the Altitude and Azimuth of *Venus*: Then in Figure XLII. in the Triangle Z P ✳ there will be given the three sides, *First*, P Z as before. *Secondly*, P ✳ the Complement of the Declination of the fix'd Star. *Thirdly*, Z ✳ the Complement of the observed Altitude of the fix'd Star: Then Z P ✳ is found the Distance of the fix'd Star from the Meridian. Again, in the Triangle Z P ♀ are given, *First*, P Z as before. *Secondly*, P Z ♀ the observed Azimuth of *Venus*. *Thirdly*, Z ♀ the Complement of the observed Altitude of *Venus*; therefore Z P ♀ is found, or the Distance of *Venus* from the Meridian. From the Angle Z P ♀ take the Angle Z P ✳, there remains the Angle ♀ P ✳, or the Difference of the Right Ascensions of *Venus* and the fix'd Star. If therefore the Right Ascension of the Sun be known at the Moment of the first Observation, add to it the difference of the Right Ascensions of the Sun and of *Venus*; so may you make up the Right Ascension of *Venus*. To this you may add the difference of the Right Ascensions of *Venus* and the fix'd Star, and the Right Ascension of the fix'd Star, which is the thing sought for, will appear. But if between the Times of both Observations *Venus* chance to change her Place, and consequently her Right Ascension, (though that cannot but be very small, *Venus* being all that while in her greatest Elongation from the Sun, and so Stationary, or very near it,) the Right Ascension of *Venus*, found by the first Observation will be to be corrected, and to be fitted to the Time of the latter Observation: And so the Declination and Right Ascension of a fix'd Star being this way found, the Longitude and Latitude of the same may be thence inferr'd by the second Converse Problem of the Spherical part of this Astronomy. Now the Right Ascension of one fix'd Star being known, it may lead us to the knowledge of the Right Ascension of all the rest; whence also their Longitudes and Latitudes may be determined.

Figure XLI.

Figure XLII.

PRO-

PROBLEM XIV.

To determine the proper Motion of the fix'd Stars, and the Siderial Year.

THE more famous Aſtronomers perceive, that the Latitudes of the fix'd Stars are not chang'd. Now therefore if by the foregoing Problem the Latitude of any fix'd Star be found, as for inſtance of *Spica Virginis*, then it was alſo the ſame in Ancient Times; now *Timocharis* did alſo of old take the Declination of the ſame fix'd Star: The Declination therefore of *Spica Virginis*, and the Latitude of the ſame being given, the Longitude of it may be found by the ſecond converſe Problem of the Spherical Part; and alſo the Longitude of the ſame fix'd Star in theſe latter years, may be found by the foregoing Problem: Therefore the Ancient Longitude compared with the Modern, will ſhew how much the fix'd Stars have in the mean time proceeded by their proper Motion of Longitude. Whence will farther appear the yearly Motion of the fix'd Stars; and moreover that Particle of Time, which the Sun ſpends in paſſing that little ſpace of the yearly Motion of the fix'd Stars, after it has return'd from the Point of the Ecliptick, which the Star was in the year before, to the ſame point again, and has indeed completed the Tropical Year; but not finding the ſame fix'd Star there, yet while it goes on to it, it completes the Siderial Year, which muſt conſequently exceed the Tropical Year, the Proportion of time now ſpoken of.

PROBLEM XV.

To take the Longitude and Latitude of the Planets.

Figure XLIII. THE Diſtance of a Planet is to be taken by two fix'd Stars, whoſe Longitude and Latitude is known. In Figure XLIII. Let the two fix'd Stars be A and B, and the Planet ♂, E C the Ecliptick, and its Poles F F. 1. In the Triangle F A B are given F A and F B, the Complements of the given Latitude of the fix'd Stars, and A F B the difference of the Longitude; and there are found A B and F A B. 2. In the Triangle A B ♂ is given A B, juſt now found, and A ♂, B ♂ by Obſervation; and then there is found ♂ A B. 3. In the Triangle ♂ A F are given F A and A ♂, as before, and the Angle ♂ A F, which is to be made up of the Marks ♂ A B and F A B; and ſo there is found F ♂, the Complement of the Latitude of the Planet, and A F ♂ the difference of the Longitude of the fix'd Star A, and of ♂. Now the Longitude of the fix'd Star A is ſuppos'd to be given, and therefore the Longitude of ♂ will be given alſo.

PROBLEM XVI.

To find the Aphelion *or Diſtance from the Sun, and the Eccentricity of the Superior Planets.*

WHen the Superior Planets are in Oppoſition to the Sun in their Acronychial Situation, they ſeem to us, who look upon them from the Earth, to be in the ſame Degree and Minute of Longitude, that they would ſeem to be in from the Sun, if the Eye was there plac'd: If therefore, about that time their Longitude be daily obſerv'd, and the true place of the Sun be computed at the very ſame Moments

Moments of the Obſervations; then the very Moment of oppoſition may be thence concluded, and beſides that the true Place of the Sun, to which will be then diametrically oppos'd the true Place of the Planet. Now three Acronychial Obſervations of this kind are neceſſary to be made of every Planet, that the *Aphelion* and Eccentricity may be thereby known. For returning to Prob. VI. and Figure XXXVIII. there will be given the Angles at *s*, which will be the Differences of the true places of the Sun, or of the Planet, at the moments of the Obſervations: And alſo the Angles at *f* will be ſet out by the middle Motions of the Planet, which it makes between the two next Obſervations. From whence by repeating over again the Calculation there ſhewn, you may gather both the *Aphelion* and the Eccentricity of the upper Planet.

PROBLEM XVII.

To find the Aphelion *and the Eccentricity of the lower Planets.*

BEcauſe the lower Planets do never go ſo far from the Sun, as to come into Oppoſition with it; therefore here there is need of another Method of finding out the *Aphelion* and the Eccentricity from what there was before; and inſtead of three Oppoſitions, three of the greateſt Elongations from the Sun are to be taken. Nor does it ſignifie much, that the Moments of theſe can ſcarce be preciſely determin'd; becauſe about that time they being Stationary, do ſcarce alter their Diſtance from the Sun: For if one do err ſomething in determining the time of the greateſt Diſtance, ſeeing this diſtance is the ſame or near upon it, it will come almoſt to the ſame thing. Let then, in Fig. XLIV. S be the Sun, D F E the three Places of the Earth in its Orbit, at its greateſt Elongations; and let the Orbit of the lower Planet be P G H I A, and let there be drawn about it the Circle P K L M A. Let the Planet in its greateſt Elongation from the Sun be in G, H, and I; and ſo the Right Lines D G, E H, F I, drawn from the Places of the Earth to theſe points, will touch the Elliptick Circle, and the ſame will cut the perfect Circle each of them twice; and the Sections or Points of their cutting, that are the nearer the Sun S, will fall upon K L M; and farther, the Lines S K, S L, S M, drawn from the Sun to theſe Points, will be perpendicular to the Tangents D G, E L, F M: And ſo the Triangles D K S, E L S, F M S will be rectangular, and the *Hypotenuſes*, or under ſides of each of them will be the Diſtances of the Earth from the Sun, and conſequently will be given; and the Angles at D E F, that lie between the right Lines, that are drawn from any place of the Earth to the Sun, and to the point of Contact, will be the greateſt Elongations of the Planet, and will be alſo given. Whence will be likewiſe not only D S K, E S L, F S M, the Complements of theſe Angles, but there will alſo be found the perpendiculars S K, S L, S M. Now theſe Lines being found, let there be imagin'd three Triangles, lying between Them and the Cords that joyn their ends together, *viz.* K S L, L S M, K S M, in every one of which there are given two ſides, (namely the perpendiculars,) and the Angle that is between them. For from D S E (which is either the Angle, or the Complement of the Angle to the four right Angles, computed according to the order of the Signs) if from that, I ſay, be taken the Angles on each ſide D S K and E S L, there remains K S L in the middle. Now D S E is the difference of the Places of the Earth in the firſt and ſecond Obſervation. So from F S M which is given, you may take F S E, the difference of the Places of the Earth, in the ſecond and third Obſervation, and there will remain E S M, whch you may alſo take from E S L which is given, and there will remain M S L. And laſtly, K S L + M S L = K S M. Wherefore there being given, as I ſaid, in every Triangle two ſides with the Angle contain'd between them, there will be found the Cords K L, L M, K M; And ſo the Triangle K L M will have two ſides given; and therefore the Angle K L M may be alſo found. And the double of that is the Arch K A M, and again, the Complement or filling up of this to four right Angles is the Arch M L K, and the perpendicular

Figure XLIV.

C N being let fall from C upon M K, there will be made the Angle N C M, the half of the Arch M L K, and the Line M K will be cut in two in N. Therefore there being now given M N and N C M, in the rectangular Triangle C N M, there will be found C M = C A, and also C N. And in the Triangle above mentioned, K S M, (whose sides K S and S M were given together, with the Angle K S M contained between them,) the Angle S K M might also be found. Wherefore now in the rectangular Triangle S O K, there being given S K and S K O, there may be found both the Legs S O and K O. From K N (= M N) take K O, and there remains O N = S Q; and from C N take Q N, (= S O,) and there remains C Q. Wherefore in the rectangular Triangle C Q S, there being given the two Legs C Q and Q S, there may be also found both S C, the Eccentricity that is sought for, and also the Angle C S Q; to which if there be added Q S M, (= S M K, found before in the Triangle S K M,) there will be made up of them C S M, which you must last of all take from F S M which is given, that there may remain F S A, or the Distance of the *Aphelion* of the Planet from the Place of the Earth when in F.

PROBLEM XVIII.

To find the Middle Distance of every Planet from the Sun.

THE middle Distance of the lower Planets was found indeed in the foregoing Problem, to wit, The Line C M or C N; but now the middle distance of the upper may be also found, if their Longitude be observed about the time of their Elongation from the Sun of 90 Degrees: For returning to Fig. XXXVI. in the Triangle S T E are given all the Angles, *viz.* E T S, the difference of the true Place of the Sun, and of the observed Longitude of the Planet; also E S T, the Difference of the true Place of the Earth, and of the Heliocentrick Place of the Planet. Lastly, S T, the Distance of the Earth from the Sun: Therefore there may be found S E, the Distance of the Planet from the Sun, agreeing with such a middle Anomaly of the Planet, as it has at that time, and this in parts, of which the middle Distance of the Earth from the Sun amounts to 100000. Then supposing for a while the middle Distance of the Planet to be of 100000 parts, if according to the same middle Anomaly you compute its distance from the Sun, you may then state the Analogy or Proportion thus: As the distance of the Planet, last found, is to the distance that was found first; so is 100000 to the middle distance of the same from the Sun, in parts, of which the middle distance of the Earth comes to 100000.

Figure LXI.

PROBLEM XIX.

To find the Nodes and greatest Inclination of every Planet.

TWO Observations are to be pitch'd upon, in which the Latitude of the Planet falls upon the same Quadrant of the Orbit: For, that you may not be ignorant of it, there is from the Ascending Node to the Northern Limit the first Quadrant, in which the Latitude is the Northern Ascendent; from the Northern Limit to the descending Node, in the second Quadrant, the Latitude is the Northern Descendent; from thence to the Southern Limit the Latitude is the Southern Descendent; lastly, from the Southern Limit to the Ascending Node, the Latitude is the Southern Ascendent: Therefore at the time of both Observations the Helio-

centrick

centrick Place of the Planet is eaſie to be found by Calculation; as alſo its Longitude and Latitude that is found by Obſervation. And then from the Latitude to the Inclination you may argue thus: As the Sign of the Angle to the Earth is to the Sign of the Angle to the Sun; ſo is the Tangent of the obſerved Latitude to the Tangent of Inclination. Let theſe Inclinations or Diſtances of a Planet from the Ecliptick, beheld from the Sun, be the Arches D A and G B in Fig. XLV. For E C repreſents the Ecliptick, whoſe Poles are F F; O R is the Orbit of the Planet plac'd in A, in the firſt Obſervation, and in the ſecond in B; and ☊ is the Aſcending Node. Now 'tis certain, that in the Triangle A F B there are given A F and B F, the Complements of the Inclinations, as alſo the Angle A F B, which is the difference of the Heliocentrick Places of the Planet; therefore there will be found F A B = ☊ A D. And moreover in the Triangle ☊ A D which is rectangular at D, there being given A D and ☊ A D; ☊ A cannot be unknown, which is the diſtance of the Node from the former Heliocentrick Place, nor the Angle A ☊ D, the meaſure of the greateſt Inclination R C.

PROBLEM XX.

To compare the Planets with reſpect to their Bulk or Magnitude.

Chriſtian Hugenius has done this as well as can be wiſh'd in his Saturnian Syſtem; I ſhall therefore give the Reader his very words from Page 77 to Page 82, which run thus. *Copernicus* has taught us in his new and divinely invented Syſtem, what Proportion the Diſtances of all the Planets from the Sun have to another; and it is known by the help of the Teleſcope, how much the Apparent Diametres of them are one bigger than another. The Proportions therefore being compar'd with one another, both of the diſtance and of the apparent Magnitude, the true Magnitude of the Planets, both with reſpect to one another, and alſo with reſpect to the Sun, is thereby known. And as for *Saturn*, firſt the Diametre of his Ring, ſeeing in his leaſt diſtance from us it is contain'd in an Angle of ſixty eight Seconds, (for we find ſuch at the moſt;) and ſeeing this leaſt diſtance of *Saturn* is to the middle diſtance of the Sun almoſt eight-fold; it follows, that if *Saturn* was as near us, as the Sun in its middle diſtance, then the Diametre of his Ring would appear eight times as much as it now appears; that is, nine Minutes and four Seconds. And the Diametre of the Sun, in its middle diſtance, is thirty Minutes and thirty Seconds; therefore that will be really the Proportion of the Diametre of the Ring or Compaſs of *Saturn* to the Diametre of the Sun, which 9 Minutes and 4 Seconds have to 30 Minutes and 30 Seconds; that is, almoſt of 11 to 37. And the Diametre of *Saturn* himſelf, which I ſaid above is to the Diametre of the Ring as 4 to 9, that is, almoſt as 5 to 11, will be to the Diametre of the Sun little leſs than 5 to 37.

But now how great the Diametre of *Saturn* is, compar'd with the Diametre of the Earth cannot be ſo certainly defin'd. Aſtronomers do go this way to work to find it: They firſt bring the ſpace between the Earth and the Sun to a certain number of the Diametres of the Earth, and thence they collect the ſought for Proportion of Magnitudes; but in determining that ſpace between the Earth and the Sun, they do vaſtly differ; and 'tis no wonder they do ſo, ſeeing no tolerable Method of meaſuring it has been as yet found: For whether they endeavour to find it by Ellipticks, or by Dichotomies of the Moon, it might be eaſily ſhown, that they labour in vain. Wherefore to me there ſeems to remain this one way, I'll tell you, by which we may determine, probably at leaſt, concerning the bigneſs and diſtance of all the Planets, with regard to the Earth. Let the apparent Diametres of the Planets be taken by the Teleſcope, and then by theſe let the Magnitude of every of them compar'd to the Sun be found out; (as I juſt now gave an Inſtance of *Saturn*;) and when all things are throughly examin'd, let that Magnitude of the Earth with reſpect to the reſt be taken, which ſhall ſeem to beſt agree with the Order and fit Diſpoſition of the whole Syſtem: So when the Proportion of the Diametres of the Earth

and of the other Planets to the Diametre of the Sun shall be determin'd; and it is moreover evident, how many of it's own Diametres the Sun is distant from us, to wit, by the Angle which it's Apparent Diametre does subtend; then the Greatness of the Earth, compar'd to the Sun, will be known; and together with that the Distance of the Sun, both from the Earth, and from the rest of the Planets, may be estimated by the Diametres of the Earth. This way therefore I think good to go, and therefore as I have just now compar'd *Saturn* with the Sun, so I will make the like Enquiries as to all the rest.

The Diametre of *Jupiter*, when he is nearest to us, does to me seem to be sixty-four Seconds: And seeing this Distance of his is to the middle Distance of the Sun, as 26 to 5; therefore if it be put, as 5 to 26, so 64 Seconds to the other number; then 5 minutes 35 Seconds will be found the Bigness of the Angle, which the Diametre of *Jupiter* would have, if it should be imagin'd to be as near us as the Sun is in its middle Distance. Now the Sun appears to be in Diametre 30 Minutes and 3 Seconds; wherefore that will be the proportion of *Jupiter*'s Diametre to the Sun's, which 5 Minutes, 55 Seconds have to 30 Minutes, 30 Seconds; that is, a little more than 1 to 5 $\frac{1}{2}$.

I also accurately measur'd the Diametre of *Venus* by the Method I shall by and by Explain; and I found that when she is nearest the Earth, it is no greater than eighty five Seconds. Now this Distance of *Venus* in her *Perigee*, to the middle distance of the Sun from the Earth, is as 21 to 82, or thereabouts: Therefore if *Venus* should be in the place of the Sun, her Diametre would appear to be only 21 Seconds, and 46 Thirds. Whence 'tis plain, that the Diametre of *Venus* is to the Sun's Diametre, as 21 Seconds and 46 Thirds, are to 30 Minutes and 30 Seconds; that is, as 1 to 84.

But as for the Diametre of *Mars*, when nearest the Earth, I found it not to exceed 30 Seconds; though not by so exact Observation, as I us'd in *Saturn*, *Jupiter*, and *Venus*; because I had not then found out its proportion at *Mars*'s nearest Approach to the Earth: Wherefore seeing the least Distance of *Mars* is to the middle Distance of the Sun, as 15 to 41, the Proportion of the Diametre of *Mars* to the Diamtre of the Sun, is gather'd to be about that of 1 to 166. Therefore this way *Mars* is made to be twice as little as *Venus*, as to his Diametre. And so it is manifest, that in the Planets that Order is not kept throughout them all, that they that are farthest from the Sun are also greatest in Bulk; for even the Compass of *Jupiter* too is found to be greater than *Saturn* without his Ring. Whence it happens, that one cannot so clearly make an Estimate of the Proportion of the Earth to the rest of the Planets: For if Bigness was to be attributed to them, according to their order, so that *Saturn* must be bigger than *Jupiter*, *Jupiter* than *Mars*, *Mars* bigger than *Venus*, and she than *Mercury*; then one might indeed almost certainly gather, that the Bigness of the Earth must be between *Mars* and *Venus*: But since the Contrary is found in some, it does not equally appear what is to be followed. Nevertheless that the Congruity of the whole System may be observ'd, as much as may be, it seems now to be most agreeable to it, that as the Earth is for place in the midst between *Mars* and *Venus*, so it is also in Magnitude. I have said above, that the Diametre of *Mars* is $\frac{1}{166}$ of the Diametre of the Sun, and the Diametre of *Venus* to be $\frac{1}{84}$. Therefore reckoning the Diametre of the Earth to be between these two, it must be $\frac{1}{111}$ of the Diametre of the Sun. Now $\frac{1}{37}$ of this are found equal to the Diametre of *Saturn*; therefore the Diametre of *Saturn* will contain fifteen times the Diametre of the Earth; And the Diametre of the Ring of *Saturn* will contain the same Diametre of the Earth about thirty four times; whereby the greatest Magnitude of these Bodies is known, which does indeed much exceed all that has been hitherto deliver'd by others.

And hence also the Space between the Earth and the Sun must necessarily be made greater than all suppose it; For if the Diametre of the Earth contain $\frac{1}{111}$ of the Diametre of the Sun, and the Diametre of the Sun equal $\frac{1}{111}$ of its middle Distance from us, (as it follows it must, because its Diametre is observ'd to be 30 Minutes, and 30 Seconds,) then the Diametre of the Earth will certainly be $\frac{1}{12543}$ of the Distance that is between it and the Sun. Again, seeing the least Distance of *Saturn* is almost eight times as much as the middle Distance of the Sun, therefore when *Saturn* is nearest the Earth, his Distance will be reckon'd to be 100344 of the Earth's Diametre's; and when it is very far off, about 122000.

I con-

I confeſs indeed, that theſe things are ſo far grounded upon a ſlippery Foundation, in as much as we have taken the Magnitude of the Earth to be between that of *Mars* and *Venus*, upon no other grounds but thoſe of probability, and ſo we may have eaſily erred ſome thouſands of Diametres from the true Eſtimate. But as we have now defin'd thoſe ſpaces to be twice, or even thrice, greater or leſs than really they are; yet it ought to be accounted no little thing, that we have thus far taken the meaſure of them, ſince there is no other way, in which ten times a greater Error is not to be fear'd; for ſo I am clearly of Opinion: But as for the reſt of the Calculation, by which we have compar'd the Diametres of the Planets to the Diametre of the Sun, there is, you muſt know, nothing in it built upon conjecture, but it proceeds upon ſuch things, as are grounded upon certain Reaſons. And ſo theſe apparent Diametres of the Planets, which I have told you, have been obſerv'd by me, being thus ſuppos'd, the Diametre of the Sun cannot but be to the Diametre of the Ring that ſurrounds *Venus*, as 37 to 11; and to the Diametre of *Saturn* himſelf, as 37 to 5; to the Diametre of *Jupiter* as 11 to 2; to that of *Mars*, as 166 to 1; to that of *Venus*, as 84 to 1. As for *Mercury* I will determine nothing about him, before I have exactly meaſured him; which I have not hitherto been able to doe ſucceſsfully, both becauſe of the littleneſs of his Body, and alſo becauſe he is for the moſt part found near the Horizon, where the Vapours that ariſe from the Earth by a ſort of trembling Refraction will not ſuffer the Compaſs of his Figure to be preciſely determin'd. But it's alſo farther manifeſt, that the Diametre of the Globe of *Saturn* has to the Diametre of *Jupiter* the Proportion of 55 to 74; and that it does contain the Diametre of *Mars* more than two and twenty times, and is to the Diametre of *Venus* as 34 to 3. Alſo that the Diametre of *Jupiter* is more than thirty times the Diametre of *Mars*, and but fifteen times more than that of *Venus*; and laſtly that the Diametre of *Mars* is about ſubduple to the Diametre of *Venus*. All which Proportions will remain fixt and ſettled, let the Diſtance between the Sun and the Earth be determin'd to be what it will, if ſo be the apparent Diametres be held to be, as they are now deliver'd.

SECTION V.

Of computing the Motions of the Planets by Tables.

THere are ſcarce any Tables give us a truer Calculation of the Moon and of the Eclipſes, than thoſe of *Tycho Brahe*; but to perfect it, there is alſo requir'd the Motion of the Sun: And therefore I give you out of *Tycho*, firſt, the *Solar*; ſecond, the *Lunar*; third, the *Lunæ-Solar* Tables. The Motions of the reſt of the Planets are beſt gather'd from the *Rudolphine* Tables, which *Kepler* compos'd chiefly out of *Tycho*'s obſervations; but which I have brought into a more commodious form, applying the Diviſion of the Circle, and of the degrees, to Decimal parts. Yet the *Philolaick* and the *Caroline* Tables, do give us more rightly the Motion of *Mercury*; becauſe the Authors of them were help'd by thoſe Obſervations which *Kepler* had wiſh'd for in vain: But that I may not curtail, or blend the Labours of divers perſons, I ſuppoſe it better to refer the curious Reader to the Authors now mention'd. Yet here let him take an Index of thoſe Tables, which I thought fit to ſubjoin to this work.

I. *Tycho*'s Solar Tables.

I. Tycho's *Solar* Epochæ, *according to the Meridian of* Urenburgh.
II. *A Table of twenty expanded Years.*
III. *A Table of Months in a common Year.*
IV. *A Table of Days.*
V. *An horary Table of the Sun.*

2. *Tycho*'s Lunar Tables.

VI. *Lunar* Epochæ, *instituted by* Tycho *himself, serving for the four next following Ages.*
VII. *A Table of twenty expanded Years.*
VIII. *A Table of Months in a common Year.*
IX. *A Table of Days.*
X. *An horary Table of the Moon.*
XI. *The* Prosthaphæreses *of the Sun and Moon jointly shewn, with the Elongation of the Moon from the Centre of the Eccentrick.*
XII. *The second and third* Prosthaphæresis *of the Moon.*

3. *Tycho*'s Lunæ-Solar Tables.

XIII. Epochæ, *or Roots, for the Meridian of* Friezland.
XIV. *A Table of twenty expanded Years.*
XV. *A Table of Months in a common Year.*
XVI. *The Canon of the Full and New Moons.*
XVII. *A Table of Lunar Months.*
XVIII. *The Reduction of the Moon to the Ecliptick.*
XIX. *The* Prosthaphæreses *or Equations of the Nodes of the Moon.*
XX. *A Table of the Moons Latitude.*
XXI. *A Table of the Parallaxes of the Sun and Moon in the vertical Circle.*
XXII. *A Table of the Refractions of the Sun and Moon.*
XXIII. *A Table of the apparent Semidiametres.*
XXIV. *A Table of the true horary Motion of the Moon in the true Syzygies, and half a day before and after the Syzygies.*

4. *Rudolphus*'s Tables, calculated in decimal parts of a Circle and Degrees.

XXV. *The* Epochæ *of the middle Motions of the Sun and Fixed Stars.*
XXVI. *The middle Motion of the Sun from* Aries, *in the expanded Years, Months, Days, Hours and Scruples.*
XXVII. *A Table of the Conversion of the decimal parts of a Circle into Degrees and decimal Parts of Degrees.*
XXVIII. *A Table of the Equation of the Centre of the Sun.*
XXIX. *A Table of the middle Motions of* Saturn.
XXX. *A Table of the Equation of the Centre of* Saturn.
XXXI. *A Table of the Inclination of* Saturn, *and of his Reduction to the Ecliptick.*
XXXII. *A Table of the middle Motions of* Jupiter.
XXXIII. *A Table of the Equation of the Centre of* Jupiter.
XXXIV. *A Table of the Inclination of* Jupiter, *and of his Reduction to the Ecliptick.*
XXXV. *A Table of the middle Motions of* Mars.
XXXVI. *A Table of the Equation of the Centre of* Mars.
XXXVII. *A Table of the Inclination of* Mars, *and of his Reduction to the Ecliptick.*
XXXVIII. *A Table of the middle Motions of* Venus.
XXXIX. *A Table of the Equation of the Centre of* Venus.
XL. *A Table of the Inclination of* Venus, *and of her Reduction to the Ecliptick.*
XLI. *A Table of the middle Motions of* Mercury.
XLII. *A Table of the Equation of the Centre of* Mercury.
XLIII. *A Table of the Inclination of* Mercury, *and of his Reduction to the Ecliptick.*
XLIV. *A Catalogue of the most eminent Fixed Stars.*
XLV. *A Table of Declinations, and of the Angles of the Ecliptick and Meridian.*
XLVI. *A Table of Right Ascensions.*
XLVII. *A Catalogue of Eminent Places, shewing their Meridian from* Urenburgh, *and Latitude from the* Æquator.

Now follows the use of the Tables, shewn by Precepts and Examples.

PRO-

PROBLEM I.

To turn usual Time into Astronomical.

THat usual Time may be fit to be admitted into the Tables, it stands in need of a threefold Correction.

1. The Current Day of usual Time falls either into Bissextile or Leap-year, after *February* is compleatly at an end, or else upon some other time. In the former case, if the Hours given be in the Forenoon, let one be taken from the given number of Days; but if the Hours be in the Afternoon, let nothing be taken away, and you shall have the compleat Astronomical Day. In the latter case, if the Hours be in the Forenoon, let two be taken from the number of the Days that are given; but if they be in the Afternoon, let one be taken away; and you shall have again the compleat Astronomical Day: Now the Hours in the Afternoon remain as they were given, but to those in the Forenoon the number of twelve is always added, and the Minutes or Seconds, *&c.* being also added, as they were given, there will appear the Astronomical Time; for *Tycho*'s Tables change nothing in Years or Months, but as both Current Months and Years are given, so also the Tables exhibit no other: But the *Rudolphine* are compil'd according to compleat Years and Months, and those too from the *Julian* Period; wherefore when a Current Year is propos'd, whether it be of a *Julian* Period, or from the Year of our Lord's Nativity, let there always be one taken from it; and to the year of our Lord, after one has been taken away, let there be farther added the number 4713, that it may be brought to the compleat year of the *Julian* Period. And now the particulars I have said of Days and Hours, let them be taken from *Tycho*'s Solar and Lunar Tables, and also from the *Rudolphine*: But also in the *Lunæ-Solar* Tables of *Tycho*, you must cautiously observe, that in every Leap-year, after *February* is at an end, you always take one from the number of the Days that is found; but at all other times, that you take away nothing; and then will be evident the Current Day of the middle *Syzygy*, with the Hours and Minutes of the same day that are pass'd after midnight. Wherefore if you examine any thing by the Solar and Lunar Tables, you must compute the Hours and Minutes from Noon-day, adding 12 hours, if they are in the Forenoon, and casting away one from the number of the days, that the compleat Astronomical day may remain: But if the Hours be in the Afternoon, then let nothing be taken from the number of the days before determin'd; but from the hours that were before computed from Midnight, let 12 be taken, that now they may be computed from the middle of the compleat Astronomical day, as I intimated above.

2. Again, Time is to be equall'd just the same way that I taught above in the little Treatise I inserted concerning my new Method, by the help of a twofold Table, which is there given; but this must be done with this Caution, that as often as the Apparent Time (which is what is commonly exhibited) is to be turn'd into Astronomical, you must always take away 7 Min. 53 Sec. after you have done all the rest; but on the contrary, when the Astromical Time is to be turn'd into Apparent, you must add 7 Min. 53 Sec. besides all other things: And you must know that this holds as well in the Solar Tables of *Tycho*, as in all the *Rudolphine*, but in the *Lunar* and *Lunæ-Solar* Tables of *Tycho*, only the first part of the Table of the Equation of time is to be made use of, taking no notice neither of the said 7 Min. and 53 Seconds.

3. Farther, the time of every place is to be reduc'd to the Meridian of the Tables, by the help of the Catalogue of Places, which follows the Series of the Tables. For those places which are more Eastwarst than *Urenburgh*, will subtract the Minutes expres'd in the Catalogue from the Hours and Minutes that are given; but those that are more Westward will add, that the time may appear which agrees with the Meridian of *Urenburgh*. And this likewise holds in *Tycho*'s Solar and Lunar Tables, and in all the *Rudolphine*; but the *Lunæ-Solar* Tables of *Tycho*, seeing they were fitted to the Meridian of *Friezland*, which is distant from *Urenburgh* Westward, hour 0. min. 25. you must first of all add always to the time of the *Syzygy*, when found,

52 Min-

25 Minutes, and then you will have the time according to the Meridian of *Urenburgh*; to which you must afterwards add or subtract the minutes found for any other place in the Catalogue, just the same way that I shew'd before.

PROBLEM II.

To compute the place of the Sun by Tycho's *Solar Tables.*

IN the Year 1662. in the Month of *March*, let the Apparent Time be given, 8 current or common days, 2 hours in the Forenoon, and 29 minutes, at *London*; at which time we are to find the true place of the Sun. The usual time given, being turn'd by the foregoing Problem into Astronomical, is the year of 1662, 6 compleat days in the Month of *March*, 15 hours, 17 minutes, and 40 Seconds. For seeing the time given does fall in Leap-year, after *February* is ended, and the hours being in the Forenoon; I take two from the given number of the days, but to the hours (because they are in the Forenoon) I add twelve; so thence come six complete days in *March*, fourteen hours, and twenty nine Minutes; then with the true place of the Sun, *viz.* twenty seven Degrees, and fifty two Minutes of ♓, I take (out of the above written Tables) the first part of the Equation of Time, *viz.* Min. 0. Sec. 43. A. and with the Anomaly of the Sun, which is 8 Signs, 19 Deg. and 25 Min. I take the latter part of the Equation of Time, that is, 7 Min. and 50 Sec. A. Therefore the absolute Equation of Time is 8 Min. 33 Sec. A. whence being cast (as is always to be done) 7 Min. 53 Sec. there remain Min. 0. Sec. 40. A. Wherefore the equall'd time for the Meridian of *London* is 6 complete days of *March*, 14 hours, 29 Min. 40 Sec. Lastly, in the Catalogue of Places, (Table XLVII.) I find the differences of the Meridians of *London* and *Urenburgh* to be 48 Min. that *London* is more Westward; which being farther added, the equall'd Time is made up for the Meridian of *Urenburgh*, as was said above, with which I go to *Tycho's* Solar Tables, setting the Numbers as follows:

	The Sun's Motion.				*Anom. of the Sun.*				
	Si.	de.	min.	sec.	Si.	de.	min.	sec.	
1601	09	20	55	12	06	15	15	27	Tab. I.
60	00	00	27	43	11	29	42	43	II.
1	11	29	45	41	11	29	44	55	II.
Mart.	01	28	59	11	01	28	09	04	III.
d. 6	00	05	04	50	00	05	54	50	IV.
h. 15	00	00	36	58	00	00	36	58	V.
m. 17	00	00	00	42	00	00	00	42	V.
s. 40	00	00	00	02	00	00	00	02	V.
	11	25	50	19	08	19	24	41	

With the collected Anomaly of 8 Signs, 19 Deg. 24 Min. 41 Sec. I take out of Table XI. the *Prosthaphæresis* of the Sun 2 Deg. 1 Min. 51 Sec. A. which being also added (as the Title directs) to the middle Motion, which is 11 Signs, 25 Deg. 50 Min. 19 Sec. it makes the true Place of the Sun 11 Signs, 27 Deg. 52 Min. and 10 Seconds.

PRO-

PROBLEM III.

To compute the Longitude of the Moon.

IN the Year of our Lord 1587. on the eighteenth of *Auguſt*, current day, ſeven hours and twenty five Minutes after Mid-night, the true Place of the Moon was by *Tycho*'s Inſtruments obſerv'd to be twenty ſix Degrees and twenty three Minutes in *Gemini*. Firſt there is to be found at the time given the true Place of the Sun by the foregoing Problem. Now the Aſtronomical Time that is fitted for that is the year 1587. *Auguſt*, ſixteen Days, nineteen Hours, and eighteen Minutes, and the middle Place of the Sun is five Signs, five Degrees, fifty Minutes, twenty one Seconds. Now the Anomaly is two Signs, 0 Degrees, twenty Minutes, four Seconds; to which anſwers the *Proſthaphæreſis*, one Degree, forty five Minutes, nine Seconds S. Wherefore the true Place of the Sun is four Degrees, five Minutes, twelve Seconds ♍. Then the ſame time that is given is to be adapted according to the Lunar Tables; and it becomes Aſtronomical, the Year 1687. *Auguſt*, ſixteen Days, nineteen Hours, twenty two Minutes, and thirty ſeven Seconds: with which I go to the Lunar Tables, placing the Numbers as follows:

		Long. of the Moon from the Sun.				*The Anomaly of the Moon.*				*The Motion of the Lat. of the Moon.*			
		Si.	de.	min.	ſec.	Sig.	de.	min.	ſec.	Si.	de.	min.	ſec.
Tab. VI.	1501	4	22	13	19	10	11	59	11	0	16	47	17
VII.	80	5	23	39	07	05	08	48	20	9	11	35	49
VII.	6	2	09	55	43	06	05	22	40	6	05	30	18
VIII.	Aug.	2	04	26	19	08	09	46	35	9	14	37	20
IX.	d. 16	6	15	03	07	06	29	02	23	7	01	40	11
X.	h. 19	0	09	39	04	00	10	20	35	0	10	28	25
X.	m. 32	0	00	16	15	00	00	17	25	0	00	17	38
X.	ſec. 37	0	00	00	19	00	00	00	20	0	00	00	20
		9	25	13	13	01	15	37	29	9	00	57	18

With the Anomaly of the Moon, 1 Sig, 15 Deg. out of Tab. XI. I take the *Proſtaphæreſis* 3 Deg. 26 Min. 59 Sec. and for 37 Min. 29 Sec. I gather the proportional part, 2 Min. 17 Sec. to be added, that the *Proſthaphæreſis* may anſwer the whole motion of the Anomaly, 3 Deg. 29 Min, 16 Sec. S. And to the ſame Anomaly does there anſwer the Elongation of the Moon from the Centre of the Eccentrick 102217, which is in Fig. XXVII. the Line E *e*, for that Figure is accomodated to this Calculation.

Then to the Motion of the Sun from *Aries* add the Longitude of the Moon from the Sun.

	Sig.	*Deg.*	*Min.*	*Sec.*	
The middle Motion of the Sun	05	05	50	21	
The Longitude of the Moon from the Sun	09	25	13	13	
The middle Diſtance of the Moon from *Aries*	03	01	03	34	
The Epicyclical *Proſtaphæreſis*	00	03	29	16	S
The Diſtance of the Moon from *Aries* firſt equall'd	02	27	34	18	Subt.
The true Motion of the Sun	05	04	05	12	
The almoſt true Diſtance of the Luminaries	09	23	29	06	
The ſame doubled	07	16	58	12	

To

To this double distance of the Luminaries does answer the Motion of the Centre of the Eccentrick in the little Circle, such as is the Circumference A L E in the Figure mention'd. With that Circumference therefore go to Tab. XII. where you shall find, agreeing with 7 Signs, 17 Deg. the Eccentricity 3987, which is the Line A E; as also the variation of the Centre of the greater Epicycle, agreeing with 29 Min. 36 Sec. S. The Excess of the Circumference A L E above the Semicircle is 46 Deg. 58 Min. 12 Sec. the half of which, *viz.* 23 Deg. 29 Min. 6 Sec. is equal to the Angle E A H or B E R, which is call'd the Angle of the second Equation. For the Angle B E C is the simple Anomaly, B E *e* is the Anomaly first equall'd, and R E *e* is the Anomaly a second time equall'd: Therefore from the

	Sig.	*deg.*	*min.*	*sec.*
Middle Anomaly B E C	01	15	37	29
Take away the Epicyclical *Prosthaphæresis* C E	00	03	29	16 S.
There rests the Anom. first equall'd B E	01	12	08	13
Take away again B E R	00	23	29	06
There rests the Anom. a second time equall'd R E *e*	00	18	38	07

Thus then there are given in the Triangle A E *e* the two sides E *e* 102217, and A E 3987, with the External Angle R E *e*; And so by Trigonometry will be found the *Prosthaphæresis* of the Eccentricity A *e* E 41 min. 21 sec. which is to be subtracted.

We have therefore now three *Prosthaphæreses* of the Moon. 1 The Epicyclical, which is of 3 deg. 29 min. 16 sec. S. 2. That of Eccentricity which is of 0 deg. 41 min. 21 sec. S. 3. That of the Variation, *viz.* 29 min. 36 sec. S. Which since they are all of the same sort, to wit, each of them subtractive; therefore being added into one Summ, they make the absolute *Prosthaphæresis* of the Moon 4 deg. 40 min. 13 sec. which is to be subtracted from the middle Distance of the Moon before found.

	Sig.	*deg.*	*min.*	*sec.*
The middle Distance of the Moon from *Aries*	03	01	03	34
The Absolute *Prosthaphæresis*	00	04	40	13 S.
The Remainder will be	02	26	23	21

Which is the true Distance of the Moon from the beginning of *Aries*. Lastly, the middle motion of Latitude being equall'd by the said Absolute *Prosthaphæresis*, the true middle Motion of Latitude will be then given.

	Sig.	*deg.*	*min.*	*sec.*
The middle motion of Latitude	09	00	57	18
The Absolute *Prosthraph*	00	04	40	13 S.
The true Motion of Latitude	08	26	17	05

With which the Table of Reduction, *viz.* the XVIII. is to be gone to, which will afford 54 sec. to be subtracted, that the true place of the Moon in the Ecliptick, *viz.* ♊ 26 deg. 22 min. 27 sec. may agree exactly enough with Observation.

PRO-

PROBLEM IV.

To Compute the Time of the middle Syzygy by the Lunæ-Solar *Tables, and the middle Motions which agree with that time,* viz. *The Anomaly of the Sun, the Anomaly of the Moon, the Motion of the Moon's Latitude, and the Sun's Motion from* Aries.

AS for Instance, let the thing sought for be the Time of the Full Moon, which happens in the year 1656 in the Month of *January*, together with the middle Motions, that agree with that time. Let the numbers be taken out of the XIII. XIV. XV. XVI. and plac'd as follow.

	The Epact.	*The Anomaly of the Sun.*	*The Anomaly of the Moon.*	*Motion of the Moon's Latitude.*	*Motion of Sol from Aries.*	
	D. ho. mi. sec.	*Si. de. mi. sec.*	*Si. de. mi. sec.*	*Si. de. mi. sec.*	*Si. de. mi. sec.*	
1601	06 20 11 45	06 08 02 20	01 24 48 14	00 01 53 27	09 13 42 04	Tab. XIII.
40	21 21 18 08	11 08 14 07	05 03 26 30	01 01 14 24	11 08 44 05	XIV.
15	14 20 10 18	11 14 33 42	03 06 05 12	09 04 02 51	11 14 44 56	XIV.
Jan.	00 00 00 00	00 00 00 00	00 00 00 00	00 00 00 00	00 00 00 00	XV.
	43 13 40 11					
	45 07 06 05	01 13 39 31	07 08 43 30	07 16 00 21	01 13 39 36	XVI.
Jan.	01 17 25 54	06 14 29 40	05 13 03 26	05 23 11 03	09 20 50 41	

Let the Epacts that are taken out of the XIII. XIV. XV. Tables be gather'd into one Summ, which will here be 43 days, 13 hours, 40 min. 11 sec. With this Summ of the Epacts go to the XVI, which is the Canon or Rule of Full Moon and New; and seeing the Full Moon is sought for in this place, the Number of days, that is next of all greater or above our Number, and that answers to the Full Moon, is found to be 45 d. 7 h. 6 min. 5 sec. which I write out, together with the middle Motions that are in the same Division; and the Epacts before gather'd from the Time writ out, I take away, but collect all the middle Motions into one: So there appears the Time of the middle Full Moon, *Jan.* 1 d. 17 ho. 25 min. 54 sec. and the Anomaly of the Sun, that agrees with that moment of time is 6 sig. 14 deg. 29 min. 40 sec. &c.

PROBLEM V.

Any Year being Propos'd, to find out the Months and Days in which Eclipses of the Sun or Moon may happen.

THe Full Moon may be Eclips'd, when in the Time of the middle Syzygy the Motion of the Moon's Latitude is between 11 Signs, 14 Degrees, 48 Minutes, and 0 Sig. 15 Deg. 12 Min. And also between 5 Signs, 14 Deg. 48 Min. and 6 Sig. 15 Deg. 12 Min. And the New Moon may Eclipse the Sun, when in the time of

the middle Syzygy the Motion of the Moon's Latitude is between 11 Sig. 18 Deg. 38 Min. and 0 Sig. 20 Deg. 41 Min. As also between 5 Sig. 9 deg. 19 min. and 6 sig. 11 deg. 12 min. And now let the year propos'd be 1668, I would know, what Days and Months of this year there will happen any Eclipses: For since the Argument of the Eclipse is taken in this place from the middle Syzygy of the Luminaries, it is not necessary, that therefore we must needs conclude, that there will certainly be an Eclipse of the Sun or Moon. For it may be, that the Moon, which at the time of the middle Syzygy was within the Bounds of Eclipsing, may at the time of the true Syzygy, be either not come to them, or may be gone beyond them, according as the true Syzygy either goes before or follows the middle one. And therefore we only say, that it is possible, that when the Moon is within the Bounds before mentioned, there may be an Eclipse of the Sun or Moon. But whether it be to be certainly expected, is to be learnt from the Time of the true Full Moon in a Lunar Eclipse, or from the time of the Apparent Conjunction in a Solar one. Let us therefore first of all enquire, what days of the year mention'd a Lunar Eclipse may be suspected to fall. Wherefore by the Method deliver'd in the Fore-going Problem; let the time of the middle Full Moon be sought for, that happens in the Month of *January*, together with the middle Motion of Latitude that agrees with it.

	The Epact.				*Motion of the Moon's Latitude.*			
	D.	ho.	min.	sec.	Sig.	deg.	min.	sec.
1601	06	20	11	45	00	01	53	27
60	03	07	13	07	02	17	31	50
7	16	08	51	30	03	27	39	55
1668	26	12	16	22				
	45	07	06	05	07	16	00	21
January	18	18	49	43	02	03	05	33

The first middle Full Moon of the said Year, is found to be *Jan.* 18 *d.* 18 *h.* 49 min. 43 sec. and the Moon's Motion of Latitude 2 sig. 3 deg. 5 min. 33 sec. which is evidently out of the Bounds of Eclipsing; whence 'tis plain, that no Lunar Eclipse is to be expected that Month. And seeing the Bounds of Eclipsing next following, which are above laid down, are these, *viz.* 5 sig. 14 deg. 48 min. which is that on this side; and the farther, that is beyond, 6 sig. 15 degr. 12 min. Therefore it is no difficult Thing to gather from hence, how much the middle Motion of Latitude above is distant from each of these Bounds.

	Sig.	deg.	min.		Si.	de.	mi.
From each of the said bounds	05	14	48	and	06	15	12
Take the Motion of Latitude before found	02	03	06		02	03	06
There rests the dist. of ☽ from the Bounds of Eclipsing	03	11	42	and	04	12	06

With both these Distances go to Table the XVII. which is of Lunar Months, seeking there under the Title of Motion of Latitude the Number which is between the 3 sig. 11. degr. 42 min. that were just now found: And you shall find 4 sig. 2 deg. 4 min. 56 sec. the number, that answers the four Lunar Months, and so the Time that agrees with so many Months. 118 *d.* 2 *h.* 56. m. 13. Sec. Therefore to the time of the first Full moon, that was found, add the time of four Months, and to the Motion of Latitude of that time reckon the Motion of Latitude of these 4 Months.

Jan.

	D.	ho.	min.	sec.	Sig.	deg.	min.	sec.
Jan.	18	18	49	43	02	03	05	33
	118	02	56	13	04	02	40	56
	136	21	45	56	06	05	46	29

So there appears the Current Day from the beginning of the Year, *viz.* 136 *d.* 21 *h.* 45 *min.* 56 *sec.* And the Motion of Latitude, that answers the said time, *viz.* 6 sig. 5. deg. 46 min. 29 sec. Which seeing it is between the Bounds of Eclipsing before assign'd, we gather thence that it is possible an Eclipse of the Moon may happen at that time. And if you enquire farther, what day and of what Month, that day is, Searching in the little Table which is added, and that in the little Colume of Leap-year (because the year 1668 is such) the number of days that is the next number less than that which was before made up by Addition; which is here found to be 121. This therefore if you take from 136 days, there remains the Current day of *May*, *viz.* The 15 d. 21 h. 45 min. 56. sec. upon which we suspect an Eclipse of the Moon may happen. But now whether it will certainly be or no, and whether it will appear in our Horizon, will be then manifest, when we have found out the time of the true Full Moon by the following Problem.

Months complete.	*Days collected into*	
	A Common Year.	*Leap-Year.*
January	31	31
February	59	60
March	90	91
April	120	121
May	151	152
June	181	182
July	212	213
August	243	244
September	273	274
October	304	305
November	334	335
December	365	366

But if you farther desire to know the time of the other Eclipse of the Moon, that will be seen the same year, (if the true Motions will allow of it,) you must add to the time just now gather'd, the time of 5 or 6 Lunar Months, and to the Motion of Latitude, the Motion that answers so many Months; so that the Motion that is made up of both, may at length be within the Bounds of Eclipsing.

	D.	ho.	min.	sec.	*Moons Mot. of Lat.* Sig.	deg.	min.	sec.
The time of the first Eclipse of ☽	136	21	45	56	06	05	46	29
The time of 6 Months ☽	177	04	24	19	06	04	01	23
	314	02	10	15	00	09	47	52

There then appears the Current day from the beginning of the year, *viz.* 314 d. 2 h. 10 m. 15 sec. which day, according to the little Table fore-going, falls in with the 9*th* of *Novemb.* Current 2 ho. 10. min. 15 sec. And the Moon's Motion of Latitude, *viz.* 0 sig. 9 deg. 47. min. 52 sec. is within the bounds of Eclipsing; wherefore

I conclude, that on that day an Eclipse of the Moon may happen, if the true Motions agree to it.

And we may also seek for the Eclipses of the Sun the same way. For if from the time of the first Full-Moon, which happens *Jan.* 18 d. 18 h. 49 min. 43 sec. you take the time of half a Lunar Month; you shall have the very minute of the first middle New-Moon.

	D. ho. mi. sec.	*The Moon's Mot. of Lat. Sig.*	*deg.*	*min.*	*sec.*
Jan.	18 18 46 43	02	03	05	33
Half Month	14 18 22 02	06	15	20	07
Jan.	04 00 27 41	07	17	45	26

The first New Moon of the Year 1668 is *Jan.* 4 d. 0 h. 27 min. 41 sec. and the Motion of the Moon's Latitude is 7 sig. 17. deg. 45 min. 26 sec. which because it is out of the Bounds of Eclipsing, it is to be taken from the Bounds that next follow, of which that on this side is 11 sig. 18 deg. 38 min. and that beyond is 0 sig. 20 deg. 41 min.

Sig. deg. min. sec.	*Sig. deg. min. sec.*
11 18 38 00	00 20 41 00
07 17 45 26	07 17. 45 26
04 00 52 34	05 02 55 34

Then in Table the XVII. under the Motion of the Moon's Latitude, let the number be sought for, that is between these two last; which is found to be 4 sig. 2 deg. 40. min. 56 sec. which agrees with 4 Lunar Months. Therefore now to the

	D. ho. min. sec.	*The Moon's Mot. of Lat. Sig.*	*deg.*	*min.*	*sec.*
Time of the first Full Moon	4 00 27 41	07	17	45	26
Add the time of 4 Months	118 02 56 13	04	02	40	56
	122 33 23 54	11	20	26	22

Then appears the current Day from the beginning of the year, *viz.* 122, which, according to the foregoing little Table, falls in with *May* the 1 d. 3 h. 23 min. 54 sec. And the Moon's Motion of Latitude 11 sig. 20 deg. 26 min. 22 sec. is within the Bounds of Eclipsing. Wherefore we say, on the first of *May* an Eclipse of the Sun may happen. Then 6 Lunar Months being added, there is found the time of another Solar Eclipse.

	D. ho. min. sec.	*The Moon's Motion of Lat. Sig.*	*deg.*	*min*	*sec.*
The time of the first Eclipse of the Sun	122 03 23 54	11	28	26	22
6 Lunar Months	177 04 24 19	06	04	01	23
	299 07 48 13	05	24	27	45

So the Current day from the beginning of the year appears to be the 299th, that is, *Octob.* 25 d. 7 h. 48 min. 13 sec. At which time the Motion of the Moon's Latitude 5 sig. 24 deg. 27 min. 45 sec. falls within the Bounds of Eclipsing, wherefore upon that day perhaps an Eclipse of the Sun may be seen.

PRO-

PROBLEM VI.

To find the time of the true Syzygy.

WE muſt firſt have found by Problem IV the time of the middle Syzygy, together with the middle Motions that agree with the ſame time. And when the Syzygy is not liable to have an Eclipſe in it, or we are not ſolicitous to examine an Eclipſe thoroughly, then the Motions of the Sun's and Moon's Anamoly are ſufficient: But when we have a mind to enquire into the Quantity, Duration, Beginning andEnding of an Eclipſe,we muſt take in alſo the Motion of the Moon's Latitude,and the Motion of the Sun from *Aries*. Let us now re-aſſume the Example us'd in the Problem, where it was found, that the firſt middle New-Moon of the year 1656 was *Jan.* 1 d. 17 ho. 25 min. 54. ſec. and at the ſame time

	Sig.	*deg.*	*min.*	*ſec.*
The Anomaly of the Sun	00	14	29	40
of the Moon	05	13	03	26
The Moon's Mot. of Lat.	05	23	11	03
The Mot. of the Sun from *Aries*	09	20	50	41

Now 'tis here apparent, that the Moon's Mot. of Lat. 5 ſig. 23 deg. 11 min. 3 ſec. falls within the Bounds of Eclipſing, whence we ſuppoſe that an Eclipſe of the Moon may happen. But that we may be more certain of this, we muſt now inquire into the exact time of the true Syzygy. With the Anomaly of the Sun 6 ſig. 14 deg. 29 min. 40 ſec. take out of the XI Table the *Proſthaphæreſis* of the Sun, 31 min. 57 ſec. A. And with the Anomaly of the Moon by the ſame Table is found the Moon's *Proſthaphæreſis*, 1 deg. 29. min. 55. ſec. S. And ſeeing the *Proſthaphæreſes* are of a different kind, the one to be added, and the other to be ſubtracted, the ſumm of them affords the Diſtance of the Luminaries 2 deg. 1 min. 52. ſec. But if they were either both to be added, or both to be ſubtracted, then the difference of the *Proſthaphæreſes* would give the Diſtance of the Luminaries. And now with the diſtance of the Luminaries juſt now found, 2 deg. 1 min. 52 ſec. go to Table X, which is the Moon's Hour Table: And under the Title of the Longitude of the Moon from the Sun, ſeek for the number, that next of all agrees with that number, and there you will find 2 deg. 1 min. 54 ſec. and over againſt it in the firſt Colume 4 hours, whence it is manifeſt,that the Moon may in the next 4 hours exceed in her middle motion that ſpace ſhe is diſtant from the Sun at the time of the middle Full-Moon. And therefore there is almoſt the ſame ſpace of time between the middle and the true Syzygy, and it is call'd the *Proſthaphæretical* Time, becauſe it is wont ſometimes to be added to, ſometimes to be taken from, the Time of the middle Syzygy, according as the Moon at the time of the middle Syzygy has either as yet not overtaken the Sun, or is paſt by it. Which thing is not hard to be gather'd from the Nature of the *Proſthaphæreſes*: For ſeeing at the time of the middle Syzygy the Lines of the middle Motions of the Sun and Moon are join'd, the *Proſthaphæreſes* ſhew how much the Lines of the true Motions do decline from them: And the Tittle of the *Proſthaphæreſes*, (Add, or Subtract,) ſhews whether the Luminary be behind or before them. In our Example the *Proſthaphæreſis* of the Sun is to be added; therefore the Sun is beyond the Lines of middle motion, that is, in the following Signs; and the Moon's *Proſthaphæreſis* being to be ſubtracted, argues it to be on this ſide, or in the Antecedent Signs. Therefore the Moon has not yet overtaken the Sun by its true Motion, and the true Syzygy follows the middle one almoſt 4 hours. And the ſame alſo holds true, when the *Proſthaphæreſis* of the Sun and Moon are both of them to be added, and the Sun's is the greater; or when they are both to be ſubtracted, and the Sun's is the leſs. For in every of theſe Caſes the Moon is on this ſide the Sun. But on the contrary the true Syzygy goes before the middle one, when the *Proſthaphæreſis* of the Sun is to be ſubtracted, and the Moon's to be added; or when the *Proſthaphæreſes* of the Sun and Moon are either both to be added, and the Moon's is the greater, or elſe both to be ſubtracted,

ed, and the Moon's is the leſs. For in each of theſe Caſes the Moon is gone beyond the Sun. Now though by this firſt way of working the *Proſthaphæretical* Time be found to be about four hours, yet if the Computation be repeated twice and again, it will be found, that 20 Minutes are to be taken from that Summ of four hours, which I thus find out. To the *Proſthaphæretical* time of four hours does anſwer (in Tab. V. which is the Sun's Hour-Table) the Anomaly of the Sun 9 Min. 51 Sec. and to the ſame *Proſthaphæretical* Time does agree (in Tab. X. which is the Moon's Hour-Table) the Anomaly of the Moon 2 Deg. 10 Min. 39 Sec. Now let theſe Portions be added every one to its own Anomaly, that is before found; let them be added, I ſay, becauſe the true *Syzygy* follows.

	The Anomaly of the Sun.				*The Anomaly of the Moon.*			
	Sig.	*Deg.*	*Min.*	*Sec.*	*Sig.*	*Deg.*	*Min.*	*Sec.*
At the time of the middle Full Moon	00	14	29	40	05	13	03	26
For the *Proſthaphæretical* time of 4 hours	00	00	09	51	00	02	10	39
The time of almoſt the true Full Moon.	06	14	39	31	05	15	14	05

And to theſe Anomalies do now anſwer the *Proſthaphæreſes* out of the XI Table, the Sun's of 32 Min. 18 Sec. A. and the Moon's of 1 Deg. 18 Min. 25 Sec. S. Wherefore the diſtance of the Lines of the middle Motion of 1 Deg. 50 Min. 43 Sec. And farther in the Tab. X. the Longitude of the Moon from the Sun, that is next of all leſs, is 1 Deg. 31 Min. 26 Seconds, anſwering to three hours.

	Deg.	*Min.*	*Sec.*	
The diſtance of the Lines of middle Motion	01	50	43	} Subt.
The Long. of the Moon from the Sun next of all leſs	01	31	26	}
The Remainder is	00	19	17	

To theſe remaining 19 Min. 17 Sec. that are found in the ſame Table under the Title of the Longitude of the Moon from the Sun, do anſwer next of all 38 Min. Therefore the truer *Proſthaphæretical* Time is 3 Hours 38 Minutes. Now why we think this *Proſthaphæretical* Time of 3 Hours 38 Min. to be truer, than that which was firſt found, *viz.* of four hours, it will be worth our while to know. For although at the Time of the middle *Syzygy*, the Lines of the true Motions of the Sun and Moon are diſtant exactly 2 Deg. 1 Min. 52 Sec. as was found above; and alſo four hours after, when the Lines of the true Motions are almoſt joyn'd, the Lines of the middle Motions are almoſt diſtant, as much as theſe *Proſthaphæreſes* did juſt now intimate, *viz.* 1 Deg. 50 Min. 43 Sec. Yet if we conſider, that when we did at firſt ſeek the *Proſthaphæretical* Time, we us'd the middle Motion of the Moon's Longitude from the Sun, or the equable Motion, which we were fore'd to doe; becauſe the Progreſs of the true Lines is inconſtant, and changes every Moment; wherefore it is no wonder, if the Lines of the true Motions (though they be really diſtant 2 Deg. 1 Min. 52 Sec.) yet do not get over the ſpace of that diſtance in four hours, as the Lines of the true Motions would indeed do, if they were ſo far diſtant. But now on the contrary, at the time of the true *Syzygy*, when the Lines of the true Motions are joyn'd, the Lines of the middle Motions are altogether diſtant, as much as the *Proſthaphæreſes* declare, *viz.* 1 Deg. 50 Min. 43 Sec. and when the Lines of the middle Motions go on equally, and according to the numbers expreſs'd in Tab. X. it is plain, that they will paſs over as much ſpace of Diſtance, *viz.* 1 Deg. 50 Min. 43 Sec. in altogether as much ſpace of Time, *viz.* in 3 Hours 38 Min. and ſo will come to be joyn'd together; that is, between the time of the joyning of the Lines of middle Motion, and the meeting of the Lines of the true Motion there muſt be 3 Hours, 38 Min. And this would plainly happen, if the Lines of the true Motions were then exactly joyn'd, when the Anomalies and *Proſthaphæreſes* were ſuch as we have now made uſe of. But becauſe they are even then diſtant ſome little ſpace, it is neceſſary to uſe a third way of working it. For we know, that the more exact ſpace between the middle *Syzygy* and the true is 3 Hours, 38 Min. in which ſpace the Anomaly of the Sun goes on 8 Min. 57 Sec. and the Anomaly of the Moon 1 Deg. 58 Min. 40 Sec. which Porti-

ons

ons are again to be added to the Anomalies that were first of all found.

	The Anomaly of the Sun.				The Anomaly of the Moon.			
	Sig.	Deg.	Min.	Sec.	Sig.	Deg.	Min.	Sec.
At the time of the middle full Moon	06	14	29	40	05	13	03	26
For the *Prosthaph.* Time of 3 h. 38′	00	00	08	57	00	01	58	40
At the time of the true Full Moon	06	14	38	37	05	15	02	06

And to these Anomalies do answer the *Prosthaphæreses* out of Tab. XI. that of the Sun's is 32 Min. 32 Sec. A. but that of the Moon 2 Deg. 19 Min. 27 Sec. S. Wherefore

	Deg.	Min.	Sec.	
The Distance of the Lines of middle Motion	01	51	59	} Subt.
Then out of Tab. X. the next less Number	01	31	26	
	00	20	33	

And to this next less Number, *viz.* of 1 Deg. 31 Min. 26 Sec. do answer in Tab. X. three hours, and to the remaining 20 Min. 33 Sec. do there agree 40 Min. Wherefore the *Prosthaphæretical* Time, which is accurate enough, is 3 Hours 40 Min.

To which Answers out of Tab. V. 9 Min. 2 Sec. and out of Tab. X. the Anomaly of the Moon 1 Deg. 59 Min. 45 Seconds.

	The Anomaly of the Sun.				The Anomaly of the Moon.			
	Sig.	Deg.	Min.	Sec.	Sig.	Deg.	Min.	Sec.
At the Time of the middle Full Moon	06	14	29	40	05	13	03	26
For the *Prosth.* Time 3 h. 40′	00	00	09	02	00	01	59	45
At the time of the true Full Moon	06	14	38	42	05	15	03	11

And now to these Anomalies do answer the *Prosthaphæreses* out of Tab. XI. that of the Sun is 32 Min. 15 Sec. A. but that of the Moon 1 Deg. 19. Min. 22 Seconds. S. Wherefore

The Distance of the Lines of middle Motion is	01	51	37	} Subt.
Then out of Tab. X. the next less Number	01	31	26	
	00	20	11	

To this next less Number, *viz.* of 1 Deg. 31 Min. 26 Sec. do answer Tab. X. three hours; and to these remaining 20 Min. 11 Sec. do there agree 39 Min. 43 Seconds. Wherefore the most exact *Prosthaphæretical* Time of all is 3 h. 39 Min. 43 Sec. which is to be added, because the Moon is on this side the Sun; Therefore to the

	D.	H.	M.	Sec.
Time of the middle Full Moon, *Jan.*	01	17	25	54
Add the *Prosthaph.* Time	00	03	39	43
The most exact Time of the true Full Moon, *Jan.*	01	21	05	37

For the finding the true Place of the Sun.

	Sig.	Deg.	Min.	Sec.
At the time of the mid. Full Moon, the mid. Mot. of ☉ from ♈	09	20	50	41
The Motion of the Sun for the *Prosth.* Time	00	00	09	01
The *Prosthaphæresis* of the Sun	00	00	32	15 A.
The true Place of the Sun at the time of the true Full Moon	09	21	31	57

With this true Place of the Sun the Table of the Equation of Time, inserted *page* 785. is to be gone to; where you shall find the Equation of Time, that answers the said Place of the Sun, to be 7 Min. 0 Sec. which are to be subtracted, because in this

this Place the Astronomical Time is to be turn'd into Apparent. Now therefore from the

	D.	H.	M.	Sec.	
Time of the true Full Moon, *Jan.*	01	21	05	37	
Take the Equation of Time	00	00	07	00	
The Time apparent of the true Full Moon, *Jan.*	01	20	58	37	After
Mid-night, or *Jan.*	01	08	58	37	After
Mid-day, or, according to the *Gregorian* Account, *Jan.*	00	08	58	37	After
Mid-day in *Friesland.*					

PROBLEM VII.

To prove the Time of the true Syzygy, *found by the foregoing Prob. out of the Solar and Lunar Tables.*

BEcause the *Epochæ* of the *Lunæ-Solar* Tables are fitted to the Meridian of *Friesland*, but the *Epochæ* of the Solar and Lunar Tables do suit with the Meridian of *Vrenburgh*; therefore the apparent Time given from *Frisian* Meridian is first to be reduc'd to that of *Vrenburgh*, by adding 25 Minutes of an hour; because the Meridian of *Vrenburgh* is more Eastward than that of *Friesland*; therefore

	D.	H.	M.	Sec.
To the apparent Time of the true Full Moon in *Friesland, Jan.*	01	08	58	37
Add the Difference of the Meridians	00	00	25	00
The apparent Time of the true Full Moon in *Denmark, Jan.*	01	09	23	37

Then the apparent Time is to be turn'd into Astronomical, and that after one way for the Sun, and after another way for the Moon; as I have taught Prob. I. of this Section, Paragraph 2. And for the Sun thus; with the true Place of the Sun ♈ 21 Deg. 31 Min. 57 Sec. Page 785. I take the first part of the Equation of Time, *viz.* 7 Min. 0 Sec. to be added. Then afterwards with the middle Anomaly of the Sun 6 Sig. 14 Deg. 38 Min. 42 Sec. I go to the other part of the Equation of Time, Page 785. which I there find to be 2 Min. 3 Sec. which are likewise to be added. And so the whole Equation of Time becomes 9 Min. 3 Sec. to be added. Hence I always take 7 Min. 53 Sec. according to what was admonish'd in the first Prob. and there remains the Equation of Time for the Sun, 1 Min. 10 Sec. A. Now therefore to

	D.	H.	M.	Sec.
The apparent Time of the true Full Moon, *Jan.*	01	09	23	37
Add the Equation of Time for the Sun	00	00	01	10
And there is made the Astronomical Time for the Sun	01	09	24	47

To which Time I gather, by Prob. II. the middle Motion of the Sun from *Aries*, 9 Sig. 20 Deg. 59 Min. 28. Sec. The middle Anomaly of the Sun, 6 Sig. 14 Deg. 38 Min. 28 Sec. and the *Prosthaphæresis* of the Sun, that agrees with this, 32 Min. 15 Sec. A. Wherefore the true Place of the Sun is 9 Sig. 21 Deg. 31 Min. 43 Sec.

The Time is likewise to be equall'd for the Moon, according to the Admonition in the first Prob. Namely, by applying only the first part of the Equation of Time, which was 7 Min. 0 Sec. A. Wherefore

	D.	H.	M.	Sec.
To the apparent Time of the true Full Moon, *Jan.*	01	09	23	37
Add the Equation of Time for the Moon	00	00	07	00
There appears the Astronomical Time for the Moon, *Jan.*	01	09	30	37

And

And to this Time I gather by Prob. III. out of the Lunar Tables, the Longitude of the Moon from the Sun, 6 Sig. 1 Deg. 51 Min. 36 Sec. The middle Anomaly of the Moon 5 Sig. 15 Deg. 3 Min. 1 Sec. and the Motion of the Moon's Latitude 5 Sig. 25 Deg. 12 Min. 10 Sec. Then the *Prosthaphæresis* of the Moon 1 Deg. 19 Min. 22 Sec. A. which agrees with the Anomaly collected. And then afterwards

	Sig.	Deg.	Min.	Sec.	
To the middle Motion of the Sun from the *Æquinox*	09	20	59	28	
I add the Longitude of the Moon from the Sun	06	01	51	36	
And there appears the mid. Mot. of the Moon from the *Æquin.*	03	22	51	04	
To which I apply the Moon's *Prosthaph.*	00	01	19	22	S.
There then comes the Moon's true Place in its Orb.	03	21	31	42	
And the true Place of the Sun above was	09	21	31	43	
Wherefore the true Distance of the Moon from the Sun	05	29	59	59	

Whence it is manifest, that the time of the true Full Moon was rightly found, which was the thing that was to be prov'd.

PROBLEM VIII.

To find the Latitude of the Moon.

	Sig.	Deg.	Min.	Sec.	
BY the foregoing Problem was found the Moon's middle Motion	05	25	12	10	
And the *Prosthaphæresis* of the Moon	00	01	19	22	S.
Therefore the true Motion of the Moon's Latitude is	05	23	52	48	

And now entring with this Motion of Latitude Tab. XX. which is the Table of the Moon's Latitude, I find the Latitude of the Moon to be 31 Min. 46 Sec. *North.*

And this is the Method of finding the Moon's Latitude in the true *Syzygies*; but out of the *Syzygies* the middle Motion of Latitude is twice to be equall'd, as in the Example of Prob. III. *viz.* in the Year 1587. *August* 18. the middle Motion of Latitude was 9 Sig. 0 Deg. 57 Min. 19 Sec. the absolute *Prosthaph.* was 4 Deg. 40 Min. 12 Sec. to be taken away: Therefore the Motion of Latitude first equall'd was 8 Sig. 26 Deg. 17 Min. 7 Sec.

But now that this same may be equall'd a second time, entring with the true Distance of the Luminaries XIX. which is the Tab. of the Equation of the Nodes of the Moon, I take from thence the Equation of the Nodes, together with the proportional Scruples: And reserving the Minutes for another use, you must add or take the *Prosthaph.* found to or from the Motion of Latitude first equall'd, that there may come out of the Motion equated a second time. As in the Example mentioned in the Year 1587. *Aug.* 18. the almost true Distance of the Luminaries was 9 Sig. 23 Deg. 29 Min. 7 Sec. the exactly true Distance, 9 Sig. 22 Deg. 18 Min. with which, when I go to the Table of the Equation of the Nodes, I meet with a *Prosthaph.* of 1 Deg. 12 Min. 57 Sec. subt. and of proportional Scruples 51. and 34 Seconds.

	Sig.	Deg.	Min.	Sec.
The Motion of Latitude first equated	08	26	17	07
The *Prosthaph.* to be taken away	00	01	12	57
The Mot. of Lat. a second time equated	00	25	04	10

From which you may thus gather the true Latitude of the Moon.

With the Motion of Latitude a ſecond time equall'd you muſt go to the Table of Latitude, as here with 8 ſig. 25 deg. 4 min. the Latitude that anſwers this Motion is this, 4 deg. 57 min. 24 ſec. South; to which is adjoin'd the Exceſs of 18 min. 55 ſec. for which you may ſafely take 19 min. Some part of this Exceſs is always to be added to the Latitude that's found; I mean the Part that anſwers in proportion to the proportional Scruples, which are reſerv'd for this uſe: For thus you muſt ſay, 60 Scruples add 19 min. how much will 51 ½ add? Suppoſe 16 min. 18 ſec. which being added to the Latitude that's found, the Summ will be the true Latitude of the Moon, *viz.* 5 deg. 13 min. 42 ſec. which was what we ſought for.

PROBLEM IX.

To compute the Beginning, Middle, and End of an Eclipſe of the Moon; as alſo the quantity of an Obſcuration.

BY the ſixth Problem was found the Apparent Time of the true Full Moon in *Friſeland*, *viz. Jan.* 1d. 8h. 58 min. 37 ſec. after Mid-day; and by the eighth Problem the Latitude of the Moon at that very moment of time was found to be 31 min. 46 ſec. North. Laſtly, By the ſeventh Problem was gather'd at the ſame time the middle Anomaly of the Sun, 6 ſig. 14 deg. 38 min. 28 ſec. and the middle Anomaly of the Moon 5 ſig. 15 deg. 3 min. 1 ſec. and the *Proſthaphæreſis* of the Moon that agrees with this, 1 deg. 19 min. 22 ſec. S. which if it be applied to the middle Anomaly, the Co-equalled Anomaly of the Moon becomes 5 ſig. 13 deg. 43 min. 39 ſec. And now being thus furniſhed with theſe things, we proceed to the Thing that's to be done. And firſt entering with the Moon's Co-equalled Anomaly of 5 ſig. 13 deg. 43 min. 39 ſec. Table XXIII. which is the Table of the apparent Semi-diametres of the Sun, Moon, and the Earth's Shadow, I take out the Semidiametre of the Full Moon 17 min. 57 ſec. and alſo the Semi-diametre of the Shadow 46 min. 51 ſec. Again, with the middle Anomaly of the Sun, 6 ſig. 14 deg. 38 min. 28 ſec. I take out of the ſame Table the Variation of the Shadow, 56 ſec. which is always taken from the Semi-diametre of the Shadow, that the ſame may remain corrected, *viz.* 45 min. 55 ſec. To this we add the Semi-diametre of the Moon: And the Summ of the Semi-diametres of the Shadow and of the Moon becomes 63 min. 52 ſec. And ſo we may at length ſafely pronounce, whether the Moon will ſuffer any Eclipſe or no. For if at the time of the true Full Moon the Moon's Latitude be leſs than that Summ of the Semi-diametres of the Shadow, and of the Moon; then you may certainly know, that there will be an Eclipſe of the Moon, otherwiſe there will be none. But now in our Example the Moon's Latitude is of 31 min. 46 ſec. North, which is much leſs than the Summ of the Semi-diametres, 63 min. 52 ſec. Therefore the Moon will certainly fall under the Shadow of the Earth.

Look now upon Figure XLVI, as if being plac'd on the Earth, you ſhould look towards the Ecliptick ♈ *u*, and the Orbit of the Moon ♈ *c*; and let ♈ mark the Deſcendent Node of the Moon, *u* the Centre of the Earth's Shadow, *l* the Moon, as far diſtant from the Node ♈ as *u* is, namely, at that we have ſaid, the true Full Moons happens. For you muſt here note three points in the Orbit of the Moon, to wit, C, L, S; and let *c u* and L *r* be perpendicular to the Ecliptick ♈ *u*. Therefore if we ſpeak of the time, at which the Moon is join'd with the Shadow according to her Longitude taken in the Ecliptick; then C is the point, in which the Moon muſt then be. But we ſhall take no notice of that moment of time, becauſe it makes nothing to our preſent purpoſe: For that time of the true Full Moon, which we found by the ſixth Problem, and repeated at the beginning of this, agrees with the Moon, when in L, and when diſtant as much from ♈, as the ſhadow *u*: And then the Latitude of the Moon is L *r*; but the greateſt Obſcuration or Immerſion of the Moon into the Shadow does not then happen, when the Moon is in L, but when it is gone on to the point S, where a perpendicular from *u* does fall upon the Orbit. And then the ſhorteſt Diſtance of the Moon from the ſhadow, that is S *u*, is equal to the Latitude L *r*, which the Moon had in *L*: but the Time of the greateſt Obſcuration follows the Conjunction

at

at so great a distance of time, as is necessary for the Moon in her true Motion to go thorough the Arch L S, which is equal to the Arch *r u*. And both of the Arches are the Difference between ☋ L the *Hypotenusa*, and ☋ *r* the greater Leg of the Rectangular Sphærical Triangle ☋ L *r*. For now in this Triangle ☋ L *r* are given besides the Right Angle at *r*, 1. The Angle ☋, which in every true *Syzygy* is of 4 deg. 58 min. 30 sec. 2. The *Hypotenusa* ☋ L of 6 deg. 7 min. 12 sec. to wit, the Complement of the true Motion of Latitude, which by the eighth Problem was found to be of 5 sig. 23 deg. 52 min. 48 sec. Now there is sought, 1. The Angle ☋ L *r*, to which ☋ *u* S is equal, namely, which the Perpendicular from the Shadow to the Orbit does make with the Ecliptick, and it is of 85 deg. 3 min. 2. The Leg ☋ *r* of 6 deg 5 min. 50 sec. Therefore the difference between ☋ L and ☋ *r* is 1 min. 22 sec. which is the Arch L S. Then afterwards we are to seek in how much time the Moon will go through this Arch L S by its true Motion. Therefore with the Co-equated Anomaly of the Moon 5 sig. 13 deg. 43 min. 39 sec. go to the XXIV Table, where you shall find the Horary Motion of the Moon in it's true *Syzygies* to be 33 min. 16 sec. Wherefore you must say by the Rule of Three: If 33 min. 16 sec. give one hour, or 3600 sec. what will 1 min. 22 sec. give? There appear 2 min. 28 sec. which is the space of Time, in which the greatest Obscuration follows the Conjunction in the Orbit. But the Apparent time of the true Conjunction in the Orbit, under the *Meridian* of *Friseland*, was *Jan.* 1 d. 8 ho. 58 min. 37 sec. and then adding 2 min. 28 sec. there is made up the Time of the greatest Obscuration 1 d. 9 h. 1 min. 5 sec. in *Friseland*. Now the difference of the *Parisian* and *Frisian Meridians* by Tab. XLVII is 16 m. For *Paris* is distant from *Urenburgh* Westward 41 min. and from *Leoward* in *Frieseland* 25. Wherefore *Paris* is 16 min. more Westward: And so there being taken out of the Time just now gather'd 16 min. the Apparent time of the greatest Obscuration is at *Paris*, *Jan.* 11 d. 8 ho. 45 min. 5 sec. after Mid-day, after the *Gregorian* Account.

It remains now, that I define the Beginning and Ending, and also the Quantity of the Obscuration. Now therefore in the XLVII Figure, let *a* be the Centre of the Shadow, *b a c* the Ecliptick; let the Angle *b a e* be of 85 deg. 3 min. equal to the Angle ☋ L *r* or ☋ *u* S in the foregoing Figure; Let *a e* be the distance of the Moon from the Centre of the Shadow at the time of the greatest Obscuration, and let it be suppos'd equal to the Latitude L *r* in the precedent Figure, being 31 min. 46 sec. and let *f e d* the Perpendicular to *a e* represent the Orbit of the Moon: Let *a g* be the Correct Semidiametre of the Shadow of 45 min. 55 sec. *g d* the Semidiametre of the Moon of 17 m. 57 sec. *a d* the Summ of the Semidiametre of the Moon and of the Shadow 63 min. 52 sec. In the plain Triangle *a e d*, there is given the *Hypotenusa a d* of 63 min. 52 sec. to the Right Angle at *e*, and the Leg *a e* of 31 min. 46 sec. The other Leg *d e* is sought by the XLVII Table and is found to be 55 min. 24 sec. which is the Line of the Moon's Course from the Beginning of the Eclipse to the middle of the Obscuration; to which is equal *e f*, the Line that is to be pass'd from the greatest Obscuration to the end. Then you must say by the Rule of Three: If 33 min. 16 sec. give one hour, or 3600 sec. what will 55 min. 24 sec. afford? There comes 1 h. 39 min. 55 sec. which is half the Duration.

	D.	*H.*	*M.*	*Sec.*	
The Appar. Time of the greatest Obscuration at *Paris*, *Jan.*	11	08	45	05	Add and
Half of the Duration	00	01	39	55	Subtract.
The beginning of the Eclipse	11	07	05	10	
The End	11	10	25	00	
The whole Duration	00	03	19	50	

Lastly, for determining the Quantity of the Eclipse,

	Min.	*Sec.*
Take the Latitude of the Moon *a e*	31	46
From the Correct Semidiametre of the Shadow *a h*	45	55
There remains *e h*	14	09
To which add the Moon's Semid. *e i*	17	57
And there appears *i h* that part of the Moon's Diam. immers'd in the Shadow	32	06

Then say by the Rule of Three: The Semidiametre of the Moon *e i* amounts to 6 Digits; how much does *i h* = 32 min. 6 sec. amount to? And there come 10 Ecliptical Digits, and 44 Scruples; which is the quantity of the Eclipse that was enquir'd after.

PROBLEM X.

To give an Account of the Calculation of an Eclipse of the Sun.

THis Calculation is perform'd in two parts: by the former the time of the true Conjunction is sought, just the same way, that was us'd in a Lunar Eclipse; by the latter time of the seeming Conjunction, and also the Beginning and Ending, and likewise the Apparent Quantity of the Eclipse, are sought for, by the help of the *Parallaxes* of Longitude and Latitude. We will take for an Example the Eclipse of the Sun, that happen'd in the year 1656, being Leap-year, *Jan.* the $\frac{16}{26}$, current Day in the Afternoon. The Calculation of it out of *Tycho*'s Tables, with respect to the *Parisian* Horizon runs thus:

	D.	H.	M.	Sec.	
The middle New Moon *Jan.*	16	11	47	55	after Midnight.
	Sig.	Deg.	M.	Sec.	
The Anomaly of ☉	06	29	02	50	
The Anomaly of ☽	11	25	57	56	
The Motion of the Moon's Latitude	00	09	20	33	
The Motion of ☉ from ♈	10	05	23	53	
The *Prosthaph.* of ☉	00	01	01	45	A.
The *Prosthaph.* of ☽	00	00	20	29	A.
The Distance of the Luminaries	00	00	41	06	
The *Prosthaphæretical* Time of hours	—	01	21	00	A.

To which does Answer

	Min.	Sec.
The Anomaly of ☉	03	19
The Anomaly of ☽	44	06

Therefore at the time of almost the true New Moon

	Sig.	Deg.	Min.	Sec.
The Anomaly of ☉ is	06	29	06	09
The Anomaly of ☽ is	11	26	42	02

	Deg.	Min.	Sec.	
The *Prosthaph.* of ☉ 11	01	01	52	A.
The *Prosthaph.* of ☽ 11	00	16	43	A.
The distance of the Lines of middle Motion	00	45	09	
The Horary *Prosthaph.* Time	—	01	29	A.

To which Answers

	Min.	Sec.
The Anomaly of ☉	03	39
The Anomaly of ☽	48	27

Therefore at the time of the true New Moon,

	Sig.	Deg.	Min.	Sec.
The Anomaly of the Sun is	06	29	06	29
The Anomaly of the Moon is	11	26	26	23

The

	Sig.	Deg.	Min.	Sec.	
The *Prosthaphæreses* of the Sun	00	01	01	52	A.
The *Prosthaphæreses* of the Moon	00	00	16	21	A.
The distance of the lines of Middle Motion	00	00	45	31	
The most exact Horary *Prosthaphæretical* Time.		01	29	37	

To which answers

	Min.	Sec.
The Anomaly of the Moon.	48	47
The Moon's Motion of Latitude.	49	24

Therefore at the most exact time of the true New Moon.

	Sig	Deg.	Min.	Sec.	
The Simple Anomaly of the Moon is	11	26	46	43	α
The middle Motion of Latitude is	00	09	20	34	β

The most exact time of the true New Moon is,

	Da.	Ho.	Min.	Sec.	
Jan. current,	16	13	17	32	
	Deg.	Min.	Sec.		
The true place of the Sun in ♒	06	29	14		
The Equation of time	00	09	37		S.
	Da.	Ho.	Min.	Sec.	
The Apparent time of the true N. M. *Jan.*	16	13	07	55	aft. Midnight
or *Jan.*	16	01	07	55	aft. Mid-day.
Or according to the *Gregorian* Account. in *Friseland.* } *Jan.*	26	01	7	55	aft Mid-day.

The Examination of this out of the *Solar* and *Lunar* Tables.

	Da.	Ho.	Min.	Sec.	
The Apparent time of the true New Moon in *Friseland*, is } *Jan.*	16	01	07	55	after Mid-day.
The difference of the Meridians of *Friseland* and *Denmark* }	00	00	25	00	A.
The Apparent time of the true New Moon in *Denmark*, is } *Jan.*	16	01	32	55	

	Min.	Sec.	
The first part of the Equation of time.	09	37	A.
The latter part.	03	56	A.
The absolute Equation of time.	13	33	A.

Min. Sec.
There being then cast away 07 53 according to what's said in Problem I.
There rests the Equation of time for the Sun. } 05 40 A.

	Da.	Ho.	Min.	Sec.	
Therefore the equable time, of the Sun is *Jan.*	16	01	38	35	after Mid-day.
or the day being compleat, *January*	15	01	38	35	after Mid-day.

	Min.	Sec.
Then the Equation of time for the Moon is	09	37 A.

	Da.	Ho.	Min.	Sec.	
Wherefore the equable time for the Moon, is *Jan.*	16	01	42	32	after Mid-day.
or the day being compleat, *January*	15	01	42	32	after Mid-day.

The

	Sig.	Deg.	Min.	Sec.	
The Motion of the Sun from ♈.	10	05	27	23	
The Anomaly of the Sun	06	29	06	24	
The *Proſthaphæreſes* of the Sun	00	01	01	52	A.
The true place of the Sun	10	06	29	15	
The Longitude of the Moon from the Sun	00	00	45	31	
The Anomaly of the Moon (α)	11	26	46	41	
The Motion of the Moon's Latitude, (β)	00	09	20	33	
The *Proſthaphæreſes* of the Moon	00	00	16	19	A.
The middle diſtance of the Moon from ♈.	10	06	12	54	
The true place of the Moon in the Zodiack	10	06	29	13	
The true place of the Sun	10	06	29	15	
The true diſtance of the Moon from the Sun.	11	29	59	58	

Thus far the Exmination, and the firſt part of the Calculation.

The other part of the Calculation belongs to the time of the Apparent Conjunction, and alſo to the beginning and ending, and the Apparent quantity of the Eclipſe, as it is ſeen at *Paris* from off the Surface of the Earth; for the true Conjunction, of which we have hitherto ſpoken, has reſpect to the Centre of the Earth. Now the Apparent time of the true New Moon, under the *Friſian* Meridian was, N. St.

	Da.	Ho.	Min.	Sec.	
Jan.	26	01	07	55	aft. Mid-day.
The difference of the *Friſian* and *Parſian* Meridian,	00	00	16	00	S.
The Apparent time of the true New Moon at *Paris*. *Jan.*	26	00	51	55	aft. Mid-day.

At which time you muſt find the Parallax of the Moon above the Sun as to Altitude and Longitude. In the XLVIII Figure, let A be the Zenith, A E D the *Pariſian* Meridian, ♈ E F the Æquinoctial, ♈ D B K the Ecliptick, B the place of the true Conjunction of the Sun and Moon, as it would be ſeen from the Centre of the Earth, if the Eye was plac'd there; A F B I is the Vertical, paſſing down through the Sun and Moon, B I is the Parallax of the Moon above the Sun as to Altitude, B K the Parallax of Longitude, I K the Parallax of Latitude. In the little Triangle B I K, we muſt find out B I and B K; ſeeing therefore the Apparent time of the true Conjunction happens 51 Min. 55 Sec. after Noonday; this time being turn'd into degrees and Minutes, gives 12 Deg. 98 Min. which is the Arch E F. with the true place of the Sun, of ♒ 6 Deg. 29 Min. 15 Sec. Out of the XLVI Table is taken the Right Aſcenſion of the Sun 308.89, which is the Point F, to which if you add the Arch E F, you will then find the Point of the *Æquator* that is plac'd under the Meridian 321.87, which is the Point E. To this point of the *Æquator* E does anſwer out of the ſame XLVI Table the Culminant point of the Ecliptick D, *viz.* 19 Deg. 26 Min. of ♒. The true place of the Moon, (the point B,) is 6 Deg. 29 Min. of ♒; therefore the Arch of the Ecliptick between the Meridian and the true place of the Moon, (*viz.* B D) is 12 Deg. 57 Min. To the point of the Ecliptick D, that is plac'd under the Meridian, does anſwer, according to the XLV Table, the Declination 15.03, which is the Arch D E; and there the Angle of the Ecliptick and of the Meridian ♈ D E = 71.72, whoſe complement 108.28 is the Angle A D B. Laſtly, to the Arch D E = 15.03 add the Latitude of *Paris* E A = 48.50, and there comes the Arch D A = 63.53.

There being therefore given in the Triangle A D B, the two ſides A D = 63.53, and D B = 12.95, with the Angle contain'd between them, A D B = 108.28; there is found A B, the diſtance of the Moon from the Zenith, to be 68.21, and the Angle A B D 66.28. The Complement of the Arch A B is the Altitude of the Moon 21.79. To the Anomaly of the Moon (α) 11 Sig. 27 Deg. does anſwer out of the XI Table, the Elongation of the Moon from the Centre 102.897. with this and the Latitude of the Moon is taken out of the XXI Table, the Parallax of the Altitude of the Moon 56 Min. and with the ſame Altitude of the Sun the Parallax

rallax of the Sun of 3 Min. Therefore the Parallax of the Moon above the Sun in 53 Min. which is B I the *Hypotenusa* of the little Triangle B I K. And in the same Triangle B I K, which is rectangular at B, there is given the Angle I B K, equal to the Angle A B D. And the Ttiangle B I K being taken (for its littleneſs) as plane, there is found B K the Parallax of Longitude = 21 Min. 18 Sec.

This the Anomaly of the Moon 11 Sig. 27 Deg. is taken out of the XXIV Table, the hourly Motion of the Moon above the Sun, 27 Deg. 44 Min. Therefore by the Golden Rule,

If the true Motion of the ☽ above ☉	give minntes of an hour.	What will the Parallax of Longit. give.	There come Min. of an hour.
27 $\frac{3}{4}$	60	21 $\frac{3}{10}$	45 $\frac{7}{10}$

Therefore the ſeeming Conjunction would follow the true 45 $\frac{7}{10}$ minutes of an hour, if the Apparent Motion of the Moon was equal to the true Motion; but becauſe that does not appear, let the Parallax of Longitude be ſought again, at the time which follows the true Conjunction 45 $\frac{7}{10}$ Minutes.

The point of the *Æquator* that is plac'd under the Meridian at the time of the true Conjunction, is 321.87.

The Intermediate time being turn'd into degrees, is 11.42. Add this to the former point of the *Æquator* that is plac'd under the Meridian at the time of the ſeeming Conjunction, 333.29, which is the point E, to which does anſwer the Culminant point of the Ecliptick ♓, 1 Deg. 15 Min. which is the point D.

	Deg. Min. Sec.
The place of ☽ at the time of the true Conjunction.	♒ 06 29 00
The Motion of ☽ in the intermediate time	00 21 18
The place of the Moon at the time of the ſeeming Conjunction which is the Point B.	♒ 06 50 18
	Sig. Deg. Min.
From the Culminant point of the Ecliptick D.	11 01 15
Take the place of the Moon B.	10 06 50
There remains the Arch B D	00 24 25

To the Culminant point of the Ecliptick does anſwer the Declination 11.06, which is the Arch D E, and the Angle of the Ecliptick and Meridian.

♈ D E.	69 14
whoſe complement to the Semicircles A D B	110 86
To the Arch D E	11 06
Add the Latitude of *Paris* E A	48 50
And there is made the Arch D A	59 56

In the Triangle A D B, there being given A D, D B, A D B, there is found A B the diſtance of the Moon from the *Zenith* 70° ½, and the Angle A B D 58° ⅝.

The compleat of the Arch A B is the Altitude of the Moon	19° ½
With which, and the Elongation of the Moon is taken,	
The Parallax of the Moon,	56 ½.
The Parallax of the Sun,	3
The Parallax of the Moon above the Sun	53 ½
which is the Arch B I.	

In the Triangle B I K, which is rectangular at K, there being given B I, and the Angle I B K equal to A D B = 58 ⅝, there is found the Parallax of Longitude B K = 27′ ⅜. θ.

	Min. Sec.
The firſt Parallax of Longitude, (ν)	21 18
The latter Parallax of Longitude, (θ)	27 40
The difference of the Parallax,	06 22

Is

	Min. Sec.
Is to be taken from the first Parallax, and there remains the Apparent Motion of the Moon in the space of the Intermediate time of 45 Min. 42 Sec. ⎓	14 56

The Apparent Motion of the Moon,	gives	Minutes of an hour.	The first Paral. of Longitude,	gives	Minutes of an hour.
14′ 56″		45′ $\frac{7}{10}$	21′ $\frac{3}{10}$		65′ $\frac{3}{10}$

Therefore the seeming Conjunction follows the true Min. 65 $\frac{3}{10}$ 1.

Let there be sought therefore the Parallax of Longitude and Latitude at the time, which follows the true Conjunction, Min. 65 $\frac{3}{10}$.

	Deg.	Min.
The point of the *Æquator* plac'd under the Meridian at the time of the true Conjunction,	321	87
To which add the Intermediate time turn'd into degrees,	15	35
The summ of this Addition shews the point of the *Æquator* that is plac'd under the Meridian at the time of the seeming Conjunction, which is the point E, ♌	337	22
To which answers out of the XLVI Table, the point of the Ecliptick plac'd under the Meridian,	♓ 5	22
The true Motion of the Moon in the time that passes between the true and seeming Conjunction, is 30 Min.		
The place of ☾ at the time of the true Conjun. was	♒ 6	29
Therefore adding these 30 Min. there is made up the true place of the Sun at the time of the seeming Coniunction, which is the point B,	♒ 6	59
The Arch of the Ecliptick between the Meridian, and the true place of the Sun is D B,	28	23
To the Culminant point of the Ecliptick D, does answer the Declination D E 9.56, and the Angle of the Ecliptick, and Meridian ♈ D E 68.44, whose compl. to the Semicircle is the Angle A D B,	111	56
To the Declination of the Culminant point, that is, to the Arch D E,	9	56
Add the Latitude of *Paris*, E A,	48	50
And there is made the Arch A D	58	06
There being therefore given in the Triangle A D B, the two sides A D, and D B, with the Angle between them A D B, there is found A B the distance of the Moon from the *Zenith*,	71	51
And the Angle A B D	56	30
The Compl. of the Arch A B is the Altitude of the Moon,	18	49

With this and the Elongation of the Moon there is taken out,

The Parallax of the Moon	57′
The Parallax of the Sun	3
The Parallax of the Moon above the Sun.	54 BI

In the Triangle B I K which is Rectangular at K, there is given B I, and I B K, the Angle equal to the Angle A D B; and there is found	Min.	Sec.	
the Parallax of Longitude B K.	30	00	K
And the Parallax of Latitude I K.	44	56	

The Examination of the seeming Conjunction.

At the time of the seeming Conjunction, the true distance of the Moon from the Sun ought to be equal to the Parallax of the Longitude of the Moon above the Sun, that is, to the Line B K which was lastly found. Let therefore the true distance of the Moon from the Sun be sought by the Rule of three, thus:

Minutes

Minutes of an hour	gives	The true Motion of the Moon.	Minutes between the true and seeming ☌	give	The dist. of ☽ from ☉ at the time of the seeming ☌
60′		27½	65′ ⅓		30′. 11″. μ

This distance of 30′ 11″ agrees with the Parallax of Longitude B K, which we had found above to be 30′. Therefore the difference of time, 65′ ⅓ between the true Conjunction and the seeming is right.

To this time that passes between the true and seeming Conjunction, *viz.* 65′ ⅓ does answer

	M.	S.	
The Anomaly of the Moon	35	33	
The Motion of the Moon's Latitude	36	00	

Therefore at the time of the seeming Conjunction

	S.	D.	M.	S.	
The simple Anomaly of the Moon is	11	27	22	16	
The middle Motion of Latitude	60	09	56	34	
The *Prosthaph.* of the Moon	00	00	13	19	A.
The true Motion of the Moon's Latitude	00	10	09	53	

To which does Answer

	M.	S.	
The true Latitude of the Moon	52	36	North.
The Parallax of Latitude I K	44	56	
The seeming Latitude of the Moon	07	40	North *a d*
The Semidiametre of the Sun	15	55	*a e*
The Semidiametre of the New Moon	12	48	*d f*
The Summ of the Semidiametres	28	43	*a g*
The cover'd part of the Sun's Semidiametre	21	03	*i e*
Which amounts to 07 55′ Eccliptical Dig.			

In the Triangle *a d g* which is as good as Rect-angular at *d*, there being given *a d* and *a g*, there is found *g d*, the Line of the Moon's Course, from the beginning to the middle of the Eclipse, = 27 min. 40 sec. Let it now be enquir'd, in how long time the Moon goes through this Line, in her Apparent Motion. Fig. XLIX

	M.	S.
The Paral. of Long. at the time of the seem. Conj. (κ.)	30	00
The Paral. of Long. at the time of the true Conj. (ι.)	21	18
The difference of the Parallaxes is	08	42 A.
The time between the true and seeming Conjunction. (ι)	65′ ⅓.	

To which Answers

The true Motion of the Moon (μ)	30	12
Take hence the Differences of the Parallaxes (λ)	08	42
There rests the Apparent Motion of the Moon for 65′ ⅓	21	30

The Appar. Mot. of the Moon	gives	Minutes of an hour.	The Line from the beginning to the middle of the Eclipse	gives	The time of Incidence.
21′½		65′ ⅓	27⅔		84′

Lastly, let the Parallax of Longitude be sought, which is 1 ho. 20 min. after the seeming Conjunction.

The point of the *Æquator*, plac'd under the Meridian at the time of the seeming Conjunction, (ξ)	337	22
The Intermediate time 1 ho. 2 min. being turn'd into deg. makes	20	add
The point of the *Æquator*, plac'd under the Meridian 80′ after the seeming Conjunction	357	22

To which Answers

	D.	M.	
The point of the Eclipt. that is plac'd under the Meridian	♓ 27	00	D.
The true place of the Moon	♒ 7	36	B.

	D.	M.	
The Arch of the Ecliptick between the Meridian and the true place of the Moon,	49	24	B D
To the point of the Ecliptick D, which is plac'd under the Meridian does answer the Inclin.	1	20	D E
And the Angle of the Ecliptick and of the Meridian,	66	51	ϒ D E
Whose Complement is	113	49	A D B
The Latitude of *Paris*,	48	50	A E
A E + D E,	49	70	
In the Triangle A D B, there being given A D, D B, A D B, there is found the distance of the Moon from the *Zenith*,	79	03	A B
And the Angle A B D,	45	43	
The compl. of the Arch A B is the Altitude of ☽,	10	97	

With this and the Elongation of the Moon there, is taken out

	Min.	
The Parallax of the Moon,	58	
The Parallax of the Sun,	3	
The Parallax of the Moon above the Sun,	55	B I

In the Triangle B I K, which is rectangular at K, there is given B I and I B K, the Angle equal to A D B; and there is found the Parallax of Longitude 38 $\frac{3}{5}$, B K ϖ: Wherefore proceed according to the Rule of Three just now deliver'd.

	Min.	*Sec.*	
The Parallax of Long. at the time of the seeming Conj. (ϰ)	30	00	
The Parallax of Long. of 80′ after the seeming Conj. (ϖ)	38	36	
The difference of the Parallaxes is	8	36	Subtract
The true Motion of the Moon in the space of 80′. is	36	59	Subtract
The Apparent Motion of the Moon in the space of 80′. is	28	23	

The Appa. Mot. of the Moon	gives	Minutes of an hour.	The line from the middle to the end of the Eclipses	gives	The Time of Emersion.
28′ 23″		80′	27 $\frac{3}{5}$		78′

	D.	*H.*	*M.*	*S.*	
The Apparent time of the true Conj. at *Paris*, *Jan.*	26	0	51	55	after Mid-day.
The time between the true and seeming Conjunction,		1	05	18	
The Apparent time of the seeming Conjunction,	26	1	57	13	after Mid-day.
The time of Insidence,		1	24		Subtract
The beginning of the Eclipse, *Jan.*	26	0	33	13	after Mid-day.
The time of Emersion,		1	18		
The end of the Eclipse,	26	3	15	13	after Mid-day.
The whole Duration in hours,		2	42		

It now remains, that I furnish you with certain Rules, how to find the Apparent Motion of the Moon from the Sun; by the Guidance of which Rules I have drawn up the fore-going Calculation, and by which we must regulate the same in other Eclipses afterwards. First of all then we must determine that Interval of Time, at which we have a mind to find the Apparent Motion of the Moon. That Interval either comes between the Moment of the true Conjunction, and that Instant, at which the Moon in it's true Motion is so far distant from the Sun, as the Parallax of Longitude amounts to, at the time of the true Conjunction; or it is the time between the seeming Conjunction and the Moment, that goes before or follows it, some Minutes. In the first Interval the point of Time, from which we begin to compute either backward or forward, is the Moment of the true Conjunction; in the other the Head or beginning of the Calculation proceeds from the seeming Conjunction

junction. For the Apparent Motion of the Moon therefore in the first Interval, let this be the first Rule: At both the Bounds of the Interval that is given, that is, at the time of the true Conjunction, and also at that point of time, that the Moon in its true Motion is distant from the Sun, as much as the Parallax of Longitude at the time of the true Conjunction amounts to, let there be in readiness the Parallax of Longitude, and let the Difference of these Parallaxes be taken. Then if the Parallax of Longitude at the time of the true Conjunction shall be greater than the Parallax that belongs to the other Bounds of the Interval, you must add the difference of Parallaxes to the Parallax that answers the very moment of the true Conjunction; and you shall have the apparent Motion of the Moon for the said Interval: But if the Parallax that belongs to the true Conjunction be less than the Parallax of the other Bound, then you must take away the difference of the Parallaxes from the Parallax that answers the true Conjunction, and again there will appear the Apparent Motion of the Moon for the said Interval. If you demand the reason of this way of working it; consider, that if the Parallaxes of both the Bounds of the given Interval were equal, then the Apparent Motion of the Moon would be equal to the true Motion: Wherefore the Apparent Motion differs as much from the true, as the difference of the Parallaxes comes to. Farthermore we must know, that all Parallaxes bring the Star nearer the Horizon, than it would be seen to be from the Centre of the Earth: Wherefore in the Eastern Quadrant of the *Zodiack*, the greater Parallaxes do the more help the proper Motion of the Moon towards the East, as the lesser Parallaxes do promote it the less. And seeing in the same Eastern Quadrant the true Conjunction does always follow the seeming one, and for the same reason also the other Bound of our Interval, which constantly tends from the true Conjunction to the seeming one; the following Bound of the true Conjunction, if it be help'd by a less Parallax than the foregoing is help'd by, must needs doe the less towards promoting the Moon towards the East, and so the Apparent Motion becomes less than the true; whereby it comes, that the difference of the Parallaxes is to be taken from the true Motion, that the Apparent Motion may be made out. The contrary happens, when the latter Bound (that is, of the true Conjunction) is help'd by a greater Parallax than the former is help'd, by which it comes, that the Moon is in its proper Motion thrust more and more Eastward. But in the Western Quadrant of the *Zodiack* the greater Parallaxes do move the Moon more towards the West, and so do retard its Motion Eastward. And since in the Western Quadrant the true Conjunction always goes before the seeming one, and so consequently before the other Bound of our Interval also, which always tends from the true to the seeming Conjunction; therefore if the foregoing Bound of the true Conjunction be retarded by the lesser Parallax, but the other following Bound by the greater, that Retardation must needs be always greater and greater; and so the Moon's Motion Eastward must be more and more hindred: Whereby the Apparent Motion does again become less than the true; and so it comes, that the difference of the Parallaxes is to be taken away from the true Motion. But if the Parallax that answers the true Conjunction in the Western Quadrant be greater than the Parallax of the following Bound; then the Retardation of the Moon, which is greater at the Beginning of the Motion, grows less and less afterwards; and so it comes to pass, that the difference of the Parallaxes is to be added to the true Motion. Now the Parallax of the Moon above the Sun in Longitude, at the time of the true Conjunction, is the same with the true Motion of the Moon from the Sun in the said Interval. And hence it is, that for shortness sake the Rule puts that Parallax for this true Motion.

Now as for the Interval taken on this side, or beyond the seeming Conjunction, the latter Rule for finding the Moon's Apparent Motion is This: Let the Moon's true Motion from the Sun at the Interval given be first of all in readiness; then at the beginning and end of that Interval let there be sought the Parallax of the Moon above the Sun in Longitude, and let the difference of these Parallaxes be taken. Afterwards consider, whether the whole given Time do pass, while the Sun is in the Eastern Quadrant of the Ecliptick, or all the while it is in the Western, or whether it be divided between both. If the whole time be spent in the Eastern Quadrant, and the Parallax at the beginning of the time be greater, than at the end, then take away the difference of the Parallaxes from the Moon's true Motion from the Sun; but if it be less, then add. But now if in all the given time the Sun be in

the Western Quadrant, and the Parallax at the beginning of this time be greater than at the end, then add the differences of them to the Moon's true Motion from the Sun; but if it be less, then take it away. Lastly, if the Time given be divided etween both Quadrants, so as that the first part is spent in the Eastern, the latter in the Western Quadrant of the Ecliptick; then you must take the Summ of both Parallaxes from the Moon's true Motion from the Sun: So there comes the Moon's Apparent Motion from the Sun for the given Interval. You may easily gather the Reason of this latter Rule from what was just now said for the Explanation of the Former.

PROBLEM XI.

To compute the true Place of the Sun, and its Distance from the Earth by the Rudolphine *Tables.*

FOr the year 1582. *Nov.* 23. Current d. 16. ho. 0 min. from Noon-day, in Apparent time, let the true place of the Sun be demanded with respect to *Urenburgh*; that is, *Nov.* 24 Current d. of usual Time, 4 ho. in the Morning. Therefore by the first Precept of the first Problem, when the given time falls not in Leap-year after the end of *February*, and the Hours are in the Fore-noon, let two be taken from the given Number of days, and let Twelve be added to the Fore-noon Hours; so there is made *Nov.* the 22 complete d. 16 ho. and seeing the true place of the Sun is in 12 deg. ♐, there is taken out of the first part of the Table of Equation of time, the first part of the Equation of time, 6 min. (omitting the Seconds) with this Title, Subtract. Where the time first Equated becomes *Nov.* 22. d. 15 ho. 54 min. Again, by the same Precept of the first Problem, the Current year of our Lord being given, *viz.* 1582, let one be taken away, and there remains the complete year of our Lord, 1581, to which is always added the number 4713, and there is made the complete year of the *Julian* Period, 6294, and above *October* complete d. 15 ho. 54 min. which is the Astronomical time fitted for our going to the *Rudolphine* Tables. For the 7 min. 53 sec. which are mention'd in the Second precept of the first Problem, do not belong to the *Rudolphine*, but only to the Solar Tables of *Tycho*; though yet the latter part of the Equation of time is by all means to be fitted to this, as well as in *Tycho*'s Solar Tables; which one thing I thought good to hint in the second Precept of Problem I. before mention'd, lest you should perchance mistake the sense of those words. But we shall then be able to find the latter part of the Equation of Time, when we have found the Sun's *Apogee*, and from that its middle Anomaly. But now omitting the *Epocha* of 6000 years, for the remaining time that has been laid open, we take out of Table XXVI. the Sun's middle Motion from the *Æquinox*, as follows:

	The mid. Mot. of ☉ from ♈.	
Years 200	0041981	
80	0016793	
14	0016628	
Octob. complete	8323241	
d. complete 22	0602340	
h. 15	0017112	* Note, that in the Table
m. 54	0001027*	of Minutes one Cypher is still
	9019122	to be prefix'd.
The Epocha of 6000	7994972	Table XXV.
Mid. Mot. ☉ from ♈	1.7014094	

There

There have past since the *Epocha* Tropical Years	294.9019	
The Logarithm of which Number is	2.4696776	α
The Log. of the mid. Mot. of the Apog. of ☉ in Trop. Y.	2.2334617	Table XXV.
The Log. of the mid. Apog. of ☉ in the time laid open is	0.7031393	

	Sig.	*Deg.*	
Whose absolute Number is	0	5.0482	
The *Epocha* of the mid. Mot. of the Apog. of ☉, 6000	3	0.3775	Table XXV.
The Place of the Apogée of the Sun	3	5.4257	β

	part. dec. circ.	*Deg. with dec. part.*	
The middle Motion of the Sun from *Aries*	70	252.0	Table XXVII.
	14	0.504	
	09	0324	
	40	1440	
		252.5073840	

That is	8	12.5073	Subt.
The Place of the Sun's Apogée (β)	3	5.4257	
The middle Anomaly of the Sun	5	7.0816	

With which is taken out of the latter part of the Table of Equation of Time, the latter part of the Equation of Time 3 Min. (omitting the Seconds) with the Title, subtract. Wherefore the true Astronomical Time is *Nov.* 22d. 15 h. 51 Min. And now to those 3 Min. does answer out of Tab. XXVI. the Particle of the Sun's Motion 0000057, which is to be subtracted from the middle Motion of the Sun before found for the time laid open above the entire Revolutions, to wit, from 9019122; and there remains the same middle Motion of the Sun more exact 9019065. γ

And the same Particle of the Sun's Motion, *viz.* 0000057 being multiplied by 36, the two last Figures being cast away, it affords 0.0021 decimal parts of a degree, which are also to be drawn from the middle Motion of the Sun from *Aries*, above turn'd, to wit, from 8 Sig. 12 Deg. 5073, and the same will remain more exact, *viz* of 8 Sig. 12 Deg. 5052. δ

With the middle Anomaly of the Sun, 5 Sig. 7 Deg. 0816 is taken out of Table XXVIII. the *Prosthaph.* of the Sun 0.8195 S. which if it be taken from the middle Motion 8 Sig. 12 Deg. 5052, there remains the true Place of the Sun 8 Sig. 11 Deg. 6857. Now to the same middle Anomaly 5 Sig. 7 Deg. 0816 does answer out of the same Table XXVIII. the Logarithm of the Distance of the Sun from the Earth, *viz.* 399277. ε

PROBLEM XII.

To find the Heliocentrick Place of the five Planets in the Ecliptick, and also the curtail'd Distance of the same from the Sun.

THE middle Motion of the Sun for the time laid open, above the entire Revolutions, and which was mark'd in the foregoing Problem by the Letter γ, was 9019065.

To

To which does answer the Log.	1.9551615	
The Log. of the mid. Mot. of ♂ in a Tropical Year	1.7256683	Table XXXV.
The Log. of the mid. Mot. of ♂ for the time laid open above the entire Revolutions	1.6808298	

To which answers the absolute Number		4795455	Table XXXV.
The Tropical Years	4	1268075	
	90	8531690	
	200	340376	
The *Epocha*	6000	426188	
The middle Motion of *Mars* from *Aries*		2.2260860	

The Log. of the Tropical Years from the *Epocha* (α)	2.4696776	
The Log. of the Mot. of the Aphel. of ♂ in a Trop. Y.	2.2694130	Table XXXV.
The Log. of the Mot. of the Aphel. of ♂ in time expan.	0.7390906	

	Sig.	*Deg.*	
Whose absolute Number is	0	5.4839	
The *Epocha* of the Aphel. of *Mars* 6000	4	23.1778	Table XXXV.
The Place of the Aphelion of *Mars*	4	28.6617	♈

Decimal Parts of a Circle.	*Deg. with Decimal Parts.*	
The middle Motion of *Mars* from *Aries* 22	079.2	Table XXVII.
60	2.160	
86	3096	
	81.39096	

	Sig.	*Deg.*	
That is	2	21.39096	Subt.
The Place of the Aphelion of *Mars* (♈)	4	28.6617	
The middle Anomaly of *Mars*	9	22.7292	

With which is taken out of Table XXXVI. the *Prosthaphæresis* of *Mars* 9.3308 A. which being therefore added to the middle Motion of *Mars* from *Aries*, to wit, 2 Sig. 21 Deg. 3909, it makes the Heliocentrick Place of *Mars* in the Orbit 3 Sig. 7217. ν

Again: The Log. of a Trop. Y. from the *Epocha* (α) is	2.4696776	
The Log. of the Mot. ☊ ♂ from ♈	2.0429783	Table XXXV.
The Log. of the Mot. ☊ ♂ in time laid open	0.5126559	

	Sig.	*Deg.*
Whose absolute Number is	0	3.2558
The *Epocha* ☊ ♂ 6000	1	13.2867
The Place ☊ ♂	1	16.5425

	Sig.	*Deg.*
From the Heliocentrick Place of ♂ in the Orbit (ν)	3	0.7217
Take the Place of ☊ ♂	1	16.5425
There remains the Argument of the Latitude of *Mars*	1	14.1792

With which, out of Table XXXVII. is taken the Inclination of *Mars* 1.2838, and the Reduction 0.0147 S.

There-

	Sig.	Deg.	
Therefore from the Helioc. Place ♂ in the Orbit	3	0.7217	
Take the Reduction	0	0.0147	S.
And there remains the Helioc. Place of ♂ in the Eclip.	3	0.7070	☿

Again With the middle Anomaly of *Mars* Sig. 9 Deg. 22. 7292. out of Table XXXVI. is taken the Distance of *Mars* from the Sun 420100; and of the Inclination of *Mars* 1.2838 the Sign of Complement is 999989, whose Arithmetical Complement is 11, which is the Curtation of the Distance of *Mars*, to be taken from the former Distance of 420100, that there may remain the curtail'd Distance of *Mars* from the Sun 420089.

PROBLEM XIII.

To determine the Geocentrick place of the Planets in the Zodiack according to Longitude and Latitude.

IN the Triangle that lies between the Sun, the Earth, and the Planet, are given the two sides, that are extended from the Sun to the Earth and the Planet, with the External Angle to the Sun; to wit, by taking away the true place of the Sun from the Heliocentrick place of the Planet in the Ecliptick; or this from that, that the Remainder may be less than the Semicircle. And here is sought the Angle to the Earth, which, in the upper Planets, is the greatest of the unknown Angles, because opposite to an upper Planet's distance from the Sun, that is greater than that of the Earth; but in the lower Planets it is less, because opposite to a less distance of a lower Planet. In our Example therefore,

	Sig.	Deg.		
The true place of the Sun found by Prob. XI. is	8	11.6857	}	Subtract
The Heliocentrick place of ♂, found by Prob. XII. (☿)	3	0.7070	}	
The External Angle to the Sun,	5	10.9787		
The half of which, is	2	20.4893		
The Curtail'd distance of ♂ from ☉ by Prob. XII. (ι)		420089	}	Subtract
The distance of the Earth from the Sun, Prob. XI. (ε)		399277	}	
		1020812		58.2314
				45
		937130		13.2314
		1077589		80.4893
		1014719		54.5283
The Angle to the Earth,				135.0176

	Sig.	Deg.
That is	4	15.0176
The true place of the Sun,	8	11.6857
The Geocentrick place of ♂.	3	26.6681

And here you must take notice of this Rule: When the Heliocentrick place of the Planet in the Ecliptick, has been taken away from the true place of the Sun, as in this example; then the Angle from the Earth is likewise taken from the Sun's true place: But if the true place of the Sun be taken from the Heliocentrick place of the Planet; then the Angle to the Earth is added to the Sun's true place, that the Geocentrick place of the Planet may appear.

For

For the finding the Latitude of a Planet.

The Rule was given Section the II. of this Book, Chap. XXIII.

As the Sign of the Angle to ☉	(160.9787)	951311
is to the Sign of the Angle to the Earth,	(135.0176)	984935
So is the Arch of Inclination	(1.2838)	010850
to the Arch of Latitude.	(2.7844)	044474

The Latitude of *Mars* is found to be 2.7844; which, becauſe the Argument of Latitude by the foregoing Problem, was 1 Sig. 14 Deg. 1792, is therefore Aſcending Northward, while the Planets tend from ☊ to the Northern limit; thence to ☋ it is North deſcending; from ☋ to the Southern Limit, it is South, deſcending; laſtly, from the Southern Limit to ☊ it is South aſcending.

PROBLEM XIV.

To find the Longitude and Latitude of the fix'd Stars at any time that is given.

IN the beginning of the Year 1676. let the Longitude of the firſt Star in *Aries* be ſought: Taking one from the year given, there remains the compleat year of our Lord 1675, and adding 4713, there is made the compleat year of the *Julian* Period, 6388.

The Logarithm of 388 is	2.58883	
The Logarithm of the middle Motion of the fix'd Stars in a Tropical year.	2.15126	Tab. XXV.
	0.74009	

	Sig. Deg.	
Whoſe abſolute number —— ——	0 5.4966	
The Epocha 6000. —— ——	0 23.1825	Tab. XXV.
The Longitude of the firſt Star of ♈, ——	0 28.6791	
The Longitude of *Sirius* from the firſt Star of ♈,	2 10.975	Tab. XLIV.
The Longitude of *Sirius* from the *Æquinox*,	3 9.6541	
The Latitude of *Sirius*, —— ——	0 39.500	Tab. XLIV.

Now the Latitudes of the fix'd Stars, are by moſt Learned Aſtronomers thought to be unchangeable, and therefore they continue always the ſame, as they are taken out of the XLIV Table; but if the Longitude of any Star be demanded at any other time after the beginning of the year, you muſt add for every Month, 0.0012, and for every week 0.0003 parts (*i e*, 10000 parts) of one degree.

And now having ſhown the uſe of *Tycho*'s, as well as of the *Rudolphine* Tables, both by Precepts and Examples, I ſhall hear to this IV. Section, and ſo to my Aſtronomical Inſtitutions put

A N E N D.

TYCHO'S
SOLAR and LUNAR
TABLES.

TABLE I.

Tycho's *Solar* Epochæ, *according to the Meridian of* Urenburgh, *at Mid-day in the Calends of* January.

Years of Christ.	*Motion of the Sun.* Si.	De.	Mi.	Se.	*Anomaly of the Sun.* Si.	De.	Mi.	Se.	*Place of the Apogee.* Si.	De.	Mi.	Se.
1401	9	19	22	47	6	16	13	02	3	03	09	45
1501	9	20	09	00	6	15	44	15	3	04	24	45
1601	9	20	55	12	6	15	15	27	3	05	39	45
1701	9	21	41	24	6	14	46	39	3	06	54	45
1801	9	22	27	36	6	14	17	51	3	08	09	45

TABLE II.

A Table of twenty expanded Years.

Years	*Motion of the Sun.* Si.	De.	Mi.	Se.	Th.	*Anomaly of the Sun.* Si.	De.	Mi.	Se.	*Motion of the Apogee.* De.	Mi.	Se.
1	11	29	45	40	38	11	29	44	55	00	00	45
2	11	29	31	21	16	11	29	29	51	00	01	30
3	11	29	17	01	55	11	29	14	47	00	02	15
4	00	00	01	50	53	11	29	58	51	00	03	00
5	11	29	47	31	31	11	29	43	46	00	03	45
6	11	29	33	12	09	11	29	28	42	00	04	30
7	11	29	18	52	47	11	29	13	38	00	05	15
8	00	00	03	41	46	11	29	57	42	00	06	00
9	11	29	49	22	24	11	29	42	37	00	06	45
10	11	29	35	03	02	11	29	27	33	00	07	30
11	11	29	20	43	40	11	29	12	29	00	08	15
12	00	00	05	32	38	11	29	56	33	00	09	00
13	11	29	51	13	17	11	29	41	28	00	09	45
14	11	29	36	53	55	11	29	26	24	00	10	30
15	11	29	22	34	33	11	29	11	20	00	11	15
16	00	00	07	23	31	11	29	55	23	00	12	00
17	11	29	53	04	09	11	29	40	19	00	12	45
18	11	29	38	44	48	11	29	25	15	00	13	30
19	11	29	24	25	26	11	29	10	10	00	14	15
20	00	00	09	14	24	11	29	54	14	00	15	00
40	00	00	18	28	48	11	29	48	29	00	30	00
60	00	00	27	43	12	11	29	42	43	00	45	00
80	00	00	36	57	37	11	29	36	58	01	00	00
100	00	00	46	12	01	11	29	31	12	01	15	00

TABLE III.

A Table of Months in a common Year.

Months.	*Motion of the Sun.* Si.	De.	Mi.	Se.	Th.	*Anomaly of the Sun.* Si.	De.	Mi.	Se.	*Ap. Mot.* Se.	Th.
January	00	00	00	00	00	00	00	00	00	00	00
February	01	00	33	18	15	01	00	33	14	03	49
March	01	28	09	11	30	01	28	09	04	07	16
April	02	28	42	29	45	02	28	42	19	11	05
May	03	28	16	39	40	03	28	16	25	14	47
June	04	28	49	57	54	04	28	49	39	18	36
July	05	28	24	07	49	05	28	23	45	22	18
August	06	28	57	26	04	06	28	57	00	26	08
September	07	29	30	44	19	07	29	30	14	29	57
October	08	29	04	54	14	08	29	04	20	33	40
November	09	29	38	12	28	09	29	37	35	37	29
December	10	29	12	22	23	10	29	11	41	41	11
Yearly Mot.	11	29	45	40	38	11	29	44	55	45	00

TABLE IV.

A Table of Days.

Days.	*Motion of the Sun.* De.	Mi.	Se.	Th.	*Days.*	*Motion of the Sun.* De.	Mi.	Se.	Th.
1	00	59	08	20	17	16	45	21	57
2	01	58	16	40	18	17	44	29	17
3	02	57	24	59	19	18	43	38	37
4	03	56	33	19	20	19	42	46	56
5	04	55	41	39	21	20	41	54	37
6	05	54	41	59	22	21	41	03	16
7	06	53	58	19	23	22	40	11	36
8	07	53	06	39	24	23	30	19	56
9	08	52	14	58	25	24	38	28	16
10	09	51	23	18	26	25	37	36	35
11	10	50	31	38	27	26	36	44	55
12	11	49	39	58	28	27	35	53	15
13	12	48	48	18	29	28	35	01	35
14	13	47	56	37	30	29	34	09	55
15	14	47	04	57	31	30	33	18	15
16	15	46	13	17					

TABLE V.

An Horary Table of the Sun.

Hours and Scru.	Motion of the Sun. Mi.	Sec.	Th.	Hour and Scru.	Motion of the Sun. Mi.	Sec.	Th.	Ho. and Scr.	Motion of the Sun. Mi.	Sec.	Scrup. and Hours.	Motion of the Sun. Min.	Sec.
1	02	27	51	17	41	53	24	32	1	19	48	1	58
2	04	55	42	18	44	21	15	33	1	21	49	2	01
3	07	23	32	19	46	49	06	34	1	24	50	2	03
4	09	51	23	20	49	16	56	35	1	26	51	2	06
5	12	19	14	21	51	44	47	36	1	29	52	2	08
6	14	47	05	22	54	12	38	37	1	31	53	2	10
7	17	14	56	23	56	40	29	38	1	34	54	2	13
8	19	42	46	24	59	08	20	39	1	36	55	2	16
9	22	10	37	Scru.	mi.	sec.	th	40	1	38	56	2	18
10	24	38	28	25	01	01	36	41	1	41	57	2	20
11	27	06	19	26	01	04	04	42	1	43	58	2	23
12	29	34	10	27	01	06	32	43	1	46	59	2	25
13	32	02	01	28	01	09	00	44	1	48	60	2	28
14	34	29	51	29	01	11	27	45	1	51			
15	36	57	42	30	01	13	55	46	1	53		Sec.	Th.
16	39	25	33	31	01	16	23	47	1	56			
	Sec.	Th.	Fo.		Sec.	Th.	Fo.		Sec.	Th.			

TABLE VI.

Lunar Epochæ, *instituted by* Tycho *himself, serving for the four next following Ages.*

Years of Christ.	Long. of the Moon from the Sun. Sig.	Deg.	Min.	Sec.	Anom. of the Moon. Sig.	Deg.	Min.	Se.	Mot. of the Moon's Lat. Sig.	Deg.	Min.	Se.
1401	06	15	09	26	03	23	28	46	09	24	47	13
1501	04	22	13	19	10	11	59	11	00	16	47	17
1601	02	29	17	13	05	00	29	35	03	08	47	04
1701	01	06	21	06	11	19	00	00	06	00	46	50
1801	11	13	25	00	06	07	30	25	08	22	46	36

TABLE VII.

A Table of twenty expanded Years.

Years	Long. of the Moon from the Sun. Sig.	Deg.	Min.	Se.	Th.	Anom. of the Moon. Sig.	Deg.	Min.	Se.	Mot. of the Moon's Lat. Sig.	Deg.	Min.	Sec.
1	04	09	37	22	40	02	28	43	08	04	28	42	45
2	08	19	14	45	19	05	27	26	15	09	27	25	31
3	00	28	52	07	59	08	26	09	23	02	26	08	16
4	05	20	40	57	20	00	07	56	25	08	08	04	47
5	10	00	18	20	00	03	06	39	33	01	06	47	33

The Residue of Table VII.

A Table of twenty expanded Years.

Years	*Long. of the Moon from the Sun.* Sig.	Deg.	Min.	Sec.	Th.	*Anom. of the Moon.* Sig.	Deg	Min.	Se.	*Mot. of the Moon's Lat.* Sig.	Deg.	Min.	Se.
6	02	09	55	42	40	06	05	22	40	06	05	30	18
7	06	19	33	05	19	09	04	05	48	11	04	13	04
8	11	11	21	54	41	00	15	52	50	04	16	09	35
9	03	20	59	17	20	03	14	35	58	09	14	52	20
10	08	00	36	40	00	06	13	19	05	02	13	35	06
11	00	10	14	02	40	09	12	02	13	07	12	17	51
12	05	02	02	52	01	00	23	49	15	00	24	14	22
13	09	11	40	14	41	03	22	32	23	05	22	57	08
14	01	21	17	37	20	06	21	15	30	10	21	39	53
15	06	00	55	00	00	09	19	58	38	03	20	22	39
16	10	22	43	49	21	01	01	45	40	09	02	19	10
17	03	02	21	12	01	04	00	28	48	02	01	01	55
18	07	11	58	34	41	06	29	11	55	06	29	44	41
19	11	21	35	57	21	09	27	55	03	11	28	27	26
20	04	13	24	46	42	01	09	42	05	05	10	23	57
40	08	26	49	33	24	02	19	24	10	10	20	47	54
60	01	10	14	20	06	03	29	06	15	04	01	11	52
80	05	23	39	06	48	05	08	48	20	09	11	35	49
100	10	07	03	53	30	06	18	30	25	02	21	59	46

TABLE VIII.

A Table of Months in a Common Year.

Months.	*Long. from the Sun.* Si.	De.	Mi.	Se.	Th.	*Anom. of the Moon.* Si.	De.	Mi.	Se.	*Mot. of the Moon's Lat.* Si.	De.	Mi.	Sec.
January	00	00	00	00	00	00	00	00	00	00	00	00	00
February	00	17	54	47	27	01	15	00	52	01	20	06	35
March	11	29	15	14	50	01	20	50	02	02	00	31	54
April	00	17	10	02	18	03	05	50	55	03	20	38	29
May	00	22	53	23	04	04	07	47	53	04	27	31	19
June	01	10	48	10	31	05	22	48	45	06	17	37	54
July	01	16	31	31	17	06	24	45	43	07	24	30	44
August	02	04	26	18	45	08	09	46	35	09	14	37	20
September	02	22	21	06	13	09	24	47	27	11	04	43	55
October	02	28	04	26	58	10	26	44	25	00	11	36	45
Novemb.	03	15	59	14	26	00	11	45	17	02	01	43	20
Decemb.	03	21	42	35	12	01	13	42	16	03	08	36	10
Years Mot.	04	09	37	22	40	02	28	43	08	04	28	42	45

TABLE IX.

A Table of Days.

Days.	*Long. of the Moon from the Sun.* Sig.	Deg.	Min.	Sec.	Th.	*Anom. of the Moon.* Sig.	Deg.	Min.	Se.	*Mot. of the Moon's Lat.* Sig.	Deg.	Min.	Se.
1	00	12	11	26	41	00	13	03	54	00	13	13	46
2	00	24	22	53	03	00	26	07	48	00	26	27	31
3	01	06	34	20	05	01	09	11	42	01	09	41	17
4	01	18	45	46	46	01	22	15	36	01	22	55	03
5	02	00	57	13	28	02	05	19	30	02	06	08	48
6	02	13	08	40	09	02	18	23	24	02	10	22	34
7	02	25	20	06	51	03	01	27	18	03	02	36	20
8	03	07	31	33	32	03	14	31	11	03	15	50	05
9	03	19	43	00	14	03	27	35	05	03	29	03	51
10	04	01	54	26	55	04	10	38	59	04	12	17	37
11	04	14	05	53	37	04	23	42	53	04	25	31	22
12	04	26	17	20	18	05	06	46	47	05	08	45	08
13	05	08	28	47	00	05	19	50	41	05	21	58	54
14	05	20	40	13	41	06	02	54	35	06	05	12	39
15	06	02	51	40	23	06	15	58	29	06	18	26	25
16	06	15	03	07	05	06	29	02	23	07	01	40	11
17	06	27	14	33	46	07	12	06	17	07	14	53	56
18	07	09	26	00	28	07	25	10	11	07	28	07	42
19	07	21	37	27	09	08	08	14	05	08	11	21	27
20	08	03	48	53	51	08	21	17	59	08	24	35	13
21	08	16	00	20	32	09	04	21	53	09	07	48	59
22	08	28	11	47	14	09	17	25	47	09	21	02	44
23	09	10	23	13	55	10	00	29	41	10	04	16	30
24	09	22	34	40	37	10	13	33	34	10	17	30	16
25	10	04	46	07	18	10	26	37	28	11	00	44	01
26	10	16	57	34	00	11	09	41	22	11	13	57	47
27	10	29	09	00	41	11	22	45	16	11	27	11	33
28	11	11	20	27	23	00	05	49	10	00	10	25	18
29	11	23	31	53	04	00	18	53	04	00	23	39	04
30	00	05	43	20	46	01	01	56	58	01	06	52	50
31	00	17	54	47	27	01	15	00	52	01	20	06	35

TABLE X.

An Horary Table of the Moon.

Ho. and Scrup.	*Long. of the Moon from the Sun.* Deg.	Min.	Sec.	Th.	*Anom. of the Moon.* Deg.	Min.	Sec.	*Mot. of the Moon's Lat.* Deg.	Min.	Sec.
1	00	30	28	37	00	32	40	00	33	05
2	01	00	57	13	01	05	19	01	06	10
3	01	31	25	50	01	37	59	01	39	14
4	02	01	54	27	02	10	39	02	12	19
5	02	32	23	04	02	43	19	02	45	23

The Residue of Table X.

An Horary Table of the Moon.

Ho. and Scrup.	*Long. of the Moon from the Sun.* Deg.	Min.	Sec.	Th.	*Anom. of the Moon.* De.	Mi.	Se.	*Mot. of the Moon's Lat.* De.	Mi.	Se.
6	03	02	51	40	03	15	58	03	18	27
7	03	33	20	17	03	48	38	03	51	32
8	04	03	48	54	04	21	18	04	24	36
9	04	34	17	30	04	53	58	04	57	41
10	05	04	46	07	05	26	37	05	30	45
11	05	35	14	44	05	59	17	06	03	49
12	06	05	43	21	06	31	57	06	36	54
13	06	36	11	57	07	04	37	07	09	58
14	07	06	40	34	07	37	16	07	43	03
15	07	37	09	11	08	09	56	08	16	07
16	08	07	37	48	08	42	36	08	49	11
17	08	38	06	24	09	15	16	09	22	16
18	09	08	35	01	09	47	55	09	55	20
19	09	39	03	38	10	20	35	10	28	25
20	10	09	32	15	10	53	15	11	01	29
21	10	40	00	51	11	25	55	11	34	33
22	11	10	29	28	11	58	34	12	07	38
23	11	40	58	05	12	31	14	12	40	42
24	12	11	26	41	12	03	54	13	13	48
Scru.	Min.		Sec.	Th.	Mi.		Se.	Mi.		Se.
25	12		41	55	13		37	13		47
26	13		12	24	14		09	14		20
27	13		43	52	14		42	14		53
28	14		13	21	15		15	15		26
29	14		43	50	15		47	15		59
30	15		14	18	16		20	16		32
31	15		44	47	16		53	17		05
32	16		15	16	17		25	17		38
33	16		45	44	17		58	18		11
34	17		16	13	18		31	18		44
35	17		46	41	19		03	19		18
36	18		17	10	19		36	19		51
37	18		47	39	20		08	20		24
38	19		18	07	20		41	20		57
39	19		48	36	21		14	21		30
40	20		19	04	21		46	22		03
41	20		49	33	22		19	22		36
42	21		20	02	22		51	23		09
43	21		50	30	23		24	24		42
44	22		20	59	23		57	24		15
45	22		51	27	24		30	24		48

The Residue of Table X.

An Horary Table of the Moon.

Ho. and Scrup.	*Long. of the Moon from the Sun.* Min.	Sec.	Th.	*Anom. of the Moon.* Mi.	Se.	*Mot. of the Moon's Lat.* Mi.	Se.
46	23	21	56	25	03	25	21
47	23	52	25	25	35	25	54
48	24	22	53	26	08	26	27
49	24	53	22	26	41	27	00
50	25	23	50	27	13	27	34
51	25	54	19	27	46	28	07
52	26	24	48	28	18	28	40
53	26	55	16	28	51	29	13
54	27	25	45	29	24	29	46
55	27	56	14	29	56	30	19
56	28	26	42	30	29	30	52
57	28	57	11	31	01	31	25
58	29	27	39	31	34	31	58
59	29	58	08	32	07	32	31
60	30	28	37	32	40	33	05

TABLE XI.

Prosthaphæreses *or* Equations.

Sign 0

Deg. dist.	*The Sun.* Subtract. De.	Mi.	Se.	*The Moon.* Subtract. De.	Mi.	Se.	*The Elongation of the Moon from the Centre of the Eccentrick.*	Deg. a/c.
0	0	00	00	0	00	00	102900	30
1	0	02	05	0	05	04	102899	29
2	0	04	10	0	10	08	102898	28
3	0	06	14	0	15	12	102896	27
4	0	08	18	0	20	16	102894	26
5	0	10	22	0	25	20	102891	25
6	0	12	26	0	30	23	102888	24
7	0	14	30	0	35	26	102884	23
8	0	16	33	0	40	28	102879	22
9	0	18	36	0	45	29	102873	21
10	0	20	39	0	50	30	102867	20
11	0	22	41	0	55	30	102860	19
12	0	24	43	1	00	28	102853	18
13	0	26	45	1	05	25	102845	17
14	0	28	47	1	10	21	102836	16
15	0	30	48	1	15	16	102826	15
16	0	32	49	1	20	10	102816	14
17	0	34	49	1	25	03	102805	13
18	0	36	48	1	29	55	102793	12
19	0	38	47	1	34	44	102781	11
20	0	40	45	1	39	32	102768	10
21	0	42	43	1	44	18	102755	9
22	0	44	40	1	49	03	102741	8
23	0	46	36	1	53	47	102726	7
24	0	48	31	1	58	29	102711	6
25	0	50	25	2	03	08	102695	5
26	0	52	18	2	07	44	102678	4
27	0	54	11	2	12	18	102660	3
28	0	56	03	2	16	50	102642	2
29	0	57	54	2	21	20	102623	1
30	0	59	44	2	25	47	102604	0
	Add			Add				

Sign 11.

The Residue of Table XI.

Prosthaphæreses or Equations.

Sign 1.

Deg. desc.	The Sun. Subtract. De.	Mi.	Se.	The Moon. Subtract. De.	Mi.	Se.	The Elongation of the Moon from the Centre of the Eccentrick.	Deg. asc.
0	0	59	44	2	25	47	102604	30
1	1	01	33	2	30	12	102584	29
2	1	03	21	2	34	34	102563	28
3	1	05	08	2	38	54	102542	27
4	1	06	54	2	43	11	102520	26
5	1	08	38	2	47	25	102497	25
6	1	10	21	2	51	37	102474	24
7	1	12	03	2	55	46	102450	23
8	1	13	44	2	59	52	102426	22
9	1	15	24	3	03	54	102401	21
10	1	17	03	3	07	53	102375	20
11	1	18	40	3	11	49	102348	19
12	1	20	16	3	15	42	102321	18
13	1	21	51	3	19	31	102293	17
14	1	23	24	3	23	17	102265	16
15	1	24	56	3	26	59	102236	15
16	1	26	27	3	30	38	102206	14
17	1	27	56	3	34	13	102176	13
18	1	29	23	3	37	44	102145	12
19	1	30	49	3	41	12	102114	11
20	1	32	13	3	44	36	102082	10
21	1	33	36	3	47	56	102049	9
22	1	34	58	3	51	12	102016	8
23	1	36	18	3	54	24	101982	7
24	1	37	36	3	57	32	101948	6
25	1	38	52	4	00	36	101913	5
26	1	40	06	4	03	35	101877	4
27	1	41	18	4	06	30	101841	3
28	1	42	29	4	09	21	101804	2
29	1	43	38	4	12	08	101767	1
30	1°	44	46	4	14	51	101729	0
	Add			Add				

Sign 10.

The Residue of Table XI.

Prosthaphæreses *or* Equations.

Sign 2.

Deg. desc.	*The Sun.* *Subtract.* De.	Mi.	Se.	*The Moon.* *Subtract.* De.	Mi.	Se.	*The Elongation of the Moon from the Centre of the Eccentrick.*	Deg. asc.
0	1	44	47	4	14	51	101729	30
1	1	45	53	4	17	29	101691	29
2	1	46	57	4	20	02	101652	28
3	1	47	59	4	22	31	101613	27
4	1	48	59	4	24	55	101573	26
5	1	49	57	4	27	14	101532	25
6	1	50	53	4	29	29	101491	24
7	1	51	47	4	31	39	101449	23
8	1	52	39	4	33	44	101407	22
9	1	53	30	4	35	44	101365	21
10	1	54	19	4	37	39	101322	20
11	1	55	07	4	39	30	101279	19
12	1	55	52	4	41	17	101235	18
13	1	56	34	4	42	59	101191	17
14	1	57	14	4	44	35	101146	16
15	1	57	52	4	46	05	101101	15
16	1	58	28	4	47	30	101055	14
17	1	59	02	4	48	50	101009	13
18	1	59	34	4	50	06	100963	12
19	2	00	04	4	51	16	100916	11
20	2	00	32	4	52	21	100869	10
21	2	00	58	4	53	21	100821	9
22	2	01	22	4	54	16	100773	8
23	2	01	43	4	55	05	100725	7
24	2	02	02	4	55	49	100676	6
25	2	02	19	4	56	28	100627	5
26	2	02	33	4	57	01	100578	4
27	2	02	45	4	57	29	100528	3
28	2	02	55	4	57	51	100478	2
29	2	03	03	4	58	08	100428	1
30	2	03	09	4	58	20	100378	0
	Add			Add				

Sign 9.

The Residue of Table XI.

Prosthaphæreses or Equations.

Sign 3

Deg. dsc.	*The Sun.* Subtract. De.	Mi.	Se.	*The Moon.* Subtract. De.	Mi.	Se.	*The Elongation of the Moon from the Centre of the Eccentrick.*	Deg. asc.
0	2	03	09	4	58	20	100378	30
1	2	03	13	4	58	26	100327	29
2	2	03	15	4	58	27	100276	28
3	2	03	13	4	58	22	100225	27
4	2	03	09	4	58	14	100174	26
5	2	03	03	4	57	59	100123	25
6	2	02	55	4	57	37	100072	24
7	2	02	45	4	57	10	100021	23
8	2	02	33	4	56	38	99969	22
9	2	02	19	4	56	01	99917	21
10	1	02	02	4	55	18	99865	20
11	2	01	43	4	54	30	99813	19
12	2	01	22	4	53	36	99761	18
13	2	00	58	4	52	37	99709	17
14	2	00	32	4	51	33	99657	16
15	2	00	04	4	50	23	99605	15
16	1	59	34	4	49	07	99553	14
17	1	59	02	4	47	46	99501	13
18	1	58	27	4	46	21	99449	12
19	1	57	50	4	44	50	99397	11
20	1	57	11	4	43	13	99343	10
21	1	56	30	4	41	31	99293	9
22	1	55	46	4	39	43	99242	8
23	1	55	00	4	37	51	99191	7
24	1	54	12	4	35	54	99140	6
25	1	53	21	4	33	51	99089	5
26	1	52	28	4	31	42	99038	4
27	1	51	33	4	29	29	98987	3
28	1	50	36	4	27	11	98937	2
29	1	49	37	4	24	48	98887	1
30	1	48	36	4	22	20	98838	0
	Add			Add				

Sign 8.

The Residue of Table XI.

Prosthaphæreses *or* Equations.

Sign 4.

Deg. def.	*The Sun.* Subtract. De. Mi. Se.	*The Moon.* Subtract. De. Mi. Se.	*The Elongation of the Moon from the Centre of the Eccentrick.*	D. def.
0	1 48 36	4 22 20	98838	30
1	1 47 33	4 19 46	98789	29
2	1 46 28	4 17 07	98740	28
3	1 45 21	4 14 24	98691	27
4	1 44 12	4 11 36	98643	26
5	1 43 01	4 08 43	98595	25
6	1 41 47	4 05 45	98547	24
7	1 40 31	4 02 42	98500	23
8	1 39 14	3 59 35	98453	22
9	1 37 55	3 56 23	98407	21
10	1 36 35	3 53 06	98361	20
11	1 35 13	3 49 45	98316	19
12	1 33 49	3 46 20	98272	18
13	1 32 23	3 42 50	98228	17
14	1 30 54	3 39 16	98185	16
15	1 29 23	3 35 38	98142	15
16	1 27 50	3 31 55	98100	14
17	1 26 16	3 28 08	98059	13
18	1 24 40	3 24 18	98018	12
19	1 23 03	3 20 24	97978	11
20	1 21 25	3 16 25	97938	10
21	1 19 46	3 12 22	97899	9
22	1 18 05	3 08 15	97861	8
23	1 16 22	3 04 05	97824	7
24	1 14 36	2 59 52	97787	6
25	1 12 48	2 55 35	97751	5
26	1 10 59	2 51 14	97716	4
27	1 09 09	2 46 49	97682	3
28	1 07 58	2 42 21	97649	2
29	1 05 26	2 37 51	97616	1
30	1 03 33	2 33 18	97584	0
	Add	Add		

Sign 7.

The Residue of Table XI.

Prosthaphæreses or Equations.

Sign 5

Deg. Anc.	The Sun. Subtract. De.	Mi.	Se.	The Moon. Subtract. De.	Mi.	Se.	The Elongation of the Moon from the Centre of the Eccentrick.	Deg. Anc.
0	1	03	33	2	33	18	97584	30
1	1	01	40	2	28	41	97553	29
2	0	59	45	2	24	01	97523	28
3	0	57	48	2	19	14	97494	27
4	0	55	49	2	14	33	97466	26
5	0	53	48	2	09	45	97439	25
6	0	51	47	2	04	55	97413	24
7	0	49	45	2	00	02	97384	23
8	0	47	43	1	55	07	97364	22
9	0	45	40	1	50	09	97341	21
10	0	43	36	1	45	08	97320	20
11	0	41	31	1	40	05	97299	19
12	0	39	25	1	35	01	97279	18
13	0	37	18	1	29	55	97260	17
14	0	35	10	1	24	47	97242	16
15	0	33	01	1	19	38	97225	15
16	0	30	52	1	14	27	97209	14
17	0	28	42	1	09	14	97194	13
18	0	26	32	1	03	59	97180	12
19	0	24	21	0	58	43	97167	11
20	0	22	10	0	53	27	97155	10
21	0	19	59	0	48	10	97144	9
22	0	17	47	0	42	52	97134	8
23	0	15	35	0	37	33	97126	7
24	0	13	22	0	32	13	97120	6
25	0	11	09	0	26	52	97115	5
26	0	08	56	0	21	30	97111	4
27	0	06	42	0	16	08	97108	3
28	0	04	28	0	10	46	97105	2
29	0	02	14	0	05	23	97102	1
30	0	00	00	0	00	00	97100	0
	Add			Add				

Sign 6.

TABLE XII.

The Second and Third Prosthaphæreses *of the Moon.*

Deg. dsc.	Sign 0. Moon's Eccen.	Add Variation. Mi.	Se.	Sign 1. Moon's Eccen.	Add Variation. Mi.	Se.	Sign 2. Moon's Eccent.	Add Variation. Mi.	Se.	Deg. asc.
0	0	0	00	1125	20	15	2174	35	04	30
1	38	0	43	1162	20	51	2207	35	25	29
2	76	1	26	1198	21	27	2240	35	45	28
3	114	2	08	1235	22	03	2272	36	05	27
4	152	2	50	1271	22	38	2304	36	24	26
5	190	3	32	1307	23	13	2336	36	42	25
6	228	4	14	1344	23	48	2368	37	00	24
7	265	4	56	1380	24	22	2400	37	17	23
8	303	5	38	1416	24	56	2432	37	33	22
9	341	6	20	1451	25	29	2463	37	48	21
10	379	7	02	1487	26	02	2494	38	03	20
11	417	7	44	1523	26	34	2525	38	17	19
12	454	8	26	1558	27	06	2556	38	30	18
13	492	9	07	1594	27	39	2587	38	42	17
14	530	9	48	1629	28	08	2617	38	55	16
15	568	10	29	1664	28	38	2647	39	07	15
16	605	11	10	1699	29	08	2677	39	18	14
17	643	11	51	1734	29	37	2707	39	28	13
18	680	12	31	1769	30	06	2737	39	37	12
19	718	13	11	1804	30	34	2766	39	45	11
20	755	13	51	1838	31	01	2795	39	53	10
21	792	14	31	1872	31	28	2824	40	00	9
22	830	15	10	1906	31	55	2853	40	06	8
23	867	15	49	1940	32	21	2881	40	12	7
24	904	16	28	1974	32	46	2909	40	17	6
25	941	17	07	2008	33	11	2937	40	21	5
26	978	17	45	2042	33	35	2965	40	25	4
27	1015	18	23	2075	33	58	2993	40	27	3
28	1052	19	01	2108	34	21	3021	40	28	2
29	1089	19	38	2141	34	43	3048	40	29	1
30	1125	20	15	2174	35	04	3075	40	30	0
		Subtract.			Subtract.			Subtract.		
	Sign 11.			Sign 10.			Sign 9.			

The Residue of Table XII.

The Second and Third Prosthaphæreses *of the Moon.*

Deg. desc.	Sign 3. Moon's Eccen.	Add Variation. Mi.	Se.	Sign 4. Moon's Eccen.	Add Variation. Mi.	Se.	Sign 5. Moon's Eccen.	Add Variation. Mi.	Se.	Deg. asc.
0	3075	40	30	3765	35	04	4200	20	15	30
1	3102	40	29	3784	34	43	4210	19	38	29
2	3128	40	28	3803	34	21	4219	19	01	28
3	3154	40	27	3821	33	58	4228	18	23	27
4	3180	40	25	3839	33	35	4236	17	45	26
5	3206	40	21	3857	33	11	4245	17	07	25
6	3231	40	17	3874	32	46	4253	16	28	24
7	3256	40	12	3891	32	21	4261	15	49	23
8	3281	40	06	3908	31	55	4268	15	10	22
9	3306	40	00	3924	31	28	4275	14	31	21
10	3331	39	53	3940	31	01	4282	13	51	20
11	3355	39	45	3956	30	34	4288	13	11	19
12	3379	39	37	3972	30	06	4294	12	31	18
13	3403	39	28	3987	29	37	4300	11	51	17
14	3426	39	18	4002	29	08	4306	11	10	16
15	3449	39	07	4017	28	38	4311	10	29	15
16	3472	38	55	4031	28	08	4316	9	43	14
17	3495	38	43	4045	27	37	4320	9	07	13
18	3518	38	30	4059	27	06	4324	8	26	12
19	3540	38	17	4073	26	34	4328	7	44	11
20	3562	38	03	4086	26	02	4332	7	02	10
21	3584	37	48	4099	25	29	4335	6	20	9
22	3605	37	33	4111	24	56	4338	5	38	8
23	3626	37	17	4123	24	22	4340	4	56	7
24	3647	37	00	4135	23	48	4342	4	14	6
25	3667	36	42	4147	23	13	4344	3	32	5
26	3687	36	24	4158	22	38	4345	2	50	4
27	3707	36	05	4169	22	03	4346	2	08	3
28	3727	35	45	4180	21	27	4347	1	26	2
29	3746	35	25	4190	20	51	4348	0	43	1
30	3765	35	04	4200	20	15	4348	0	00	0
		Subtract.			Subtract.			Subtract.		
	Sign 8.			Sign 7.			Sign 6.			

TYCHO'S
LUNAR and SOLAR
TABLES

TABLE XIII.

Epochæ, *or Roots, for the Meridian of* Friſeland, *and at Midnight, which precedes the Kalends of* January.

Years of Chriſt.	*Epact.* Da. H. M. S.	*Sun's Anomaly.* Si. De. Mi. Se.	*Moon.s Anom.* Si. De. Mi. Se.	*Motion of the Moon's Lat.* Si. De. Mi. Se.	*Sun's Motion.* Si. De. Mi. Se.
1401	15 12 37 26	06 00 26 22	08 24 19 37	02 23 00 30	09 03 36 02
1501	11 04 24 35	06 04 14 20	05 09 33 55	07 12 26 58	09 08 39 04
1601	06 20 11 45	06 08 02 20	01 24 48 14	00 01 53 27	09 13 42 04
1701	02 11 59 00	06 11 50 20	10 10 02 28	04 21 19 55	09 18 45 05

TABLE XIV.

A Table of twenty expanded Years.

Abſolute Years.	*Epact.* Da. Ho. Mi. Se.	*Sun's Anomaly.* Si. De. Mi. Se.	*Moon's Anom.* Si. De. Mi. Se.	*Motion of the Moon's Lat.* Si. De. Mi. Se.	*Sun's Motion.* Si. De. Mi. Se.
1	10 15 11 22	11 19 16 08	10 09 48 01	00 08 02 47	11 19 16 52
2	21 06 22 45	11 08 32 16	08 19 36 01	00 16 05 33	11 08 33 43
3	02 08 50 04	11 26 54 44	07 25 13 02	01 24 48 34	11 26 56 59
4	14 00 01 26	11 16 10 52	06 05 01 03	02 02 51 21	11 16 13 51
5	24 15 12 48	11 05 17 00	04 14 49 04	02 10 54 08	11 05 30 42
6	05 17 40 07	11 23 49 29	03 20 26 05	03 19 37 08	11 23 53 58
7	16 08 51 30	11 13 05 37	02 00 14 05	03 27 39 55	11 13 10 50
8	28 00 02 52	11 02 21 45	00 10 02 06	04 05 42 42	11 02 27 42
9	09 02 30 11	11 20 44 13	11 15 39 07	05 14 25 42	11 20 50 57
10	19 17 41 33	11 10 00 21	09 25 27 08	05 22 28 29	11 10 07 49
11	00 20 08 52	11 28 22 50	09 01 04 08	07 01 11 30	11 28 31 05
12	12 11 20 15	11 17 38 58	07 10 52 09	07 09 14 16	11 17 47 57
13	23 02 31 37	11 06 55 06	05 20 40 10	07 17 17 03	11 07 04 48
14	04 04 58 56	11 25 17 34	04 26 17 11	08 26 00 04	11 25 28 04
15	14 20 10 18	11 14 33 42	03 06 05 12	09 04 02 51	11 14 44 56
16	26 11 21 41	11 03 49 50	01 15 53 12	09 12 05 37	11 04 01 47
17	07 13 49 00	11 22 12 19	00 21 30 13	10 20 48 38	11 22 25 03
18	18 05 00 22	11 11 28 26	11 01 18 14	10 28 51 25	11 11 41 55
19	28 20 11 44	11 00 44 34	09 11 06 15	11 06 54 12	11 00 58 46
20	10 22 39 03	11 19 07 03	08 16 43 16	00 15 37 12	11 19 22 03
40	21 21 18 08	11 08 14 07	05 03 26 30	01 01 14 24	11 08 44 05
60	03 07 13 07	11 26 27 30	02 15 58 48	02 17 31 50	11 27 12 33
80	14 05 52 12	11 15 34 33	11 02 42 04	03 03 09 02	11 16 34 36

TABLE XV.

A Table of Months in a common Year.

Years of Christ.	*Epacts.* Da. Ho. Mi. Se.	*Sun's Anomaly.* Si. De. Mi. Se.	*Moon's Anom.* Si. De Mi. Se.	*Motion of the Moon's Lat.* Si. De. Mi. Se.	*Sun's Motion.* Si. De. Mi. Se.
January	00 00 00 00	00 00 00 00	00 00 00 00	00 00 00 00	00 00 00 00
Febru.	01 11 15 57	00 29 06 21	00 25 49 00	01 00 40 14	00 29 06 24
March	29 11 15 57	00 29 06 21	00 25 49 00	01 00 40 14	00 29 06 24
April	01 09 47 51	02 27 19 02	02 17 27 00	03 02 00 42	02 27 19 13
May	01 21 03 47	03 26 25 23	03 13 16 00	04 02 40 56	03 26 25 37
June	03 08 19 44	04 25 31 43	04 09 05 00	05 03 21 09	04 25 32 02
July	03 19 35 41	05 24 38 04	05 04 54 00	06 04 01 23	05 24 38 26
Auguſt	05 06 51 38	06 23 44 25	06 00 43 00	07 04 41 37	06 23 44 50
Septem.	06 18 07 35	07 22 50 45	06 26 32 01	08 05 21 51	07 22 51 14
October	07 05 23 32	08 21 57 06	07 22 21 01	09 06 02 05	08 21 57 39
Novem.	08 16 39 29	09 21 03 27	08 18 10 01	10 06 42 19	09 21 04 03
Decem.	09 03 55 25	10 20 09 47	09 13 59 01	11 07 22 33	10 20 10 27
Years.	10 15 11 22	11 19 16 08	10 09 48 01	00 08 02 47	11 19 16 52

TABLE XVI.

The Canon of the Full and New Moons.

Syzyg.	*The Time from which the Epacts are ſubtracted.* Da. H. M. S.	*Sun's Anomaly.* Si. De. Mi. Se.	*Moon's Anom.* Si. De. Mi. Se.	*Motion of the Moon's Lat.* Si. De. Mi. Se.	*Sun's Motion.* Si. De. Mi. Se.
Full ☾.	15 18 22 02	00 14 33 10	06 12 54 30	06 15 20 07	00 14 33 12
New ☾.	30 12 44 03	00 29 06 21	00 25 49 00	01 00 40 14	00 29 06 24
Full ☾.	45 07 06 05	01 13 39 31	07 08 43 30	07 16 00 21	01 13 39 36
New ☾.	60 01 28 06	01 28 12 41	01 21 38 00	02 01 20 28	01 28 12 48
Full ☾.	74 19 50 08	02 12 45 52	08 04 32 30	08 16 40 35	02 12 46 01
New ☾.	89 14 12 09	02 27 19 02	02 17 27 00	03 02 00 42	02 27 19 13

TABLE XVII.

A Table of Lunar Months.

Lunar Months.	Time. Da. Ho Mi. Se. Th.	Sun's Anomaly. Si. De. Mi. Se. Th.	Moon's Anomaly. Si. De. Mi. Se. Th.	Motion of the Moon's Latitude. Si De Mi. Se. Th.	Motion of the Sun from the Æquin. Si. De. Mi. Se Th
1	029 12 44 03 09	00 29 06 20 40	00 25 49 00 04	01 00 40 13 54	00 29 06 24 18
2	059 01 28 06 17	01 28 12 41 19	01 21 38 00 07	02 01 20 27 47	01 28 12 48 36
3	088 14 12 09 26	02 27 19 01 59	02 17 27 00 11	03 02 00 41 41	02 27 19 12 54
4	118 02 56 12 35	03 26 25 22 38	03 13 16 00 15	04 02 40 55 35	03 26 25 37 12
5	147 15 40 15 43	04 25 31 43 18	04 09 05 00 19	05 03 21 09 29	04 25 32 01 31
6	177 04 24 18 52	05 24 38 03 58	05 04 54 00 23	06 04 01 23 22	05 24 38 25 49
7	206 17 08 22 01	06 23 44 24 37	06 00 43 00 26	07 04 41 37 16	06 23 44 50 07
8	236 05 52 25 09	07 22 50 45 17	06 26 32 00 30	08 05 21 51 10	07 22 51 14 25
9	265 18 36 28 18	08 21 57 05 57	07 22 21 00 34	09 06 02 05 03	08 21 57 38 43
10	295 07 20 31 27	09 21 03 26 36	08 18 10 00 38	10 06 42 18 56	09 21 04 03 01
11	324 20 04 34 35	10 20 09 47 16	09 13 59 00 41	11 07 22 32 51	10 20 10 27 19
12	354 08 48 37 44	11 19 16 07 56	10 09 48 00 45	00 08 02 46 45	11 19 16 51 37
13	383 21 32 40 53	00 18 22 28 35	11 05 37 00 49	01 08 43 00 38	00 18 23 15 56
	The half Month and Quadrant.				
½	014 18 22 01 34	00 14 33 10 20	06 12 54 30 02	06 15 20 06 57	00 14 33 12 09
¼	007 09 11 00 47	00 07 16 35 10	03 06 27 15 01	03 07 40 03 28	00 07 16 36 04

TABLE XVIII.

The Reduction of the Moon to the Ecliptick.

Subtract.

Degrees.	Sign 0. / Sign 6. Min.	Sec.	Sign 1. / Sign 7. Min.	Sec.	Sign 2. / Sign 8. Min.	Sec.	Degrees.
0	00	00	06	06	06	05	30
1	00	15	06	12	05	57	29
2	00	30	06	18	05	48	28
3	00	45	06	24	05	39	27
4	00	59	06	29	05	30	26
5	01	13	06	35	05	21	25
6	01	27	06	40	05	12	24
7	01	42	06	44	05	01	23
8	01	56	06	47	04	51	22
9	02	10	06	51	04	40	21
10	02	24	06	54	04	29	20
11	02	38	06	56	04	18	19
12	02	52	06	57	04	07	18
13	03	06	06	58	03	55	17
14	03	19	06	59	03	42	16
15	03	32	07	00	03	31	15
16	03	43	06	59	03	18	14
17	03	56	06	58	03	05	13
18	04	08	06	57	02	51	12
19	04	19	06	56	02	38	11
20	04	30	06	54	02	23	10
21	04	41	06	51	02	09	9
22	04	52	06	47	01	55	8
23	05	02	06	44	01	41	7
24	05	13	06	40	01	26	6
25	05	22	06	35	01	12	5
26	05	31	06	28	00	58	4
27	05	40	06	25	00	45	3
28	05	49	06	17	00	30	2
29	05	58	06	11	00	15	1
30	06	06	06	05	00	00	0
	Sign 11. / Sign 5.		Sign 10. / Sign 4.		Sign 9. / Sign 3.		

Add.

TABLE XIX.

The Prosthaphæreses *or Equations of the Nodes of the Moon.*

Degrees	Sign 0 and 6, Equations Add De. Mi. Se.	Scruples propor. Mi. Se.	Sign 1 and 7, Equations Add De. Mi. Se.	Scrup. propor. Mi. Se.	Sign 2 and 8, Equations Add De. Mi. Se.	Scrup. propor. Mi. Se.	Degrees
0	0 00 00	00 00	1 33 28	15 22	1 30 32	45 20	30
1	0 03 50	00 01	1 35 12	16 18	1 28 34	46 13	29
2	0 07 39	00 04	1 36 47	17 15	1 26 30	47 05	28
3	0 11 27	00 09	1 38 12	18 13	1 24 21	47 54	27
4	0 15 14	00 16	1 39 31	19 11	1 22 06	48 43	26
5	0 19 00	00 26	1 40 42	20 10	1 19 45	49 31	25
6	0 22 46	00 41	1 41 45	21 09	1 17 18	50 19	24
7	0 26 29	00 56	1 42 44	22 09	1 14 46	51 04	23
8	0 30 09	01 13	1 43 38	23 10	1 12 09	51 47	22
9	0 33 47	01 32	1 44 29	24 11	1 09 27	52 28	21
10	0 37 23	01 53	1 45 08	25 13	1 06 41	53 08	20
11	0 40 56	02 16	1 45 34	26 16	1 03 49	53 46	19
12	0 44 26	02 41	1 45 50	27 19	1 00 51	54 23	18
13	0 47 52	03 08	1 45 56	28 22	0 57 50	54 59	17
14	0 51 14	03 38	1 45 59	29 25	0 54 47	55 33	16
15	0 54 32	04 10	1 46 00	30 28	0 51 42	56 06	15
16	0 57 47	04 43	1 45 53	31 30	0 48 32	56 36	14
17	1 00 46	05 18	1 45 36	32 33	0 45 18	57 03	13
18	1 04 00	05 54	1 45 13	33 35	0 42 00	57 28	12
19	1 06 59	06 32	1 44 41	34 37	0 38 40	57 52	11
20	1 09 53	07 12	1 44 00	35 39	0 35 18	58 14	10
21	1 12 42	07 54	1 43 10	36 41	0 31 53	58 35	9
22	1 15 25	08 38	1 42 14	37 42	0 28 26	58 53	8
23	1 18 02	09 24	1 41 10	38 43	0 24 57	59 09	7
24	1 20 33	10 13	1 39 59	39 43	0 21 26	59 22	6
25	1 22 58	11 02	1 38 42	40 41	0 17 54	59 33	5
26	1 25 16	11 51	1 37 18	41 38	0 14 21	59 43	4
27	1 27 28	12 41	1 35 46	42 35	0 10 46	59 50	3
28	1 29 34	13 33	1 34 08	43 31	0 07 11	59 55	2
29	1 31 34	14 27	1 32 23	44 26	0 03 36	59 58	1
30	1 33 28	15 22	1 30 32	45 20	0 00 00	60 00	0
	Sign 11 and 5, Subtract		Sign 10 and 4, Subtract		Sign 9 and 3, Subtract		

TABLE XX.

A Table of the Moon's Latitude.

Deg.	Sign 0 North / Sign 6 South — Moon's Lat. De. Mi. Se.	Excess M. S.	Sign 1 North / Sign 7 South — Moon's Lat. De. Mi. Se.	Excess M. S.	Sign 2 North / Sign 8 South — Moon's Lat. De. Mi. Se.	Excess Mi. Se.	Deg.
0	00 00 00	00 00	02 29 06	09 28	04 18 26	16 25	30
1	00 05 13	00 20	02 33 36	09 45	04 20 59	16 35	29
2	00 10 25	00 40	02 38 03	10 02	04 23 28	16 45	28
3	00 15 36	00 59	02 42 26	10 28	04 25 53	16 55	27
4	00 20 47	01 19	02 46 46	10 35	04 28 13	17 04	26
5	00 25 58	01 39	02 51 04	10 51	04 30 28	17 12	25
6	00 31 09	01 59	02 55 19	11 07	04 32 38	17 20	24
7	00 36 19	02 19	02 59 30	11 23	04 34 43	17 28	23
8	00 41 29	02 38	03 03 38	11 39	04 36 43	17 36	22
9	00 46 38	02 57	03 07 47	11 55	04 38 38	17 43	21
10	00 51 46	03 17	03 11 44	12 11	04 40 27	17 51	20
11	00 56 53	03 36	03 15 42	12 26	04 42 11	17 58	19
12	01 01 59	03 55	03 19 36	12 41	04 43 50	18 05	18
13	01 07 04	04 15	03 23 26	12 56	04 45 23	18 12	17
14	01 12 08	04 34	03 27 13	13 10	04 46 52	18 18	16
15	01 17 10	04 53	03 30 56	13 24	04 48 18	18 23	15
16	01 22 11	05 13	03 34 35	13 38	04 49 35	18 27	14
17	01 27 10	05 32	03 38 10	13 52	04 50 49	18 31	13
18	01 32 08	05 51	03 41 42	14 05	04 51 58	18 34	12
19	01 37 04	06 10	03 45 07	14 18	04 53 00	18 38	11
20	01 41 58	06 29	03 48 30	14 31	04 53 57	18 42	10
21	01 46 51	06 47	03 51 52	14 44	04 54 49	18 45	9
22	01 51 41	07 06	03 55 09	14 57	04 55 36	18 48	8
23	01 56 30	07 24	03 58 19	15 09	04 56 17	18 51	7
24	02 01 17	07 42	04 01 23	15 21	04 56 52	18 53	6
25	02 06 01	08 00	04 04 24	15 32	04 57 22	18 55	5
26	02 10 43	08 18	04 07 21	15 43	04 57 46	18 56	4
27	02 15 23	08 35	04 10 15	15 54	04 58 05	18 57	3
28	02 20 00	08 53	04 13 05	16 05	04 58 18	18 58	2
29	02 24 34	09 11	04 15 49	16 15	04 58 26	18 59	1
30	02 29 06	09 28	04 18 26	16 25	04 58 30	19 00	0
Deg.	Sign 11 South / Sign 5 North		Sign 10 South / Sign 4 North		Sign 9 South / Sign 3 North		Deg.

TABLE XXI.

A Table of the Parallaxes of the Sun and Moon in the vertical Circle.

Semidiametres		*The Moon.*									
Degrees of Altitude.	*Sun* Min.	52 92028 Mi.	53 93798 Mi.	54 95568 Mi.	55 97337 Mi.	56 99107 Mi.	57 100877 Mi.	58 102647 Mi.	59 104416 Mi.	60 106186 Mi.	61 107956 Mi.
0	3	66	65	64	63	61	60	59	58	57	56
3	3	66	65	64	62	61	60	59	58	57	56
6	3	66	65	63	62	61	60	59	58	57	56
9	3	65	64	63	62	61	60	59	58	57	56
12	3	65	64	62	61	60	59	58	57	56	55
15	3	64	63	62	61	60	59	58	57	56	55
18	3	63	62	61	60	59	58	57	56	55	54
21	3	62	61	60	59	58	57	56	55	54	53
24	3	61	60	59	58	56	55	54	54	53	52
27	3	60	58	57	56	55	54	53	52	51	51
30	3	58	57	56	55	54	53	52	51	50	49
33	3	56	55	54	53	52	51	50	49	48	48
36	2	54	53	52	51	50	49	48	48	47	46
39	2	52	51	50	49	48	47	47	46	45	44
42	2	50	49	48	47	46	45	45	44	43	42
45	2	47	46	46	45	44	43	42	42	41	40
48	2	45	44	43	42	42	41	40	39	39	38
51	2	42	41	41	40	39	38	38	37	37	36
54	2	40	39	38	37	37	36	35	35	34	34
57	2	37	36	35	35	34	33	33	32	32	31
60	2	34	33	32	32	31	31	30	30	29	29
63	1	31	30	29	29	28	28	27	27	26	26
66	1	27	27	26	26	25	25	25	24	24	23
69	1	24	24	23	23	22	22	22	21	21	20
72	1	21	20	20	20	19	19	19	18	18	18
75	1	17	17	17	16	16	16	16	15	15	15
78	1	14	14	13	13	13	13	13	12	12	12
81	0	11	10	10	10	10	10	9	9	9	9
84	0	7	7	7	7	7	6	6	6	6	6
87	0	4	3	3	3	3	3	3	3	3	3
90	0	0	0	0	0	0	0	0	0	0	0

The Moon's Distance.

In this Table I have purposely omitted the Seconds, because they are wont to cause more trouble than the thing it self is worth.

TABLE XXII.

A Table of the Refractions of the Sun and Moon.

Degrees of Altitude.	Refractions	
	Of the Sun.	Of the Moon.
	Min.	Min.
0	34	33
1	26	25
2	20	20
3	17	17
4	15	15
5	14	14
6	13	14
7	13	13
8	11	12
9	10	11
10	10	11
11	09	10
12	09	10
13	08	09
14	08	08
15	07	08
16	07	07
17	06	07
18	06	06
19	05	06
20	04	05
21	04	05
22	03	05
23	03	04
24	03	04
25	02	03
26	02	03
27	02	03
28	02	02
29	02	02
30	01	02
31	01	01
32	01	01
33	01	01
34	01	01
35	01	01
36	01	01
37	00	01
38	00	01
39	00	00
40	00	00
41	00	00

TABLE XXIII.

A Table of the Apparent Semediametres.

The middle Anomaly of the Sun shews the Semidiametre of the Sun, with the Variation of the Shadow. The co-equall'd Anomaly of the Moon gives the Semidiametre of the Moon, and the Semidiametre of the Shadow, from which the Variation is taken.

Anomaly of the ☉ or ☾.		*Of the Sun.*		*Of the Full ☾.*		*Of the New ☾.*		*Of the Shadow.*		*Part.*	*Anomaly of the ☉. or ☾.*	
Si.	De.	Mi.	Se.	Mi.	Se.	Mi.	Se.	Mi.	Se.	Se.	Si.	De.
00	00	15	00	16	00	12	48	43	00	00	12	00
00	06	15	00	16	00	12	48	43	01	00	11	24
00	12	15	01	16	01	12	49	43	02	01	11	18
00	18	15	02	16	03	12	50	43	06	02	11	12
00	24	15	03	16	05	12	52	43	09	03	11	06
01	00	15	04	16	08	12	54	43	14	04	11	00
01	06	15	06	16	12	12	58	43	19	06	10	24
01	12	15	08	16	16	13	01	43	25	09	10	18
01	18	15	10	16	21	13	05	43	33	12	10	12
01	14	15	12	16	25	13	08	43	41	15	10	06
02	00	15	15	16	30	13	12	43	49	18	10	00
02	06	15	18	16	36	13	17	43	58	21	09	24
02	12	15	21	16	42	13	22	44	09	24	09	18
02	18	15	24	16	48	13	26	44	20	27	09	12
02	24	15	26	16	54	13	31	44	31	30	09	06
03	00	15	29	17	00	13	35	44	43	34	09	00
03	06	15	32	17	05	13	41	44	55	37	08	24
03	12	15	35	17	12	13	46	45	07	40	08	18
03	18	15	38	17	17	13	50	45	19	43	08	12
03	24	15	41	17	22	13	54	45	31	46	08	06
04	00	15	43	17	26	13	57	45	44	49	08	06
04	06	15	46	17	31	14	01	45	56	51	07	24
04	12	15	49	17	36	14	05	46	07	52	07	18
04	18	15	51	17	41	14	09	46	17	53	07	12
04	24	15	53	17	46	14	13	46	27	54	07	06
05	00	15	55	17	50	14	16	46	36	55	07	00
05	06	15	57	17	54	14	19	46	44	55	06	24
05	12	15	58	17	57	14	22	46	50	55	06	18
05	18	15	59	17	58	14	22	46	55	56	06	12
05	24	15	59	17	59	14	23	46	58	56	06	06
06	00	16	00	18	00	14	24	47	00	56	06	00

TABLE XXIV.

A Table of the true horary Motion of the Moon in the true Syzygies, and half a day before and after the Syzygies.

Anomalies.

Deg.	Sign 0. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Sign 1. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Sign 2. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Deg.
0	27	43	27	12	28	04	27	34	29	01	28	46	30
3	27	44	27	12	28	08	27	39	29	09	28	57	27
6	27	45	27	13	28	13	27	45	29	17	29	08	24
9	27	45	27	14	28	18	27	51	29	25	29	19	21
12	27	47	27	15	28	23	27	58	29	33	29	29	18
15	27	48	27	16	28	29	28	05	29	41	29	41	15
18	27	51	27	19	28	35	28	12	29	49	29	52	12
21	27	53	27	22	28	41	28	20	29	58	30	04	9
24	27	57	27	26	28	47	28	29	30	06	30	17	6
27	28	00	27	30	28	54	28	38	30	15	30	29	3
30	28	04	27	34	29	01	28	46	30	24	30	42	0
	Sign 11.				Sign 10.				Sign 9.				

Anomalies.

Deg.	Sign 3. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Sign 4. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Sign 5. In the true Syz. Min.	Sec.	Before and after the Syz. Min.	Sec.	Deg.
0	30	24	30	42	31	51	32	53	32	58	34	45	30
3	30	33	30	55	31	58	33	05	33	03	34	53	27
6	30	41	31	07	32	06	33	18	33	07	35	00	24
9	30	50	31	20	32	14	33	30	33	11	35	08	21
12	30	58	31	33	32	21	33	41	33	14	35	14	18
15	31	07	31	46	32	28	33	54	33	17	35	18	15
18	31	17	32	00	32	35	34	05	33	20	35	24	12
21	31	26	32	15	32	41	34	17	33	21	35	29	9
24	31	34	32	28	32	47	34	26	33	23	35	32	6
27	31	43	32	41	32	53	34	36	33	24	35	34	3
30	31	51	32	53	32	58	34	45	33	24	35	37	0
	Sign 8.				Sign 7.				Sign 6.				

RUDOLPHUS'S TABLES,

Calculated in Decimal Parts of a Circle and Degrees.

TABLE XXV.

The Epochæ *of the middle Motions of the Sun and fixed Stars in the Years of the* Julian *Period, according to the Meridian of* Urenburgh, *in the Kalends of* January, *and the Mid-day of Aſtronomical Time,* Julian *Account.*

Mid. Mot. of the Sun from the Æquinox,		*Mid. Mot. of the Sun's Apogee from the* Æquin.			*Mid. Mot. of the firſt Star of* ♈ *from the* Æquinox.		
The Periodical Year according to the Julian *account.*	*Decimal parts of a Circle.*	*The Periodical Year according to the* Julian *account.*	*Signs*	*Degrees of decimal parts.*	*The Periodical Year according to the* Julian *account.*	*Signs*	*Degrees of decimal parts.*
0	6735528	0	11	17.6658	0	01	28.1825
1000	6945435	1000	00	4.7844	1000	10	12.3492
2000	7155342	2000	00	21.9031	2000	10	26.5158
3000	7365250	3000	01	9.0217	3000	11	10.6825
4000	7575157	4000	01	26 1403	4000	11	24.8492
5000	7785065	5000	02	13.2589	5000	00	9.0158
6000	7994972	6000	03	0.3775	6000	00	23.1825
		The Logarithm of the mid. Mot. of the Sun's Apogee in the Tropical Year. 2,23346.17			*The Logarithm of the mid. Motion of the fixed Stars in the Tropical Year.* 2,15126.24		

TABLE XXVI.

The middle Motion of the Sun from Aries *in the expanded Years, Months, Days, Hours, and Scruples.*

Mid. Met. of the Sun from the Æquinox *in expanded Years.*		*Mid. Motion of the Sun from the* Æqu. *in the Months of a common Year.*		*Middle Motion of the Sun from the* Æquinox *in Days.*	
Years.	*Decimal parts of a Circle.*	*Months compleat.*	*Decimal parts of a Circle.*	*Days compl.*	*Decimal parts of a Circle.*
B 1	0020744	*January*	0848752	1	0023379
2	0014109	*Febru.*	1615366	2	0054758
3	0007474	*March*	2464117	3	0082137
4	0000840	*April*	3285490	4	0109516
B. 5	0021584	*May*	4134241	5	0136895
6	00 4949	*June*	4955614	6	0164274
7	0008314	*July*	5804366	7	0191654
8	0001679	*August*	6653117	8	0219033
B 9	0022423	*Septem.*	7474490	9	0246412
10	0015789	*October*	8323241	10	0273791
11	0009154	*Novem.*	9144614	11	0301170
12	0002519	*Decem.*	9993365	12	0328549
B 13	0023263			13	0355928
14	0016628	*Mid. Mot. ☉. from the Æq. in the Months of a biſſextile Year.*		14	0383307
15	0009993			15	0410686
16	0003359	*January*	0848752	16	0438065
B 17	0024103	*Febru.*	1642745	17	0465444
18	0017468	*March*	2491497	18	0492823
19	0010833	*April*	3312869	19	0520203
20	0004198	*May*	4161621	20	0547582
40	0008396	*June*	4982993	21	0574961
60	0012594	*July*	5831745	22	0602340
80	0016793	*August*	6680496	23	0629719
100	0020991	*Septem.*	7501869	24	0657098
200	0041981	*October*	8350620	25	0684477
300	0062972	*Novem.*	9171993	26	0711856
400	0083963	*Decem.*	1.0020744	27	0739235
500	0104954			28	0766614
600	0125944	Note, That the Table of expanded Years agrees only with the Epoch. in the year of the *Julian* Period, in which year 1, 5, 9, 13, 17, above the Complement of twenty or an hundred, are biſſext.		29	0793993
600	0146935			30	0821372
800	0167926			31	0848752
900	0188917				
1000	0209907				

The Residue of Table XXVI.

The middle Motion of the Sun from Aries *in the expanded Years, Months, Days, Hours, and Scruples.*

Mid. Mot. of the Sun from the Æquinox in Hours.		*Mid. Motion of the Sun from the Æqu. in Minutes.*		*Middle Motion of the Sun from the Æquinox in Minutes.*	
Hours.	*Decimal parts of a Circle.*	*Minut.*	*Decimal parts of a Circle.*	*Minut.*	*Decimal parts of a Circle.*
1	0001141	1	0000190	31	0005894
2	0002282	2	0000380	32	0006084
3	0003422	3	0000570	33	0006274
4	0004563	4	0000761	34	0006465
5	0005704	5	0000951	35	0006655
6	0006845	6	0001141	36	0006845
7	0007986	7	0001331	37	0007035
8	0009126	8	0001521	38	0007225
9	0010267	9	0001711	39	0007415
10	0011408	10	0001901	40	0007605
11	0012549	11	0002091	41	0007796
12	0013690	12	0002282	42	0007986
13	0014830	13	0002472	43	0008176
14	0015971	14	0002662	44	0008366
15	0017112	15	0002852	45	0008556
16	0018253	16	0003042	46	0008746
17	0019394	17	0003232	47	0008936
18	0020534	18	0003422	48	0009126
19	0021675	19	0003613	49	0009317
20	0022816	20	0003803	50	0009507
21	0023957	21	0003993	51	0009697
22	0025097	22	0004183	52	0009887
23	00 6238	23	0004373	53	0010077
24	0027379	24	0004563	54	0010267
		25	0004753	55	0010457
		26	0004943	56	0010648
		27	0005134	57	0010838
		28	0005324	58	0011028
		29	0005514	59	0011218
		30	0005704	60	0011408

TABLE XXVII.

A Table of the Conversion of the decimal parts of a Circle into Degrees, and decimal parts of Degrees.

Decimal parts of a Circle. (1) (2) (3) (4) (5) (6)	Degrees or decimal parts of Degrees. 210.1 0.123 0.12345	Decimal parts of a Circle. (1) (2) (3) (4) (5) (6)	Degrees or decimal parts of Degrees. 210.1 0.123 0.12345	Decimal parts of a Circle. (1) (2) (3) (4) (5) (6)	Degrees, or decimal parts of Degrees. 210.1 0.123 0.12345
01	0036	34	1224	67	2412
02	0072	35	1260	68	2448
03	0108	36	1296	69	2484
04	0144	37	1332	70	2520
05	0180	38	1368	71	2556
06	0216	39	1404	72	2592
07	0252	40	1440	73	2628
08	0288	41	1476	74	2664
09	0324	42	1512	75	2700
10	0360	43	1548	76	2736
11	0396	44	1584	77	2772
12	0432	45	1620	78	2808
13	0468	46	1656	79	2844
14	0504	47	1692	80	2880
15	0540	48	1728	81	2916
16	0576	49	1764	82	2952
17	0612	50	1800	83	2988
18	0648	51	1836	84	3024
19	0684	52	1872	85	3060
20	0720	53	1908	86	3096
21	0756	54	1944	87	3132
22	0792	55	1980	88	3168
23	0828	56	2016	89	3204
24	0864	57	2052	90	3240
25	0900	58	2088	91	3276
26	0936	59	2124	92	3312
27	0972	60	2160	93	3348
28	1008	61	2196	94	3384
29	1044	62	2232	95	3420
30	1080	63	2268	96	3456
31	1116	64	2304	97	3492
32	1152	65	2340	98	3528
33	1188	66	2376	99	3564

TABLE XXVIII.

A Table of the Equation of the Centre of the Sun.

Subtract.

Degrees.	*Sign* 0. De. dec. p.	Log.di.	*Sign* 1. De. dec. p.	Log. di.	*Sign* 2. De. dec. p.	Log.di.	Degrees.
0	00.0000	400775	01.0114	400677	01.7667	400398	30
1	00.0353	400775	01.0419	400668	01.7847	400389	29
2	00.0706	400775	01.0717	400659	01.8022	400376	28
3	00.1058	400775	01.1019	400655	01.8192	400363	27
4	00.1406	400775	01.1317	400647	01.8358	400350	26
5	00.1758	400771	01.1619	400638	01.8517	400342	25
6	00.2106	400771	01.1897	400634	01.8669	400329	24
7	00.2453	400771	01.2181	400625	01.8819	400316	23
8	00.2806	400766	01.2464	400617	01.8961	400303	22
9	00.3158	400766	01.2747	400608	01.9122	400290	21
10	00.3506	400762	01.3025	400599	01.9239	400277	20
11	00.3856	400762	01.3303	400591	01.9356	400264	19
12	00.4200	400758	01.3567	400582	01.9475	400252	18
13	00.4547	400753	01.3833	400574	01.9592	400238	17
14	00.4919	400751	01.4097	400565	01.9703	400225	16
15	00.5236	460749	01.4353	400557	01.9806	400212	15
16	00.5606	400745	01.4603	400548	01.9903	400199	14
17	00.5906	400740	01.4850	400535	01.9992	400186	13
18	00.6181	400736	01.5194	400527	02.0078	400173	12
19	00 6581	400732	01.5336	400518	02.0156	400160	11
0	00.6917	400728	01.5569	400509	02.0225	400147	10
21	00.7244	400723	01.5805	400497	02.0292	400134	9
22	00.7575	400719	01.6031	400488	02 0358	400121	8
23	00.7883	400715	01.6256	400475	02.0414	400108	7
24	00.8219	400711	01.6497	400467	02.0467	400095	6
25	00.8542	400702	01.6675	400454	02.0506	400078	5
26	00.8861	400698	01.6900	400445	02.0542	400065	4
27	00.9178	400694	01.7094	400432	02.0572	400052	3
28	00.9503	400685	01.7292	400419	02.0597	400039	2
29	00.9794	400682	01.7483	400411	02.0614	400026	1
30	01.0114	400677	01.7667	400398	02.0625	400013	0
	Sign 11.		*Sign* 10.		*Sign* 9.		

Add

TABLE XXVIII.

A Table of the Equation of the Centre of the Sun.

Subtract							
Degrees.	*Sign* 3,		*Sign* 4,		*Sign* 5,		Degrees.
	D. dec. p.	Log.di.	D. dec. p.	Log.di.	D. dec. p.	Log.di.	
0	02.0625	400013	01.8064	399616	01.0511	399321	30
1	02.0628	400000	01.7886	399607	01.0189	399313	29
2	02.0622	399987	01.7703	399594	00.9869	399308	28
3	02.0619	399974	01.7514	399585	00.9547	399299	27
4	02.0606	399961	01.7319	399572	00.9222	399295	26
5	02.0575	399943	01.7119	399559	00.8889	399290	25
6	02.0553	399930	01.6908	399550	00.8553	399282	24
7	02.0517	399917	01.6700	399537	00.8222	399277	23
8	02.0483	399904	01.6481	399524	00.7886	399273	22
9	02.0436	399891	01.6258	399515	00.7544	399269	21
10	02.0381	399874	01.6028	399506	00.7200	399264	20
11	02.0331	399865	01.5808	399493	00.6856	399260	19
12	02.0267	399852	01.5572	399480	00.6506	399255	18
13	02.0192	399835	01.5331	399471	00.6156	399250	17
14	02.0114	399822	01.5083	399462	00.5806	399246	16
15	02.0031	399808	01.4828	399449	00.5450	399242	15
16	01.9947	399795	01.4569	399441	00.5194	399238	14
17	01.9850	399782	01.4308	399432	00.4739	399233	13
18	01.9747	399769	01.4042	399423	00.4381	399229	12
19	01.9639	399756	01.3744	399412	00.4022	399226	11
20	01.9525	399743	01.3497	399401	00.3661	399224	10
21	01.9408	399730	01.3222	399392	00.3297	399224	9
22	01.9283	399717	01.2936	399383	00.2939	399220	8
23	01.9153	399704	01.2647	399378	00.2572	399220	7
24	01.9014	399695	01.2353	399367	00.2206	399220	6
25	01.8867	399682	01.2056	399360	00.1839	399216	5
26	01.8692	399669	01.1756	399352	00.1469	399216	4
27	01.8564	399656	01.1447	399344	00.1106	399216	3
28	01.8400	399642	01.1139	399335	00.0736	399216	2
29	01.8231	399629	01.0825	399326	00.0369	399216	1
30	01.8064	399616	01.0511	399321	00.0000	399216	0
	Sign 8,		*Sign* 7,		*Sign* 6,		
Add							

TABLE XXIX.

The Epochæ *of the middle Motions of* Saturn, *in the Years of the Julian Period, for the Meridian of* Urenburgh, *in the Kalends of* January, *the Mid-day of Aſtronomical Time, according to the Julian Account.*

Middle Motion of Saturn *from the* Æquinox.		*Middle Motion of the Aphelion of* Saturn *from the* Æquinox.			*Middle Mot. of the northern Node of* Saturn *from the* Æquinox.		
In a comp. Year of the Julian Period.	*Decimal parts of a Circle.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees with decimal parts.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees with decimal parts.*
0	026400	0	04	13.2536	0	11	15.7004
1000	012229	1000	05	4.2748	1000	00	5.5485
2000	998059	2000	05	25.2959	2000	00	25.3966
3000	983888	3000	06	16.3170	3000	01	15.2447
4000	969718	4000	07	7.3381	4000	02	5.0928
5000	955547	5000	07	28.3592	5000	02	24.9409
6000	941377	6000	08	19.3803	6000	03	14.7890
Logarithm of the mid. Mot. of Saturn *in the Tropical Year.* $\bar{2}.53128.87$		*Logarithm of the middle Motion of the Aphelion of* Saturn *in the Tropical Year.* $\bar{2}.32268.65$			*Logarithm of the middle Motion of the northern Node of* Saturn *in a Tropical Year.* $\bar{2}.29771.49$		

Middle Motion of Saturn *from the* Æquinox *in the Tropical Years.*		*The Bounds of the Stations of* Saturn:				
		The Anomaly of the Eccentrick.	*The Angle of Commutation, or the Angle to the Sun.*			
			First.		*Second.*	
Trop. Years	*Decimal parts of a Circle.* 0.12345.678	Sig.	Sig.	De. d. p. cir.	Sig.	De. d. p. cir.
1	03398.512					
2	06797.023					
3	10195.535					
4	13594.046					
5	16992.558					
6	20391.070	00	03	23.80	03	23.95
7	23789.581	03	03	25.45	03	24.78
8	27188.093	06	03	26.88	03	26.83
9	30586.604	09	03	24.62	03	25.33

TABLE XXX.

A Table of the Equation of the Centre of Saturn.

Subtract.

Degrees.	Sign 0. De.p.d.	Log. di.	Sign 1. De. p.d.	Log.di.	Sign 2. De.p. d.	Log. di.	Degrees.
0	00.0000	500223	03.0736	499939	05.4528	499136	30
1	00.1064	500223	03.1681	499920	05 5128	499101	29
2	00.2119	500222	03.2614	499900	05.5711	499067	28
3	00.3183	500220	03.3539	499880	05.6283	499031	27
4	00.4242	500218	03.4456	499859	05.6836	498996	26
5	00.5314	500214	03.5367	499838	05.7372	498960	26
6	00.6372	500211	03.6267	499817	05.7889	498923	24
7	00.7353	500207	03.7158	499794	05.8394	498886	23
8	00.8494	500201	03.8042	499772	05.8878	498849	22
9	00.9544	500197	03.8914	499747	05.9347	498812	21
10	01.0653	500191	03.9775	499724	05.9800	498774	20
11	01.1647	500184	04.0625	499696	06.0239	498736	19
12	01.2694	500173	04.1464	499675	06.0664	498697	18
13	01.3739	500169	04.2292	499649	06.1072	498658	17
14	01.4778	500160	04.3108	499623	06.1464	498619	16
15	01.5814	500151	04.3917	499596	06.1833	498580	15
16	01.6844	500142	04.4708	499570	06.2183	498540	14
17	01.7872	500131	04.5486	499542	06.2517	498500	13
18	01.8894	500121	04.6256	499513	06.2831	498460	12
19	01.9919	500008	04.7014	499484	06.3131	498419	11
20	02.0928	500095	04.7753	499455	06.3411	498378	10
20	02.2100	500082	04.8483	499426	06.3669	498337	9
22	02.2939	500071	04.9206	499395	06.3911	498296	8
23	02.3933	500056	04.9942	499364	06.4133	498254	7
24	02.4925	500041	05.0611	499333	06.4336	498212	6
25	02.5908	500026	05.1297	499302	06.4522	498170	5
26	02.6889	500009	05.1972	499269	06.4686	498128	4
27	02.7864	499993	05.2636	499236	06.4842	498086	3
28	02.8828	499973	05.3281	499203	06.4978	498044	2
29	02.9786	499956	05.3914	499170	06.5092	498002	1
30	03.0736	499939	05.4528	491136	06.5183	497958	0
	Sig. 11.		Sign 10.		Sign 9.		

Add

The Residue of Table XXX.

Subtract,

Degrees.	*Sign* 3. De. p. d.	Log. di.	*Sign* 4. De. p. d.	Log. di.	*Sign* 5. De. p. d.	Log. di.	Degrees.
0	06.5183	497958	05.8539	496677	03.4775	495662	30
1	06.5256	497916	05.8008	496637	03.3714	495637	29
2	06.5306	497873	05.7458	496596	03.2689	495614	28
3	06.5331	497829	05.6889	496557	03.1631	495590	27
4	06.5339	497787	05.6294	496517	03.0553	495568	26
5	06.5331	497744	05.5686	496478	02.9469	495547	25
6	06.5306	497700	05.5058	496439	02.8372	495525	24
7	06.5256	497657	05.4417	496401	02.7267	495505	20
8	06.5189	497613	05.3747	496364	02.6153	495485	22
9	06.5097	497570	05.3061	496327	02.5036	495466	21
10	06.4983	497526	05.2353	496390	02.3906	495449	20
11	06.4856	497482	05.1628	496254	02.2769	495432	19
12	06.4711	497439	05.0886	496217	02.1619	495416	18
13	06.4539	497396	05.0131	496181	02.0469	495400	17
14	06.4353	497352	04.9361	496143	01.9308	495386	16
15	06.4142	497309	04.8572	496112	01.8139	495372	15
16	06.3914	497266	04.7769	496077	01.6967	495359	14
17	06.3664	497223	04.6947	496044	01.5786	495347	13
18	06.3333	497180	04.6103	496011	01.4597	495336	12
19	06.3097	497137	04.5247	495978	01.3400	495326	11
20	06.2786	497094	04.4372	495946	01.2197	425317	10
21	06.2450	497051	04.3478	495914	01.0989	495308	9
22	06.2097	497009	04.2572	495883	00.9778	495301	8
23	06.1725	496967	04.1647	495854	00.8567	495294	7
24	06.1325	496925	04.0703	495824	00.7350	495288	6
25	06.0908	496882	03.9750	495796	00.6111	495283	5
26	06.0472	496841	03.8783	495767	00.4900	495279	4
27	06.0022	496799	03.7803	495740	00.3678	495276	3
28	05.9547	496758	03.6808	495713	00.2453	495273	2
29	05.9053	496715	03.5797	495688	00.1228	495272	1
30	05.8539	496677	03.4775	495662	00.0000	495272	0
	Sign 8.		*Sign* 7.		*Sign* 6.		

Add

TABLE XXXI.

A Table of the Inclination of Saturn, *and of his Reduction to the Ecliptick.*

Degrees	Inclination. Sign 0, Sign 6, D. dec. p.	Sign 1, Sign 7, D. dec. p.	Sign 2, Sign 8, D. dec. p.	Reduction, Subtract. Sign 0, Sign 6, D. dec. p.	Sign 1, Sign 7, D. dec. p.	Sign 2, Sign 8, D. dec. p.	Degrees
0	00.0000	01.2667	02.1942	00.0000	00.0242	00.0242	30
1	00.0442	01.3050	02.2161	00.0011	00.0247	00.0236	29
2	00.0883	01.3431	02.2375	00.0022	00.0250	00.0231	28
3	00.1325	01.3806	02.2581	00.0031	00.0256	00.0225	27
4	00.1767	01.4175	02.2778	00.0042	00.0261	00.0219	26
5	00.2206	01.4539	02.2967	00.0050	00.0264	00.0214	25
6	00.2644	01.4897	02.3147	00.0061	00.0267	00.0208	24
7	00.3083	01.5250	02.3322	00.0069	00.0269	00.0203	23
8	00.3519	01.5600	02.3492	00.0078	00.0272	00.0197	22
9	00.3956	01.5944	02.3653	00.0089	00.0272	00.0189	21
10	00.4389	01.6283	02.3806	00.0097	00.0275	00.0181	20
11	00.4819	01.6619	02.3953	00.0106	00.0275	00.0175	19
12	00.5250	01.6950	02.4092	00.0117	00.0278	00.0167	18
13	00.5681	01.7275	02.4222	00.0125	00.0278	00.0158	17
14	00.6108	01.7194	02.4347	00.0133	00.0281	00.0150	16
15	00.6536	01.7908	02.4464	00.0142	00.0281	00.0142	15
16	00.6961	01.8217	02.4575	00.0150	00.0281	00.0133	14
17	00.7386	01.8519	02.4678	00.0158	00.0278	00.0125	13
18	00.7808	01.8817	02.4775	00.0167	00.0278	00.0117	12
19	00.8228	01.9108	02.4864	00.0175	00.0275	00.0106	11
20	00.8644	01.9394	02.4947	00.0181	00.0275	00.0097	10
21	00.9061	01.9675	02.5022	00.0189	00.0272	00.0089	9
22	00.9475	01.9950	02.5089	00.0197	00.0272	00.0078	8
23	00.9886	02.0219	02.5147	00.0203	00.0269	00.0069	7
24	01.0292	02.0483	02.5197	00.0208	00.0267	00.0061	6
25	01.0697	02.0742	02.5239	00.0214	00.0264	00.0050	5
26	01.1097	02.0994	02.5275	00.0219	00.0261	00.0042	4
27	01.1494	02.1242	02.5303	00.0225	00.0256	00.0031	3
28	01.1889	02.1481	02.5322	00.0231	00.0250	00.0022	2
29	01.2281	02.1714	02.5333	00.0236	00.0247	00.0011	1
30	01.2667	02.1942	02.5333	00.0242	00.0242	00.0000	0
	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	

Add

TABLE XXIX.

The Epochæ *of the middle Motions of* Jupiter, *in the Years of the Julian Period, for the Meridian of* Urenburgh, *in the Kalends of* January, *the Mid-day of Astronomical Time, according to the Julian Account.*

Middle Motion of Jupiter *from the* Æquinox.

In a comp. Year of the Julian Period.	*Decimal parts of a Circle.*
0	996112
1000	337982
2000	679851
3000	021721
4000	363590
5000	705460
6000	047330

Logarithm of the mid. Mot. of Jupiter *in the Tropical Year.* 2.92603.41

Middle Motion of the Aphelion of Jupiter *from the* Æquinox.

In a comp. Year of the Julian Period.	*Signs.*	*Degrees with decimal parts.*
0	03	14.1282
1000	03	27.2343
2000	04	10.3404
3000	04	23.4465
4000	05	6.5526
5000	05	19.6588
6000	06	0.7649

Logarithm of the middle Motion of the Aphelion of Jupiter *in the Tropical Year.* 2.11746.48

Middle Mot. of the northern Node of Jupiter *from the* Æquinox.

In a comp. Year of the Julian Period.	*Signs.*	*Degrees with decimal parts.*
0	12	29.3005
1000	03	0.2719
2000	03	1.2433
3000	03	2.2147
4000	03	3.1861
5000	03	4.1575
6000	03	5.1289

Logarithm of the middle Motion of the northern Node of Jupiter *in a Tropical Year.* 4.98735.64

Middle Motion of Jupiter *from the* Æquinox *in the Tropical Years.*

Trop. Years	*Decimal parts of a Circle.* 0.12345.678
1	08434.010
2	16868 020
3	25302.030
4	33736.040
5	42170.150
6	50604.060
7	59038.069
8	67472.079
9	75906.089

The Bounds of the Stations of Jupiter.

The Anomaly of the Eccentrick.	*The Angle of Commutation, or the Angle to the Sun.*			
	First.		*Second.*	
Sig.	Sig.	De. d. p. cir.	Sig.	De. d. p. cir.
00	04	3.95	04	4.90
03	04	6.37	04	6.40
06	04	8.25	04	7.25
09	04	5.63	04	5.68

TABLE XXXIII.

A Table of the Equation of the Centre of Jupiter.

Subtract.

Degrees.	Sign 0. De. dec. p.	Log. di.	Sign 1. De. dec. p.	Log. di.	Sign 2. De. dec. p.	Log. di.	Degrees.
0	00.0000	473645	02.6250	473400	04.6394	472706	30
1	00.0911	473645	02.7047	473385	04.6892	472677	29
2	00.1819	473644	02.7839	473366	04.7381	472648	28
3	00.2728	473642	02.8628	473349	04.7856	472616	27
4	00.3639	473640	02.9408	473331	04.8317	472586	26
5	00.4547	473639	03.0183	473312	04.8764	472555	25
6	00.5453	473636	03.0947	473294	04.9194	472524	24
7	00.6356	473631	03.1703	473272	04.9617	472485	23
8	00.7261	473626	03.2450	473255	05.0022	472462	22
9	00.8158	473620	03.3189	473235	05.0417	472429	21
10	00.9050	473615	03.3922	473214	05.0794	472397	20
11	00.9947	473609	03.4642	473193	05.1158	472368	19
12	01.0839	473605	03.5356	473169	05.1511	472331	18
13	01.1731	473599	03.6056	473148	05.1844	472296	17
14	01.2619	473592	03.6750	473126	05.2167	472263	16
15	01.3503	473584	03.7439	473102	05.2475	472230	15
16	01.4386	473575	03.8111	473079	05.2764	472197	14
17	01.5264	473565	03.8772	473056	05.3036	472162	13
18	01.6139	473556	03.9428	473031	05.3300	472127	12
19	01.7008	473546	04.0072	473006	05.3553	472093	11
20	01.7875	473536	04.0703	472981	05.3781	472058	10
21	01.8739	473524	04.1328	472955	05.3983	472023	9
22	01.9597	473513	04.1936	472929	05.4178	471988	8
23	02.0453	473500	04.2539	472903	05.4356	471953	7
24	02.1300	473487	04.3128	472875	05.4519	471917	6
25	02.2144	473474	04.3706	472848	05.4672	471883	5
26	02.2978	473460	04.4269	472821	05.4803	471847	4
27	02.3806	473446	04.4822	472792	05.4919	471810	3
28	02.4569	473431	04.5358	472764	05.5022	471774	2
29	02.5439	4734 7	04.5883	472735	05.5103	471737	1
30	02.6250	473400	04.6394	472706	05.5175	471700	0
	Sig. 11.		Sign 10.		Sign 9.		

Add

The Residue of Table XXXIII.

A Table of the Equation of the Centre of Jupiter.

Degrees	Subtract						Degrees
	Sign 3.		*Sign* 4.		*Sign* 5.		
	De. dec. p.	Log. di.	De. dec. p.	Log. di.	De. dec. p.	Log. di.	
0	05.5175	471704	04.9286	470621	02.9125	469776	30
1	05.5219	471657	04.8828	470587	02.8261	469755	29
2	05.5250	471631	04.8353	470554	02.7389	469736	28
3	05.5267	471594	04.7861	470519	02.6506	469717	27
4	05.5264	471558	04.7353	470487	02.5606	469698	26
5	05.5250	471521	04.6828	470455	02.4697	469680	25
6	05.5217	471484	04.6286	470423	02.3783	469663	24
7	05.5167	471445	04.5733	470390	02.2861	469645	23
8	05.5100	471406	04.5164	470358	02.1925	469630	22
9	05.5017	471372	04.4581	470327	02.0981	469615	21
10	05.4911	471336	04.3983	470297	02.0022	469600	20
11	05.4794	471299	04.3367	470266	01.9064	469586	19
12	05.4656	471262	04.2742	470236	01.8097	469572	18
13	05.4500	471225	04.2097	470207	01.7122	469560	17
14	05.4331	471188	04.1442	470177	01.6142	429547	16
15	05.4142	471152	04.0775	470149	01.5158	469536	15
16	05.3939	471116	04.0094	470121	01.4167	469526	14
17	05.3711	471079	03.9392	470092	11.3175	469516	13
18	05.3469	4710 3	03.8678	470065	11.2178	469507	12
19	05.3208	471007	03.7950	470037	11.1178	469499	11
20	05.2933	470970	03.7208	470012	01.0172	469491	10
21	05.2653	470935	03.6453	469986	00.9164	469484	9
22	05.2350	470899	03.5683	469960	00.8153	469478	8
23	05 2019	470863	03.4903	469934	00.7139	469472	7
24	05.1681	470828	03.4108	469910	00.6125	469468	6
25	05.1322	470793	03.3303	469887	00.5108	469463	5
26	05.0947	470759	03.2489	469863	00.4089	469460	4
27	05.0558	470724	03.1664	469841	00.3067	469457	3
28	05.0147	470689	03.0831	469819	00.2044	469455	2
29	04.9725	470655	02.9983	469797	00.1025	469454	1
30	04.9286	470621	02.9125	469776	00.0000	469453	0
	Sign 8.		*Sign* 7.		*Sign* 6.		
	Add						

TABLE XXXIV.

A Table of the Inclination of Jupiter, *and of his Reduction to the Ecliptick.*

Degrees.	Inclination, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Reduction, Subtract, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Degrees.
0	00.0000	00.6611	01.1953	00.0000	00.0067	00.0067	03
1	00.0231	00.6808	01.1564	00.0006	00.0069	00.0067	29
2	00.0461	00.7003	01.1672	00.0008	00.0069	00.0064	28
3	00.0692	00.7194	01.1778	00.0011	00.0072	00.0064	27
4	00.0922	00.7386	01.1878	00.0014	00.0072	00.0061	26
5	00.1153	00.7575	01.1978	00.0017	00.0072	00.0061	25
6	00.1383	00.7764	01.2072	00.0019	00.0075	00.0058	24
7	00.1611	00.7950	01.2164	00.0022	00.0075	00.0056	23
8	00.1842	00.8136	01.2253	00.0025	00.0075	00.0056	22
9	00.2069	00.8319	01.2339	00.0028	00.0078	00.0053	21
10	00.2297	00.8500	01.2422	00.0028	00.0078	00.0150	20
11	00.2525	00.8681	01.2503	00.0031	00.0078	00.0050	19
12	00.2750	00.8858	01.2578	00.0033	00.0078	00.0047	18
13	00.2975	00.9033	01.2650	00.0036	00.0078	00.0044	17
14	00.3200	00.9206	01.2717	00.0039	00.0078	00.0042	16
15	00.3422	00.9372	01.2778	00.0039	00.0071	00.0039	15
16	00.3642	00.6539	01.2836	00.0042	00.0078	00.0039	14
17	00.3867	00.9700	01.2889	00.0044	00.0078	00.0036	13
18	00.4086	00.9856	01.2936	00.0047	00.0078	00.0033	12
19	00.4306	01.0008	01.2981	00.0050	00.0078	00.0031	11
20	00.4522	01.0158	01.3019	00.0050	00.0078	00.0028	10
21	00.4739	01.0303	01.3056	00.0053	00.0078	00.0028	9
22	00.4953	01.0444	01.3086	00.0056	00.0075	00.0025	8
23	00.5167	01.0583	01.3114	00.0056	00.0075	00.0022	7
24	00.5378	01.0717	01.3139	00.0058	00.0075	00.0019	6
25	00.5589	01.0847	01.3161	00.0061	00.0072	00.0017	5
26	00.5797	01.0975	01.3181	00.0061	00.0072	00.0014	4
27	00.6003	01.1100	01.3197	00.0064	00.0072	00.0011	3
28	00.6208	01.1222	01.3211	00.0064	00.0069	00.0008	2
29	00.6411	01.1339	01.3219	00.0067	00.0069	00.0006	1
30	00.6611	01.1953	01.1453	00.0067	00.0067	00.0000	0
	Sign 11, Sign 5.	Sign 10, Sign 4,	Sign 9, Sign 3,	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	

TABLE XXXV.

The Epochæ *of the middle Motions of* Mars, *in the Years of the Julian Period, for the Meridian of* Urenburgh, *in the Kalends of* January, *the Mid-day of Astronomical Time, according to the Julian Account.*

Middle Motion of Mars *from the* Æquinox.		*Middle Motion of the Aphelion of* Mars *from the* Æquinox.			*Middle Mot. of the northern Node of* Mars *from the* Æquinox.		
In a comp. Year of the Julian Period.	*Decimal parts of a Circle.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees with decimal parts.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees with decimal parts.*
0	147958	0	01	1.6012	0	11	7.0439
1000	860996	1000	01	20.1973	1000	11	18.0844
2000	574035	2000	02	8.7934	2000	11	29.1248
3000	287073	3000	02	27.3895	3000	00	10.1652
4000	0001 2	4000	03	15.9856	4000	00	21.2058
5000	713150	5000	04	4.5817	5000	01	2.2462
6000	426188	6000	04	23.1778	6000	01	13.2867
Logarithm of the mid. Mot. of Mars *in the Tropical Year.* 1.72566.83		*Logarithm of the middle Motion of the Aphelion of* Mars *in the Tropical Year.* $\bar{2}$.26941.30			*Logarithm of the middle Motion of the northern Node of* Mars *in a Tropical Year.* $\bar{2}$.04297.83		

Middle Motion of Mars *from the* Æquinox *in the Tropical Years.*		*The Bounds of the Stations of* Mars.			
		The Anom. of the Eccentrick.	*The Angle of Commutation, or the Angle to the Sun.*		
			First.		*Second.*
Trop. Years	*Decimal parts of a Circle.* 0.12345.678	Sig.	Sig.	De. d. p. cir.	Sig. De. d. p. cir.
1	53170.188	00	05	7.67	05 8.22
2	06340 376	02	05	10.37	05 11.22
3	59510 563	03	05	14.00	05 14.38
4	12680.751	04	05	17.28	05 17.28
5	65850.939	06	05	20.33	05 19.82
6	19021.127	08	05	15.13	05 15.13
7	72191.314	09	05	12.77	05 12.45
8	25361.502	10	05	6.93	05 9.90
9	78531.690				

TABLE XXXVI.

A Table of the Equation of the Centre of Mars.

Subtract.

Degrees.	*Sign* 0.		*Sign* 1.		*Sign* 2.		Degrees.
	De. dec. p.	Log. di.	De. dec. p.	Log. di.	De. dec. p.	Log. di.	
0	00.0000	422131	04.8175	421714	08.6575	420499	30
1	00.1661	422131	04.9653	421685	08.7728	420447	29
2	00.3319	422130	05.1142	421654	08.8544	420392	28
3	00.4978	422128	05.2614	421625	08.9497	420339	27
4	00.6633	422123	05.4069	421593	09.0447	420284	26
5	00.8319	422121	05.5517	421564	09.1344	420230	25
6	00.9947	422115	05.6939	421529	09.2231	420172	24
7	01.1600	422110	05.8361	421498	09.3086	420115	23
8	01.3253	422102	05.9778	421463	09.3906	420058	22
9	01.4897	422094	06.1186	421426	09.4731	420028	21
10	01.6544	422084	06.2572	421392	09.5517	419940	20
11	01.8186	422076	06.3928	421354	09.6278	419882	19
12	01.9822	422066	06.5275	421317	09.7014	419821	18
13	02.1458	422053	06.6603	421277	09.7722	419761	17
14	02.3089	422040	06.7925	421240	09.8406	419700	16
15	02.4708	422027	06.9217	421200	09.9061	419637	15
16	02.6319	422011	07.0494	421157	09.9717	419576	14
17	02.7931	421995	07.1761	421115	10.0328	419512	13
18	02.9533	421979	07.3000	421072	10.0900	419449	12
19	03.1133	421964	07.4272	421029	10.1447	419388	11
20	03.2722	421945	07.5450	420984	10.1969	419324	10
21	03.4306	421924	07.6647	420938	10.2464	419257	9
22	03.6022	421906	07.7822	420892	10.2931	419190	8
23	03.7450	421885	07.8997	420847	10.3364	419125	7
24	03.9011	421864	08.0169	420798	10.3778	419058	6
25	04.0561	421840	08.1311	420752	10.4147	418993	5
26	04.2111	421817	08.2375	420701	10.4494	418927	4
27	04.3644	421790	08.3456	420653	10.4814	418859	3
28	04.5106	421767	08.4519	420602	10.5100	418789	2
29	04.6675	421740	08.5556	420550	10.5356	418721	1
30	04.8175	421714	08.6575	420499	10.5583	418653	0
	Sig. 11.		*Sign* 10.		*Sign* 9.		

Add

The Residue of Table XXVIII.

A Table of the Equation of the Centre of the Mars.

Subtract.

Degrees.	*Sign* 3. De. dec. p.	Log. di.	*Sign* 4. De. dec. p.	Log. di.	*Sign* 5. De. dec. p.	Log. di.	Degrees.
0	10.5583	418653	09.7106	416536	05.8881	414768	30
1	10.5756	418585	09.6300	416468	05.7156	414721	29
2	10.5947	418515	09.5458	416400	05.5408	414681	28
3	10.6075	418446	09.4586	416331	05.3639	414638	27
4	10.6178	418375	09.3675	416265	05.1853	414597	26
5	10.6242	418307	09.2739	416197	04.9972	414557	25
6	10.6281	418236	09.1783	416131	04.8211	414520	24
7	10.6281	418164	09.0731	416065	04.6361	414485	23
8	10.6264	418096	08.9683	415999	04.4489	414448	22
9	10.6244	418024	08.8617	415936	04.2606	414414	21
10	10.6111	417952	08.7511	415872	04.0794	414383	20
11	10.5947	417880	08.6367	415809	03.8769	414351	19
12	10.5781	417811	08.5181	415746	03.6828	414323	18
13	10.5600	417739	08.3989	415682	03.4872	414295	17
14	10.5381	417667	08.2753	415622	03.2900	414270	16
15	10.5125	417595	08.1456	415560	03.0917	414245	15
16	10.4833	417525	08.0153	415500	02.8908	414220	14
17	10.4517	417452	07.8822	415442	02.6906	414198	13
18	10.4158	417383	07.7458	415385	02.4886	414179	12
19	10.3761	417310	07.6069	415327	02.2856	414161	11
20	10.3333	417240	07.4647	415269	02.0811	414145	10
21	10.2867	417170	07.3200	415217	01.8756	414129	9
22	10.2367	417097	07.1722	415161	01.6689	414117	8
23	10.1906	417026	07.0219	415109	01.4614	414107	7
24	10.1269	416556	06.8717	415057	01.2536	414098	6
25	10.0667	416885	06.7119	415005	01.0458	414092	5
26	09.9994	416814	06.5528	414956	00.8375	414085	4
27	09.9319	416747	06.3917	414907	00.6283	414079	3
28	09.8617	416681	06.2278	414857	00.4189	414073	2
29	09.7881	416605	06.0600	414811	00.2097	414068	1
30	09.7106	416536	05.8881	414768	00.0000	414060	0
	Sign 8		*Sign* 7.		*Sign* 6.		

Add

TABLE XXXVII.

A Table of the Inclination of Mars, *and of his Reduction to the Ecliptick.*

Degrees.	Inclination, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Reduction, Subtract, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Degrees.
0	00 0000	00.9208	01.5950	00.0000	00.0131	00.0131	30
1	00 0322	00.9483	01.6108	00.0003	00.0133	00.0128	29
2	00 0644	00.9756	01.6261	00.0008	00.0133	00.0125	28
3	00.0964	01.0025	01 6411	00.0014	00.0136	00.0122	27
4	00.1283	01.0292	01.6556	00.0019	00.0136	00.0119	26
5	00.1606	01 0558	01.6994	00.0025	00.0136	00.0117	25
6	00.1925	01.0819	01.6828	00.0031	00.0139	00.0114	24
7	00.2244	01.1078	01.6656	00.0036	00.0139	00.0108	23
8	00.2564	01 1333	01.7078	00.0042	00.0142	00.0106	22
9	00.2881	01.1586	01.7194	00.0044	00.0142	00.0103	21
10	00.3197	01.1836	01.7306	00.0050	00.0144	00.0100	20
11	00.3514	01.2083	01.7411	00.0056	00.0144	00.0094	19
12	00 3828	01.2325	01.7514	00.0061	00.0144	00 0092	18
13	00 4142	01.2564	01.7611	00 0067	00.0144	00.0089	17
14	00.4453	01.2797	01.7703	00.0072	00.0147	00.0083	16
15	00 4764	01.3028	01 7789	00.0078	00.0147	00.0078	15
16	00.5072	01.3253	01.7869	00 0083	00.0147	00.0072	14
17	00.5381	01.3472	01.7944	00.0089	00.0147	00.0067	13
18	00.5686	01.3689	01.8014	00.0092	00.0144	00.0061	12
19	00.5992	01.3903	01.8078	00.0094	00.0144	00.0056	11
20	00 6294	01.4111	01.8136	00.0100	00.0144	00 0050	10
21	00 6594	01.4317	01.8189	00.0103	00 0142	00.0044	9
22	00 6894	01 4517	01.8 36	00.0106	00.0142	00.0042	8
23	00.7192	01.4711	01.8278	00.0108	00.0139	00.0036	7
24	00.7486	01.4903	01.8314	00.01 4	00.0139	00.0031	6
25	00.7778	01 5189	01.8344	00.0117	00.0136	00.0025	5
26	00.8069	01.5269	01.8369	00 0119	00 0136	00.0019	4
27	00.8358	01.5447	0 .8389	00.0122	00.0136	00.0014	3
28	00 8644	01.5619	01.8403	00.0125	00.0133	00.0008	2
29	00.8928	01.5786	01.8411	00.0128	00 0133	00.0003	1
30	00 9208	01.5950	01.8417	00.0131	00.0131	00.0000	0
	Sign 11, Sign 5.	Sign 10, Sign 4,	Sign 9, Sign 3,	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	

Add

TABLE XXXVIII.

The Epochæ *of the middle Motions of* Venus, *in the Years of the Julian Period, for the Meridian of* Urenburgh, *in the Kalends of* January, *the Mid-day of Astronomical Time, according to the Julian Account.*

Middle Motion of Venus *from the* Æquinox.		*Middle Motion of the Aphelion of* Venus *from the* Æquinox.			*Middle Mot. of the northern Node of* Venus *from the* Æquinox.		
In a comp. Year of the Julian Period.	*Decimal parts of a Circle.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees, with decimal parts.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees, with decimal parts.*
0	949785	0	05	14.3789	0	11	20.5925
1000	488472	1000	06	6.0580	1000	00	3.6480
2000	027158	2000	06	27.7372	2000	00	16.7036
3000	565845	3000	07	19.4164	3000	00	29.7591
4000	104532	4000	08	11.0955	4000	01	12.8147
5000	643219	5000	09	2.7747	5000	01	25.8703
6000	181905	6000	09	24.4539	6000	02	8.9258
Logarithm of the mid. Mot. of Venus *in the Tropical Year.* 0.21098.80		*Logarithm of the middle Motion of the Aphelion of* Venus *in the Tropical Year.* $\bar{2}$.33603.34			*Logarithm of the middle Motion of the northern Node of* Venus *in a Tropical Year.* $\bar{2}$.11578.61		

Middle Motion of Venus *from the* Æquinox *in the Tropical Years.*		*The Bounds of the Stations of* Mars.				
			The Angle of Commutation, or the Angle to the Sun.			
Trop. Years	*Decimal parts of a Circle.* 0.12345.678	*The Anom. of the Eccentrick.*	*First.*		*Second.*	
		Sig.	*Sig.*	*De. d. p. cir.*	*Sig.*	*De. d. p. cir.*
1	62550.457					
2	25100.913					
3	87651.370					
4	50201.827					
5	12752.283					
6	75302.740	00	05	17.78	05	17.97
7	37853.196	03	05	17.97	05	17.58
8	00403.653	06	05	16.92	05	16.72
9	62954.110	09	05	16.72	05	17.12

TABLE XXXIX.

A Table of the Equation of the Centre of Venus.

Subtract

Degrees.	Sign 0. De. dec. p.	Log. di.	Sign 1. De. dec. p.	Log. di.	Sign 2. De. dec. p.	Log. di.	Degrees.
0	00.0000	386279	00.3931	386243	00.6828	386136	30
1	00.0133	386279	00.4047	386237	00.6897	386130	29
2	00.0267	386279	00.4164	386237	00.6961	386124	28
3	00.0400	386279	00.4278	386231	00.7028	386118	27
4	00.0542	386279	00.4392	386231	00.7092	386118	26
5	00.0678	386279	00.4505	386225	00.7153	386112	25
6	00.0814	386279	00.4617	386225	00.7211	386106	24
7	00.0950	386279	00.4725	386219	00.7264	386100	23
8	00.1083	386279	00.4836	386219	00.7322	386094	22
9	00.1219	386279	00.4944	386213	00.7372	386094	21
10	00.1353	386279	00.5053	386213	00.7425	386088	20
11	00.1486	386278	00.5164	386207	00.7478	386082	19
12	00.1622	386273	00.5264	386207	00.7517	386076	18
13	00.1758	386273	00.5369	386201	00.7561	386070	17
14	00.1892	386273	00.5469	386201	00.7606	386064	16
15	00.2025	386273	00.5567	386195	00.7644	386058	15
16	00.2158	386273	00.5664	386189	00.7678	386057	14
17	00.2292	386267	00.5761	386189	00.7711	386052	13
18	00.2419	386267	00.5856	386183	00.7742	386046	12
19	00.2553	386267	00.5947	386177	00.7769	386040	11
20	00.2681	386261	00.6039	386177	00.7792	386034	10
21	00.2808	386261	00.6128	386171	00.7814	386028	9
22	00.2933	386261	00.6211	386165	00.7836	386027	8
23	00.3058	386255	00.6294	386165	00.7858	386022	7
24	00.3186	386255	00.6375	386159	00.7875	386016	6
25	00.3311	386255	00.6456	386153	00.7892	386010	5
26	00.3436	386249	00.6533	386153	00.7906	386004	4
27	00.3558	386249	00.6608	386147	00.7914	385998	3
28	00.3681	386249	00.6681	386141	00.7922	385997	2
29	00.3806	386249	00.6756	386136	00.7931	385992	1
30	00.3931	386243	00.6828	386136	00.7931	385986	0
	Sign 11.		Sign 10.		Sign 9.		

Add

The Residue of Table XXXIX.

A Table of the Equation of the Centre of Venus.

Subtract

Degrees.	Sign 3. De. dec. p.	Log. di.	Sign 4. De. dec. p.	Log. di.	Sign 5. De. dec. p.	Log. di.	Degrees.
0	00.7931	385986	00.6897	385830	00.4000	385722	30
1	00.7933	385981	00.6831	385829	00.3878	385715	29
2	00.7933	385973	00.6761	385824	00.3756	385714	28
3	00.7931	385968	00.6686	385818	00.3633	385713	27
4	00.7925	385962	00.6611	385812	00.3508	385709	26
5	00.7914	385956	00.6533	385811	00.3383	385708	25
6	00.7900	385950	00.6453	385806	00.3256	385708	24
7	00.7883	385944	00.6375	385800	00.3125	385703	23
8	00.7867	385943	00.6292	385799	00.3000	385703	22
9	00.7850	385938	00.6208	385794	00.2869	385702	21
10	00.7828	385932	00.6125	385788	00.2739	385697	20
11	00.7803	385926	00.6036	385787	00.2606	385696	19
12	00.7778	385920	00.5944	385781	00.2472	385695	18
13	00.7750	385914	00.5847	385775	00.2336	385691	17
14	00.7719	385908	00.5753	385774	00.2203	385691	16
15	00.7686	385907	00.5656	385769	00.2067	385690	15
16	00.7650	385902	00.5556	385763	00.1931	385685	14
17	00.7614	385896	00.5467	385762	00.1797	385684	13
18	00.7572	385890	00.5353	385757	00.1661	385684	12
19	00.7528	385884	00.5247	385754	00.1525	385684	11
20	00.7483	385878	00.5139	385751	00.1389	385683	10
21	00.7436	385872	00.5031	385745	00.1253	385682	9
22	00.7386	385871	00.4919	385743	00.1117	385682	8
23	00.7333	385866	00.4808	385739	00.0978	385679	7
24	00.7275	385860	00.4697	385737	00.0836	385679	6
25	00.7219	385854	00.4583	385735	00.0694	385679	5
26	00.7157	385848	00.4469	385733	00.0556	385679	4
27	00.7097	385842	00.4356	385731	00.0417	385679	3
28	00.7033	385842	00.4236	385727	00.0275	385679	2
29	00.6967	385836	00.4119	385724	00.0136	385679	1
30	00.6897	385830	00.4000	385722	00.0000	385679	0
	Sign 8.		Sign 7.		Sign 6.		

Add

TABLE XL.

A Table of the Inclination of Venus, *and of her Reduction to the Ecliptick.*

Degrees.	Inclination, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Reduction, Subtract, Sign 0, Sign 6, De. p. d.	Sign 1, Sign 7, De. p. d.	Sign 2, Sign 8, De. p. d.	Degrees.
0	00.0000	01.6833	02.9156	00.0000	00.0431	00.0431	30
1	00.0589	01.7342	02.9447	00.0019	00.0439	00.0422	29
2	00.1178	01.7842	02.9731	00.0036	00.0450	00.0411	28
3	00.1764	01.3333	03.0003	00.0053	00.0458	00.0403	27
4	00.2353	01.8822	03.0264	00.0072	00.0464	00.0392	26
5	00.2936	01.9308	03.0514	00.0089	00.0472	00.0378	25
6	00.3519	01.9789	03.0756	00.0106	00.0478	00.0367	24
7	00.4100	02.0264	03.0987	00.0125	00.0483	00.0356	23
8	00.4681	02.0731	03.1214	00.0142	00.0486	00.0342	22
9	00.5261	02.1189	03.1431	00.0158	00.0489	00.0331	21
10	00.5842	02.1642	03.1636	00.0172	00.0489	00.0317	20
11	00.6419	02.2089	03.1831	00.0189	00.0492	00.0306	19
12	00.6969	02.2531	03.2017	00.0203	00.0494	00.0292	18
13	00.7575	02.2964	03.2194	00.0217	00.0494	00.0278	17
14	00.8147	02.3389	03.2361	00.0233	00.0497	00.0261	16
15	00.8714	02.3806	03.2519	00.0247	00.0497	00.0247	15
16	00.9278	02.4217	03.2667	00.0261	00.0497	00.0233	14
17	00.9839	02.4622	03.2778	00.0278	00.0494	00.0217	13
18	01.0400	02.5019	03.2933	00.0292	00.0494	00.0203	12
19	01.0958	02.5408	03.3050	00.0306	00.0492	00.0189	11
20	01.1514	02.5792	03.3158	00.0317	00.0489	00.0172	10
21	01.2067	02.6167	03.3256	00.0331	00.0489	00.0158	9
22	01.2617	02.6533	03.3342	00.0342	00.0486	00.0142	8
23	01.3158	02.6889	03.3419	00.0356	00.0483	00.0125	7
24	01.3694	02.7236	03.3486	00.0367	00.0478	00.0106	6
25	01.4225	02.7578	03.3539	00.0378	00.0472	00.0089	5
26	01.4753	02.7911	03.3586	00.0392	00.0464	00.0072	4
27	01.5278	02.8233	03.3619	00.0403	00.0458	00.0053	3
28	01.5800	02.8550	03.3644	00.0411	00.0450	00.0036	2
29	01.6319	02.8856	03.3661	00.0422	00.0439	00.0019	1
30	01.6833	02.9156	03.3667	00.0431	00.0431	00.0000	0
	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	Sign 11, Sign 5,	Sign 10, Sign 4,	Sign 9, Sign 3,	

Add

TABLE XLI.

The Epochæ *of the middle Motions of* Mercury, *in the Years of the Julian Period, for the Meridian of* Urenburgh, *in the Kalends of* January, *the Mid-day of Astronomical Time, according to the Julian Account.*

Middle Motion of Mercury *from the* Æquinox.		*Middle Motion of the Aphelion of* Mercury *from the* Æquinox.			*Middle Mot. of the northern Node of* Mercury *from the* Æquinox.		
In a comp. Year of the Julian Period.	*Decimal parts of a Circle.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees, with decimal parts.*	*In a comp. Year of the Julian Period.*	*Signs.*	*Degrees, with decimal parts.*
0	181505	0	02	9.0214	0	8	12.9378
1000	247954	1000	03	8.1372	1000	09	6.6167
2000	314403	2000	04	7.2530	2000	10	0.2956
3000	380852	3000	05	6.3689	3000	10	23.9745
4000	447301	4000	06	5.4847	4000	11	17.6534
5000	513750	5000	07	4.6005	5000	00	11.3322
6000	580199	6000	08	3.7164	6000	01	5.0111
Logarithm of the mid. Mot. of Mercury *in the Tropical Year.* 0.61825.50		*Logarithm of the middle Motion of the Aphelion of* Mercury *in the Tropical Year.* $\bar{2}$.46412.00			*Logarithm of the middle Motion of the northern Node of* Mercury *in a Trop. Year.* $\bar{2}$.37435.21		

Middle Motion of Merc. *from the* Æquinox *in the Tropical Years.*		*The Bounds of the Stations of* Mercury.				
		The Anom. of the Eccentrick.	*The Angle of Commutation, or the Angle to the Sun.*			
			First.		*Second.*	
Trop. Years	*Decimal parts of a Circle.* 0.12345.678	Sig.	Sig.	De.d.p.cir.	Sig.	De.d.p.cir.
1	15197.930					
2	30395.859	00	05	3.80	05	4.17
3	45593.789	02	05	0.77	05	0.22
4	60791.719	03	04	24.97	04	25.15
5	75989.648	04	04	22.03	04	20.95
6	91187.578	06	04	16.77	04	16.38
7	06385.507	08	04	21.45	04	22.28
8	21583.437	09	04	25.32	04	26.47
9	36781.367	10	05	00.95	05	1.57

TABLE XLII.

A Table of the Equation of the Centre of Mercury.

Subtract

Degrees.	*Sign* 0. De. dec. p.	Log. di.	*Sign* 1. De. dec. p.	Log. di.	*Sign* 2. De. dec. p.	Log. di.	Degrees.
0	00.0000	367173	09.7475	366464	18.1139	364316	30
1	00.3322	367164	10.0572	366417	18.3500	364217	29
2	00.6644	367164	10.3644	366361	18.5825	364118	28
3	00.9967	367163	10.6697	366313	18.8125	364018	27
4	01.3286	367154	10.9739	366257	19.0383	363919	26
5	01.6608	367145	11.2772	366200	19.2603	363809	25
6	01.9919	367136	11.5786	366143	19.4792	363709	24
7	02.3228	367128	11.8781	366087	19.6942	363599	23
8	02.6536	367117	12.1756	366030	19.9053	363498	22
9	02.9842	367108	12.4706	365973	20.1125	363387	21
10	03.3144	367089	12.7642	365906	20.3156	363276	20
11	03.6439	367070	13.0556	365839	20.5144	363165	19
12	03.9731	367052	13.3444	365782	20.7094	363043	18
13	04.3014	367034	13.6311	365706	20.8994	362931	17
14	04.6292	367015	13.9161	365639	21.0858	362808	16
15	04.9561	366987	14.1983	365571	21.2683	362696	15
16	05.2825	366969	14.4781	365495	21.4456	362572	14
17	05.6081	366941	14.7556	365418	21.6178	362449	13
18	05.9331	366922	15.0311	365350	21.7858	362324	12
19	06.2567	366885	15.3036	365273	21.9483	362201	11
20	06.5797	366857	15.5733	365186	22.1069	362076	10
21	06.9019	366820	15.8408	365108	22.2603	361941	9
22	07.2228	366792	16.1058	365031	22.4086	361815	8
23	07.5425	366755	16.3658	364943	22.5511	361679	7
24	07.8614	366717	16.6256	364856	22.6894	361542	6
25	08.1789	366680	16.8817	364777	22.8219	361405	5
26	08.4956	366633	17.1347	364689	22.9458	361267	4
27	08.8106	366596	17.3839	364591	23.0683	361140	3
28	09.1242	366549	17.6306	364503	23.1842	361991	2
29	09.4374	366502	17.8739	364404	23.2936	361853	1
30	09.7475	366464	18.1139	364316	23.3981	360713	0
	Sign 11		*Sign* 10.		*Sign* 9.		

Add

The Residue of Table XLII.

A Table of the Equation of the Centre of Jupiter.

Subtract

Degrees.	*Sign* 3. De. dec. p.	Log. di.	*Sign* 4. De. dec. p.	Log. di.	*Sign* 5. De. dec. p.	Log. di.	Degrees.
0	23.3981	360713	23.2986	355883	15.3294	350947	30
1	23.4958	360573	23.1711	355642	14.9017	350812	29
2	23.5878	360423	23.0344	355534	14.4931	350678	28
3	23.6739	360271	22.8883	355352	14.0617	350556	27
4	23.7533	360130	22.7339	355182	13.5892	350420	26
5	23.8267	359977	22.5697	354998	13.1739	350297	25
6	23.8994	359824	22.3967	354851	12.7167	350174	24
7	23.9542	359671	22.2131	354667	12.2514	350051	23
8	24.0078	359517	22.0203	354494	11.7786	349941	22
9	24.0547	359362	21.8178	354320	11.2989	349831	21
10	24.0947	359207	21.6069	354145	10.8117	349734	20
11	24.1278	359051	21.3850	353970	10.3175	349624	19
12	24.1539	358894	21.1539	353794	09.8161	349527	18
13	24.1725	358737	20.9125	353631	09.3086	349443	17
14	24.1839	358580	20.6617	353453	08.7958	349346	16
15	24.1881	358410	20.4008	353288	08.2758	349262	15
16	24.1847	358252	20.1308	353122	07.7497	349192	14
17	24.1742	358092	19.8506	352956	07.2194	349122	13
18	24.1558	357921	19.5614	352789	06.6836	349052	12
19	24.1294	357749	19.2614	352621	06.1433	348982	11
20	24.0953	357588	18.9525	352466	05.5983	348925	10
21	24.0536	357426	18.6336	352310	05.0500	348883	9
22	24.0036	357252	18.3047	352140	04.4981	348827	8
23	23.9453	357078	17.9656	351983	03.9422	348786	7
24	23.8789	356914	17.6178	351825	03.3842	348756	6
25	23.8039	356738	17.2597	351680	02.8233	348728	5
26	23.7203	356573	16.8922	351534	02.2611	348700	4
27	23.6278	356396	16.5164	351375	01.6975	348671	3
28	23.5269	356229	16.1297	351228	01.1325	348657	2
29	23.4172	356050	15.7339	351095	00.6481	348657	1
30	23.2986	355883	15.3294	350947	00.0000	348650	0
	Sign 8		*Sign* 7.		*Sign* 6.		

Add

TABLE XLIII.

A Table of the Inclination of Mercury, *and of his Reduction to the Ecliptick.*

Degrees.	Inclination,			Reduction, Subtract,			Degrees.
	Sign 0, *Sign* 6,	*Sign* 1, *Sign* 7,	*Sign* 2, *Sign* 8,	*Sign* 0, *Sign* 6,	*Sign* 1, *Sign* 7,	*Sign* 2, *Sign* 8,	
	De. p. d.	De. p. d.	De. p. d.	De. p. d.	De. p. d.	De. p. d.	
0	00.0000	03.4500	05.9756	00.0000	00.1803	00.1803	30
1	00.1203	03.5536	06.0350	00.0072	00.1839	00.1767	29
2	00.2408	03.6561	06.0925	00.0144	00.1872	00.1728	28
3	00.3611	03.7578	06.1481	00.0217	00.1903	00.1686	27
4	00.4814	03.8583	06.2017	00.0289	00.1931	00.1642	26
5	00.6014	03.0578	06.2536	00.0361	00.1956	00.1594	25
6	00.7211	04.0558	06.3036	00.0431	00.1981	00.1547	24
7	00.8406	04.1525	06.3517	00.0503	00.2003	00.1497	23
8	00.9600	04.2481	06.3975	00.0572	00.2022	00.1447	22
9	01.0792	04.3422	06.4414	00.0642	00.2039	00.1394	21
10	01.1981	04.4350	06.4833	00.0711	00.2053	00.1339	20
11	01.3164	04.5264	06.5233	00.0778	00.2064	00.1283	19
12	01.4344	04.6167	06.5617	00.0844	00.2072	00.1225	18
13	01.5519	04.7056	06.5981	00.0911	00.2078	00.1164	17
14	01.6689	04.7931	06.6325	00.0975	00.2081	00.1103	16
15	01.7856	04.8792	06.6647	00.1039	00.2083	00.1039	15
16	01.9017	04.9636	06.6950	00.1103	00.2081	00.0975	14
17	02.0172	05.0467	06.7231	00.1164	00.2078	00.0911	13
18	02.1322	05.1281	06.7492	00.1225	00.2072	00.0844	12
19	02.2464	05.2078	06.7733	00.1283	00.2064	00.0778	11
20	02.3597	05.2858	06.7653	00.1339	00.2053	00.0711	10
21	02.4725	05.3622	06.8150	00.1394	00.2039	00.0642	9
22	02.5847	05.4369	06.8328	00.1447	00.2022	00.0572	8
23	02.6961	05.5103	06.8486	00.1497	00.2003	00.0503	7
24	02.8067	05.5819	06.8650	00.1547	00.1981	00.0431	6
25	02.9161	05.6519	06.8736	00.1594	00.1956	00.0361	5
26	03.0247	05.7203	06.8831	00.1642	00.1931	00.0289	4
27	03.1325	05.7869	06.8906	00.1686	00.1903	00.0217	3
28	03.2394	05.8517	06.8958	00.1728	00.1872	00.0144	2
29	03.3453	05.9144	06.8989	00.1767	00.1839	00.0072	1
30	03.4500	05.9756	06.9000	00.1803	00.1803	00.0000	0
	Sign 11, *Sign* 5,	*Sign* 10, *Sign* 4,	*Sign* 9, *Sign* 3,	*Sign* 11, *Sign* 5,	*Sign* 10, *Sign* 4,	*Sign* 9, *Sign* 3,	

TABLE XLIV.

A Catalogue of the most noted fixed Stars.

The Names of the Stars.	Signs.	*Longitude.* D. p. d.	*Latitude.* D. p. d.	Signs.
The Southern Star in the Fore-horn of *Aries*, called the first.	00	0.000	7.142 *B*	4
The left Foot of *Andromeda.*	00	11.033	27.775 *B*	2
The bright Star of the Jaw of the Whale.	00	11.167	12.617 *A*	2
The Head of *Meduſa.*	00	23.000	22.367 *B*	3
The bright Star of *Pleiades.*	00	26.783	4.000 *B*	3
The right Side of *Perſeus.*	00	28.667	30.083 *B*	2
The Southern Eye of *Taurus*, *Aldebara.*	01	6.592	5.517 *A*	1
The left Foot of *Orion*, *Regel.*	01	13.667	31.183 *A*	1
The left Shoulder of *Orion.*	01	17.767	16.883 *A*	2
The Kid, or Hee-goat.	01	18.650	22.842 *B*	1
The firſt in the Girdle of *Orion.*	01	19.225	23.633 *A*	2
The right Ankle of *Auriga.*	01	19.375	5.333 *B*	2
The middlemoſt in the Girdle of *Orion.*	01	20.283	24.555 *A*	2
The laſt in the Girdle of *Orion.*	01	21.492	25.355 *A*	2
The laſt in the Tail of the leſſer Bear, called the Pole Star.	01	25.425	66.033 *B*	2
The right Shoulder of *Orion.*	01	25.583	16.100 *A*	2
The bright Star in the right Shoulder of *Auriga.*	01	28.250	21.455 *B*	2
In the bottom of the fore Foot of the Dog.	02	4.092	41.305 *A*	2
The bright Star of the left Foot of *Gemini.*	02	5.900	6.805 *A*	2
The greater Dog, *Sirius.*	02	10.975	39.500 *A*	1
The Northern Head of *Gemini*, *Caſtor.*	02	17.067	10.033 *B*	2
The Southern Head of *Gemini*, *Pollux.*	02	20.100	6.633 *B*	2
The leſſer Dog, *Procyon.*	02	22.692	15.950 *A*	2
Præſepe, the Breaſt of *Cancer.*	03	4.155	1.233 *B*	*ne.*
The Northern Aſs.	03	4.333	3.133 *B*	4
The Southern Aſs.	03	5.517	0.067 *B*	4
The Northern Fore-Star of *Charles's* Wane.	03	11.950	49.667 *B*	2
The Southern Fore-Star of *Charles's* Wane.	03	16.105	45.055 *B*	2
The Heart of *Hydra.*	03	24.142	22.400 *A*	1
The Neck of *Leo.*	03	26.367	8.783 *B*	2
The Heart of *Regulus.*	03	26.667	0.442 *B*	1
The Southern hinder Star of *Charles's* Wane.	03	27.133	47.105 *B*	2
The Northern hinder Star of *Charles's* Wane.	03	27.805	51.617 *B*	2
The third from the laſt in the Tail of the Dragon.	04	4.555	66.600 *B*	2
The Root of the Tail of the greater Bear.	04	5.550	54.300 *B*	2
The Back of *Leo.*	04	8.067	14.333 *B*	2

The Reſidue of Table XLIV.

A Catalogue of the moſt noted fixed Stars.

The Names of the Stars.	*Signs.*	*Longitude.* D. p.d.	*Latitude.* De. p. d.		
The laſt ſave one in the Tail of the greater Bear.	04	12.325	56.367	*B*	2
The Tail of *Leo*.	04	18.433	12.300	*B*	1
The irregular Star in *Charles*'s Wain about the Tail.	04	20.105	40.100	*B*	2
The laſt in the Tail of the greater Bear.	04	23.583	54.417	*B*	2
The Wing of the Virgin *Vindemiatrix*.	05	6.767	16.255	*B*	3
Spica Virginis.	05	20.650	1.983	*A*	1
The Star by the Tail of the greater Bear, *Arcturus*.	05	21.033	31.042	*B*	1
The bright Star in the *Gnoſian* Crown.	06	9.025	44.383	*B*	2
The Southern Scale of *Libra*.	06	11.900	0.433	*B*	2
The Northern Scale of *Libra*.	06	16.183	8.583	*B*	2
The bright Star in the neck of the Serpent.	06	18.883	25.583	*B*	2
The left hand of *Ophiuchus*.	06	29.125	17.317	*B*	3
The uppermoſt Star in the Forehead of *Scorpio*.	06	29.983	1.083	*B*	2
The Heart of *Scorpio*, *Antares*.	07	6.600	4.450	*A*	1
The Head of *Hercules*.	07	12.900	37.383	*B*	3
The Head of *Ophiuchus*.	07	19.217	35.950	*B*	3
The bright Star of *Lyra*.	08	12.100	61.792	*B*	1
The Bill of the Swan.	08	28.117	49.033	*B*	3
The more Northern of the three in the Horn of *Capricorn*.	09	0.683	7.033	*B*	3
The more Southern of the three in the Horn of *Capricorn*.	09	0.900	4.683	*B*	3
The foremoſt of the two bright Stars in the Tail of *Capricorn*.	09	18.617	2.433	*A*	3
The left Shoulder of *Aquarius*.	09	20.233	8.700	*B*	3
The hinder part of the Tail of *Capricorn*.	09	20.383	2.483	*A*	3
The Breaſt of the Swan.	09	21.800	57.150	*B*	3
The Mouth of *Pegaſus*.	09	28.750	22.125	*B*	3
The right Shoulder of *Aquarius*.	10	0.200	10.700	*B*	3
Fomahant.	10	0.575	21.000	*A*	1
The Tail of the Swan.	10	2.275	59.942	*B*	2
Marchab Pegaſus.	10	20.325	19.433	*B*	2
Scheat Pegaſus.	10	26.200	31.125	*B*	2
The Tail of the Whale.	10	29.317	20.783	*A*	2
The laſt in the Wing of *Pegaſus*.	11	6.013	12.583	*B*	2
The Head of *Andromeda*.	11	11.167	25.700	*B*	2
The knot of the Fiſh-net.	11	26.175	9.075	*A*	3
The Girdle of *Andromeda*.	11	27.200	25.983	*B*	2

TABLE XLV.

A Table of Declinations, and of the Angles of the Ecliptick and Meridian.

Declinations.				*Angles of the Ecliptick and Meridian.*			
Degrees.	♈ ♎	♉ ♏	♊ ♐	♈ ♎	♉ ♏	♊ ♐	Degrees.
	De. p. d.	De. p. d.	De. p. d.	De. p. d.	De. p. d.	De. p. d.	
0	00 0000	11.5003	20.2017	66.50	69.37	77 73	30
1	00.3986	11.8511	20.4111	66.50	69.57	78.10	29
2	00.7975	12.1989	20.6142	66.52	69.77	78.47	28
3	01.1958	12.5431	20 8111	66.53	69.97	78.83	27
4	01.5939	12.8839	21.0017	66.55	70.18	79.20	26
5	01.9917	13.2214	21.1856	66.58	70.40	79.58	25
6	02.3889	13.5550	21.3631	66.62	70.62	79.97	24
7	02.7853	13.8850	21.5339	66.65	70.85	80.35	23
8	03.1814	14.2111	21.6981	66.70	71.08	80.75	22
9	03.5761	14.5333	21.8553	66.77	71.33	81.15	21
10	03.9706	14.8514	22.0058	66.82	71.58	81.55	20
11	04.3636	15.1653	22.1494	66.88	71.83	81.95	19
12	04.7556	15.4747	22.2861	66.97	72.10	82.35	18
13	05.1464	15.7800	22.4158	67.03	72.37	82.75	17
14	05.5358	16.0808	22.5383	67.12	72.63	83.17	16
15	05.9236	16.3772	22.6539	67.22	72.92	83.58	15
16	06.3103	16.6686	22.7619	67.32	73.20	84.00	14
17	06.6950	16.9553	22.8631	67.42	73.48	84.42	13
18.	07.0781	17.2372	22.9567	67.53	73.78	84.83	12
19	07.4592	17.5142	23.0431	67.65	74.08	85.25	11
20	07 8383	17.7858	23. 1219	67.78	74.38	85.68	10
21	08 2156	18.0525	23·1936	67.90	74 70	86.12	9
22	08 5906	18 3136	23.2578	68.05	75.02	86.53	8
23	08.9633	18.5694	23.3144	68.18	75.33	86.97	7
24	09 3339	18.8200	23.3636	68.33	75.67	87.40	6
25	09.7017	19.0647	23.4053	68.50	76.00	87.83	5
26	10.0669	19.3039	23.4394	68.65	76 33	88.27	4
27	10.4297	19.5372	23.4658	68.82	76.68	88.70	3
28	10.7894	19.7647	23.4847	69.00	77.02	89.13	2
29	11.1464	19.9864	23.4961	69.18	77.38	89.57	1
30	11.5003	20.2017	23.5000	69.37	77.73	90.00	0
	♓ ♍	♒ ♌	♑ ♋	♍ ♓	♒ ♌	♑ ♋	

TABLE XLVI.

A Table of Right Ascensions.

Degrees.	Sign ♈	♉	♊	♋	♌	♍
	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.
0	000.0000	027.8997	057.8069	090.0000	122.1931	152.1203
1	000.9169	028.8558	058.8494	091.0906	123.2331	153.0542
2	001.8352	029.8144	059.8950	092.1808	124.2700	154.0058
3	002.7517	030.7758	060.9419	093.2708	125.3039	154.9550
4	003.6692	031.7394	061.9939	094.3606	126.3350	155.9019
5	004.5872	032.7058	063.0475	095.4497	127.3631	156.8469
6	005.5056	033.6747	064.1036	096.5381	128.3881	157.7897
7	006.4244	034.6467	065.1622	097.6258	129.4103	158.7306
8	007.3442	035.6211	066.2233	098.7128	130.4292	159.6694
9	008.2644	036.5983	067.2867	099.7989	131.4453	160.6067
10	009.1853	037.5783	068.3525	100.8836	132.4581	161.5419
11	010.1072	038.5614	069.4203	101.9672	133.4681	162.4756
12	011.0303	039.5472	070.4906	103.0497	134.4750	163.4075
13	011.9542	040.5361	071.5625	104.1306	135.4789	164.3381
14	012.8792	041.5281	072.6367	105.2097	136.4794	165.2669
15	013.8056	042.5228	073.7125	106.2875	137.4772	166.1944
16	014.7331	043.5206	074.7903	107.3633	138.4719	167.1208
17	015.6619	044.5211	075.8694	108.4375	139.4639	168.0458
18	016.5925	045.5250	076.9503	109.5094	140.4528	168.9697
19	017.5244	046.5319	078.0328	110.5797	141.4386	169.8928
20	018.4553	047.5419	079.1164	111.6475	142.4217	170.8147
21	019.3933	048.5547	080.2011	112.7133	143.4017	171.7356
22	020.3306	049.5708	081.2872	113.7767	144.3789	172.6558
23	021.2694	050.5897	082.3742	114.8378	145.3533	173.5756
24	022.2103	051.6119	083.4619	115.8992	146.3253	174.4944
25	023.1531	052.6369	084.5503	116.9525	147.2942	175.4128
26	024.0981	053.6650	085.6394	118.0061	148.2606	176.3308
27	025.0450	054.6961	086.7292	119.0569	149.2242	177.2483
28	025.9942	055.7300	087.8192	120.1050	140.1856	178.1658
29	026.9458	056.7669	088.9094	121.1506	151.1442	179.0831
30	027.8997	057.8069	090.0000	122.1931	152.1003	180.0000

The Residue of Table XLVI.

A Table of the Right Ascension.

Degrees.	Sign ♎	♏	♐	♑	♒	♓
	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.	De. dec. p.
0	180.0000	207.8997	237.8069	270.0000	302.1931	332.1003
1	180.9169	208.8558	238.8494	271.0906	303.2331	333.0542
2	181.8342	209.8144	239.8950	272.1808	304.2700	334.0058
3	182.7517	210.7758	240.9419	273.2708	305.3039	334.9550
4	183.6692	211.7399	241.9939	274.3606	306.3350	335.9019
5	184.5872	212.7058	243.0475	275.4497	307.3631	336.8469
6	185.5056	213.6747	244.1036	276.5381	308.3881	337.7897
7	186.4244	214.6467	245.1622	277.6258	309.4103	338.7306
8	187.3442	215.6211	246.2233	278.7128	310.4292	339.6694
9	188.2644	216.5983	247.2867	279.7989	311.4453	340.6067
10	189.1853	217.5773	248.3525	280.8836	312.4581	341.5419
11	190.1072	218.5614	249.4203	281.9672	313.4681	342.4756
12	191.0303	219.5472	250.4906	283.0497	314.4750	343.4075
13	191.9542	220.5361	251.5625	284.1306	315.4789	345.3381
14	192.8792	221.5281	252.6267	285.2097	316.4794	345.2669
15	193.8056	222.5228	253.7125	286.2875	317.4772	346.1944
16	194.7331	223.5206	254.7903	287.3633	318 4719	347.1208
17	195.6619	224.5211	255.8694	288.4375	319.4639	348.0458
18	196.5925	225.5250	256.9503	289.5094	320.4528	348.9697
19	197.5244	226.5319	258.0328	290.5797	321.4386	349.8928
20	198.4553	227.5419	259.1164	291.6475	322.4217	350.8147
21	199.3933	228.5547	260.2011	292.7133	323.4017	351.7356
22	200.3306	229.5708	261.2872	293.7767	324.3789	352.6558
23	201.2694	230.5897	262.3742	294.8378	325.3533	353.5756
24	202.2103	231.6119	263.4619	295.8992	326.3253	354.4944
25	203.1531	232.6369	264.5503	296.9525	327.2942	355.4128
26	204.0981	233.6650	265.6394	298.0061	328.2606	356.3308
27	205.0450	234.6961	266.7292	299.0569	329.2242	357.2483
28	205.9942	235.7300	267.8191	300.1050	330.1856	358.1658
29	206.9458	236.7669	268.9094	301.1506	331.1442	359.0831
30	207.8997	237.8069	270.0000	302.1931	332.1003	360.0000

AN APPENDIX.

CONTAINING

Divers neceſſary Rules and Tables

Relating to

I. Gauging of Brewers Tuns and Coppers, and of Wine or Ale Veſſels.

II. Meaſuring of Timber Growing, and of the Works of the ſeveral Artificers belonging to the Building of Houſes or other Edifices.

Procœme.

I Intend not here to treat of the whole Art of Practical *Gauging*; neither of the Meaſuring of Timber growing or Un-hewn; nor of the Works of the ſeveral *Artificers* belonging to *Building*: As the *Bricklayers*, *Carpenters*, *Maſons*, *Plaiſterers*, *Joyners*, *Painters*, *Glaſiers*, *Paviers*, &c. For he that can meaſure Superficies and Solids, both Regular and Irregular, (the performance whereof is taught at large in the Second Book of Geometry, in the Third Part thereof, from Page 301 to Page 330, and that both *Arithmetical* and *Inſtrumentally*) cannot be ignorant of the performance of any of theſe things. And therefore, I ſhall here only give a ſhort Account of ſuch *Cuſtoms* as are uſed in the *Menſurations* of theſe *Particular Works*; and exhibit to view ſome Tables ready Calculated which may be of good uſe to ſuch Gentlemen and others who may have occaſion to make uſe of them.

SECT.

SECTION I.

Of the Gauging of Wine or Ale Vessels, and to find the content of them in Wine or Ale Gallons, and parts of Gallons: Also of Brewers Tuns and Coppers, and to find their Contents in Barrels, and parts of Barrels.

A Table for the exact and ready Gageing of any close Cask or Vessel, and to give the content thereof in Wine Gallons, and 1000 parts of a Gallon.

The Number of whole Inches in the Diametre of any Cask.	Quarters of Inches. 0 Quar.	1 Quar.	2 Quar.	3 Quar.
10	0.1133	0.1190	0.1250	0.1330
11	0.1371	0.1434	0.1499	0.1565
12	0.1632	0.1701	0.1771	0.1842
13	0.1915	0.1990	0.2066	0.2143
14	0.2221	0.2300	0.2383	0.2466
15	0.2550	0.2636	0.2723	0.2811
16	0.2991	0.2993	0.3086	0.3180
17	0.3275	0.3372	0.3471	0.3570
18	0.3672	0.3775	0.3879	0.3984
19	0.4091	0.4200	0.4319	0.4420
20	0.4533	0.4647	0.4763	0.4880
21	0.4998	0.5117	0.5272	0.5362
22	0.5485	0.5609	0.5734	0.5866
23	0.5995	0.6126	0.6259	0.6393
24	0.6528	0.6665	0.6803	0.6943
25	0.7083	0.7226	0.7370	0.7515
26	0.7661	0.7808	0.7969	0.8110
27	0.8262	0.8416	0.8571	0.8727
28	0.8885	0.9044	0.9206	0.9368
29	0.9531	0.9696	0.9863	1.0031
30	1.0200	1.0370	1.0543	1.0716
31	1.0891	1.1064	1.1245	1.1425
32	1.1605	1.1787	1.1971	1.2156
33	1.2342	1.2530	1.2719	1.2910
34	1.3101	1.3298	1.3490	1.3686
	1.3833			
35	1.4688	1.4083	1.4283	1.4485
36	1.5513	1.4893	1.5099	1.5306
37	1.6365	1.5726	1.5938	1.6184
38	1.7238	1.6581	1.6799	1.7018
39	1.8133	1.7460	1.7683	1.7908
40	1.9051	1.8361	1.8589	1.8819
41	1.9992	1.9285	1.9519	1.9755
42	2.0955	2.0231	2.0471	2.0713
43	2.1941	2.1199	2.1445	2.1693
44	2.2950	2.2291	2.2443	2.2696
45	2.3981	2.3206	2.3463	2.3722
46	2.5035	2.4243	2.4506	2.4770
47	2.6112	2.5303	2.5571	2.5840
48	2.7211	2.6385	2.6659	2.6934
49	2.8333	2.7489	2.7736	2.8051
50	2.9478	2.8617	2.8867	2.9190
51	3.0645	2.9768	3.0059	3.0352
52	3.1835	3.0941	3.1237	3.1536
53	3.3048	3.2136	3.2439	3.2743
54	3.4283	3.3355	3.3663	3.3972
55	3.5541	3.4596	3.4910	3.5224
56	3.6822	3.5859	3.6179	3.6500
57	3.8125	3.7146	3.7471	3.7797
58	3.9451	3.8454	3.8786	3.9118
59	4.0800	9786	4.0123	4.0461
60		4.1141	4.1483	4.1826

A Table for the exact and ready Gageing of any close Cask or Vessel, and to give the Content thereof in Ale Gallons, and 1000 parts of a Gall.

The Number of whole Inches in the Diametre of any Cask or Vessel.	Quarters of Inches. 0 Qua.	1 Qua.	2 Qua.	3 Qua.
10	0.092	0.098	0.102	0.107
11	0.112	0.117	0.123	0.128
12	0.134	0.139	0.145	0.151
13	0.157	0.163	0.169	0.176
14	0.182	0.188	0.193	0.202
15	0.209	0.216	0.223	0.230
16	0.238	0.243	0.253	0.260
17	0.267	0.274	0.284	0.292
18	0.301	0.308	0.318	0.326
19	0.335	0.342	0.353	0.362
20	0.371	0.381	0.357	0.399
21	0.409	0.419	0.429	0.439
22	0.449	0.459	0.470	0.480
23	0.491	0.501	0.512	0.524
24	0.534	0.546	0.557	0.568
25	0.580	0.592	0.604	0.616
26	0.628	0.639	0.652	0.664
27	0.677	0.689	0.702	0.725
28	0.728	0.741	0.754	0.767
29	0.781	0.794	0.808	0.822
30	0.836	0.849	0.864	0.878
31	0.892	0.907	0.921	0.935
32	0.951	0.963	0.980	0.996
33	1.011	1.026	1.042	1.057
34	1.073	1.089	1.105	1.121
35	1.137	1.153	1.170	1.187
36	1.203	1.220	1.233	1.254
37	1.271	1.288	1.305	1.323
38	1.341	1.358	1.376	1.394
39	1.712	1.430	1.445	1.467
40	1.485	1.504	1.523	1.541
41	1.561	1.579	1.599	1.618
42	1.638	1.657	1.697	1.697
43	1.717	1.737	1.757	1.777
44	1.797	1.817	1.838	1.859
45	1.880	1.901	1.922	1.943
46	1.964	1.986	2.007	2.029
47	2.051	2.072	2.095	2.117
48	2.139	2.161	2.183	2.206
49	2.229	2.252	2.274	2.298
50	2.321	2.344	2.367	2.391
51	2.414	2.438	2.462	2.486
52	2.510	2.534	2.553	2.583
53	2.601	2.631	2.657	2.682
54	2.707	2.732	2.757	2.782
55	2.908	2.834	2.856	2.885
56	2.911	2.934	2.964	2.990
57	3.016	3.043	3.069	3.096
58	3.123	3.150	3.177	3.204
59	3.232	3.292	3.287	3.314
60	3.342	3.373	3.398	3.426

The Use of the Tables *for the* Gauging *of* Wine *or* Ale *Vessels.*

To Gauge any *Wine* or *Ale Cask* or *Vessel*, you must first take the *Diametres* at the *Head* and *Bung* of the *Vessel*, and also the *length* of the Vessel within; in Inches, and quarters of Inches, which being truly taken, the *Content* of the *Cask* in *Wine* or *Ale Gallons*, and parts of a *Gallon*, may exactly be found by these Tables.

Example 1. *Let there be a Wine Vessel, whose Diametre at the Bung is* 32 *Inches, at the Head* 17 *Inches, and the length thereof* 54 *Inches; and let the Content thereof be required in Wine Gallons, and parts of a Gallon.*

Look into the Table, and find 32 Inches (the Diametre at the Bung) in the first Column, against which stands 1.1605, which number you must set down twice. Also look in the first Column for 17 the Diametre at the Head, against which stands 0.3275, which number set down under the two former, and add all three together, and their Summ will be 2.6485, which Summ you must multiply by 54, the length of the Vessel, and the Product will be 143.0190, from which cut off four Figures to the right hand, and there will remain 143 Gallons and .019 thousand parts of a Gallon, which is not a quarter of a Pint. For

.125		1
.250		2
.375		3
.500	Parts cut off are equal to	4 Pints.
.625		5
.750		6
.875		7

See the following Work.

Diametre at the Bung 32 Inches	1.1605
The same again	1.1605
Diametre at the Head 17 Inches	0.3275
	2.6485
The length of the Cask 54 Inches	54
	105940
	132425
The Content of the Vessel	143.0190

143 Gallons, and .019 parts of a Gallon, which is not a quarter of a Pint.

Example 2. *Let there be a Wine Cask, whose Diametre at the Bung is* 29 *Inches and a half, at the Head* 23 *Inches and a quarter, and the length* 47 *Inches and a half, to find the Content in Wine Gallons.*

Look for 29 Inches in the first Column of the Table, and for 2 quarters (which is half an Inch) at the head of the Table, and against 29 and under 2 quarters, you shall find this number, 0.9863, which set down twice; Also look for 23 Inches in the first Column, and 1 quarter at the head of the Table; and against 23, and under 1 quarter you shall find this Number, 0.6126, which set under other two, and these three numbers added together, make 2.5852, which number multiplyed by 47 Inches and a half, (that is, by 47 5, for 25.8 for a quarter of an Inch, 5 for half, and 75 for 3 quarters of an Inch) the Product will be 122.79700, from which cut off the five last Figures to the right hand, and there will be 122 Gallons, and 797 thousand parts of a Gallon, which is somewhat above six Pints.

Diametre

Diametre at the Bung 29 ½ Inches	0.9863
The same again	0.9863
Diametre at the Head 23 ¼ Inches	0.6126
The Summ	2.5852
The length of the Vessel 47 ½ Inches	47.5
	129260
	180964
	103408
	122.79700

The reason why five Figures were now cut off is, because of the parts of an Inch in the length of the Cask, for had the length been only 47 Inches, you should then have cut off only 4 Figures as before; and had the length been 47 ¼ or 47 ¾ which are thus to be expressed 47.25, and 47.75, you must then have cut off 6 Figures, &c.

The Table for the Gauging of Beer or Ale Vessels, is to be used in all respects as that for the Gauging of Wine Cask. And therefore one *Example* shall serve to shew the Use of it. Which let be this following.

Example. *There is a Vessel, whose Diametre at the Bung is* 34 *Inches and a half, at the Bung* 26 *Inches and a quarter, and is long* 54 *Inches, How many Ale Gallons are contained therein?*

The Diametre at the Bung 34½	1.105
The same again	1.105
The Diametre at the Head 26 ¼	0.639
The Summ	2.849
The length of the Vessel	54
	11396
	14245
The Content	153 | 846

That is 153 Gallons, and almost 7 Pints.

A Table

A Table shewing the Content of any Round Brewers Tun in Barrels, and 10000 Parts of a Barrel.

The Diametre of the Tun in Inches.	B. Parts	The Diametre &c.	B. Parts	The Diametre &c.	B. Parts
1	0.0001	41	0.1300	81	0.5077
2	.0003	42	.1364	82	.5203
3	.0007	43	.1430	83	.5331
4	.0010	44	.1498	84	.5466
5	.0019	45	.1568	85	.5591
6	0.9028	46	0.1639	86	0.5723
7	.0038	47	.1711	87	.5857
8	.0050	48	.1784	88	.5992
9	.0063	49	.1858	89	.6129
10	.0077	50	.1934	90	.6268
11	0.0094	51	0.2012	91	0.6408
12	.0112	52	.2092	92	.6549
13	.0131	53	.2173	93	.6692
14	.0152	54	.2256	94	.6836
15	.0174	55	.2340	95	.6982
16	0.0198	56	0.2426	96	.7130
17	.0223	57	.2514	97	.7279
18	.0256	58	.2603	98	.7430
19	.0279	59	.2693	99	.7584
20	.0308	60	.2785	100	.7737
21	0.0341	61	0.2879	101	0.7893
22	.0374	62	.2974	102	.8050
23	.0409	63	.3071	103	.8209
24	.0445	64	.3169	104	.8369
25	.0484	65	.3269	105	.8530
26	0.0523	66	0.3371	106	.8693
27	.0564	67	.3474	107	.8858
28	.0606	68	.3578	108	.9034
29	.0650	69	.3684	109	.9192
30	.0696	70	.3791	110	.9361
31	0.0743	71	0.3900	111	0.9532
32	.0794	72	.4011	112	0.9705
33	.0843	73	.4124	113	0.9880
34	.0896	74	.4238	114	1.0056
35	.0959	75	.4353	115	1.0233
36	0.1005	76	0.4469	116	1.0411
37	.1061	77	.4588	117	1.0591
38	.1118	78	.4708	118	1.0772
39	.1177	79	.4830	119	1.0955
40	.1238	80	.4953	120	1.1140

The use of this Table.

Example, *There is a Round Tun, whose Diametre is* 81 *Inches, and it is deep in the Liquor* 48 *Inches; how many Barrels is there in this Tun?*

Look for 81, the Diametre of the *Tun*, in the first Column of this *Table*, and right against it, you shall find this *Number*, 0.5077, which being Multiplyed by the *depth* of the *Liquor*, produceth 24.3696, which is 24 Barrels, and .3696 thousand parts of a *Barrel*, and so many *Barrels* is in the *Tun*.

See the Work.

The Diametre 81, —	0.5077
Depth of Liquor 48 Inch.	48
	40616
	20308
The Content in *Barrels*, and Parts,	24.3696

Now how much this .3696 Parts of a *Barrel* is, you are to take notice, That

1.0000	is	1 *Barrel*,
.5000		1 *Kilderkin*,
.2500		1 *Firkin*,
.0278		1 *Gallon*,
.0139		1 *Pottle*,
.0069½		1 *Quart*,
.0035		1 *Pint*,

Now look in this *Table* for .3696, or the neareſt number to it, which is leſs, and that is .2500 and ſignifies 1 *Firkin*.— Subtract 2500 from .3696, and there remains .0096, which number look for in this *Table*, and the neareſt to it is 69½, which is 1 *Quart*; ſo that the *Liquor* contained in this *Tun*, is 24 *Barrels*, 1 *Firkin*, and 1 *Quart*, and ſomewhat more.

This is exactly true, when the *Tun* is all of one bigneſs, both at the top and bottom; but if it be narrower at the top than at the bottom, then the eaſieſt way, (and exact enough) is, to take the Diametre of the *Tun* about the middle of it, or rather (if you can) at the middle of the *Liquor*: This may be called the *mean Diametre*, which being found in the firſt Column of this *Table*, and the Number ſtanding againſt it, Multiplied by the depth of the *Liquor*, ſhall give the Content in *Barrels*.

Example 2, *Let the mean Diametre of a Tun be found to be (about the middle of the Liquor,)* 98 Inches, *and the depth of the Liquor* 76 Inches, *how many Barrels of Liquor is here in that Tun?*

The mean Diametre 98 Inches,	
The number againſt 98, is —	0.7430
The depth of the *Liquor* 76 Inches —	76
	44580
	52010
The Content, — —	56.4680

Barrels	*Kil.*	*Fir.*	*Gal.*	*Pot.*	*Qu.*	*Pi.*
56	0	1	0	1	0	1

And ſo much *Liquor* is in the *Tun*.

A

A Table shewing the Content of any exact Square, or Oblong Brewer's Tun, in Barrels, and 10000 Parts of a Barrel.

The Length or Breadth of the Tun in Feet.

	XXI	XIX	XVIII	XVII	XVI	XV	XIV	XIII	XII	XI	
	B.parts	B.parts	B.parts	B.parts	B.parts	B.parts	B.parts	B.parts	B.parts	B.parts	
20	5.6739	5.3990	5.1164	4.8228	4.5392	4.2554	3.9716	3.6880	3.4044	3.1206	20
		5.1205	4.8511	4.5817	4.3122	4.0426	3.7731	3.5036	3.2340	2.5645	19
			4.5958	4.3406	4.0852	3.8298	3.5746	3.3192	3.0603	2.8084	18
	I			4.0994	3.8883	3.6172	3.3760	3.1348	2.8934	2.6524	17
1	0.0142	II			3.6314	3.4044	3.1774	2.9504	2.7232	2.4964	16
2	0.0284	0.0567	III			3.1916	2.9788	2.7660	2.5530	2.3303	15
3	0.0426	0.0851	0.1277	IV			2.7862	2.5816	2.3828	2.1842	14
4	0.0567	0.1135	0.1708	0.2270	V			2.3972	2.2126	2.0282	13
5	0.0709	0.1418	0.2128	0.2837	0.3546	VI			2.0244	1.8722	12
6	0.0851	0.1702	0.2553	0.3404	0.4255	0.5106	VII			1.7162	11
7	0.0993	0.1986	0.2979	0.3972	0.4905	0.5950	0.6951	VIII			
8	0.1135	0.2270	0.3424	0.4539	0.5674	0.6808	0.7944	0.9080	IX		
9	0.1277	0.2553	0.3830	0.5106	0.6383	0.7660	0.8937	1.0214	1.1489	X	
10	0.1418	0.2837	0.4255	0.5674	0.7092	0.8511	0.9929	1.1348	1.2766	1.4184	10
11	0.1560	0.3120	0.4680	0.6241	0.7801	0.9361	1.0921	1.2402	1.4042	1.5603	11
12	0.1702	0.3404	0.5106	0.6808	0.8510	1.0212	1.1914	1.3616	1.5318	1.7022	12
13	0.1844	0.3688	0.5532	0.7376	0.9220	1.1064	1.2908	1.4752	1.6596	1.8440	13
14	0.1986	0.3972	0.5958	0.7944	0.9929	1.1915	1.3901	1.5887	1.7873	1.9858	14
15	0.2128	0.4254	0.6383	0.8511	1.0639	1.2766	1.4894	1.7022	1.9149	2.1277	15
16	0.2269	0.4538	0.6809	0.9078	1.1348	1.3618	1.5887	1.8157	2.0426	2.2696	16
17	0.2411	0.4823	0.7234	0.9640	1.2057	1.4468	1.6880	1.9291	2.1703	2.4114	17
18	0.2533	0.5106	0.7659	1.0212	1.2766	1.5319	1.7872	2.0425	2.2979	2.5532	18
19	0.2695	0.5390	0.8085	1.0780	1.3475	1.6170	1.8865	2.1560	2.4255	2.6970	19
20	0.2837	0.5674	0.8511	1.1348	1.4184	1.7032	1.9859	2.2696	2.5532	2.8368	20
	I	II	III	IV	V	VI	VII	VIII	IX	X	

The Length or Breadth of the Tun in Feet.

The use of this Table.

Example 1. *Let there be a square Tun, whose four sides are each of them 9 Foot, and it is 46 inches full of Liquor, how many Barrels of Liquor is there in this Tun?*

Look in the *top*, *bottom*, or *middle* of the *Table* for IX Foot, among the *Numeral Letters*, and also for 9 in either side of the *Table*, where you can find it among the *common figures*, and you shall find 9 in the left hand *Column*; then carry your finger along that Line from 9, till you come under IX in the middle of the *Table*, and the number under IX is 1.1489, which multiplied by 46 inches, (the depth of the Liquor,) the Product is 52.8494, which is 52 *Barrels*, and .8494 ten thousand parts of a *Barrel*, or 52 *Barrels*, 1 *Kilderkin*, 1 *Firkin*, 1 *Quart*, and a little more.

The number against 9, and under IX is	1.1489
The depth of the Liquor 46 inches,	46
	68934
	45956
Barrels,	52.8494

Example 2. *Let there be an Oblong, or Long-Square Tun, whose Length is 18 Foot, and Breadth is 12 Foot, and it is 35 Inches full of Liquor; how many Barrels is therein contained?*

Look for XII (the breadth of the Tun) at the head of the *Table*, and look down that Column till you come against 18 in the right side of the *Table*, and under XII, and against 18, you shall find this number 3.0003, which being multiplied by 35 Inches, the depth of the Liquor, the Product will be 105.0105, which (4 figures being cut off) leaves 105 *Barrels*, and .0105 ten thousand parts of a *Barrel*, which is 1 Quart and 1 Pint; and so much Liquor is in the Tun.

The number between 18 and XII is	3.0003
The depth of Liquor 35 Inches,	35
	150015
	90009
The Content in *Barrels*,	105.0105

A Table shewing how much is wanting of a Beer Barrel, at every Inch, and a quarter of an Inch of Emptiness, both in Wine and Ale Gallons.

Inches, Dry Quart.	The Barrel wants of Beer Gall. Ga. Pi. 100 pts.	Wine Gall. Ga. Pi. 100 pts.	Inches, Dry Quart.	The Barrel wants of Beer Gallons Gal. Pi. 100 pts.	Wine Gallons Gal. Pi. 100 pts.
	0 0 00				
0 0	0 0 00	0 0 49	12 0	19 6 16	24 1 48
1	0 0 40	0 1 47	1	20 1 22	24 7 30
2	0 1 20	0 2 57	2	20 7 40	25 4 60
3	0 2 10	0 3 80	3	21 3 10	26 1 36
1 0	0 3 10	0 5 30	13 0	21 7 40	26 6 38
1	0 4 33	0 7 35	1	22 3 00	27 3 36
2	0 6 00	1 1 29	2	22 7 00	28 2 18
3	0 7 60	1 4 00	3	23 3 00	25 5 18
2 0	1 1 80	1 6 56	14 0	23 7 30	29 2 20
1	1 3 90	2 1 22	1	24 3 70	29 7 40
2	1 6 10	2 4 34	2	24 7 40	30 3 90
3	2 0 66	2 7 98	3	25 3 60	31 1 00
3 0	2 3 50	3 3 10	15 0	25 7 50	31 5 80
1	2 6 16	3 6 20	1	26 3 30	32 2 60
2	3 0 70	4 2 00	2	26 7 00	32 7 00
3	3 3 80	4 5 30	3	27 3 00	33 3 80
4 0	3 6 50	5 1 35	16 0	27 6 40	34 0 30
1	4 1 80	5 5 60	1	28 2 20	34 4 80
2	4 5 25	6 1 45	2	28 5 80	35 0 80
3	5 0 42	6 5 70	3	29 1 40	35 5 34
5 0	5 3 90	7 1 70	17 0	29 4 80	36 1 80
1	5 7 20	7 6 20	1	30 0 40	36 6 29
2	6 2 80	8 2 65	2	30 4 10	37 2 29
3	6 6 50	8 7 20	3	30 7 50	37 6 54
6 0	7 2 20	9 3 20	18 0	31 3 00	38 2 39
1	7 5 50	9 7 70	1	31 6 10	38 6 64
2	8 1 10	10 4 20	2	32 1 80	39 2 70
3	8 4 80	10 1 00	3	32 5 00	39 6 00
7 0	9 0 70	11 5 40	19 0	32 7 80	40 1 80
1	9 4 50	12 2 20	1	33 2 10	40 4 90
2	10 0 40	12 7 00	2	33 4 80	41 0 10
3	10 4 30	13 4 10	3	33 7 40	41 3 65
8 0	11 0 50	14 0 80	20 0	34 2 00	41 6 77
1	11 4 30	14 5 80	1	34 4 30	42 1 43
2	12 0 30	15 2 80	2	34 6 20	42 4 00
3	12 4 29	15 7 70	3	35 0 10	42 6 70
9 0	13 0 30	16 4 60	21 0	35 2 00	43 0 64
1	13 4 30	17 1 60	1	35 3 60	43 2 70
2	14 0 40	17 6 60	2	35 4 80	43 4 19
3	14 4 60	18 3 40	3	35 6 00	43 5 42
10 0	15 0 50	19 0 30	22 0	35 6 80	43 6 51
1	15 4 48	19 6 50	1	35 7 40	43 7 50
2	16 0 80	20 3 25	2	36 0 00	44 0 00
3	16 5 50	21 1 00			
11 0	17 2 20	22 0 00			
1	17 7 90	22 6 98			
2	18 5 49	23 4 31			
3	19 2 00				

The use of this Table.

The use of this Table is twofold.

1. *To know how much Liquor is in a Barrel, being partly full:* And

2. *To know how much the Barrel wants of being full.*

Example, Suppose the depth of the Liquor in the Barrel be 14 Inches, this taken from 22 Inches and a half, (the whole Diametre of the Barrel at the bung,) leaves 8 inches and half for the empty part. Wherefore, look in the Table for 8 inches and a half, against which you shall find 12 Gallons, 0 Pints, and 30 parts of a Pint; and so much doth the Barrel want of being full.— Also look in the Table for 14 inches, against which you shall find 23 Gallons, 7 Pints, and 30 parts of a Pint, and so much Liquor there is in the Barrel.— And these two being added together, do make 35 Gallons, 7 Pints, and 60 parts of a Pint, which is the whole content of the barrel in Ale Gallons within less than half a Pint.

Now if you would know the same in Wine Gallons, against 8 and a half (the empty part) here stands 14 Gallons, 5 Pints, 80 parts, and against 14 the full part, there stands 29 Gallons, 2 Pints, 20 parts, which added together, make 44 Gallons, the Content of the Barrel in Wine Gallons.

SECT.

SECTION II.

Of the Menſuration of Timber-Trees, *Growing, or* Un-ſquared; *And of the* Works *of the ſeveral* Artificers *Relating to the Building of* Houſes *or other* Edifices.

I. *Of Timber Growing, or Un-ſquared.*

IN the third Part of the Second Book hereof is taught how to meaſure all ſorts of Regular Solids, and ſuch are Timber and Stone when ſquared: But for Timber Growing or Un-ſquared there are ſeveral Cuſtoms uſed in the meaſuring of them upon Buying or Selling of them, as of Oak, Aſh, Elm, *&c.*

I. In Buying of Oak Timber (where the Bark is pieled off) the Cuſtom is, To take a Line, and girt about the middle of the Piece, and then to double the Line four Times, and account that 4*th* part of the Girt, to be the Side of a Square equal to that Tree or Piece: But this way of meaſuring is egregiouſly falſe, (as hath been often detected;) for it makes the quantity of the Tree or Piece (be it great or little,) one fifth part leſs than it would be, were it truly meaſured.

Alſo in the meaſuring of Elm, Beech and Aſh, whoſe Bark is on it, the Cuſtom is, to caſt away one Inch out of the 4*th* part of the Girt, which may well be allowed when the Bark is three quarters of an Inch thick, and the Tree 40 Inches about; But if the Bark be thin, and the Tree leſs then 30 or 32 Inches about, then an Inch is too much to be allowed.

Illuſtration.

Suppoſe a Tree to be 4 Foot, or 48 Inches about, then the Diametre of that Tree will be 15 $\frac{1}{4}$ Inches, and one 4*th* part thereof is 12 Inches: Now if you abate one Inch and a half of the Diametre for the Bark, then the Tree will be but 43 $\frac{1}{2}$ Inches about, whoſe 4*th* part is not full 11 Inches; ſo that in this Caſe, one Inch may very well be abated from the 4*th* part. So again, if the Bark be thinner, and the Tree leſſer, a ſmaller allowance will ſerve: And if the Bark be thicker, and the Tree larger, there ought to be more allowed.

But this way of meaſuring by the 4*th* part of the Girt, and allowing for the Bark, I ſay (as I did before) will prove to be juſt one fifth part over Meaſure: This being premiſed, and that the difference being as 4 to 5, let them look to it whom it may concern. And for the benefit of ſuch as ſhall have occaſion thereof, I have here inſerted a Table, *&c.*

A Table shewing how much in length of any round Timber Tree, whose Circumference (or Girt) is known, doth make a Solid Foot of Timber, according to true Measure.

Inch.	*The length of a Foot Solid in* F.	*in.*	10p.	*Inch.*	*The length, &c.* F.	*in.*	10p.	*Inch.*	*The length, &c.* F.	*in.*	10p.	*Inches.*	*The length, &c.* F.	*in.*	10p.
10	18	11	2	33	1	7	9	56	0	6	9	79	0	3	5
11	14	11	5	34	1	6	8	57	0	6	7	80	0	3	4
12	12	6	8	35	1	5	7	58	0	6	4	81	0	3	3
13	10	8	5	36	1	4	7	59	0	6	2	82	0	3	2
14	9	3	7	37	1	3	8	60	0	6	0	83	0	3	2
15	7	10	3	38	1	3	0	61	0	5	8	84	0	3	1
16	7	0	8	39	1	2	3	62	0	5	6	85	0	3	0
17	6	3	0	40	1	1	6	63	0	5	5	86	0	2	9
18	5	7	0	41	1	0	9	64	0	5	2	87	0	2	9
19	5	0	2	42	1	0	3	65	0	5	1	88	0	2	8
20	4	6	3	43	0	11	7	66	0	4	9	89	0	2	7
21	4	1	2	44	0	11	1	67	0	4	8	90	0	2	7
22	3	8	9	45	0	10	7	68	0	4	7	91	0	2	6
23	3	4	9	46	0	10	2	69	0	4	6	92	0	2	6
24	3	1	7	47	0	9	9	70	0	4	4	93	0	2	5
25	2	10	7	48	0	9	4	71	0	4	3	94	0	2	5
26	2	8	1	49	0	9	0	72	0	4	2	95	0	2	4
27	2	5	8	50	0	8	7	73	0	4	1	96	0	2	4
28	2	3	7	51	0	8	3	74	0	3	9	97	0	2	3
29	2	1	8	52	0	8	0	75	0	3	8	98	0	2	3
30	2	0	1	53	0	7	8	76	0	3	7	99	0	2	2
31	1	10	6	54	0	7	4	77	0	3	7	100	0	2	2
32	1	9	2	55	0	7	2	78	0	3	6				

The Circumference of the Tree in Inches. — *The Circumference, &c.* — *The Circumference, &c.* — *The Circumference, &c.*

The Use of this Table.

Example. *If a Timber Tree be* 48 *Inches about, how much in length thereof will make a Solid Foot of Timber?*

Look in the Column towards your left Hand for 48 Inches, againſt which ſtands 0. 9. 4. that is, no Feet, 9 Inches, and 4 tenth parts of an Inch: And ſo much in length, 48 Inches about, will make a Foot Solid.

Now if this Tree were 20 Foot (or 240 Inches) long, 9 Inches and 4 tenths of an Inch would be found 25 times and a half in that length; and ſo many Foot doth that Tree contain.

But if you take one fourth part of 48, which is 12 for the ſide of the Square equal, then this Tree would contain but juſt 20 foot, which is 5 foot and a half too little, which is above a fifth part, as was ſaid before.

SECTION III.

Of the Meaſuring of the Works of the ſeveral Artificers relating to Building; and what Method and Cuſtoms are obſerved therein,

THE Principal Artificers relating to the Building of Houſes and other Edifices, are the *Carpenter*, *Bricklayer*, *Plaiſterer*, *Joyner*, *Painter*, *Glaſier*, &c. of which, ſome of their Works are meaſured by the Square of Ten Foot (or 100 Foot.) Some by

by the Rod, Pole, or Pearch, (or 272 foot ſquare,) ſome by the yard Superficial, (or 9 foot ſquare,) or yard Solid, containing 27 foot; ſome by the foot Superficial (of 144 inches,) or Solid, (of 1728 inches.)

Of the Carpenter's Work.

The Carpenters Works which are meaſurable, are Flooring, Partitioning, and Roofing: All which are meaſured by the ſquare of 12 foot, ſo that one ſquare contains 100 ſquare feet.

1. *Of* **Flooring,**] *If a Floor be* 57 *foot* 3 *inches long, and* 28 *foot* 6 *inches broad, how many ſquare of Flooring is there in that Room?*

Multiply 57 foot, 3 inches, by 28 foot, 6 inches, the Product will be 1631 foot, 7 ½ inches; that is, 16 ſquare, 31 foot, or 16 ſquare, 1 quarter, and 6 foot; for the 7 ½ inches, (in theſe works,) they are not to be regarded.

2. *Of* **Partitioning,**] *A Partition between Rooms being in length* 82 *foot,* 6 *inches, and in height* 12 *foot,* 3 *inches, how many ſquare is therein?*

Multiply 82.5 foot, by 12.25 foot, the Product will be 1010.62 foot; that is, 10 ſquare and 10 foot, the 62 or 7 ½ inches is neglected.

3. Of **Roofing,**] It is a Rule received among Work-men, that the flat of any houſe, and half the flat thereof, taken within the Walls, is equal to the meaſure of the *Roof* of the ſame houſe.

Example, *If a houſe within the Walls be* 46 *foot deep, and* 18 *foot broad; how many ſquare of* Roofing *will cover that houſe?*

Multiply 46 by 28, the Product will be 828, and the half thereof is 414, which added to 128, the Summ is 1242; that is, 12 ſquare, 42 foot, or 12 ſquare, 1 quarter and 17 foot, and ſo many ſquare will there be in the Roof.

Note, That 25 foot is 1 quarter, 50 is half, and 75 foot is 3 quarters of a ſquare.

There are other Works about a Building done by the Carpenter, that are meaſured by the foot (Running meaſure;) that is, by the number of Feet in length only: As

Contaliver Cornice,	*Guttering,*	*Rails* and *Balaſters,*
Modellion Cornice,	*Lintales,*	*Timber Front,*
Plain Cornice,	*Breſt Somers,*	*Shelves, Dreſſers,*
Painthouſe Cornice,	*Skirt board,*	*&c.*

There are alſo

Doors and Caſes,	*Balcony Doors and Caſes,*	*Stairs and Caſes,*
Window Lights,	*Celler Doors and Caſes,*	*Mantle-Trees and Taſſels,*
Lutheren Lights,	*Columbs and Pilaſters,*	*Pediments, &c.*

All which are Rated at —— *per* piece.

Note 1, In the meaſuring of flooring, after that you have meaſured the whole Floor, you muſt deduct out of it the Well-holes for the Stairs and Chimneys.—And in Partitioning for the Doors, *&c.* except, (by agreement) they are to be included.

Note 2. In meaſuring of Roofing, ſeldom any deductions are made for the holes for the Chimney Shafts, the Vacancies for Lutheren Lights, and Sky-Lights; they are more trouble to the Workman, then the Stuff which would cover them is worth.

Of the Bricklayer's Work.

The Principal are Tiling, Walling, and Chimney work,

1. Of **Tiling,**] Tiling is meaſured by the ſquare of 10 foot: As Flooring, Partitioning, and Roofing in the Carpenters Work were; ſo that between the Roofing and the Tiling, the difference will not be much, yet the Tiling will be the moſt: For the Bricklayer ſometimes will require to have running meaſure for Hyps and Valleys, which in ſome Caſes may be allowed, but in moſt plain Roofs not.

An Example in this is needleſs.

2. For

2. For Walling,] Bricklayers do measure all their work by the Rod square, each Pole or Rod containing 16 foot and an half in length; so that one Rod in length, and one in breadth, do make a Rod square, containing upon the Superficies of the Wall 272 ¼ square feet; for 16,5 multiplied by 16.5, produceth 272.25.

Some other things are to be observed in the measuring of Brickwork; as the reducing thereof to a Standard thickness, for in Walls and Foundations of Houses, the Walls are of different thicknesses, which must be reduced to one, *viz.* to one brick and a half thick, as in *Examples* following.

Example 1, *If a Brick wall be 192 foot long, and 12 foot high, how many Rod of work is contained therein?*

Multiply 192 the length, by 12 the height, the Product will be 2304; and so many square feet are contained in the Superficies of the Wall, which divided by 272, (the number of Superficial feet in one Rod, for the quarter of 9 foot, we always reject,) the Quotient will be 8, and 128 remaining, which is 8 Rod, and the 128 foot, which divided by 68, (the feet in one quarter of a Rod,) the Quotient will be 1 Quarter of a Rod, and 60 foot remaining; so that the whole Wall contains 8 Rod, 1 quarter, and 60 foot, which is 8 Rod and a half wanting 8 foot.

Example 2. *If a Wall (or side of a House) be 24 foot, 6 inches long, and 37 foot, 8 inches broad; how many Rod is contained therein?*

Multiply 24 6 by 37.66, the Product will be 922.67, that is 922 foot, 9 inches, which we will call 923 foot; this divided by 272, the Quotient will be 3 Rod, and 107 remaining, which divided by 68, gives in the Quotient, 1 quarter, and 39 foot remaining: And this is the true content of this Wall, if it be all of one brick and a half thick from the top to the bottom.

But there is something else to be considered in the measuring of Brick-work, namely, the thickness of the Wall; for the thicker the Wall is, the more Rods are contained therein; for if the Wall be thicker then one Brick and a half, a square Rod (or 272 foot, will contain more than a Rod of Brick work, and a Rod of Wall which is less then one brick and half thick, will contain less than a Rod, when it is reduced to the Standard of one brick and a half thick.— So that every Rod measured upon the Superficies of a Wall that is 3 bricks thick, will contain two Rods being reduced; and one Rod upon a Wall of 4 bricks and a half thick, will contain three Rods, *&c.* wherefore for the ready reducing of brick work of any number of bricks thick, to the Standard of one and a half, take this general

RULE.

Multiply the number of Superficial feet, that are found to be contained upon the Superficies of any Wall, by the number of half bricks, which that Wall is in thickness; one third part of that Product shall be the content thereof reduced to the Standard of one brick and a half.

Example 3. *If a Wall be 72 foot long, 19 foot high, and 5 bricks and a half thick; how many Rods of brick work is contained therein, when reduced to Standard.*

Multiply 72 by 19, the Product will be 1368, and so many Superficial feet doth that Wall contain.— This multiplyed by 11, (the number of half bricks, that the Wall is thick) produceth 15048, one third part whereof is 5016; and so many feet doth the Wall contain, it being reduced to one brick and a half.— Lastly, Divide 5016, by 272, the Quotient will be 18 Rod, and 120 remaining, which divided by 68, the Quotient is 1 quarter, and 52 foot remaining; and that is the true content of that Wall, when reduced to a brick and a half thick, and so of any other.

	Long.	*High*	*Thick*		*R.*	*Q.*	*F.*
Thus if a Wall be	273	9	2	It will be found to contain Reduced.	12.	1.	44.
	45	16	3½		18.	2.	08.
	117	23	1		19.	3.	10.

Other

Other Bricklayers Works, viz.

Chimneys.] The Chimneys in moſt Building are agree'd for by the Hearth in each Room, and ſometimes they are included in the Building, and paid for by the Rod, and meaſured with the reſt of the Building, in this manner.—If the Chimney ſtand ſingle, and alone, the uſual way is to girt it about; and if the Jaums are but one Brick thick, and wraught upright over the Mantletree to the next Floor, then girt it about for a length, and the height of the ſtory for a breadth, at one brick thick.—But if the Chimney ſtand againſt a Wall that is before meaſured, then the breadth of the breaſt, and the depth of the two Jaums is the length, and the height of the ſtory the breadth, at one brick and half, if the Jaums be ſo thick; and nothing to be deducted for the Area between the Hearth and Mantletree, becauſe of the Wyths and thickning for the next Hearth above. For the Chimney Shafts, girt them about in the ſmalleſt place, for the breadth, and the height of the Shaft for the length, at one brick thick, in conſideration of the Wyths, Pargetting, and Scaffolding, which is required for bringing them up.

Other Bricklayers Works which are meaſured by the Foot, running meaſure, are;

Cornices of all ſorts, Facioes, } { Streight Arches, Skeen Arches, } { Hyps and Valleys, Water Courſes.

Other Works valued by the Piece, viz.

Pers, Pilaſters, Ruſtick Work, Pediments, *&c.* There comes alſo to the Bricklayers Hands the Paving of Cellers, or the like, with Brick or Tiles, and for thoſe they are paid for by the yard ſquare.

Example 4. *If a Celler be paved with Brick or Tiles, which is 32 foot long, and 19 foot broad, How many yards of Paving is there in it?*

Multiply 32 by 19, the product will be 608, which divided by 9, the Quotient is 67 yards and 5 foot, which is above half a yard for the Content.

Note, That in the meaſuring of Brick work, if you take the Dimenſions of the Sides of the Houſe on the out-ſide, you muſt take the Dimenſions of the Ends in the inſide. And note alſo, that you muſt make Deductions for all Doors, Windows, *&c.*

Of the Plaiſterers Works.

The Plaiſterers Works are principally of two kinds, Namely, (1.) Work Lathed or Plaiſtered, which they call Cieling. (2.) Work Rendered, which is of two kinds, *viz.* Upon Brick Walls; or between Quarters in the Partitions between Room and Room; all which are meaſured by the Yard Square, or the Square of 3 foot; which is 9 foot.

1. *Of* **Cieling**] *If a Cieling be 58 foot 9 inches long, and 23 foot 7 inches broad, How many yards doth that Cieling contain?*

Multiply 58 75 foot, by 23 58 Foot, the Product will be 1385.32 foot, which divided by 9, the Quotient will be 154 yards *fere*.

2. Of **Partitioning,**] *If the Partitions, between Rooms be 132 foot about, and 12 foot high, How many yards in that Partition?*

Multiply 132 by 12, the Product is 1584, which divided by 9, gives in the Quotient 176 yards for the Content of the Partitions.

Note 1. If there be any Doors, Windows, or the like in your Partitioning, you muſt make deduction for them.

Note 2. When you meaſure rendring upon brick Walls, you are to make no Deductions, but when you meaſure rendring between Quarters, you may very well deduct one fifth part for the Quarters, Braces and Enterteices.

Note 3. That Whitning and Colouring are both meaſured by the yard, as the Ceiling and Rendring were; and as in Rendring between Quarters you deducted one fifth part, ſo in Whiting and Colouring you muſt add one fourth or fifth part at the leaſt.

Of

Of the Joyners Works.

Joyners do measure their Work by the Yard Square or 9 Foot; but in the taking of their Dimensions they differ, for the Joyners have a Custom, and say, *We ought to measure where our Plane touches.* Wherefore, in the taking of the height of any Room, where there is a Cornice about, and swelling Pannels and Moldings downwards, they use, with a Line to girt over every Member of the Cornice, and Swellings of the Pannels and Moldings, which will make the Room much higher thna it is: Then for measuring about the Room they only take it as it is upon the Floor.

Example 1. *If a Room of Wainscot (being girt downwards) do contain in height 15 foot, 7 Inches, and be in compass about 286 foot, How many yards doth that Room contain?*

Multiply 286 foot by 15.58 foot, the Product will be 4456, which divided by 9, giveth in the Quotient 495 yards, and 1 foot, for the content of the Room in yards.

In Joyners Work there is another thing to be observed, that is in the measuring of Doors, Window-shutters, Cupboard-doors and Drawers, and such like, as are wrought on both sides; for these, they account to be paid for Work and half Work; for indeed the Work is (though not half, yet) more, although the Stuff be the same.

Example 2. *Let the Window Shutters about a Room be 78 foot 4 Inches, and the height of them 6 foot 6 Inches, How many yards are contained in these Shutters, at Work and half.*

Multiply 78.33 foot, dy 6.5 foot, the Product will be 50914, the half whereof is 254.57, which added to 509.14, the Summ is 763.71 (or 764) which divided by 9, the Quotient is 85 yards *feré*, for the Content at Work and half Work.

Note, That you are to make Deductions for all Window Lights, but you must measure the Window-boards, the Sopheta-boards and Cheeks by themselves.

Of the Painters Works.

The taking of the Dimensions for the Painters Works is the same as that for the Joyners, by Girting of the Moldings, *&c.* And it is but reason that they should be paid for that upon which their Time and Colours are both expended. The Dimentions thus taken, the casting up, and reducing the Feet into Yards, is altogether the same as that of the Plaisters and Joyners: but the Painter never acquires Work and half Work, but reckons his Work Once, Twice or Thrice Primed, or Coloured over: I need give no Examples hereof, the foregoing being sufficient; Only take notice, that Window Lights, Window Bars, Casements, and such like things, they do them by the number at so much *per* Piece.

Of the Glaisiers Works.

Glasiers do measure their Work by the Foot Square; so that the length and breadth being multiplyed into each other, produceth the quantity of that Pain of Glass.

Example, *If a Pain of Glass be 4 foot 9 Inches long, and 3 foot 3 Inches broad, How many foot of Glass is contained in that Pain?*

Multiply 4.75 foot by 3.25 foot, the Product will be 15.44 foot, that is 15 foot 4 Inches and a quarter, and so many foot is contained in that Plain.

Note, That when Windows have half rounds at the top, they measure them at the full height as if they were square. Also Round or Oval Windows are measured at the full length and breadth of their Diametres. Likewise Crocket Windows in Stone-work are all measured by their full Squares: And there is reason for so doing: For the trouble in taking their Dimensions to work by, the waste of Glass in working, and the time expended in setting up, is far more than the Glass can be valued at.

Of the Masons Works.

Masons do measure all their Works by the Foot, either Superficial or Solid; and therefore I need give no Examples in their Works, for the Rules before delivered in Lib. 2. Part 1. hereof, for the measuring of Superficies and Solids, are sufficient to perform any Work done by the Mason in any case commensurable; And therefore I will give over Measuring for this time.

FINIS.